Classical Physics

Module A: Classical Mechanics

Module B: Classical Electrodynamics

Exam. type	Module - A	Module - B
Two Combined Quizzes	5%	5%
Midterm Examination	20%	20%
End term Examination	25%	25%
Total	50%	50%

Mathematical Tools	5 lectures
Electrostatics	3 lectures
Special techniques	3 lectures
Concepts of Multipole Expansion	2 lectures
Electric Field in Materials	3 lectures
Magnetostatics	2 lectures
Magnetic Field in Materials	2 lectures
Electrodynamics	1 lectures
Maxwell's Equation	1 lectures

Reference Books

1. **Introduction to Electrodynamics by David. J. Griffiths**

2. **Classical Electrodynamics by John David Jackson**

3. **Electricity and Magnetism by Edward M. Purcell**

Mathematical Tools

1. Vector Calculus

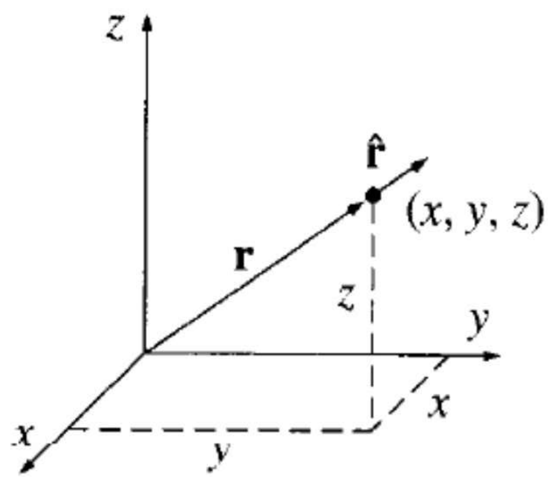
2. Cylindrical Co-ordinate System

3. Spherical Co-ordinate System

4. Delta Function

Vector Calculus

Position Vector, Separation Vector and Infinitesimal Displacement vector



$$\mathbf{r} \equiv x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}.$$

$$r = \sqrt{x^2 + y^2 + z^2},$$

$$\hat{\mathbf{r}} = \frac{\mathbf{r}}{r} = \frac{x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}}{\sqrt{x^2 + y^2 + z^2}}$$

$$\boldsymbol{\mathscr{r}} \equiv \mathbf{r} - \mathbf{r}'$$

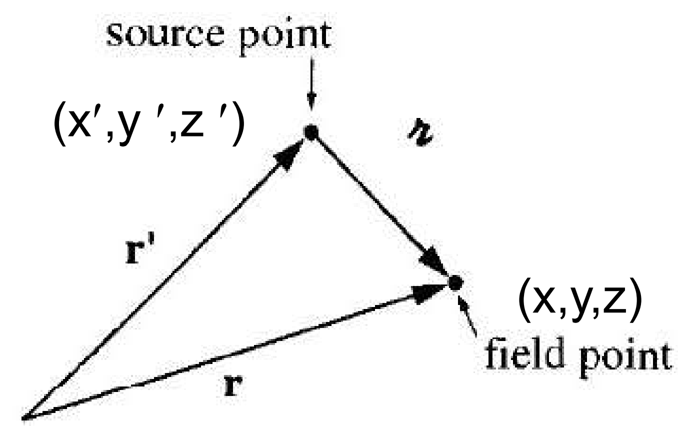
$$\mathscr{r} = |\mathbf{r} - \mathbf{r}'|$$

$$\hat{\boldsymbol{\mathscr{r}}} = \frac{\boldsymbol{\mathscr{r}}}{\mathscr{r}} = \frac{\mathbf{r} - \mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|}$$

$$\boldsymbol{\mathscr{r}} = (x - x')\hat{\mathbf{x}} + (y - y')\hat{\mathbf{y}} + (z - z')\hat{\mathbf{z}},$$

$$\mathscr{r} = \sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2},$$

$$\hat{\boldsymbol{\mathscr{r}}} = \frac{(x - x')\hat{\mathbf{x}} + (y - y')\hat{\mathbf{y}} + (z - z')\hat{\mathbf{z}}}{\sqrt{(x - x')^2 + (y - y')^2 + (z - z')^2}}$$



infinitesimal displacement vector

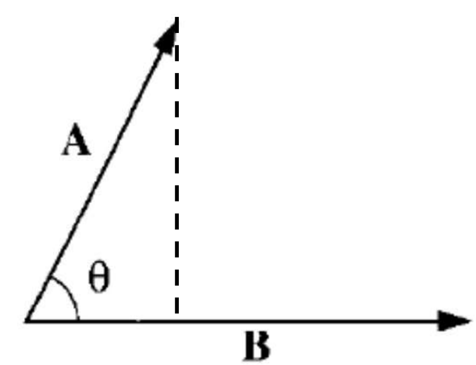
$$(x, y, z) \text{ to } (x + dx, y + dy, z + dz),$$

$$d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$$

Dot product of two vectors

$$\mathbf{A} \cdot \mathbf{B} \equiv AB\cos\theta$$

$$\vec{A} \bullet \widehat{B} = \left|\vec{A}\right| \cos\theta$$



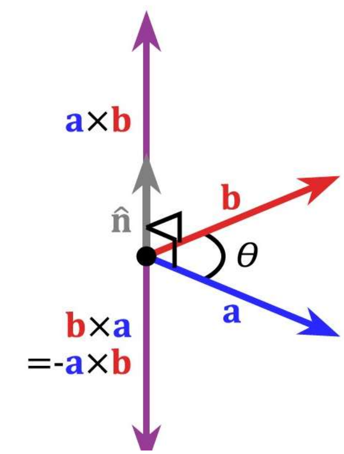
$$\begin{aligned} \mathbf{A} \cdot \mathbf{B} &= (A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}) \cdot (B_x\hat{\mathbf{x}} + B_y\hat{\mathbf{y}} + B_z\hat{\mathbf{z}}) \\ &= A_x B_x + A_y B_y + A_z B_z. \end{aligned}$$

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$$

$$\hat{\mathbf{x}} \cdot \hat{\mathbf{x}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{z}} \cdot \hat{\mathbf{z}} = 1; \quad \hat{\mathbf{x}} \cdot \hat{\mathbf{y}} = \hat{\mathbf{x}} \cdot \hat{\mathbf{z}} = \hat{\mathbf{y}} \cdot \hat{\mathbf{z}} = 0$$

Cross product of two vectors

$$\mathbf{A} \times \mathbf{B} \equiv AB \sin\theta \, \hat{\mathbf{n}}$$

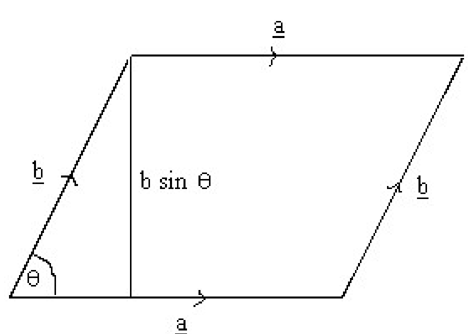


$$\mathbf{A} \times \mathbf{B} = (A_x \hat{\mathbf{x}} + A_y \hat{\mathbf{y}} + A_z \hat{\mathbf{z}}) \times (B_x \hat{\mathbf{x}} + B_y \hat{\mathbf{y}} + B_z \hat{\mathbf{z}})$$

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_y B_z - A_z B_y)\hat{\mathbf{x}} + (A_z B_x - A_x B_z)\hat{\mathbf{y}} + (A_x B_y - A_y B_x)\hat{\mathbf{z}}$$

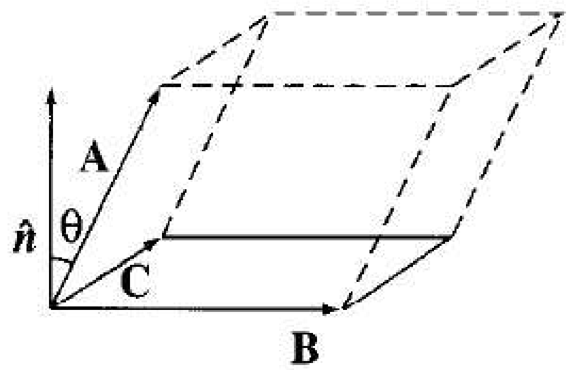
$$(\mathbf{B} \times \mathbf{A}) = -(\mathbf{A} \times \mathbf{B})$$



$$\begin{aligned} \hat{\mathbf{x}} \times \hat{\mathbf{x}} = \hat{\mathbf{y}} \times \hat{\mathbf{y}} &= \hat{\mathbf{z}} \times \hat{\mathbf{z}} = 0, \\ \hat{\mathbf{x}} \times \hat{\mathbf{y}} = -\hat{\mathbf{y}} \times \hat{\mathbf{x}} &= \hat{\mathbf{z}}, \\ \hat{\mathbf{y}} \times \hat{\mathbf{z}} = -\hat{\mathbf{z}} \times \hat{\mathbf{y}} &= \hat{\mathbf{x}}, \\ \hat{\mathbf{z}} \times \hat{\mathbf{x}} = -\hat{\mathbf{x}} \times \hat{\mathbf{z}} &= \hat{\mathbf{y}}. \end{aligned}$$

Triple Products

Scalar triple product: $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$



$|\mathbf{B} \times \mathbf{C}|$ is the area of the base

$|\mathbf{A} \cos\theta|$ is the altitude

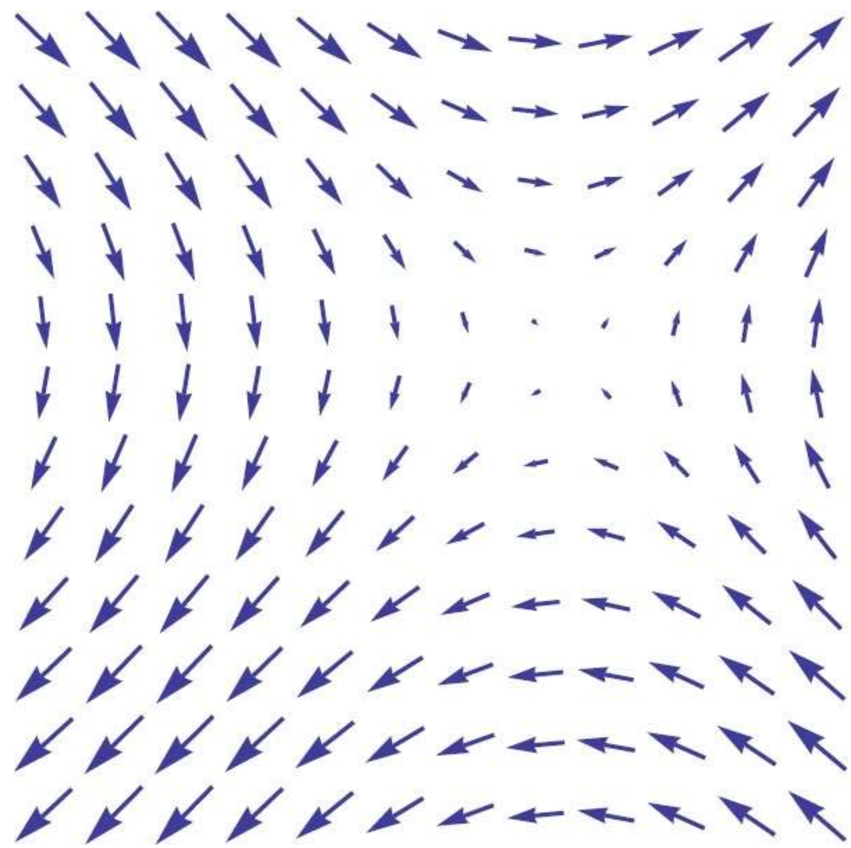
$$\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = \mathbf{B} \cdot (\mathbf{C} \times \mathbf{A}) = \mathbf{C} \cdot (\mathbf{A} \times \mathbf{B})$$

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

Vector calculus

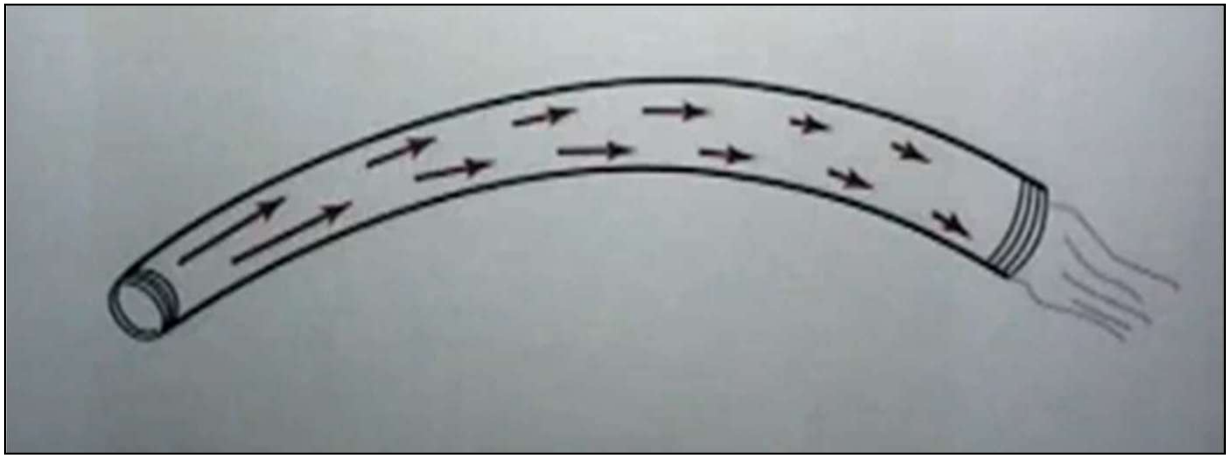
Scalar Function / Scalar Field

Vector Function / Vector Field



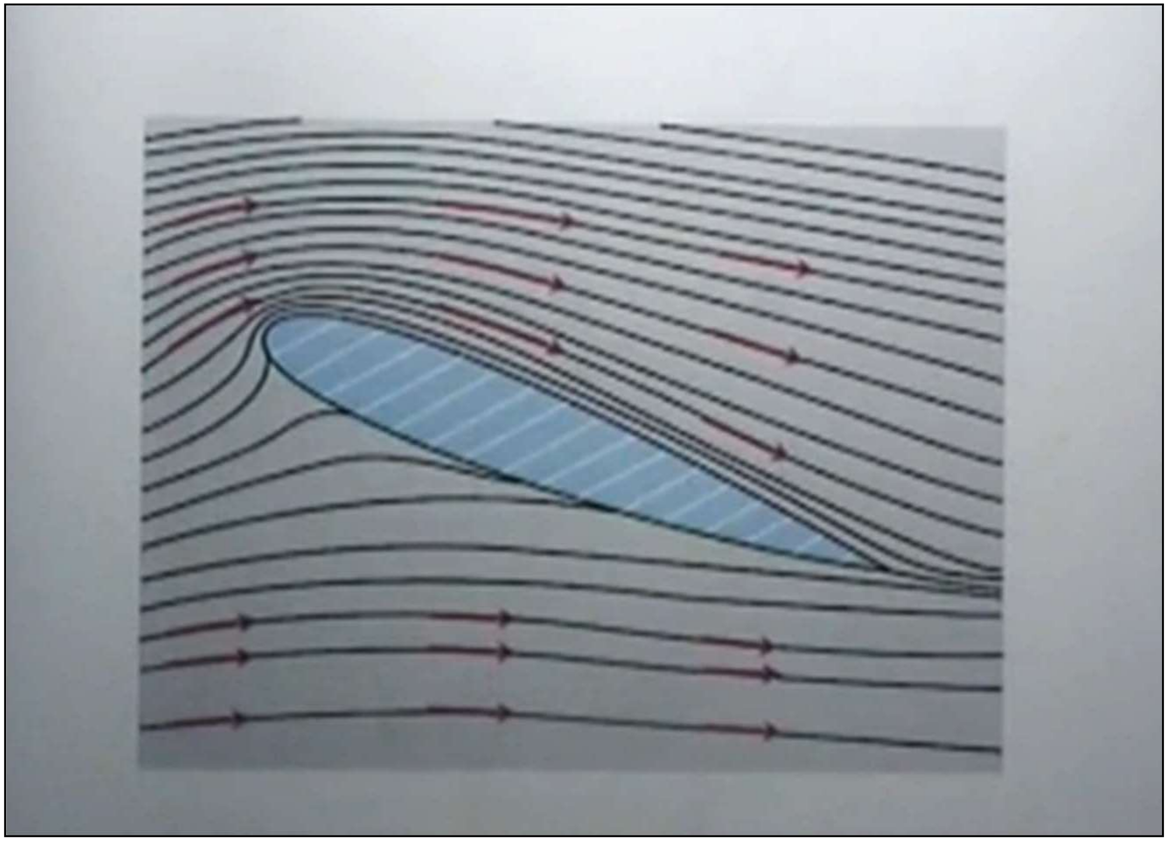
$$\vec{F} = \sin y\hat{i} + \sin x\hat{j}$$

http://en.wikipedia.org/wiki/Vector_field

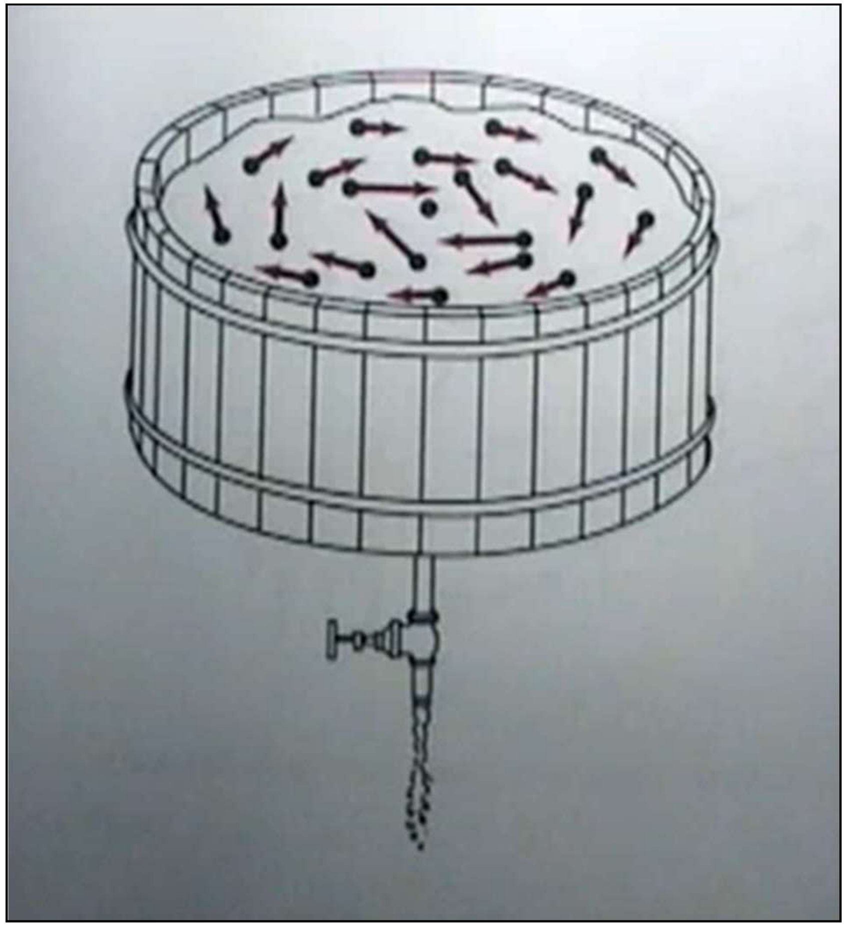


A vector field describing the velocity of a flow in a pipe

Note: All the figures related of vector field has been taken from a lecture series given by Dr. Chris Tisdell, UNSW

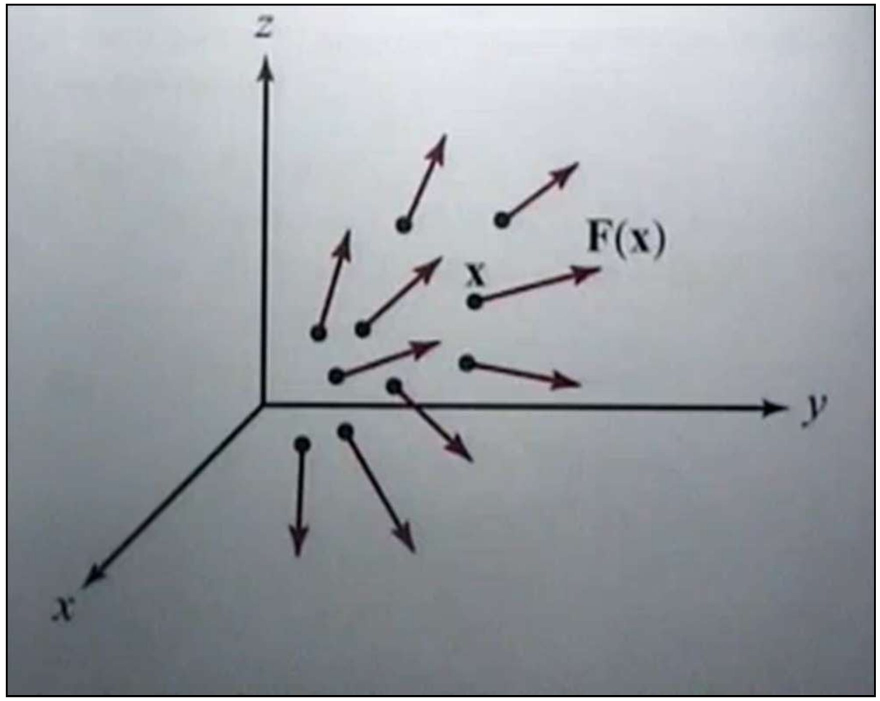


Velocity vector field of a flow around a aircraft wing



Circular flow in a tub

Vector Field or Vector function



$$\vec{F}(x,y,z) = F_1(x,y,z)\hat{i} + F_2(x,y,z)\hat{j} + F_3(x,y,z)\hat{k}$$

$$\vec{F}(x,y,z) = xyz\hat{i} - x^2 z^4 \hat{j} + x\hat{k}$$

Sketching of Vector Function/Field

$$\vec{V} = x\hat{i}$$

$$\vec{V} = x^2\hat{i}$$

$$\vec{V} = -y\hat{i} + x\hat{j}$$

$$\vec{\boldsymbol{V}} = (\boldsymbol{x} + \boldsymbol{y})\hat{\boldsymbol{\imath}} + (\boldsymbol{x} - \boldsymbol{y})\hat{\boldsymbol{\jmath}}$$

Gradient Operator (∇)

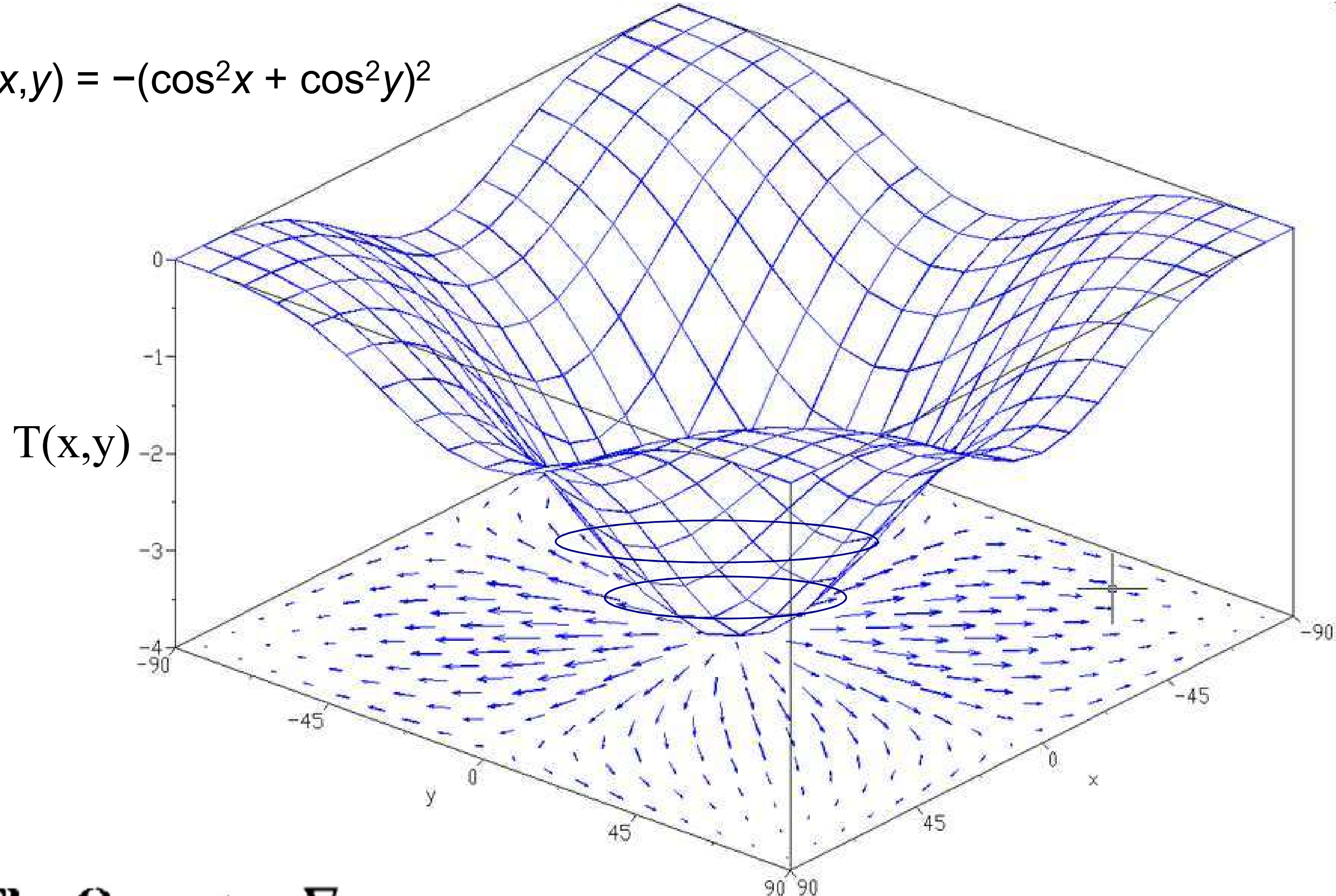
In Cartesian Coordinate

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}\right)\hat{i} + \left(\frac{\partial}{\partial y}\right)\hat{j} + \left(\frac{\partial}{\partial z}\right)\hat{k}$$

Find out gradient of the following function

$$T(x,y) = -(\cos^2 x + \cos^2 y)^2$$

$T(x,y) = -(\cos^2 x + \cos^2 y)^2$



The Operator ∇

$$\boxed{\nabla = \hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}.}$$

http://en.wikipedia.org/wiki/Gradient

$$\nabla T = \underbrace{\left(\hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}\right)}_{\text{Vector Field}} T$$

Scalar Field

T=T(x,y,z)

$$dT = \left(\frac{\partial T}{\partial x}\right) dx + \left(\frac{\partial T}{\partial y}\right) dy + \left(\frac{\partial T}{\partial z}\right) dz.$$

$$\begin{aligned} dT &= \left(\frac{\partial T}{\partial x}\hat{\mathbf{x}} + \frac{\partial T}{\partial y}\hat{\mathbf{y}} + \frac{\partial T}{\partial z}\hat{\mathbf{z}}\right) \cdot (dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}) \\ &= (\nabla T) \cdot (d\mathbf{l}), \end{aligned}$$

$$\nabla T \equiv \frac{\partial T}{\partial x}\hat{\mathbf{x}} + \frac{\partial T}{\partial y}\hat{\mathbf{y}} + \frac{\partial T}{\partial z}\hat{\mathbf{z}}$$

Geometrical Interpretation of the Gradient.

$$dT = \nabla T \cdot d\mathbf{l} = |\nabla T||d\mathbf{l}| \cos\theta$$

when $\theta = 0$ (for then $\cos\theta = 1$)

The gradient ∇T points in the direction of maximum increase of the function T.

The magnitude $|\nabla T|$ gives the slope (rate of increase) along this maximal direction.

The Divergence

$$\nabla \cdot \mathbf{v} = \left(\hat{\mathbf{x}} \frac{\partial}{\partial x} + \hat{\mathbf{y}} \frac{\partial}{\partial y} + \hat{\mathbf{z}} \frac{\partial}{\partial z} \right) \cdot (v_x \hat{\mathbf{x}} + v_y \hat{\mathbf{y}} + v_z \hat{\mathbf{z}})$$

$$= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

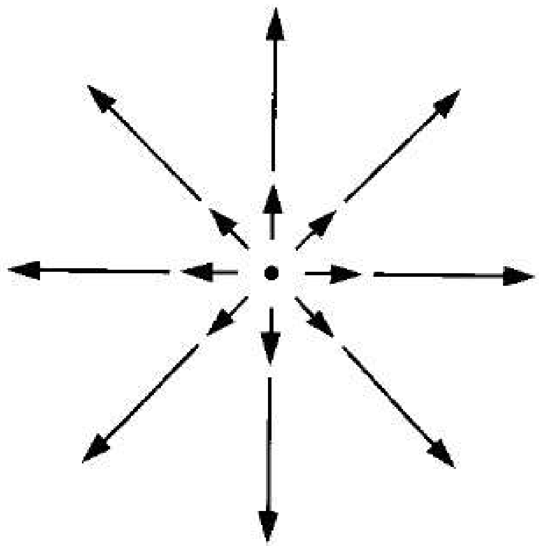
$$\nabla \cdot \vec{V} \neq \vec{V} \cdot \nabla$$

(a) $\mathbf{v}_a = x^2 \, \hat{\mathbf{x}} + 3xz^2 \, \hat{\mathbf{y}} - 2xz \, \hat{\mathbf{z}}$.

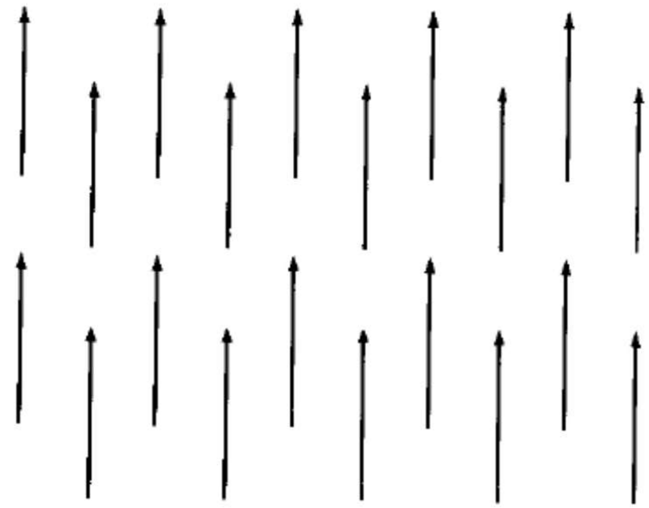
(b) $\mathbf{v}_b = xy \, \hat{\mathbf{x}} + 2yz \, \hat{\mathbf{y}} + 3zx \, \hat{\mathbf{z}}$.

(c) $\mathbf{v}_c = y^2 \, \hat{\mathbf{x}} + (2xy + z^2) \, \hat{\mathbf{y}} + 2yz \, \hat{\mathbf{z}}$.

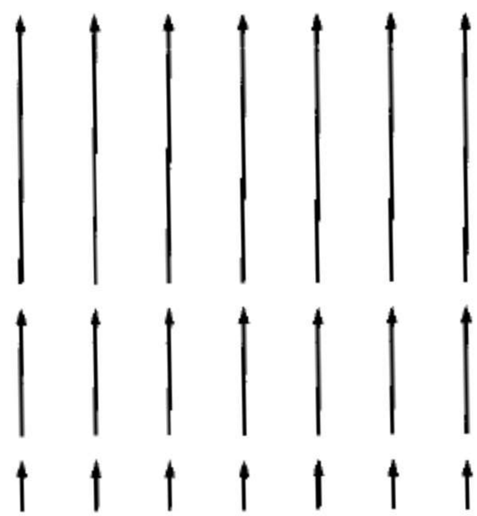
$$\mathbf{v}_a = \mathbf{r} = x\,\hat{\mathbf{x}} + y\,\hat{\mathbf{y}} + z\,\hat{\mathbf{z}}$$



$$\mathbf{v}_b = \hat{\mathbf{z}}$$

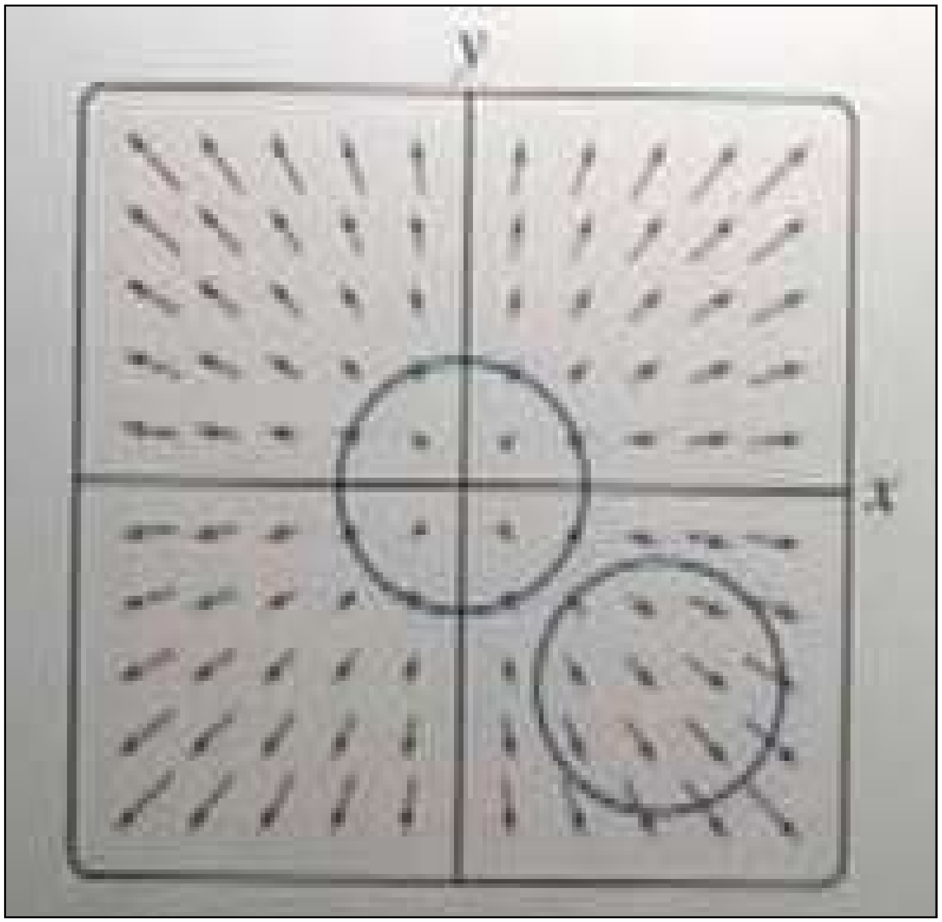


$$\mathbf{v}_c = z\,\hat{\mathbf{z}}$$



In many cases, the divergence of a vector function at point P may be predicted by considering a closed surface surrounding P and analyzing the flow over the boundary, keeping in mind that at P:

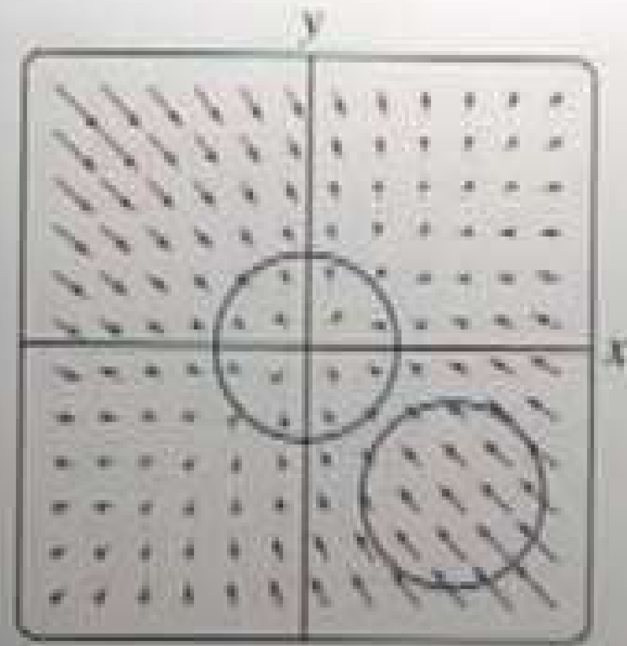
$$\nabla \cdot \vec{F} = \text{outflow} - \text{inflow}$$



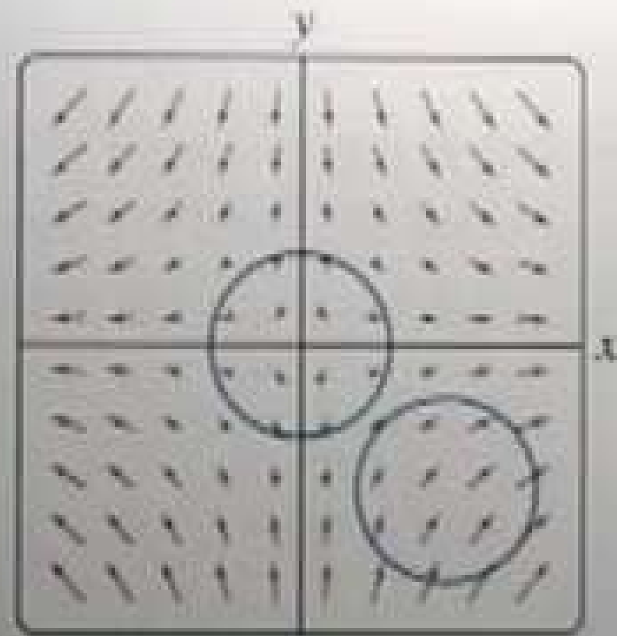
$$\vec{V} = x\hat{i} + y\hat{j}$$

$$\vec{\nabla} \cdot \vec{V} = 2$$

In 3-D, divergence is a measure of change of flux per unit volume



(B) The force field $\mathbf{F} = \langle y - 2x, x - 2y \rangle$ with $\operatorname{div}(\mathbf{F}) = -4$. There is a net inflow into every circle.

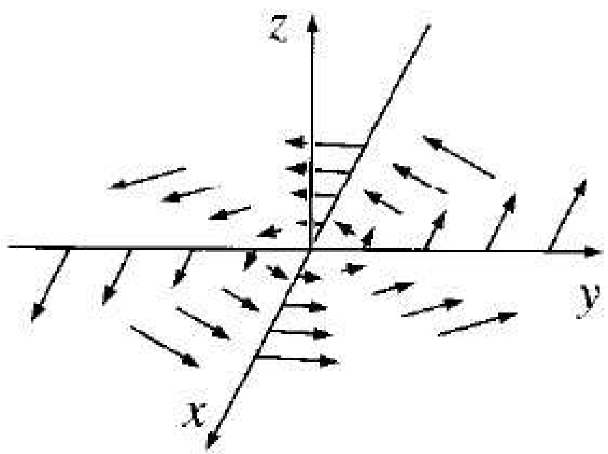


(C) The force field $\mathbf{F} = \langle x, -y \rangle$ with $\mathrm{div}(\mathbf{F}) = 0$. The flux through every circle is zero.

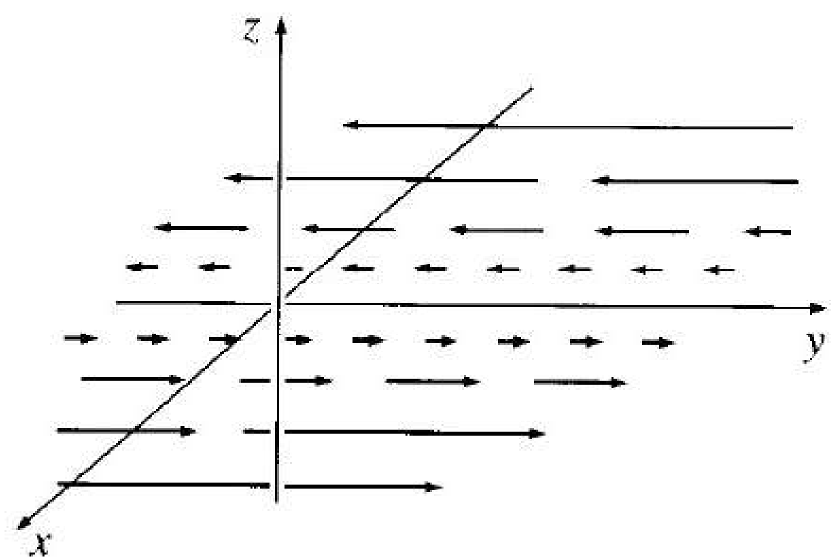
The Curl

$$\nabla \times \mathbf{v} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ v_x & v_y & v_z \end{vmatrix}$$

$$= \hat{\mathbf{x}}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)$$

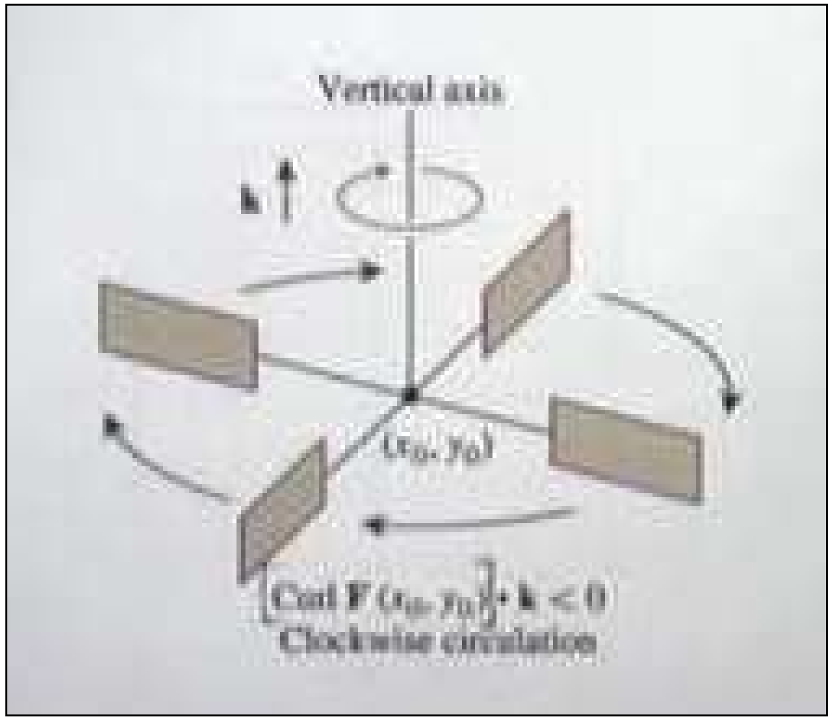


$\mathbf{v}_a = -y\hat{\mathbf{x}} + x\hat{\mathbf{y}}$



$\mathbf{v}_b = x\hat{\mathbf{y}}$

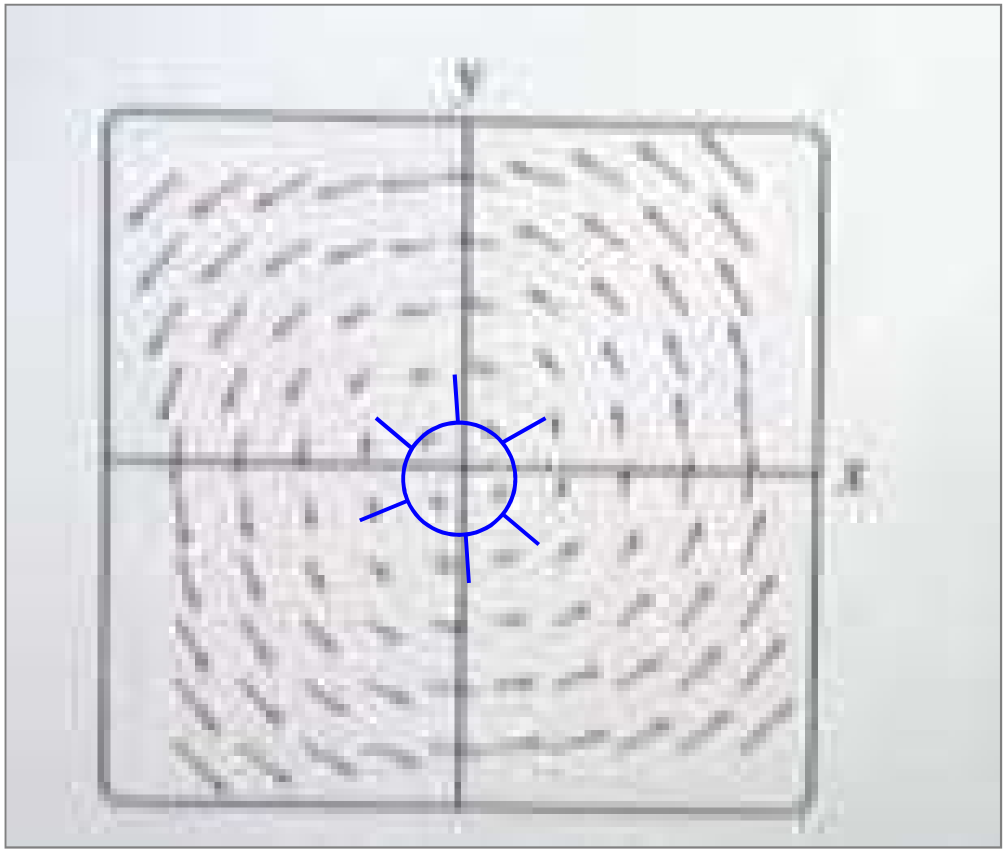
Paddle wheel analysis



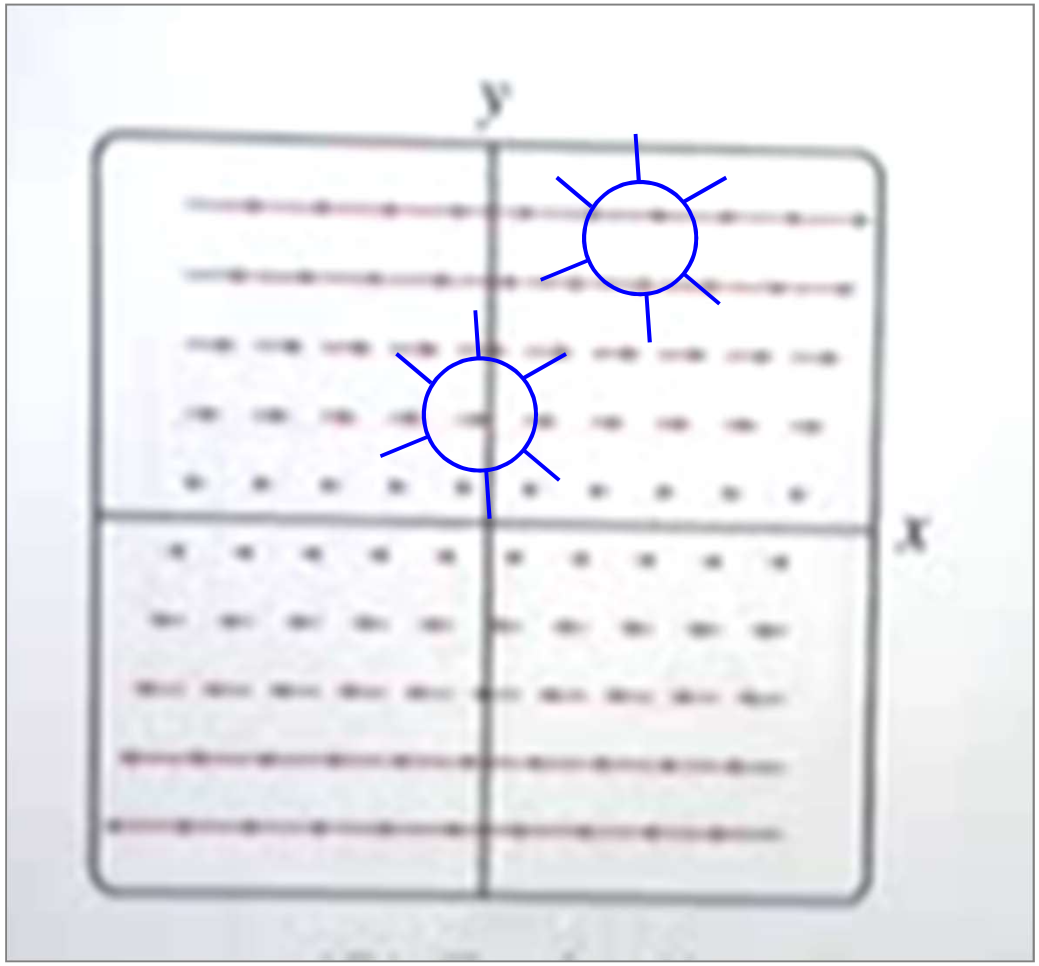
$$[\nabla \times \vec{\boldsymbol{F}}(\boldsymbol{x}_0, \boldsymbol{y}_0)] \cdot \hat{\boldsymbol{k}} < 0$$

$$\vec{F}(x,y) = -y\hat{i} + x\hat{j}$$

$$(\vec{\nabla} \times \vec{F}) \cdot \hat{k} = 2$$



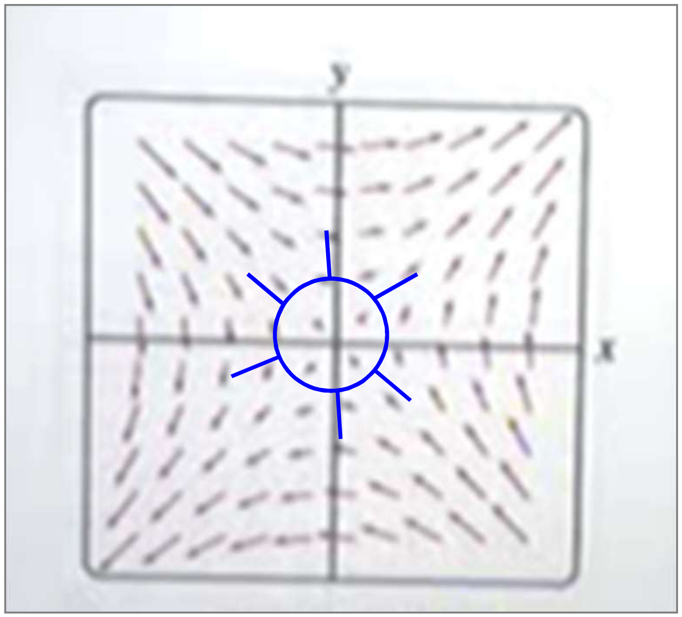
$$\vec{F}(x, y) = y\hat{i}$$

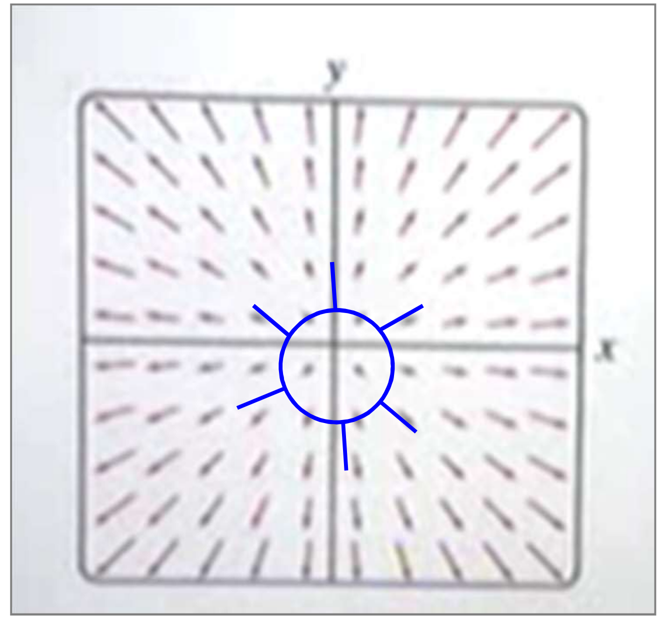


$$(\nabla \times \vec{F}) \cdot \hat{k} = -1$$

$$\vec{\boldsymbol{F}}(\boldsymbol{x},\boldsymbol{y}) = \boldsymbol{y}\hat{\boldsymbol{i}} + \boldsymbol{x}\hat{\boldsymbol{j}}$$

$$\vec{F}(x,y) = x\hat{i} + y\hat{j}$$





$$\nabla \times \vec{\boldsymbol{F}} = 0$$

Summary

Gradient of a Scalar Field $\longrightarrow$ **Vector Field**

$$\underset{\text{Scalar Field}}{\nabla T} = \underbrace{\left(\hat{\mathbf{x}}\frac{\partial}{\partial x} + \hat{\mathbf{y}}\frac{\partial}{\partial y} + \hat{\mathbf{z}}\frac{\partial}{\partial z}\right)}_{\text{Vector Field}} T$$

Divergence of a Vector Field $\longrightarrow$ **Measure of Change of flux per unit volume**

Curl of a Vector Field $\longrightarrow$ **Measure of degree of rotation of the field**