Numerical Method

Course Title: Numerical Method
Course No.: CSC207
Nature of the Course: Theory + Lab
Semester: III

Full Marks: 60 + 20 + 20
Pass Marks: 24 + 8 + 8
Credit Hrs: 3

Course Description: This course contains the concepts of numerical method techniques for solving linear and nonlinear equations, interpolation and regression, differentiation and integration, and partial differential equations.

Course Objectives: The main objective of the course is to provide the knowledge of numerical method techniques for mathematical modeling.

Course Detail:

Unit 1: Solution of Nonlinear Equations (10 Hrs.)

- Review of relation between computer programming and Numerical Methods
- Errors in numerical calculations – truncation, round off, errors in original data, blunders, propagated errors and floating point arithmetic, error in converting values, relative – absolute errors
- Trial and Error Method
- Half-Interval Method – bisection method, algorithm, implementation and convergence
- Secant Method – the method, algorithm, implementation and convergence
- Newton's method – the method, algorithm, relating Newton’s method to other methods, implementation and convergence
- Fixed point iteration – the method, different rearrangements, algorithm, implementation and its convergence
- Newton's method for polynomials – the method, synthetic division algorithm and remainder theorem, Horner's method and algorithm, implementation and convergence

Unit 2: Interpolation and Approximation (8 Hrs.)

- Interpolation – definition, application and definition of extrapolation
- Lagrange’s Interpolation - Lagrange's polynomials, error, algorithms, numerical applications and implementations
- Newton's interpolation - divided differences, algorithm for constructing divided difference table, divided difference for a polynomial, error of interpolation, Newton’s

forward difference interpolation and Newton's backward difference interpolation, differences Vs. divided differences, algorithm and implementation

- Cubic Spline interpolation – definition, derivation, algorithm, examples illustrating cubic spline interpolation
- Introduction to regression, Regression Vs. Interpolation, Least Squares Approximation – definition and application, derivation, algorithm and implementation of least square approximation for linear, non- linear and polynomial data

Unit 3: Numerical Differentiation and Integration (5 Hrs.)

- Numerical Differentiation – definition, application, derivatives from divided difference table , error term, algorithm, Derivatives for evenly spaced data, forward difference formula, central difference formula, error terms, Second Order Derivatives, Maxima and minima of tabulated function
- Numerical Integration- definition, application, Newton-Cote's quadrature formulas, Trapezoidal rule, Simpson's 1/3 rule, Simpson's 3/8 rule, Composite formulas for trapezoidal and Simpson's rules, algorithms and implementations, Numerical Double Integration, Gaussian integration algorithm, its derivation, Romberg integration formulas, derivation and algorithm,

Unit 4: Solution of Linear Algebraic Equations (10 Hrs.)

- Review of the existence and uniqueness of solutions of systems of linear equations, and properties of matrices, ill conditioning, types of solution of linear algebraic method
- Gaussian elimination method and algorithm, Gaussian elimination with partial pivoting and its algorithm, Gauss-Jordan method and its algorithm, Inverse of matrix using Gauss Jordan method
- Matrix factorization – Solving system of linear equation using Dolittle algorithm, Cholesky's algorithm
- Iterative methods – Jacobi method and Gauss Seidel Method, and their algorithms
- Eigen values and eigen vectors problems with practical examples, Solving eigen value problems using Lagrangian and power method

Unit 5: Solution of Ordinary Differential Equations (7 Hrs.)

- Review of differential equations, definition of ordinary differential equations and examples, order and degree, initial value problem

- Taylor series method and error terms, Picard’s method, Euler's method and its accuracy, Heun's method, Runge-Kutta methods (4^{th} order method: formula and problem solutions), algorithm and implementations
- Solution of the higher order equations – definitions and examples of higher order difference equations, solution of system of differential equations, representation of higher order equations into system of equations
- Boundary value problems: Definition and examples, shooting method and its algorithm

Unit 6: Solution of Partial Differential Equations (5 Hrs.)

- Review of partial differential equations, Classification of partial differential equation, Deriving difference equations, Laplacian equation and Poisson's equation and their applications, their solution techniques, algorithms and examples

NOTE:

- Relations between different related methods, their advantages, disadvantages and comparisions are essential
- Each method should be implemented and compared with each other drawing conclusions of their efficiency, accuracy and errors.

Laboratory Works:

The laboratory exercise should consists of program development and testing of non-linear equations, interpolation, numerical integration and differentation, linear algebraic equations, ordinary and partial differential equations numerical solutions using appropriate languages like C, C++ or Matlab.

List topics to be included in Laboratory Exercises:

- Solution of non-linear equations using Bisection Method and Secant Method
- Solution of non-linear equations using Newton’s Raphson Method and Fixed Point Iteration Method
- Solution of polynomial using Newton’s Method and Horner’s Rule to evaluate polynomial

- Polynomial interpolation using Lagrange's Interpolation and Newton's Divided Difference Interpolation, Newton's forward and backward difference interpolation
- Fitting of linear (straight line , $y=ax + b$) and non-linear (exponential $y=ae^{bx}$, quadratic $y=ax^2+bx+c$) function using least square method
- Derivatives from divided difference table
- Integration using Trapezoidal rule, Simpson's 1/3 rule and Simpson's 3/8 rule, Line and Double Integration.
- Solution of system of linear equations using Gauss Elimination method and Gauss Jordan Method
- Gauss Seidel Method, Jacobi Method and Power Method
- Solution of ordinary differential equation using Euler's Method, Heun's Method and 4th order Runge-Kutta Method
- Boundary value problems using Shooting Method
- Laplacian Equation, Poison's Equation

Model Questions:

Bachelor/Second Year/Third Semester/Science	**FM:60**
Computer Science and Information Technology CSc.207	**PM:24**
Numerical Method	**Time: 3hrs**

Candidates are required to give their answers in their own words as far as practicable. Assume suitable data if necessary.

The figure in the margin indicates full mark.

Group A

Attempt any TWO questions. **(10X2=20)**

1. Compare Gauss Elimination method and Gauss Jordan method of solving simultaneous equation. Use Gauss Elimination to solve the following system of equation and also write its algorithm.

   $2x+3y+4z=5$
   $3x+4y+5z=6$
   $4x+5y+6z=7$

2. Write an algorithm and a program to compute the interpolation value at a specific point, given a set of data points, using Lagrange interpolation method.
3. Derive Composite Simpson's 1/3 Rule for numerical integration. How does it improve the accuracy of integration?

Group B

Attempt any EIGHT questions. **(5X8=40)**

4. Write a program to solve a non linear equation using bisection method.
5. Find the roots of the following equations using Newton's method.

   $\log x - \cos x = 0$
6. Estimate the value of ln(3.5) using Newton's backward difference formula, given the following data.

x	1.0	2.0	3.0	4.0
ln	0.0	0.6931	1.0986	1.3863

7. Fit a straight line to the following set of data.

x	1	2	3	4	5
y	3	4	5	6	8

8. Estimate the first derivative of $f(x)=\ln x$ at $x=1$ using the second order central difference formula.
9. Compare and contrast between Jacobi iterative method and Gauss Seidal method.
10. Explain about boundary value problem with example? Differentiate it with initial value problem.
11. Use the Heun's method to estimate y(0.4) when

    $y'(x)=x^2+y^2$ with $y(0)=0$.

    Assume h=0.2
12. Solve the Poisson's equation $\nabla^2 f=2x^2y^2$ over the square domain $0 <= x <= 3$ and $0 <= y <= 3$ with f=0 on the boundary and h=1.