12/11
# Magnetostatics

$$\boxed{\vec{\nabla}\cdot\vec{J} + \frac{\partial \rho}{\partial t} = 0} \Rightarrow \text{continuity equation.}$$

where, $\vec{J}$ = volume current density

$\rho$ = volume charge density.

$$\vec{\nabla}\cdot\vec{J} = -\frac{\partial \rho}{\partial t}$$

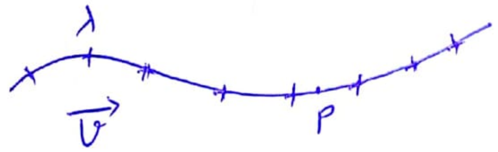
$$\boxed{\vec{\nabla}\cdot\vec{J} = 0}$$ ⇒ no change in volume charge density at that point.

↳ ⇒ steady current.

Different types of current

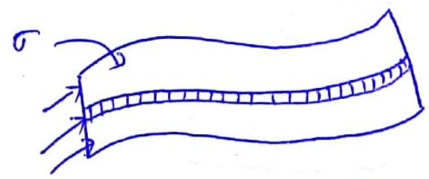
where $v$ = velocity

(1) Line current :



$$I = \frac{dQ}{dt} = \frac{\lambda(v\,\Delta t)}{\Delta t} = \lambda v$$

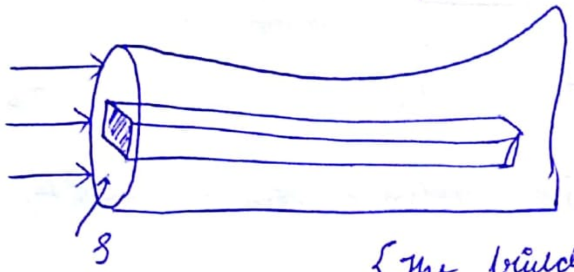
(2) Surface current : Current passing per unit length perpendicular to the direction of current.



$$K = \frac{dI}{dl_\perp} = \frac{\sigma(\Delta t\, v)}{\Delta t} = \sigma v$$

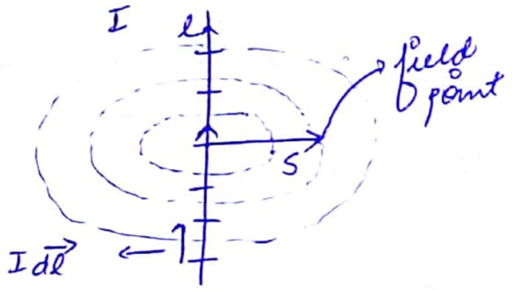
Volume current density:

$$\vec{J} = \frac{dI}{da_\perp} = \frac{\rho(\Delta t)v}{\Delta t} = \rho v$$




{The building block of magnetism is a current carrying loop not a current element becoz a loop can produce steady current but element cannot}

# Biot - Savart law.

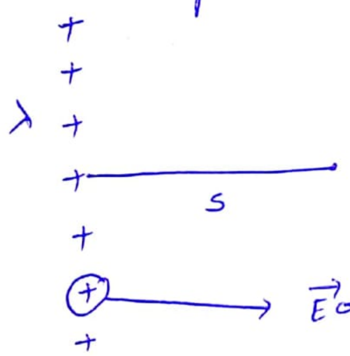



$$|\vec{B}| \propto I$$

$$|\vec{B}| \propto \frac{1}{S}$$

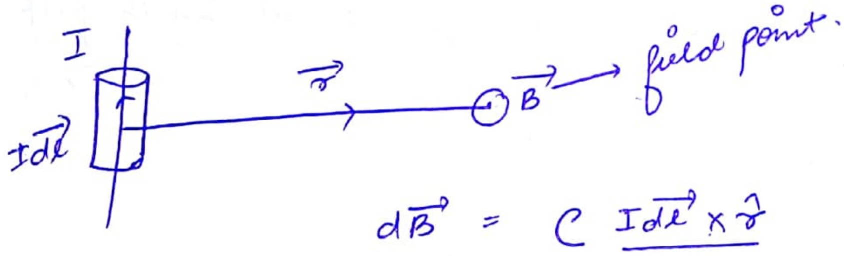
$$\vec{B} \perp \text{plane } \vec{S} \,\&\, \vec{l}$$

concept used:




$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 S}\hat{S}$$

$$\vec{E} \propto \frac{dq}{r^2}$$




$$d\vec{B} = C\frac{I\vec{dl} \times \hat{r}}{r^2}$$

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,\vec{dl} \times \hat{r}}{r^2}}$$

$\vec{B}$ due to line current distribution :

$$\vec{B} = \frac{\mu_0}{4\pi} \int \frac{\vec{I} \times \hat{r}}{r^2} \, dl$$

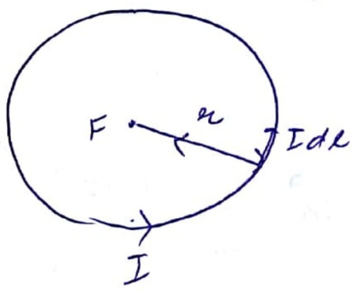
surface current distribution :

$$\vec{B} = \frac{\mu_0}{4\pi} \iint \frac{\vec{K} \times \hat{r}}{r^2} \, da$$

volume- current distribution

$$\vec{B} = \frac{\mu_0}{4\pi} \iiint \frac{\vec{J} \times \hat{r}}{r^2} \, d\tau.$$

eg. $\vec{B}$ at centre ?




$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\,dl \times \hat{r}}{r^2} \quad (\because \theta = 90^\circ)$$

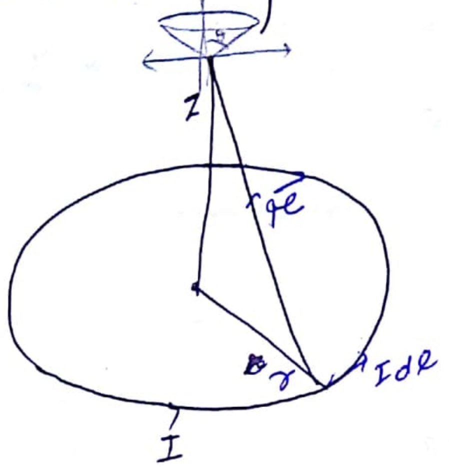
$$\vec{dB} = \frac{\mu_0 I \, dl}{4\pi r^2}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi r^2} \int dl$$

$$= \frac{\mu_0 I}{4\pi r^2} \times 2\pi r$$

$$\vec{B} = \frac{\mu_0 I}{2r} \hat{z}$$

eg. find $\vec{B}$ at a point z.




$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\vec{dl} \times \vec{\mathscr{r}}}{\mathscr{r}^2}$$

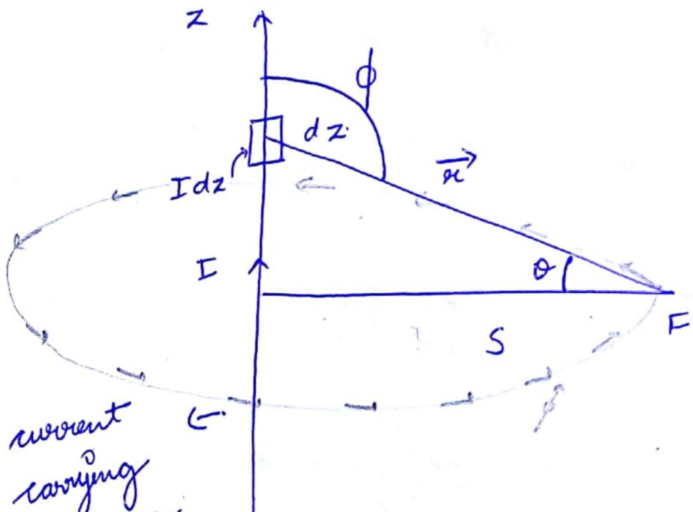
$$= \frac{\mu_0}{4\pi} \frac{I\, dl \cos\theta}{\mathscr{r}^2}$$

$$\vec{dB} = \frac{\mu_0 I\, dl}{4\pi} \frac{r}{(z^2+r^2)^{3/2}}$$

$$\vec{B} = \int dB = \frac{\mu_0 I r}{4\pi (z^2+r^2)^{3/2}} \int dl$$

$$B = \frac{\mu_0 I r^2}{2(z^2+r^2)^{3/2}}.$$

#




$$\vec{dB} = \frac{\mu_0}{4\pi} \frac{I\, dz\, \hat{z} \times \hat{\mathscr{r}}}{\mathscr{r}^2}$$

$$= \frac{\mu_0}{4\pi} \frac{I\, dz \sin\phi}{\mathscr{r}^2}$$

$$= \frac{\mu_0}{4\pi} \frac{I\, dz \cos\theta}{\mathscr{r}^2}$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{2\pi s} \hat{\phi}}$$

Magnetic force :

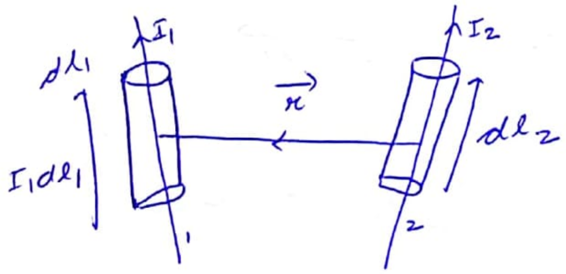
$$\vec{F}_q = q(\vec{v}\times\vec{B})$$
$$= (I\,\vec{dl})\times\vec{B}$$

[like $\vec{F}_E = q\vec{E}$]

$\cancel{\vec{F}_E}$

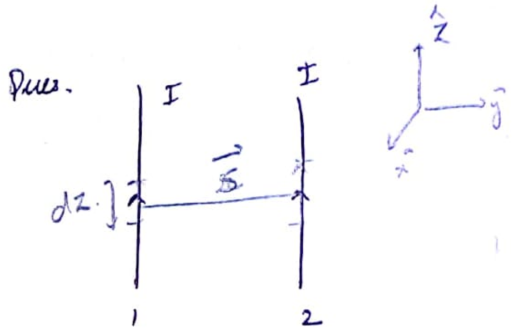
Work done by magnetic force $= \int \vec{F}_q \cdot \vec{dl}$

$$= \int (I\vec{dl}\times\vec{B})\cdot\vec{dl} = 0 \quad (\because \vec{F}\perp\vec{dl})$$




For 1 due to 2 $\leftarrow F_{12} = (I_1\vec{dl_1})\times\frac{\mu_0}{4\pi}\frac{I_2\vec{dl_2}\times\hat{r}}{r^2}$

Ques.




Prove that the force of attraction is acting on 1.

$$F = I\vec{dl}\times\vec{B}$$

$$\vec{B} = \frac{\mu_0 I}{2\pi s}\hat{\phi}$$

$$= I\,dz\,\hat{z}\times\frac{\mu_0 I}{2\pi s}\hat{\phi}$$

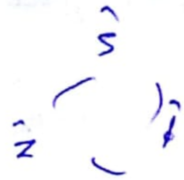
$$= \frac{\mu_0 I^2 dz}{2\pi s}(-\hat{s})$$

$$= -\frac{\mu_0 I^2 dz}{2\pi s}\hat{s}$$

HP.

14/11

$\vec{\nabla}\cdot\vec{J} = 0 \rightarrow$ steady current




$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{I\vec{dl}\times\hat{r}}{r^2}$$

$$\vec{F_m} = I\vec{dl}\times\vec{B}$$

↑ Test current element

$$dE = \frac{1}{4\pi\epsilon_0}\frac{dq\,\hat{r}}{r^2}$$

$$\vec{dB} = \frac{\mu_0}{4\pi}\frac{(I\vec{dl})\times\hat{r}}{r^2}$$

$$\vec{F_e} = q\vec{E} \qquad \vec{F_M} = (I\vec{dl})\times\vec{B}$$

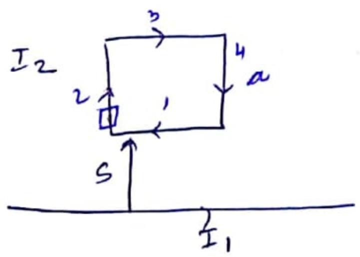
$\rightarrow$ $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$ $\quad \vec{E} = -\vec{\nabla}V.$

$\vec{\nabla}\times\vec{E} = 0$ $\quad \nabla^2 V = -\frac{\rho}{\epsilon_0}$

→ In electrostatics.

Today ⇒ $\vec{\nabla}\cdot\vec{B} = ?$ $\quad \vec{\nabla}\times\vec{B} = ?$ , magnetic potential function = ?

Ques.




$$\vec{B_1} = \frac{\mu_0 I_1}{2\pi s}\hat{\phi}$$

$$\vec{F_1} = I_2 a(-\hat{z})\times\frac{\mu_0 I_1}{2\pi s}\hat{\phi}$$

$\rightarrow \hat{z}$

$$= \frac{I_1 I_2 \mu_0 a}{2\pi s}\hat{s}$$

$$\vec{F_3} = I_2 a(\hat{z})\times\frac{\mu_0 I_1}{2\pi(s+a)}\hat{\phi} = \frac{\mu_0 I_1 I_2 a}{2\pi(s+a)}(-\hat{s})$$

$$dF_2 = I_2 ds\,(\hat{s}) \times \frac{\mu_0 I_1}{2\pi s}(\hat{\phi})$$

$$= \frac{\mu_0 I_1 I_2}{2\pi} \frac{ds}{s}\,\hat{z}$$

$$F_2 = \frac{\mu_0 I_1 I_2}{2\pi} \int_s^{s+a} \frac{ds}{s}\,\hat{z}$$

$$= \frac{\mu_0 I_1 I_2}{2\pi} \ln \frac{s+a}{s}\,\hat{z}$$

→

$\boxed{\vec{\nabla}\cdot\vec{B} \equiv 0}$ always zero. $\boxed{\oiint \vec{B}\cdot d\vec{a} \equiv 0}$

meaning: we cannot seperate the poles of magnet i.e. it is always dipole. but in electro we can seperate +ve & -ve.

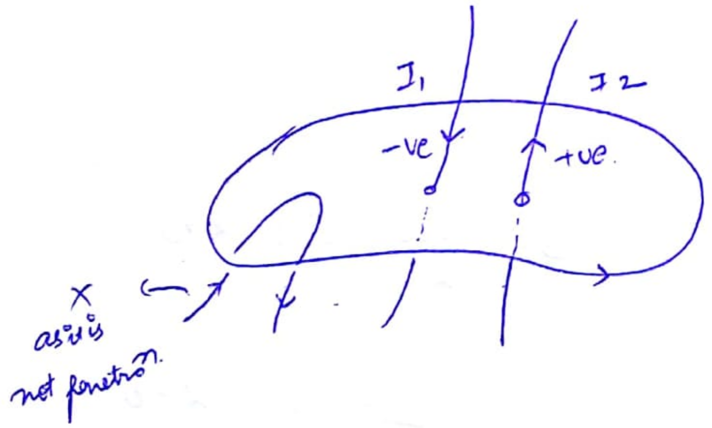
→ Magnetic monopole does not exist.

$\boxed{\vec{\nabla}\times\vec{B} = \mu_0 \vec{J}}$ where $\vec{J}$ = volume current density.

$$\iint (\vec{\nabla}\times\vec{B})\cdot d\vec{a} = \mu_0 \iint \vec{J}\cdot d\vec{a}$$

$\boxed{\oint \vec{B}\cdot d\vec{l} = \mu_0 I}$ Ampere's law.

$I = I_{enclosed}$.
↓
current that penetrates the surface

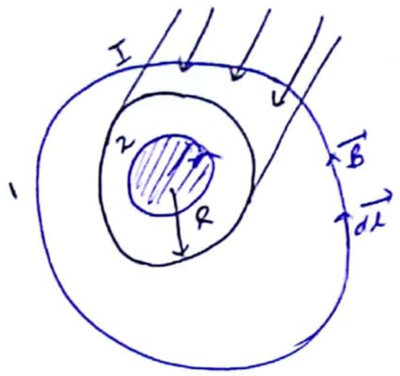



↑+ve

$$\oint \vec{B}\cdot d\vec{l} = \mu_0 (I_2 - I_1)$$

Ques.

magnetic field inside & outside?




$$B_1 \oint d\vec{l} = \mu_0 I$$

$$B_1 (2\pi r) = \mu_0 I$$

$$B_1 = \frac{\mu_0 I}{2\pi r}.$$

choose loop parallel to magnetic field.

$$B_2 \oint dl = \mu_0 \frac{I}{\pi R^2} (\pi r^2)$$

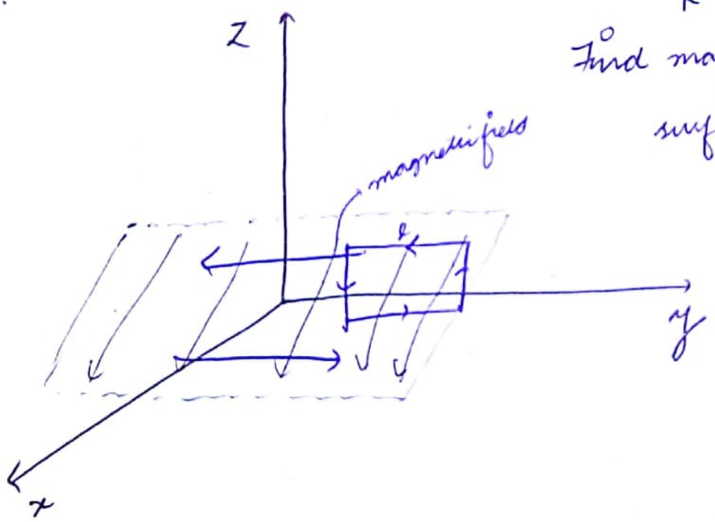
$$B_2 (2\pi r) = \frac{\mu_0 I r^2}{R^2}$$

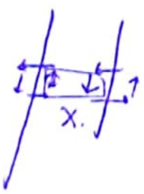
$$B_2 = \frac{\mu_0 I r}{2\pi R^2}$$

Ques.

$$\vec{K} = c\hat{x}$$

Find magnetic field produced by surface current.




$$Bl + Bl = \mu_0 (cl)$$

$$B = \frac{\mu_0 c}{2} (-\hat{y}) \quad z > 0$$

$$\frac{\mu_0 c}{2} (\hat{y}) \quad z < 0$$

$$\vec{B} = \frac{\mu_0 c}{2} (\hat{x} \times \hat{z})$$

# $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ $\quad \vec{\nabla} \cdot \vec{B} = 0$

$$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{F}) \equiv 0.$$

$$\boxed{\vec{B} = \vec{\nabla} \times \vec{A}}$$ $\Rightarrow$ magnetic potential func$^n$.

$$\vec{A} \rightarrow \vec{A} + \vec{\nabla}\psi$$

$$\vec{B} = \vec{\nabla} \times \vec{A} = \vec{\nabla} \times \vec{A} + \vec{\nabla} \times (\cancel{\vec{\nabla}} \cdot \psi)^{0}$$

no changen in magnetic field.

$$\vec{\nabla} \times \vec{A} = \vec{B}$$

$$\vec{\nabla} \cdot \vec{A} = 0$$

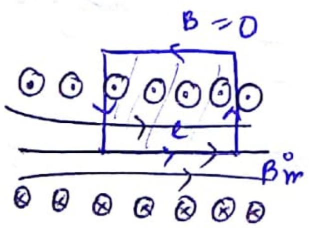
$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \mu_0 \vec{J}$$

$$\vec{\nabla} \underbrace{(\vec{\nabla} \cdot \vec{A})}_{0} - \nabla^2 A = \mu_0 J.$$

$$\boxed{\nabla^2 A = -\mu_0 J}$$

Ques.




$L, N, I$ $\quad$ magnetic field = ?

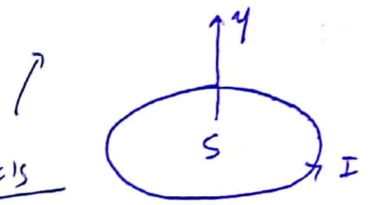
$$B_{in} \times l = \mu_0 \, E\left(\frac{N}{L} \times l \times I\right)$$

$$B_{in} = \frac{\mu_0 N I}{L}$$

10/11.

## Magnetic Dipole moment :

$\vec{\mu} = I\vec{S}$  S= surface area.




CIF's

## Magnetic field in magnetic material :

what we know?

Dielectric

$\boxed{|\vec{E}|_{net} < |\vec{E}_{ext}|}$

$\longrightarrow$ Eext.

Magnetic material

Diamagnetic $\rightarrow |\vec{B}_{net}| < |\vec{B}_{ext}|$

Paramagnetic $\rightarrow |\vec{B}_{net}| > |\vec{B}_{ext}|$

Ferromagnetic $\rightarrow |\vec{B}_{net}| >> |\vec{B}_{ext}|$.

$\longrightarrow$ Bext.

The natural behaviour of all material is diamagnetic.

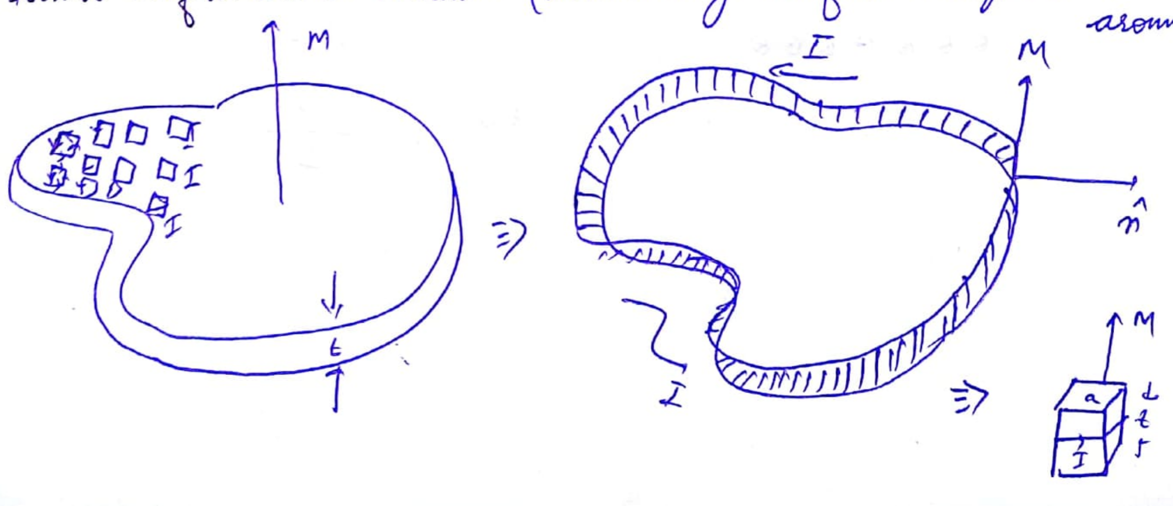
## Magnetisation :

$$\boxed{\vec{M} = \frac{\sum \vec{\mu}_m}{\Delta V}}$$

$\vec{M} \longleftrightarrow \vec{K} \Rightarrow$ surface current.

unit $= \dfrac{q}{l}$.

## Bound Surface ~~current~~ current : (considering uniform magnetisation all around)



$|\vec{\mu_m}| = aI$

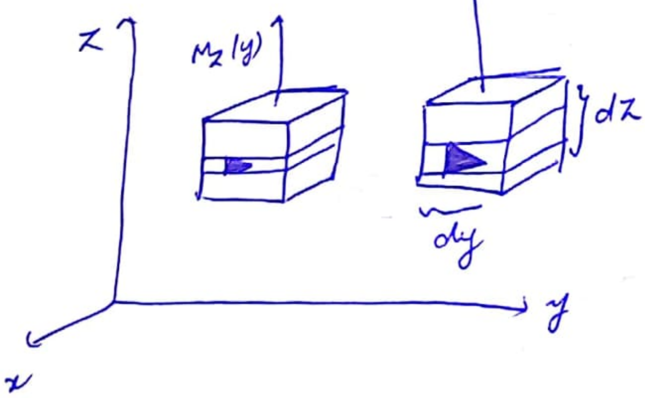
$|\vec{\mu_m}| = Mat$ (M × volume)

$M = \frac{I}{t}$

$|\vec{M}| = |\vec{K}|$

$\boxed{\vec{K} = \vec{M} \times \hat{n}}$ ← Bound surface current density.

→ When magnetisation is non uniform :-




Bound volume current density $(\vec{J_b})$ arises due to non-uniform magnetisation.

$$\boxed{\vec{J_b} = \vec{\nabla} \times \vec{M}}$$

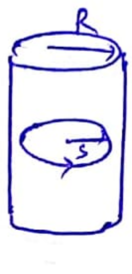
Ques. $\vec{M} = cs^2\hat{\phi}$

Find $\vec{B}$ inside & outside.

The object is magnetised itself.


→ no free current

{How to consider a loop: direction of magnetic field should be in the direction of $\vec{dl}$}




$$\oint \vec{B_{in}} \cdot \vec{dl} = B_{in} \int dl$$

$$= B_{in}\, 2\pi s = \mu_0 I_{enc}.$$

As there is no surface inside we will choose Bound volume current

$$\vec{J}_b = \vec{\nabla} \times \vec{M} = \frac{1}{s}\frac{\partial}{\partial s}\left(sM_\phi\right)\hat{z}$$

$$= \frac{1}{s}\frac{\partial}{\partial s}\left(cs^3\right)\hat{z}$$

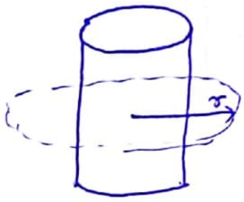
$$\vec{J}_b = 3sc\,\hat{z}$$

$$I_{enc.} = \int_0^{2\pi}\int_0^{s} 3sc\, s\, d\phi\, ds = 2\pi c s^3$$

$$B_{in} \times \cancel{2\pi s} = \mu_0 \times \cancel{2\pi} c s^{\cancel{3}\,2}$$

$$\boxed{B_{in} = \mu_0 c s^2}$$

Bout:→ we will take surface current density.



$$\vec{K}_b = cs^2\hat{\phi} \times \hat{s}\Big|_{s=R}$$

$$= cR^2(-\hat{z})$$

Total surface current $= 2\pi R \times cR^2 \hat{z}$.

$$= 2\pi c R^3(-\hat{z})$$

Total volume current $= 2\pi c R^3(\hat{z})$

$$B_{out}(2\pi R) = 0 \quad (\hat{z} - \hat{z})$$

for magnetic field at $s=R$:

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

$$\frac{\vec{\nabla} \times \vec{B}}{\mu_0} = \vec{J} = \vec{J}_{free} + J_b = \vec{J}_{free} + \vec{\nabla} \times \vec{M}$$

$$\vec{\nabla} \times \left( \frac{\vec{B}}{\mu_0} - \vec{M} \right) = \vec{J}_{free}.$$

$$\left\{ \frac{\vec{B}}{\mu_0} - \vec{M} = \vec{H} \right\} \text{ (called)}$$

[ $\vec{H}$ is like $\vec{M}$ ]

$$\vec{B} = \underbrace{\mu_0 \vec{M}}_{\substack{\text{Magnetic field} \\ \text{due to bound current.}}} + \underbrace{\mu_0 \vec{H}}_{\text{Magnetic field due to free current.}}$$

$$\boxed{\vec{\nabla} \times \vec{H} = \vec{J}_{free}.} \Rightarrow \text{Integral form.}$$

$$\Downarrow$$

$$\boxed{\oint \vec{H} \cdot \vec{dl} = I_{free}.}$$

$H_{in} (2\pi r) = 0$ (no free current)

$H_{in} = 0.$

then $\vec{B}_{in} = \mu_0 \vec{M}$

$= \mu_0 c s^2 \hat{\phi}$

$\vec{B}_{out} = 0.$ (since $\vec{M} = 0$, $\vec{H} = 0$)

17/11 Magnetic field in magnetic materials.

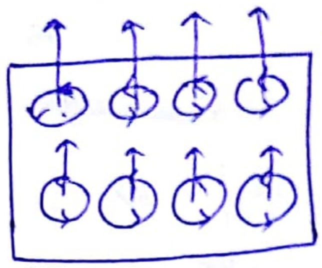
Magnetisation $\leftarrow (\vec{M})$

$|\vec{B}_{net}| < |\vec{B}_{ext}|$

$|\vec{B}_{net}| > |\vec{B}_{ext}|$

$|\vec{B}_{net}| >> |\vec{B}_{ext}|$

$\longrightarrow B_{ext}$



Bound surface current density

$\vec{K} = \vec{M} \times \hat{n}$

Bound volume current density

$\vec{J_b} = \vec{\nabla} \times \vec{M}$

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 I_{enc.}$$

$$\oint \vec{H} \cdot \vec{dl} = I_{free}.$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

$$\vec{B} = \mu_0 \vec{H} + \mu \vec{M}$$

Linear magnetic Material

$$\left[ \vec{B} = \mu_0 \vec{H} + \chi_m \mu_0 \vec{H} = \mu_0 (1 + \chi_m) \vec{H}. \right]$$

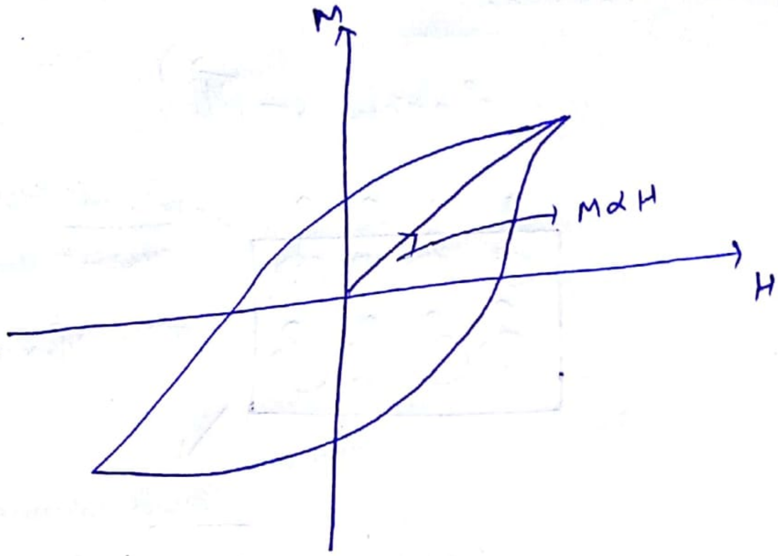
$\vec{M} = \chi_e \vec{H}$

$\hookrightarrow$ magnetic susceptibility

$\chi_e < 0 \sim -10^{-3 \to 5} \to$ diamagnetic

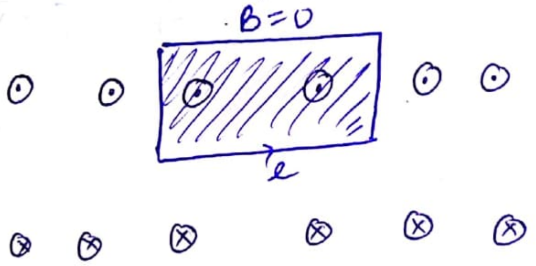
$\chi_e > 0 \sim +10^{-4 \to 5} \to$ Paramagnetic

$\chi_e >> 0 \sim +10^{2 \to 6} \to$ Ferromagnetic.




Ques.

$N, I, L$.

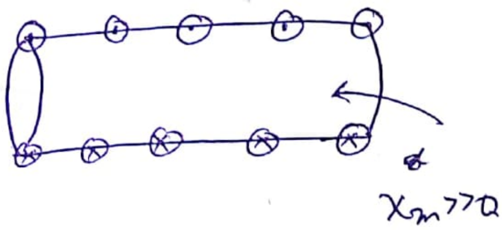



$$\oint \vec{B}\cdot\vec{dl} = \mu_0 I$$

$$Bl = \mu_0 \frac{N}{L} l I$$

$$B = \mu_0 \frac{NI}{L}$$

now.




$$\vec{K_b} = \vec{M} \times \hat{n}$$

$$\vec{J_b} = \vec{\nabla} \times \vec{M}$$

$$\oint \vec{H}\cdot\vec{dl} = I_{free}$$

$$Hl = I \times \frac{N}{L} \times l$$

$$H = \frac{NI}{L}$$

$$\vec{B} = \mu_0(1+\chi_m) H = \mu_0(1+\chi_m)\frac{NI}{L}$$

$$\vec{B} = \underbrace{\left(\frac{\mu_0 N I}{L}\right)}_{\text{free}} + \left(\mu_0 \chi_m \frac{NI}{L}\right) \rightarrow \text{bound.}$$

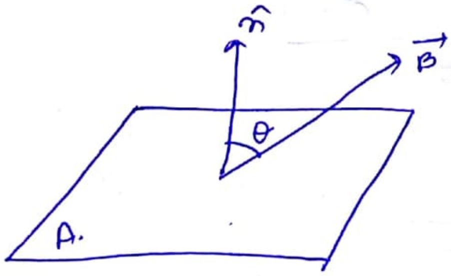
$\chi_m >> 0.$

## Electrodynamics:

$$\mathcal{E} = -\frac{d\phi_B}{dt}$$

$\phi_B \leftarrow$ magnetic flux.

$\mathcal{E}$ ← e.m.f.

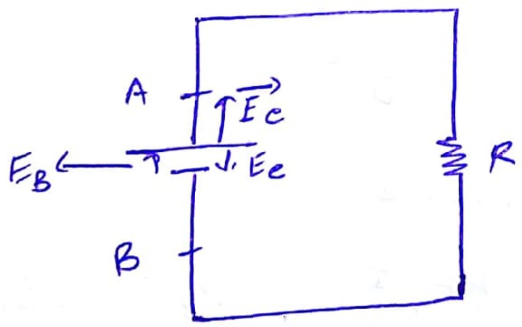



$$\phi_m = \vec{A} \cdot \vec{B}$$
$$= |\vec{A}||\vec{B}|\cos\theta.$$

$$\mathcal{E} = \oint \vec{E} \cdot \vec{dl}$$

$\vec{E}$ ← force / charge.

This electric field is not just electrostatic field.




$$\mathcal{E} = \oint \vec{E} \cdot \vec{dl}$$

$$= \oint (\vec{E_B} + \vec{E_e}) \cdot \vec{dl}$$

$$= \oint \vec{E}_B \cdot \vec{dl} + \underbrace{\oint \vec{E_e} \cdot \vec{dl}}_{\rightarrow 0.}$$

$$= \int_B^A \vec{E}_B \cdot \vec{dl} = -\int_B^A \vec{E_e} \cdot dl$$

$$V(A) - V(B) =$$

$$\mathcal{E} = -\frac{d\phi_m}{dt}.$$

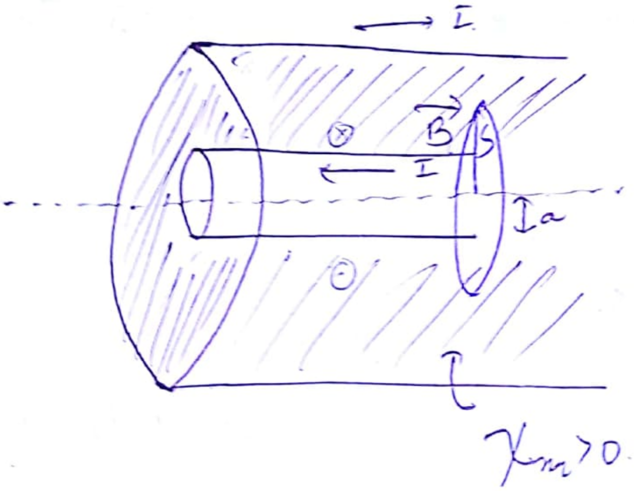
$$\oint \vec{E}\cdot\vec{dl} = -\frac{d}{dt}\iint \vec{B}\cdot\vec{da}$$

$$\iint (\vec{\nabla}\times\vec{E})\cdot\vec{da} = -\iint \frac{dB(t)}{dt}\cdot\vec{da}$$

non-conservation $\vec{E}$

$$\boxed{\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}}$$

Ques.




$\vec{B} = ?$

$$\oint \vec{H}\cdot\vec{dl} = I_{free}$$

$$H(2\pi s) = I$$

$$H = \frac{I}{2\pi s}. \qquad \vec{M} = \frac{\chi_m I}{2\pi s}\hat{\phi}$$

$$\vec{B} = \mu_0(1+\chi_m)H = \mu_0(1+\chi_m)\frac{I}{2\pi s}.$$

$$\vec{J_b} = \vec{\nabla} \times \vec{M}$$

$$\vec{K_b} = \vec{M} \times \hat{n}$$

$$\vec{J_b} = \frac{1}{s}\left[\frac{\partial}{\partial s}(s.M\phi)\right]\hat{z} = 0.$$

$$\vec{K_b} = \frac{\chi_m I}{2\pi s}\hat{\phi} \times (-\hat{s})\Big|_{s=a.}$$

$$= \frac{\chi_m I}{2\pi a}\hat{z}$$

Total Bound surface current $= \frac{\chi_m I}{2\pi a} \times 2\pi a = \chi_m I.$

$$B \cdot (2\pi s) = \mu_0 I + \mu_0 \chi_m I$$

$$B = \frac{\mu_0 I(1+\chi_m)}{2\pi s.}$$

19/11

~~Gauss's law of electrostatics~~

$$\oint\!\!\oint \vec{E} \cdot d\vec{a} = \frac{q_{encl}}{\epsilon_0} \qquad \vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\oint\!\!\oint \vec{B} \cdot d\vec{a} = 0 \qquad \vec{\nabla} \cdot \vec{B} = 0$$

$$\mathcal{E} = \oint \vec{E} \cdot \vec{dl} = -\frac{d}{dt}\iint \vec{B} \cdot d\vec{a}. \qquad \vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 J. \qquad \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

we know that:

$$\bar{\nabla}\cdot(\bar{\nabla}\times\vec{F})=0$$

for the exp: $\bar{\nabla}\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t}.$

$$\bar{\nabla}(LHS) = \bar{\nabla}\cdot(\bar{\nabla}\times E) \equiv 0$$

$$\bar{\nabla}(RHS) = -\frac{\partial}{\partial t}(\cancel{\bar{\nabla}\cdot\vec{B}}^{0}) \equiv 0$$

for the exp: $\bar{\nabla}\times\vec{B} = \mu_0\vec{J}$

$$\bar{\nabla}\cdot(LHS) = \bar{\nabla}\cdot(\bar{\nabla}\times\vec{B}) \equiv 0$$

$$\bar{\nabla}\cdot(RHS) = \mu_0\,\bar{\nabla}\cdot\vec{J}$$

[ Continuity equation $\bar{\nabla}\cdot\vec{J} + \dfrac{\partial\rho}{\partial t} = 0.$ ]

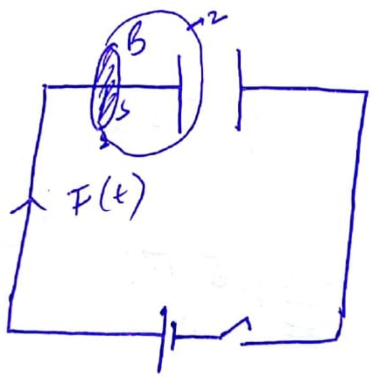
$\therefore$ for the 2nd exp.

LHS is always 0 but RHS is not always 0.

for loop 2.

$B(2\pi s) = 0$

$B = 0.$




for loop 1

$B(2\pi s) = \mu_0 I(t)$

$$B(t) = \frac{\mu_0 I(t)}{2\pi s}\hat{\phi}$$

continue —