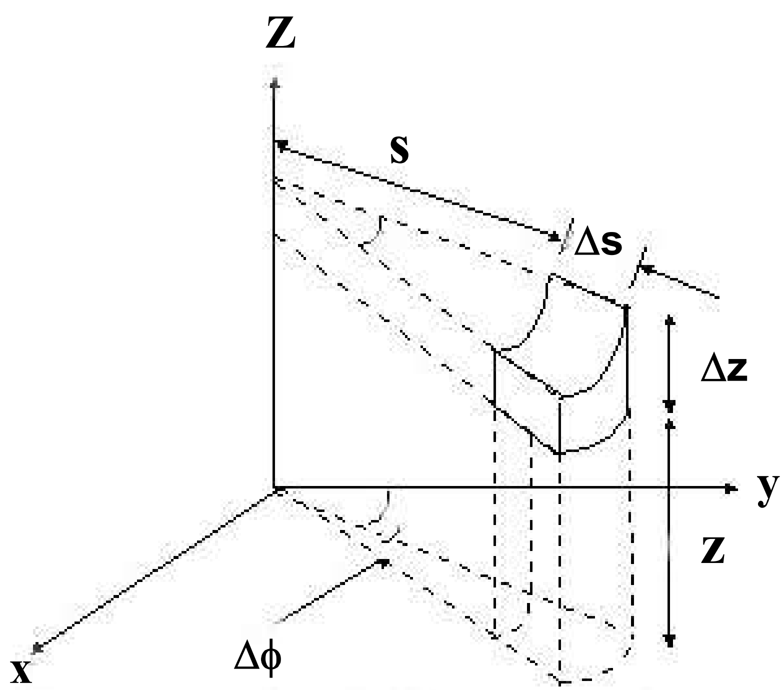
Infinitesimal vector

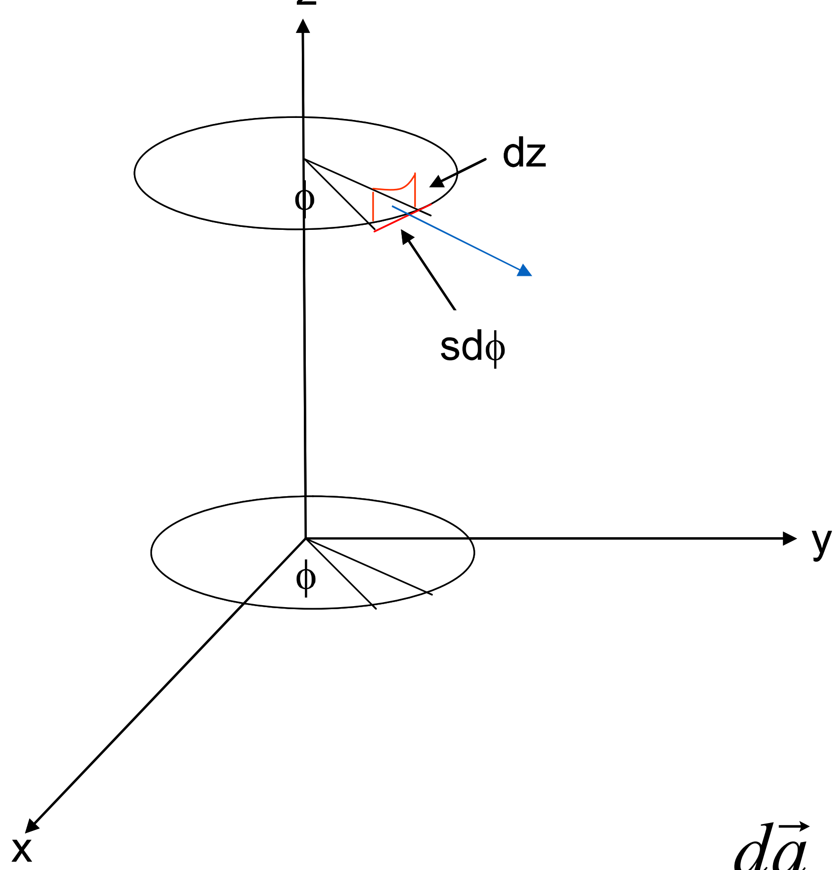
$$dl_s = ds, \qquad dl_\phi = s\,d\phi, \qquad dl_z = dz$$

$$d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}$$



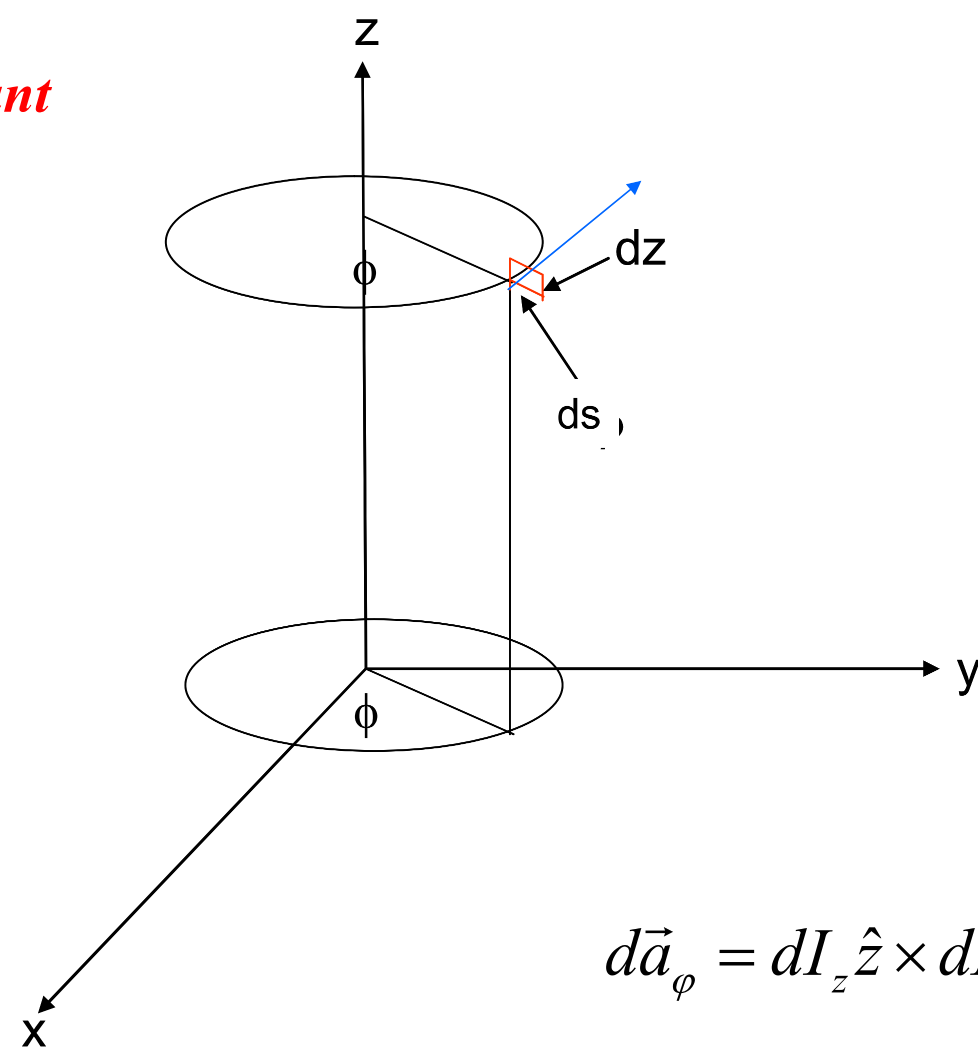
Differential Surface Elements

When s is constant



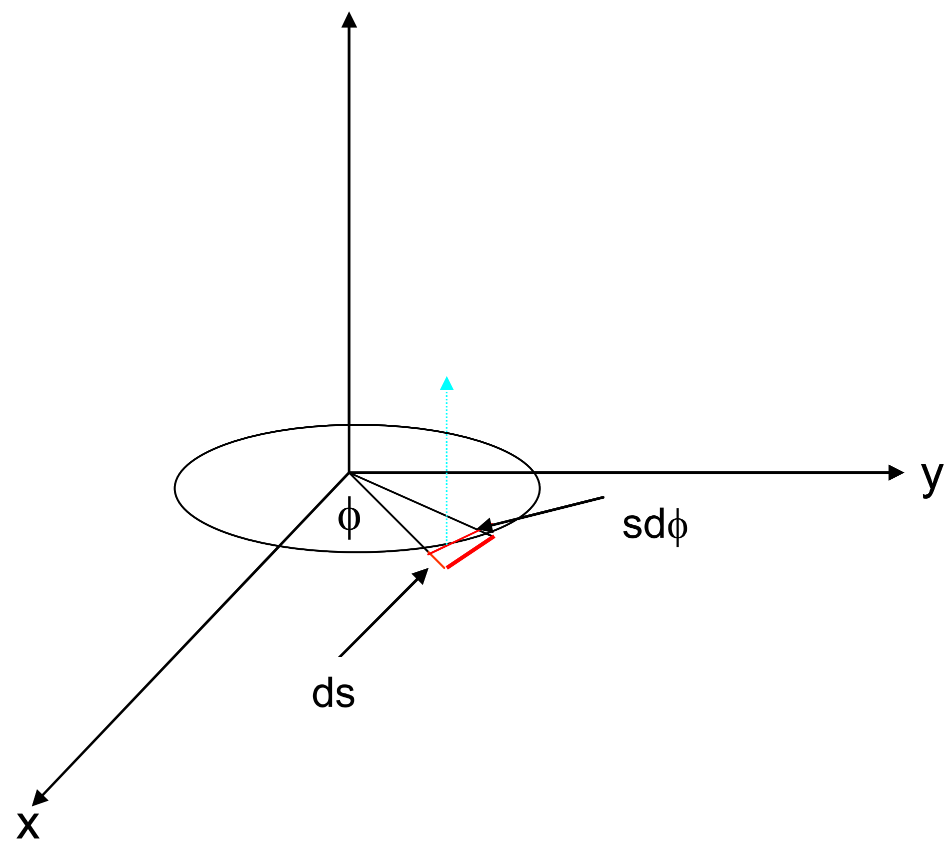
$$d\vec{a}_s = dI_z\hat{z} \times dI_\phi\hat{\phi} = s\,d\phi\,dz\,\hat{s}$$

When ϕ is constant



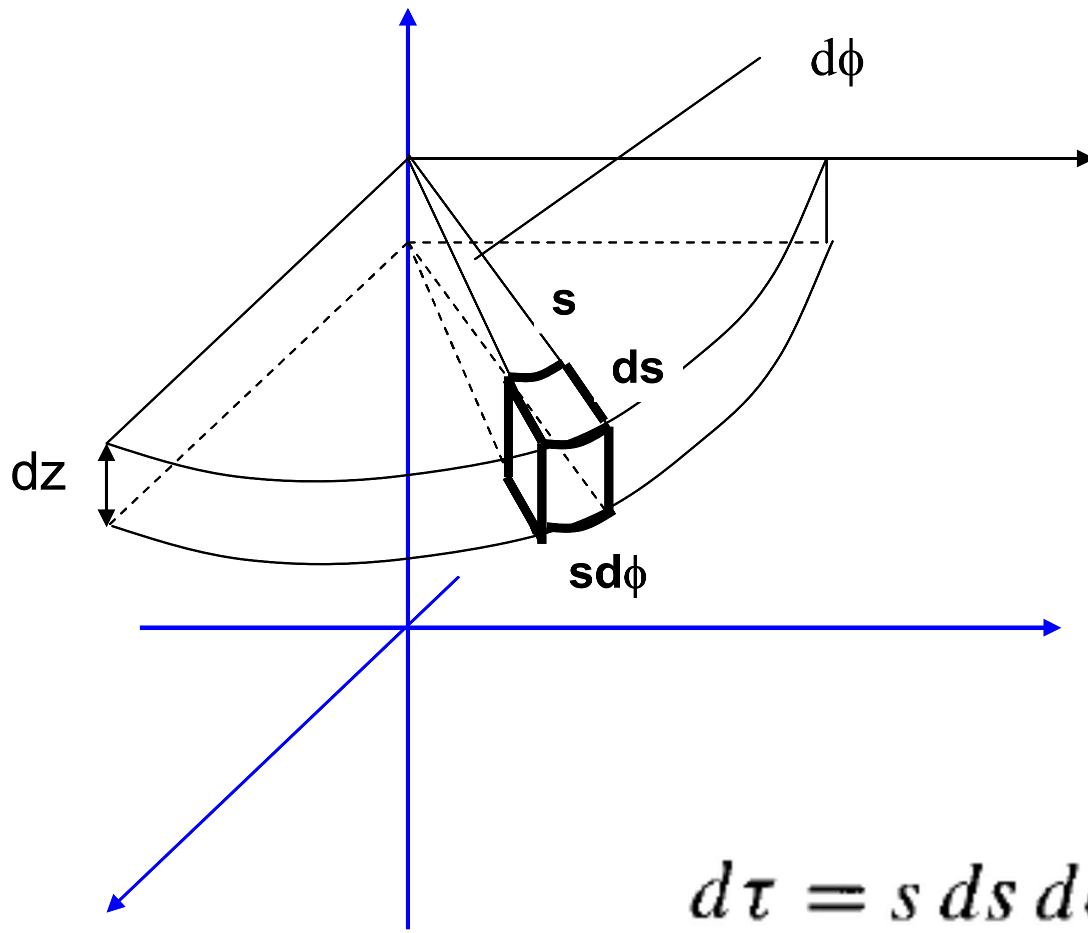
$$d\vec{a}_{\varphi} = dI_z \hat{z} \times dI_s s = dz ds \hat{\varphi}$$

When z is constant



$$d\vec{a}_z = dI_s s \times dI_\phi \hat{\phi} = s\,d\phi\,ds\,\hat{z}$$

Volume element in cylindrical coordinate system



$$d\tau = s\, ds\, d\phi\, dz$$

Example

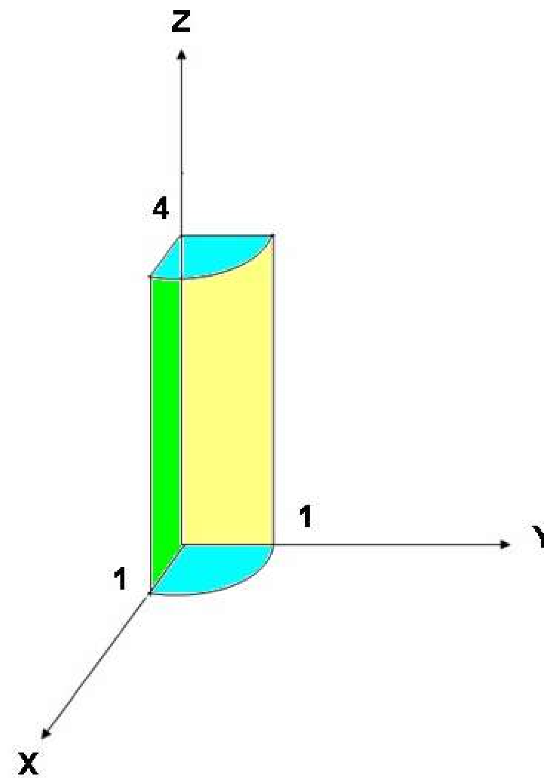
$$(i) \quad \vec{V} = \hat{s} + \hat{\varphi} + z\hat{z}$$

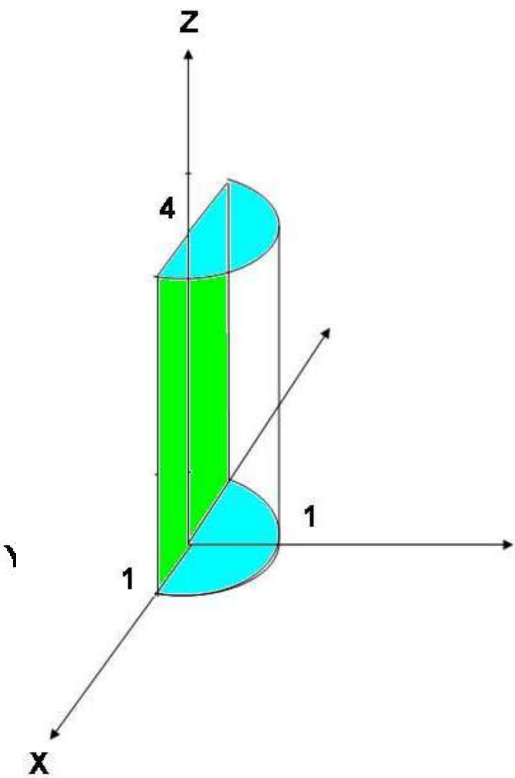
$$(ii) \quad \vec{V} = s\hat{s} + \sin\varphi\hat{\varphi} + z\hat{z}$$

$$d\vec{a}_s = dI_z\hat{z} \times dI_\phi\hat{\phi} = sd\phi dz\hat{s}$$

$$d\vec{a}_\varphi = dI_z\hat{z} \times dI_s s = dzds\hat{\varphi}$$

$$d\vec{a}_z = dI_s s \times dI_\phi\hat{\phi} = sd\phi ds\hat{z}$$





Gradient, Curl, Divergence and Laplacian operator in Cylindrical Coordinate

$$\nabla T = \frac{\partial T}{\partial s}\,\hat{\mathbf{s}} + \frac{1}{s}\frac{\partial T}{\partial \phi}\,\hat{\boldsymbol{\phi}} + \frac{\partial T}{\partial z}\,\hat{\mathbf{z}}.$$

$$\nabla \cdot \mathbf{v} = \frac{1}{s}\frac{\partial}{\partial s}(s v_s) + \frac{1}{s}\frac{\partial v_\phi}{\partial \phi} + \frac{\partial v_z}{\partial z}.$$

$$\nabla \times \mathbf{v} = \left(\frac{1}{s}\frac{\partial v_z}{\partial \phi} - \frac{\partial v_\phi}{\partial z}\right)\hat{\mathbf{s}} + \left(\frac{\partial v_s}{\partial z} - \frac{\partial v_z}{\partial s}\right)\hat{\boldsymbol{\phi}} + \frac{1}{s}\left[\frac{\partial}{\partial s}(s v_\phi) - \frac{\partial v_s}{\partial \phi}\right]\hat{\mathbf{z}}$$

$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2}.$$

s is ρ (*the coordinate of Cylindrical Coordinate*)

$$\mathbf{A}(\mathbf{r}) = \mathbf{A}(\rho, \phi, z) = \hat{\boldsymbol{\rho}} A_\rho(\rho, \phi, z) + \hat{\boldsymbol{\phi}} A_\phi(\rho, \phi, z) + \hat{\mathbf{k}} A_z(\rho, \phi, z)$$

$$\nabla \cdot \mathbf{A} = \left(\hat{\boldsymbol{\rho}} \frac{\partial}{\partial \rho} + \frac{\hat{\boldsymbol{\phi}}}{\rho} \frac{\partial}{\partial \phi} + \hat{\mathbf{k}} \frac{\partial}{\partial z}\right) \cdot (\hat{\boldsymbol{\rho}} A_\rho + \hat{\boldsymbol{\phi}} A_\phi + \hat{\mathbf{k}} A_z)$$

$$= \left[\hat{\boldsymbol{\rho}} \cdot \frac{\partial}{\partial \rho}(\hat{\boldsymbol{\rho}} A_\rho + \hat{\boldsymbol{\phi}} A_\phi + \hat{\mathbf{k}} A_z)\right] + \left[\frac{\hat{\boldsymbol{\phi}}}{\rho} \cdot \frac{\partial}{\partial \phi}(\hat{\boldsymbol{\rho}} A_\rho + \hat{\boldsymbol{\phi}} A_\phi + \hat{\mathbf{k}} A_z)\right]$$

$$+ \left[\hat{\mathbf{k}} \cdot \frac{\partial}{\partial z}(\hat{\boldsymbol{\rho}} A_\rho + \hat{\boldsymbol{\phi}} A_\phi + \hat{\mathbf{k}} A_z)\right]$$

$$= \left[\frac{\partial A_\rho}{\partial \rho}\right] + \left[\frac{\hat{\boldsymbol{\phi}}}{\rho} \cdot \left(\frac{\partial \hat{\boldsymbol{\rho}}}{\partial \phi} A_\rho + \hat{\boldsymbol{\rho}} \frac{\partial A_\rho}{\partial \phi} + \frac{\partial \hat{\boldsymbol{\phi}}}{\partial \phi} A_\phi + \hat{\boldsymbol{\phi}} \frac{\partial A_\phi}{\partial \phi}\right.\right.$$

$$\left.\left. + \hat{\mathbf{k}} \frac{\partial A_z}{\partial \phi}\right)\right] + \left[\frac{\partial A_z}{\partial z}\right]$$

$$= \frac{\partial A_\rho}{\partial \rho} + \frac{\hat{\boldsymbol{\phi}}}{\rho} \cdot \left(\hat{\boldsymbol{\phi}} A_\rho + \hat{\boldsymbol{\rho}} \frac{\partial A_\rho}{\partial \phi} - \hat{\boldsymbol{\rho}} A_\phi + \hat{\boldsymbol{\phi}} \frac{\partial A_\phi}{\partial \phi} + \hat{\mathbf{k}} \frac{\partial A_z}{\partial \phi}\right) + \frac{\partial A_z}{\partial z}$$

$$= \frac{\partial A_\rho}{\partial \rho} + \frac{1}{\rho}\left(A_\rho + \frac{\partial A_\phi}{\partial \phi}\right) + \frac{\partial A_z}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{\rho} \frac{\partial}{\partial \rho}(\rho A_\rho) + \frac{1}{\rho} \frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$