DCS (340)
Name: Sagar Suman
Roll No. 2019197

Ans 2 :- we want to find out symbol error probability in terms of SNR-per-bit for M-ary PAM signalling.

↳ In General M-ary PAM, M signal amplitudes are usually selected to be symmetric about origin, That is →

$$A_m = \pm(2i+1)A$$
$$\text{for } i = 0, 1, 2 \ldots \left(\frac{M}{2} - 1\right)$$

with above lets first find avg. symbol energy E_s →

By symmetric we can neglect -ve part and can write E_s as →

$$E_s = \frac{1}{\frac{M}{2}} \sum_{i=0}^{(M/2-1)} \underbrace{(2i+1)^2 A^2}_{\text{as } A_m = (2i+1)A}$$

Further,

$$E_s = \frac{2}{m} \sum_{i=0}^{(m/2-1)} (2i+1)^2 A^2$$

$$= \frac{2A^2}{m} \sum_{i=0}^{(m/2-1)} (4i^2 + 4i + 1)$$

$$= \frac{2A^2}{m} \left[\sum_{i=0}^{(m/2-1)} 4i^2 + \sum_{i=0}^{(m/2-1)} 4i + \sum_{i=0}^{(m/2-1)} 1 \right]$$

$$= \frac{2A^2}{m} \left[\frac{4\left(\frac{m}{2}-1\right)\frac{m}{2}(m-1)}{6} + \frac{4\left(\frac{m}{2}-1\right)\frac{m}{2}}{2} + \frac{m}{2} \right]$$

Further simplying above expression we get →

$$E_s = \frac{2A^2}{m} \left[\frac{m^3}{6} - \frac{m}{6} \right]$$

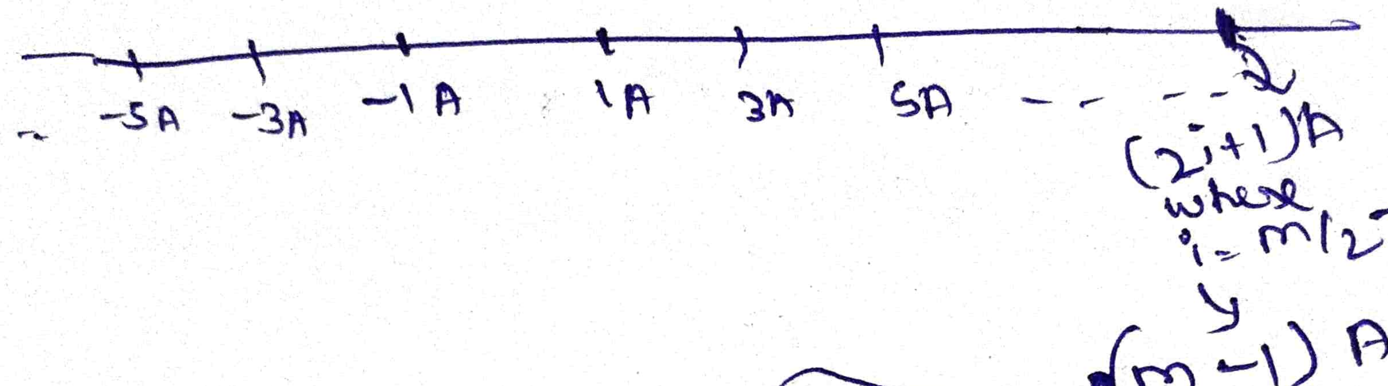
$$E_s = \frac{\cancel{2}A^2}{\cancel{m}} \frac{\cancel{m}}{\cancel{6}\,3} \left[m^2 - 1 \right]$$

$$\boxed{E_s = \frac{A^2}{3} [m^2 - 1]}$$ → avg symbol energy

We will require above in later steps

Continuing further,

A general M-ary PAM can be represented as →



Now according to law of total probability, we can write →

$$P[\text{symbol Error}] = \sum_{i=0}^{M-1} P[\text{symbol Error} \mid H_i]\, P[H_i]$$

where H_i represent i^{th} hypothesis.

$$= \sum_{i=0}^{M-1} P[\text{symbol Error} \mid H_i]\, P[H_i]$$

Now there can be two cases for H_i

H_i → first case :→ H_i is end points [H_{iend}]

→ second case :→ H_i is interior [H_{imid}] points.

So, we can write →

$P[\text{Symbol Error}] = P[\text{Symbol Error} \mid H_i\text{end}]\, P[H_i\text{end}]$
$+ P[\text{Symbol Error} \mid H_i\text{mid}]\, P[H_i\text{mid}]$

If we assume equiprobable hypothesis
i.e, →

$$P[H_i] = \frac{1}{M} \quad \text{for all } i$$

$$P[\text{Symbol Error}] = \frac{1}{M}\Big[\underline{P[\text{Symbol Error} \mid H_i\text{end}]} + \underline{P[\text{Symbol Error} \mid H_i\text{mid}]}\Big]$$

Now, finding.

$$\underline{P[\text{Symbol Error} \mid H_i\text{end}]}$$

$$= P[x > -(M-2)A]$$

$$+ P[x < (M-2)A]$$

Error Boundary for left end

Error Boundary for right end

As, By symmetry

$$= 2\,P[x > -(M-2)A]$$

Normalizing,

$$= 2\,P\left[\frac{x - (-(M-1)A)}{\sqrt{N_0/2}} > \frac{-(M-2)A - (-(M-1)A)}{\sqrt{N_0/2}}\right]$$

$$= 2\,P\left[z > A/\sqrt{N_0/2}\right] = \boxed{2\,Q\left[\frac{A}{\sqrt{N_0/2}}\right]}$$

As there are $M-2$ internal points $\rightarrow$

$$P\begin{bmatrix}\text{symbol} \\ \text{Error}\end{bmatrix} \Big| \text{Mimid}\Big] = (M-2)\begin{bmatrix} P[x < -(M-2)A] \\ + P[x > (M-2)A] \end{bmatrix}$$

$$= (M-2)\left[P\left[z < \frac{-(M-2)A - (-(M-3))A}{\sqrt{N_0/2}}\right] + P\left[z > \frac{(M-2)A - (+(M-3))A)}{\sqrt{N_0/2}}\right]\right]$$

By symmetry

$$= \boxed{(M-2)\left[2Q\left[\frac{A}{\sqrt{N_0/2}}\right]\right]}$$

So. finally we can write $\rightarrow$

$$P\begin{bmatrix}\text{symbol} \\ \text{Error}\end{bmatrix} = \frac{1}{M}\left[2Q\left[\frac{A}{\sqrt{N_0/2}}\right] + (M-2)\left[2Q\left[\frac{A}{\sqrt{N_0/2}}\right]\right]\right]$$

$$= \boxed{\frac{2(M-1)}{M} Q\left(\frac{A}{\sqrt{N_0/2}}\right)} \quad \text{——— (1)}$$

As we know,

Energy per bit → $\boxed{E_b = \frac{1}{\log_2 M} E_s}$

or,

$$E_b = \frac{1}{\log_2 M}\left[\frac{(M^2-1)A^2}{3}\right]$$

so,

$$\boxed{A^2 = \frac{3(\log_2 M)\,E_b}{M^2-1}}$$

eq. (1) can be written as →

$$\boxed{P\begin{bmatrix}\text{symbol}\\ \text{error}\end{bmatrix}} = \frac{2(M-1)}{M} Q\left(\sqrt{\frac{A^2}{N_0/2}}\right)$$

Putting A^2

we get

$$= \boxed{\frac{2(M-1)}{M} Q\left(\sqrt{\frac{6(\log_2 M)}{(M^2-1)}\frac{E_b}{N_0}}\right)}$$

where $\left(\frac{E_b}{N_0}\right)$ is SNR per bit