Basic cutset A(1,6), B(2,6), C(3,5,6) & D(4,5)

B =

element \ Basic cutset	A	B	C	D	
1	1	0	0	0	$B_b = U_b$
2	0	1	0	0	
3	0	0	1	0	
4	0	0	0	1	
5	0	0	1	-1	B_l
6	1	1	1	0	

$B_{ij} = 1$

If ith element is a part of jth cutset & direction of there is same

$B_{ij} = -1$

If ith element is a part of jth cutset & their directions are opposite

$B_{ij} = 0$

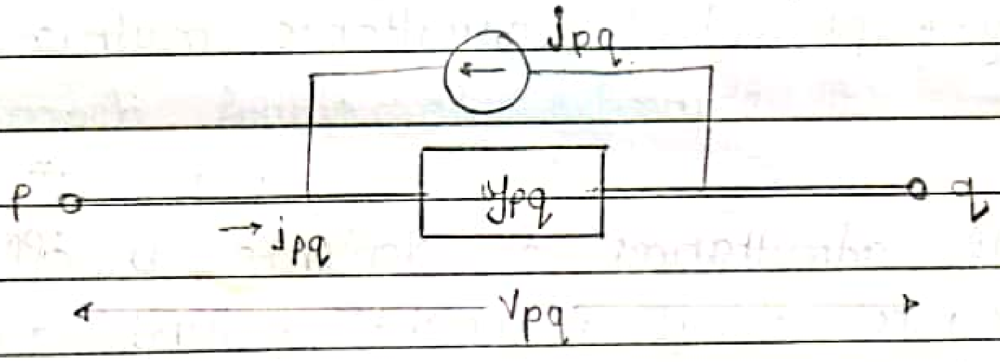
If ith element is not a part of jth cutset

The total power in primitive n/w is given by ~~$(j^*)^t . e$~~

Diagonal entries of Z represents self impedance of elements & off diagonal entry of Z represents mutual impedances between the elements

2) Admittance form of representing an element

In admittance form element connected between p & q represented as,



y_{pq} → Admittance of the element
j_{pq} → Impress current (source) current across the element p & q
v_{pq} & i_{pq} → Voltage & current through element

Performance eq^n

$$i_{pq} + j_{pq} = y_{pq} \cdot v_{pq}$$

Hence the performance equation in premitive

✯ Incidence Matrices

1) Element node incidence matrix [$\hat{A}$]

An element is said to be an incident on a node if the node is one terminal of the element. Thus each element is incident on two nodes

property : Addition of Any Row is zero

$$\hat{A}_{ij} = 1$$

If ith element is incident on jth node & is directed away from the node

$$\hat{A}_{ij} = -1$$

If ith element is incident on jth node & its directed towards the node.

$$\hat{A}_{ij} = 0$$

If ith element is not incident on jth node

elements \ nodes	①	②	③	④	
1	-1	0	0	0	Ab
2	1	-1	0	0	
3	0	1	0	-1	
4	0	1	-1	0	
5	0	0	1	-1	Al
6	0	0	0	-1	

from Bus incidence matrix make graph of original power system

elements \ nodes	①	②	③	④	⑤
1	-1	0	0	0	1
2	1	-1	0	0	0
3	0	1	0	-1	0
4	0	1	-1	0	0
5	0	0	1	-1	0
6	0	0	0	-1	1

Proof :-

i) $A_b . K^t = U$

$$K = \begin{bmatrix} -1 & -1 & -1 \\ 0 & -1 & -1 \\ 0 & 0 & -1 \end{bmatrix} \qquad K^t = \begin{bmatrix} -1 & 0 & 0 \\ -1 & -1 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

$$A_b . K^t = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 \\ -1 & -1 & 0 \\ -1 & -1 & -1 \end{bmatrix}$$

$$A_b . K^t = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = U$$ Hence proved

ii) $B_l . A_b = A_l$

$$B_l . A_b = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}_{2\times3} \begin{bmatrix} -1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}_{3\times3}$$

$$B_l . A_b = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} = A_l$$

Henced proved

property $B_b [U_b] \rightarrow$ It is a sequre & identity matrix

$$B_1 . A_b = \begin{bmatrix} 0 & 0 & 1 & -1 \\ 1 & 1 & 1 & 0 \end{bmatrix}_{2\times4} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix}_{4\times4}$$

$$= \begin{bmatrix} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & -1 \end{bmatrix} = A_1$$

Dimension of B-matrix $e \times (n-1)$

5) Augmented cutset incidence matrix (B')
dimension $(e \times e)$

To form this matrix fictitious (dummy) cutsets are taken on fictitious cutset involves only one link & no any branch. The coloumn corresponding to fictitious cutset are introduce after the basic cutset coloumn

no. of fictitious cutset = no. of link

$$k_{ij} = 1$$

If ith branch is a part of jth path & its direction is same as that of path

$$k_{ij} = -1$$

If ith branch is part of jth path & its direction is opposite to path

$$k_{ij} = 0$$

If ith branch is not part of jth path

A_b - Part of incidence matrix include Branch

A_l = part of incidence matrix include links

property of A_b - It is always sqeure matrix

Relation : $\boxed{A_b \cdot K^t = U}$

	Basic loop / element	D	E	
	1	-1	-1	C_b
	2	-1	-1	
$C =$	3	0	-1	
	4	1	0	U_l
	5	0	1	

⑦ Augmented loop incidence matrix ($\hat{C}$)

	Basic loop / element	A	B	C	D	E
	1	1	0	0	-1	-1
	2	0	1	0	-1	-1
$\hat{C} =$	3	0	0	1	0	-1
	4	0	0	0	1	0
	5	0	0	0	0	1

Direction of fictitious cutset is same as the direction of link involved in it.

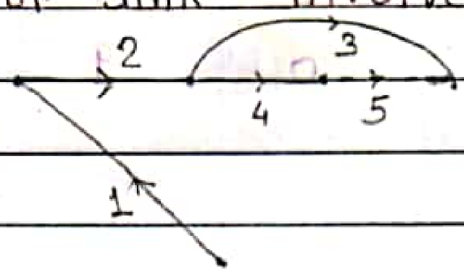
Basic cutset element	A	B	C	D	E	F
1	1	0	0	0	0	0
2	0	1	0	0	0	0
3	0	0	1	0	0	0
4	0	0	0	1	0	0
5	0	0	1	-1	1	0
6	1	1	1	0	0	1

$$\hat{B} = \begin{bmatrix} U_b & 0 \\ B_l & U_l \end{bmatrix}$$

[This is formed by keep previous same & adding (no. of links) column & it is always two to crresp. link]

5) Basic loop incidence matrix (C)

The basic loop is formed by restoring one link at a time into the tree. Direction of basic loop is decided by the direction of link involved in it.



Bus :-

$$\text{power} = (I_{bus}^{*})^{t} \cdot E_{bus}$$

Branch :-

$$\text{power} = (I_{br}^{*})^{t} \cdot E_{br}$$

loop :-

$$\text{power} = (I_{loop}^{*})^{t} \cdot E_{loop}$$

* Property of matrix

If A & B are two matrices then

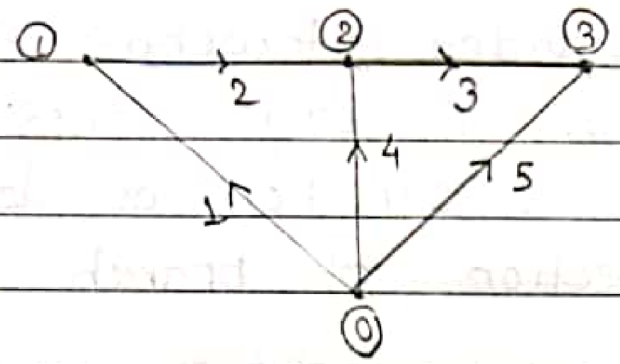
1. $(A.B)^{*} = A^{*}.B^{*}$
2. $(A.B)^{t} = B^{t}.A^{t}$
3. $(A.B)^{-1} = B^{-1}.A^{-1}$

Also $A^{*} = A$ if is real matrix

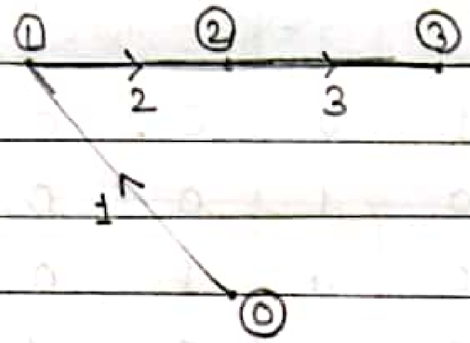
Scanned with CamScanner

H.W.

Oriented graph of a power system is give
below. select 0 as node referance. 1, 2.
3 is tree form matrices A, $\hat{A}$, K, B, $\hat{B}$,
C, $\hat{C}$. And S.T. (i) $A_b K^t = U$ (ii) $B_l \cdot A_b = A_l$



Tree →



(1) Element node incidence matrix ($\hat{A}$)

$\hat{A} =$

nodes / elements	0	1	2	3
1	1	-1	0	0
2	0	1	-1	0
3	0	0	1	-1
4	1	0	-1	0
5	1	0	0	-1

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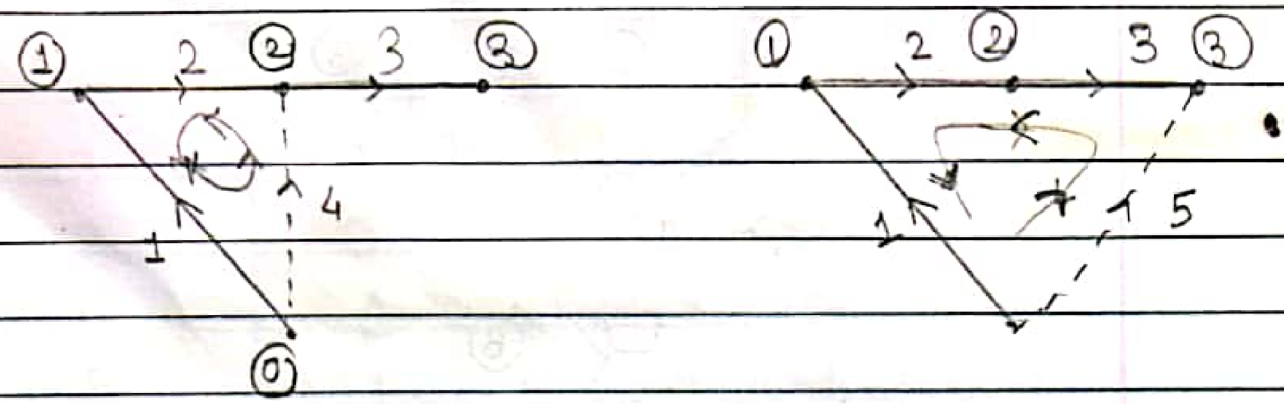
	Basic cutset element	A	B	C	
	1	1	0	0	B_b
	2	0	1	0	
$B =$	3	0	0	1	
	4	1	1	0	B_l
	5	1	1	1	

⑤ Augment cutset incidence matrix ($\hat{B}$)

	Basic cutset element	A	B	C	D	F
	1	1	0	0	0	0
	2	0	1	0	0	0
$\hat{B} =$	3	0	0	1	0	0
	4	1	1	0	1	0
	5	1	1	1	0	1

⑥ Basic loop incidence matrix (C)



elements \ nodes	⓪	①	②	③	④
1	1	-1	0	0	0
2	0	1	-1	0	0
3	0	0	1	0	-1
4	0	0	1	-1	0
5	0	0	0	1	-1
6	1	0	0	0	-1

Dimension of $(\hat{A}) = e \times n$

e : no. of element

n : no. of nodes

2) Bus incidence matrix [A]

If we remove any one coloumn of element node incidence matrix then we get 'Bus incidence matrix'. Generally prefer ground or zero node

Dimension of $A = e \times (n-1)$

network of admittance form

$$\boxed{i + j = [y]\, v}$$

where

$i \rightarrow$ vector of current through element

$v \rightarrow$ —"— voltage across element

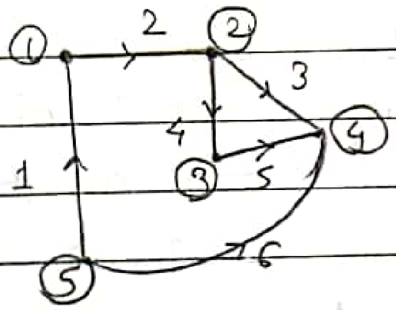
$j \rightarrow$ —"— impressed current in ||el with the elements

$y \rightarrow$ primitive admittance matrix

Diagonal entries of y represent self admittances of elements & off diagonal entries of y represent mutual admittances betn the elements

Total power in primitive network is

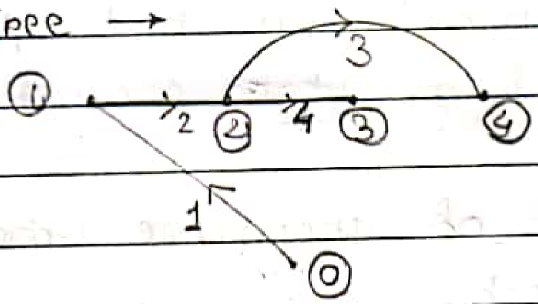
$$(j^*)^t \cdot v$$



3) Branch path incidence matrix (K)

path is formed by moving from a node to reference node in a tree

Select Tree →



If n nodes are present then (n-1) path will be available

Dimension : (n-1) x (n-1)

path / Branch	1	2	3	4
1	-1	-1	-1	-1
2	0	-1	-1	-1
3	0	0	0	-1
4	0	0	-1	0

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☆ Network performance eq^n

1) Bus frame of peferance

$$E_{bus} = Z_{bus} \cdot I_{bus}$$

↓ Bus impedance matrix

└→ Impress bus current

$$I_{bus} = Y_{bus} \cdot E_{bus}$$

└→ Bus admittance matrix

2) Branch frame of Referance

$$E_{br} = Z_{br} \cdot I_{br}$$

└→ Impress branch current

$$I_{br} = Y_{br} \cdot E_{br}$$

└→ Branch admittance matrix

3) Loop frame of Referance

$$E_{loop} = Z_{loop} \cdot I_{loop}$$

↓ Vector of Voltage

└→ loop impedance matrix

$$I_{loop} = Y_{loop} \cdot E_{loop}$$

└→ loop admittance matrix

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(2) Bus incidence matrix

nodes / elements	(1)	(2)	(3)	
1	-1	0	0	A_b
2	1	-1	0	
3	0	1	-1	
4	0	-1	0	A_l
5	0	0	-1	

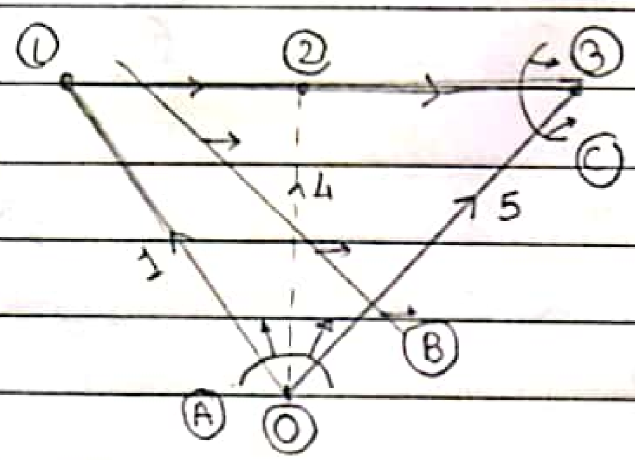
A =

(3) Branch path incidence matrix (K)

Branch \ path	(1)	(2)	(3)
1	-1	-1	-1
2	0	-1	-1
3	0	0	-1

K =

(4) Basic cutset incidence matrix (B)



6) Augmented loop incidence matrix ($\hat{C}$)

Formation of this matrix fictitiou loop (open loop) are taken. An open loop involves only one brach & no any link. coloumn corresponding to open loop are added towords the left of the coloumn of basic loops

Direction of loop is decided by the direction of branch

Basic loop element	A	B	C	D	E	F
1	1	0	0	0	0	-1
2	0	1	0	0	0	-1
3	0	0	1	0	-1	-1
4	0	0	0	1	1	0
5	0	0	0	0	1	0
6	0	0	0	0	0	1

$$\hat{C} = \left[\begin{array}{c|c} U_b & C_b \\ \hline 0 & U_l \end{array}\right]$$

proof :-

$$\begin{bmatrix} -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix} = A_b$$

$$K = \begin{bmatrix} -1 & -1 & -1 & -1 \\ 0 & -1 & -1 & -1 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

$$K^t = \begin{bmatrix} -1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ -1 & -1 & 0 & -1 \\ -1 & -1 & -1 & 0 \end{bmatrix}$$

$$A_b . K^t = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 1 & -1 & 0 \end{bmatrix} . \begin{bmatrix} -1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 \\ -1 & -1 & 0 & -1 \\ -1 & -1 & -1 & 0 \end{bmatrix}$$

$$A_b . K^t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4) Basic cutset incidence matrix (B)

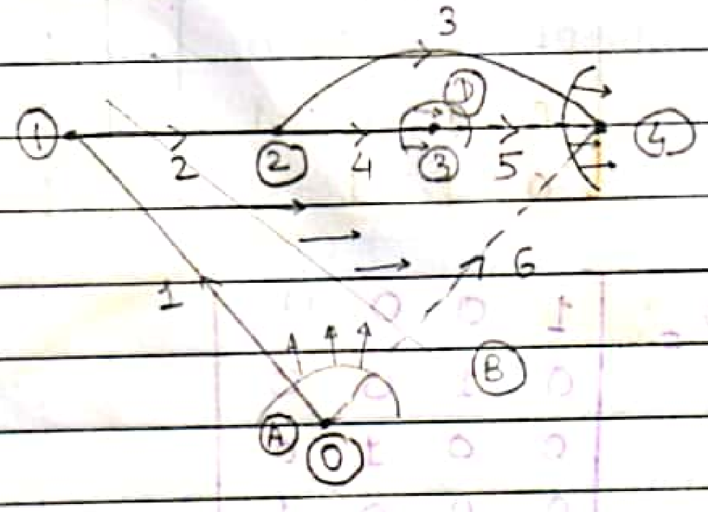
A cutset is group of minimum no. of elements which when removed from the graph divides the graph into two parts, such that restoration of any one element [of cutset] makes the two part connected.

ex. of cutset (1, 2), (1, 6) & etc

A cutset which involves only one branch is called 'Basic cutset' (n-1) from which (1, 6) is Basic cutset. corresponding to each brach, thre will be one basic cutset

no. of branches = nodes - 1

The direction of basic cutset is decided by, the direction of branch Involved in it.



$Z_{pq} \rightarrow$ impedance of the element

performance equation $\rightarrow$

Apply KVL to circuit,

$$E_p + e_{pq} - Z_{pq} \cdot i_{pq} = E_q$$

$$E_p - E_q + e_{pq} = Z_{pq} \cdot i_{pq}$$

$$\boxed{v_{pq} + e_{pq} = Z_{pq} \cdot i_{pq}}$$

This is a performance eq^n in impedance form

performance eq^n for primitive n/w in impedance form can be written as,

$$v + e = [Z] \cdot i$$

where

$v \rightarrow$ vector of voltage accross the elements

$e \rightarrow$ vector of Source voltages in series with the elements

$i \rightarrow$ vector of current through elements

$Z \rightarrow$ primitive impedance matrix

[The total power in primitive n/w is given by $(i^*)^t \cdot e$

$c_{ij} = 1$

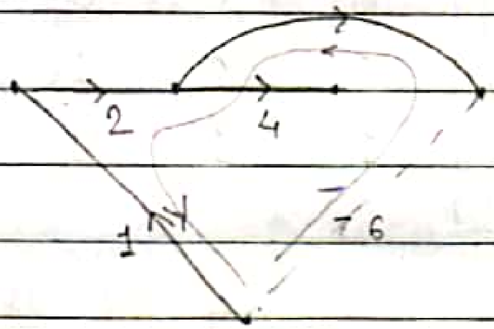
If ith element is a part jth loop & their direction are same (element & loop)

$c_{ij} = -1$

If ith element is a part jth loop & their direction are opposite

$c_{ij} = 0$

If ith element is not a part of jth loop



	Basic loop element	F	F	
	1	0	-1	C_b
or ex.l	2	0	-1	
C =	3	-1	-1	
ex(e-n+1)	4	1	0	
ex[e-(n-1)]	5	1	0	U_l
	6	0	1	

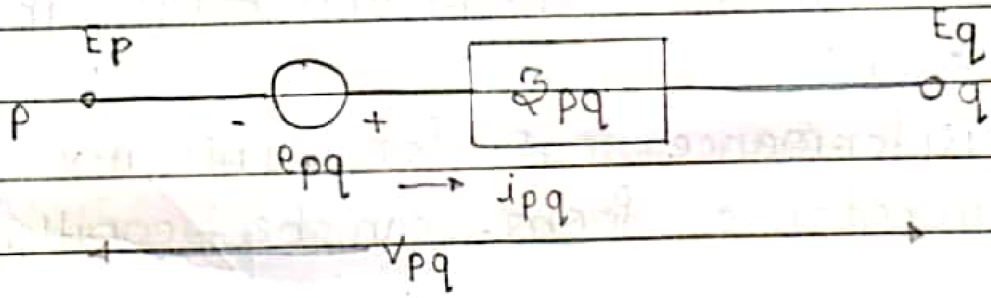
* Primitive Network

A group of unconnected elements is called primitive network.

An element can be represented either in impedance form or in admittance form

1) Impedance form of representing an element

In impedance form an element connected between points p & q represented as,



Here,

E_p, E_q → Voltage at point p & q resp

V_{pq} → Voltage across the element connected betn p & q

i_{pq} → current through the element

e_{pq} → source voltage in series with the element