Classical Mechanics

(A few Problems on Variable mass, Work and Energy, and Collisions)

(For practice and for your own satisfictation)

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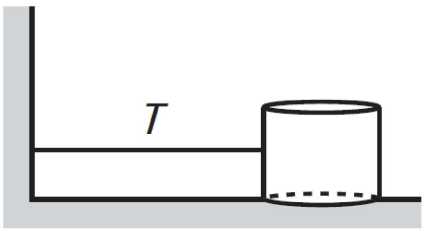
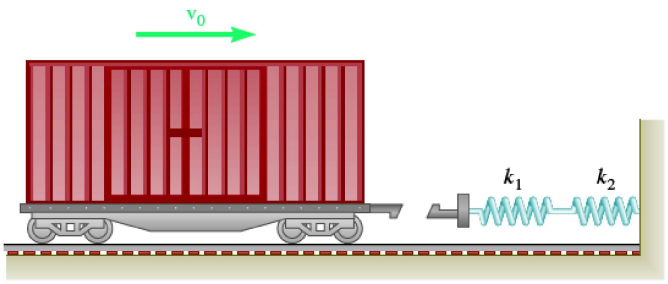
1. At $t = 0$, a massless bucket contains a mass M of sand. It is connected to a wall by a massless spring with constant tension T (that is, independent of length). See Fig. The ground is frictionless, and the initial distance to the wall is L. At later times, let x be the distance from the wall, and let m be the mass of sand in the bucket. The bucket is released, and on its way to the wall, it leaks sand at a rate $\frac{dm}{dx} = \frac{M}{L}$. In other words, the rate is constant with respect to distance, not time; and it ends up empty right when it reaches the wall. Note that dx is negative, so dm is also.



   - What is the kinetic energy of the (sand in the) bucket, as a function of x? What is its maximum value?
   - What is the magnitude of the momentum of the bucket, as a function of x? What is its maximum value?

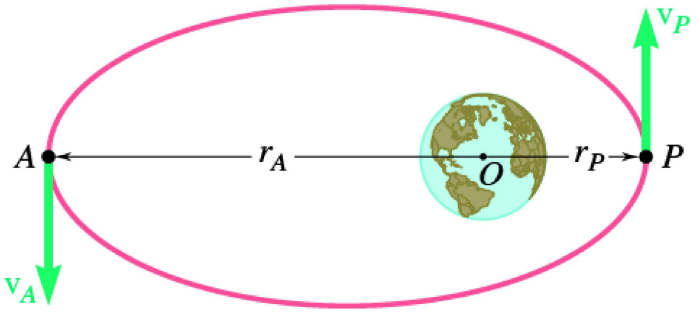
2. Consider the setup in the above problem, but now let the sand leak at a rate proportional to the buckets acceleration. That is, $\frac{dm}{dx} = \frac{d^2x}{dt^2}$.. Note that $\frac{d^2x}{dt^2}$is negative, so is dm.

   - Find the mass as a function of time, m(t).
   - Find v(t) and x(t) during the time when the bucket contains a nonzero amount of sand. Also find v(m) and x(m). What is the speed right before all the sand leaves the bucket (assuming it hasnt hit the wall yet)?
   - What is the maximum value of the buckets kinetic energy, assuming it is achieved before it hits the wall?
   - What is the maximum value of the magnitude of the buckets momentum, assuming it is achieved before it hits the wall?
   - For what value of b does the bucket become empty right when it hits the wall?



3. A loaded railroad car of mass m is rolling at a constant velocity v_0 when it couples with a massless bumper system. Determine the maximum deflection of the bumper assuming the two springs are (a) in series (as shown), (b) in parallel.

4. Show that the values v_A and v_P of the speed of an earth satellite at the apogee A and the perigee P of an elliptic orbit are defined by the relations,



$$v_A^2 = \frac{2GM}{r_P + r_A}\frac{r_P}{r_A}, \quad v_P^2 = \frac{2GM}{r_P + r_A}\frac{r_A}{r_P}$$

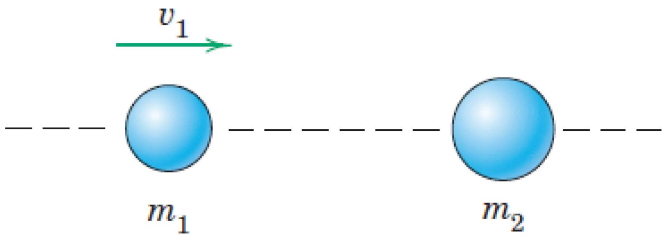
where M is the mass of the earth, and r_A and r_P represent, respectively, the maximum and minimum distances of the orbit to the center of the earth. Also show that the total energy E of an earth satellite of mass m describing an elliptic orbit is $E = GMm/(r_P + r_A)$.

5. A projectile proton with a speed of 500 m/s collides elastically with a target proton initially at rest. The two protons then move along perpendicular paths, with the projectile path at 60 from the original direction. After the collision, what are the speeds of (a) the target proton and (b) the projectile proton?

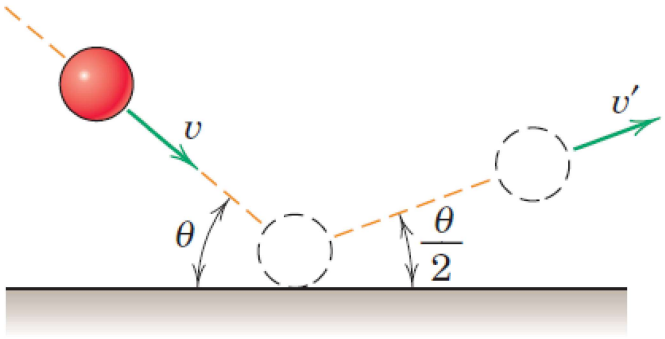
6. **Coefficient of Restitution:** The capacity of the contacting bodies to recover from the impact can be expressed by the ratio e of the magnitude of the restoration impulse to the magnitude of the deformation impulse. This ratio is called the coefficient of restitution and is given as,

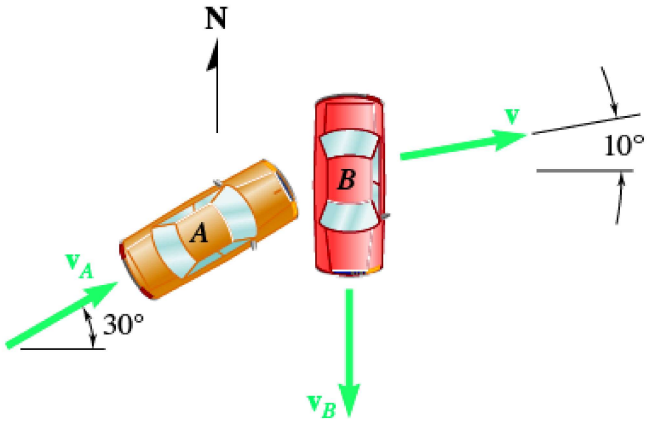
$$e = \frac{|\text{relative velocity of separation}|}{|\text{relative velocity of approach}|}$$

Problem: The sphere of mass m_1 travels with an initial velocity v_1 directed as shown and strikes the sphere of mass m_2. For a given coefficient of restitution e, determine the mass ratio m_1/m_2 which results in m_1 being motionless after the impact.



7. Determine the value of the coefficient of restitution e for which the outgoing angle is one-half of the incoming angle as shown. Evaluate your general expression for $\theta = 40^o$.





8. At an intersection, car B was traveling south and car A was traveling 30^o north of east when they slammed into each other. Upon investigation, it was found that after the crash the two cars got stuck and skidded off at an angle of 10^o north of east. Each driver

claimed that he was going at the speed limit of 50 km/h and that he tried to slow down but couldnt avoid the crash because the other driver was going a lot faster. Knowing that the masses of cars A and B were 1500 kg and 1200 kg, respectively, determine (a) which car was going faster, (b) the speed of the faster of the two cars if the slower car was traveling at the speed limit.

Successful investigator of problem 8 will get a chocolate upon explaining it on the board!

Prepared by Nabyendu Das
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