GRADE 10 MATH
CLASS NOTES
UNIT E – ALGEBRA

In this unit we will study exponents, mathematical operations on polynomials, and factoring.

Much of this will be an extension of your studies from Math 10F. This unit doesn't cover the '*solving for x*' type of algebra that you should already be a bit familiar with.

EXPONENTS AND RADICALS

1. Exponents and radicals are important and simple ways to express numbers. You have probably seen them often. An exponent is the superscript number in this expression: $\mathbf{x^2}$. A radical symbol is the funny looking √ , used in $\sqrt{5}$ for example.

2. Exponents were a very big part of Grade 9. Mathematics (Math 10F). There will be a very quick review in these notes of the basic ideas of exponents.

3. Radicals are the 'roots of numbers', they are really like doing exponents backwards. For example $\mathbf{3^2}$ is an exponential expression but $\sqrt{9} = 3$ is a root expression. They both really say the same thing. In Grade 12 you will call these 'inverse operations'.

Exponents			Roots	
x ⇨	x^2		x ⇨	$\sqrt{x}$
2	4		4	2
3	9		9	3
5	25		25	5
7	49		49	7
10	100		100	10

4. Maybe you already see the relationship between exponents and radicals. We will study radicals also in Unit B.

EXPONENTS

5. Recall the basic definitions and 'rules' of exponents

An ***exponent*** shows how many base **factors** are multiplied. Eg: $\mathbf{3^4 = 3*3*3*3}$. The 3 is the ***base***; the 4 is the ***exponent***. The full $\mathbf{3^4}$ is called a ***power***. Factors are numbers that multiply together.

Rules (easily derived from logic and the basic definition).

Product Rule of Exponents Add exponents of powers when two powers having same base are multiplied	$\mathbf{x^a * x^b = x^{(a+b)}}$ $\mathbf{2^3 * 2^5 = 2^8}$ $\mathbf{x^2 * x^3 = x^5}$	Also: $\mathbf{(ab)^2 = a^2b^2}$
Quotient Rule of Exponents: Subtract exponent in denominator from exponent in numerator for powers of same base.	$\frac{x^4}{x^3} = x^{(4-3)} = x^1 = x$ $\frac{4x^4y^3}{2x^2y} = 2x^2y^2$	Also: $\left(\frac{3x}{y}\right)^2 = \frac{3^2x^2}{y^2} = \frac{9x^2}{y^2}$
POWER Rule of Exponents. A **power** of a **power** is the **product** of the exponents. Multiply the exponents together.	$\mathbf{(x^2)^3 = x^6}$ $\mathbf{(3x^3y^2)^4 = 81x^{12}y^8}$	

EXAMPLES of EXPONENT RULES

Example Product Rule: $3^2 * 3^3 = 3^{2+3} = 3^5 = 243$. Sometimes on computers $\mathbf{3^5}$ is written **3^5**.

This is no majick law delivered by the creator, it is common sense.

If 3^2 means 3*3 and if 3^3 means 3*3*3 then by logic:

$3^2 * 3^3 = (3*3) * (3*3*3)$ which is really five **factors** of three multiplied together or 3^5. Notice the rule only works if the bases of the powers are the same.

You try a few. Simplify using the power rule then 'evaluate' when you can; obviously you cannot evaluate if it is a variable as the base. Do them without a calculator but check your answers with a calculator

a. $2^2 * 2^5$	b. $4^1 * 4^4$	c. $x^3 * x^7$
d. $\left(\frac{1}{2}\right)^2 * \left(\frac{1}{2}\right)^2$	e. $x^4 + x^2$ *Trick*!	f. $t^2 * t^{10}$

QUOTIENT RULE

Example Quotient Rule. $\frac{3^5}{3^2} = 3^3$.

Well of course it is true; look at what it means: $\frac{3^5}{3^2} = \frac{3*3*3*3*3}{3*3} = \frac{3*3*3*1*1}{1*1} = 3^3$

POWER RULE

Example Power Rule. $\mathbf{(2^3)^4 = 2^{12}}$. Again sort of obvious since:

$\mathbf{(2^3)^4}$ *means* $\mathbf{(2^3)\ (2^3)\ (2^3)\ (2^3)}$ which is **(2*2*2) (2*2*2) (2*2*2) (2*2*2)**
which is more simply: $\mathbf{2^{12}}$.

You try to simplify using the quotient or power laws. It is not necessary to completely evaluate the result.

a. $\frac{x^4}{x^2}$	b. $\frac{t^5}{t}$	c. $\frac{5^4}{5^2}$
d. $(x^3)^2$	e. $(2x^2)^3$	f. $(2^2)^4$

6. **Integer Exponents**. What happens when we use other ***integer*** numbers like zero or negative numbers as exponents, not just natural numbers {1, 2, 3, 4,…} ? What does $\mathbf{3^0}$ mean for example? What does $\mathbf{5^{-2}}$ mean for example?

7. **The Zero Exponent**. The rule is simple; **any power with an exponent of zero is 1**. You can see this by a simple pattern in a table:

Exponent m	**5**	**4**	**3**	**2**	**1**	**0**
$\mathbf{2^m}$	$\mathbf{2^5 = 32}$	$\mathbf{2^4 = 16}$	$\mathbf{2^3 = 8}$	$\mathbf{2^2 = 4}$	$\mathbf{2^1 = 2}$	$\mathbf{2^0 =}$

Notice the values of $\mathbf{2^m}$ keeps 'halving' every time exponent **m** decreases by one.

8. Evaluate the following:

a. $3^2=$	b. $3^1=$	c. $3^0=$	d. $7^0=$
e. $52^0=$	f. $(37xy^2)^0=$	g. $\left(\frac{3\pi}{4z2}\right)^0 =$	

9. **Negative Exponents**. Negative exponents using integers are possible. The rule about basic negative exponents is that: $a^{-m} = \frac{1}{a^m}$. So for example $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$. Check it on a calculator if you know how, of course calculators don't show fractions as answers.

10. A demonstration of why we say that $a^{-m} = \frac{1}{a^m}$ is in table form again. Follow the pattern:

m	5	4	3	2	1	0	-1	-2	-3
2^m	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$	$2^{-1}=1/2$	$2^{-2}=1/4$	??

11. Evaluate the following: Check with a calculator if you know how.

a. $\mathbf{2^{-1}}$	b. $\mathbf{4^{-2}}$	c. $\mathbf{x^{-3}}$ **Check with a couple of your own x's**	d. $\left(\frac{1}{7}\right)^{-1}$
e. $\mathbf{4^2 * 4^{-2}}$	f. $\mathbf{5^{-3}}$	g. $\left(\frac{2}{3}\right)^{-2}$	h. $\mathbf{7\pi^0}$

12. The big picture for integer exponents. Complete the table below:

$\mathbf{2^3}$	**=**			$\mathbf{2^{-1}}$	**=**	
$\mathbf{2^2}$	**=**			$\mathbf{2^{-2}}$	**=**	
$\mathbf{2^1}$	**=**			$\mathbf{2^{-3}}$	**=**	
$\mathbf{2^0}$	**=**					

13. **Scientific Notation**. You likely have seen scientific notation used with negative exponents and a base of 10. A quick review.

$\mathbf{3.1*10^2=}$	**310**	$\mathbf{27.6*10^4=}$	**276,000**
$\mathbf{4.5*10^{-2}=}$	$\frac{4.5}{100}$ **0.045**	$\mathbf{280*10^{-9}=}$	$\frac{280}{1000000000} =$ **0.00000028**

14. **Fractional Exponents**. Can you have fractional exponents? Like $\mathbf{4^{1/2}}$? Yes! If you understand the conceptual idea of an exponent. Fractional exponents are also called rational exponents because a fraction is a ratio of two numbers. The rule about fraction exponents is this monster:

$$a^{m/n} = \sqrt[n]{a^m}$$

The symbol $\sqrt{\ }$ is called a radical sign. $\sqrt{9}$ means to find that number that multiplied by itself gives you 9. You likely guess that since 3*3 = 9 that the answer is 3.

In some books or videos or countries you will find that the radical is called 'surd' especially in proper English texts.

15. Example of a fractional exponent.
$3^{4/2}$ means $\sqrt[2]{3^4}$ or $\sqrt{81}$ or 9.

If you have never seen the little '2' in the $\sqrt[2]{\ }$ it is because whenever you had used $\sqrt{\ }$ before, there was really an understood and assumed 'little two'. So the symbol $\sqrt[2]{\ }$ means exactly $\sqrt{\ }$.

16. **Explaining Fractional Exponents**. We already know what a^2 means. But what does $a^{1/2}$ mean?

If $\mathbf{5^2 = 25}$ then what does $\mathbf{(5^2)^{1/2}}$ mean? We know from the power 'law' that we can ***multiply*** the exponents together so:

$\mathbf{(5^2)^{1/2}}$ = _______

So $\mathbf{25^{½}}$ = ________. So raising 25 to the power of one-half is the same as __________

($\mathbf{25^{1/2}}$ is ***exactly*** the same as $\sqrt{25}$)[1]

See the pattern of fractional powers. Complete the table to find $4^{1/2}$:

4^4	4^2	4^1	$4^{1/2}$	$4^{1/4}$
256	16	4	??	??

We are halving the exponent of the power each step and the resultant value is the square root of the previous value.

[1] Be aware that there is one other thing to learn later about roots, especially *even* roots. You may already have wondered, but what is if $\mathbf{x^2 = 4}$, what is **x** for example. There are actually two answers: one answer is **2**, but **–2** works also!!

17. Another example of fractional (ie: rational) exponents.

$4^{3/2} = (4^3)^{1/2} = (64)^{1/2} = \sqrt{64} = 8$ or you might have done the fractional power first:

$4^{3/2} = (4^{1/2})^3 = (\sqrt{4})^3 = 2^3 = 8$

18. ***Evaluate*** the following. Give an exact answer where possible:

a. $4^{3/2} =$	b. $27^{2/3} =$	c. $8^{4/3}$
d. $4^{-3/2}$ (this is good!)	e. $\sqrt[3]{27}^{\,2}$	f. $2^{5/2}$

19. **Exponents on your Calculator**. In the event you need to use a calculator to get decimal answers to some problems you can do exponents as follows.

a. TI 83 and EXCEL and some other graphing tools on Line. To show that you want to apply an exponent to a number you use the '**^**' key;

b. on more simple calculators you may have to use a **x^y** key.

POLYNOMIALS

20. Polynomials are mathematical expressions. *You* are really a polynomial!

21. Polynomials are made up of **terms** added to together. So for example: **4 + 7 + 1** is a polynomial expression. The example given is made up of '**constants**'.

22. Polynomial terms can also have **variables** (unknowns) as **terms**. x + y + z is a polynomial. And so is **3x + 5y + 7z**. The constant in front to the variable is called a **co-efficient**. We generally put the *co-efficient* in front of the variable to be consistent.

23. What is the 'co-efficient' on the '*z*' term of **3x + 5y + 7z**? ________________

ADDING LIKE TERMS

24. Say you had **three** pigs and **two** cows in one pasture, and **four** pigs and **one** cow in another pasture. If someone asked you how many different animals you had you would say ' *7 pigs and 3 cows*'. You would combine like animals. Like in a polynomial; we '*combine like terms*'.

25. Try these:

a. **2x + 7 x + 4 y + 3y =** ?? _________________

b. **5z + 4t – 2z + 7t =** ?? _________________

26. Try these more complicated ones:

a. **4xz + 7yz – 2xz – 9yz =** _____________________

b. **5qt – 4qr + 7tq** = _________________ (trick!)

c. $\frac{2}{3}x + \frac{1}{2}y - \frac{1}{3}x + \frac{3}{4}y =$ __________________

d. $x * 1.4 + \frac{3}{10}x =$ ______________

ADD AND SUBTRACT POLYNOMIALS

27. Simple Add Example. **(2x + 3y) + (7x – y) = 9x + 2y**

28. You try adding the two polynomials:

a. **(5x + 7y) + (3y + 2x)**	b. **(4xz + 2qt) + (xz + 2qt)**

*Note we tend to generally not write ‘3y + 2x’, but rather ‘2x + 3y’ in alphabetic order, just so that we are consistent. The first is not wrong, just the second is more right.

29. Simple Subtract Example: (9x + 5y) – (7x + y) = 2x + 4y

30. These are trickier. The subtract applies to the entire ‘bouquet’. You try:

a. **(5x + 7y) – (8y + 2x)**	b. **(4xz + 2qt) – (xz + 2qt)**
c. **$(3x^2 – 7y) – (5x^2 – y)$**	d. **(–3x + 2y) – (x – 2y)**

TYPES OF POLYNOMIALS NAMES

31. Monomials are just simple individual ‘terms’. Eg: 4, x, 7x, 3rt, …Notice when we write monomial terms we write them so the variables are in alphabetical order and the coefficient at the front. It would be technically not wrong to come up with answer of ‘**z5t**’ to something but we would standardize it so everyone got the same answer as: ‘**5tz**’.

32. **Bi**nomials have **two** terms. Eg: $\mathbf{2x^2 + 7}$, **x – y**, **6 – x**,

33. **Tri**nomials have **three** terms: Eg: $\mathbf{3x^2 + 2x + 7}$, **2xz + 7yz – 3qt**, … *etc.*

MULTIPLY POLYNOMIALS

34. What does **4*(2x + 7)** mean? What does **(x + 5)*(x + 4)** mean? That is what we need to explore.

Multiply Monomial by Binomial

35. Eg: **4*(x + 7)**. Doesn’t that really mean four bunches (‘*bouquets*’ for the ladies) of ‘x + 7’ added together? So doesn’t it really mean **(x + 7) + (x + 7) + (x + 7) + (x + 7)**. Which is really **4x + 28** isn’t it??

36. **You try**; multiply:

a. 3*(7q + 4t)	b. 2(3x – 7)
c. -5 (5x + y)	d. $\frac{1}{3}(6x-9)$

37. Notice how this form of multiplying is possible because of the ‘***Distributive Law***’ of arithmetic which you may recall says: ***a*(b + c) = ab + ac***. In other words a quantity ‘***a***’ bouquets of ***b + c*** is really ‘***a***’ bunches of ***b*** plus ‘***a***’ bunches of ***c***.

38. Jasmine loves her ***x***'s and ones. Her b/f likes to buy her a bouquet that is always *four* **x**'s and *three* **ones**. This month, Jasmine has received *five* bouquets like that! What has Jasmine received?
$5*(4x + 3)$; so Jasmine has received how many ***x***s and **ones**?

____

Multiply Monomial by Polynomial

39. Example: **$4*(2w - x + 2y + 3z) = 8w - 4x + 8y + 12z$**

You try: **$2*(3w - 2x + 3y + 7) =$** ______________________

Multiply Binomial By Binomial

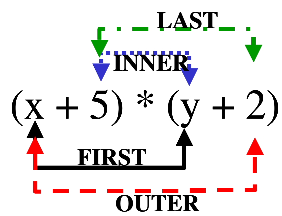
40. **Example**: **$(x + 5) * (y + 2) = xy + 2x + 5y + 10$**.

41. **Explanation**. Think of Jasmine and her binomial bouquets. This time Jasmine expects bouquets consisting of a '***y***' and **two** ones ie: **$(y + 2)$**. Her b/f buys her **5 this week** and **last week** he forgets how many he bought, so call it an unknown '**x**' number of bouquets he bought her last week. So he bought her '**$x + 5$**' bouquets of '**$y + 2$**', right?

What did Jasmine get total??

THE FOIL METHOD OF MULTIPLYING BINOMIALS

42. FOIL stands for the '*mnemonic*' (meaning a 'memory aid'): **FIRST, OUTER, INNER, LAST**. Memorize it. It is the order in which you multiply the terms in each binomial.



43. So **(x + 5) * (y + 2)** is calculated as follows:

x*y + x*2 + 5*y + 5*2 , or simplified: **xy + 2x + 5y +10**

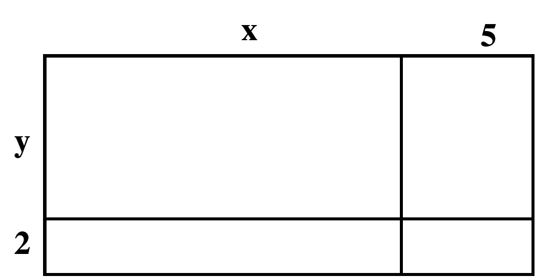
More recently some textbooks and websites have started to call the foil method the '*Each With Each*' (***EWE***) method. In other works multiply each term in each factor by each term in the other factor and add them together. Same thing!

44. You try a few using **FOIL** to multiply:

a. **(x + 4)*(y + 1)**	b. **(t + 3) * (t – 3)**
c. **(2x + 3)*(3t – 4)**	d. **(2x + 1)*(0.5x – 2)**

Multiply Binomials Using Geometry

45. Some people prefer this method to explain how to multiply binomials. If you recall the area of a rectangle it is length times width.



46. What is the length? ______________ . What is the height: ________________
So multiplying the Length by Height should give the total area. But the total area is also given by adding together the four separate rectangles.

	x	**5**
y	**xy**	**5y**
2	**2x**	**10**

47. The area of the four separate rectangles all added together is simply: **xy + 2x + 5y + 10**

48. You try a couple using the geometry method:

a. **(x + 1) * (x +3)**	b. (x + 4)* (z – 2) *(using negative amounts, subtracting, is a bit more difficult to show with geometry)*

Evaluating Polynomials

49. To ‘***evaluate***’ means to ‘*give a value*’. Example: given the binomial expression **2x + 3y**; and given that **x = 2** and **y = –5**; evaluate. (or ‘*plug in*’ we sometimes say)

2x + 3y = 2(*2*) + 3(*-5*) = 4 – 15 = -11

50. **You Try**. Evaluate the following polynomial expressions:

a. $2x - 7y + 3$; given $x = 1$, $y = 0$.

b. $3x^2 - 2z$; given $x = 2$; $z = -5$.

Note: Recall that x^2 means ***x*x*** from exponents lessons in previous lessons.

APPLICATIONS

51. When would you ever use this stuff? Here are some examples, some are pretty advanced.

52. **Application 1**. In two years my sister will be **19**, how old is she now?
Let '***a***' represent her age now. In two years her age will be her age now *plus 2*, so '***a + 2***'. So if ***a + 2 = 19*** then, using algebra, ***a = 17***.

a + 2 was a rather simple binomial and we knew its value, so we solved to find the ***a***.

53. **Application 2**. The height of a ball above ground that falls out our classroom window is given by the polynomial expression: $45 - 5t^2$. This is the actual expression (on *earth*) that you would learn in Physics! Evaluate the expression for different times; *t,* in the table below:

T (seconds)	**height $45 - 5t^2$**
0	
1	
2	
3	

FACTORING

54. In the final section of these notes you will learn how to go from the polynomial form like $x^2 + 5x + 6$; several terms *added* together, and turn it **back into** two binomials *multiplied* together.
$(x + 2)(x + 3)$.

55. There are more practice problems in Appendix 1 to these notes.

FACTORING

FACTORING

60. Factoring is really *un-multiplying*. So technically it is dividing! So if you understand multiplying of polynomials just think backwards.

61. For example if $2(x + 7) = 2x + 14$, then taking $2x + 14$. Turning it back into the original factors that multiply together; ie: $2(x + 7)$ is called factoring.

FACTORING BY 'DIVIDING OUT' A GREATEST COMMON FACTOR

62. Example: $\mathbf{8xy+4xy^2}$ can be factored into $\mathbf{4xy(2 + y)}$. You can confirm this is true by multiplying it out.

63. The method of removing a Greatest Common Factor is just to divide each term by their GCF and to make that GCF multiply that result. Dividing by something and then multiplying by the same thing doesn't change the expression.

64. **Example**: Factor completely $\mathbf{9x^2y + 3xy^2}$.

65. Break everything into basic primes: $\mathbf{9x^2y + 3xy^2 = 3*3*x*x*y + 3*x*y*y}$
Both terms have **3*x*y** in common; so that common factor can come outside the brackets when we write it as factors.

$\mathbf{3*3*x*x*y + 3*x*y*y}$ $\boxed{\mathbf{= 3xy(3x + y)}}$

66. The '$\mathbf{3x + y}$' cannot be factored any further so we are done. The expression $\mathbf{9x^2y + 3xy^2}$ has been *completely factored*. We have turned an expression that was *some terms added* together into something that is some *factors multiplied* together. This becomes essential later when solving equations using algebra.

67. **Examples**: You Try. Factor completely.

a. $\mathbf{12x^2y + 8xy^2}$	b. $\mathbf{3x^2 + 5xy + 9xy^2}$
c. $\mathbf{8x^2 + 4x + 2}$	d. $\mathbf{8x^2 + 4x}$

FACTORING BY GROUPING

68. This is much more difficult. Example: $\mathbf{px + py + qx + qy = (x + y)(p + q)}$.

The secret is to group the terms like this $\mathbf{[px + py] + [qx + qy]}$ and factor each group

$\mathbf{[px + py] + [qx + qy] \;\; = p[x + y] + q[x + y]}$
Now $\mathbf{[x + y]}$ is a common factor to each term so
$\mathbf{= [x + y] [p + q]}$
It gets to be simple after a while, just figuring out which terms to group together takes some experience.

69. We will not spend much more effort on this general grouping method but will use it shortly in a related method with trinomials. One final example though.

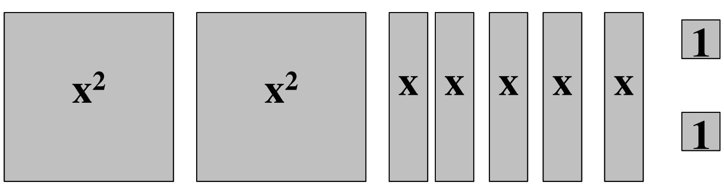
Factor Completely: $\mathbf{2x^2 + 5x + 2}$.

70. There is no common factor to remove. But notice it could be regrouped as:

$\mathbf{2x^2 + 4x + x + 2}$ **or** $\mathbf{[2x^2 + 4x] + [x + 2]}$
but: $\mathbf{[2x^2 + 4x] + [x + 2] = 2x[x + 2] + [x + 2]}$ and $\mathbf{[x + 2]}$ is common to both terms so:

$\mathbf{2x[x + 2] + [x + 2] = [x + 2][2x + 1]}$

71. It is often easier to see using algebra tiles.



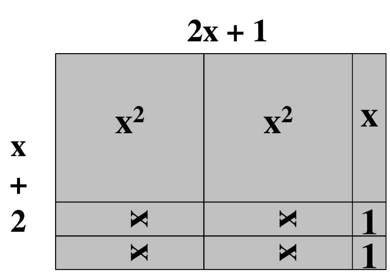


Illustration of : $\mathbf{2x^2 + 5x + 2 = (2x + 1)(x + 2)}$

FACTORING QUADRATIC TRINOMIALS

72. Quadratic Trinomials are polynomials with three terms in them and are a *second degree* expression (they have a '**x^2**' in them). They are a big part of Grade 11. Factoring quadratic trinomials (and some related special binomials of second degree) is the subject of the remainder of this unit.

Quadratic Trinomials have this form: $\mathbf{ax^2 + bx + c}$

73. **Example**: Factor the following quadratic trinomial completely: $\mathbf{x^2 + 3x + 2}$

74. There is no common factor to each term. So that won't work.
It can actually be grouped though as $[x^2 + 2x] + [x + 2]$ and then a common factor of $[x + 2]$ divided out. But that is too hard to see.

75. The simplest method to factor a quadratic trinomial is as follows:

a. Write the quadratic trinomial properly in terms in descending order of degree: ie: $\mathbf{x^2 + 3x + 2}$

b. know that any second degree trinomial can be turned into two binomials multiplied together and vice versa. Eg: $(x + m)(x + n)$

c. Find the possible pairs of integers $\{m, n\}$ that multiply to make the constant term '**2**': the only possibilities are $\{1, 2\}$ and $\{-1,-2\}$

d. Of these pairs, which pair will uniquely add to make the '**3**' coefficient in front of the x term. Only $\{1, 2\}$.

e. So '1' and '2' multiply to make '2' and add to make '3'.

f. so the trinomial factors into two binomials: $\boxed{\mathbf{(x + 1)*(x + 2)}}$.

g. check that it works by multiplying it out with FOIL. $\mathbf{(x + 1)*(x + 2) = x^2 + 3x +2}$

76. You try. Factor the quadratic trinomials below: (if they can be done)

a. $\mathbf{x^2 + 7x + 10}$	b. $\mathbf{x^2 + 3x - 10}$
c. $\mathbf{x^2 + 5x + 4}$	d. $\mathbf{x^2 + 5x + 5}$
e. $\mathbf{x^2 + 4x + 4}$	f. $\mathbf{x^2 + 0x - 25}$

77. Notice that d) above cannot be factored (well not using simple numbers at least). So only certain expressions contrived by your teacher or a text book can actually be fully factored.

Also notice that e) and f) suggest an interesting pattern also.

FACTORING PERFECT SQUARE TRINOMIALS

78. Trinomial like $\mathbf{x^2 + 6x + 9}$ are called perfect square trinomials. It is because they can be turned into a single binomial squared ie: $\mathbf{(x + 3)^2}$.

79. The pattern to recognize is that the constant **9** is a perfect square number formed by two 3s **multiplied** together, but the **6** coefficient in front of the x term is formed by two threes **added** together. So for $\mathbf{x^2 + bx + c}$ if half of the 'b' squared is the 'c' then it is a perfect square trinomial.

80. You try. State if these are perfect square trinomials (Y/N) then factor them to show it.

a. $\mathbf{t^2 + 8t + 16}$	b. $\mathbf{x^2 - 10x + 25}$
c. $\mathbf{x^2 + 10x + 25}$	d. $\mathbf{v^2 - 2v + 1}$

81. Notice the perfect square trinomials given so far had a leading coefficient 'a' of 1. Thepattern also works for expressions like:

$\mathbf{4x^2 + 12\,x + 9}$

Notice that $\mathbf{4x^2}$ is a perfect square **2x*2x** and that **9** is a perfect square: **3*3**.

And if you multiply $\sqrt{4}\ by\ \sqrt{9}$ and double it you get the '12'
Consequently $\mathbf{4x^2 + 12\,x + 9}$ can be written by inspection as $\mathbf{(2x + 3)^2}$ which of course is easily checked by FOIL.

82. You try. Factor by inspection these perfect square trinomials with leading coefficient not equal to one then check your answer.

a. **$4x^2 + 4x + 1$**	b. **$9x^2 + 12x + 4$**
c. **$25x^2 – 30x + 9$**	d. **$4x^2 – 40x + 100$**

DIFFERENCE OF SQUARES

83. Another special arrangement is binomials of the possible form:

$px^2 – q$ where **p** and **q** are perfect squares. So it is a difference (a subtraction) of two squares. For example

$x^2 – 9 = (x – 3)(x + 3)$

$4x^2 – 16 = (2x – 4)(2x + 4)$

84. **You try**. Factor by inspection these differences of squares then check your answer.

a. **$x^2 - 64$**	b. **$9x^2 - 100$**
c. **$t^2 - 1$**	d. **$25z^4 - 16x^2$**

THE AC METHOD OF FACTORING

85. The AC Method will work ***some times*** for many trinomial polynomial expressions to help you more easily find the *four* terms that can be readily factored by grouping.
Let's try **$4x^2 + 5x - 6$**

- **Step 1**: Multiply the **a*c: ac = –24**
- **Step 2**: Find the factors (that **multiply** together) of **–24** and that **add** to give **+5**. Notice that **–3** and **+8** work.
- **Step 3**: Rewrite the original expression with the smaller number*x first (without consideration of its sign) and the larger number*x next in place of the original x term; **$4x^2 - 3x + 8x - 6$.** Group them: **$[4x^2 - 3x] + [8x - 6]$**
- **Step 4**: Factor the first two terms: **$x(4x - 3)$** and factor the second two terms **$2(4x - 3)$**
- **Step 5**: Do the final factoring. **$x(4x - 3) + 2(4x - 3)$** both have **$(x - 3)$** in common as a factor; so **$(x + 2)(4x - 3)$** is the final answer.
- **Step 6**: Check your answer by multiplying out again (F.O.I.L)
  $(x + 2)(4x - 3) = 4x^2 + 5x - 6.$

PRACTICE EXAMPLES:

86. Factor: **$6x^2 + 17x + 12$**

- ac = 72 which has factors of 36 and 2 , 9 and 8, etc.

- 9 and 8 ***add*** to give 17 though!

- So rewrite as: **$6x^2 + 8x + 9x + 12$** (smallest factor: '8' first)

- Factor the first two terms and the last two terms:

$2x(3x + 4) + 3(3x + 4)$

- Do the final factoring since both parts have a **$3x + 4$** in common; thus:
  $(3x + 4)*(2x + 3)$

- Check the answer by multiplying!

87. **Example 2**: Factor **$10x^2 – x – 3$**

- ac = –30 which has factors of 15, –2 and –5, 6 and –6, 5, etc...

- –6 and 5 ***add*** to give the '**b**' of **–1** though!

- So rewrite as: **$10x^2 + 5x – 6x – 3$**

- Factor the first two terms and the last two terms:
  $5x(2x + 1) + -3(2x + 1)$

- Do the final factoring since both parts have a $3x + 4$ in common; thus:
  $(5x – 3)*(2x + 1)$

- Check the answer by multiplying! It works!

88. One for you to try. Factor: $\mathbf{16x^2 + 34x + 4}$ (hint: factor out a GCF of 2 firstwill makeit easier)

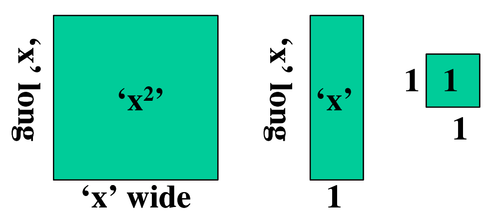
The answer is: $\mathbf{(8x + 1)(2x + 4)}$ but notice you should have factored out a two first, so really the fully factored answer is: $\mathbf{2(8x + 1)(x + 2)}$.

FACTORING USING GEOMETRY (ALGEBRA TILES)

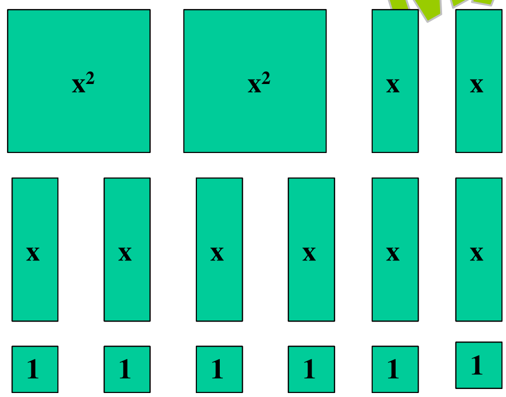
89. Some students like to see a more visual way of factoring. Your nieces and nephews in Grade 5 and 6 probably learn it this way using Algebra Tiles. In fact this is the original way that ancient mathematicians factored trinomials. This is just a demonstration of the method, you can research it further on your own if you want.

90. Factor the trinomial $\mathbf{2x^2 + 8x + 6}$

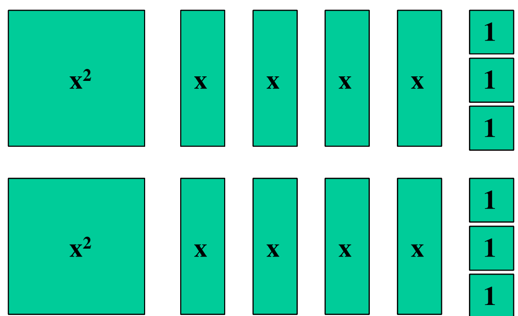
91. The ancients would draw squares of size x*x (ie: x^2) in the dirt, and rectangles of length x and width one, tiny squares of length one by one. You have to pretend you do not know how long a length of **x** is.



92. So $\mathbf{2x^2 + 8x + 6}$ would be scratched in the dirt as:

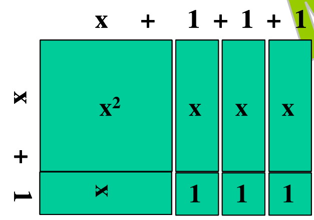


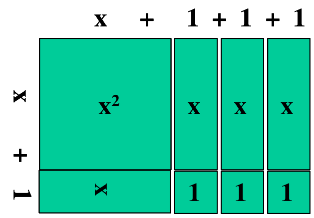
93. And then they noticed they could group them together like this:



Which is really two groups of $\mathbf{x^2 + 4x + 3}$

94. But both of these groups could be re-grouped like this:





Which can be expressed mathematically as **2** bunches of **(x+3) by (x+1)**; in other words: **2((x+3)(x+1))** or **2(x+3)(x+1)**

So: $\mathbf{2x^2 + 8x + 6 = 2(x+3)(x+1)}$

Of course, negative numbers were a bit perplexing for the ancients! So they didn't bother with negative numbers. But you may find your nieces and nephews are taught that red tiles are negatives, and green are positive and that a red cancels a green because they are opposites. It just complicates things, but it can be done.

FACTORING IN TWO UNKNOWNS

95. Factoring in two unknowns is also possible using the **AC Method** with some adaptation.

96. **Example**. Factor: $\mathbf{2x^2 + 5xy + 2y^2}$. Pretend this is in a form $\mathbf{ax^2 + bxy + cy^2}$

'**ac**' = **4** and factors 4 and 1 add to 5
so: $\mathbf{2x^2 + 1xy + 4xy + 2y^2}$.
So: $\mathbf{x(2x + y) + 2y(2x + y)}$
So: $\mathbf{(x + 2y)(2x + y)}$

SOME PRACTICE ON YOUR OWN

97. Try these on your own. The answers are given. Factor:

a. $\mathbf{3x^2 + 7x + 4}$

Ans: (3x + 4)(x + 1)

b. $\mathbf{15x^2 - 7x - 2}$

Ans: (3x – 2)(5x + 1)

c. $\mathbf{-12x^2 + 4x + 21}$

Ans: (6x + 7)(–2x + 3)

d. $\mathbf{15x^2 - 44x - 20}$

Ans: (5x + 2)(3x –10)

Simplify by factoring the numerator

e. $\dfrac{x^2 + x - 6}{x - 2}$

Ans: (x + 3)

f. $\dfrac{3x^2 - 8x - 3}{x - 3}$

Ans: (3x + 1)

SOME EXTRA PRACTICE PROBLEMS

1. Add or subtract the Polynomials

a. $(2x + 3y) + (x + 7y)$	b. $(2x + 4y) + (x + 7y) - (x + y)$
c. $(x^3 + 3x) - (4x^3 + x) + x^2$	d. $(3x + 2y + z) + (x - y + 8z)$

2. Simplify by multiplying and combining like terms:

a. $3(xy + z) + 2(xy)$	b. $4*(2x - 8) - 3*(x + 7)$
c. $4(2x + 1) - 3(x - 7)$	d. $\frac{1}{4}(12x-8)+\frac{1}{2}(x-4)$

3. **Evaluate** the expressions in question 2 for x = 2, y = 1, z = 0

a. $3(xy + z) + 2(xy)$	b. $4*(2x - 8) - 3*(x + 7)$
c. $4(2x + 1) - 3(x - 7)$	d. $\frac{1}{4}(12x-8)+\frac{1}{2}(x-4)$

4. Multiply the Binomials

a. $(x + 9)(x - 9)$	b. $(x + 4)(x + 4)$ aka: $(x + 4)^2$
c. $\left(\frac{2}{3}x+1\right)\left(\frac{3}{4}y-2\right)$	d. $(1.6t + z)*(0.4t - 1)$