System of equations

i) $5a + b = 17$ — ①

$4a - 3b = 6$ — ②

s-① Eliminate 'a' from eqⁿ

Divide by coefficients of a on both sides of the eqⁿ

$\Rightarrow a + \frac{1}{5}b = \frac{17}{5}$ — ③

$a - \frac{3}{4}b = \frac{6}{4}$ — ④

Subtract eqⁿ ③ from ④

$a - 0.75b = 1.5$

$a + 0.2b = 3.4$

$0a - 0.95b = -1.9$

$\Rightarrow b = \frac{-1.9}{0.95} = 2$

$a + 0.2(2) = 3.4$

$\Rightarrow a = 3$

Quiz :

Solve :

$$\begin{cases} 2a + 5b = 46 \\ 8a + b = 32 \end{cases}$$

=) Divide both sides by coefficients of a

$$a + 2.5b = 23 \quad - (1)$$

=) $$a + 0.125b = 4 \quad - (2)$$

Subtract (1) from (2)

$$\begin{array}{r} a + 0.125b = 4 \\ -\ a + 2.5b = 23 \\ \hline 0a - 2.375b = -19 \end{array}$$

$$b = \frac{-19}{-2.375} = 8$$

Substitute in (2)

$$a + 0.125(8) = 4$$

=) $$a = 4 - 1 = 3$$

$$a = 3, \quad b = 8$$

② Solve:

$$\begin{cases} 5a + b = 11 \\ 10a + 2b = 22 \end{cases}$$

=) Divide both sides by coefficient of a

$$a + 0.2b = 2.2 \quad -①$$

$$a + 0.2b = 2.2 \quad -②$$

Subtract ① from ②

$$\begin{array}{r} a + 0.2b = 2.2 \\ a + 0.2b = 2.2 \\ \hline 0 = 0 \end{array}$$

Infinitely many solutions

Elimination for more variables:

System = $\begin{cases} a + b + 2c = 12 \\ 3a - 3b - c = 3 \\ 2a - b + 6c = 24 \end{cases}$

S-① Divide each row by coefficient of a

⇒ $a + b + 2c = 12$ — ①

$a - b - \frac{1}{3}c = 1$ — ②

$a - \frac{1}{2}b + 3c = 12$ — ③

Subtract ① from ② & ③

⇒ $a + b + 2c = 12$

$-2b - \frac{7}{3}c = -11$ — ④

$-\frac{3}{2}b + c = 0$ — ⑤

Divide ④ & ⑤ with coefficient of b

⇒ $a + b + 2c = 12$ — ⑥

$b + \frac{7}{6}c = 11/2$ — ⑦

$b - \frac{2}{3}c = 0$

Subtract ⑥ from ⑦

$$\Rightarrow \quad b - \frac{2}{3}c = 0$$

$$b + \frac{7}{6}c = 11/2$$

$$-1.8c = -11/2$$

$$c = 3.085$$

$$6$$

Substitute c in

$$b + \frac{7}{6_2}(3) = 11/2$$

$$b = 11/2 - 7/2 = 2$$

Substitute b & c in ①

$$a = 12 - 2 - 6 = 4$$

$$\Rightarrow a = 4, \quad b = 2, \quad c = 3$$

Assignment

① Solve using method of elimination :

$$x + y = 4$$
$$-6x + 2y = 16$$

=1 Divide by coefficient of y

$$x + y = 4 \quad -①$$
$$-3x + y = 8 \quad -②$$

Subtract ① from ②

$$-3x + y = 8$$
$$-x + y = 4$$

$$-4x = 4$$
$$x = -1$$

Substitute in ①

$$y = 4 + 1 = 5$$

$x = -1, y = 5$

② Calculate determinant and determine if matrices are singular

i) $\begin{bmatrix} 4 & -3 \\ 7 & -8 \end{bmatrix}$

$= -32 - (-21)$

$= -32 + 21 = -11$

Non-singular

ii) $\begin{bmatrix} -3 & 8 & 1 \\ 2 & 2 & -1 \\ -5 & 6 & 2 \end{bmatrix}$

$= 1.2.-5 + 8.2.2 + -3.-1.6$

$-(-3.2.2) - 8.-1 - 5 - 1.2.6$

$= -10 + 32 + 18 + 12 - 40 - 12$

$= 50 - 50 = 0$

Singular

4. Determine if matrix has linear dependent rows

$$\begin{bmatrix} a & b & c \\ d & e & f \\ 2a-d & 2b-e & 2c-f \end{bmatrix}$$

$$R_3 = 2R_1 - R_2$$

$\Rightarrow$ Dependent

⑥ $$\begin{bmatrix} a & a \\ b & c \end{bmatrix}$$

$$ac - ab \neq 0$$

$$c \neq b$$

Rank of Matrix

↳ Useful for image compression

Amount of information a system carries is its rank.

i) Determine rank of

$$\begin{bmatrix} 5 & 1 \\ -1 & 3 \end{bmatrix}$$

⇒ Rank 2 (Independent)

ii) $$\begin{bmatrix} 2 & -1 \\ -6 & 3 \end{bmatrix}$$

⇒ Rank 1

$R_2 = -3R_1$

Solution space: Number of solutions of a matrix when constants are 0

Rank = No. of Rows (2×2) − Solution space dimensio

Non-singular if rank = rows

Row echelon form

Original matrix

$$\begin{bmatrix} 5 & 1 \\ 4 & -3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0.2 \\ 1 & -0.75 \end{bmatrix}$$

Divide each row by left-most coefficient

Subtract $R_2 - R_1$

$$\begin{bmatrix} 1 & 0.2 \\ 0 & -0.95 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}$$

Divide R_2 by left-most non-zero coefficient

Rank of matrix is no. of 1's in diagonal of row echelon form

Rules about row echelon:

i) Zero rows at bottom

ii) Each row has a pivot (left-most non-zero entry)

iii) Every pivot is to the right of pivots on rows above

Rank = Number of pivots

Reduced row echelon form:

→ All values above pivot is 0

→ pivots are 1

Assignment

i) $7f + 5a + 3c = 120$

$3f + 2a + 5c = 70$

$1f + 2a + 1c = 20$

$$\begin{bmatrix} 7 & 5 & 3 \\ 3 & 2 & 5 \\ 1 & 2 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 5/7 & 3/7 \\ 3 & 2 & 5 \\ 1 & 2 & 1 \end{bmatrix}$$

Subtract R_1 from R_3,

Divide R_2 by 3

$$\Rightarrow \begin{bmatrix} 1 & 5/7 & 3/7 \\ 1 & 2/3 & 5/3 \\ 0 & 9/7 & 4/ \end{bmatrix}$$

Solve : $7f + 5a + 3c = 120$

$3f + 2a + 5c = 70$

$f + 2a + c = 20$

$f = 20 - 2a - c$

Substitute in (2)

$3(20 - 2a - c) + 2a + 5c = 70$

$60 - 6a - 3c + 2a + 5c = 70$

$-4a + 2c = 10$

$c = \frac{10 + 4a}{2}$

$c = 5 + 2a$

Substitute in (1)

$7(20 - 2a - 5 - 2a) + 5a + 3(5+2a) = 120$

$140 - 14a - 35 - 14a + 5a + 15 + 6a = 120$

$-28a + 11a = 120 - 140 + 35 - 15$

$-17a = 0$

$a = 0, c = 5, f = 15$

6 $\begin{bmatrix} 7 & 5 & 3 \\ 3 & 2 & 5 \\ 1 & 2 & 1 \end{bmatrix}$

$$7\begin{bmatrix} 2 & 5 \\ 2 & 1 \end{bmatrix} - 5\begin{bmatrix} 3 & 5 \\ 1 & 1 \end{bmatrix} + 3\begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$$

$$= 7(2-10) - 5(3-5) + 3(6-2)$$

$$= 7(-8) - 5(-2) + 3(4)$$

$$= -56 + 10 + 12 = -34$$

⑧ $\begin{bmatrix} 0 & 1 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 0 & 1 & 1 \end{bmatrix}$

2) $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 2 & 4 & 2 \end{bmatrix} R_3 - 2R_1$

$= \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} R_2$

$a = 5$

$b = 2$

$a = \frac{a+b}{5+2} = 7$

$b = \frac{a-b}{a-b}$

$a = a - b$