CS-328
HW-2

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(1)

(1) Suppose there are n number of users and let interest vectors of all the users are like:-

$$v_1 = \{a_{11}, a_{12} \dots\}$$
$$v_2 = \{a_{21}, a_{22} \dots\}$$
$$v_3 = \{a_{31}, a_{32} \dots\}$$

As per the question, V (computed vectors of all users will be :-

$$V = \{v_1, v_2, v_3, v_4 \dots v_n\}$$

Since we now that any personalized Page Rank vector can be written in the form of linear combination of $\{v_1, v_2 .. v_n\}$

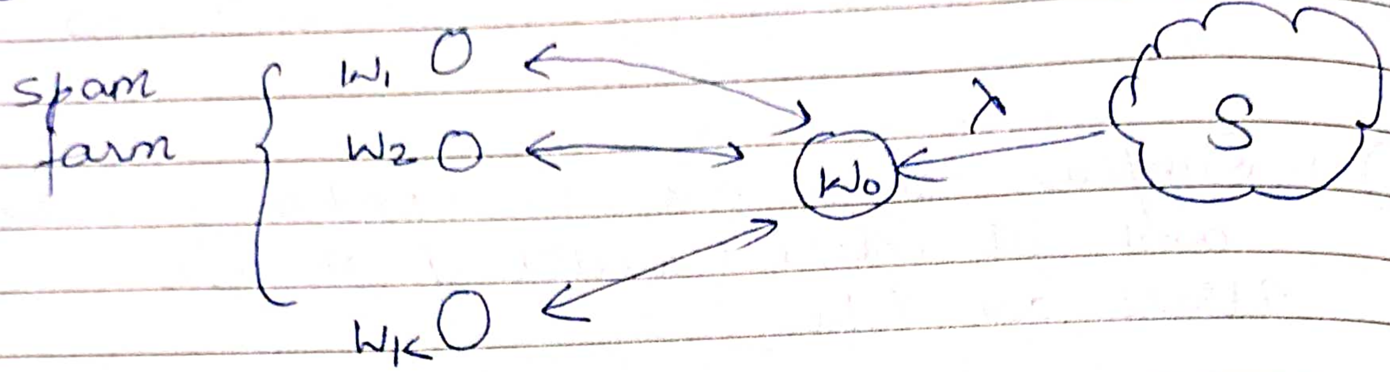
$$I = \alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 + \dots \alpha_n v_n$$

$\alpha \in R$

Therefore $I \in \text{span}(V)$

Set of all personalized page rank vectors $= \text{span}(V)$

(2) Given :~



Parametes

N (total pages) $= |S| + K + 1$

λ (pagerank flows from $S \to w_0$) $= \sum_{j \in S} \frac{p_j}{d_j}$ → pagerank of page j; d_j → outdegree

p_0 → page rank of page w_0

We need to calculate p_0 using λ, K, N and α (teleportation parameter)

Now Page rank of w_0 (p_0) depends on 3 factors:

1. Page rank recieved from $S \to \underline{\alpha\lambda}$
2. Page ranks recieved from $\{w_0 \dots w_k\} \to \alpha \sum_{i=1}^{K} p_i$
3. Pagerank recieve from random jump $\to (1-\alpha) \times \frac{1}{N}$

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Combining all three factors

$$p_0 = \alpha\lambda + \alpha \sum_{i=1}^{K} p_i + \frac{(1-\alpha)}{N} \quad - \text{ i}$$

(1) (2) (3)

Also $1 \le i \le K$ $\quad p_i = \frac{\alpha p_0}{K} + \frac{(1-\alpha) \times 1}{N} \quad - \text{ ii}$

(uniform distribution of p_0 onto all p_i) (random jump)

Substitute ii in i

$$p_0 = \alpha\lambda + \alpha \sum_{i=1}^{K} \left(\frac{\alpha p_0}{K} + \frac{(1-\alpha)}{N} \right) + \frac{1-\alpha}{N}$$

$$p_0 = \alpha\lambda + \frac{\alpha(1-\alpha)K}{N} + \alpha^2 p_0 + \frac{1-\alpha}{N}$$

~~Pos~~

$$p_0(1-\alpha^2) = \alpha\lambda + \frac{(1-\alpha)}{N}(K\alpha+1)$$

$$p_0 = \frac{\alpha\lambda}{1-\alpha^2} + \frac{(\alpha K+1)}{(1+\alpha) N}$$

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(3) Given a turnstile stream of n distinct items.

→ no. of distinct items $= n$

no. of items with frequency $k = \frac{c}{k^3}$

no. of items with freq $1 = \frac{c}{1^3}$

with freq $2 = \frac{c}{2^3}$

with freq $3 = \frac{c}{3^3}$

Hence $n = \frac{c}{1^3} + \frac{c}{2^3} + \frac{c}{3^3} + \dots$

$$n = \sum_{k=1}^{\infty} \frac{c}{k^3}$$

$$n = c \sum_{k=1}^{\infty} \frac{1}{k^3}$$

let $\lambda = \sum_{k=1}^{\infty} \frac{1}{k^3}$, which converges and is a constant

Hence $c \simeq \frac{n}{\lambda}$ $\lambda \simeq 1.20$

$$c \simeq \frac{n}{1.20}$$

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$$c \approx \frac{n}{1.2} \approx \frac{n}{1}$$

Hence $c \approx O(n)$

On fixing w, and d CS ~~it~~ will give better guarantee for the given distribution.

Collaborated with :-

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