



Today's agenda

- ↳ No. of iterations
- ↳ Big O Notations

↳ The magic you are looking for is in the work you are avoiding.



AlgoPrep



Quiz

↳ How many numbers are in range $[3, 10]$ (corners included)?

$\{3, 4, 5, 6, 7, 8, 9, 10\} \rightarrow 8$

$\begin{matrix} \text{inc} \\ \downarrow \\ [a, b] \end{matrix} \rightarrow b - a + 1$

$\begin{matrix} \text{inc.} \\ \downarrow \\ [a, b) \end{matrix} \rightarrow b - a$

$\begin{matrix} \text{enc.} \\ \downarrow \\ (a, b) \end{matrix} \rightarrow b - a - 1$

//log basics

$$n * 2 = 10 \Rightarrow n = 10/2$$

$$n + 2 = 10 \Rightarrow n = 10 - 2$$

$$n^3 = 27 \Rightarrow n = \sqrt[3]{27}$$

$$n^2 = 16 \Rightarrow n = \sqrt{16}$$

$$n^2 = 16 \rightarrow 2 = \log_n 16$$

$$\text{ans} = \log_2 16$$



→ $\log_a b$
representation number
base of log

$$\rightarrow \log_2 10 = \text{ans}$$

$$10 = 2^{\text{ans}} \rightarrow 2^{3.33}$$

$$\downarrow$$
$$\text{ans} = 3.33 \dots$$

(a & b are given)
* $\log_b a = \text{ans} \Rightarrow a = b^{\text{ans}} \rightarrow \text{then hit \& total for ans.}$

$$\textcircled{i} \log_2 64 = \text{ans}$$

$$64 = 2^{\text{ans}} \rightarrow 2^6 = 64 \Rightarrow \text{ans} = 6$$

$$\textcircled{ii} \log_3 343 = \text{ans}$$

$$343 = 3^{\text{ans}} \Rightarrow \text{ans} = 5.2 \dots$$

$$\textcircled{iii} \log_2 33 = \text{ans}$$

$$33 = 2^{\text{ans}} \Rightarrow \text{ans} = 5. \text{ something}$$



Properties:

$$\hookrightarrow \textcircled{I} \log_a a^n = n$$

$$\hookrightarrow \textcircled{II} \log_c (a*b) = \log_c a + \log_c b$$

$$\text{ex: } \log_3 \underset{(4*2)}{10} = \log_3 5 + \log_3 2$$



AlgoPrep



Quiz

↳ No. of steps for $N \rightarrow \frac{N}{2} \rightarrow \frac{N}{4} \rightarrow \dots \rightarrow 1$

$$\left(\left(N * \frac{1}{2} \right) * \frac{1}{2} \right) * \frac{1}{2} \dots = 1$$

No. of steps == How many times you can multiply by $\frac{1}{2}$ before reaching 1.

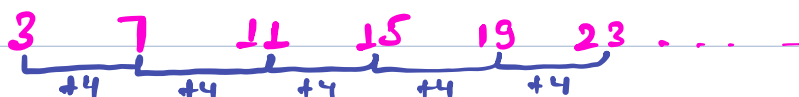
$$\frac{N}{2^K} = 1$$

$$\hookrightarrow N = 2^K \Rightarrow 2^K = N$$

$$K = \log_2 N$$



A.P → Arithmetic Progression



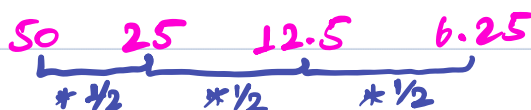
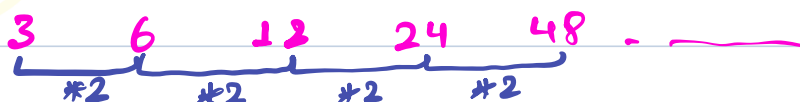
first term = a = starting number, $a = 3$

Common diff = d = difference betⁿ cons. no.s, $d = 4$

$$\overset{1}{(a+d)} \quad \overset{2}{(a+d)} \quad \overset{3}{(a+2d)} \quad \overset{4}{(a+3d)} \quad \dots \quad \overset{n\text{th term}}{a+(n-1)d}$$

∴ Sum of first N terms of A.P. = $\frac{N}{2} [2a + (N-1)d]$

G.P → Geometric Progression



first term = a = starting number $a = 3$

Common ratio = r = multiple to get next no.

$$\overset{1}{a r^0} \quad \overset{2}{a r^1} \quad \overset{3}{a r^2} \quad \overset{4}{a r^3} \quad \dots \quad \overset{n\text{th term}}{a r^{n-1}}$$

∴ Sum of first N terms of G.P. = $a * \frac{r^N - 1}{r - 1}$



Quiz

```
int Sum = 0;
for (int i = 1; i <= N; i++) {
    Sum = Sum + 1;
}
```

$[1, N] \rightarrow N - 1 + 1$
 \sim iterations
 $O(N)$

Quiz

```
void func (int N, int m) {
    for (int i = 1; i <= N; i++) {
        Print(i);
    }
    for (int i = 1; i <= m; i++) {
        Print(i);
    }
}
```

$\rightarrow [1, N] = N - 1 + 1$
 N iterations
 $\rightarrow [1, m] = m - 1 + 1$
 m iterations

Total = $N + m$ iterations

3

$O(N+m)$

- $N > m \rightarrow O(N)$
- $N < m \rightarrow O(m)$



Quiz

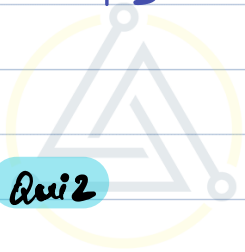
```
int fun (int n){  
    int s = 0;  
    for (int i = 0; i <= 100; i++){  
        s = s + i^2;  
    }  
    return s;  
}
```

→ [0, 100]

↳ $100 - 0 + 1$

↳ 101 iterations

↓
 $O(1)$



AlgoPrep

Quiz

```
void fun (int n){  
    int s = 0;  
    for (int i = 1; i <= n; i++){  
        s = s + i^2;  
    }  
    return s;  
}
```

→ [1, \sqrt{n}] → $\sqrt{n} - 1 + 1$

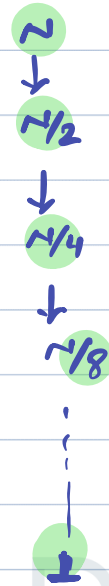
= \sqrt{n} iterations

↓
 $O(\sqrt{n})$



Quiz

```
void fun (int n) {  
    int i = n;  
    while (i >= 1) {  
        i = i / 2;  
    }  
}
```



$\rightarrow \log n$ iterations
 \downarrow
 $O(\log n)$

Quiz

```
void fun (int n) {
```

```
    int s = 0;
```

```
    for (int i = 0; i <= n; i = i * 2) {  
        s = s + i;  
    }
```

$\rightarrow [0, 0, 0, 0, \dots]$
infinite

}



Quiz

```
void fun (int n) {
```

```
    int S = 0;
```

```
    for (int i = 1; i <= n; i = i * 2) {
```

```
        S = S + i;
```

```
    }
```

```
}
```

$\rightarrow 1 \rightarrow 2 \rightarrow 4 \rightarrow 8 \dots$

$\dots \dots \dots \sim n$

$\sim \log n$ iterations

$O(\log n)$

1

2

4

8

...

...

...

...

1

2

4

8

...

...

...

...

Break till 9:25 pm

P1 \rightarrow

1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 + 1024 + 2048 + 4096 + 8192 + 16384 + 32768 + 65536 + 131072 + 262144 + 524288 + 1048576 + 2097152 + 4194304 + 8388608 + 16777216 + 33554432 + 67108864 + 134217728 + 268435456 + 536870912 + 1073741824 + 2147483648 + 4294967296 + 8589934592 + 17179869184 + 34359738368 + 68719476736 + 137438953472 + 274877906944 + 549755813888 + 1099511627776 + 2199023255552 + 4398046511104 + 8796093022208 + 17592186044416 + 35184372088832 + 70368744177664 + 140737488355328 + 281474976710656 + 562949953421312 + 1125899906842624 + 2251799813685248 + 4503599627370496 + 9007199254740992 + 18014398509481984 + 36028797018963968 + 72057594037927936 + 144115188075855872 + 288230376151711744 + 576460752303423488 + 1152921504606846976 + 2305843009213693952 + 4611686018427387904 + 9223372036854775808 + 18446744073709551616 + 36893488147419103232 + 73786976294838206464 + 147573952589676412928 + 295147905179352825856 + 590295810358705651712 + 1180591620717411303424 + 2361183241434822606848 + 4722366482869645213696 + 9444732965739290427392 + 18889465931478580854784 + 37778931862957161709568 + 75557863725914323419136 + 151115727451828646838272 + 302231454903657293676544 + 604462909807314587353088 + 1208925819614629174706176 + 2417851639229258349412352 + 4835703278458516698824704 + 9671406556917033397649408 + 19342813113834066795298816 + 38685626227668133590597632 + 77371252455336267181195264 + 154742504910672534362390528 + 309485009821345068724781056 + 618970019642690137449562112 + 1237940039285380274899124224 + 2475880078570760549798248448 + 4951760157141521099596496896 + 9903520314283042199192993792 + 19807040628566084398385987584 + 39614081257132168796771975168 + 79228162514264337593543950336 + 158456325028528675187087900672 + 316912650057057350374175801344 + 633825300114114700748351602688 + 1267650600228229401496703205376 + 2535301200456458802993406410752 + 5070602400912917605986812821504 + 10141204801825835211973625643008 + 20282409603651670423947251286016 + 40564819207303340847894502572032 + 81129638414606681695789005144064 + 162259276829213363391578010288128 + 324518553658426726783156020576256 + 649037107316853453566312041152512 + 1298074214633706907132624082305024 + 2596148429267413814265248164610048 + 5192296858534827628530496329220096 + 10384593717069655257060992658440192 + 20769187434139310514121985316880384 + 41538374868278621028243970633760768 + 83076749736557242056487941267521536 + 166153499473114484112975882535043072 + 332306998946228968225951765070086144 + 664613997892457936451903530140172288 + 1329227995784915872903807060280344576 + 2658455991569831745807614120560689152 + 5316911983139663491615228241121378304 + 10633823966279326983230456482242756608 + 21267647932558653966460912964485513216 + 42535295865117307932921825928971026432 + 85070591730234615865843651857942052864 + 170141183460469231731687303715884105728 + 340282366920938463463374607431768211456 + 680564733841876926926749214863536422912 + 1361129467683753853853498429727072845824 + 2722258935367507707706996859454145691648 + 5444517870735015415413993718908291383296 + 10889035741470030830827987437816582766592 + 21778071482940061661655974875633165533184 + 43556142965880123323311949751266331066368 + 87112285931760246646623899502532662132736 + 174224571863520493293247799005065324265472 + 348449143727040986586495598010130648530944 + 696898287454081973172991196020261297061888 + 1393796574908163946345982392040522594123776 + 2787593149816327892691964784081045188247552 + 5575186299632655785383929568162090376495104 + 11150372599265311570767859136324180752990208 + 22300745198530623141535718272648361505980416 + 44601490397061246283071436545296723011960832 + 89202980794122492566142873090593446023921664 + 178405961588244985132285746181186892047843328 + 356811923176489970264571492362373784095686656 + 713623846352979940529142984724747568191373312 + 1427247692705959881058285969449495136382746624 + 2854495385411919762116571938898990272765493248 + 5708990770823839524233143877797980545530986496 + 11417981541647679048466287755595961091061972992 + 22835963083295358096932575511191922182123945984 + 45671926166590716193865151022383844364247891968 + 91343852333181432387730302044767688728495783936 + 182687704666362864775460604089535377456991567872 + 365375409332725729550921208179070754913983135744 + 730750818665451459101842416358141509827966271488 + 1461501637330902918203684832716283019655932542976 + 2923003274661805836407369665432566039311865085952 + 5846006549323611672814739330865132078623730171904 + 11692013098647223345629478661730264157247460343808 + 23384026197294446691258957323460528314494920687616 + 46768052394588893382517914646921056628989841375232 + 93536104789177786765035829293842113257979682750464 + 187072209578355573530071658587684226515959365500928 + 374144419156711147060143317175368453031918731001856 + 748288838313422294120286634350736906063837462003712 + 1496577676626844588240573268701473812127674924007424 + 2993155353253689176481146537402947624255349848014848 + 5986310706507378352962293074805895248510699696029696 + 11972621413014756705924586149611790497021399392059392 + 23945242826029513411849172299223580994042798784118784 + 47890485652059026823698344598447161988085597568237568 + 95780971304118053647396689196894323976171195136475136 + 191561942608236107294793378393788647952342390272950272 + 383123885216472214589586756787577295904684780545900544 + 766247770432944429179173513575154591809369561091801088 + 1532495540865888858358347027150309183618739122183602176 + 3064991081731777716716694054300618367237478244367204352 + 6129982163463555433433388108601236734474956488734408704 + 12259964326927110866866776217202473468949912977468817408 + 24519928653854221733733552434404946937899825954937634816 + 49039857307708443467467104868809893875799651909875269632 + 98079714615416886934934209737619787751599303819750539264 + 196159429230833773869868419475239575503198607639501078528 + 392318858461667547739736838950479151006397215279002157056 + 784637716923335095479473677900958302012794430558004314112 + 1569275433846670190958947355801916604025588861116008628224 + 3138550867693340381917894711603833208051177722232017256448 + 6277101735386680763835789423207666416102355444464034512896 + 12554203470773361527671578846415332832204710888928069025792 + 25108406941546723055343157692830665664409421777856138051584 + 50216813883093446110686315385661331328818843555712276103168 + 100433627766186892221372630771322662657637687111424552206336 + 200867255532373784442745261542645325315275374222849104412672 + 401734511064747568885490523085290650630550748445698208825344 + 803469022129495137770981046170581301261101496891396417650688 + 1606938044258990275541962092341162602522202993782792835301376 + 3213876088517980551083924184682325205044405987565585670602752 + 6427752177035961102167848369364650410088811975131171341205504 + 12855504354071922204335696738729300820177623950262342682411008 + 25711008708143844408671393477458601640355247900524685364822016 + 51422017416287688817342786954917203280710495801049370729644032 + 102844034832575377634685573909834406561420991602098741459288064 + 205688069665150755269371147819668813122841983204197482918576128 + 411376139330301510538742295639337626245683966408394965837152256 + 822752278660603021077484591278675252491367932816789931674304512 + 1645504557321206042154969182557350504982735865633579863348609024 + 3291009114642412084309938365114701009965471731267159726697218048 + 6582018229284824168619876730229402019930943462534319453394436096 + 13164036458569648337239753460458804039861886925068638906788872192 + 26328072917139296674479506920917608079723773850137277813577744384 + 52656145834278593348959013841835216159447547700274555627155488768 + 105312291668557186697918027683670432318895095400549111254310977536 + 210624583337114373395836055367340864637790190801098222508621955072 + 421249166674228746791672110734681729275580381602196445017243910144 + 842498333348457493583344221469363458551160763204392890034487820288 + 1684996666696914987166688442938726917102321526408785780068975640576 + 3369993333393829974333376885877453834204643052817571560137951281152 + 6739986666787659948666753771754907668409286105635143120275902562304 + 13479973333575319897333507543509815336818572211270286240551805124608 + 26959946667150639794667015087019630673637144422540572481103610249216 + 53919893334301279589334030174039261347274288845081144962207220498432 + 107839786668602559178668060348078522694548577690162289924414440996864 + 215679573337205118357336120696157045389097155380324579848828881993728 + 431359146674410236714672241392314090778194310760649159697657763987456 + 862718293348820473429344482784628181556388621521298319395315527974912 + 1725436586697640946858688965569256363112777243042596638790631055949824 + 3450873173395281893717377931138512726225554486085193277581262111899648 + 6901746346790563787434755862277025452451108972170386555162524223799296 + 13803492693581127574869511724554050904902217944340773110325048447598592 + 27606985387162255149739023449108101809804435888681546220650096895197184 + 55213970774324510299478046898216203619608871777363092441300193790394368 + 110427941548649020598956093796432407239217743554726184882600387580788736 + 220855883097298041197912187592864814478435487109452369765200775161577472 + 441711766194596082395824375185729628956870974218904739530401550323154944 + 883423532389192164791648750371459257913741948437809479060803100646309888 + 1766847064778384329583297500742918515827483896875618958121606201292619776 + 3533694129556768659166595001485837031654967793751237916243212402585239552 + 7067388259113537318333190002971674063309935587502475832486424805170479104 + 14134776518227074636666380005943348126619871175004951664972849610340958208 + 28269553036454149273332760011886696253239742350009903329945699220681916416 + 56539106072908298546665520023773392506479484700019806659891398441363832832 + 113078212145816597093331040047546785012958969400039613319782796882727665664 + 226156424291633194186662080095093570025917938800079226639565593765455331328 + 452312848583266388373324160190187140051835877600158453279131187530910662656 + 904625697166532776746648320380374280103671755200316906558262375061821325312 + 1809251394333065553493296640760748560207343510400633813116524750123642650624 + 3618502788666131106986593281521497120414687020801267626233049500247285301248 + 7237005577332262213973186563042994240829374041602535252466099000494570602496 + 14474011154664524427946373126085988481658748083205070504932198000989141204992 + 28948022309329048855892746252171976963317496166410141009864396001978282409984 + 57896044618658097711785492504343953926634992332820282019728792003956564819968 + 115792089237316195423570985008687907853269984665640564039457584007913129639936 + 231584178474632390847141970017375815706539969331281128078915168015826259279872 + 463168356949264781694283940034751631413079938662562256157830336031652518559744 + 926336713898529563388567880069503262826159877325124512315660672063305037119488 + 1852673427797059126777135760139006525652319754650249024631321344126610074238976 + 3705346855594118253554271520278013051304639509300498049262642688253220148477952 + 7410693711188236507108543040556026102609279018600996098525285376506440296955904 + 14821387422376473014217086081112052205218558037201992197050570753012880593911808 + 29642774844752946028434172162224104410437116074403984394101141506025761187823616 + 5928554968950589205686834432444820882



Nested loops

Quiz

```
void fun (int n) {
```

```
    int s = 0;
```

```
    for (int i = 1; i <= 10; i++) {
```

```
        for (int j = 1; j <= n; j++) {
```

```
            s = s + 10;
```

```
        }
```

```
    }
```

```
}
```

i	j	Count
1	[1, n]	n
2	[1, n]	n
3	[1, n]	n
⋮		⋮
10	[1, n]	n

$10 * n$

↓ iteration
 $O(n)$

Quiz

```
void fun (int n) {
```

```
    int s = 0;
```

```
    for (int i = 1; i <= n; i++) {
```

```
        for (int j = 1; j <= n; j++) {
```

```
            s = s + 10;
```

```
        }
```

```
    }
```

```
}
```

i	j	Count
1	[1, n]	n
2	[1, n]	n
3	[1, n]	n
⋮		⋮
n	[1, n]	n

$(n * n)$

iterations

↓
 $O(n^2)$



Quiz

```

void fun (int n) {
    int s = 0;
    for (int i = 1; i <= n; i++) {
        for (int j = 1; j <= i; j++) {
            s = s + 10;
        }
    }
}

```

i	j	Count
1	[1, 1]	1
2	[1, 2]	2
3	[1, 3]	3
⋮	⋮	⋮
⋮	⋮	⋮
⋮	⋮	⋮
N	[1, N]	N

3

$$\frac{N * (N+1)}{2} \text{ iterations}$$

$$\hookrightarrow O(N^2)$$

Quiz

```

void fun (int n) {

```

```

    for (int i = 1; i <= 2^n; i++) {
        print(i);
    }
}

```

3

3

$$\hookrightarrow O(2^n)$$



Quiz

```
void fun (int N) {  
    int s = 0;  
    for (int i = 1; i <= N; i++) {  
        for (int j = 1; j <= 2i; j++) {  
            s = s + 10;  
        }  
    }  
}
```

i	j	count
1	[1, 2 ¹]	2 ¹
2	[1, 2 ²]	2 ²
3	[1, 2 ³]	2 ³
⋮	⋮	⋮
⋮	⋮	⋮
⋮	⋮	⋮
N	[1, 2 ^N]	2 ^N

3

∴ Sum of first N terms of G.P = $a * \frac{r^N - 1}{r - 1}$

$$2^1 + 2^2 + 2^3 + 2^4 + \dots + 2^N$$

*2 *2 *2

$$= 2 * \frac{2^N - 1}{2 - 1} = 2 * (2^N - 1)$$

iterations

↓

$$O(2^N)$$



Comparison of iteration

→

$$N = 10^5$$

$$1 < \log_2 N < \sqrt{N} < N < N \log N < N\sqrt{N} < N^2 < 2^N$$

Time Complexity

↳ Big O Notation → Approximate iteration count

- ① Calculate iteration.
- ② around '+' sign, neglect lower order term.
- ③ Neglect Constants.

iteration count

$$\text{ex: } \cancel{10}N^2 + \cancel{20}N + \cancel{30}N^0$$

$$\Downarrow \\ O(N^2)$$

$$\text{ex: } \cancel{15}N^2 + \cancel{15}N\sqrt{N} + \cancel{20}N$$

$$\Downarrow \\ O(N^2)$$



ex: ~~$N \log N$~~ + ~~$15N$~~ + ~~O~~

\Downarrow

$O(N \log N)$

↳ No. of iterations for same code is different

$N=20$, $K=100$

```
for (int i=1; i<=N; i++) {  
    if (i==K) {  
        break;  
    }  
}
```

→ 2 iterations

→ 10 iterations

→ N iterations

Big O notation will consider the worst case scenario.

↳ $O(N)$