Pruning Decision Tree Problem

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1 CS 536: Pruning Decision Trees Assignment

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```
In [32]: #Importing libraries
    import numpy as np
    import pandas as pd
    from pprint import pprint
    import matplotlib.pyplot as plt
    plt.rcParams['figure.figsize'] = [15, 7]
In [28]: #Initializing global variables
    k = 20 # so at to take i values from 0-20
    m = 30
    epsilon = 0.0000001
```

2 Answer 1

Write a function to generate m samples of (X, Y), and another to fit a tree to that data using ID3. Write a third function to, given a decision tree f, estimate the error rate of that decision tree on the underlying data, err(f). Do this repeatedly for a range of m values, and plot the 'typical' error of a tree trained on m data points as a function of m. Does this agree with your intuition?

Function to generate dataset >>

```
else: # when x == 1
                          X.append(int(np.random.choice([1,0], 1, p=[0.5, 0.5])))
                  if X[0] == 0:
                      counts = np.bincount(X[1:7])
                      X.append(np.argmax(counts))
                  else:
                      counts = np.bincount(X[8:14])
                      X.append(np.argmax(counts))
                  return X
             for x in range(1, m+1):
                  data.append(vector_generation_function(k))
              #Create header list
             headers = ['X'+str(x) \text{ for } x \text{ in } range(0,k+1)] + ['Y']
             dataframe = pd.DataFrame(data, columns=headers)
             return dataframe
   Function to calculate Information Gain >>
In [38]: def information_gain(subset_dataframe):
                  classes = list(subset_dataframe)
                  x0_y0 = len(subset_dataframe.loc[(subset_dataframe[classes[0]] == 0) & (subset_dataframe)
                  x0_y1 = len(subset_dataframe.loc[(subset_dataframe[classes[0]] == 0) & (subset_dataframe)
                  x1_y0 = len(subset_dataframe.loc[(subset_dataframe[classes[0]] == 1) & (subset_dataframe)
                  x1_y1 = len(subset_dataframe.loc[(subset_dataframe[classes[0]] == 1) & (subset_dataframe)
                  py0 = (x0_y0 + x1_y0) / (x0_y0 + x1_y0 + x0_y1 + x1_y1 + epsilon)
                  py1 = (x0_y1 + x1_y1) / (x0_y0 + x1_y0 + x0_y1 + x1_y1 + epsilon)
                  px0 = (x0_y0 + x0_y1) / (x0_y0 + x1_y0 + x0_y1 + x1_y1 + epsilon)
                  px1 = (x1_y0 + x1_y1) / (x0_y0 + x1_y0 + x0_y1 + x1_y1 + epsilon)
                  py0_x0 = x0_y0 / (x0_y0 + x0_y1 + epsilon)
                  py1_x0 = x0_y1 / (x0_y0 + x0_y1 + epsilon)
```

X.append(int(np.random.choice([prev_X, 1-prev_X], 1, p=[0.75, 0.25]))

```
py1x1 = x1_y1 / (x1_y0 + x1_y1 + epsilon)
                 pyx0 = (-1 * py0_x0 * np.log(py0_x0 + epsilon)) + (-1 * py1_x0 * np.log(py1_x)
                 pyx1 = (-1 * py0x1 * np.log(py0x1 + epsilon)) + (-1 * py1x1 * np.log(py1x1 + epsilon))
                 hy = (-1 * py0 * np.log(py0 + epsilon)) + (-1 * py1 * np.log(py1 + epsilon))
                 hyx = (px1 * pyx1) + (px0 * pyx0)
                 igx = hy - hyx
                 return igx
   Function to split feature based on Entropy .... Splitting Variable function :
In [5]: def splitting_variable(dataframe):
            """This function take a dataframe as input and returns apt splitting variable base
            #Fetch information gain for every X
            columns = list(dataframe)
            ig = [information_gain(dataframe[[x, 'Y']]) for x in columns[:-1]]
            split_on_variable = columns[np.argmax(ig)]
            return split_on_variable #splitting variable
   Function to split feature based on Chi-Squared test
In [6]: #Splitting function based on Chi-Squared Test
        def chi_squared_split(dataframe):
            def chi_squared_function(subset_df):
                """Extract the count when
                       / x=0 / x=1
                   y=0 / x0y0 / x1y0
                   y=1 \ / \ x0y1 \ / \ x1y1
                classes = list(subset_df)
                x0y0 = len(subset_df.loc[(subset_df[classes[0]] == 0) & (subset_df[classes[1]]
                x0y1 = len(subset_df.loc[(subset_df[classes[0]] == 0) & (subset_df[classes[1]]
                x1y0 = len(subset_df.loc[(subset_df[classes[0]] == 1) & (subset_df[classes[1]]
                x1y1 = len(subset_df.loc[(subset_df[classes[0]] == 1) & (subset_df[classes[1]]
                total\_count = x0y0 + x0y1 + x1y0 + x1y1
```

 $py0x1 = x1_y0 / (x1_y0 + x1_y1 + epsilon)$

 $py0 = (x0y0 + x1y0) / (total_count + epsilon)$ $py1 = (x0y1 + x1y1) / (total_count + epsilon)$

```
px0 = (x0y0 + x0y1) / (total_count + epsilon)
                                                                               px1 = (x1y0 + x1y1) / (total_count + epsilon)
                                                                               t_x0y0 = (((px0 * py0 * total_count) - x0y0) ** 2)/((px0 * py0 * total_count
                                                                               t_x0y1 = (((px0 * py1 * total_count) - x0y1) ** 2)/((px0 * py1 * total_count
                                                                               t_x1y0 = (((px1 * py0 * total_count) - x1y0) ** 2)/((px1 * py0 * total_count) - x1y0)
                                                                               t_x1y1 = (((px1 * py1 * total_count) - x1y1) ** 2)/((px1 * py1 * total_count
                                                                               T = t_x0y0 + t_x0y1 + t_x1y0 + t_x1y1
                                                                               return T
                                                            \#Fetch information gain for every X
                                                            columns = list(dataframe)
                                                            chi = [chi_squared_function(dataframe[[x, 'Y']]) for x in columns[:-1]]
                                                            split_on_variable = columns[np.argmax(chi)]
                                                           return split_on_variable, chi[np.argmax(chi)] #splitting variable
              Function to generate decision tree based on ID3 algorithm
In [21]: #generate decision tree
                                            def generate_decision_tree(dataframe, tree=None):
                                                                  #fetch the features X1, X2...Xn
                                                                classes = list(dataframe)[:-1]
                                                                  #qet the node to split on
                                                                 split_var = splitting_variable(dataframe)
                                                                 #initialize tree in form of dictionary if not already initialized
                                                                if tree is None:
                                                                                    tree = {}
                                                                                    tree[split_var] = {}
                                                                  #Explore when split_var is 0 & 1
                                                                for value in (0,1):
                                                                                     split_dataframe = dataframe[dataframe[split_var] == value]
                                                                                     class_value, value_count = np.unique(split_dataframe['Y'], return_counts=True
                                                                                     #check if split_dataframe has only single class to consider, if not then expl
                                                                                     if len(value_count) == 1:
                                                                                                        tree[split_var][value] = class_value[0]
                                                                                     else:
                                                                                                         #recursively call the tree
                                                                                                         tree[split_var][value] = generate_decision_tree(split_dataframe)
```

```
return tree
In [22]: #predict function using the decision tree
         def predict(X_dataframe, tree):
             for value in tree.keys():
                 sub_tree = int(X_dataframe[value])
                 try:
                     tree = tree[value][sub_tree]
                 except KeyError:
                     return sub_tree
                 if type(tree) is not dict:
                     return tree
                 else:
                     return predict(X_dataframe, tree)
In [23]: #Create function for computing ERR_train
         def compute_ERR(dataframe, tree):
             err = 0
             length = len(dataframe)
             for row_index in range(0, length):
                 row = dataframe[row_index: row_index+1]
                 try:
                     if 'Y' not in tree.keys() and int(row['Y']) != predict(row, tree):
                         err += 1
                     elif int(row['Y']) != tree['Y']:
                         err += 1
                 except:
                     pass
             return err/length
In [34]: k = 20
         m = [10, 50, 100, 200, 400, 600, 800, 1000, 2000, 3000, 4000, 5000, 6000, 7000, 8000,
         split_ratio = 0.8
         err = \{\}
         for values in m:
             df = dataframe_function(k, values)
             train_df = df.iloc[:int(split_ratio*values)]
             test_df = df.iloc[int(split_ratio*values):]
```

#return the generated tree

```
tree = generate_decision_tree(train_df)

ERR = compute_ERR(test_df, tree)

err[values] = ERR

err_list = sorted(err.items())

x,y = zip(*err_list)

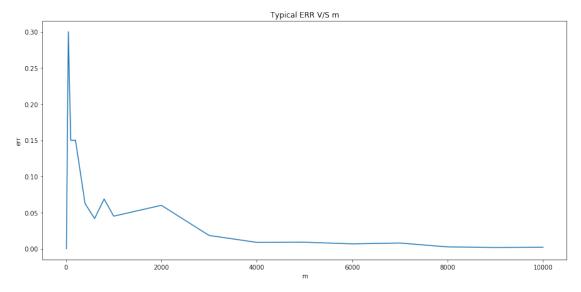
plt.plot(x,y)

plt.title('Typical ERR V/S m')

plt.xlabel('m')

plt.ylabel('err')

plt.show()
```

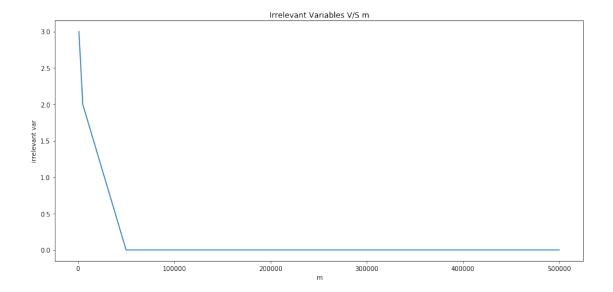


We can see that as the data set increases, typical error decreases. But after a point in m value the error rate does not improve much. As there is not much significant improvement in error rate from m = 7000 to m = 10000.

4 Answer 2

Note that X15 through X20 are completely irrelevant to predicting the value of Y . For a range of m values, repeatedly generate data sets of that size and fit trees to that data, and estimate the average number of irrelevant variables that are included in the fit tree. How much data would you need, typically, to avoid fitting on this noise?

```
In [36]: #Check for irrelevant variables
                         def get_irrelv_var_count(decision_tree_generator = generate_decision_tree):
                                      irrelevant = ["'X15'", "'X16'", "'X17'", "'X18'", "'X19'", "'X20'"]
                                     k = 20
                                      \#m = [10, 50, 100, 200, 400, 600, 800, 1000, 2000, 3000, 4000, 5000, 6000, 7000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 10000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 1000, 
                                     m = [1000, 5000, 50000, 100000, 500000]
                                     split ratio = 0.8
                                     irr_count = {}
                                     for values in m:
                                                 df = dataframe_function(k, values)
                                                 train_df = df.iloc[:int(split_ratio*values)]
                                                 test_df = df.iloc[int(split_ratio*values):]
                                                 tree = decision_tree_generator(train_df)
                                                 #count irrelevant variable in current decision tree
                                                 variable_list = str(tree)
                                                 count = 0
                                                 for irr in irrelevant:
                                                            if irr in variable list:
                                                                        count += 1
                                                 irr_count[values] = count
                                     irr_list = sorted(irr_count.items())
                                     x,y = zip(*irr_list)
                                     plt.plot(x,y)
                                     plt.title('Irrelevant Variables V/S m')
                                     plt.xlabel('m')
                                     plt.ylabel('irrelevant var')
                                     plt.show()
                                     average_irrelevant_variable = sum(y)/len(y)
                                     print('Average number of Irrelevant Variables: ', average_irrelevant_variable)
                                     return average_irrelevant_variable
In [37]: get_irrelv_var_count()
```



Average number of Irrelevant Variables: 1.0

Out[37]: 1.0

5 Conclusion

On an average we are getting around 1-2 irrelevant variable when m is 50000 and as the value of m increases the count of irrelevant variables in our decision tree goes down to 0

6 Answer 3

Generate a data set of size m = 10000, and set aside 8000 points for training, and 2000 points for testing. The remaining questions should all be applied to this data set.

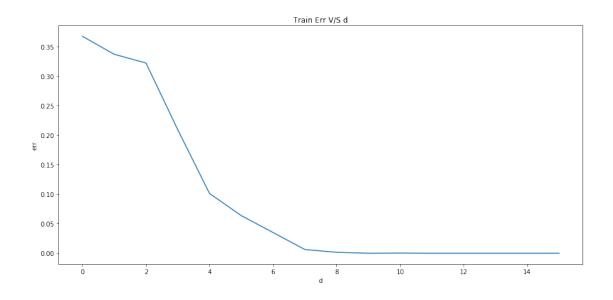
Part a)

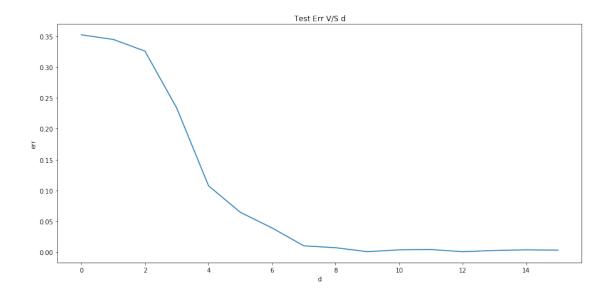
Pruning by Depth: Consider growing a tree as a process - running ID3 for instance until all splits up to depth d have been performed. Depth d = 0 should correspond to no decisions - a prediction for Y is made just on the raw frequencies of Y in the data. Plot, as a function of d, the error on the training set and the error on the test set for a tree grown to depth d. What does your data suggest as a good threshold depth?

```
split_var = splitting_variable(dataframe)
             #initialize tree in form of dictionary if not already initialized
             if tree is None:
                 tree = {}
                 tree[split_var] = {}
             if depth == 0 and count == 0:
                 tree = {}
                 counts = np.bincount(dataframe['Y'])
                 if counts[0] == counts[1]:
                     tree['Y'] = int(np.random.choice([1,0], 1, p=[0.5, 0.5]))
                     tree['Y'] = np.argmax(counts)
                 return tree
             #Explore when split_var is 0 & 1
             for value in (0,1):
                 split_dataframe = dataframe[dataframe[split_var] == value]
                 class_value, value_count = np.unique(split_dataframe['Y'], return_counts=True
                 #check if split_dataframe has only single class to consider, if not then expl
                 if len(value_count) == 1:
                     tree[split_var][value] = class_value[0]
                 else:
                     #recursively call the tree
                     if depth == 0:
                         """print(split_var)
                         print(split\_dataframe)"""
                         counts = np.bincount(split_dataframe['Y'])
                         if counts[0] == counts[1]:
                             tree[split_var][value] = int(np.random.choice([1,0], 1, p=[0.5, 0
                         else:
                             tree[split_var][value] = np.argmax(counts)
                     tree[split_var][value] = depth_pruning_decision_tree(split_dataframe, dep
             #return the generated tree
             return tree
In [42]: pprint(depth_pruning_decision_tree(dataframe_function(k=20, m=100), depth=2))
{'X3': {0: {'X1': {0: {'X0': {0: 0, 1: 0}}}, 1: {'X11': {0: 0, 1: 1}}}},
        1: {'X10': {0: {'X6': {0: 0, 1: 0}}, 1: {'X5': {0: 0}}}}}
In [43]: k = 20
```

#get the node to split on

```
m = 10000
split_ratio = 0.8
err_train_list = {}
err_list = {}
sample_list = [0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]
\#sample_list = [0, 1, 2]
for values in sample_list:
    df = dataframe_function(k, m)
    train_df = df.iloc[:int(split_ratio*m)]
    test_df = df.iloc[int(split_ratio*m):]
    tree = depth_pruning_decision_tree(train_df, depth=values)
    """print(tree)"""
    err_train_list[values] = compute_ERR(train_df, tree)
    err_list[values] = compute_ERR(test_df, tree)
err = sorted(err_train_list.items())
x,y = zip(*err)
plt.plot(x,y)
plt.title('Train Err V/S d')
plt.xlabel('d')
plt.ylabel('err')
plt.show()
err = sorted(err_list.items())
x,y = zip(*err)
plt.plot(x,y)
plt.title('Test Err V/S d')
plt.xlabel('d')
plt.ylabel('err')
plt.show()
```





As the depth of the tree increases the error rate on Train set and Test set decreases. As we can see from above plots that there is no much significant change in error rate in d=8 and d=10 so we can take d=8 as a good threshold for d. Which will also improve the computation cost versus when d=10.

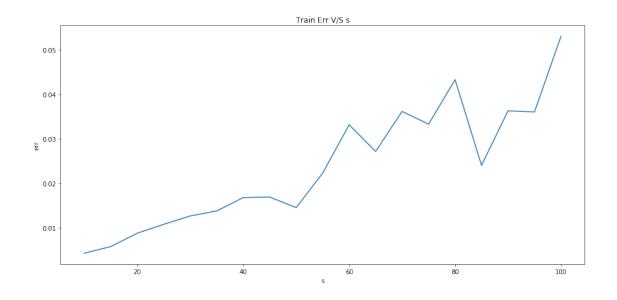
Part b)

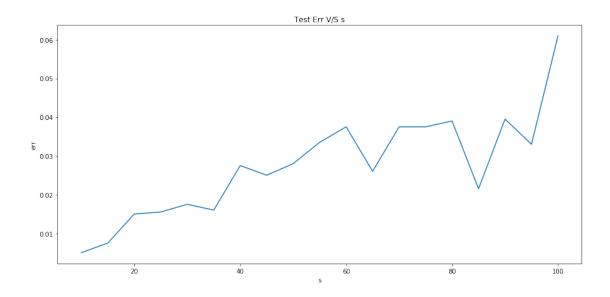
Pruning by Sample Size: The less data a split is performed on, the less 'accurate' we expect the result of that split to be. Let s be a threshold such that if the data available at a node in your

decision tree is less than or equal to s, you do not split and instead decide Y by simple majority vote (ties broken by coin flip). Plot, as a function of s, the error on the training set and the error on the testing set for a tree split down to sample size s. What does your data suggest as a good sample size threshold?

```
In [44]: #generate decision tree using pruning by sample
         def sample_pruning_decision_tree(dataframe, tree=None, s=0):
             #fetch the features X1, X2...Xn
             classes = list(dataframe)[:-1]
             #get the node to split on
             split_var = splitting_variable(dataframe)
             #initialize tree in form of dictionary if not already initialized
             if tree is None:
                 tree = {}
                 tree[split_var] = {}
             #Explore when split_var is 0 & 1
             for value in (0,1):
                 if len(dataframe) <= s:</pre>
                     counts = np.bincount(dataframe['Y'])
                     if counts[0] == counts[1]:
                         tree[split_var][value] = int(np.random.choice([1,0], 1, p=[0.5, 0.5])
                     else:
                         tree[split_var][value] = np.argmax(counts)
                     return tree
                 split_dataframe = dataframe[dataframe[split_var] == value]
                 class_value, value_count = np.unique(split_dataframe['Y'], return_counts=True
                 #check if split_dataframe has only single class to consider, if not then expl
                 if len(value_count) == 1:
                     tree[split_var][value] = class_value[0]
                 else:
                     #recursively call the tree
                     tree[split_var][value] = sample_pruning_decision_tree(split_dataframe, s=
             #return the generated tree
             return tree
In [45]: pprint(sample_pruning_decision_tree(dataframe_function(k=20, m=100), s=10))
{'X10': {0: {'X13': {0: 0,
                     1: {'X3': {0: 0,
                                1: {'X8': {0: {'X16': {0: {'X6': {0: 1}}},
```

```
1: 0}},
                                           1: 1}}}}},
         1: {'X5': {0: {'X0': {0: 0,
                               1: {'X13': {0: {'X7': {0: 0}},
                                           1: {'X12': {0: {'X8': {0: 1}}},
                                                        1: 1}}}}},
                    1: {'X0': {0: {'X1': {0: 1}}, 1: 1}}}}
In [46]: k = 20
        m = 10000
         split_ratio = 0.8
         err_train_list = {}
         err_list = {}
         sample_list = [10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95
         \#sample_list = [0, 1, 2]
         for values in sample_list:
             df = dataframe_function(k, m)
             train_df = df.iloc[:int(split_ratio*m)]
             test_df = df.iloc[int(split_ratio*m):]
             tree = sample_pruning_decision_tree(train_df, s=values)
             """print(tree)"""
             err_train_list[values] = compute_ERR(train_df, tree)
             err_list[values] = compute_ERR(test_df, tree)
         err = sorted(err_train_list.items())
         x,y = zip(*err)
         plt.plot(x,y)
         plt.title('Train Err V/S s')
         plt.xlabel('s')
        plt.ylabel('err')
        plt.show()
         err = sorted(err_list.items())
         x,y = zip(*err)
        plt.plot(x,y)
         plt.title('Test Err V/S s')
        plt.xlabel('s')
         plt.ylabel('err')
         plt.show()
```





In the case when we prune the tree based on the sample size of the input, we get the above results for Train and Test Error. We can see that our algorithm best performs when sample size taken is 10, ie having lowest Test error and a negligible amount of training error. Thus we can conclude that our sample size threshold is 10.

Part c)

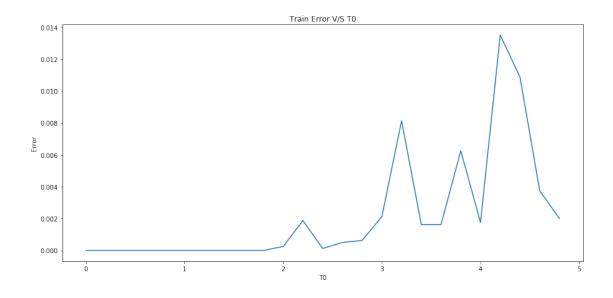
Pruning by Significance: If a variable X is independent of Y, then X has no value as a splitting variable. We can use something like the 2-test to estimate how likely a potential splitting variable

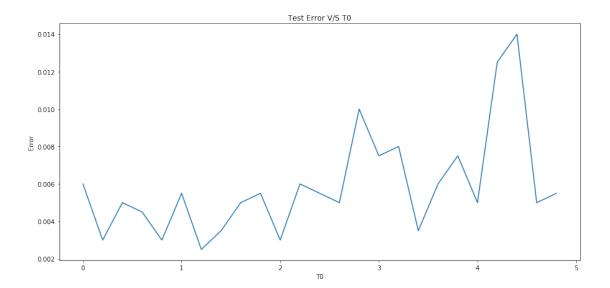
is to be independent, based on the test statistic T compared to some threshold T0 (in the usual 2-outcome case, T0 = 3.841 is used to test at a significance level of p = 5% - see notes for more explanation). Given T0, if given the data for X the value of T is less than T0, it is deemed not significant and is not used for splitting. If given the data for X the value of T is greater than T0, it is deemed significant, and used for splitting. Plot, as a function of T0, the error on the training set and the error on the testing set for a tree split at significance threshold T0. What does your data suggest as a good threshold for significance?

```
In [48]: #generate decision tree using pruning by significance
         def significance_pruning_decision_tree(dataframe, tree=None, t0=0):
             #fetch the features X1, X2...Xn
             classes = list(dataframe)[:-1]
             #get the node to split on
             split_var, t = chi_squared_split(dataframe)
             #initialize tree in form of dictionary if not already initialized
             if tree is None:
                 tree = {}
                 tree[split_var] = {}
             #Explore when split_var is 0 & 1
             for value in (0,1):
                 if t < t0:
                     counts = np.bincount(dataframe['Y'])
                     if counts[0] == counts[1]:
                         tree[split_var][value] = int(np.random.choice([1,0], 1, p=[0.5, 0.5])
                     else:
                         tree[split_var][value] = np.argmax(counts)
                     return tree
                 split_dataframe = dataframe[dataframe[split_var] == value]
                 class_value, value_count = np.unique(split_dataframe['Y'], return_counts=True
                 #check if split_dataframe has only single class to consider, if not then expl
                 if len(value_count) == 1:
                     tree[split_var][value] = class_value[0]
                 else:
                     #recursively call the tree
                     tree[split_var][value] = significance_pruning_decision_tree(split_dataframetree)
             #return the generated tree
             return tree
In [49]: pprint(significance_pruning_decision_tree(dataframe_function(k=20, m=100), t0=3.841))
```

{'X5': {0: {'X0': {0: 0,

```
1: {'X13': {0: {'X12': {0: 0, 1: {'X17': {0: 0, 1: 1}}}},
                               1: {'X9': {0: 1}}}}},
        1: {'X6': {0: {'X2': {0: 0}}},
                   1: {'X4': {0: {'X1': {0: {'X2': {0: 0}}}, 1: 1}},
                              1: {'X0': {0: 1,
                                         1: {'X8': {0: {'X16': {0: 0, 1: 1}},
                                                    1: 1}}}}}}
In [55]: k = 20
        m = 10000
         split ratio = 0.8
         err_train_list = {}
         err_list = {}
         sample_list = [0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2, 1.4, 1.6, 1.8, 2.0, 2.2, 2.4, 2.6, 2.5]
         \#sample_list = [0, 1, 2]
         for values in sample_list:
             df = dataframe_function(k, m)
             train_df = df.iloc[:int(split_ratio*m)]
             test_df = df.iloc[int(split_ratio*m):]
             tree = significance_pruning_decision_tree(train_df, t0=values)
             """print(tree)"""
             err_train_list[values] = compute_ERR(train_df, tree)
             err_list[values] = compute_ERR(test_df, tree)
         err = sorted(err_train_list.items())
         x,y = zip(*err)
         plt.plot(x,y)
         plt.title('Train Error V/S TO')
         plt.xlabel('T0')
        plt.ylabel('Error')
        plt.show()
         err = sorted(err_list.items())
         x,y = zip(*err)
        plt.plot(x,y)
         plt.title('Test Error V/S TO')
        plt.xlabel('T0')
         plt.ylabel('Error')
         plt.show()
```





By using Chi-Squared Test, when we prune by significance, to check the dependence value between variable and output vector. We train this for various threshold of T value for Chi-Squared Test and the results for Train and Test set are plotted above. From the plot we can see that for value of T0 = 2.15 we have the least Train and Test Error. Hence we can conclude that our algorithm works best for threshold value of T0 = 2.15

10 Answer 5

Repeat the computation of Problem 2, growing your trees only to depth d as chosen in 3.a. How does this change the likelihood or frequency of including spurious variables in your trees?

```
In []: irrelevant = ["'X15'", "'X16'", "'X17'", "'X18'", "'X19'", "'X20'"]
       k = 20
       m = [1000, 5000, 50000, 100000, 500000]
       split_ratio = 0.8
       irr_count = {}
       depth = 8
       for values in m:
           df = dataframe_function(k, values)
           train_df = df.iloc[:int(split_ratio*values)]
           test_df = df.iloc[int(split_ratio*values):]
           tree = depth_pruning_decision_tree(train_df, depth=depth)
           #count irrelevant variable in current decision tree
           variable_list = str(tree)
           count = 0
           for irr in irrelevant:
              if irr in variable_list:
                  count += 1
           irr_count[values] = count
       irr_list = sorted(irr_count.items())
       x,y = zip(*irr_list)
       plt.plot(x,y)
       plt.title('Irrelevant Variables V/S m')
       plt.xlabel('m')
       plt.ylabel('#Irrelevant variable')
       plt.show()
       average_irrelevant_variable = sum(y)/len(y)
       print('Average number of Irrelevant Variables: ', average_irrelevant_variable)
```

11 Conclusion

On an average we are getting around 0.8 irrelevant variable when m is less than 10000 and as the value of m increases the count of irrelevant variables in our decision tree drops to 0. This is observed when we prune our decision tree using depth d = 8.

12 Answer 6

Repeat the computation of Problem 2, splitting your trees only to sample size s as chosen in 3.b. How does this change the likelihood or frequency of including spurious variables in your trees?

```
In [ ]: irrelevant = ["'X15'", "'X16'", "'X17'", "'X18'", "'X19'", "'X20'"]
       k = 20
       m = [1000, 5000, 50000, 100000, 500000]
       split_ratio = 0.8
       irr_count = {}
       sample = 10
       for values in m:
           df = dataframe_function(k, values)
           train_df = df.iloc[:int(split_ratio*values)]
           test_df = df.iloc[int(split_ratio*values):]
           tree = sample_pruning_decision_tree(train_df, s=sample)
           #count irrelevant variable in current decision tree
           variable_list = str(tree)
           count = 0
           for irr in irrelevant:
              if irr in variable_list:
                  count += 1
           irr_count[values] = count
       irr_list = sorted(irr_count.items())
       x,y = zip(*irr_list)
       plt.plot(x,y)
       plt.title('Irrelevant Variables V/S m')
       plt.xlabel('m')
       plt.ylabel('#Irrelevant variable')
       plt.show()
       average_irrelevant_variable = sum(y)/len(y)
       print('Average number of Irrelevant Variables: ', average_irrelevant_variable)
```

13 Conclusion

On an average we are getting around 1 irrelevant variable when m is less than 5000 and as the value of m increases the count of irrelevant variables in our decision tree drops to 0. This is observed when we prune our decision tree using sample size = 10.

14 Answer 7

Repeat the computation of Problem 2, splitting your trees only at or above threshold level T0 as chosen in 3.c. How does this change the likelihood or frequency of including spurious variables in your trees?

```
In []: irrelevant = ["'X15'", "'X16'", "'X17'", "'X18'", "'X19'", "'X20'"]
       k = 20
       m = [1000, 5000, 50000, 100000, 500000]
       split_ratio = 0.8
       irr_count = {}
       t0 = 2.2
       for values in m:
           df = dataframe_function(k, values)
           train_df = df.iloc[:int(split_ratio*values)]
           test_df = df.iloc[int(split_ratio*values):]
           tree = significance_pruning_decision_tree(train_df, t0=t0)
           #count irrelevant variable in current decision tree
           variable_list = str(tree)
           count = 0
           for irr in irrelevant:
              if irr in variable_list:
                  count += 1
           irr_count[values] = count
       irr_list = sorted(irr_count.items())
       x,y = zip(*irr_list)
       plt.plot(x,y)
       plt.title('Irrelevant Variables V/S m')
       plt.xlabel('m')
       plt.ylabel('#Irrelevant variable')
       plt.show()
       average_irrelevant_variable = sum(y)/len(y)
       print('Average number of Irrelevant Variables: ', average_irrelevant_variable)
```

15 Conclusion

On an average we are getting around 1.8 irrelevant variable when m is less than 10000 and as the value of m increases the count of irrelevant variables in our decision tree slowly drops to 0. This is observed when we prune our decision tree using T0 = 2.2.