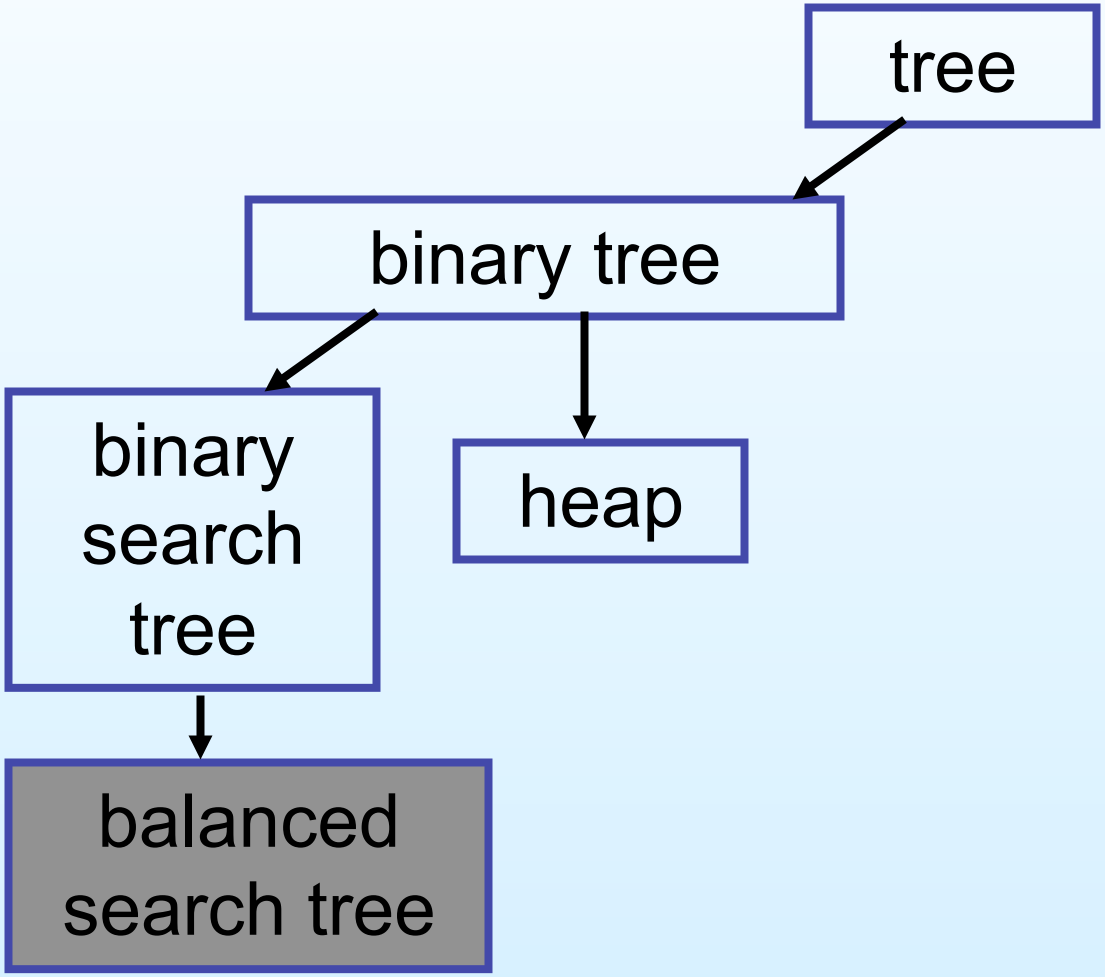
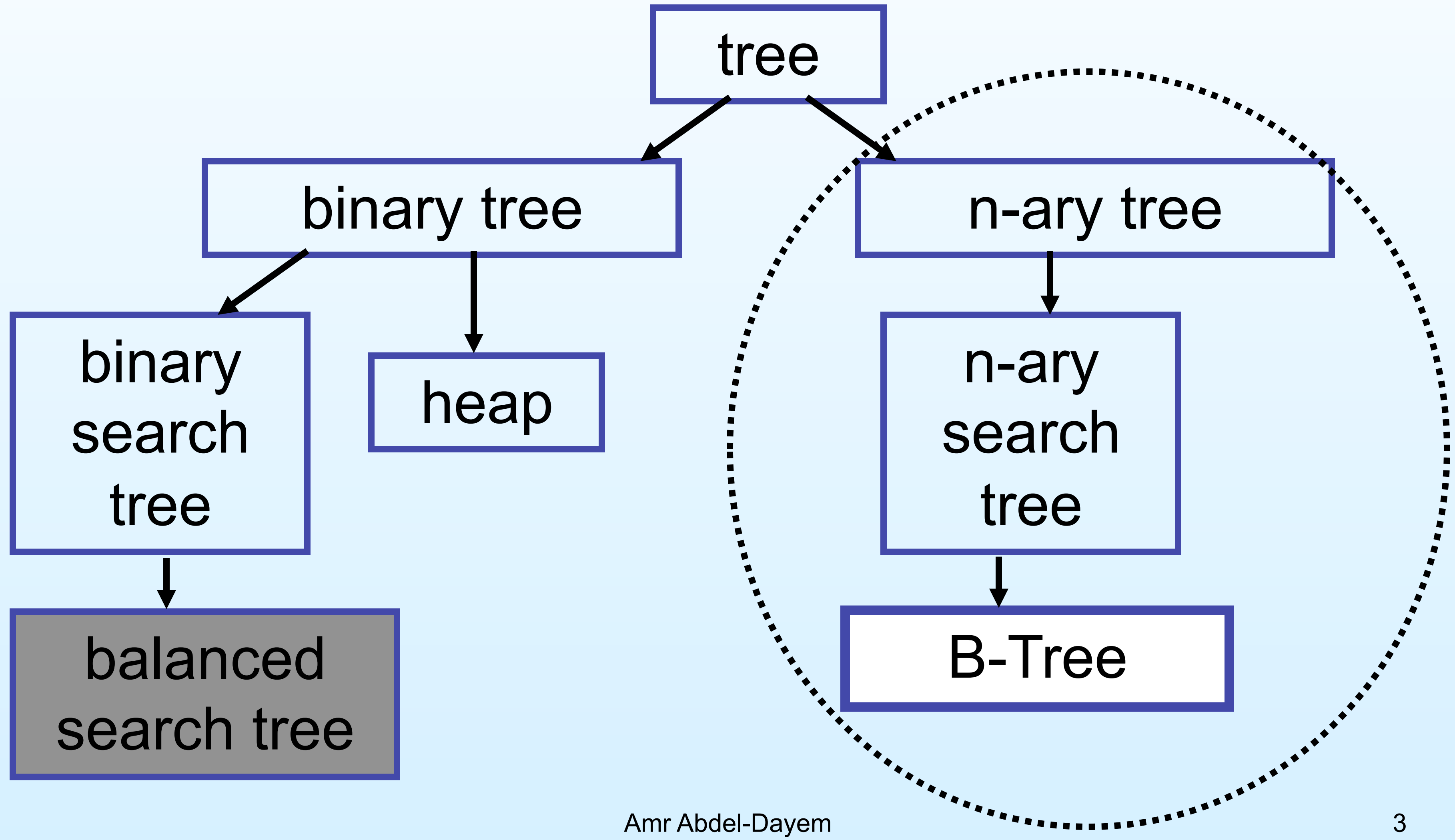
B-Trees

World of trees so far…



World of trees so far…



Motivation for B-Trees

- So far we have assumed that we can store an entire data structure in main memory
- What if we have so much data that it won't fit?
- We will have to use disk storage but when this happens our time complexity fails

Motivation for B-Trees

- The problem is that Big-Oh analysis assumes that all operations take roughly equal time
- This is not the case when disk access is involved
- It is worth executing lots of instructions to avoid a disk access

Motivation (cont.)

- Assume that we use an AVL tree to store all the Health card details in CANADA (about 28 million records)
- We still end up with a **very** deep tree with lots of different disk accesses; $\log_2$ 28,000,000 is about 25
- We know we can't improve on the log *n* for a binary tree

Motivation (cont.)

- But, the solution is to use more branches and thus less height!
- As branching increases, depth decreases
- If we use 10-ary tree:

  Log_{10} 28,000,000 ~= 8

The B-Tree Basics

- Similar to a binary search tree (BST)
  - where the implementation requires the ability to compare two entries via a ***less-than operator (<)***
- But a B-tree is NOT a BST – in fact it is not even a binary tree
  - *B-tree nodes have many (more than two) children*
- Another important property
  - *each node contains more than just a single entry*
- Advantages:
  - *Not too deep*

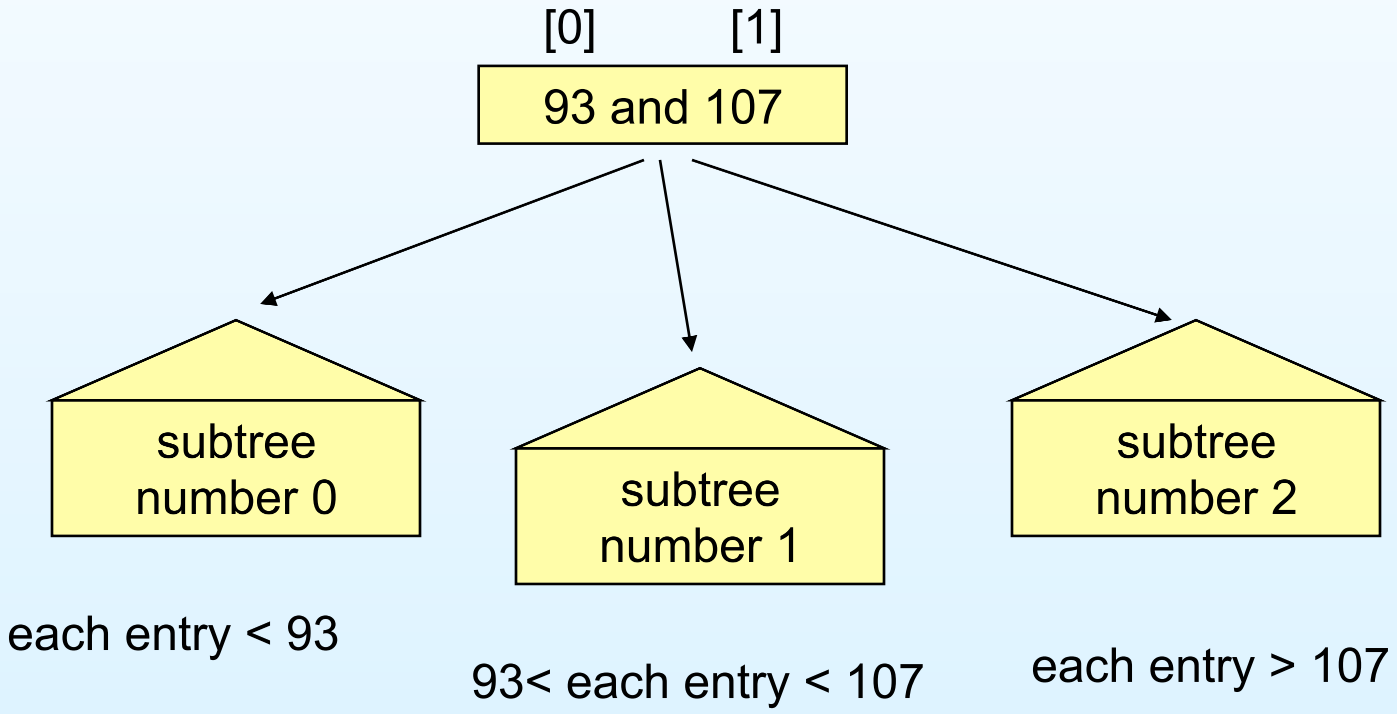
The B-Tree Rules

- The entries in a B-tree node
  - **B-tree Rule 1:** The root may have as few as one entry (or 0 entry if no children); every other node has at least MINIMUM entries
  - **B-tree Rule 2:** The maximum number of entries in a node is 2* MINIMUM.
  - **B-tree Rule 3:** The entries of each B-tree node are stored in a partially filled array, sorted from the smallest to the largest.

The B-Tree Rules (cont.)

- The subtrees below a B-tree node
  - **B-tree Rule 4:** The number of the subtrees below a non-leaf node with n entries is always n+1
  - **B-tree Rule 5:** For any non-leaf node:
    - (a). An entry at index i is greater than all the entries in subtree number i of the node
    - (b) An entry at index i is less than all the entries in subtree number i+1 of the node

An Example of B-Tree



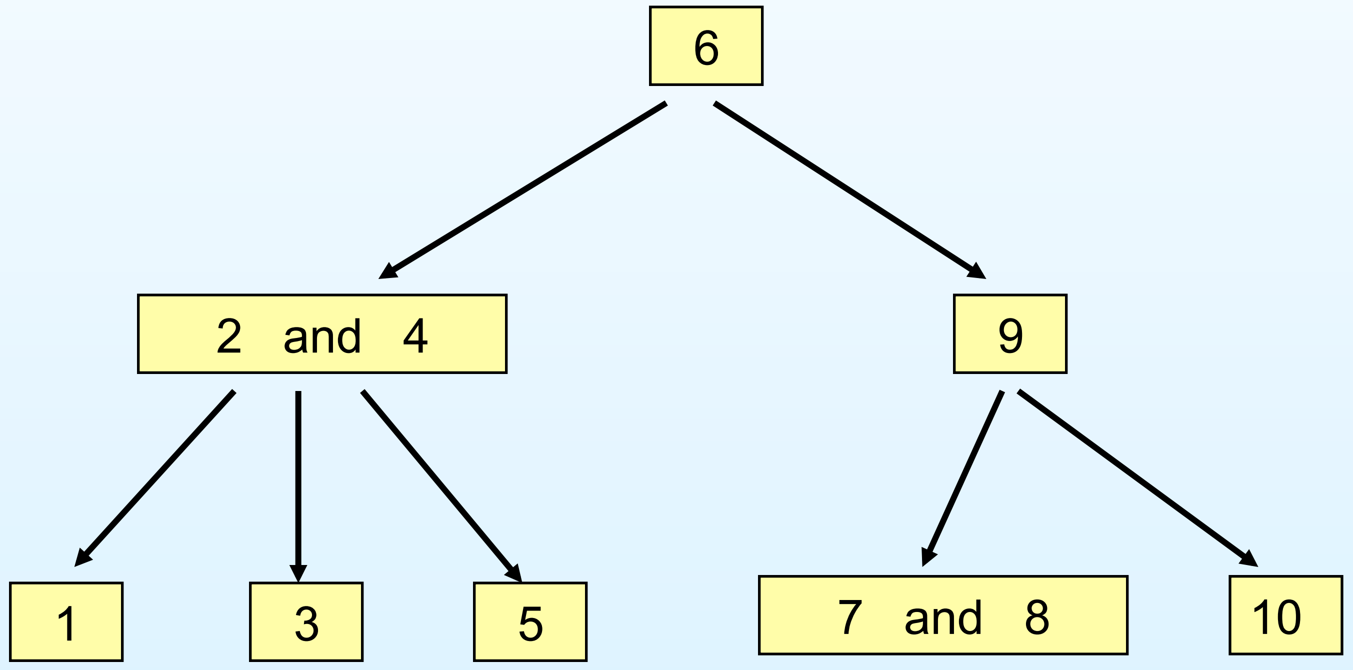
What kind of traversal can print a sorted list?

The B-Tree Rules (cont.)

- A B-tree is balanced
  - **B-tree Rule 6:** Every leaf in a B-tree has the same depth

- This rule ensures that a B-tree is balanced

Another Example, MINIMUM = 1

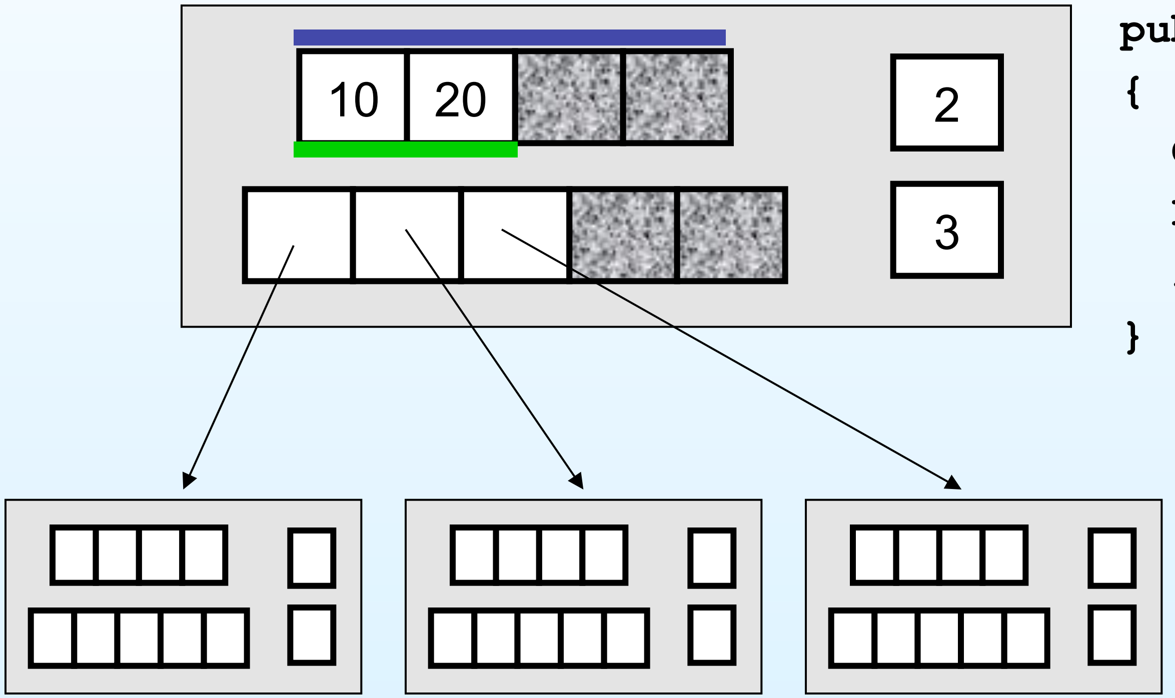


Can you verify that all 6 rules are satisfied?

B-Tree Implementation

```
public class BTNode
{
   Object[] data; //comparable?
   BTNode[] subset;
   int dataCount, childCount;
}
```

B-Tree Implementation



```
public class BTNode
{
   Object[] data;
   BTNode[] subset;
   int dataCount, childCount;
}
```

MAXIMUM: **`data`** array size

MINIMUM: MAXIMUM/2

B-Tree Capacity

- 1000 element `data` array (1001 subtree)
- depth 0: 1000
- depth 1: 1000+1001*1000 = 1002000
- depth 2: ~1 Billion

Minimum population of depth 2 subtree:

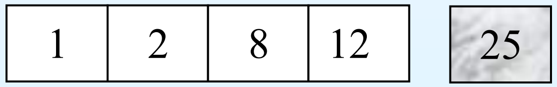
~500,000 (500*501*2 + 500*2 + 1)

Inserting into a B-Tree

- Attempt to insert the new key into a leaf
- If this would result in that leaf becoming too big, split the leaf into two, promoting the middle key to the leaf's parent
- If this would result in the parent becoming too big, split the parent into two, promoting the middle key
- This strategy might have to be repeated all the way to the top
- If necessary, the root is split in two and the middle key is promoted to a new root, making the tree one level higher

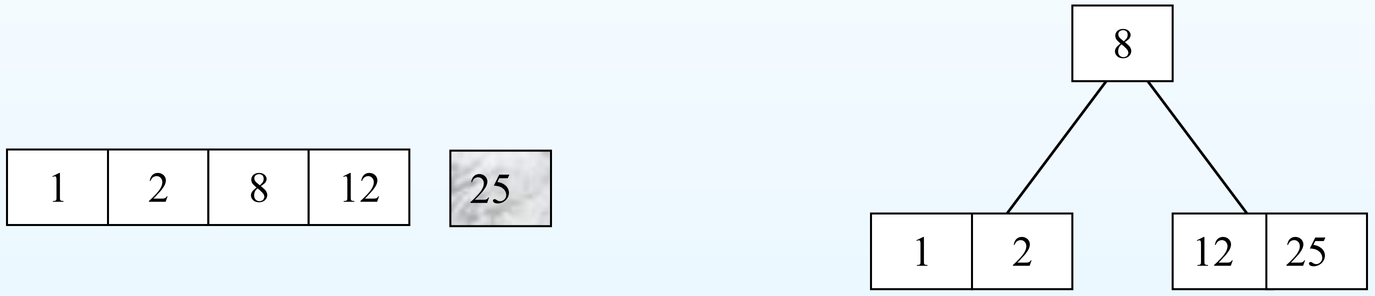
Inserting into a B-Tree

- Suppose we start with an empty B-tree and keys arrive in the following order:1 12 8 2 25 5 14 28 17 7 52 16 48 68 3 26 29 53 55 45
- We want to construct a B-tree of order 5 (min = 2, max = 2*2+1)
- The first four items go into the root:

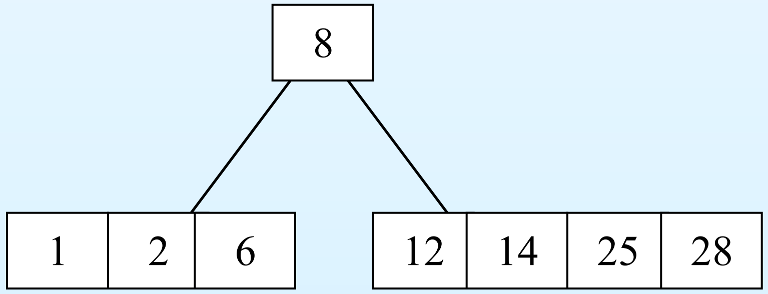


- To put the fifth item in the root would violate condition 5
- Therefore, when 25 arrives, pick the middle key to make a new root

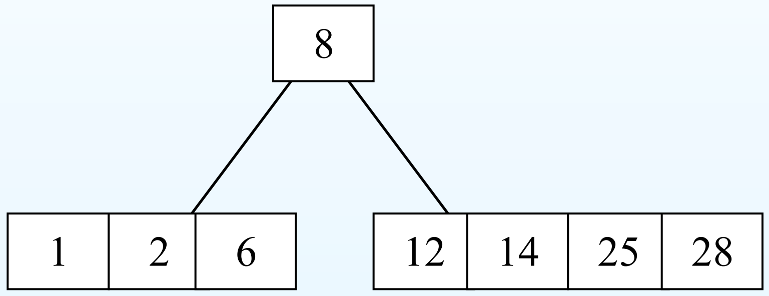
Inserting into a B-Tree



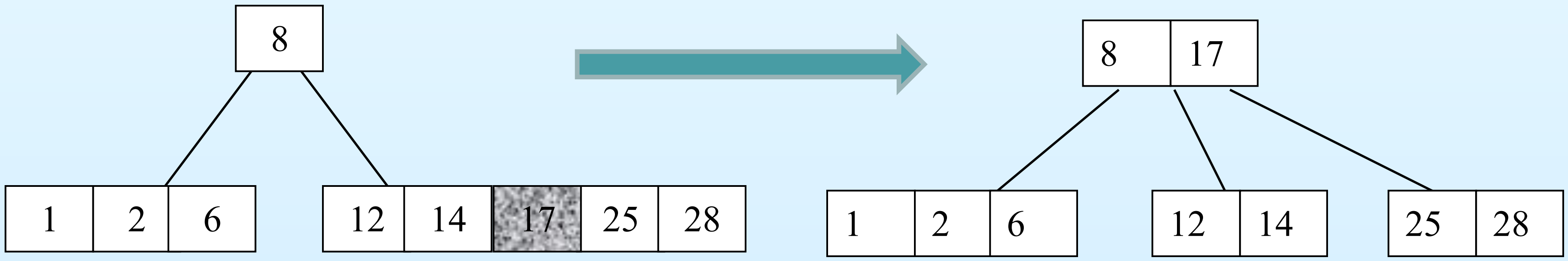
6, 14, 28 get added to the leaf nodes:



Inserting into a B-Tree



Adding 17 to the right leaf node would over-fill it, so we take the middle key, promote it (to the root) and split the leaf



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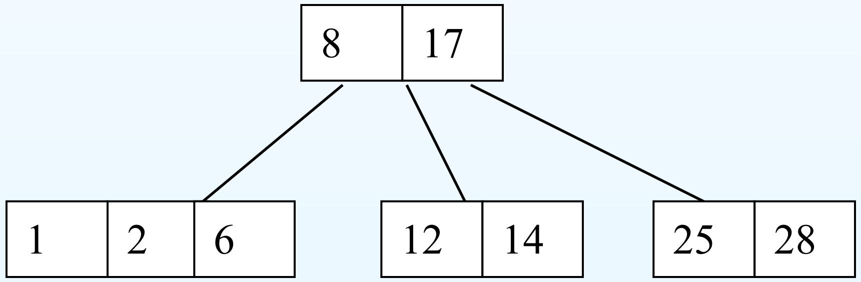
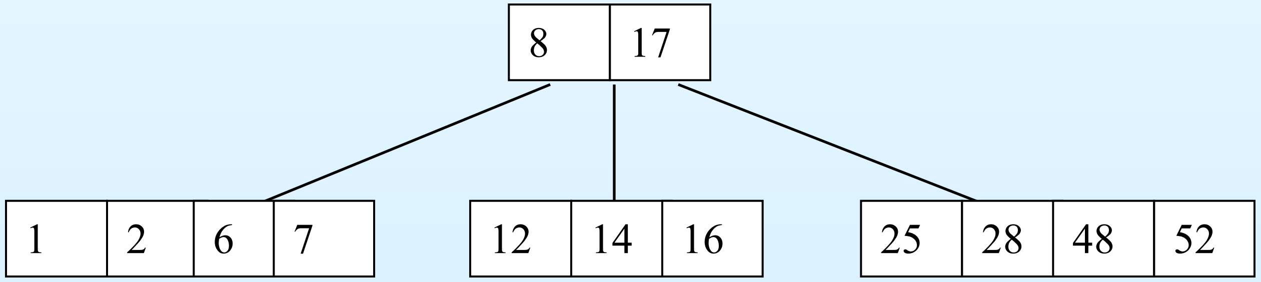
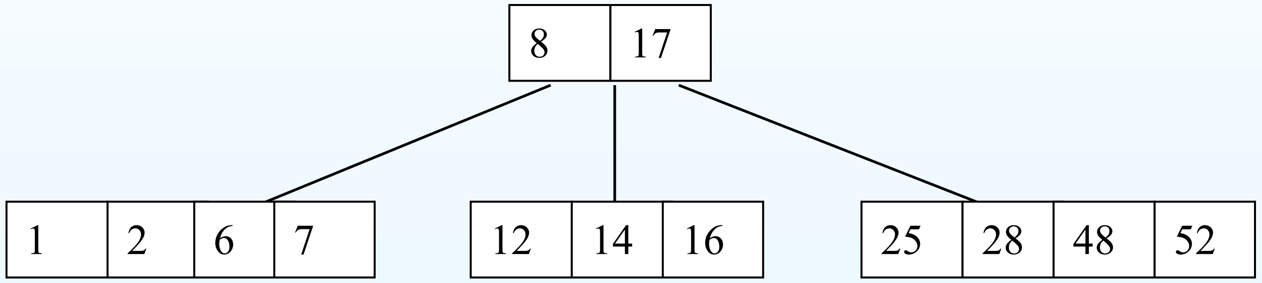


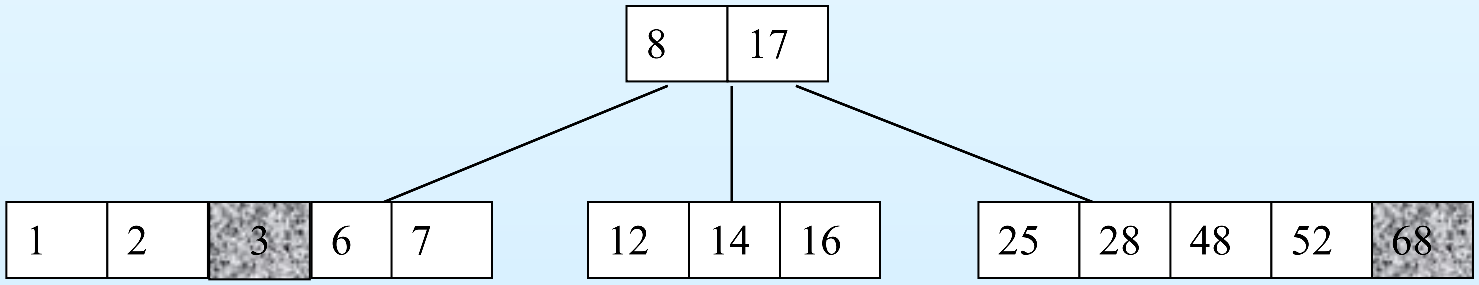
7, 52, 16, 48 get added to the leaf nodes



Inserting into a B-Tree



Adding 68 causes us to split the right most leaf, promoting 48 to the root, and adding 3 causes us to split the left most leaf, promoting 3 to the root



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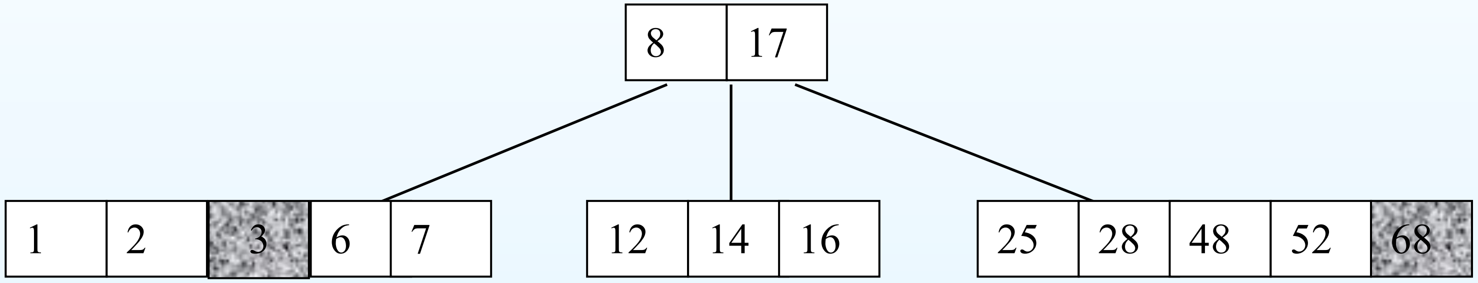

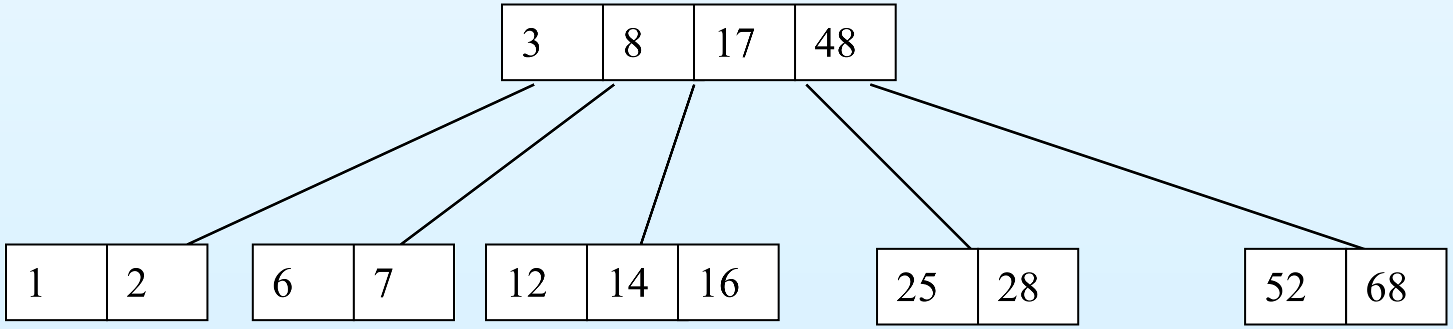

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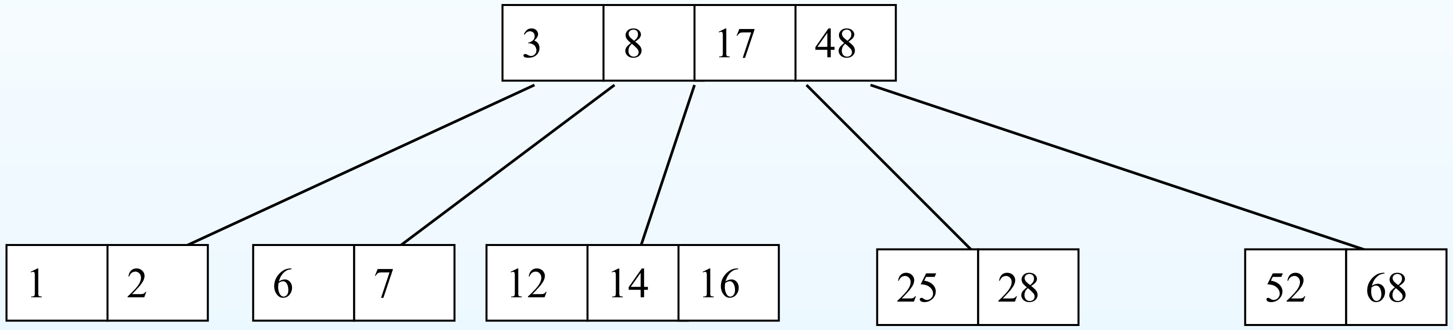
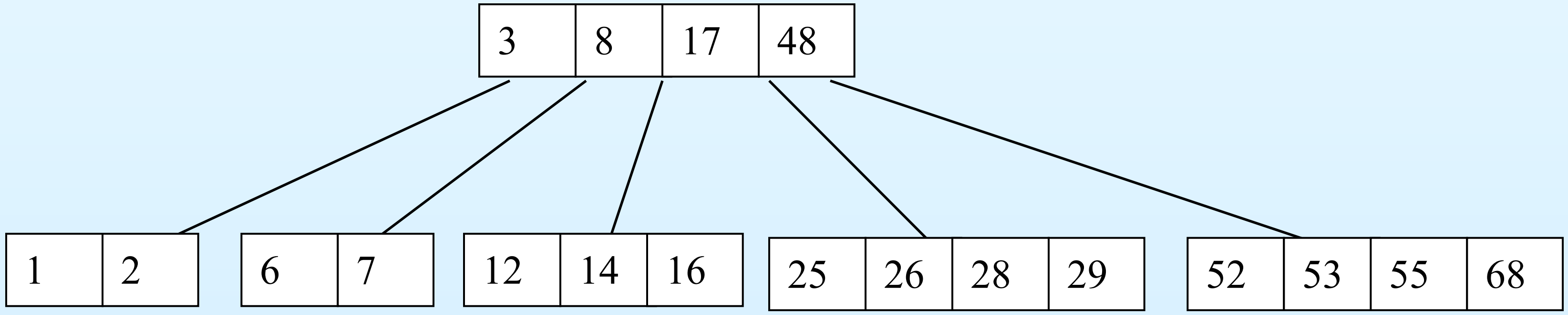
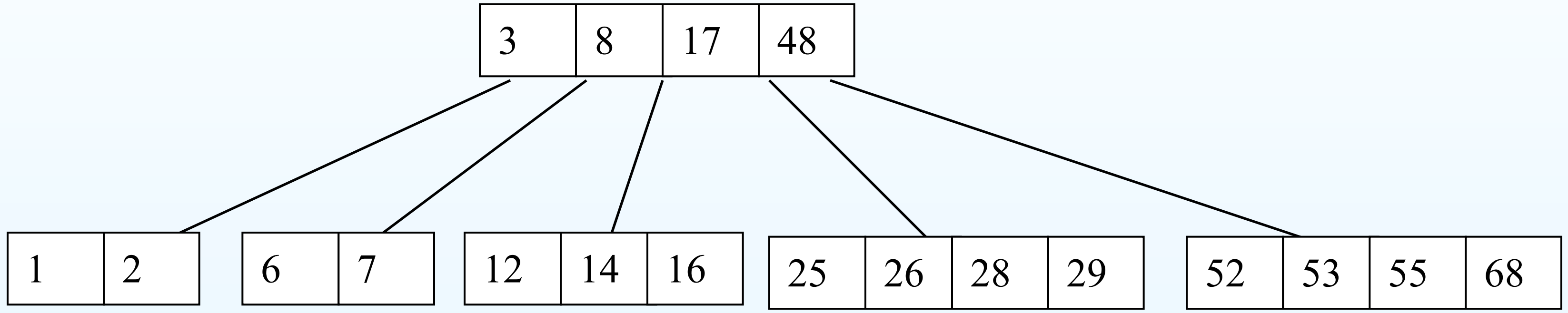


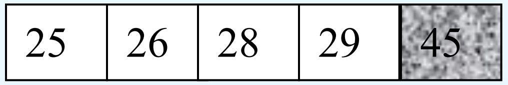
Adding 26, 29, 53, 55: go into the leaves



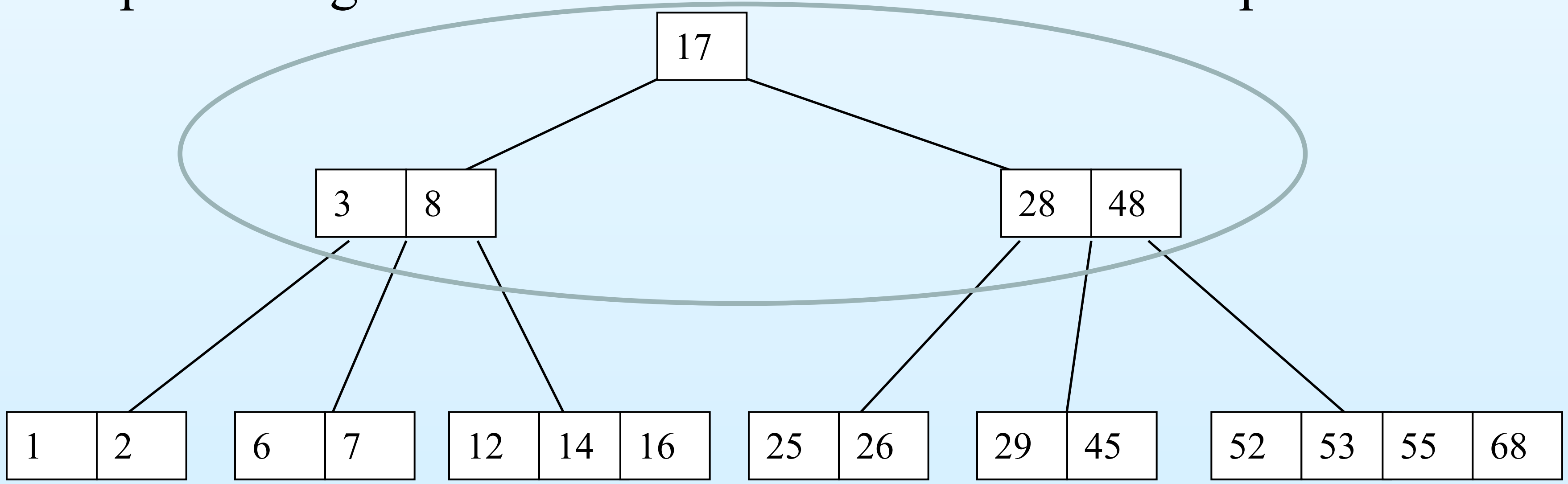
Inserting into a B-Tree



Adding 45 causes a split of



and promoting 28 to the root then causes the root to split



Sample Exam Question

- Insert the following keys to a 5-way B-tree:
- 3, 7, 9, 23, 45, 1, 5, 14, 25, 24, 13, 11, 8, 19, 4, 31, 35, 56

- Check your approach with a friend and discuss any differences.

Removal from a B-Tree

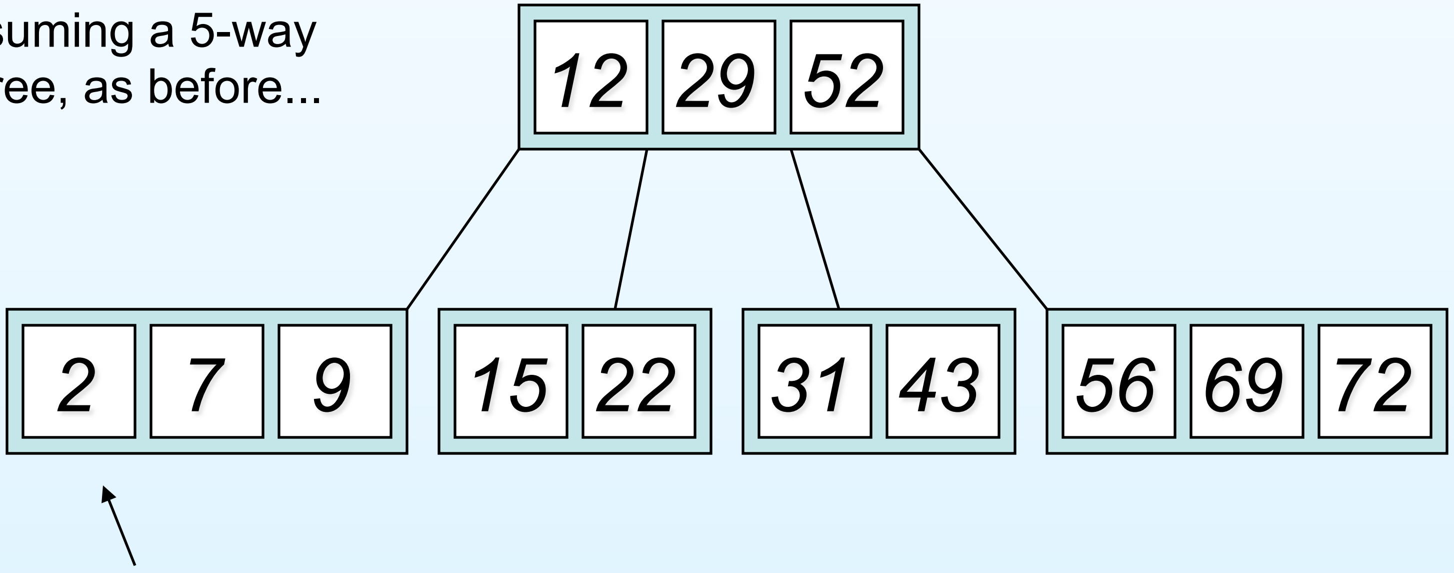
- During insertion, the key always goes *into* a *leaf*. For deletion we wish to remove *from* a leaf. There are three possible ways we can do this:
- 1 - If the key is already in a leaf node, and removing it doesn't cause that leaf node to have too few keys, then simply remove the key to be deleted.
- 2 - If the key is *not* in a leaf then it is guaranteed (by the nature of a B-tree) that its predecessor or successor will be in a leaf -- in this case can we delete the key and promote the predecessor or successor key to the non-leaf deleted key's position.

Removal from a B-tree (2)

- If (1) or (2) lead to a leaf node containing less than the minimum number of keys then we have to look at the siblings immediately adjacent to the leaf in question:
  - 3: if one of them has more than the min' number of keys then we can promote one of its keys to the parent and take the parent key into our lacking leaf
  - 4: if neither of them has more than the min' number of keys then the lacking leaf and one of its neighbours can be combined with their shared parent (the opposite of promoting a key) and the new leaf will have the correct number of keys; if this step leave the parent with too few keys then we repeat the process up to the root itself, if required

Type #1: Simple leaf deletion

Assuming a 5-way B-Tree, as before...



Delete 2: Since there are enough keys in the node, just delete it

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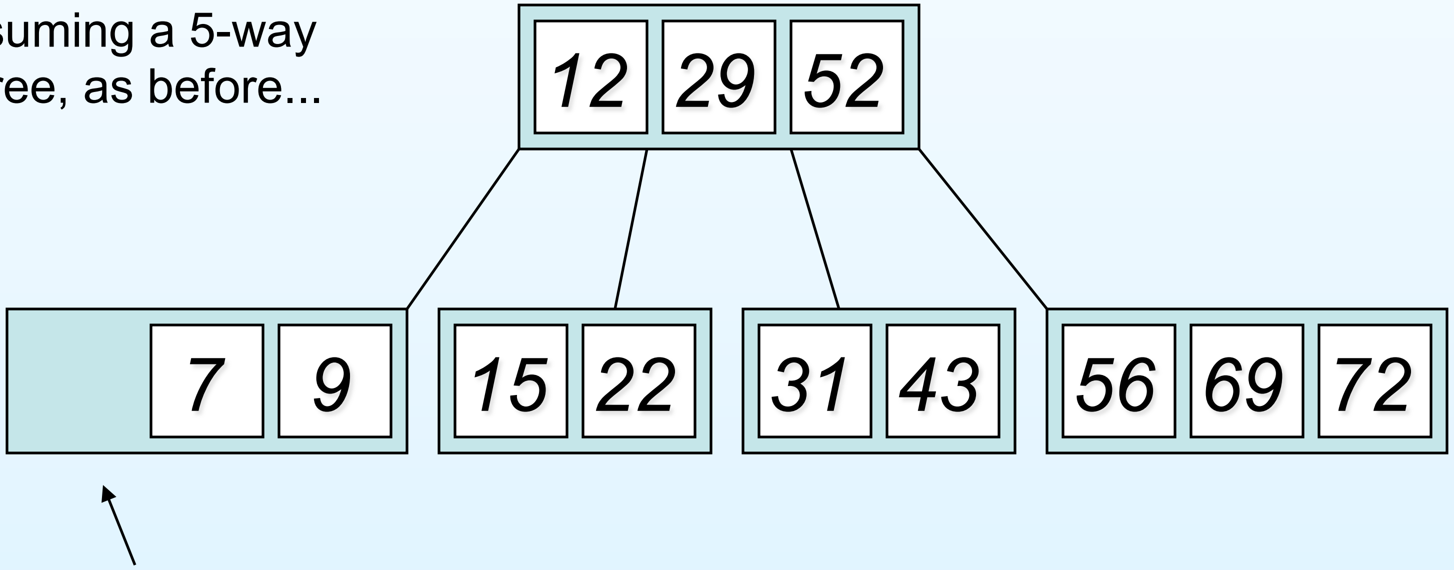
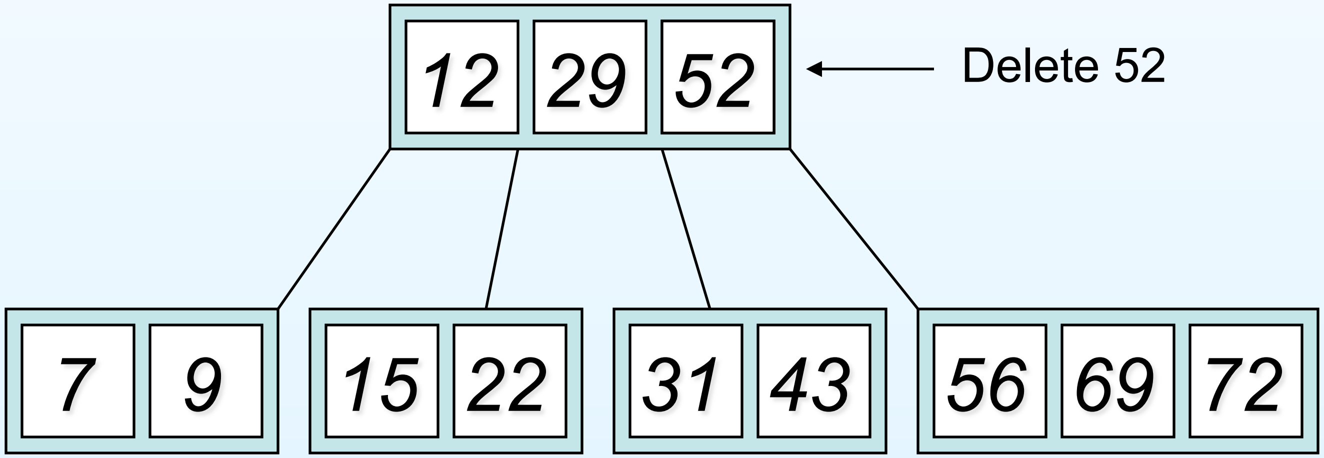
Assuming a 5-way B-Tree, as before...



Delete 2: Since there are enough keys in the node, just delete it

Type #2: Simple non-leaf deletion



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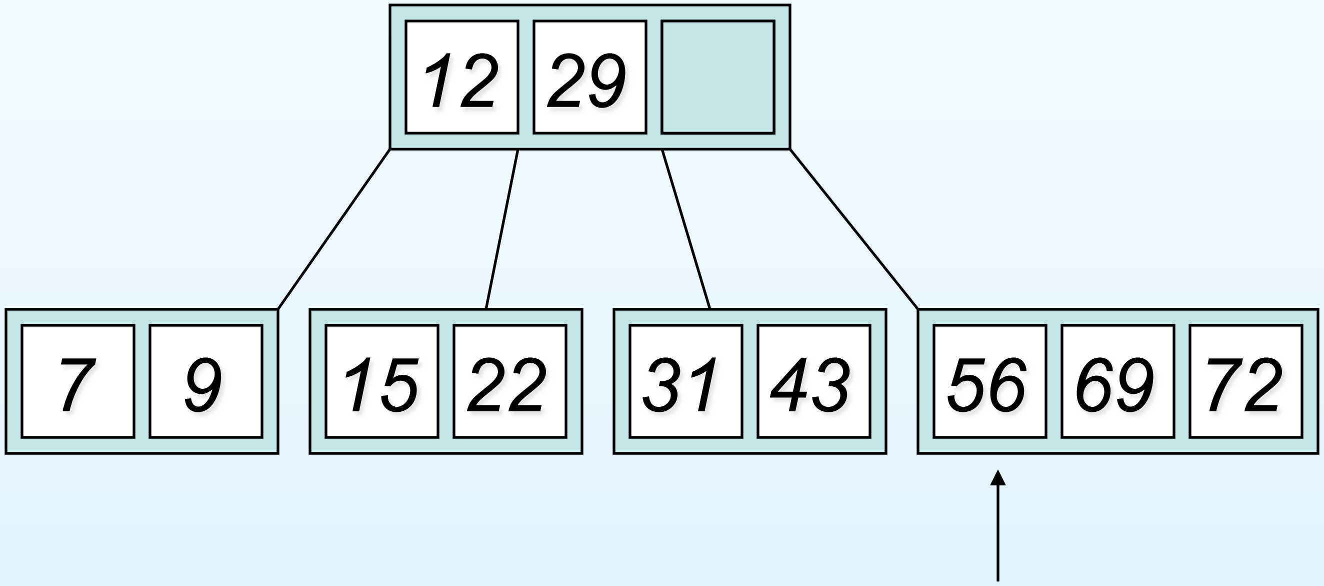
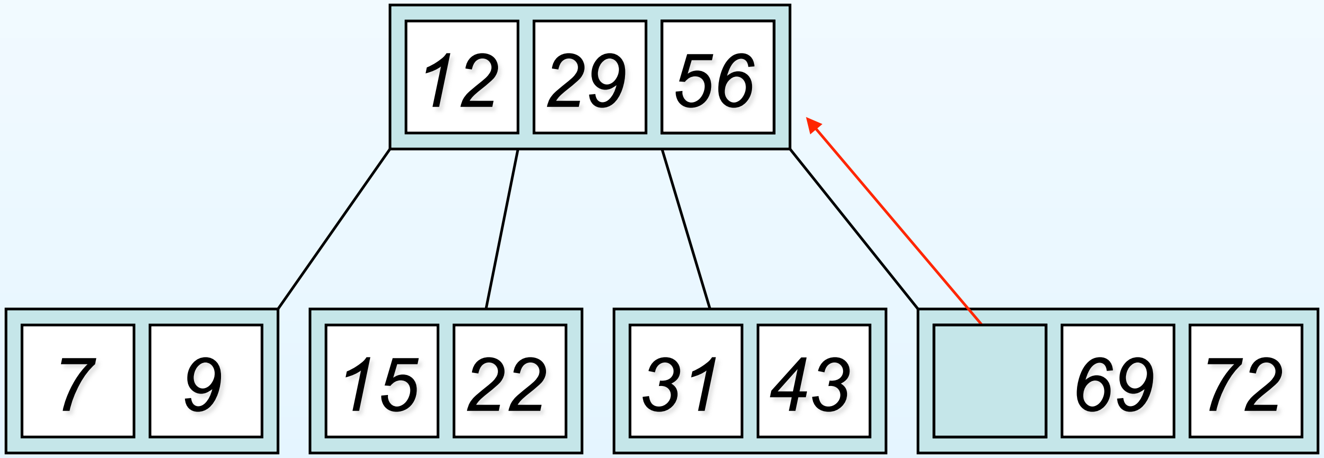
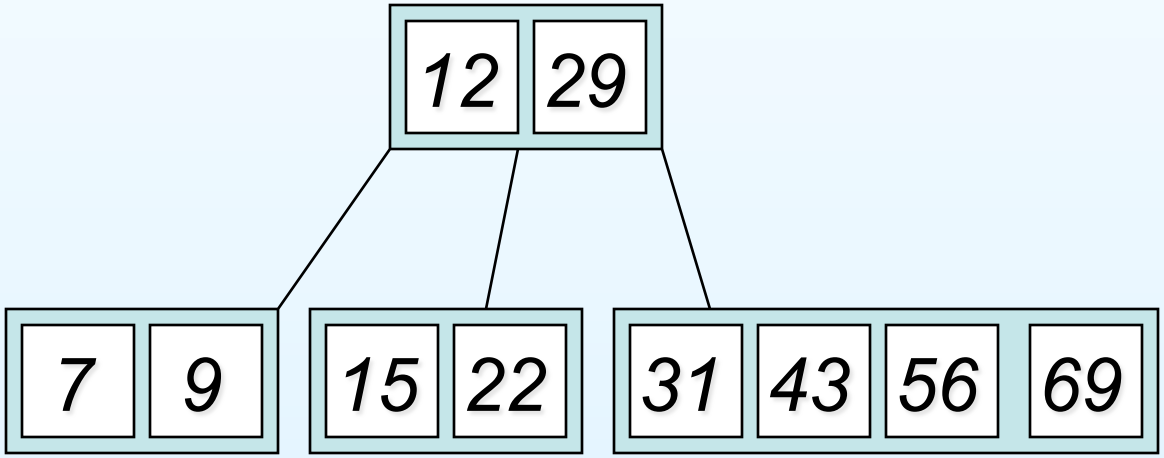


Borrow the predecessor
or (in this case) successor

Type #2: Simple non-leaf deletion

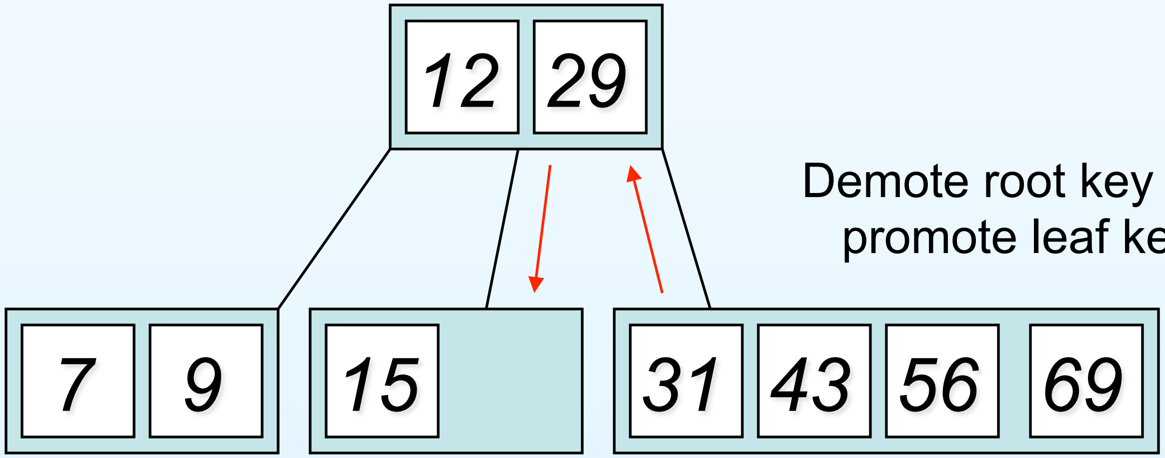


Type #3: Enough siblings



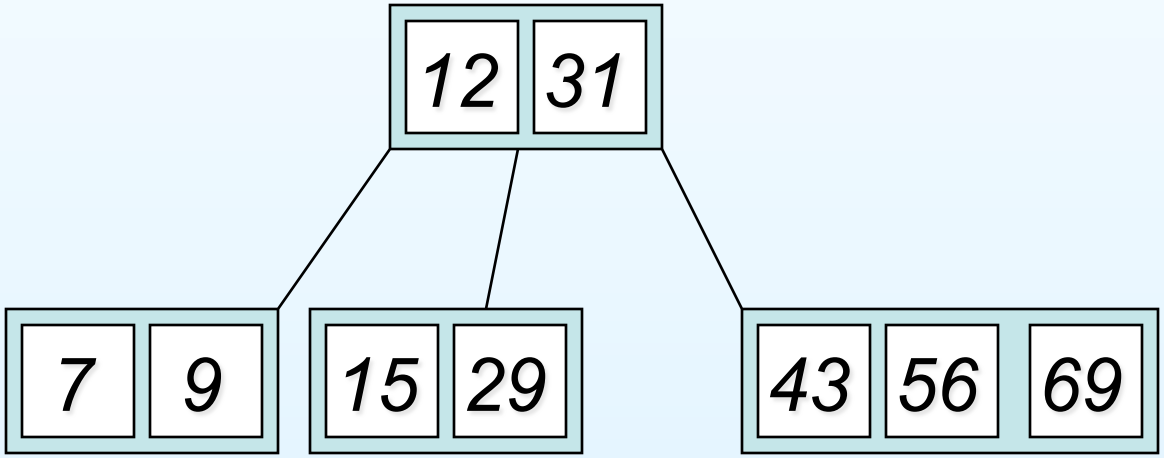
Delete 22

Type #3: Enough siblings

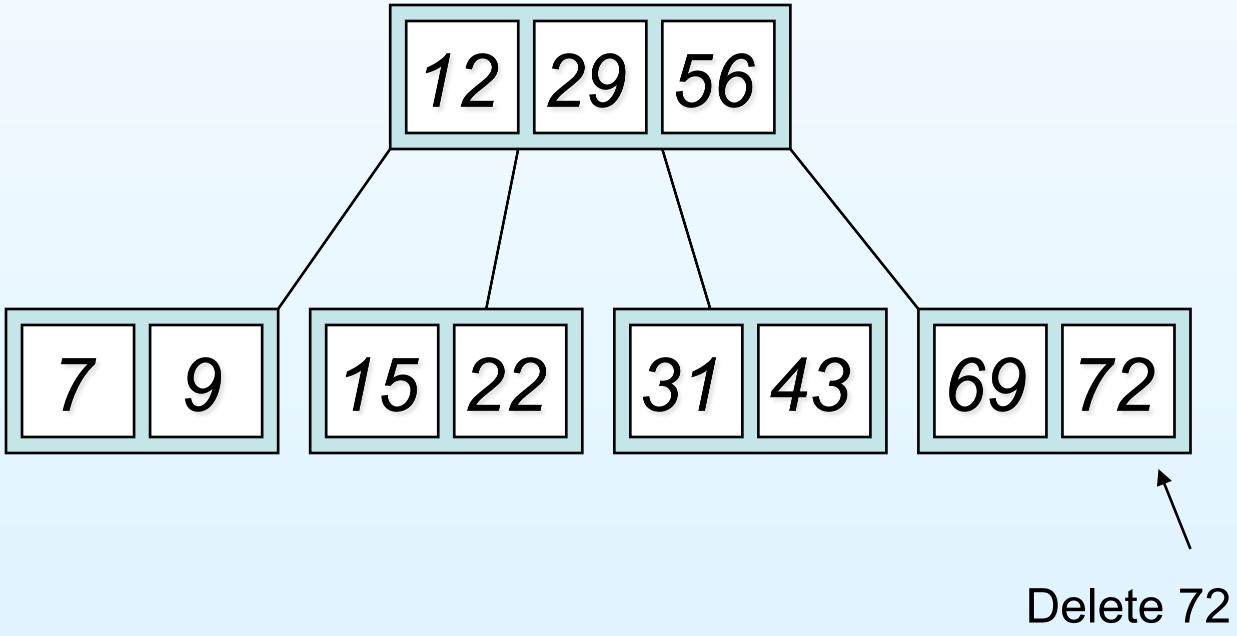


Demote root key and
promote leaf key

Type #3: Enough siblings



Type #4: Too few keys in node and its siblings



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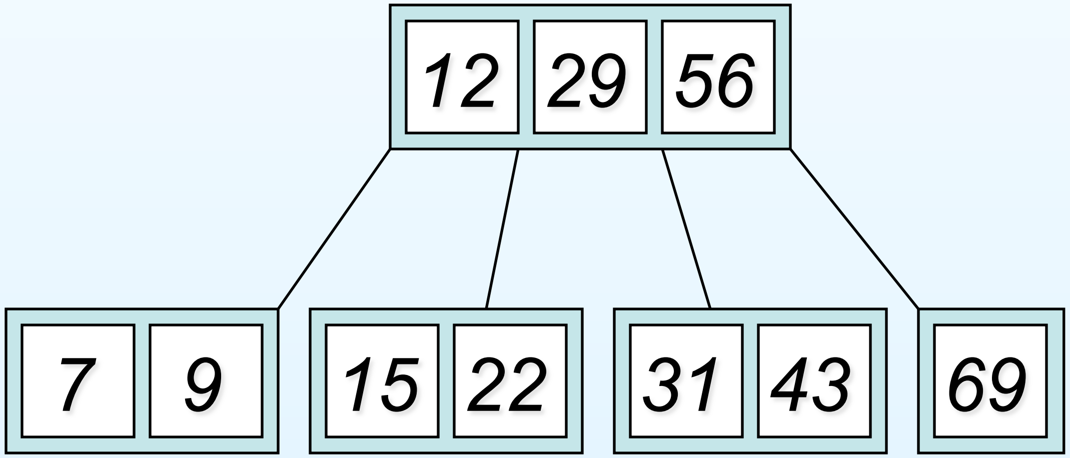
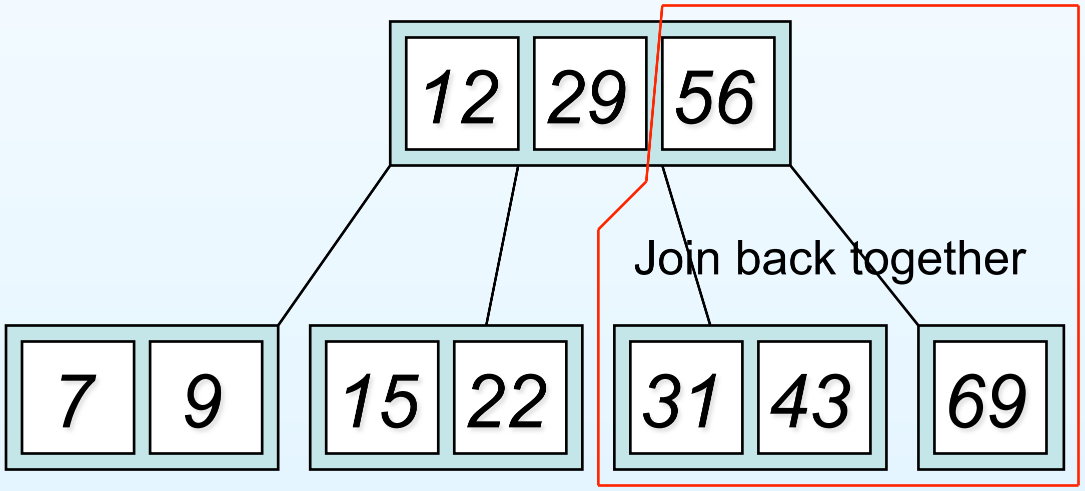


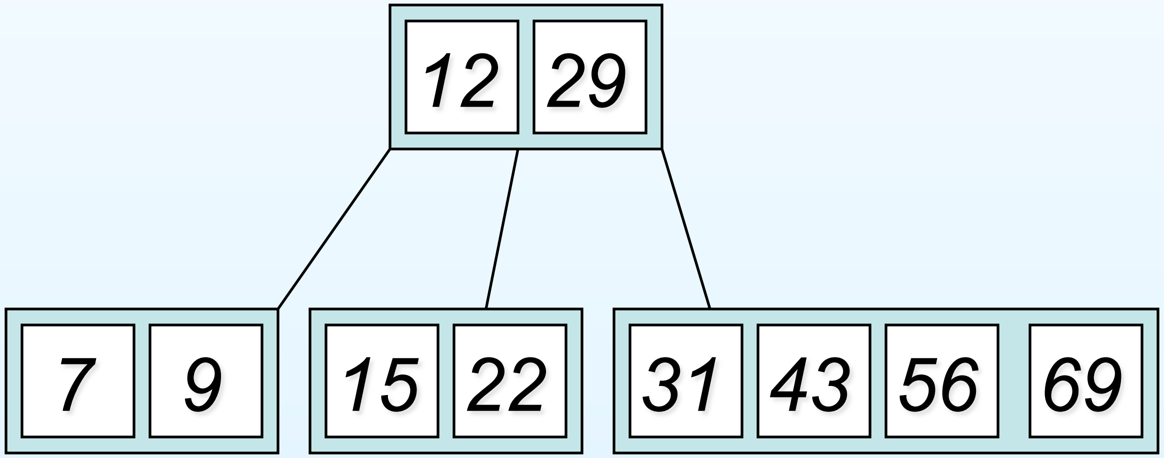
Too few keys!

Type #4: Too few keys in node and its siblings



Too few keys!

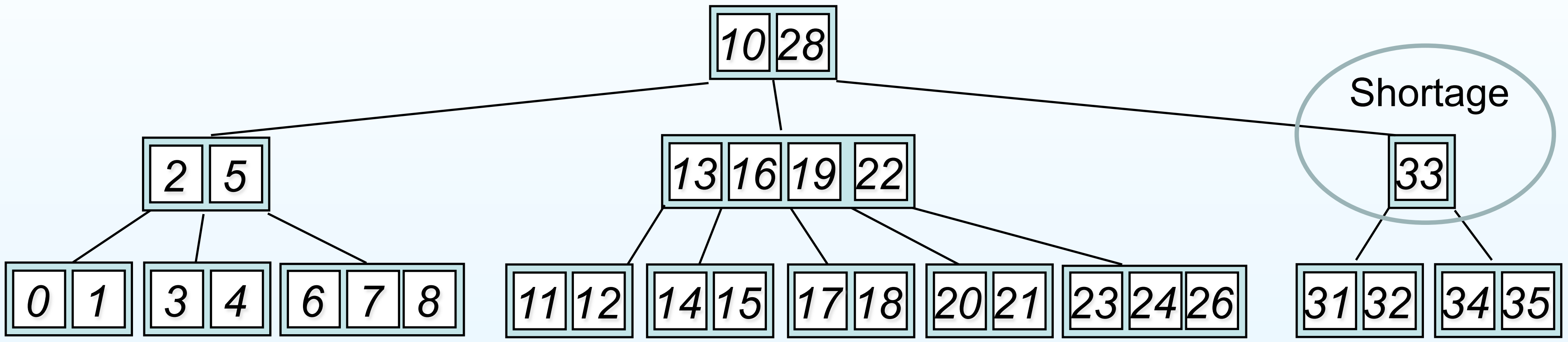
Type #4: Too few keys in node and its siblings

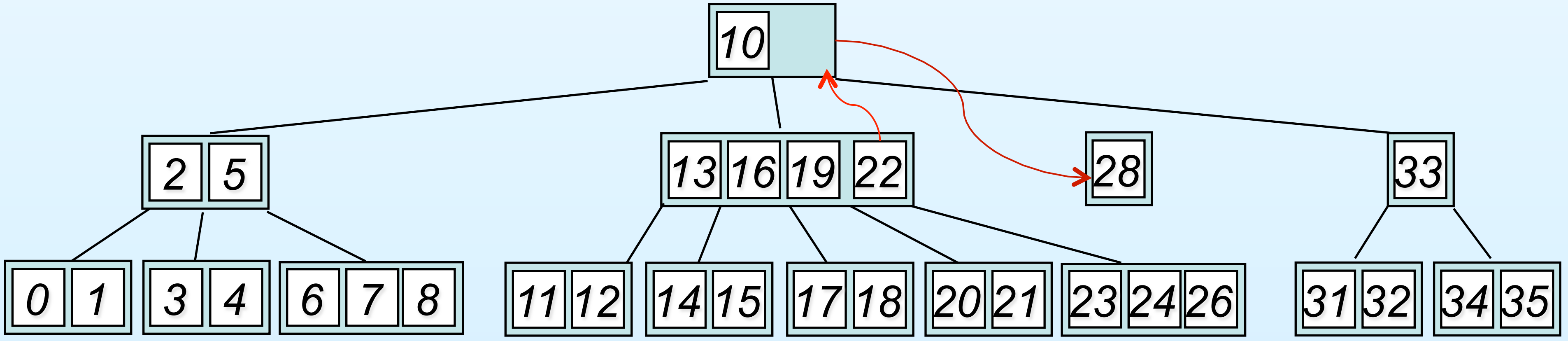


Sample Exam Question

- Given 5-way B-tree created by these data (last exercise):
- 3, 7, 9, 23, 45, 1, 5, 14, 25, 24, 13, 11, 8, 19, 4, 31, 35, 56
- Add these further keys: 2, 6,12
- Delete these keys: 4, 5, 7, 3, 14
- Again, check your approach with a friend and discuss any differences.

Example: Remove a key from a B-Tree





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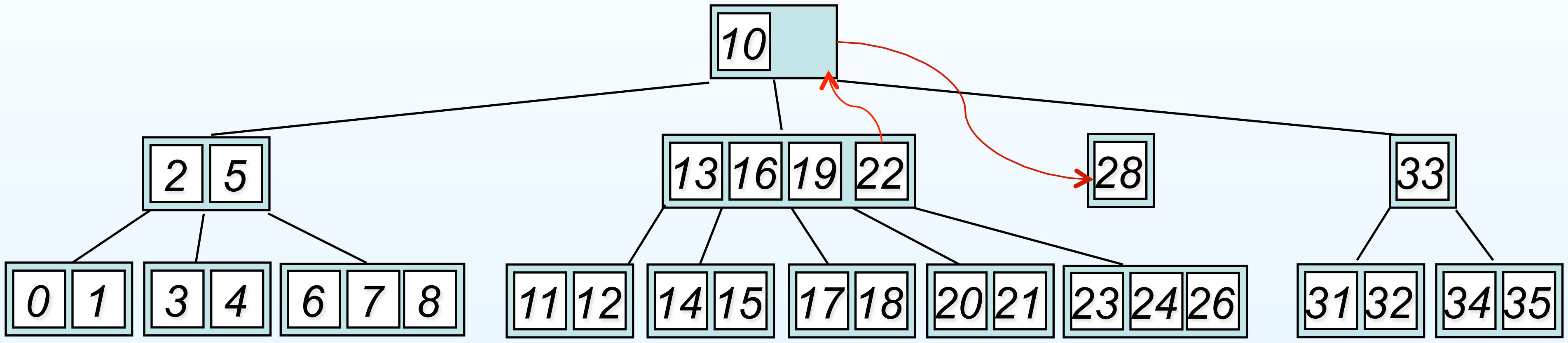

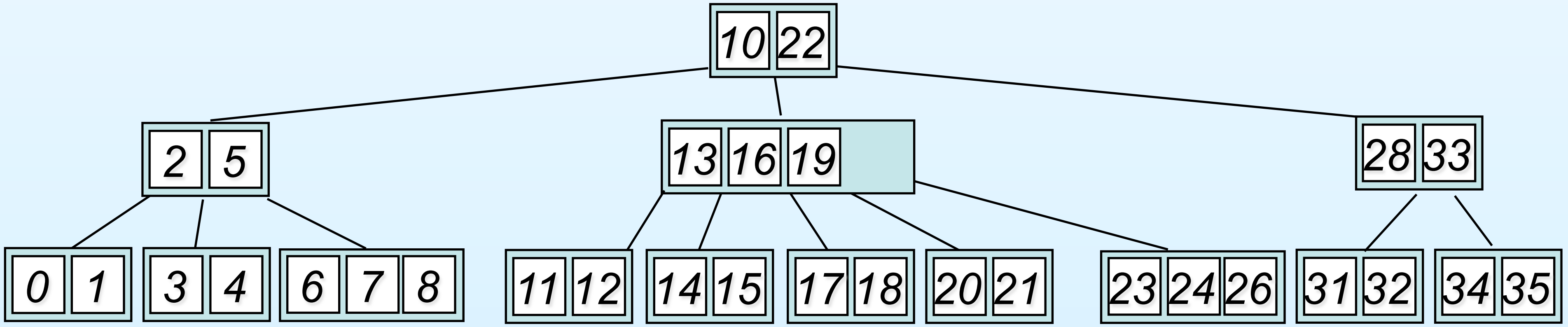

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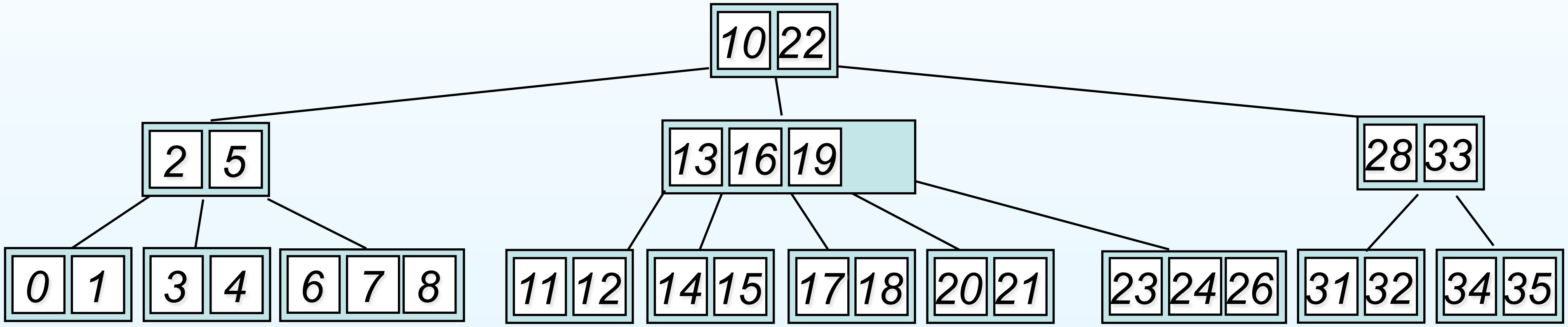
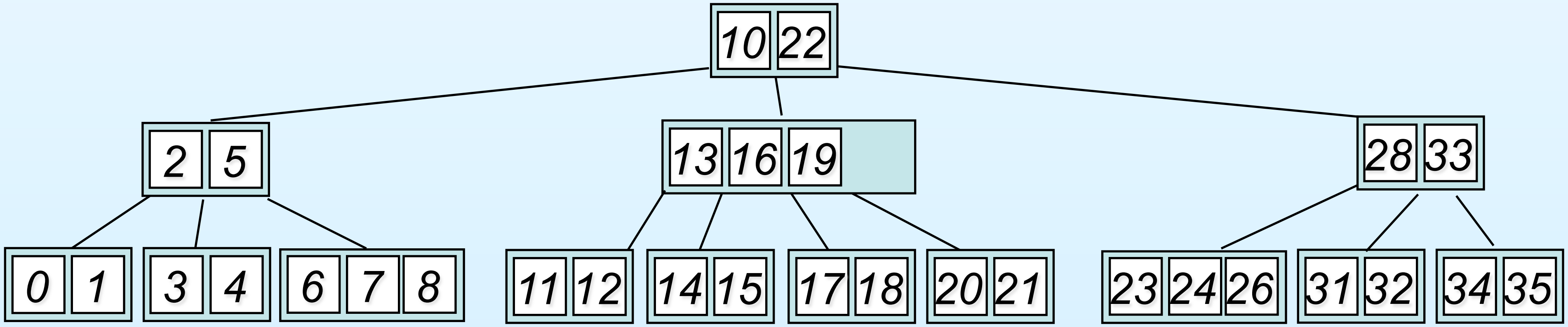

10 22
2 5
13 16 19
28 33
0 1
3 4
6 7 8
11 12
14 15
17 18
20 21
23 24 26
31 32
34 35


10 22
2 5
13 16 19
28 33
0 1
3 4
6 7 8
11 12
14 15
17 18
20 21
23 24 26
31 32
34 35