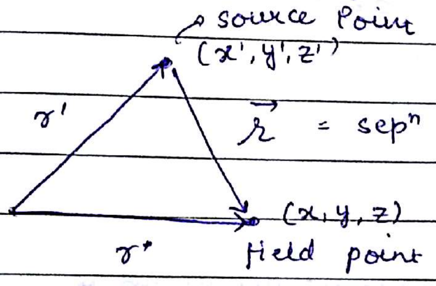
# ELECTRODYNAMICS

Reference Books :

1. Introd$^n$ to Electrodynamics by David J. Griffiths
2. Classical Electrodynamics by John David Jackson
3. Electricity & Magnetism by

## Vector Calculus




$\vec{\mathscr{r}} = \cancel{x}\ \vec{r} - \vec{r}'$

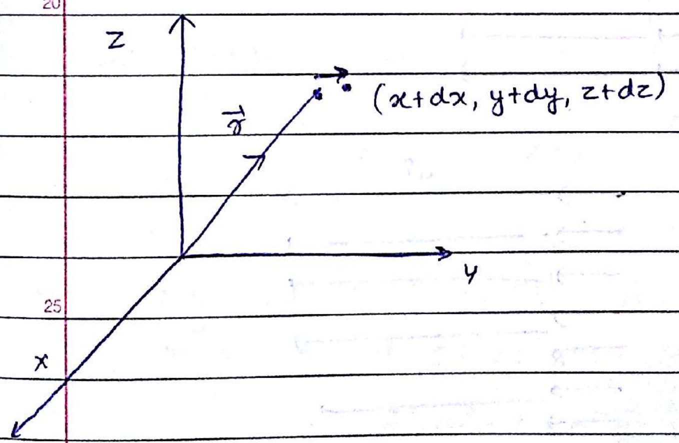
$\vec{\mathscr{r}}$ ↓ sep$^n$ vector

$\vec{r}$ ↓ position vector

→ infinitesimal displacement vector :

$(x, y, z)$ to $(x+dx,\ y+dy,\ z+dz)$

$$dl = dx\,\hat{x} + dy\,\hat{y} + dz\,\hat{z}$$




→ If $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$ ⇒ Dot product exist

→ $\vec{A}$ in projection of $\vec{B}$ = $\vec{A}\cdot\hat{B}$

$A \cdot (B \times C) = B \cdot (C \times A) = C \cdot (A \times B)$

$A \times (B \times C) = B\,[A \cdot C] - C\,[A \cdot B]$



→ scalar func$^n$ : varies with certain parameters.

Eg. Temperature

## Ordinary Derivative

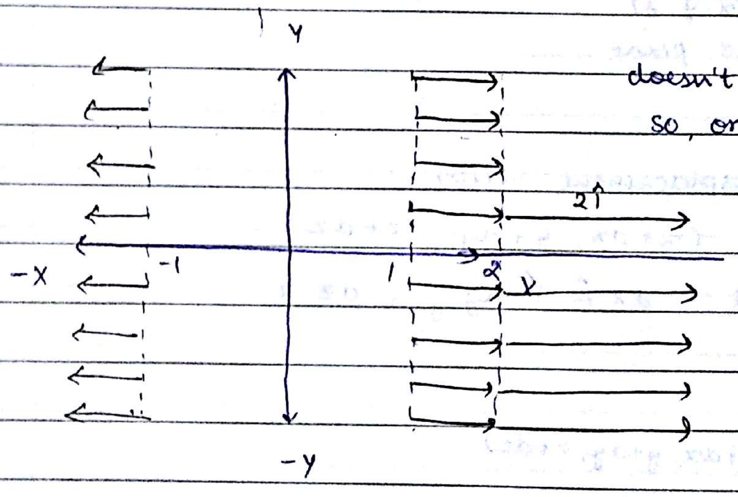
→ $T = f(x)$

$$\frac{df}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \Rightarrow df = \left(\frac{df}{dx}\right) dx$$

→ $T = T(x, y, z)$

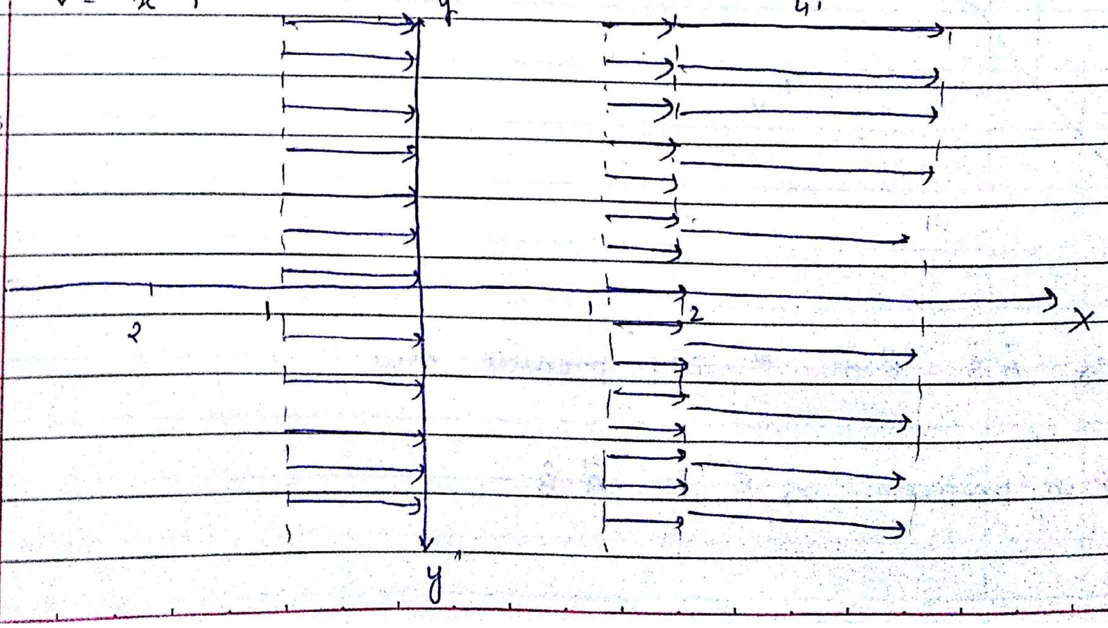
$$dT = \left(\frac{\partial T}{\partial x}\right) dx + \left(\frac{\partial T}{\partial y}\right) dy + \left(\frac{\partial T}{\partial z}\right) dz$$

⇒ $V = x\hat{i}$ → Not constant vector.



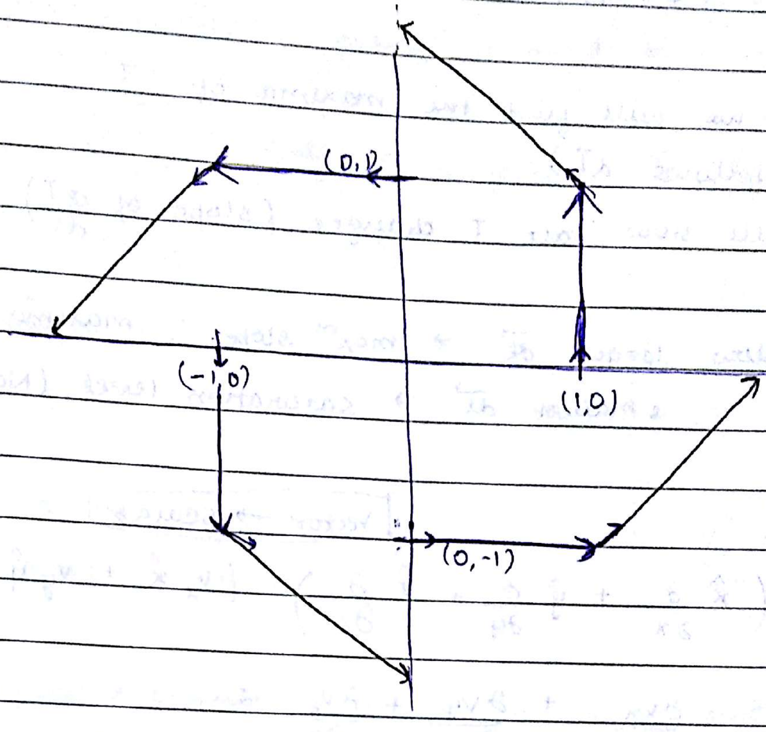

doesn't depend on $y$
so, on $x = 1$ & $x = -1$

→ $V = x^2\,\hat{i}$



→ $\vec{v} = -y\hat{i} + x\hat{j}$

(1,0) (0,1) (-1,0) (0,-1)




→ $\vec{v} = x^2\hat{i} + y^2\hat{j}$

1-3-17 **Gradient Operator ($\vec{\nabla}$)**

→ It is a gradient operator.

In cartesian co-ordinate

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}\right)\hat{i} + \left(\frac{\partial}{\partial y}\right)\hat{j} + \left(\frac{\partial}{\partial z}\right)\hat{k}$$

→ $T(x,y) = -(\cos^2 x + \cos^2 y)^2$ → $\boxed{\text{salar} \to \text{vector}}$

$$\vec{\nabla} T = +2(\cos^2 x + \cos^2 y)2\cos x \sin x\,\hat{i} + 4\cos y \sin y\,(\cos^2 x + \cos^2 y)$$

$$= 2(\cos^2 x + \cos^2 y)\,[\sin 2x\,\hat{i} + \sin 2y]\,\hat{j}$$

⇒ We get a vector func$^n$

→ Let $T = T(x,y,z)$

$$dT = \frac{\partial T}{\partial x}dx + \frac{\partial T}{\partial y}dy + \frac{\partial T}{\partial z}dz$$

$$= \left(\frac{\partial T}{\partial x}\hat{i} + \frac{\partial T}{\partial y}\hat{j} + \frac{\partial T}{\partial z}\hat{k}\right)\cdot\left(dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}\right)$$

$= \vec{\nabla} T \cdot \vec{dl}$

$= |\vec{\nabla} T| |\vec{dl}| \cos\theta$

⇒ if $\theta = 0°$, we will find the maxima of $T$
( $\vec{\nabla} T$ follows $\vec{dl}$ )

•) it also tells how fast $T$ changes (slope of $\frac{dT}{dl}$)

•) If you follow largest $\vec{dl}$ ⇒ max$^m$ slope, maxima
smallest $\vec{dl}$ ⇒ saturation level (No change)

Divergence: $\boxed{\text{Vector} \rightarrow \text{Scalar}}$

$\nabla \cdot V = \left( \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z} \right) \cdot (V_x\hat{x} + V_y\hat{y} + V_z\hat{z})$

$= \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$

$\nabla \cdot \vec{V} \neq \vec{V} \cdot \nabla$
($\nabla$ is not vector, it is a vector operator)

(a) $V_a = x^2\hat{x} + 3xz^2\hat{y} - 2xz\hat{z}$
$\nabla \cdot V_a = (2x - 2x) = (0)$

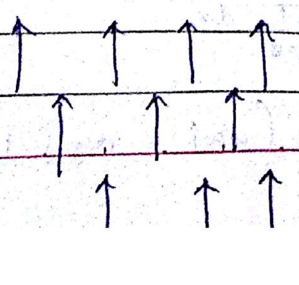
(c) $V_c = y^2\hat{x} + (2xy + z^2)\hat{y} + 2yz\hat{z}$
$\nabla \cdot V_c = 2x + 2y$

•) $\vec{V} \cdot \vec{\nabla} = V_x\frac{\partial}{\partial x} + V_y\frac{\partial}{\partial y} + V_z\frac{\partial}{\partial z}$, scalar operator.

⇒ $V_a = r = x\hat{x} + y\hat{y} + z\hat{z}$
Divergence $= 1 + 1 + 1 = 3$

⇒ $V_b = \hat{z}$
Divergence $= 0$

$V_c = z\hat{z}$
Divergence $= 1$



•) The divergence of a vector func$^n$ at Point P may be predicted by considering a closed surface surrounding P & analyzing the flow over the boundary, keeping in mind that at P :

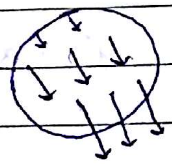
$$\vec{\nabla} \cdot \vec{F} = \text{outflow} - \text{inflow}. \Rightarrow \text{change of flux per unit volume}$$

Eg. $\vec{V} = x\hat{i} + y\hat{j}$

Divergence $= 2$

Eg. $F = (y-2x)\hat{i} + (x-2y)\hat{j}$

Divergence $= -4$



magnitude of outflow > magnitude of inflow

•) If outflow > inflow ⇒ there must be a source (+ive)

•) " < " ⇒ " " sink (−ive)

<u>The curl</u> :

$$\nabla \times V = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ V_x & V_y & V_z \end{vmatrix} = \hat{x}\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) + \hat{y}\left(\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right) + \hat{z}\left(\frac{\partial V_y}{\partial x}\right.$$

⇒ $V_a = -y\hat{x} + x\hat{y}$

$$\nabla \times V = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 0 & 0 & \\ -y & x & \end{vmatrix} \cancel{=0}$$

$$= \hat{z}\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right) = \hat{z}(2) = 2\hat{z}$$

From field, it implies something is rotating about z axis.

•) If curl > 0 ⇒ field is rotational field.

$\rightarrow$ $v_b = x\hat{y}$

$\nabla \times v = \hat{z}$

•) $[\text{curl } F(x_0, y_0)] \cdot k < 0 \rightarrow$ Clockwise dirⁿ

Eg. $\vec{F} = -y\hat{i} + x\hat{j}$

$(\vec{\nabla} \times \vec{F}) \cdot \hat{k} = 2 \Rightarrow$ Anti clockwise $dir^n$

(Force is unbalanced)

Eg. $\vec{F} = y\hat{i} + x\hat{j}$ or $\vec{F} = x\hat{i} + y\hat{j}$

$\rightarrow \vec{\nabla} \cdot \vec{F} = 0$

Identities related to Gradient :

$\rightarrow$ $\nabla(f+g) = \nabla f + \nabla g$

$\rightarrow$ $\nabla \times (A+B) = (\nabla \times A) + (\nabla \times B)$

$\rightarrow$ $\nabla \cdot (\underbrace{f}_{\text{scalar}} \underbrace{A}_{\text{vector}}) = f(\nabla \cdot A) + A \cdot (\nabla f)$

$\rightarrow$ $\nabla \cdot (A \times B) = B \cdot (\nabla \times A) - A \cdot (\nabla \times B)$

$\rightarrow$ $\nabla \times (fA) =$

$\nabla T$ is a vector
1.) Divergence of gradient $\nabla \cdot (\nabla T)$
2.) curl " " $\nabla \times (\nabla T)$

$\nabla \cdot V$ is scalar
1.) ~~Div~~ Gradient of div. $\nabla(\nabla \cdot V)$

$\nabla \times V$ is vector
1.) Div of curl $\nabla \cdot (\nabla \times V)$
2.) curl of curl $\nabla \times (\nabla \times V)$

Always. $\boxed{(\bar{\nabla} \times (\bar{\nabla} T)) = 0}$

Always $\boxed{\nabla \cdot (\nabla \times V) = 0}$

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→ $\nabla \times (\nabla \times v) =$

Laplacian of a vector:

$$\nabla^2 V = (\nabla^2 V_x)\hat{x} + (\nabla^2 V_y)\hat{y}$$

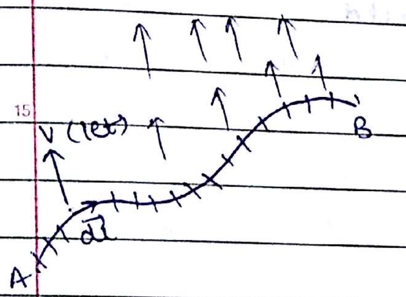
→ $\nabla \cdot (\nabla T) = \nabla^2 T$.

$$\nabla \times (\nabla \times V) = \nabla(\nabla \cdot V) - \nabla^2 V$$

15-3-17

## Integration

Vector line Integration.




$$\lim_{\Delta l \to 0} \sum_{i=A}^{B} \vec{V_i} \cdot \Delta\vec{l}$$

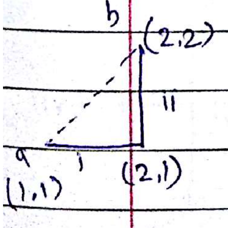
→ let $\vec{V}$: force → $\int \vec{V} \cdot d\vec{l}$ = Work done

→ work done from A to B ≠ B to A.

(i) $\vec{V} = -y\hat{i} + x\hat{j}$

$$\int \vec{V} \cdot d\vec{l} = \cancel{\int_1^2 -y \cdot dx + \int_1^2 x\,dx}$$

$$\cancel{= -\frac{yx}{2} + \frac{x^2}{2} = -\frac{1}{2}[4-1] + \frac{1}{2}[4-1]} = 0$$




(i) → $\int_1^2 (-y\hat{i} + x\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$, $y \to 1$, $dy \to 0$

$$= -\int_1^2 dx = -1$$

(ii) → $\int_1^2 (2)\,dy = 2$

If we directly go from a to b.

$$\int \vec{v} \cdot \vec{dl} = \int (\overset{-y}{\cancel{y}}\hat{i} + \overset{x}{\cancel{y}}\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$$

$$= \int_{(1,1)}^{(2,2)} -y\,dx + x\,dy$$

$y = x$

$dy = dx$

$$= \int_{x=1}^{2} (x-y)\,dx = \left[\cancel{\frac{x^2}{2}} - \cancel{xy}\right]_1^2$$

$$= 0$$

$\Rightarrow$ Work done in both cases gives different values

$\Rightarrow$ It is non-conservative vector field.

(ii.) $\vec{v} = x\hat{i} + y\hat{j}$

(i) $\rightarrow$ $\int (x\hat{i} + y\hat{j}) \cdot (dx\hat{i} + \underset{0}{dy}\hat{j})$

$$= \int_1^2 x\,dx = \frac{3}{2}$$

(ii.) $\rightarrow$ $\int_1^2 y\,dy = \frac{3}{2}$ $\Rightarrow$ $\int \vec{v} \cdot \vec{dl} = 3$

a–b :

$$\int x\,dx + y\,dy$$

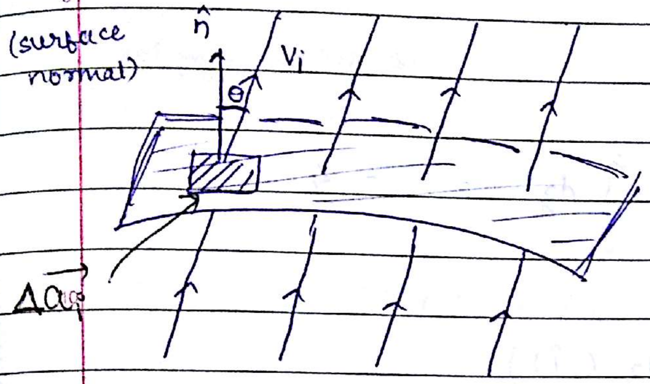
$y = x$

$$\int_1^2 2x\,dx = \left[x^2\right]_1^2 = 3$$

$\Rightarrow$ It is conservative vector field.

## Surface Integration :

$\int_S \vec{V} \cdot \vec{da}$ or $\iint \vec{V} \cdot \vec{da}$ = flux going through a certain surface.




dir$^n$ of surface : normal to that surface

$$\lim_{\Delta a_i \to 0} \sum V_i \cdot \Delta a_i$$

→ If $\vec{V}$ : $\vec{J}$

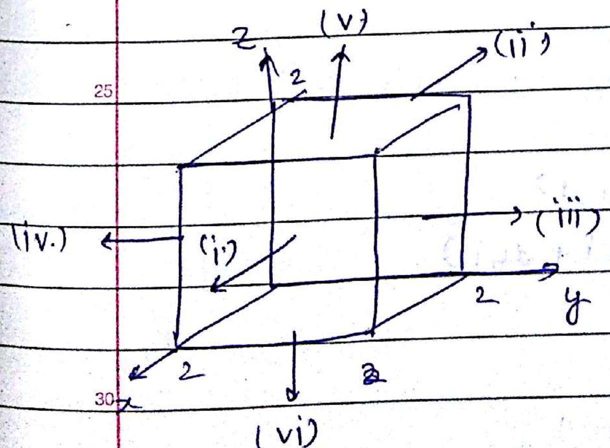
$\int \vec{V} \cdot \vec{da} = I$ : no. of charges crossing surface per unit time

$\int \vec{v} \cdot \vec{da}$ : $\begin{cases} (V\cos\theta)\, da \\ V\,(da\cos\theta) \end{cases}$

→ Two types of surfaces :

i) Open surface

ii) Closed surface : consider outward dir$^n$ as dir$^n$ of surface.



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(i) $v_1 = -y\hat{i} + x\hat{j}$

(i) : $\iint_{(i)} (-y\hat{i} + x\hat{j}) \cdot dy\, dz\, \hat{i}$ (x remains constant)

$\int_0^2\int_0^2 (-y\, dy)dz$

$\int_0^2 \left[-\frac{y^2}{2}\right]_0^2 dz = -2\int_0^2 dz = -4$

(ii) : $\iint_{(ii)} (-y\hat{i} + x\hat{j}) \cdot (dy\, dz\, (-\hat{i}))$

$= \int_0^2\int_0^2 y\, dy\, dz = 4$

→ we know that $-y\hat{i} + x\hat{j}$ is rotational field ⇒ flux $\hat{i}$, $-\hat{i}$ are opposite

(v) : $\iint_{(v)} (-y\hat{i} + x\hat{j}) \cdot (dy\, dx)\, \hat{k} = 0$

Physically, surface is ∥ to V ⇒ flux = 0

(iii) $\vec{V}_3 = xy\hat{i} + yz\hat{j} + zx\hat{k}$

(i.)

17-3-17

⇒ $V = x\hat{i} + y\hat{j}$ $(0,0) \to (1,1)$

$\int \vec{v} \cdot \vec{dl} = \int (x\hat{i} + y\hat{j}) \cdot (dx\hat{i} + dy\hat{j})$

$= \int x\,dx + y\,dy$

$= \int_{(0,0)}^{(1,1)} 2x\,dx = 1$

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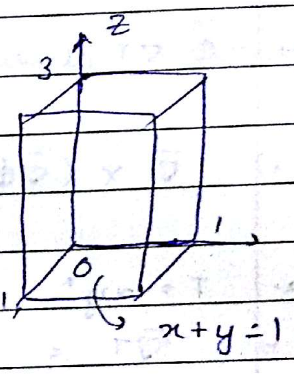
- if go from (1,1) to (0,0) don't change $dx\hat{i} + dy\hat{j}$ to $-dx\hat{i} - dy\hat{j}$. Only change the limit to $\int_{(1,1)}^{(0,0)}$. Don't change both the things.

## Volume Integral

$\int_V T\,d\tau \qquad d\tau = dx\,dy\,dz.$

$$\equiv \iiint T(x,y,z)\,dx\,dy\,dz.$$




$T = xyz^2$

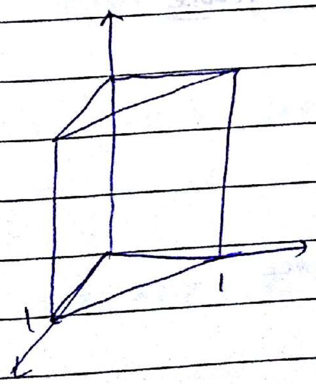
$$= \int\left(\int\left(\int x\,dx\right) y\,dy\right) dz$$

$$= \int\int_0 y \int_0^{1-y} \cancel{x\,dx\,dy}$$

$$= \int_0^1 x\left[\int_0^1 y\left(\int_0^3 z^2\,dz\right)dy\right]dx \qquad \frac{z^3}{3}$$

$$= 9\int_0^1 x\left(\int_0^1 y\,dy\right)dx = 9 \times \frac{1}{2} \times \frac{1}{2} = \frac{9}{4}$$

→




$$\int_0^3 z\left[\int_0^1 y \int_0^{1-y} (x\,dx)\,dy\right]dz$$

## The fundamental theorem for Gradients

$$\int_a^b (\vec{\nabla} T)\cdot \vec{dl} = T(b) - T(a)$$ → doesn't depend on path

$\Downarrow$

we know that $dT = (\vec{\nabla} T)\cdot \vec{dl}$

$\vec{F} = \nabla\phi$ If $\vec{F}$ can be written in terms of gradient of scalar, it is conservative vector field.

→ $\oint \nabla T\cdot dl = 0$ $\{ (b = a),\ T(b) - T(a) = 0 \}$

·) $\vec{\nabla} \times (\vec{\nabla}\phi) \equiv 0$

Ex. $T = xy^2$ $a: (0,0,0)$ $b: (2,1,0)$

$\vec{\nabla} T = y^2\,\hat{i} + 2xy\,\hat{j}$ $(0,0)$ $(2,1)$

$2y = x$

$T(b) - T(a) = 2(1) = 2$ $2dy = dx$

$$\int_a^b \vec{\nabla} T\cdot dl = \int_a^b \cancel{xy^2}\, y^2 dx + 2xy\, dy = \int_0^1 2y^2 dy + 4y^2 dy$$

$$= 2$$

## The fundamental theorem for Divergence

$$\iiint_V (\vec{\nabla}\cdot\vec{F})\, d\tau = \oiint \vec{F}\cdot\vec{da}$$

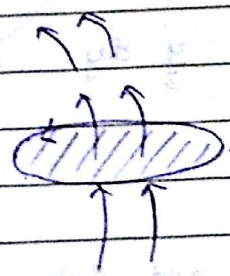
↓ change of flux per unit volume at a particular point

↓ Total flux going through closed surface

Q. $v = y^2\hat{x} + (2xy + z^2)\hat{y} + (2yz)\hat{z}$

## The Fundamental Theorem for Curls — Stoke's Theorem

$$\iint_S (\vec{\nabla}\times\vec{F})\cdot d\vec{a} = \oint \vec{F}\cdot d\vec{l}$$



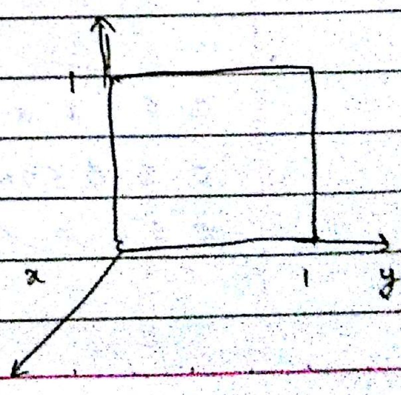

If you move in ACW dir$^n$ : dir$^n$ of surface : upwards
CW dir$^n$ : : downwards

Corollary 1: $\int(\bar{\nabla}\times\bar{V})\,d\vec{a}$ depends only on boundary line, not on the particular surface used.

Corollary 2: $\oint(\bar{\nabla}\cdot\bar{V})\cdot da = 0$ for closed surface

Q. $v = (2xz + 3y^2)\hat{y} + (4yz^2)\hat{z}$

$$\bar{\nabla}\times\bar{V} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & 2xz+3y^2 & 4yz^2 \end{vmatrix}$$



$$= \hat{x}\left(4z^2 - 2x\right) - \hat{y}\left(0+0\right) + \hat{z}\left(2z\right)$$

$$= (4z^2 - 2x)\hat{x} + (2z)\hat{z}$$

$$\iint_S (\vec{\nabla}\times\vec{F})\cdot d\vec{a} = \iint (4z^2 - 2x)\, d\vec{a}$$

$$d\vec{a} = \hat{x}\, dy\, dz$$

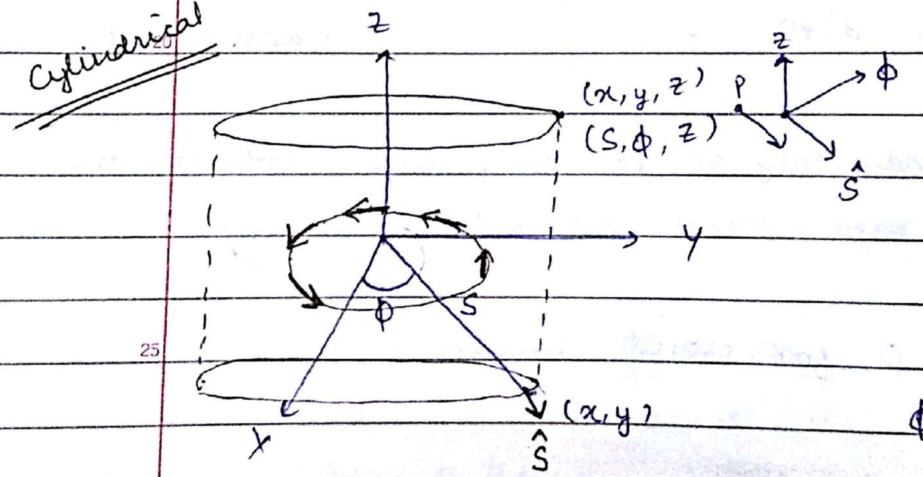
$$= \iint (4z^2 - 2x)\, \cancel{\hat{z}}\, dy\, dz\ \hat{\imath}$$

$$= \int_0^1 \left( \int_0^1 (4z^2 - 2\overset{0}{\cancel{x}})\, dz \right) dy$$

$$= \int_0^1 \frac{4}{3}\, dy = \boxed{\frac{4}{3}}$$

## Cylindrical and Spherical Co-ordinate System

Cylindrical




If we rotate $\phi$ keeping $s$ const, we will get cylindrical.

$\phi$ : always defined wrt +ive x-axis, rotates in ACW dirⁿ

| | | | |
|---|---|---|---|
| $0 \le s \le \infty$ | $x = s\cos\phi$ | $y = s\sin\phi$ | $z = z$ |
| $0 \le \phi \le 2\pi$ | $s = \sqrt{x^2+y^2}$ | $\phi = \tan^{-1}\left(\frac{y}{x}\right)$ | $z = z$ |
| $-\infty \le z < \infty$ | | | |

$z$ : const.

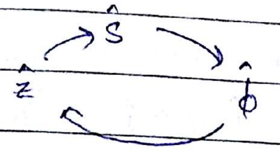
$\hat{s}$ : $\hat{s}(\phi)$ ~~($s$ is changing as $\phi$ changes)~~

$\hat{\phi}$ : $\hat{\phi}(\phi)$ (only change $\phi$. keep $\hat{s}$ & $\hat{z}$ const. also!)

dir$^n$ of $\hat{\phi}$ : dir$^n$ of tangent ~~dir~~ of circle drawn as projection with fixed $\hat{s}$ & $\hat{z}$.



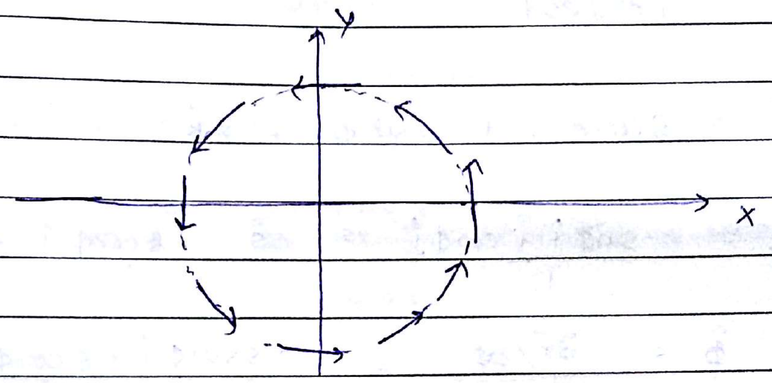
* $\hat{s}$, $\hat{z}$, $\phi$ are orthogonal to each other



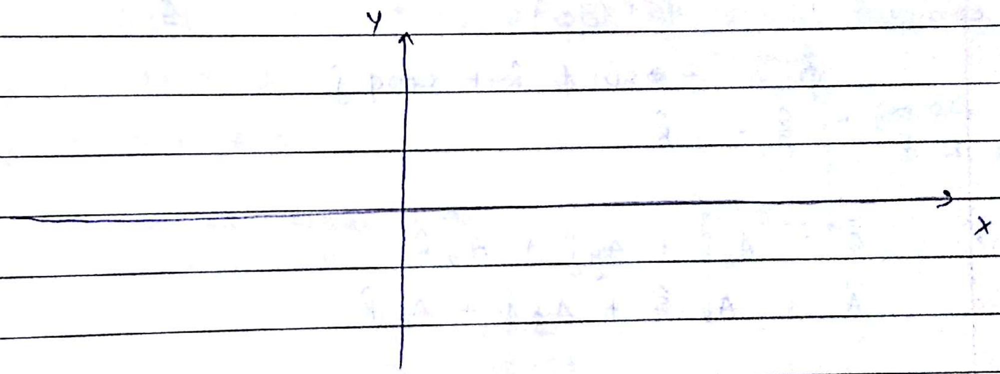

→ i.) $V = s\hat{\phi}$ : Draw vector field

ii) $V = \hat{\phi}$

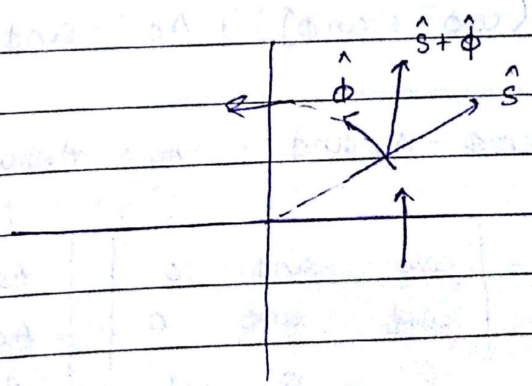
ii) $\vec{V} = \hat{\phi}$




i.) $\vec{V} = s\hat{\phi}$




iii) $\vec{V} = \hat{\phi} + \hat{s}$



## Unit vectors

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\hat{i} = \frac{\partial \vec{r}/\partial x}{|\partial \vec{r}/\partial x|} \Rightarrow \hat{i} = \cancel{\frac{x}{x}}\ \frac{\hat{i}}{(1)} = \hat{i}$$

Similarly, $\hat{s} = \dfrac{\partial \vec{r}/\partial s}{|\partial \vec{r}/\partial s|}$

$$\vec{r} = s\cos\phi\,\hat{i} + s\sin\phi\,\hat{j} + z\hat{k}$$

$$\hat{s} = \cancel{\frac{-\sin\phi\,\hat{i} + \cos\phi\,\hat{j}}{1}} \Rightarrow \hat{s} = \frac{\cos\phi\,\hat{i} + \sin\phi\,\hat{j}}{1}$$

$$\hat{\phi} = \frac{\partial \vec{r}/\partial \phi}{|\partial \vec{r}/\partial \phi|} = \frac{-s\sin\phi\,\hat{i} + s\cos\phi\,\hat{j}}{s}$$

$$\hat{\phi} = -\sin\phi\,\hat{i} + \cos\phi\,\hat{j}$$

$$\hat{z} = \hat{k}$$

$$\rightarrow \vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$$\vec{A} = A_s\hat{s} + A_\phi\hat{\phi} + A_z\hat{k}$$

$$= A_s(\cos\phi\,\hat{i} + \sin\phi\,\hat{j}) + A_\phi(-\sin\phi\,\hat{i} + \cos\phi\,\hat{j}) + A_z\hat{k}$$

$$\Rightarrow \boxed{A_x = A_s\cos\phi - A_\phi\sin\phi \ ; \ A_y = A_s\sin\phi + A_\phi\cos\phi}$$

$$\Rightarrow \begin{bmatrix} A_x \\ A_y \\ A_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} A_s \\ A_\phi \\ A_z \end{bmatrix}$$

$$\boxed{\begin{aligned} A_s &= A_x\cos\phi + A_y\sin\phi \\ A_\phi &= -A_x\sin\phi + A_y\cos\phi \\ A_z &= A_z \end{aligned}}$$

## Infinitesimal vector

$\vec{dl} = dx\hat{i} + dy\,\hat{j} + dz\,\hat{k}$

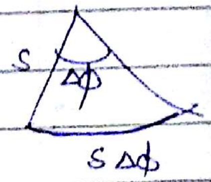
$\vec{dl} = dl_s\,\hat{s} + dl_\phi\,\hat{\phi} + dl_z\,\hat{z}$

~~$dx = dl_s\cos\phi - dl_\phi \sin\phi$, $dy = dl_s \sin\phi + dl_\phi \cos\phi$, $dz = dl_z$~~

$(s, \phi, z) \longrightarrow (s+ds,\ \phi + d\phi,\ z + dz)$

learn

$$\boxed{\vec{dl} = ds\,\hat{s} + s\,d\phi\,\hat{\phi} + dz\,\hat{z}}$$




$\vec{dl} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

$da_x$ (keeping x const.) : (taking cross product of other 2)

$= dy\,dz\,\hat{i}$

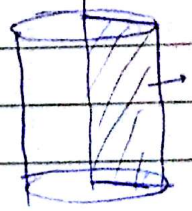
Here, $\vec{dS}_s = s\,d\phi\,dz\,\hat{s}$ → we will get cylindrical surface

$\vec{dS}_\phi = dz\,ds\,\hat{\phi}$ ⟶

$\vec{dS}_z = s\,ds\,d\phi\,\hat{z}$

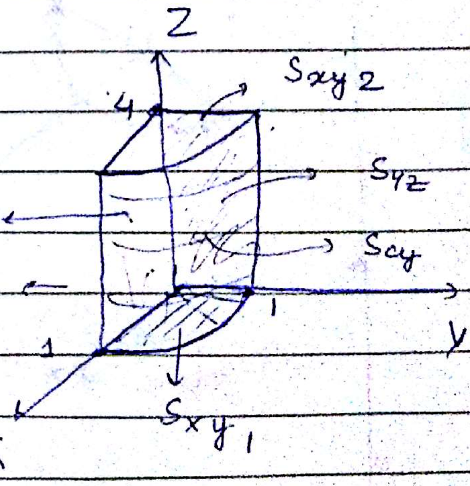
⇓

circular plane ∥ to x-y plane.



for all surfaces, $\phi$ is same

$$\text{volume element} = \int \partial\tau = \int_0^R s\,ds \int_0^{2\pi} d\phi \int_0^{h} dz$$




eg. i) $\vec{V} = \hat{s} + \sin\phi\,\hat{\phi} + z\,\hat{z}$

ii) $\vec{V} = s\,\hat{s} + \hat{\phi} + z\,\hat{z}$

No. of surfaces = 5

→ $dS_{xz} = -ds\,dz\,\hat{\phi}$ (dirⁿ: outwards → $-\hat{\phi}$)

$$\int \vec{V}\cdot dS_{xz} = \int \overset{0}{\cancel{\sin\phi}}\,(-ds\,dz)\hat{\phi}$$

$= 0$

→ $dS_{yz} = s\,d\phi\,dz\,(-\hat{s})$ ⟹ $\phi$ : const.

$$\int \vec{V}\cdot dS_{yz} = -\iint s\,\overset{0}{d\phi}\,dz = 0$$

$\rightarrow$ $dS_{xy_1}$ : z const $\Rightarrow$ z : const.

$\Rightarrow s\,ds\,d\phi\,(-\hat{z})$

$$\int \vec{V}\cdot d\vec{S}_{xy_1} = -\iint \overset{0}{\cancel{z}}\cdot ds\,d\phi = 0$$

$\rightarrow$ $dS_{xy_2} = s\,ds\,d\phi\,(\hat{z})$ $\Rightarrow$ z : const.

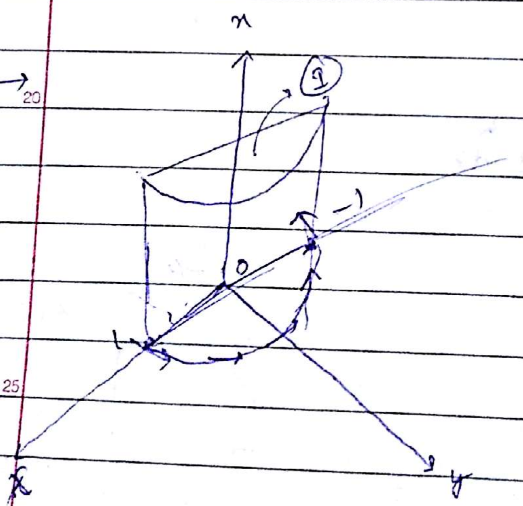
$$\int \vec{V}\cdot d\vec{S}_{xy_2} = \int_0^{\pi/2}\int_0^{1} z\,4s\,ds\,d\phi = \frac{4}{2}\frac{\pi}{2} = 2\pi$$

$\rightarrow$ ~~$\phi_{sxyz} = \iint \vec{V}\, s\,ds\,d\phi\,dz$~~

$\rightarrow$ $S_{cy} = s\,d\phi\,dz\,\hat{s}$ $\Rightarrow$ s : const

$$\int \vec{V}\cdot d\vec{s} = \int_0^{4}\int_0^{\pi/2} \overset{1}{\cancel{s}}\,d\phi\,dz = \frac{\pi}{2}\times 4 = 2\pi$$

$\rightarrow$




$\vec{V} = \hat{\phi}$

Don't integrate from -1 to +1 ~~directly~~. Do it from -1 → 0 & 0 → 1

+ive zx plane : $dS_\phi = dz\,d\phi(\hat{\phi}_{-})$

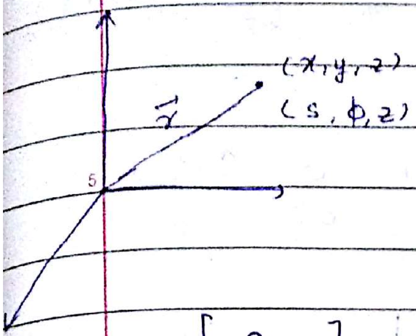
$$\int \vec{V}\cdot d\vec{s} = \int dz\,d\phi$$

- ive zx plane : $dz\,d\phi\,(\hat{\phi})$

total through zx plane = 0

→ $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ → same vector in cylindrical coordinate system

$$\vec{r} = a_s\hat{s} + a_\phi\hat{\phi} + a_z\hat{z}$$




$$\begin{bmatrix} a_s \\ a_\phi \\ a_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$a_s = x\cos\phi + y\sin\phi$ $\qquad x = s\cos\phi \quad y = s\sin\phi$

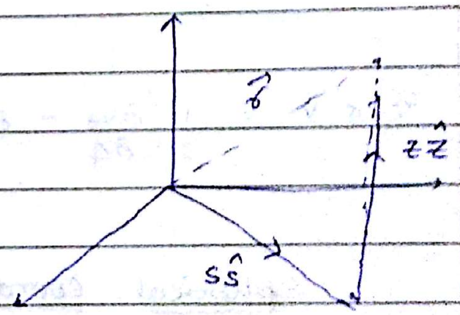
$a_\phi = -x\sin\phi + y\cos\phi$

$a_z = z$

$a_s = s$

$a_\phi = 0$

$\boxed{\vec{r} = s\hat{s} + z\hat{z}}$ :




* But we need 3 terms to define a point, so here, $\phi$ is hidden in $\hat{s}$.
  ($\hat{s} = \cos\phi\,\hat{i} + \sin\phi\,\hat{j}$)
  we get correct dir$^n$ due to $\hat{s}$.

→ $\bar{\nabla}$ in cylindrical coordinate system :-

$T = T(s, \phi, z)$

$$dT = \left(\frac{\partial T}{\partial s}\right) ds + \left(\frac{\partial T}{\partial \phi}\right) d\phi + \left(\frac{\partial T}{\partial z}\right) dz \quad \text{—— (1)}$$

$dT = \bar{\nabla}T \cdot \bar{dl}$

$\bar{\nabla}T \cdot \bar{dl} = \left[(\text{grad } T)_s\hat{s} + (\text{grad } T)_\phi\hat{\phi} + (\text{grad } T)_z\hat{z}\right] \cdot \left[ds\,\hat{s} + s\,d\phi\,\hat{\phi} + dz\,\hat{z}\right]$

$= (\text{grad } T)_s\, ds + s(\text{grad } T)_\phi\, d\phi + (\text{grad } T)_z\, dz$ —— (2)

* Never find curl like $\begin{vmatrix} \hat{s} & \hat{\phi} & \hat{z} \\ \partial & \cdot & \cdot \end{vmatrix}$ bcoz $\hat{s}$ & $\hat{\phi}$ are not const, they are functions of $\phi$.

Comparing ~~one~~ eq$^n$ 1 and 2, we get

$$\frac{\partial T}{\partial s} = (\text{grad } T)_s \qquad \frac{\partial T}{\partial \phi} = s(\text{grad } T)_\phi \qquad \frac{\partial T}{\partial z} = (\text{grad } T)_z$$

$$\Rightarrow (\text{grad } T) = \left( \frac{\partial}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial}{\partial \phi}\hat{\phi} + \frac{\partial}{\partial z}\hat{z} \right) T$$

$$\Rightarrow \boxed{\bar{\nabla} = \frac{\partial}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial}{\partial \phi}\hat{\phi} + \frac{\partial}{\partial z}\hat{z}}$$

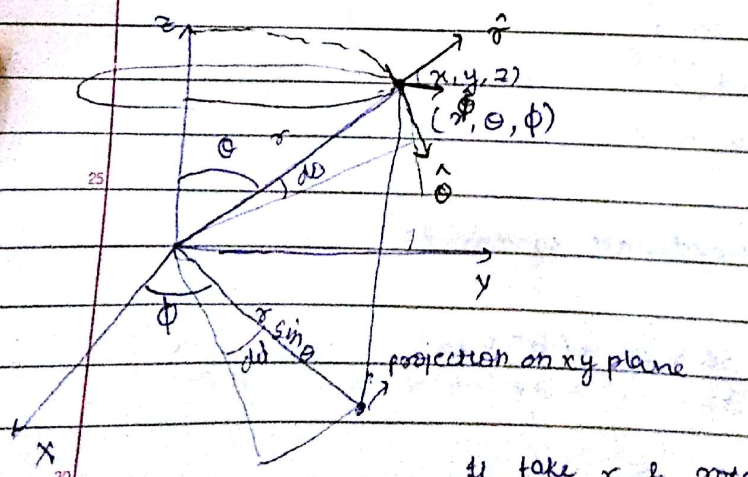
Eg. $\vec{V} = \sin\phi\, \hat{s} + \cos\phi\, \hat{\phi}$ $\qquad \bar{\nabla}\cdot\bar{V} = ?$

$$\bar{\nabla}\cdot\bar{V} = \frac{1}{s}\frac{\partial}{\partial s}(sV_s) + \frac{1}{s}\frac{\partial V_\phi}{\partial \phi} + \frac{\partial V_z}{\partial z}$$

$=$

$$\rightarrow \bar{\nabla}\times\bar{V} = \frac{1}{s}\left(\frac{\partial V_z}{\partial \phi} - \frac{\partial V_\phi}{\partial z}\right)\hat{s} + \left(\frac{\partial V_s}{\partial z} - \frac{\partial V_z}{\partial s}\right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial}{\partial s}(sV_\phi) - \frac{\partial V_s}{\partial \phi}\right)\hat{z}$$

## Spherical Coordinate system.




(1) → distance from origin (r)

$\theta$ → wrt z

If take r & rotate with changing $\theta$ → cone

$0 \le r \le \infty \qquad 0 \le \theta \le \pi \qquad 0 \le \phi \le 2\pi$

↳ After rotating with $\theta$ to $\pi$, rotate $\phi$ to get comp. sphere

$r = \sqrt{x^2+y^2+z^2}$

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

$z = r\cos\theta$

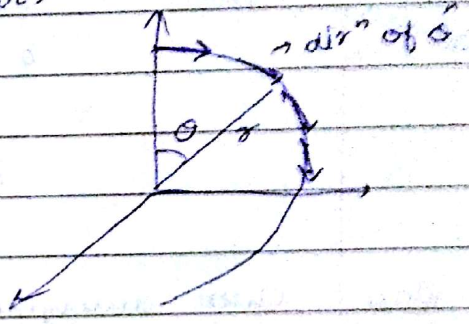
$\theta = \cos^{-1}\left(\frac{z}{r}\right)$

$\phi = \tan^{-1}\left(\frac{y}{x}\right)$

→ dirⁿ of $\hat{r}$ : outwards (radially)

→ dirⁿ of $\hat{\phi}$ : tangent (same as earlier)

→ dirⁿ of $\hat{\theta}$ :




•) $\hat{r}, \hat{\theta}, \hat{\phi}$ : orthogonal

$\hat{r} \to \hat{\theta} \to \hat{\phi} \to \hat{r}$

→ $\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$

$$\hat{r} = \frac{\partial\vec{R}/\partial r}{\left|\frac{\partial\vec{R}}{\partial r}\right|}$$

•) $\hat{r} = \sin\theta\cos\phi\,\hat{i} + \sin\theta\sin\phi\,\hat{j} + \cos\theta\,\hat{k}$

$\hat{\theta} = \cos\theta\cos\phi\,\hat{i} + \cos\theta\sin\phi\,\hat{j} - \sin\theta\,\hat{k}$

$\hat{\phi} = -\sin\phi\,\hat{i} + \cos\phi\,\hat{j}$

Infinitesimal ~~dis~~ displacement vector :

$dl = dI_r\hat{r} + d\theta I_\theta\,\hat{\theta} + dI_\phi\,\hat{\phi}$

only change r

$$\boxed{\vec{dl} = dr\,\hat{r} + (r\,d\theta)\,\hat{\theta} + (r\sin\theta\,d\phi)\,\hat{\phi}}$$

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Surfaces :

$\vec{da}_r = r^2 \sin\theta \, d\theta \, d\phi \, \hat{r}$ : surface of a sphere

$\vec{da}_\theta = r \sin\theta \, dr \, d\phi \, \hat{\theta}$ : surface of cone

$\vec{da}_\phi = r \, dr \, d\theta \, \hat{\phi}$ : semi-circular for particular $\phi$


$$d\tau = r^2 \sin\theta \, dr \, d\theta \, d\phi$$

To check whether correct :-

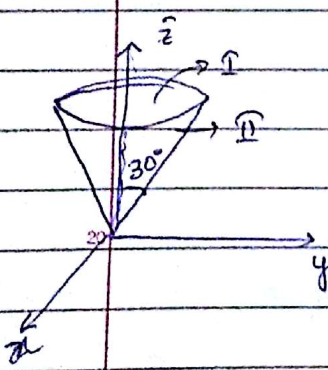
$$\int d\tau = \int_0^r r^2 dr \int_0^\pi \sin\theta \, d\theta \int_0^{2\pi} d\phi$$

$-\cos$

$$= \int_0^R \frac{r^3}{3} \times 2\pi \times 2 = \frac{4\pi R^3}{3}$$ ✓ (volume of sphere)

Ques. Check divergence theorem (find flux)

$$V = r^2 \sin\theta \, \hat{r} + 4r^2 \cos\theta \, \hat{\theta} + r^2 \tan\theta \, \hat{\phi}$$




I $\Rightarrow$ r const

II $\Rightarrow$ $\theta$ const

I : flux $= \iint \bar{V} \cdot \vec{da}_r$

$$= \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi/6} \bar{V} \cdot r^2 \sin\theta \, d\theta \, d\phi \, \hat{r}$$

$$= \iint r^2 \sin^2\theta \, d\theta \, d\phi = r^2 \times$$

II : flux $= \int_{\phi=0}^{2\pi} \int_{R=0}^{R} \bar{V} \cdot r \sin\theta \, dr \, d\phi \, \hat{\theta}$

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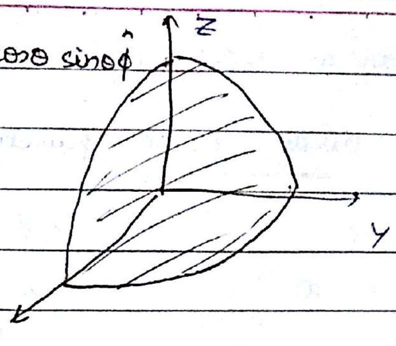
Ques $V = r^2\cos\theta\,\hat{r} + r^2\cos\phi\,\hat{\theta} - r^2\cos\theta\sin\phi\,\hat{\phi}$




→ for XZ plane : $\phi$ const $= 0$

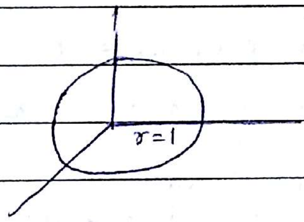
→ XY plane : $\phi$ " $= \pi/2$

→ xy plane : $\theta$ const.

→ surface of sphere : R const.

Ques. verify divergence theorem :

** $\boxed{\vec{F} = \dfrac{\hat{r}}{r^2}}$




$\iiint (\bar{\nabla}\cdot\bar{V})\,d\tau = \oiint \bar{V}\cdot d\bar{a}$

$r$ : const.

$\cancel{\bar{\nabla}\cdot\bar{F} = \frac{\partial}{\partial}}$ $\quad d\bar{a}_r = r^2\sin\theta\,d\theta\,d\phi\,\hat{r}$

$\nabla\cdot V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\cdot\frac{1}{r^2}\right) = \frac{1}{r^2}(0) = 0$

LHS $= 0$

RHS : $\iint \frac{\hat{r}}{r^2}\cdot\left(r^2\sin\theta\,d\theta\,d\phi\right)\hat{r}$ $\quad [-\cos\theta]_0^{\pi} \quad -(-1-1)$

$= \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \sin\theta\,d\theta\,d\phi = 4\pi$

LHS $\neq$ RHS

* Something is wrong (valid for all possible vector func$^n$s)

It is bcoz func$^n$ is not defined at $r = 0$.

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This can be dealt by :

**Dirac Delta function**

•) $\delta(x) = 0 \qquad x \neq 0$

$\quad = \infty \qquad x = 0$

(i) $\displaystyle\int_{-\infty}^{\infty} \delta(x) \cdot dx = 1$

(ii) $\displaystyle\int_{-\infty}^{\infty} \delta(x) f(x)\, dx = f(0)$

•) $\delta(x-a) = 0 \qquad x \neq a$

$\delta(x-a) = \infty \qquad x = a$

(i) $\displaystyle\int_{-\infty}^{+\infty} \delta(x-a)\, dx = 1$

(ii) $\displaystyle\int_{-\infty}^{\infty} \delta(x-a) f(x)\, dx = f(a)$

In 3-D :-

$\delta^3(r) = 0 \qquad r \neq 0$

$\quad = \infty \qquad r = 0$

(i) $\displaystyle\iiint_{\text{all space}} \delta^3(r)\, d\tau = 1$

(ii) $\displaystyle\iiint_{\text{all space}} \delta^3(r) f(r)\, d\tau = f(0)$

$\bar{\nabla} \cdot \bar{F} = 4\pi\, \delta^3(r)$

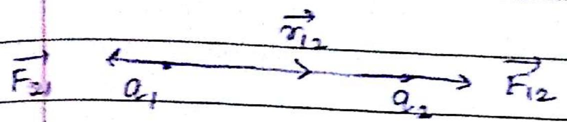
In previous question (L.H.S. is wrong as func$^n$ is N.D. at $x=0$, divergence is 0 everywhere except at $x=0$ (func$^n \to \infty$))

L.H.S. $= \displaystyle\iiint 4\pi\, \delta^3(r)\, d\tau$

$= 4\pi$

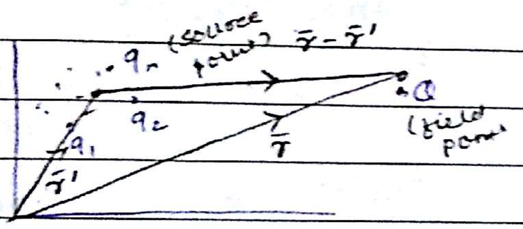
# ELECTROSTATICS

## Coulomb's law :-




$$\vec{F}_{12} = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}^2}\, \hat{r}_{12}$$

→ Superposition Principle : True for certain cases
(Not true for dielectric medium)




$$F_Q = \bar{F}_{q_1Q} + \bar{F}_{q_2Q} + \cdots + \bar{F}_{q_nQ}$$

$$\bar{F}_Q(r) = \frac{q_1 Q}{4\pi\epsilon_0 |\bar{r}-\bar{r}_1'|^3}(\bar{r}-\bar{r}_1') + \frac{q_2 Q}{4\pi\epsilon_0 |\bar{r}-\bar{r}_2'|^3}(\bar{r}-\bar{r}_2') \ldots\ldots$$

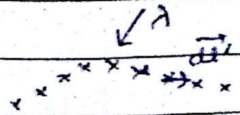
$$\boxed{\bar{F}(r) = \frac{Q}{4\pi\epsilon_0}\sum_{i=0}^{n} \frac{q_i(\bar{r}-\bar{r}_i')}{|\bar{r}-\bar{r}_i'|^3}}$$

$$\bar{F}(r) = Q\,\bar{E}(r)$$

⇒ For continuous charges :-

$$\bar{E} = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r^2}\,\hat{r}$$

1.)




$$E(r) = \frac{1}{4\pi\epsilon_0}\int \frac{\lambda\, dl'\,(\bar{r}-\bar{r}')}{|\bar{r}-\bar{r}'|^3}$$

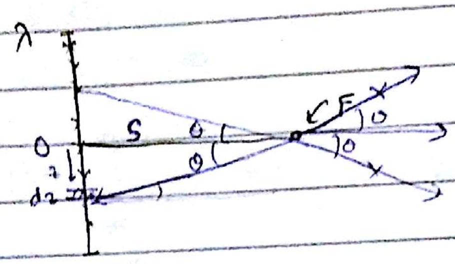
2.) Surface charge Density $(\sigma)$ : $\vec{E}(r) = \dfrac{1}{4\pi\epsilon_0}\displaystyle\iint \frac{\sigma\, da'\,(\bar{r}-\bar{r}')}{|\bar{r}-\bar{r}'|^3}$

* $E$ is always in terms of $r$, not $r'$.



3.) Volume charge: $\bar{E}(r) = \dfrac{1}{4\pi\epsilon_0}\iiint \dfrac{\rho\, dz'\,(\bar{r}-\bar{r}')}{|\bar{r}-\bar{r}'|^3}$

Ques




1.) $d\bar{E} = \dfrac{dz\,\lambda\cos\theta}{4\pi\epsilon_0\,(z^2+s^2)} = 2\displaystyle\int_0^L \dfrac{dz\,\lambda\, s}{4\pi\epsilon_0\,(z^2+s^2)^{3/2}}$

$z^2+s^2=t^2$
$2dz = 2t\,dt$
$z = s\tan\theta$

$= 2\displaystyle\int_0^L \dfrac{\lambda s\, dt}{4\pi\epsilon_0\, t^3} = \dfrac{\lambda s}{4\pi\epsilon_0}\,\dfrac{1}{t^2} = 2\displaystyle\int \dfrac{\lambda}{s\,4\pi\epsilon_0}\,\dfrac{\cos\theta\, d\theta}{\sec^2\theta}$

$E = \dfrac{2\lambda L}{4\pi\epsilon_0 s\sqrt{s^2+L^2}}$

2.) $\bar{r} = s\hat{s}$ $\quad \bar{r}' = z\hat{z}$

$\bar{r}-\bar{r}' = s\hat{s} - z\hat{z}$

$|\bar{r}-\bar{r}'| = \sqrt{s^2+z^2}$

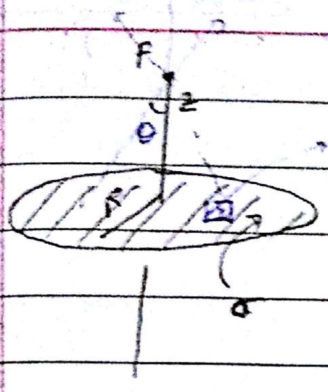
substitute in formula

$\displaystyle\int d\bar{E} = \dfrac{1}{4\pi\epsilon_0}\int \dfrac{\lambda\, dz\,(s\hat{s} - z\hat{z})}{(s^2+z^2)^{3/2}}$

$= \dfrac{\lambda}{4\pi\epsilon_0}\left[\displaystyle\int_{-L/2}^{L/2} \dfrac{dz\, s\hat{s}}{(s^2+z^2)^{3/2}} - \int_{-L}^{+L} \cancelto{0}{\dfrac{z\, dz\,\hat{z}}{(s^2+z^2)^{3/2}}}\right]$

Camlin Page
Date / /

Ans




$da' = s\,ds\,d\phi$

$$E = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]$$

$$2\int dE\cos\theta = \frac{2}{4\pi\epsilon_0}\iint_{\phi=0}^{\pi} \frac{\sigma\, s\,ds\,d\phi\cos\theta}{(s^2+z^2)}$$

$s = z\tan\theta$

$ds = z\sec^2\theta\,d\theta$

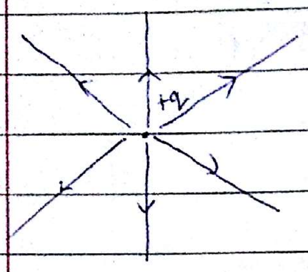
$$= \frac{\sigma}{2\epsilon_0}\int \frac{(z\tan\theta)(z\sec^2\theta\,d\theta)\cos\theta}{z^2\sec^2\theta}$$

$$= \frac{\sigma}{2\epsilon_0}\int \frac{\sin\theta}{\cos\theta}\,d\theta$$

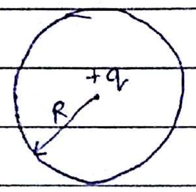
$$= -\frac{\sigma}{2\epsilon_0}\Big[\cos\theta\Big]_0^{\cos^{-1}\left(\frac{z}{\sqrt{R^2+z^2}}\right)}$$

$$\boxed{\vec{E} = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{R^2+z^2}}\right]}$$

Field line :




Flux :



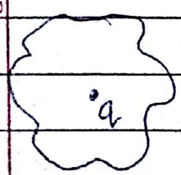

$$\phi = \oint\!\!\oint \bar{E} \cdot d\bar{a}$$

$$= \oint\!\!\oint \left( \frac{q}{4\pi\epsilon_0 r^2} \hat{r} \right) \cdot \left( r^2 \sin\theta \, d\theta \, d\phi \, \hat{r} \right)$$

$$= \frac{q}{4\pi\epsilon_0} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \sin\theta \, d\theta \, d\phi$$

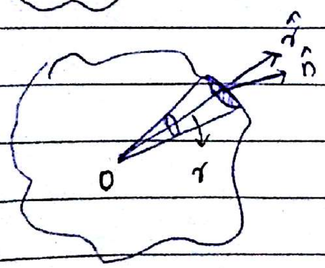
$$= \frac{q}{4\pi\epsilon_0} (2\pi)(2) = \boxed{\frac{q}{\epsilon_0}}$$

$$\boxed{\phi = \oint\!\!\oint \bar{E} \cdot d\bar{a} = \frac{q}{\epsilon_0}}$$ : Gauss's law




Here also, $\phi = \frac{q}{\epsilon_0}$

Proof :-




$$d\Omega = \frac{\hat{n} \cdot \hat{r} \, da}{r^2} \quad \text{(solid angle)}$$

Total solid angle $= 4\pi$ ( ◯ )

$$\oint\!\!\oint d\Omega = 4\pi$$

In Gauss's law,

$$\oint\!\!\oint \frac{q}{4\pi\epsilon_0} \underbrace{\frac{\hat{r}}{r^2} \cdot \hat{n} da}_{d\Omega} = \frac{4\pi}{4\pi} \frac{q}{\epsilon_0} = \frac{q}{\epsilon_0}.$$

: (Independent of shape of closed surface.)

→ $Q_1 \quad Q_2 \quad Q_3$

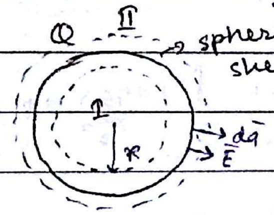
$q_1 \quad q_2 \quad q_3 \quad q_4$ → I

$$E_I = \frac{q_1+q_2+q_3+q_4}{\epsilon_0}$$

$Q_4 \quad Q_5$

* Charges which are outwards don't affect flux (all going inside also comes out, net flux is zero)
* This formula is not mostly used
  Bcoz of formula, $\bar{E} \parallel d\bar{a}$ & $\bar{E}$ should be uniform everywhere on the surface.

(i.) Q — spherical shell




$\Phi_I = 0 \Rightarrow \bar{E}\cdot d\bar{a} = 0$

$E_I \Rightarrow (\bar{E})($

(I) : $\oiint \bar{E}_1 \cdot d\bar{a} = 0$

$\oiint E_1 \, da = 0$

$E_1 \oiint da = 0$

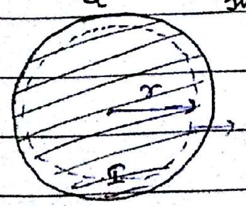
$E_1 (4\pi R^2) = 0 \Rightarrow \boxed{E_1 = 0}$

(II) : $\oiint \bar{E}_2 \cdot d\bar{a} = \dfrac{Q}{\epsilon_0}$

$E_2 (4\pi R^2) = Q/\epsilon_0$

$$\boxed{E_2 = \frac{Q}{4\pi\epsilon_0 R^2}\hat{R}}$$

(ii.) Q Insulated solid sphere




(I) : $\oiint \bar{E}_I \cdot d\bar{a} = \dfrac{Q \times \frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3 \epsilon_0} = \dfrac{Q r^3}{\epsilon_0 R^3}$

II

$\bar{E}_I (4\pi r^2) = \dfrac{Q r^3 / R^3}{\epsilon_0}$

$$\boxed{\bar{E}_1 = \frac{Q r}{4\pi\epsilon_0 R^3}\hat{r}}$$

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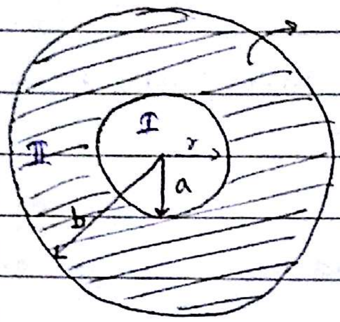
$$\boxed{\bar{E}_2 = \frac{Q}{4\pi\epsilon_0 R^2}}$$

(iii)?



$\rho = \frac{K}{r^2}$ $a \le r \le b$

$\int \rho \cdot$ (we can use $4\pi r^2 dr$ if $\rho$ is not a func$^n$ of $\theta$ & $\phi$)

$$\oint \bar{E}_I \cdot d\bar{a} = 0 \Rightarrow \bar{E}_I (4\pi r^2) = 0 \Rightarrow \bar{E}_I = 0$$

$$\oint \bar{E}_{II} \cdot d\bar{a} = \int \frac{\rho \cdot 4\pi (r^2)}{\epsilon_0} dr = \frac{4\pi K}{\epsilon_0} \int_{r=a}^{r} 1 \, dr$$

$$\bar{E}_{II} (4\pi r^2) = \frac{4\pi K}{\epsilon_0} (r-a)$$

$$\boxed{E_{II} = \frac{K (r-a)}{\epsilon_0 \, r^2}}$$

~~$\bar{E}_{III} (4\pi r^2) =$~~

$$E_{II} (4\pi r^2) = \iiint_{r=a}^{r} \frac{K}{r^2} r^2 \sin\theta \, d\theta \, d\phi \, dr$$

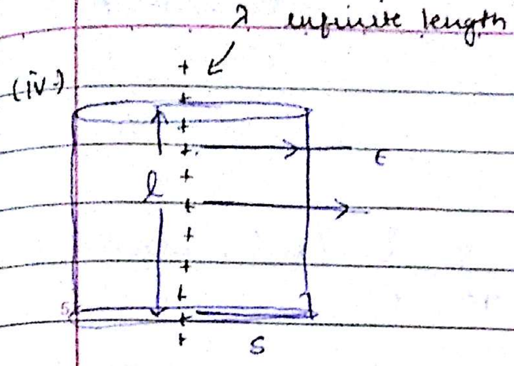
$$= \frac{K (4\pi)(r-a)}{\epsilon_0}$$

$$\bar{E}_{III} (4\pi r^2) = \iiint_{r=b}^{a} \rho \, r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$\boxed{E_{III} = \frac{K (b-a)}{\epsilon_0 r^2}}$$

Quiz : 29th March

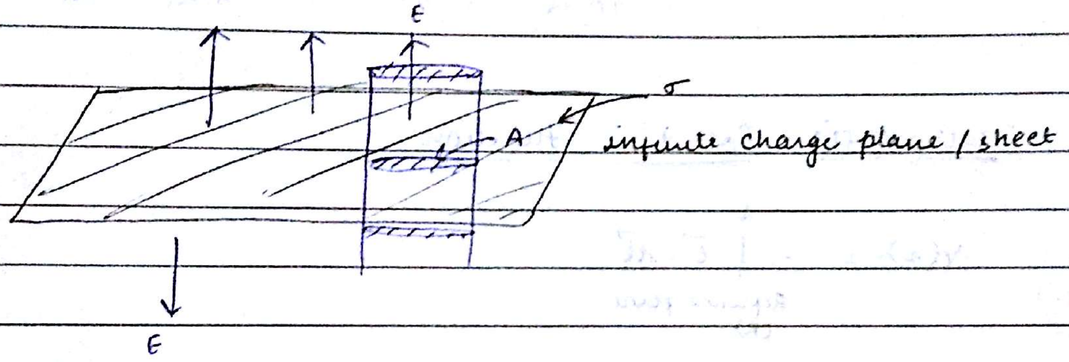
(iv)




$$(E)(2\pi s l) = \frac{\lambda l}{\epsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi s \epsilon_0}\hat{s}}$$

(v)




$$E(2A) = \frac{\sigma A}{\epsilon_0} \Rightarrow \boxed{E = \frac{\sigma}{2\epsilon_0}\hat{n}}$$

⇒

$$\oiint \bar{E}\cdot d\bar{a} = \frac{q}{\epsilon_0}$$

Using divergence theorem

$$\iiint (\bar{\nabla}\cdot\bar{E})\, d\tau = \frac{\iiint \rho(r)\, d\tau}{\epsilon_0}$$

$$\boxed{\bar{\nabla}\cdot\bar{E}(r) = \frac{\rho(r)}{\epsilon_0}}$$ $E(r)$ : +ive 4 +ive charge

→

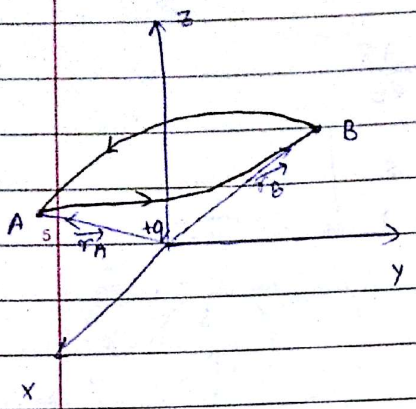
+q → E here ≠ 0 ($\rho(r) \neq 0$)

• → $\bar{\nabla}\cdot\bar{E}$ here would be 0 ($\rho(r) = 0$)

but E won't be 0.

Any conservative vector field can be written as gradient of a scalar funcⁿ.

Date 24 / 03 / 17




$$\oint \vec{E}\cdot d\vec{l}$$

$$\int_A^B \vec{E}\cdot d\vec{l} = \int_A^B \frac{q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}\cdot(dr\,\hat{r} + r\,d\theta\,\hat{\theta} + r\sin\theta\, d\phi\,\hat{\phi})$$

$$= \frac{q}{4\pi\epsilon_0}\int_A^B \frac{1}{r^2}\,dr = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

## Electrostatic Potential Function

$$V(r) = -\int_{\text{Refrence point (R)}}^{r} \vec{E}\cdot d\vec{l}$$

$$V(A) = -\int_R^A \vec{E}\cdot d\vec{r}$$

$$V(B) = -\int_R^B \vec{E}\cdot d\vec{r}$$

$$V(B) - V(A) = -\int_A^B \vec{E}\cdot d\vec{r}$$

$$\int_A^B \nabla V\cdot d\vec{r} = V(B) - V(A)$$

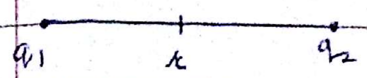
$$\boxed{\vec{E} = -\vec{\nabla} V}$$

V — scalar function

$$V(r) = -\int_R^r \vec{E}\cdot d\vec{l}$$

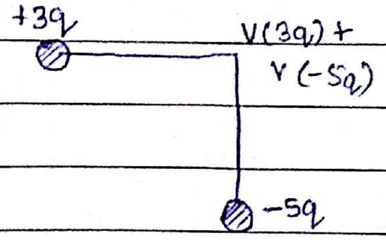
$R \rightarrow R'$

$$V(r') = -\int_{R'}^{r} \vec{E}\cdot d\vec{l} = -\int_R^r \vec{E}\cdot d\vec{l} + \int_{R'}^{R} \vec{E}\cdot d\vec{l} = V(r) + C$$

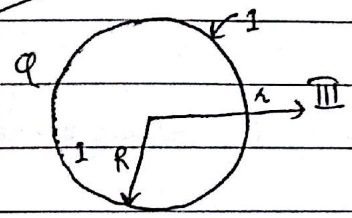



$$= \int_R^r \vec{F}_{me}\cdot\vec{dl} = -\int_R^r \vec{F}_{elec}\cdot\vec{dl}$$

$$= -\int_R^r q_2\vec{E}\,\vec{dl} = q_2\left\{-\int_R^r \vec{E}\cdot\vec{dl}\right\} = \underline{q_2 V(r)}$$




Hollow sphere

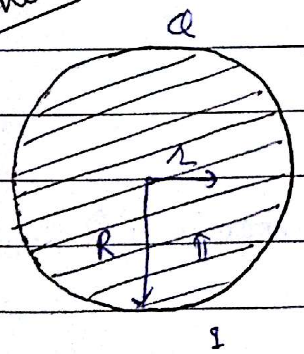



VIN $V_{III}(r) = -\int_\infty^r \frac{Q}{4\pi\epsilon_0 r^2}\,dr = \frac{Q}{4\pi\epsilon_0 r}$

$$V_{II}(R) = \frac{Q}{4\pi\epsilon_0 R}$$

$$V_I(r) = -\int_\infty^r \bar{E}\cdot d\bar{r} = -\int_\infty^R \bar{E}\cdot dr - \int_R^r \cancel{\bar{E}\cdot dr} = \frac{Q}{4\pi\epsilon_0 R}$$

Solid sphere




$$dq = \frac{Q}{\frac{4}{3}\pi R^3}\times 4\pi r^2 dr = \frac{3Qr^2 dr}{R^3}$$

$$E = \frac{dQ}{4\pi\epsilon_0}$$

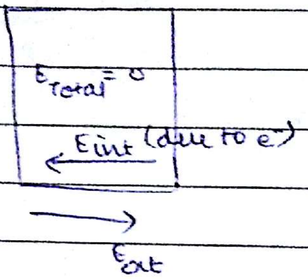
$$V_I = \frac{Q}{4\pi\epsilon_0 R}$$

$$V_{II} = \int \vec{E}\cdot\vec{dl} = \frac{kQ}{2R^3}\left[3R^2 - r^2\right]$$

## Conductors

**Properties :**

(i)



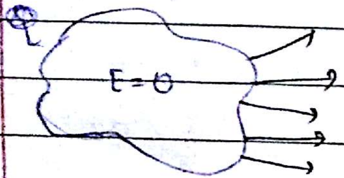

If conductor is kept in uniform external $E_{ext}$, $e^-$ adjust in such a way that $\bar{E}_{net}$ inside $= 0$

(ii) We know that

$$\bar{\nabla} \cdot \bar{E} = \frac{\rho}{\epsilon_0}$$

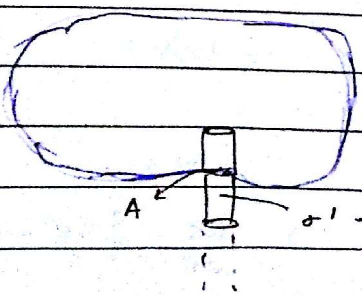
if $\bar{E} = 0 \Rightarrow \rho = 0 \Rightarrow$ charge density (net) $= 0$ everywhere

(iii) If we've some excess charge it will reside on surface. It will be distributed in such a way that $E = 0$ inside (max at sharper edges)




(iv) Everywhere, potential difference is zero. (Equipotential $\exists$ surface)
$\rightarrow$ E is always $\perp^r$ outside. ($E_{\parallel} = 0$)

(v)



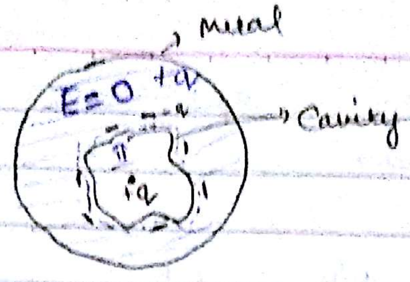

$\sigma' \rightarrow$ local surface charge density.

$$EA = \frac{\sigma' A}{\epsilon_0} \Rightarrow \boxed{\vec{E} = \frac{\sigma'}{\epsilon_0}\hat{n}}$$

$\downarrow$ electric field close to the surface.

* Electrostatic field is conservative field



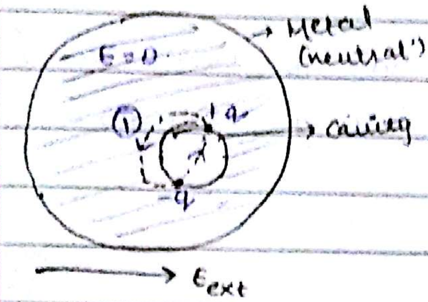
→ 


Gaussian surface

$$\oiint \vec{E}\cdot d\vec{a} = \frac{q_{enc}}{\epsilon_0}$$

$0 = +q + (-q)$ ⇒ $-q$ will be induced on inner surface

⇒ $+q$ will be induced on outer metal surface.

→ 


faraday cage

Let cavity induce some charge ($q_{net} = 0$ as $E = 0$)

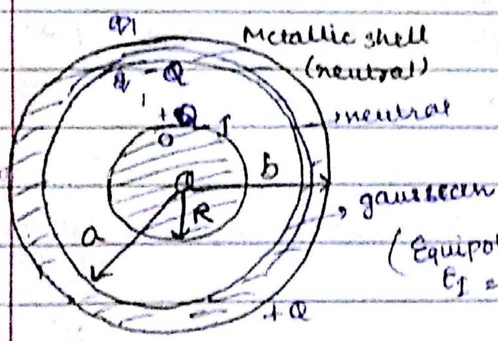
Gaussian says that:

$0 \neq \oint \vec{E}\cdot d\vec{l} = 0$

We can take a path like ①

inside the cavity, $\oint \vec{E}\cdot d\vec{l} \neq 0$ ($\because \vec{E}$ due to $+q$, $-q$)

Hence, the cavity has no affect of external electrostatic field

Eg. 


Find surface charge density on different surfaces. Also find potential at the centre. $\sigma_R = ?$ $\sigma_a = ?$ $\sigma_b = ?$

(Equipotential)

$E_f = 0 \Rightarrow \frac{KQ}{R} = \frac{-Kq}{R} - \frac{Kq}{a}$ $\quad \frac{2KQ}{R} = \frac{\cancel{K}q}{a}$

$q = -2Q/R$

$$\sigma_R = \frac{Q}{4\pi R^2} \qquad \sigma_a = \frac{-Q}{4\pi a^2} \qquad \sigma_b = \frac{Q}{4\pi b^2}$$

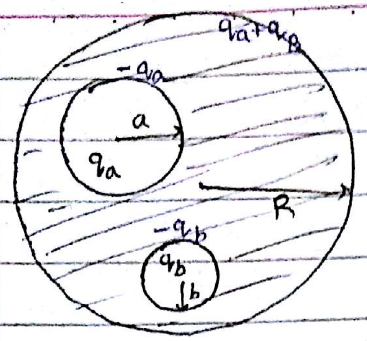
Potential at centre : $-\int_{\infty}^{0} \vec{E}\cdot d\vec{l} = -\int_{\infty}^{b} \vec{E}\cdot d\vec{l} - \int_{b}^{a} \cancel{\vec{E}}\cdot d\vec{l} - \int_{a}^{r} \vec{E}\cdot d\vec{l} - \underbrace{\int_{R}^{0} \vec{E}\cdot d\vec{l}}_{0}$

$$= \cancel{\frac{KQ}{2R^3}\left[3R^2\right]} = \frac{3Q}{2\cdot 4\pi\epsilon_0 R}$$

$$= -\int_{\infty}^{b} \vec{E}\cdot d\vec{l} = -\int_{\infty}^{b} \frac{KQ}{r^2}\,dr = \frac{KQ}{b}$$

$$-\int_{a}^{r} \vec{E}\cdot d\vec{l} = -\int_{a}^{R} \frac{KQ}{r^2}\,dr = \frac{-KQ}{a} + \frac{KQ}{R}$$

Ques




$\sigma_a = -q_a/4\pi a^2$ $\quad\sigma_R = (q_a+q_b)/4\pi R^2$

$\sigma_b = -q_b/4b^2\pi$

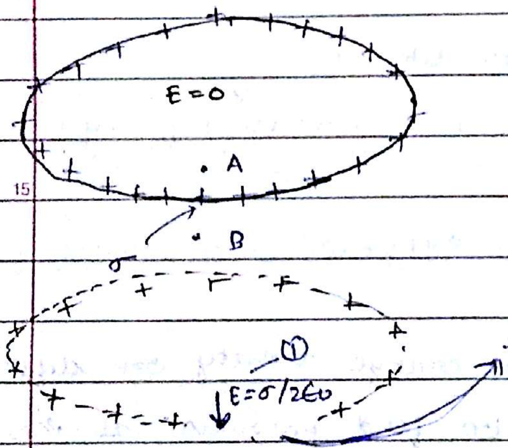
What is force b/w a & b?

0 (a & b are isolated as E b/w them is 0)

$\cdot Q(c)$

If bring $+Q_c$ near the conductor, $q_a + q_B$ won't be distributed uniformely, hence, $\sigma_R$ will get changed.

## Electrostatic Pressure




A and B are very close to surface

$\vec{E}_B = \dfrac{\sigma}{\epsilon_0}\hat{n}$ $\qquad \vec{E}_A = 0$

$\cdot\ E_{above} = \sigma/2\epsilon_0 \uparrow$

$+ + + + + + + +$ (treat as infinite charge plate)

$\cdot\ E_{below} = \sigma/2\epsilon_0 \downarrow$

① Here $E = \dfrac{\sigma}{2\epsilon_0}\downarrow$ such that when that segment is added (as in previous figure), the $E = 0$.

$$\boxed{\underset{\Downarrow}{|E|\sigma} = \frac{\sigma^2}{2\epsilon_0}}$$ : Electrostatic pressure

Electric field × charge per unit area = ~~Fo~~ force per unit are = pressure

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