

Partial derivative notation:

$$\frac{\partial}{\partial x}$$

Gradient notation:

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$f(x, y) = 85 - \frac{1}{90} x^3 y^3 + \frac{1}{15} x^3 y^2 - \frac{2}{5} x^2 y^2$$

$$\frac{\partial f}{\partial x} = -\frac{3x^2 y^3}{90} + \frac{3x^2 y^2}{15} - \frac{4xy^2}{5}$$

$$= -\frac{1}{30} x^2 y^3 + \frac{1}{5} x^2 y^2 - \frac{4}{5} xy^2$$

Practice Quiz:

$$f(x, y) = xy^2 + 2x + 3y$$

$$\nabla f(x, y) = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix}$$

$$= \begin{bmatrix} y^2 + 2 \\ 2xy + 3 \end{bmatrix}$$

Find gradient of

$$f(x, y, z) = x^2 + 2xyz + z^2$$

$$= \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \\ \frac{\partial f}{\partial z} \end{bmatrix} = \begin{bmatrix} 2x + 2yz \\ 2xz \\ 2xy + 2z \end{bmatrix}$$

$$f(x, y) = x^2 + 2y^2 + 8y$$

$$\frac{\partial f}{\partial x} = 2x = 0$$

$$\frac{\partial f}{\partial y} = 4y + 8 = 0$$

$$x = 0$$

$$y = -2$$

$$\begin{aligned} f(0, -2) &= 0 + 2(-2)^2 + 8(-2) \\ &= 8 - 16 = -8 \end{aligned}$$

$$f(x, y) = 2x^2 + 3y^2 - 2xy - 10x$$

find min.

$$\frac{\partial f}{\partial x} = 4x - 2y - 10 = 0$$

$$\frac{\partial f}{\partial y} = 6y - 2x = 0$$

$$y = \frac{2x}{6} = \frac{1}{3}x$$

$$4x - \frac{2}{3}x - 10 = 0$$

$$x \left(\frac{12 - 2}{3} \right) = 10$$

$$x \left(\frac{10}{3} \right) = 10$$

$$x = 3$$

$$6y - 6 = 0$$

$$y = 1$$

$$f(x, y) = 2(3)^2 + 3 - 6 - 30$$

$$= -15$$

Gradient Descent

$$(x, y) = (x, y) - a \cdot \nabla f(x, y)$$

$$\Rightarrow 1 - 0.01 \times \begin{bmatrix} -6 \\ 2 \end{bmatrix}$$

$$\Rightarrow 0.04, 0.02$$

