The Impact of Topology on Cooperation Games

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1 Introduction

In Bargaining with Neighbors, Alexander and Skyms propose that in a square lattice model, bargaining with neighbors or strangers almost always leads to justice in the sense of equity. We can easily see this for ourselves by applying the Bargaining Game to the Replicator Dynamic models provided by NetLogo. However, both of these relax the assumptions that come with how humans interact spatially. This assumption, I show, could be fatal to the argument made by Alexander and Skyrms without modifications.

2 Cooperation

Cooperation is a fundamental mechanism in evolutionary dynamics. Many cases of cooperation have been observed in communities of high relatedness but even in communities where that is not true, evidence of cooperation exists [2]. Alexander and Skyrms suggest that cooperation in communities of low relatedness can arise in spacial embedding of games [1]. Cooperation, according to Alexander and Skyrms as well as Axelrod and Hamilton, is inherently tied to notions of altruism and justice. Of course, both terms - justice and altrusim - are interpreted in different ways by different schools of philosophy based on their respective concepts in fairness. Alexander and Skyrms in their paper settle on "fair division" (most likely read equal division assuming normalization over social indices) as their ideal for fairness and thereby the standard for cooperation and altruism. Axelrod and Hamilton suggest a Tit-For-Tat strategy in the Prisoner's Dilemma in which cooperation is brought about through reciprocity. It also holds as an Evolutionarily Stable Strategy and is shown to be Robust to tremors. The fundamental idea here being that optimal payoffs come from mutually beneficial strategies and so these strategies, over time, are adopted and hold true. Alexander and Skyrms take this idea further, saying that in repeated games played a population, these are the strategies which evolve to be adopted for their optimality. Thereby cooperation (read mutually beneficial

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strategies) should not only spread through populations over time, but also be Evolutionarily Stable and Robust to tremors.

3 The Ultimatum Game

In their paper, "Bargaining with Neighbors," (hereafter refered to as "the paper") Alexander and Skyrms use a repeated model of The Ultimate Game - also known as Divide the Dollar. The premise of The Ultimatum Game (hereafter referenced to as simply "the game") is two players deciding how to split a finite number of dollars/utiles/etc. In its singular form, the game can be modeled as a 3 x 3 x 2 tensor and simplified to the payoff matrix:

$$G = \begin{bmatrix} (0,0) & (0,0) & (n,m) \\ (0,0) & (P/2,P/2) & (P/2,m) \\ (m,n) & (m,P/2) & (m,m) \end{bmatrix} | n > m, n+m = P$$

where P is the total amount being divided, the set of indices $i = 1, 2, 3$ map to the strategies $S = Greedy, Fair, Weak$, respectively and the first in the pair of payoffs maps to the payoff of Player 1, while the second does so to Player 2. In the paper, two polymorphisms are tested - both of which hold P = 10, and one which holds n = 9 and the other which holds n = 6. Alexander and Skyrms embed this in a square lattice spacial model and allow the game to be played repeatedly by each node against its eight neighbors, and allows the node to change strategies based on imitating the success of its neighbors in each round, t, (or in evolutionary terms, generation). In this model they find that justice arises because the greatest basin of attraction comes at the (Fair, Fair) strategy pair. This occurs because the Weak strategies often see both other strategies faring better, creating more Fair and Greedy players. The nodes playing Fair sometimes see the Greedy players faring better - depending on the strategies of the neighbors. However, it is the very dominance of the Greedy strategy which leads to its downfall because as more players become Greedy, the total payoff for each player decreases because the default to the incongruous payoffs in G which give both players $\pi = 0$). Their models show that in theses replicator dynamics, the game tends towards all of the nodes playing Fair strategies as $\lim_{t\to\infty}$.

While the validity of this logic seems feasible, the issue arises that the foundational premise upon which the argument is based on is topology. While this in and of itself is not an issue, there exists the problem of the assumption regarding *choice* of topology. Because the game is being embedded in a social network, one must respect the topology of most social networks and that has been shown to deviate far from a square lattice, and closer to a scale-free network [3].

4 A New Model

Naturally, to alleviate this issue, the topological assumption must relaced. To do this, the scale-free network is introduced to replace the squate lattice. While in a square lattice, each node has a degree $k = 8$, a scale-free network is one in which the second moment of node degree diverges to infinity. To achieve this type of network, a Barabási-Albert network is created by adding nodes to the network until the network reaches a given size, N, and adding edges based to each node using the probability

$$p_i = \frac{k_i}{\sum_j k_j}. \tag{1}$$

This results in a network that looks more like which includes a degree distri-

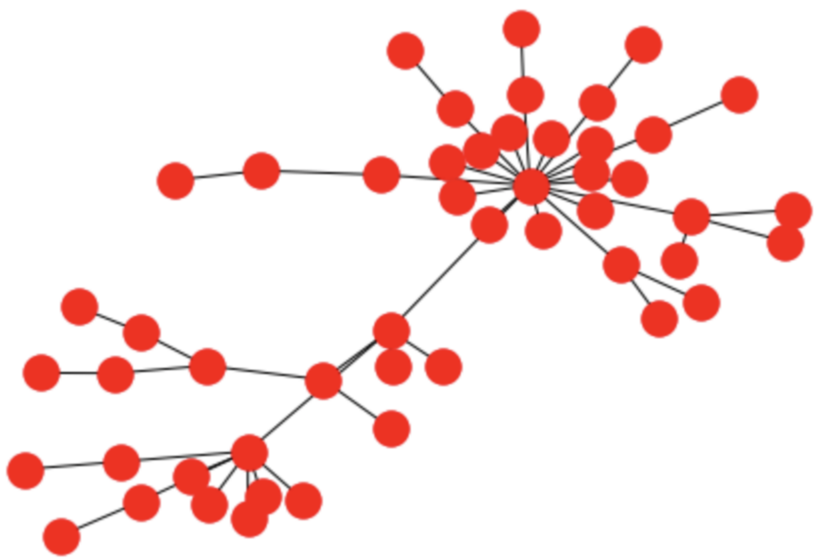

Figure 1: A Barabási-Albert Network of $N = 50$, $m = 1$

bution that allows for nodes of degree $k = 1$ as well as very large degree nodes known as hubs and is modeled as

$$P(k)\ k^{-\gamma} \tag{2}$$

Where γ is the degree exponent and in the case of scale-free networks, $2 < \gamma < 3$. As you can see, this allows for a non-zero probability of finding a node of arbitrary size whereas in the square lattice model, it is impossible to find a node of degree that is not 8.

m	polymorphism	intial conditions	results	time steps
1	1-9	G = 4999, F = 1, W = 5000	G=10000	1
1	1-9	G = 4999, F = 1, W = 5000	G = 9997, F = 3, W = 0	4
1	1-9	G = 4999, F = 1, W = 5000	G = 9999, F = 1	2
5	1-9	G = 4999, F = 1, W = 5000	G = 10000	2
5	1-9	G = 4999, F = 1, W = 5000	G = 10000	2
5	1-9	G = 4999, F = 1, W = 5000	G = 10000	12
1	1-9	G = 3333, F = 3333, W = 3334	G = 3601, F = 6399	27
1	1-9	G = 3333, F = 3333, W = 3334	G = 800, F = 1650, W = 7550	50[1]
1	1-9	G = 3333, F = 3333, W = 3334	G = 122, F = 9878	7
5	1-9	G = 3333, F = 3333, W = 3334	F = 10000	14
5	1-9	G = 3333, F = 3333, W = 3334	F = 10000	5
5	1-9	G = 3333, F = 3333, W = 3334	F = 10000	19
1	4-6	G = 4999, F = 1, W = 5000	G=10000	1
1	4-6	G = 4999, F = 1, W = 5000	G=10000	1
1	4-6	G = 4999, F = 1, W = 5000	W=10000	1
5	4-6	G = 4999, F = 1, W = 5000	W=10000	2
5	4-6	G = 4999, F = 1, W = 5000	W=10000	2
5	4-6	G = 4999, F = 1, W = 5000	W=10000	2
1	4-6	G = 3333, F = 3333, W = 3334	W = 10000	6
1	4-6	G = 3333, F = 3333, W = 3334	W = 10000	1
1	4-6	G = 3333, F = 3333, W = 3334	G = 4045, F = 5955	6
5	4-6	G = 3333, F = 3333, W = 3334	W = 10000	2
5	4-6	G = 3333, F = 3333, W = 3334	F = 10000	3
5	4-6	G = 3333, F = 3333, W = 3334	W = 10000	2

Figure 2: Results from a Barabási-Albert network of $N = 10^4$

5 Results

This network results in games that deviate very quickly from Alexander and Skyrms' paper. In the paper, Alexander and Skyrms build a network of $N = 10^4$ so that is held constant as well as the strategy adoption model of imitating the most successful neighbor. The Barabási-Albert model, in addition to node size requires an intitial condition, l, describing the number of nodes at time $t = 0$ which also defines the number of links made by a new node. To begin , $l = 1$ is chosen to model a social network in which every new node makes one "new friend" or "new neighbor". These results show that the only situation in which the model results in all-Fair is in under the polymorphism 1-9 when m=5. The results otherwise seem to be rather stochastic. Each permutation of the values of m, two of the polymorphisms used in the paper, and starting conditions which favored greedy and weak as well as nearly even distributions was tested three times in teh model. Of course, there is room to test other values of m, other polymorphisms of the game, and other starting conditions but it was thought to be sufficient for purposes of argument to show these. Further analysis would most likely show that the stark difference between these results and Alexander and Skyrms' is due to the degree distribution of the network not being constant. In a model using imitation adoption strategies, low degree nodes see hubs scoring significantly higher using a strategy that may be suboptimal for them and adopt that strategy despite the fact that hubs see large payoffs due simply to their large degree.

6 Conclusion

In contrast to the findings made by Alexander and Skyrms, the scale-free network shows no consistent basin of attraction attached to cooperative strategies, no stability, and no robustness. This is crucial to understanding evolutionary behavior both in animals and people. There exists a very good reason that large profit-making corporations still exist, certain tweets go viral, and there still exist territorial disputes within animals and it would be naïve to assume that these will fade out in time as evolution draws us all to fairness. If anything, this imitation model has shown that Greedy and Weak strategies are exacerbated in evolutionary dynamics. Of course, it must be considered that this model uses imitation for strategy adoption and modern economic theory suggests that perhaps Rational Choice Theory is a more accurate adoption strategy for humans [5] and that remains to be put under rigorous testing in this game. For the scope of this study, however, the semantics of results do remain relevant to real world situations. In situations like the Brain Worm, or the Vampire Bat, the dissemination and dissipation of cooperative strategies is certainly seen. It would be my position that these communities, however, likely show initial conditions closer to those in the 1-9 polymorphism where $m = 5$. This would mean a tighly knit social network in which the the average path length between two individuals is relatively short and the benfits of cooperation greatly outweight the Weak Strategy and the Greedy Strategy is largely imbalanced.

Perhaps Alexander and Skyrms were, however, on the right track. For none other than ethical reasons, a Rational Choice Adoption Model which results in nearly global fairness would be a much more hopeful conclusion. Perhaos the very fact that that is my hope says more than this model ever did. Ultimately, the question now becomes - how might we be able to enable cooperation in social networks? What would that look like?

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