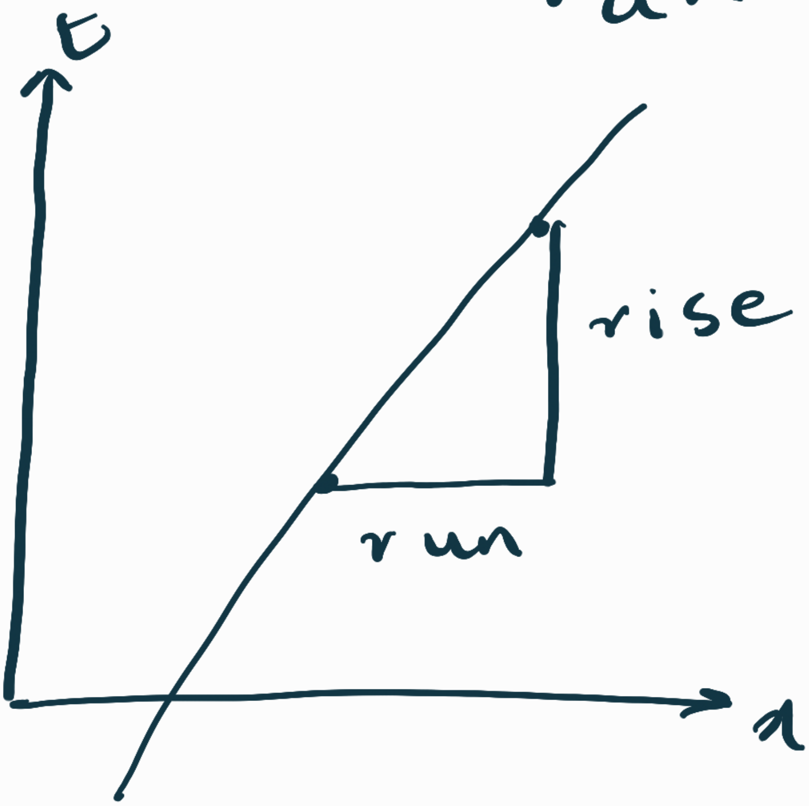
Calculus for ML

Derivatives:

Slope = $\frac{\text{rise} \uparrow}{\text{run} \rightarrow}$



Velocity = $\frac{\Delta x}{\Delta t}$

Notation:

$y = f(x)$

Derivative is:

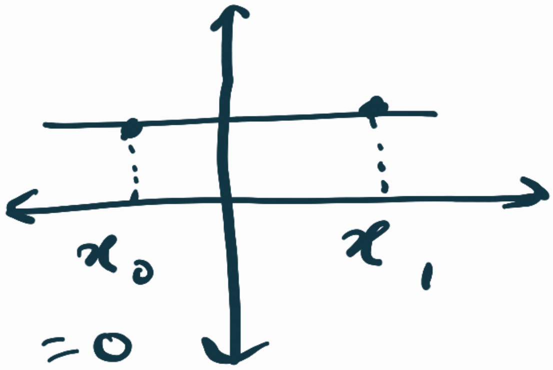
$f'(x)$ → Lagrange's notation

$\frac{dy}{dx} = \frac{d}{dx} f(x)$ → Leibniz's notation

Common derivatives:

i) Constant

$$y = f(x) = c$$

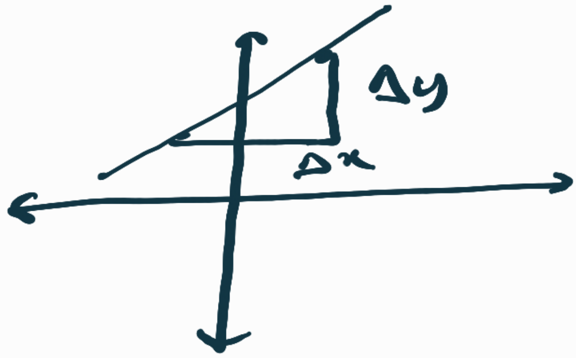


slope: $\dfrac{\Delta y}{\Delta x} = \dfrac{c - c}{x_1 - x_0} = 0$

$\Rightarrow f'(x) = 0$

ii) Line

$$f(x) = ax + b$$



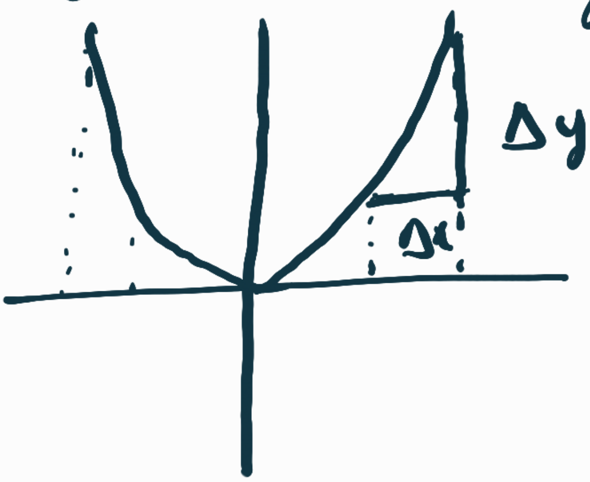
$$\frac{\Delta y}{\Delta x} = \frac{\text{rise}}{\text{run}} = a$$

$$\frac{\Delta y}{\Delta x} = \frac{\cancel{ax} + a\Delta x + \cancel{b} - (\cancel{ax} + \cancel{b})}{\Delta x}$$

$$\frac{\Delta y}{\Delta x} = a \frac{\cancel{\Delta x}}{\cancel{\Delta x}}$$

Derivative of quadratics

Quadratic = $y = f(x) = x^2$



slop = $\dfrac{\Delta y}{\Delta x} = \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

$$\frac{\Delta y}{\Delta x} = \frac{(x+\Delta x)^2 - x^2}{\Delta x}$$

$$= \frac{\cancel{x^2} + \Delta x^{\cancel{2}} + 2x\cancel{\Delta x} - \cancel{x^2}}{\cancel{\Delta x}}$$

$$\frac{\Delta y}{\Delta x} = 2x + \Delta x$$

$\Rightarrow f(x) = x^2 \Rightarrow \dfrac{d}{dx} f(x) = 2x$

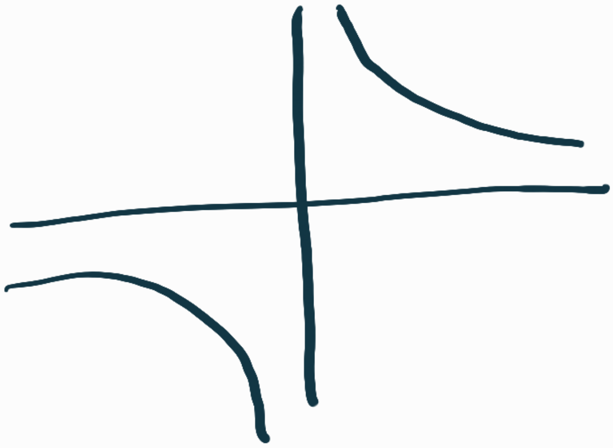
Cubic : $f(x) = x^3$

$$\frac{\Delta f}{\Delta x} = \frac{(x+\Delta x)^3 - x^3}{\Delta x}$$

$$= \frac{x^3 + \Delta x^3 + 3x\Delta x^2 + 3x^2\Delta x - x^3}{\Delta x}$$

$$= 3x\Delta x + 3x^2 + \Delta x$$

$$\frac{\Delta f}{\Delta x} = 3x^2$$



$$y = f(x) = \frac{1}{x} = x^{-1}$$

$$\frac{\Delta f}{\Delta x} = \frac{(x+\Delta x)^{2} - x^{-1}}{\Delta x}$$

$$= -x^{-2}$$

Derivative of power functions

$$f(x) = x^n$$

$$f'(x) = \frac{d}{dx} f(x) = \boxed{n x^{n-1}}$$

$g(x) = f^{-1}(x)$ [Notation]

$$g(f(x)) = x$$

Trigonometric derivatives:

$f(x) = \sin(x)$

$f'(x) = \cos(x)$

$f(x) = \cos(x)$

$f'(x) = -\sin(x)$

$\sin\theta = \dfrac{opp}{hyp} = \dfrac{-\Delta\cos x}{h}$

$\cos\theta = \dfrac{adj}{hyp} = \dfrac{\Delta\sin x}{h}$

$e = 2.718$

$f(x) = e^x$

$f'(x) = e^x$

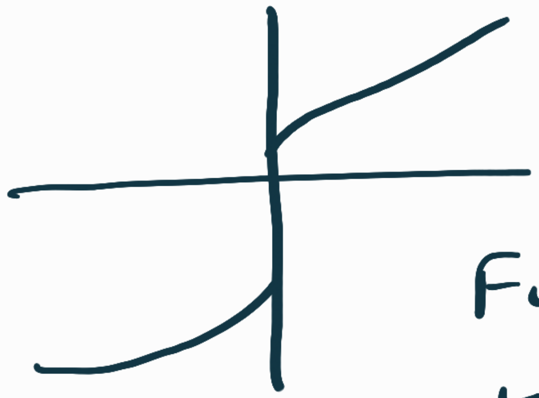
A function when plotted does not have a derivative at corners

e.g.

No derivative here

Non-continuous functions are not differentiable.

e.g



Function with vertical tangent are not differentiable

Also, function × scalar
$\Rightarrow$ derivative × scalar

$f(x) = xe^x$

$f'(x) = e^x + xe^x$

—

chain rule

$f(t) = g(h(t))$

$$f'(t) = \frac{dg}{dh} \times \frac{dh}{dt}$$

can scale

$= f(g(h(t)))$

$$= \frac{df}{dg} \times \frac{dg}{dh} \times \frac{dh}{dt}$$

Practice Quiz

i) Slope (Line 1) $= \frac{0.8}{0.4}$

Slope (Line 2) $= \frac{0.8}{0.4}$

ii) Slope $= \frac{0.8}{0.4} = 2$

Optimization

Power-line problem

$$\underbrace{(x-a)^2 + (x-b)^2}_{\text{Quadratic and look like}} \quad \text{(Minimize)}$$

Differentiating,

$$\Rightarrow 2(x-a) + 2(x-b) = 0$$

$$x - a + x - b = 0$$

$$\Rightarrow 2x - a - b = 0$$

$\Rightarrow$ Minimize $(x-a_1)^2 + (x-a_2)^2 + \dots$

$$\Rightarrow x = \frac{a_1 + a_2 + \dots + a_n}{n} \quad \text{(squared-loss)}$$