The LNM Institute of Information Technology
Jaipur, Rajasthan

MATH-I ■ Assignment #6

(Riemann Integration)

Q1. Let $f : [0,1] \to \mathbb{R}$ be defined as $f(x) = \begin{cases} 1 & \text{if} \quad 0 \leq x < 1 \\ 10^9 & \text{if} \quad x = 1 \end{cases}$. Show that f is integrable on $[0,1]$ and that $\displaystyle\int_0^1 f = 1$.

Q2. Define $f : [-1,1] \to \mathbb{R}$ by

$$f(x) = \begin{cases} -1, & -1 \leq x \leq 0 \\ 1, & 0 < x \leq 1. \end{cases}$$

Is the function continuous on $[-1,1]$? Is the function Riemann integrable?

Q3. Does there exist a continuous function f on $[0,1]$ such that

$$\int_0^1 x^n f(x) dx = \frac{1}{\sqrt{n}} \quad \text{for all} \quad n \in \mathbb{N}.$$

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Q4. For each $n \in \mathbb{N}$, let $g_n : [0,1] \to \mathbb{R}$ be defined as $g_n(x) := \begin{cases} \frac{(n+1)x^n}{1+x}, & \text{if } 0 \leq x < 1 \\ 0, & x = 1. \end{cases}$.

Then prove that $\displaystyle\lim_{n\to\infty} \int_0^1 g_n(x)dx = \frac{1}{2}$ whereas $\displaystyle\int_0^1 \lim_{n\to\infty} g_n(x)dx = 0.$

Q5. Let $f : [-1,1] \to \mathbb{R}$ such that

$$f(x) = \begin{cases} a, & \text{if } -1 \leq x < 0 \\ 0, & \text{if } x = 0 \\ b, & 0 < x \leq 1 \end{cases}$$

For each $\epsilon > 0$, find a partition P of $[-1,1]$ such that $U(P,f) - L(P,f) < \epsilon$.

Q6. Consider $\displaystyle a_n := \sum_{i=1}^{n} \frac{1}{\sqrt{n^2 + in}}$ for $n \in \mathbb{N}$. Find $\displaystyle\lim_{n\to\infty} a_n$.

Q7. Find the intervals in which the function $f(x) = 2x^3 + 2x^2 - 2x - 1$ is convex, concave, increasing, decreasing. Also find local maxima, local minima and point of inflection.