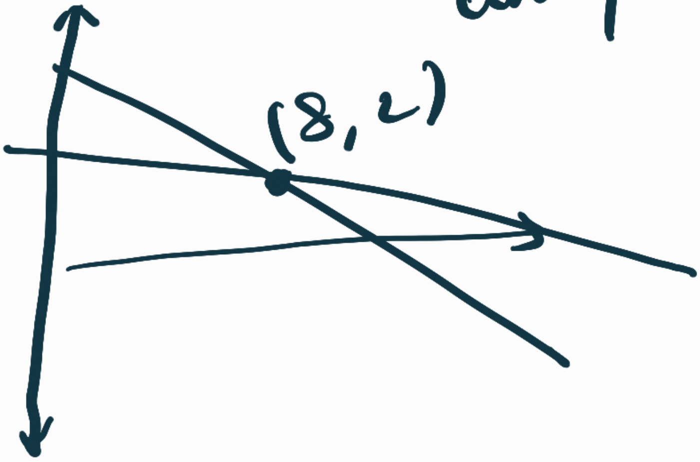
1) System of equations as lines

System 1

$a+b=10$
$a+2b=24$

unique solution

Complete &
Singular

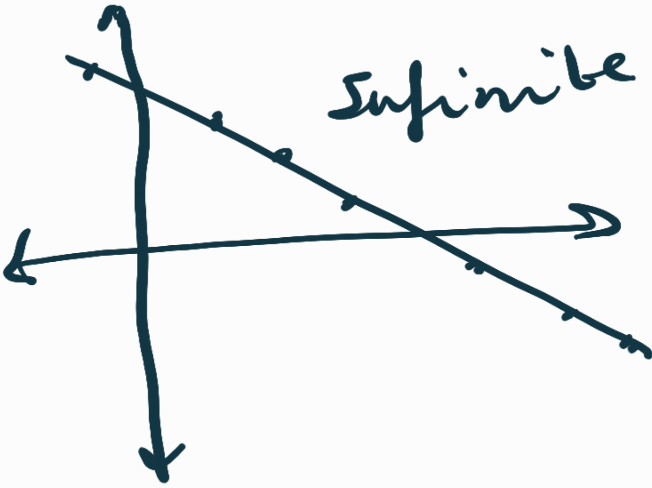


System 2

$a+b=10$
$2a+2b=20$

Infinite solutions

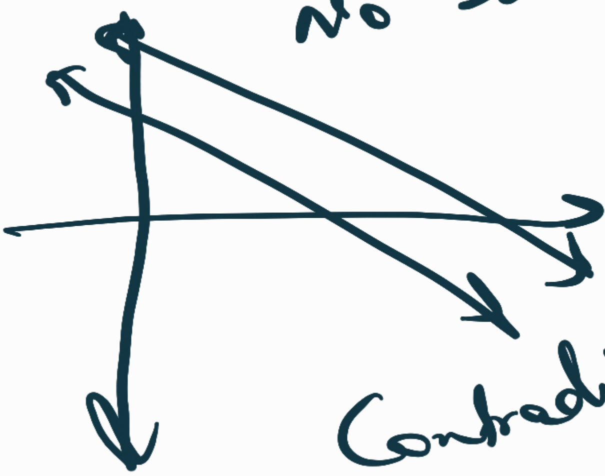
Redundant
&
Singular



System 3

No solutions

$a+b=10$
$2a+2b=24$



Contradictory

Quiz 2

$$\begin{cases} 3a + 2b = 8 \\ 2a - b = 3 \end{cases}$$

$$3(1) + 2(3.5) = 8$$

$$3(2) + 2(1) = 8$$

$$2(1) - 5 = 3$$

$$2(2) - 7 = 3$$

$$\frac{16}{3} - 3 = \frac{4}{3}b + b$$

$$\frac{16}{3} - 3 = b\left(\frac{4}{3} + 1\right)$$

$$\frac{\frac{\cancel{16}^{2}}{\cancel{3}_{1}} - 3}{\frac{\cancel{4}}{\cancel{3}} + 1} = b$$

$$= 1$$

$a + b = 0$
$a + 2b = 0$

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$$ Non singular matrix

↳ No row is multiple of other

If same info is carried in both equations ⇒ Singular System

e.g
$a + b = 10$
$2a + 2b = 20$

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

Linearly dependent

Determinant

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Matrix is singular if

$[a \quad b] \times k = [c \quad d]$

$\Rightarrow ak = c \quad ; \quad bk = d$

$\Rightarrow \dfrac{c}{a} = \dfrac{d}{b} = k$

$\Rightarrow ad = bc$

$\Rightarrow ad - bc = 0$

→ Determinant

If matrix is singular

$\Rightarrow$ Determinant $= 0$

Quizz :

Find determinant of

i) $\begin{bmatrix} 5 & 1 \\ -1 & 3 \end{bmatrix}$ Non singular

$\Rightarrow 5 \times 3 - 1 \times -1$

$\Rightarrow 15 + 1 = 16$

ii) $\begin{bmatrix} 2 & -1 \\ -6 & 3 \end{bmatrix}$

$\Rightarrow 2 \times 3 - (-1 \times -6)$

$6 - 6 = 0$

Singular

Assignment - 1

1) $2x + 3y = 15$ — ①

$2x + 4y = 16$ — ②

→ Determinant

$$= \begin{bmatrix} 2 & 3 \\ 2 & 4 \end{bmatrix} = 8 - 6 = 2$$

⇒ Non-singular, 2

$$2x + 3y = 15$$

$$x = \frac{15 - 3y}{2}$$

Substitute in ②

$$2\left(\frac{15 - 3y}{2}\right) + 4y = 16$$

$$15 - 3y + 4y = 16$$

$$y = 1$$

$$\Rightarrow 2x = 15 - 3$$

$$\Rightarrow x = 6$$

$(6, 1)$

$$1a + 1b + 1c = 10 \quad (1)$$
$$1a + 2b + c = 15 \quad (2)$$
$$1a + 1b + 2c = 12 \quad (3)$$

$$a = 10 - b - c$$

Substitute in (2)

$$10 - b - \cancel{c} + 2b + \cancel{c} = 15$$

$$b = \frac{15 - 10}{2} = \frac{5}{2}$$

Substitute into (3)

$$10 - b - c + \frac{5}{2} + 2c = 12$$

$$\Rightarrow 10 - \cancel{\frac{5}{2}} - \cancel{c} + \cancel{\frac{5}{2}} + 2c = 12$$

$$c = 12 - 10 = 2$$

Substitute in (1)

$$\Rightarrow a = 10 - \frac{5}{2} - 2 = 5$$

1

$a+b+c=10 \quad -①$

$a+b+2c=15 \quad -②$

$a+b+3c=20 \quad -③$

$a=10-b-c$

Substitute in ②

$10-\cancel{b}-\cancel{c}+\cancel{b}+\cancel{2}c=15$

$c=5$

Substitute in ③

$10-\cancel{b}-5+\cancel{b}+15=20$

$\Rightarrow$ There are ∞ many sol^n

2

$a+b+c=10 \quad -①$

$a+b+2c=15 \quad -②$

$a+b+3c=18 \quad -③$

$\rightarrow \quad a=10-b-c$

$10-\cancel{b}-\cancel{c}+\cancel{b}+\cancel{2}c=15$

$c=5$

$10-\cancel{b}-5+\cancel{b}+15=18$

$20\neq 18$

* Linear dependence & independence

$$a = 1 \qquad\qquad a + 0b + 0c = 1$$
$$b = 2 \quad \Rightarrow \quad 0a + b + 0c = 2$$
$$a + b = 3 \qquad\qquad \overline{a + b + 0c = 3}$$

$\Rightarrow$ Row 1 + Row 2 = Row 3

$\Rightarrow$ Rows are linearly dependent

2A Row 2 is Average of Row1 + R3

No relation b/w rows $\Rightarrow$ linearly independent

Quiz :
Determine if the matrix has linearly dependent or independent rows :

i)
$$\begin{array}{c} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 3 & 2 & 3 \end{bmatrix} \end{array} \begin{array}{l} R_1 \\ R_2 \\ R_3 \end{array}$$

$\rightarrow$ Dependent as $R_3 = 3R_1 + 2R_2$

ii) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix}$ $R_3 = R_1 - R_2$

→ Dependent

iii) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$

→ Independent

iv) $\begin{bmatrix} 1 & 2 & 5 \\ 0 & 3 & -2 \\ 2 & 4 & 10 \end{bmatrix}$

→ Dependent
$R_3 = 2R_1$

* Determinant in a 3×3 matrix

$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$ $\Rightarrow 1.2.2 + 1.1.1 - 1.2.1 + 1.1.1 - 1.1.2 - 1.1.1$

$\Rightarrow 4 + 1 + 1 - 2 - 2 - 1 = 1$

Quiz:

$$M_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 3 & 2 & 3 \end{bmatrix}$$

$\Rightarrow 1.1.3 + 0.0.3 + 0.2.1 - 1.2.0 - 0.0.3 - 1.1.3$

$= 0$ $\Rightarrow$ Singular

$$M_2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix}$$

$\Rightarrow 1.1.-1 + 1.2.0 + 1.1.0 - 1.1.0 - 1.1.-1 - 1.2.0$

$\Rightarrow -1 + 1 = 0$

$$M_3 = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

$\Rightarrow 1.2.3 + 1.2.0 + 1.0.0 - 1.2.0 - 1.0.3 - 1.2.0$

$= 6$

$$\det|M_3| = 1 \times \begin{bmatrix} 2 & 2 \\ 0 & 3 \end{bmatrix} - 1 \begin{bmatrix} 0 & 2 \\ 0 & 3 \end{bmatrix} + 1 \begin{bmatrix} 0 & 2 \\ 0 & 0 \end{bmatrix}$$

$$= 1 \times (6-0) - 1(0) + 1(0)$$

$$= 6$$

$$M_4 = \begin{bmatrix} 1 & 2 & 5 \\ 0 & 3 & -2 \\ 2 & 4 & 10 \end{bmatrix}$$

$$\det|M_4| = 1 \times \begin{bmatrix} 3 & -2 \\ 4 & 10 \end{bmatrix} - 2 \begin{bmatrix} 0 & -2 \\ 2 & 10 \end{bmatrix} + 5 \begin{bmatrix} 0 & 3 \\ 2 & 4 \end{bmatrix}$$

$$= 1 \times (30+8) - 2(4) + 5(-6)$$

$$= 38 - 8 - 30 = 0$$

If all values below diagonal are 0, $\det|M| = \text{product (diagonal)}$

$$= 1 \cdot 2 \cdot 3 = 6$$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

(Includes if 0 in diagonal)

Assignment Solutions

① $2b + 1m + 5c = 20$

$1b + 2m + 1c = 10$

$2b + 1m + 3c = 15$

② $$\begin{bmatrix} 2 & 1 & 5 \\ 1 & 2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$$

③ $$\det = 2\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} - 1\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix} + 5\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

$$= 2(6-1) - 1(3-2) + 5(1-4)$$

$$= 10 - 1 - 15$$

$$= -6,$$ Non-singular

④ Linearly independent

5 Solve the equations

$\Rightarrow$ $2b + m + 5c = 20$ — 1

$b + 2m + c = 10$ — 2

$2b + m + 3c = 15$ — 3

$m = 20 - 5c - 2b$

Substitute m in 3

$\cancel{2b} + 20 - 5c - \cancel{2b} + 3c = 15$

$-2c = -5$

$c = \frac{5}{2} = 2.5$

Substitute in 2

$b + 2\left(20 - 5\left(\frac{5}{2}\right) - 2b\right) + \frac{5}{2} = 10$

$\cancel{b} + 40 - \cancel{4}^{3}b - 25 + \frac{5}{2} = 10$

$40 - 25 - 10 + \frac{5}{2} = 3b$

$5 + \frac{5}{2} = 3b$

$\frac{15}{6} = b = 2.5$

$$m = 15 - 2\left(\frac{15}{6}\right) - 3\left(\frac{5}{2}\right)$$

$$= 15 - \frac{\cancel{15}^{5}}{\cancel{3}} - \frac{15}{2}$$

$$m = 10 - \frac{15}{2} = 2.5$$

$\Rightarrow$ each $= 2.5$

⑥ $\begin{bmatrix} 2 & 1 & 5 \\ 1 & 2 & 1 \\ x & y & z \end{bmatrix}$

$\Rightarrow x = 3, \ y = 3, \ z = 6$

⑦ Find determinant of

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 2 \\ 1 & 4 & 5 \end{bmatrix}$$

$=)$

$1 \cdot 2 \cdot 5 + 2 \cdot 2 \cdot 1 + 3 \cdot 0 \cdot 4 - 1 \cdot 2 \cdot 4 - 0 \cdot 2 \cdot 5 - 3 \cdot 2 \cdot 1$

$= 10 + 4 - 6 - 8$

$= 14 - 14 = 0$