CS 7646 Machine Learning for Trading

Project 1: Martingale Report

1. **Probability of winning $80 within 1000 sequential bets in exp.1**

Ans: The probability of winning $80 within 1000 sequential bets is 1 because we have assumed infinite bankroll.

By doubling the betting amount after each loss, it is made sure that even a single win would cover all the previous losses, and win a profit of $1, which was the initial stake.

This means that for $80 in winnings, we need at least 80 instances of wins.

Given:
Probability of win, P(win) = 18/38
No. of spins required for 80 wins: 18 / 38 = 80 / x

$$x = 80 * 38 / 18$$
$$\sim 169$$

Thus, the probability of winning $80 within 1000 sequential bets is 1.

Also, as seen from the first graph, winnings in all 10 episodes reached $80 after around 200 spins.

2. **Estimated expected value of winnings after 1000 sequential bets in exp.1**

Ans: In exp. 1, since the player has to quit playing after he has reached the limit of $80 in winnings, the expected value of winnings after 1000 sequential bets is $80. The player will reach $80 in winnings by around 200 spins, after which his winnings remain constant at $80.

Also, since P(winning $80 after 1000 bets)=1,
Expected value of winnings after 1000 bets = $80

3. **In exp.1, does the standard deviation reach a maximum value then stabilize or converge as the number of sequential bets increases?**

Ans: In exp. 1, as the winning stabilizes at $80 after around 200 spins, the standard deviation also converges to zero and then stabilizes. Since the winnings values have stabilized at $80, there is no deviation from the mean value, hence the standard deviation is constant at zero. The same can be seen from the graph as well.

4. **Probability of winning $80 within 1000 sequential bets in exp. 2**

Ans: In order to win $80, a person needs to be able to keep playing until he has had 80 wins.

For a person to be able to keep playing, he needs to avoid 8 consecutive losses, because after 8 consecutive losses his winnings reach -$256, which means that his bankroll is empty, and he cannot bet anymore.

Given:

Probability of loss, P(loss) = 20/38 = 0.526

P(8 consecutive losses) = 0.526^8 = 0.00586

A person can win $80 if he wins 80 times avoiding the possibility of 8 consecutive losses

Thus P($80 winnings) = (1-0.00586)^80

= 0.99414^80

= 0.625

5. **Estimated expected value of winnings after 1000 sequential bets in Exp. 2.**

Ans: Given the rules of this simulation, after 1000 sequential bets, a person would have either won $80 and stopped at that, or lost $256, gone bankrupt and stopped playing.

Thus, Expected value of winnings after 1000 sequential bets =

P($80 winnings) * $80 + P(going bankrupt) * -($256)

= 0.625 * $80 – (1-0.625)*$256

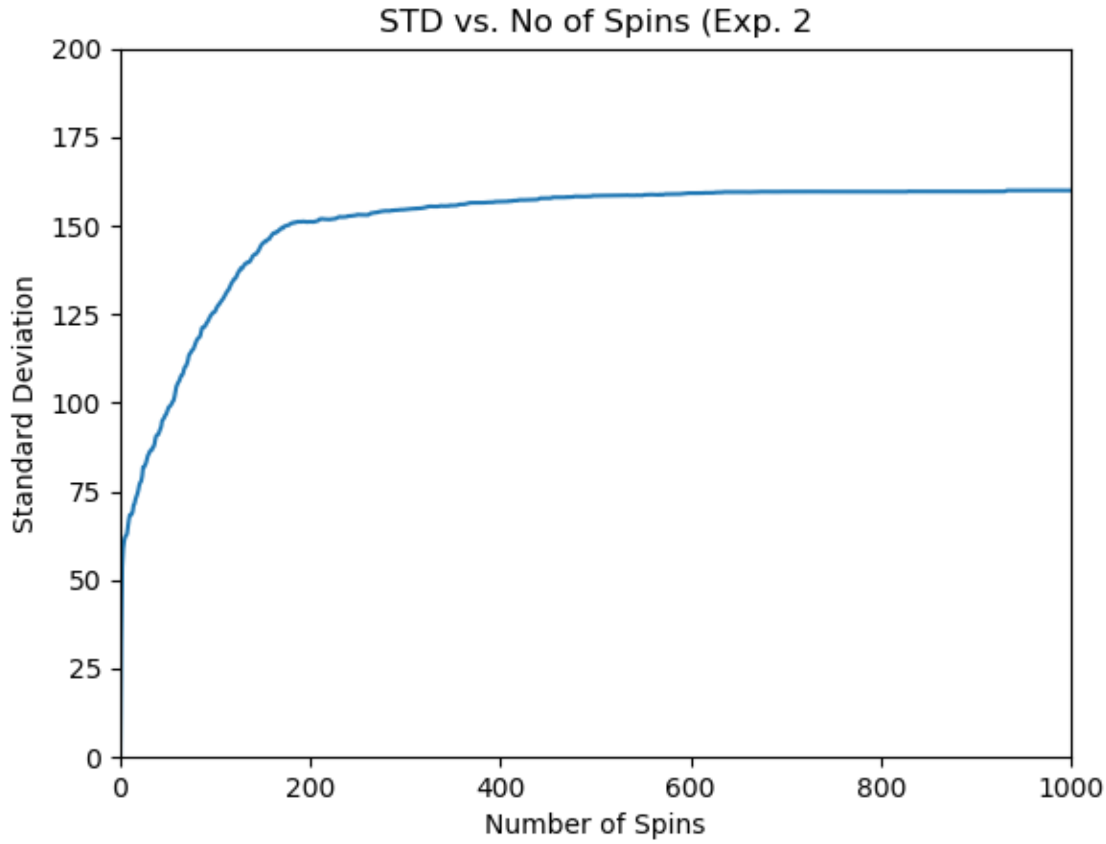
= $50 – 0.375*256

= $50 - $96

= -$46

6. **In Experiment 2, does the standard deviation reach a maximum value then stabilize or converge as the number of sequential bets increases?**

Ans: In exp. 2, the standard deviation reaches a maximum value and then stabilizes. This can be seen from the graph as well.



The value of standard deviation reaches a constant value because for each episode, after a given number of spins, the winnings will become constant at either \$80 or -\$256.

7. **Include figures 1 through 5.**

Ans.

(a) Exp. 1 part (a): Plots of winnings in 10 episodes vs Number of spins

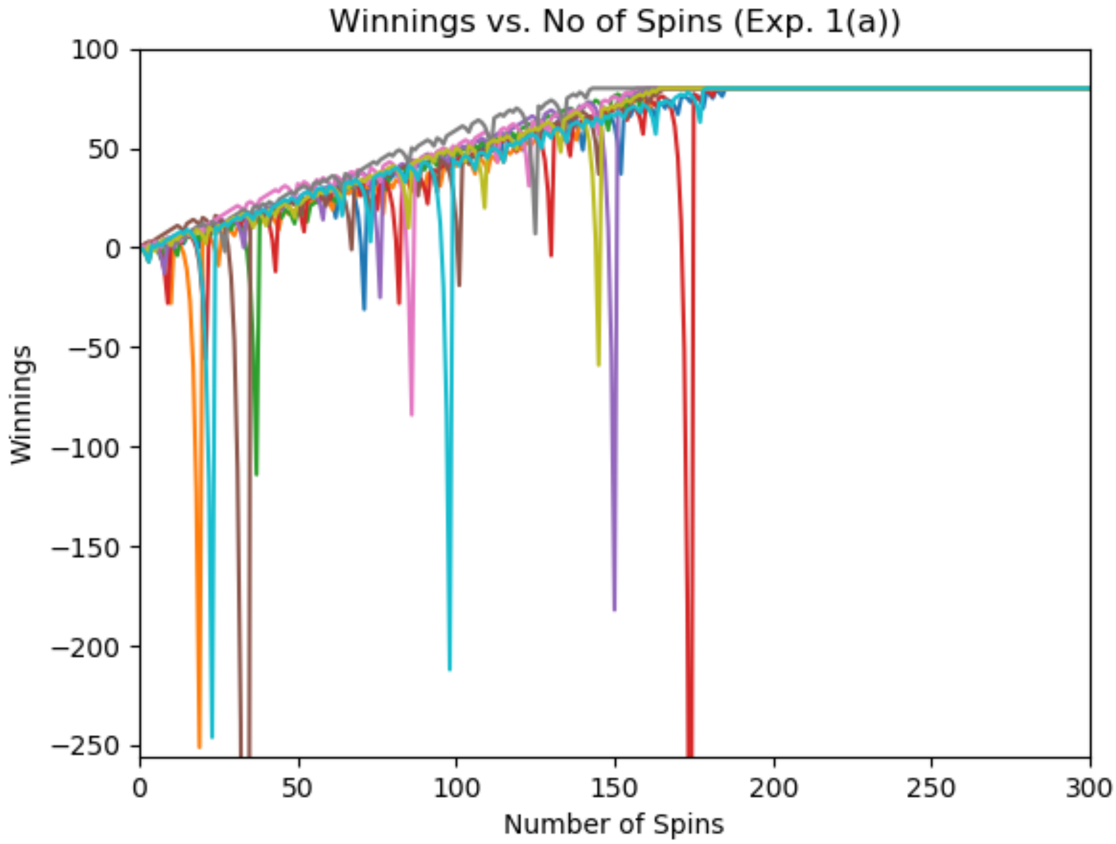


Fig 1: Winnings in 10 episodes vs. Number of spins

(b) Exp. 1 part (b): Mean value of winnings in 1000 episodes vs. Number of spins

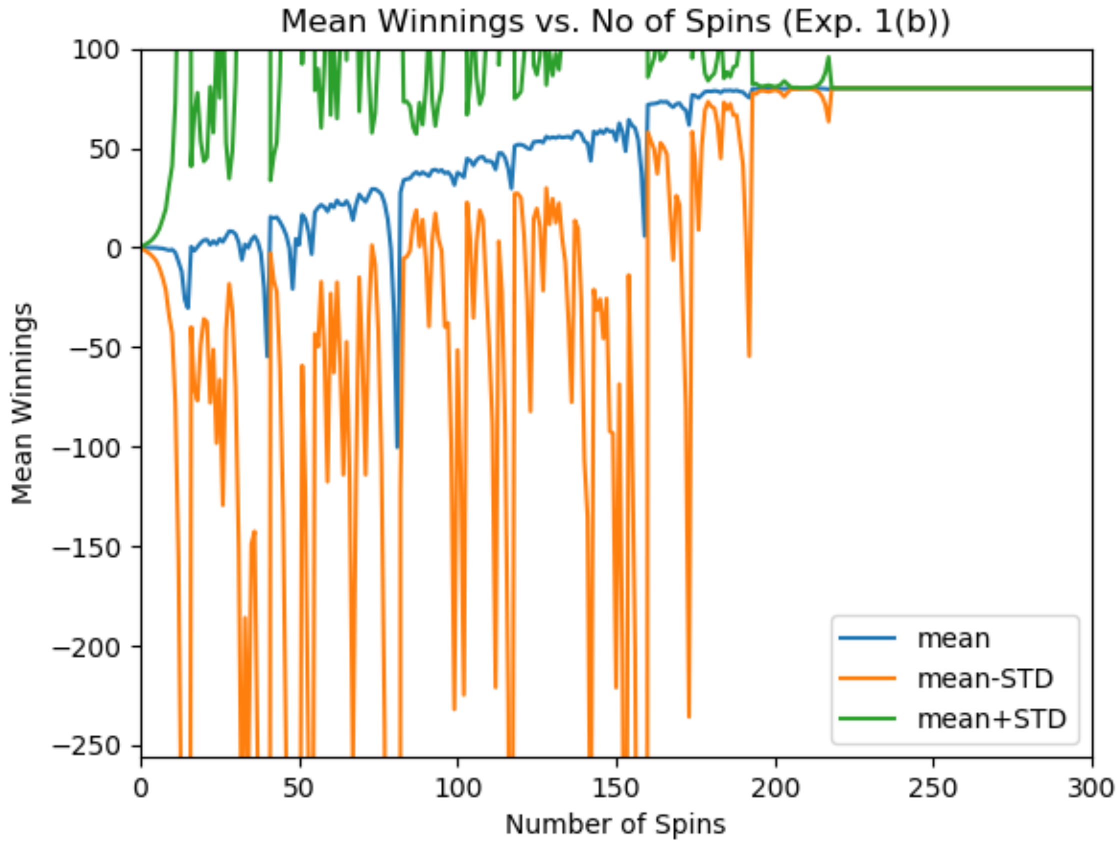


Fig. 2: Mean, (Mean-standard deviation), and (mean+standard deviation) of winnings in 1000 episodes vs. Number of spins

(c) Exp. 1 part (b): Median value of winnings in 1000 episodes vs. Number of spins

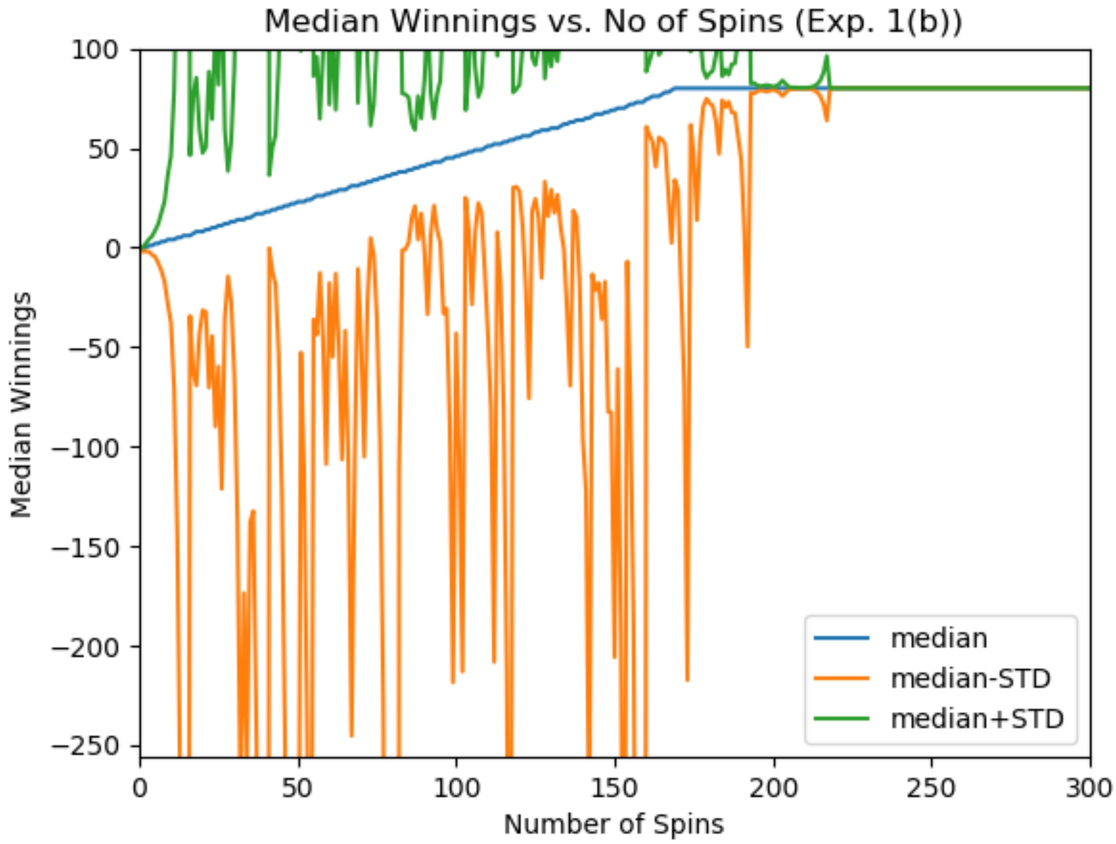


Fig. 3: Median, (median-standard deviation), and (median+standard deviation) of winnings in 1000 episodes vs. Number of spins

(d) Exp. 2 part (a): Mean value of winnings in 1000 episodes vs. Number of spins

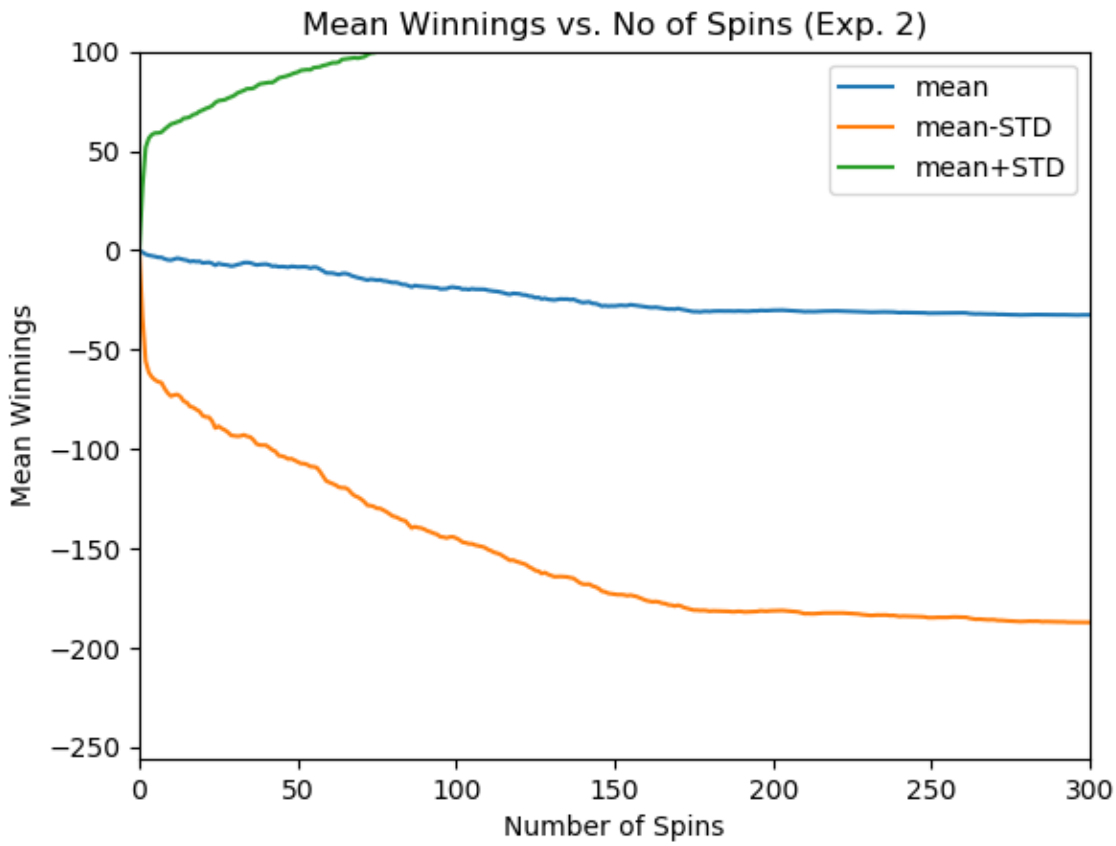


Fig. 4: Mean, (Mean-standard deviation), and (mean+standard deviation) of winnings in 1000 episodes vs. Number of spins

(e) Exp. 2 part (b): Median value of winnings in 1000 episodes vs. Number of spins

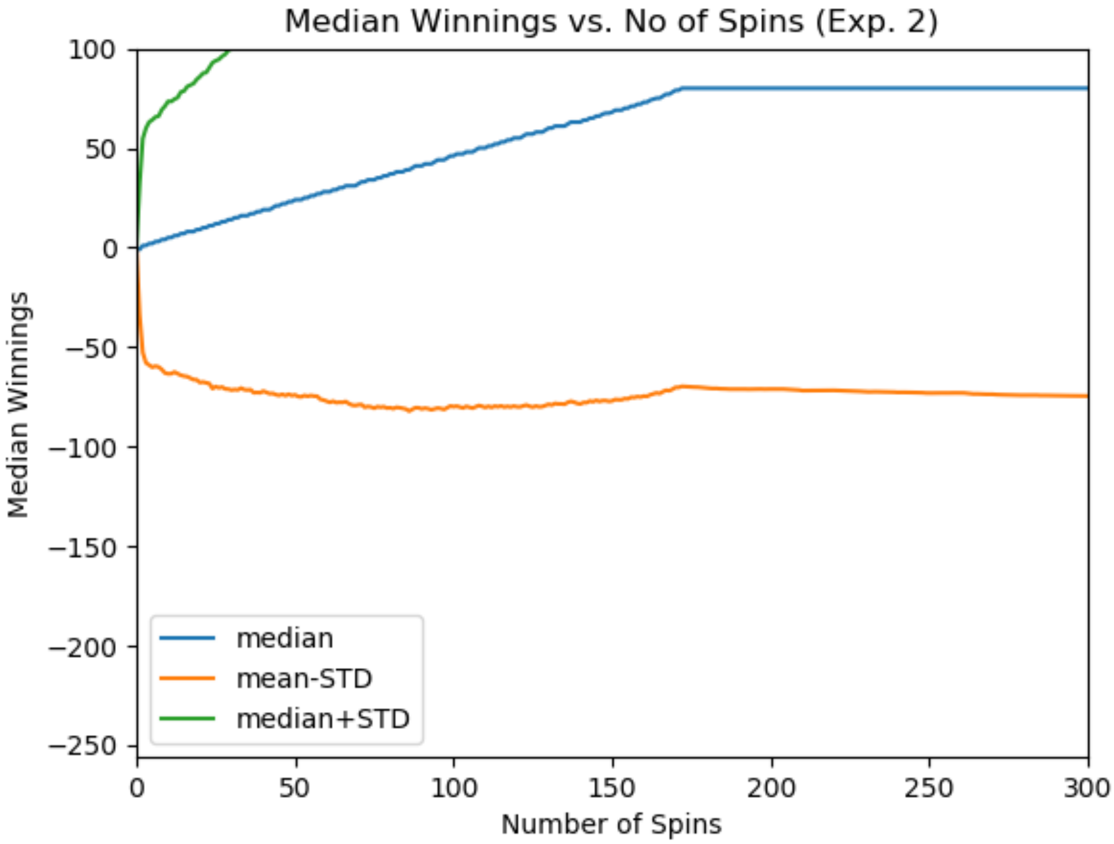


Fig. 5: Median, (median-standard deviation), and (median+standard deviation) of winnings in 1000 episodes vs. Number of spins