

```

1  import random
2  from tqdm.notebook import tqdm
3  from HW2.utils import distributionBin
4  from scipy.sparse import csr_matrix, triu, tril
5  import scipy.sparse
6  import numpy as np
7
8  """
9  HyperNetworkModel.py
10
11  Written by Sagar Kumar, April 2023 for Problem 3 in Homework 2 in the course NETS6116.
12
13  All references to "HER" stand for Hypercanonical Erdos-Renyi Model as described in the
14  homework and all references to
15  "HSCM" refer to the Hypersoft Configuration Model, as described in the homework.
16  """
17  class HyperNetworkModel:
18
19      def __init__(self,
20                  n,
21                  kmean,
22                  gamma):
23          self.n = n
24          self.kmean = kmean
25          self.gamma = gamma
26
27          self.ensemble = list()
28
29          """
30          HyperNetworkModel is a class which holds the percolating functions for both the HER and
31          HSCM models and
32          supporting functions.
33
34          :param n: [Int] Number of nodes
35          :param kmean: [Float] scale parameter (m in the numpy documentation)
36          :param gamma: [Float] shape (tail) parameter (a+1 in the numpy documentation)
37          :param ensemble: [Tuple] List of adjacency matrices which sample the model
38          """
39
40      def paretoDistribution(self,
41                          samples):
42
43          """
44          paretoDistribution() creates a list of n samples from a Pareto Distribution, as described in
45          HW2 for NETS 6116.
46
47          This is done using Numpy's random.pareto, which takes provides a Lomax distribution. Adding
48          one and multiplying
49
50          by m=kmax, with a=gamma - 1, we obtain the Pareto Distribution described in the homework.

```

```

46
47     
$$p(\bar{k}, \gamma) = (\gamma - 1) (x_m)^{\gamma - 1} x^{-\gamma}$$
 where
48     
$$x_m = \frac{(\gamma - 2) \bar{k}}{\gamma - 1}$$

49
50     :param samples: number of samples to take
51
52     :return: [1 x N Array] samples
53     """
54
55     s = (np.random.pareto(self.gamma-1, samples) + 1) * self.kmean
56
57
58     return s
59
60
61     def HSCM(self):
62
63         """
64         HSCM() percolates a hypersoft configuration graph model by taking in the three parameters
65         below which correspond
66         to the necessary statistics for generation, and outputs the adjacency matrix as a scipy sparse
67         matrix.
68         The latter two variables are fed into paretoDistribution() to sample the hidden variables
69         which determine
70         a node's connection probabilities.
71
72         :return: [N x N Array]
73         """
74
75         # sampling n values from the distribution for the n nodes
76         distribution = self.paretoDistribution(samples=self.n)
77
78         row = list()
79         column = list()
80         data = list()
81
82         # iterating over each edge
83         for i in range(self.n):
84             xi = distribution[i]
85             for j in range(i+1, self.n):
86                 xj = distribution[j]
87                 pij = (1 + (self.kmean*self.n)/(xi*xj))**-1 # connection probability in HSCM
88
89                 r = random.random() # coin flip
90
91                 if r <= pij:
92                     row.append(i)
93                     column.append(j)
94                     data.append(1)
95                 else:

```

```

92         pass
93
94     # placing the values in a SciPy Sparse Matrix
95     M = csr_matrix((data, (row, column)), shape=(self.n, self.n))
96
97     # Only upper diagonal has been filled out, so we return a symmetrized version
98     return triu(M) + tril(M.T)
99
100
101 def HER(self):
102     """
103     HER() percolates the Hypercanonical ER model as described in HW2.
104
105     :return:
106     """
107
108     kappa = self.paretoDistribution(samples=1) # sampling the Pareto
109     p = min(1, kappa/self.n)
110
111     row = list()
112     column = list()
113     data = list()
114
115     # creating an NxN matrix filled with values (0,1)
116     rand_m = scipy.sparse.random(self.n, self.n, density=1, format='csr')
117
118     # Converting that matrix to a boolean matrix where values correspond to whether or not the
119 value in the
120     # element is less than the probability sampled from the pareto distribution
121     M = (rand_m <= p)
122
123     # Graph is undirected, so lower triangle is discarded and upper triangle is reflected over the
124 main diagonal
125     return triu(M) + tril(M.T)
126
127 def create_ensemble(self,
128                     model,
129                     num_graphs):
130     """
131     create_ensemble() samples to create [num_graphs] sample graphs, appending them to self.
132     ensemble
133
134     :param model: [String] either "HER" or "HSCM"
135     :param num_graphs: [Int] number of sample graphs
136     :return: [N X N x num_graphs Array] ensemble of graphs
137     """
138
139     if model == "HSCM":

```

```

138     for _ in tqdm(range(num_graphs), desc="Generating HSCM Ensemble: "):
139         m = self.HSCM()
140         self.ensemble.append(m)
141
142     elif model == "HER":
143         for _ in tqdm(range(num_graphs), desc="Generating HER Ensemble: "):
144             m = self.HER()
145             self.ensemble.append(m)
146     else:
147         raise ValueError("Graph model must be either HSCM or HER")
148
149     def degree_distribution(self,
150                             binning,
151                             num_bins,
152                             graph=None):
153
154         """
155         degree_distribution() calculates the degree distribution of the
156
157         :param binning: [String] either "log" or "linear"
158         :param num_bins: [Int] number of bins for the graph
159         :param graph: [Optional, Array] If graph is not none, this is run over the ensemble attribute
160         of the object. If
161         graph is provided, it is run for that graph.
162         :return: Tuple[Array, Array] list of bin-midpoint x values, and a list of probability density
163         values
164         """
165         degrees = list()
166
167         if graph is not None:
168             G = [graph]
169         else:
170             G = self.ensemble
171
172         for g in G:
173             # compressing ensemble by concatenating all degrees of all graphs into one degree
174             sequence
175             degrees.extend(g.sum(axis=0).tolist()[0])
176
177         if binning == "log":
178             dist_x, dist_y = distributionBin(degrees, num_bins)
179
180         elif binning == "linear":
181             bins = np.linspace(min(degrees), max(degrees), num_bins)
182             dist_x = (bins[1:] + bins[:-1])/2
183             dist_y, _ = np.histogram(degrees, bins=bins)

```

```
184     else:
185         raise ValueError("Data binning must be either log or linear.")
186
187     return dist_x, dist_y
188
```