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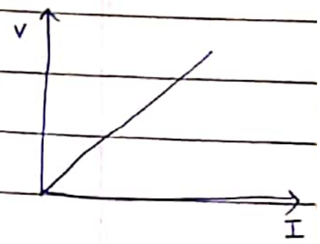
Ohm's Law

$V \propto I$

$\tan\theta = \frac{dV}{dI}$



$V = RI$

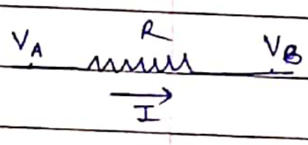
R = Resistance

$V_d = \mu E$

$I = nqAV_d$

$= nqA\mu E$

$= nqA\mu \frac{V}{l}$



$I = \frac{V_A - V_B}{R}$

$\frac{V}{I} = \left(\frac{1}{nq\mu}\right)\frac{l}{A} = R$

$R = \rho \frac{l}{A}$ $\rho = \frac{1}{nq\mu}$ = resistivity

For ohm's law -

l = same A = same T = Temp.

Kirchoff's Laws

Kirchoff's Current Law (KCL) -

$\sum I = 0$

$I_{entering} = I_{leaving}$ at a node

When two passive elements are connected and there is a division of current then it is node.

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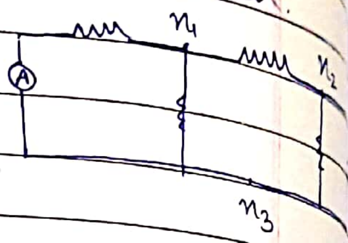
Path connecting 2 nodes is a branch.

$n = 3$ $b = 5$ $l = 3$

$b = l + n - 1$

$5 = 3 + 3 - 1$



Kirchoff's Voltage Law

Summation of voltage drop across the circuit is equal to zero.

Passive Components

1) Inductor (L) - Unit is Henry

$$V = L\frac{di}{dt}$$

$$i = \frac{1}{L}\int_0^t V dt$$

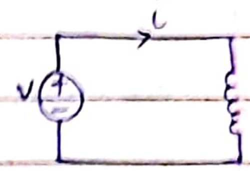
$$P = V \times I$$

$$\frac{dw}{\cancel{dt}} = Li\frac{di}{\cancel{dt}}$$

$$dw = Li\,di$$

$$W = \frac{1}{2}LI^2$$

Q. Calculate P & W ?

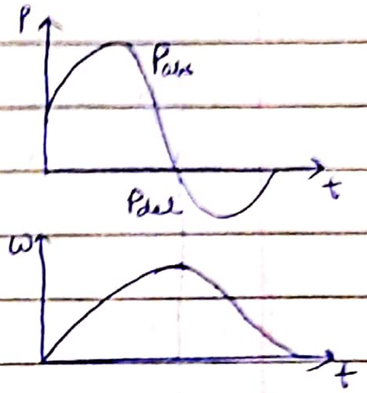


Ans. $i = 10e^{-5t}$ A

$V = e^{-5t}(1-5t)$ V

$P = 10t\,e^{-10t} - 50t^2 e^{-10t}$

$W = 5t^2 e^{-10t}$



Conclusion -

1) Inductor opposes instantaneous change in current.
2) Inductor only works with A.C. current. It will short-circuit with D.C. current.
3) Inductor does not dissipate energy as heat. It stores energy and then delivers it.

2) Capacitor (C) - Unit is Farad

$$i = \frac{C\,dv}{dt}$$

$$V = \frac{1}{C}\int i\,dt$$

$$P = V \times I$$

$$\frac{dw}{dt} = V \times C\frac{dv}{dt}$$

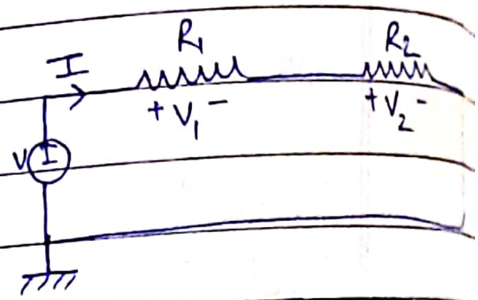
$$dw = C\,V\,dv$$

$$W = \frac{1}{2}CV^2$$

Conclusion

1) Acts as open circuit to D.C current
2) Opposes sudden change in voltage
3) Does not dissipate energy.

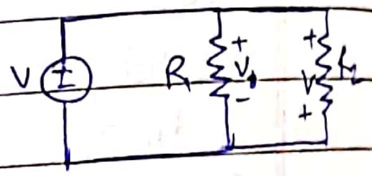
Voltage Division



$V_1 = IR_1$

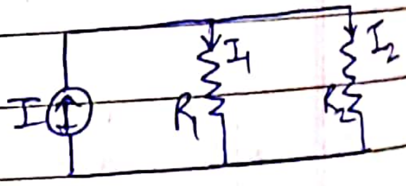
$V_2 = IR_2$

$V = V_1 + V_2 = I(R_1+R_2)$



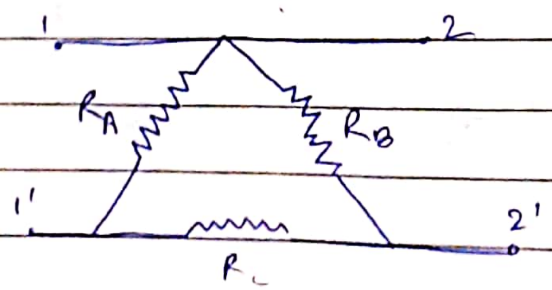
$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$

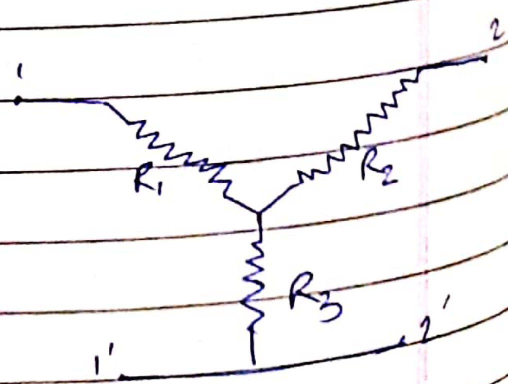
Current Division



$I_1 = \frac{IR_2}{I_1+I_2}$ $\quad$ $I_2 = \frac{IR_1}{I_1+I_2}$

Delta to Y Conversion





$R_{11'} = R_a/(R_b+R_c)$

$R_{22'} = R_b/(R_a+R_c)$

$R_{12} = R_c/(R_a+R_b)$

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$$R_{11'} = R_1 + R_3$$
$$R_{22'} = R_2 + R_3$$
$$R_{12} = R_1 + R_2$$

$$R_1 + R_3 = R_a /(R_b + R_c) \quad - (1)$$
$$R_2 + R_3 = R_b /(R_a + R_c) \quad - (2)$$
$$R_1 + R_2 = R_c /(R_a + R_b) \quad - (3)$$

(2) − (1)

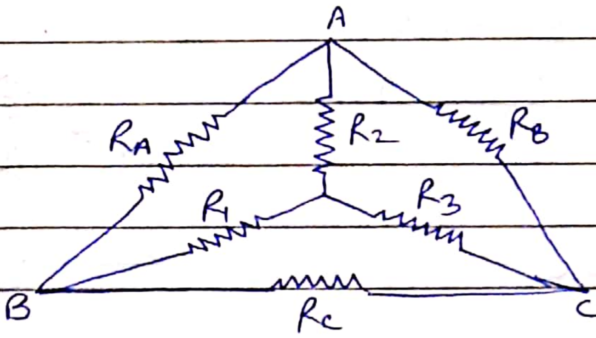
$$R_1 - R_2 = R_a/(R_b + R_c) - R_b(R_a + R_c) \quad - (4)$$

(3) + (4)

$$2R_1 = R_c/(R_a+R_b) + R_a/(R_b+R_c) - R_b(R_a+R_c)$$
$$= \frac{R_c(R_a+R_b)}{R_a+R_b\,R_c} + \frac{R_a(R_b+R_c)}{R_a+R_b+R_c} - \frac{R_b(R_a+R_c)}{R_a+R_b+R_c}$$

$$2R_1 = \frac{R_cR_a + \cancel{R_cR_b} + \cancel{R_aR_b} + R_aR_c - \cancel{R_aR_b} - \cancel{R_bR_c}}{R_a+R_b+R_c}$$

$$R_1 = \frac{R_aR_c}{R_a+R_b+R_c}$$



$$R_1 = \frac{R_aR_c}{R_a+R_b+R_c}$$
$$R_2 = \frac{R_aR_b}{R_a+R_b+R_c}$$
$$R_3 = \frac{R_bR_c}{R_a+R_b+R_c}$$

(1) $\dfrac{R_1}{R_2} = \dfrac{R_c}{R_b}$ (2) $\dfrac{R_2}{R_3} = \dfrac{R_a}{R_c}$ (3) $\dfrac{R_1}{R_3} = \dfrac{R_a}{R_b}$

(4) $R_1R_2 = \dfrac{R_a^2R_bR_c}{(R_a+R_b+R_c)^2}$ (5) $R_2R_3 = \dfrac{R_aR_b^2R_c}{(R_a+R_b+R_c)^2}$

(6) $R_3R_1 = \dfrac{R_aR_bR_c^2}{(R_a+R_b+R_c)^2}$

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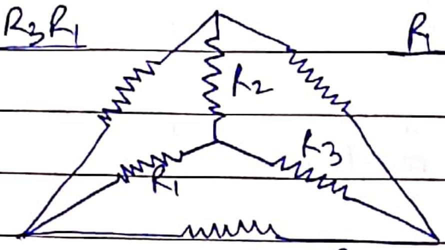
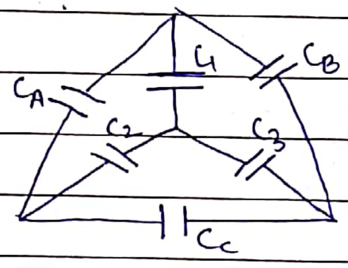
$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_aR_bR_c}{R_a+R_b+R_c} \quad — (7)$$

$$R_b = \frac{R_1R_2 + R_3R_2 + R_3R_1}{R_1}$$

$$\frac{R_1R_2+R_2R_3+R_3R_1}{R_3} \qquad \frac{R_1R_2+R_2R_3+R_3R_1}{R_1}$$



$$\frac{R_1R_2+R_2R_3+R_3R_1}{R_2}$$

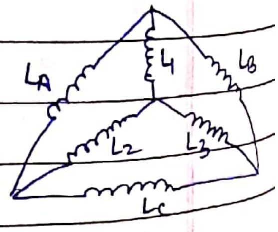


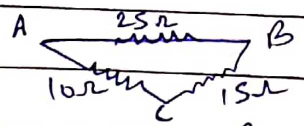
$$\frac{1}{\omega C_1} = \frac{\frac{1}{\omega C_A} \times \frac{1}{\omega C_B}}{\frac{1}{\omega C_A} + \frac{1}{\omega C_B} + \frac{1}{\omega C_c}}$$

$$\frac{1}{C_1} = \frac{C_c}{C_BC_c + C_AC_c + C_AC_B}$$

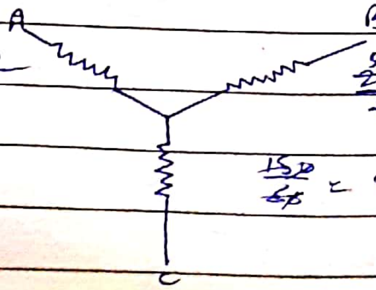
$$C_1 = \frac{C_BC_c + C_AC_c + C_AC_B}{C_c}$$

$$L_1 = \frac{L_AL_B}{L_A+L_B+L_C}$$



d. 

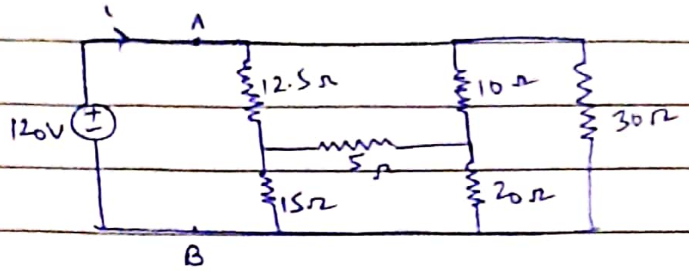
Ans. $\frac{250}{25+10+15} = 5\Omega$



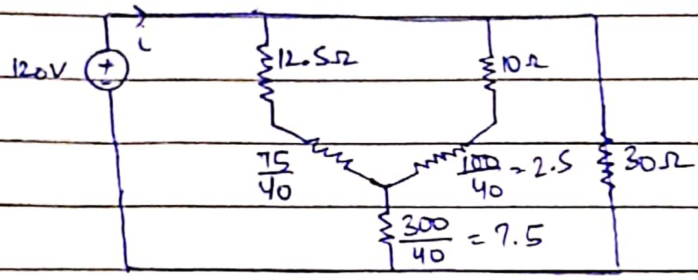
$\frac{25\times15}{50} = 7.5\Omega$

$\frac{150}{50} = 3\Omega$

Q. Find R_{ab} & i ?



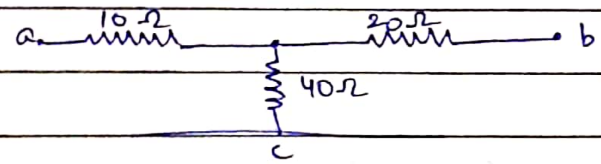
Ans.



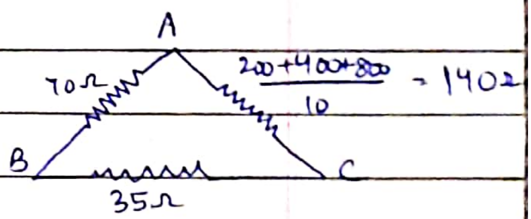
$R_{ab} = 9.63\,\Omega$

$i = 12.45\,A$

Q.



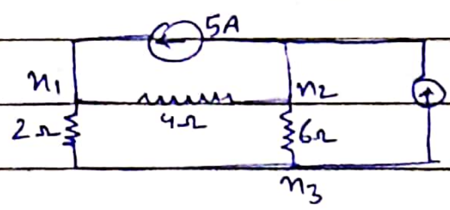
Ans.

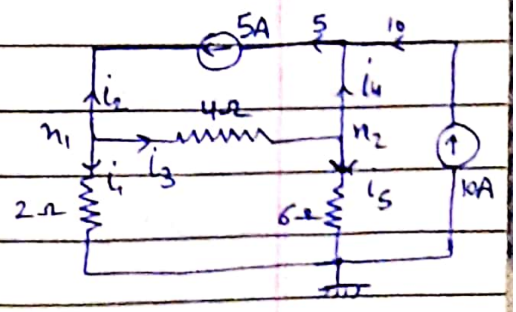


Nodal Analysis

→ Based on KCL
→ Node variable
→ Among all one will be reference node and others will be variable ($V_1, V_2, V_3, \quad V_{n-1}$)

Q.





Ans. let $V_{n_3} = 0$

At node n_1

$i_1 + i_2 + i_3 = 0$

$$\frac{V_1 - 0}{2} + (-5) + \frac{V_1 - V_2}{4} = 0$$

$V_1 - 5 + V_1 - V_2 = 0$

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$$\frac{3}{4}V_1 - \frac{V_2}{4} = 5$$

$$3V_1 - V_2 = 20 \quad -(1)$$

At node n_2

$$i_4 + i_5 = i_3$$

$$5+(-10) + \frac{V_2 - 0}{6} = \frac{V_1 - V_2}{4}$$

$$\frac{V_2}{6} - \frac{V_1}{4} + \frac{V_2}{4} = 10-5$$

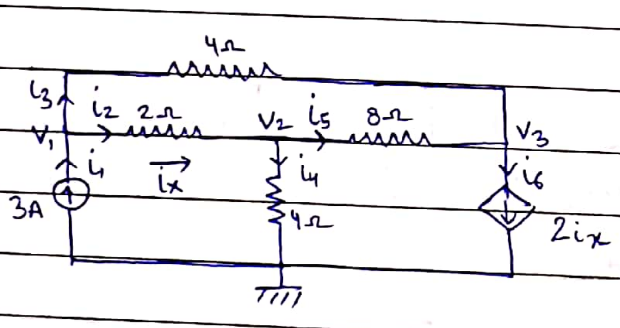
$$\frac{5}{12}V_2 - \frac{V_1}{4} = 10-5$$

$$5V_2 - 3V_1 = 60 \quad -(2)$$

(2) + (1)

$$5V_2 - \cancel{3V_1} = 60$$
$$+\ \cancel{3V_1} - V_2 = 20$$
$$4V_2 = 80 \Rightarrow V_2 = 20V, \quad V_1 = \frac{40}{3}V$$

Q.



Ans. At node n_1,

$$i_1 + i_2 + i_3 = 0$$

$$-3 + \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} = 0 \quad -(1)$$

At node n_2,

$$i_4 + i_5 - i_2 = 0$$

$$\frac{V_2}{4} + \frac{V_2 - V_3}{8} - \frac{V_1 - V_2}{2} = 0 \quad -(2)$$

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At node n_3,

$$-i_3 - i_5 + i_6 = 0$$

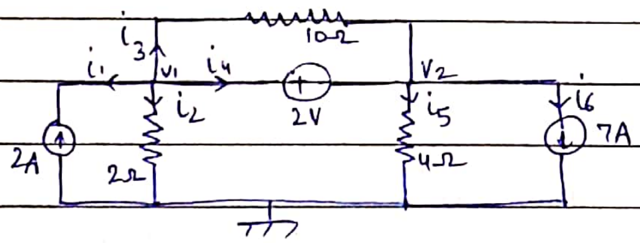
$$-\frac{V_1 - V_3}{4} - \frac{V_2 - V_3}{8} + 2i_x = 0 \quad — ③$$

$$i_x = i_2 = \frac{V_1 - V_2}{2}$$

$$-\frac{V_1 - V_3}{4} - \frac{V_2 - V_3}{8} + V_1 - V_2 = 0 \quad — ④$$

$V_1 = 4.8\,V$ $\quad V_2 = 2.4\,V$ $\quad V_3 = -2.4\,V$

Q.



Ans. At node n_1,

$$i_1 + i_2 + i_3 + i_4 = 0$$

$$-2 + \frac{V_1}{2} + \frac{V_1 - V_2}{10} + i_4 = 0 \quad — ①$$

At node n_2,

$$i_5 + i_6 - i_4 - i_3 = 0$$

$$\frac{V_2}{4} + 7 - i_4 - \frac{V_1 - V_2}{10} = 0 \quad — ②$$

① + ②

$$-2 + \frac{V_1}{2} + \cancel{\frac{V_1 - V_2}{10}} + \cancel{i_4} + \frac{V_2}{4} + 7 - \cancel{i_4} - \cancel{\frac{V_1 - V_2}{10}} = 0$$

$$\frac{V_1}{2} + \frac{V_2}{4} = -5$$

$$2V_1 + V_2 = -20 \quad — ③$$

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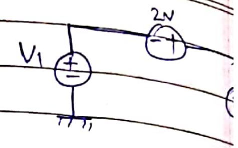
Using KVL 

$+V_1 + 2 - V_2 = 0$

$V_1 - V_2 = -2$ — (4)

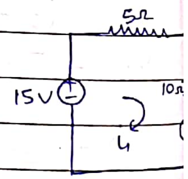
$V_1 = -2.33V$ $V_2 = -5.33V$

- Supernode - whenever a voltage exist two non-reference nodes than that is known as super-node.

Mesh Analysis

- Circuit variable current
- Applied only for planar N/W (Network)
- Variant of loop analysis
- Special case - Super mesh (current source two mesh)
- Apply KVL around loop

Q. Find i_1 & i_2 ?



Ans. For loop 1

$15 - 5i_1 - 10(i_1 - i_2) - 10 = 0$ — (1)

For loop 2

$10 - 10(i_2 - i_1) - 6i_2 - 4i_2 = 0$ — (2)

$i_1 = 1A$

$i_2 = 1A$

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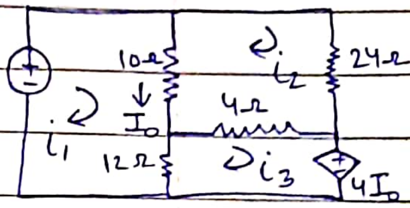
Q. Find i_1, i_2 & i_3 ?



Ans. For loop ①

$24 - 10(i_1 - i_2) - 12(i_1 - i_3) = 0$ — ①

For loop ②

$-10(i_2 - i_1) - 24i_2 - 4(i_2 - i_3) = 0$ — ②

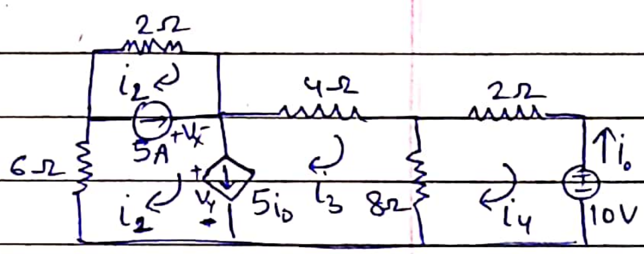
For loop ③

$-12(i_3 - i_1) - 4(i_3 - i_2) - 4I_o = 0$ — ③

$I_o = i_1 - i_2$

$i_1 = 2.25A$ $i_2 = 0.75A$ $i_3 = 1.5A$

Q. Find currents ?



Ans. For loop 1

$-6i_1 + V_x - V_y = 0$ — ①

For loop 2

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$\Rightarrow -6i_1 - 2i_2 - 4i_3 - 8(i_3 - i_4) = 0$

$-6i_1 - 2i_2 - 12i_3 + 8i_4 = 0$

$i_1 = -7.5A$ $i_2 = -2.5A$ $i_3 = 3.93A$

$i_4 = 2.14A$

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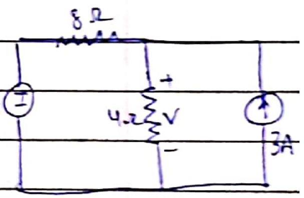
Network Theorems

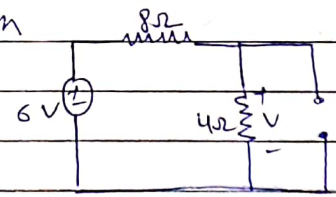
Superposition	Thevenin	Norton
• Not for dependent sources • Voltage & current across an element in a circuit (linear circuit) is the algebraic sum of voltage & current resp. in that element due to each independent source acting alone. • Steps - 1) Turn off all the	• Any linear circuit can be transformed into an equivalent circuit consisting of a voltage source with a series resistance. Steps - 1) The equivalent voltage source is the open circuit voltage across an element of	• It is equivalent to Thevenin theorem where the representation is in the form of a current source in parallel with a resistance. The current source is the short circuit current flowing through the component across which we need to calculate.

2) Repeat the above step with other sources.	2) Turn off all the independent sources.
3) Find the total contribution by doing algebraic sum obtained by each independent sources.	

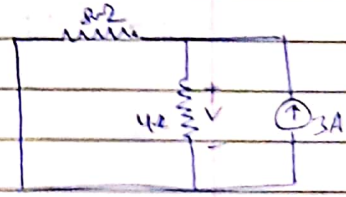
Q. Calculate V ?



Ans. By superposition



$V_1 = \frac{6}{8+4} \times 4 = 2V$

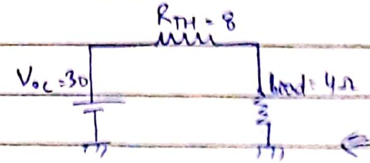


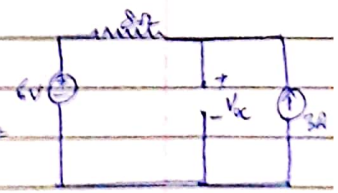
$V_2 = \left(\frac{8}{8+4} \times 3\right) \times 4 = 8V$

$V = V_1 + V_2$

$= 2 + 8 = 10V$

By Thevenin





$V_{oc} = 6 - (-24)$

$= 30V$

$R_{TH} = 8Ω$

$V = \frac{30}{8+4} \times 4 = 10V$

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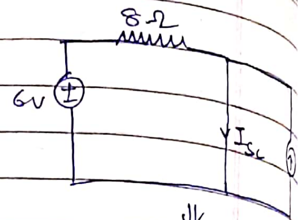
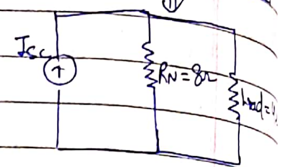
By Norton



$I_{sc} = \frac{6}{8} + 3$

$= \frac{15}{4}$



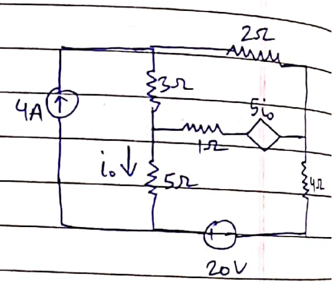
$R_N = 8Ω$

$I_2 = \frac{8}{8+4} \times \frac{15}{4}$

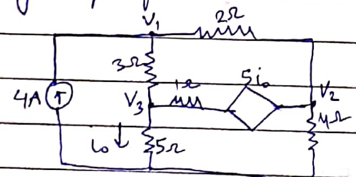
$= \frac{5}{2}$

$V = \frac{5}{2} \times 4 = 10V$

Q. Calculate i_o using superposition?



Ans. By superposition



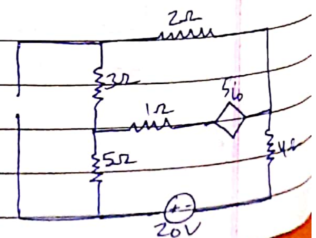
At n_1

$\frac{V_1 - V_3}{3} + \frac{V_1 - V_2}{2} - 4 = 0$

Node 2 - $\frac{V_2 - V_1}{2} + \frac{V_2}{4} + \frac{V_2 + 5i_o - V_3}{1} = 0$

Node 3 - $\frac{V_3}{5} + \frac{V_3 + 5i_o - V_2}{1} + \frac{V_3 - V_1}{3} = 0$

$i_o = \frac{V_3}{5}$



$i_o = \frac{52}{17} A$

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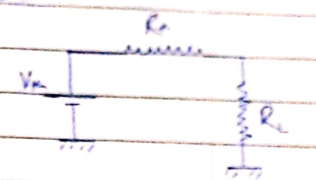
Maximum Power Transfer Theorem



$P = i^2 R_L$

$P = \left(\frac{V_{th}}{R_{th} + R_L}\right)^2 R_L$

$\frac{dP}{dR_L} = \left[\frac{(R_L + R_{th})^2 \times 1 - R_L\, 2\,(R_L + R_{th})}{(R_{th} + R_L)^4}\right]$

$= \frac{R_{th} + R_L - 2R_L}{(R_{th} + R_L)^3} = \frac{R_{th} - R_L}{(R_{th} + R_L)^3} = 0$

$\Rightarrow R_{th} = R_L$

$P_{va} = V_{th} \times I_{th}$

Available Power $= V_{th} \times \frac{V_{th}}{R_{th} + R_L}$

$= \frac{V_{th}^2}{R_{th} + R_L}$

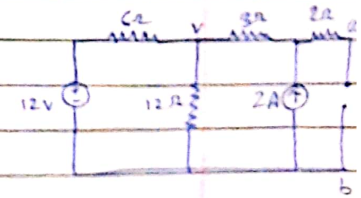
$P_{RL} = i^2 R_L$

$= \frac{V_a^2}{(R_{th} + R_L)^2} \times R_L = \frac{V_{th}^2}{4R_L^2} \times R_L$

$= \frac{V_{th}^2}{4R_L}$

$P_{RL} = \frac{1}{2} P_{va}$

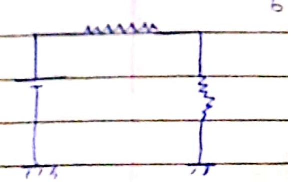
Q. Find V_{ab}?



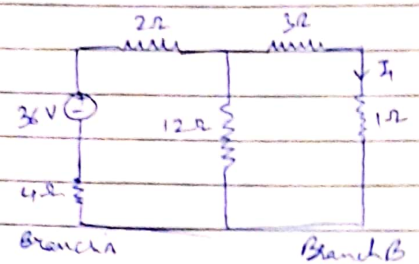
Ans. $\frac{V-12}{6} + \frac{V}{12} + (-2) = 0$

$V = 16V$

$V_{ab} = 16 - (-6) = 22V$



Reciprocity (V & I Interchangable)



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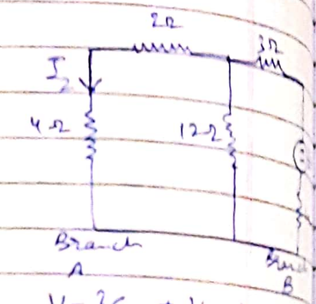


$$\frac{V-36}{6} + \frac{V}{12} + \frac{V}{4} = 0$$

$$V = 12V$$

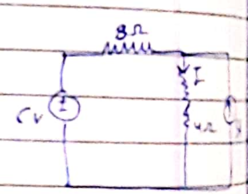
$$I_1 = \frac{V-0}{4} = \frac{12}{4} = 3A$$

$$\frac{V-36}{4} + \frac{V}{12} + \frac{V}{6} = 0$$

$$V = 18V$$

$$I_2 = \frac{V}{6} = \frac{18}{6} = 3A$$

Q.

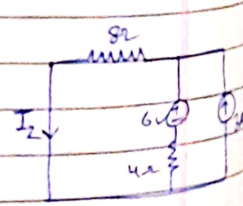


Ans

$$\frac{V-6}{4} + \frac{V}{8} - 3 = 0$$

$$V = 12V$$

$$I_1 = \frac{12-6}{4} = \frac{6}{4} V$$

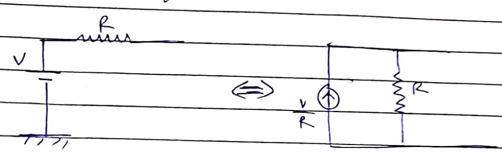


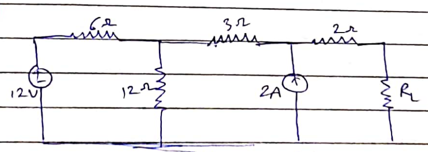
$$\frac{V-6}{8} + \frac{V}{4} - 3 = 0$$

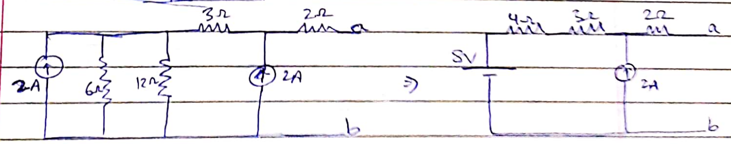
$$V = 10V$$

$$I_2 = \frac{V}{4} = \frac{10}{4} V$$

Source Transformation

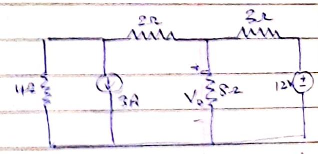


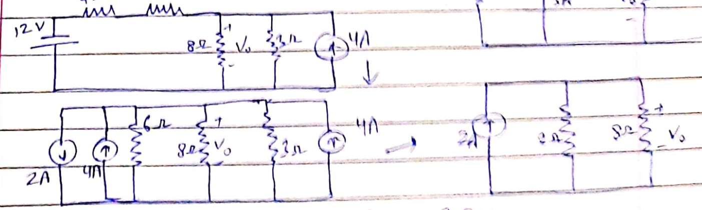
Q. 

Ans. 

$8 - (-14) \rightarrow 22V$

Q. Find V_o



Ans. 

$V_o = I \times 8 = \frac{2 \times 6 \times 8}{10} = 9.6V$

Transients

D.C. Transients
- R.C
- R.L.

R.C / R.L. → Source free
R.C / R.L. → Forced Source → AC, DC

$R \rightarrow V = IR \rightarrow$ Eq. $\rightarrow$ Algebraic

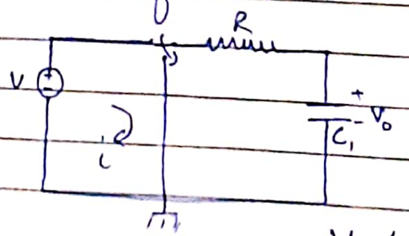
$L \rightarrow V = L\frac{di}{dt}$
$C \rightarrow i = C\frac{dv}{dt}$
} 1st order differential eq.

	Transient	Steady State
	$t = 0^+ 0^-$	∞
Capacitor	V_c	
Inductor	I_o	

$X_c = \frac{1}{2\pi f c}$ $\quad t = 0 \rightarrow X_c = \infty$

$X_L = 2\pi f L$ $\quad t = 0 \rightarrow X_L = 0$

Source from R.C.



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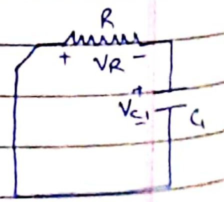


$V_{c_1}(0^-) = V_{c_1}(0^+) = V_0$

$$iR + V_{C_1} = 0 \Rightarrow C_1\frac{dV_{C_1}}{dt}R + V_{C_1} = 0$$

$$RC\frac{dV_{C_1}}{dt} + V_{C_1} = 0 \Rightarrow \frac{dV_{C_1}}{dt} = \frac{-V_{C_1}}{RC}$$

$$\int_0^{V(t)} \frac{dV_{C_1}}{V_{C_1}} = \int_0^t \frac{-1}{RC}\,dt$$

$$\Rightarrow \left[\ln V_{C_1}\right]_0^{V(t)} = \frac{-1}{RC}\left[t\right]_0^t + \ln A$$

$$\ln V(t) = \frac{-1}{RC}t + \ln A$$

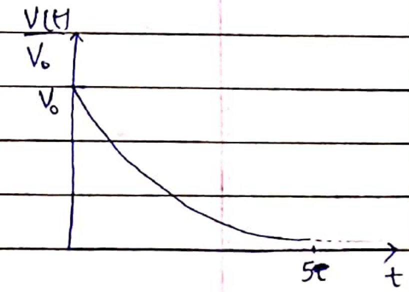
$$\frac{\ln V(t)}{A} = \frac{-t}{RC} \Rightarrow V(t) = Ae^{-t/RC}$$

At $t = 0$ $V(t) = V_0$

$$V_0 = Ae^{-t/RC} \Rightarrow A = V_0$$

$$V(t) = V_0 e^{-t/RC}$$

$$V(t) = V_0 e^{-t/\tau}, \quad \tau = RC \text{ (Time constant)}$$



$$i(t) = \frac{V(t)}{R} = \frac{V_0 e^{-t/RC}}{R}$$

$$P_R = i^2(t)R$$

$$= \frac{V_0^2 e^{-2t/RC}}{R^2} \times R$$

$$P_R = \frac{V_0^2}{R} e^{-2t/RC}$$

$$W_R(t) = \int_0^t P_R\,dt$$

$$= \int_0^t \frac{V_0^2}{R} e^{-2t/RC}\,dt$$

$$= \frac{V_0^2}{R}\left[\frac{e^{-2t/RC}}{-2/RC}\right]_0^t = \frac{-V_0^2 RC}{2R}\left[e^{-2t/RC}\right]_0^t$$

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$W_R(t) = -\frac{1}{2} C V_0^2 (e^{-2t/RC} - 1)$

$W_R(t) = \frac{1}{2} C V_0^2 (1 - e^{-2t/RC})$

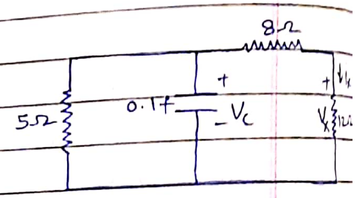
$V(t) = V_0 e^{-t/\tau}$

$V(0) = V_0$

$W_c(0) = \frac{1}{2} C V_0^2$

$W_R(\infty) = \frac{1}{2} C V_0^2 = W_R(0)$

Q. Find V_c, V_x, i_x for $t > 0$



Ans. $V_c = V_0 e^{-t/RC}$

$R_{eq} = 5 \| (8+12) = 4\Omega$

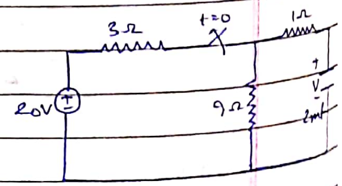
$RC = 4 \times 0.1 = 0.4$

$V_c = 15 e^{-t/0.4}$

$V_x = \frac{12}{12+8} \cdot V_c = 9 e^{-2.5t}$ V

$i_x = \frac{V_x}{12} = 0.7 e^{-2.5t}$ A

Q. Find $V(t)$ for $t > 0$.



Ans. $V_0 = 15V$

$V_t = 15 e^{-t/RC}$

$RC = 10 \times 2 \times 10^{-3} = 2 \times 10^{-2}$

$V(t) = 15 e^{-t/2\times10^{-2}}$

$W_c(0) = \frac{1}{2} C V_0^2 = 2.25 J$

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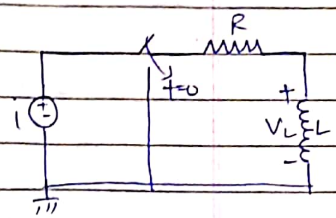
Source Free R.L. Circuit



$C \rightarrow$ Open Circuit
$L \rightarrow$ Short Circuit
$i(0^-) = i(0^+) = I_0$
$W_0(L) = \frac{1}{2} L I_0^2$

$$V_R + V_L = 0$$

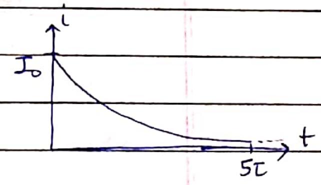
$$iR + L\frac{di}{dt} = 0 \;,\quad L\frac{di}{dt} = -iR$$

$$\frac{di}{i} = -\frac{R}{L}dt \rightarrow \int_{i_0}^{i(t)} \frac{di}{i} = \int_0^t -\frac{R}{L}dt$$

$$\ln \frac{i(t)}{i_0} = -\frac{R}{L}t$$

$$i(t) = i_0 e^{-R/L\, t}$$

$$i(t) = I_0 e^{-t/\tau} \quad \text{where } \tau = \frac{L}{R}$$



$$V(t) = i(t)R = I_0 R e^{-t/\tau}$$

$$P = i^2 R = I_0^2 R e^{-2t/\tau}$$

$$W_R(t) = \int_0^t P\,dt = I_0^2 R \int_0^t e^{-2t/\tau} dt = I_0^2 R\left(-\frac{1}{2} \times \frac{L}{R}\right)\left(e^{-2t/\tau}\right)_0^t$$

$$W_R(t) = \frac{1}{2} L I_0^2 \left(1 - e^{-2t/\tau}\right)$$

$$W_R(\infty) = \frac{1}{2} L I_0^2 = W_R(0)$$

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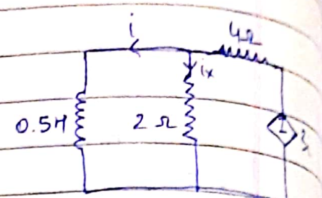
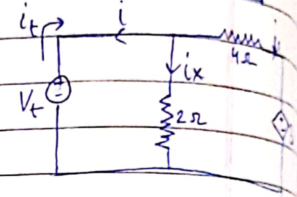
Q. $I_0 = 10A$ Calculate $i(t), i_x(t)$



Ans. $i(t) = I_0 e^{-t/\tau}$

$\tau = \frac{L}{R} = \frac{0.5}{R_{th}}$



$R_{th} = \frac{V_t}{i_t}$

$i + i_x + i_1 = 0$

$i + \frac{V_t}{2} + \frac{V_t - 3i}{4} = 0$

$-i_t + \frac{1}{2}V_t + \frac{1}{4}V_t + \frac{3}{4}i_t = 0$

$\left(\frac{1}{2} + \frac{1}{4}\right) V_t = \left(-\frac{3}{4} + 1\right) i_t$

$\frac{3}{4} V_t = \frac{1}{4} i_t$

$\frac{V_t}{i_t} = \frac{1}{3} = R_{th}$

$i(t) = 10 e^{-t/1.5}$

$i_x(t) = \frac{V_{2Ω}}{2}$

$= \frac{L\, di/dt}{2} = \frac{0.5}{2} \times \frac{d}{dt}\left(10 e^{-\frac{2}{3}t}\right)$

$= \frac{0.5}{2} \times 10 e^{-2/3 t} \left(-\frac{2}{3}\right)$

$= -\frac{5}{3} e^{-2/3 t}$

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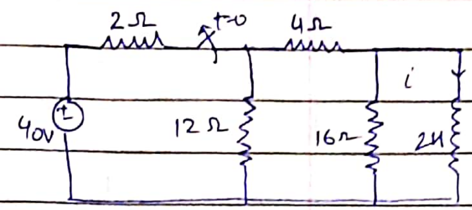
Q. Switch Open at $t = 0$.
Calculate $i(t)$, $t > 0$



Ans. $I_0 = 6A$

$i(t) = I_0 e^{-t/\tau}$

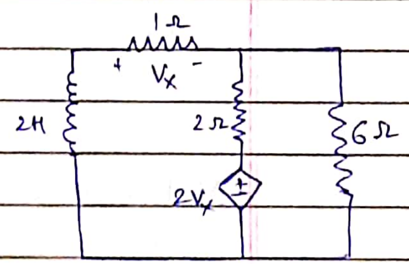
$\tau = \frac{L}{R} = \frac{2}{8} = \frac{1}{4}$

$R + R_{th} = 8\Omega$

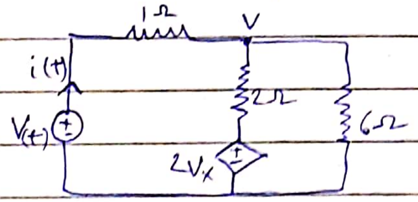
$i(t) = 6e^{-t/\frac{1}{4}}$

$= 6e^{-4t} A$

Q. Calculate V_x, $i(t)$, $i(0) = 7A$?



Ans.



$$\frac{V - 2V_x}{2} + \frac{V - V_t}{1} + \frac{V}{6} = 0$$

$V - V_t = -i_t \Rightarrow V = V_t - i_t$

$V_x = i_t$

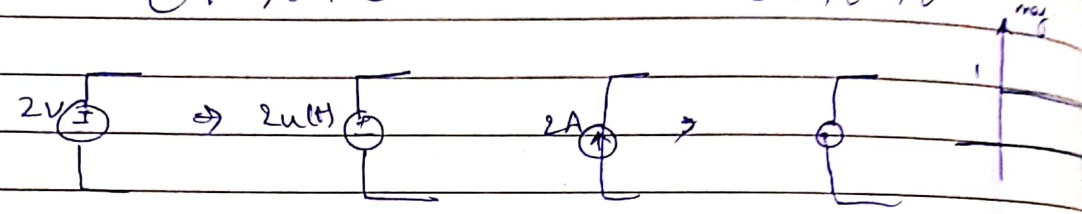
$$\frac{V_t - i_t - 2i_t}{2} + -i_t + \frac{V_t - i_t}{6} = 0$$

Forced Responce

Singularity func. → Unit step signal $u(t)$
→ Impulse $\delta(t)$
→ Ramp $r(t)$

Discontinuous in nature
Deruvaton (Derivative) is discontinuous

$$u(t) \begin{cases} 0, & t<0 \\ 1, & t>0 \end{cases} \quad , \quad u(t) = \begin{cases} 0, & t<0 \\ 1, & t \geq 0 \end{cases}$$

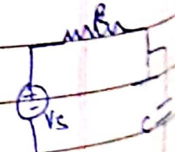


Natural Response

$i(t) = I_0 e^{-t/\tau}$, $\tau = \frac{L}{R}$ ⇒ L

$V(t) = V_0 e^{-t/\tau}$, $\tau = RC$ ⇒ C

Step response of RC Circuit



$V_s\, u(t) = V_R + V_C$
$= iR + V$

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$$\ln [V_{(t)} - V_s) - \ln [V_0 - V_s] = \frac{-1}{RC} t$$

$$\ln \frac{V_t - V_s}{V_0 - V_s} = \frac{-t}{RC}$$

$$\frac{V_t - V_s}{V_0 - V_s} = e^{-t/RC}$$

$$V_t - V_s = (V_0 - V_s) e^{-t/RC}$$

$$V_t = V_s + (V_0 - V_s) e^{-t/RC}$$

$$V_t = V_s + (V_0 - V_s) e^{-t/\tau}$$

Total responce Steady state response Transient response

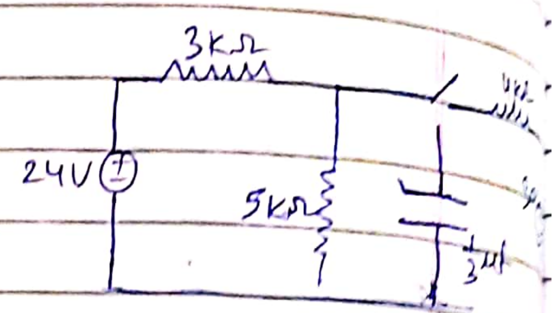
$$V_t = V_\infty + [V_0 - V_\infty] e^{-t/\tau}$$

$$V_t = V_0 e^{-t/\tau} + V_s (1 - e^{-t/\tau})$$

Total responce Natural responce Forced Response

if $V_0 = 0$

Q. At $t=0$ switch changes to B. Calculate $V(t)$ $t>$ $v(t)$ for $t = 1, 4$ sec.



Ans. $V_0 = 15V$

$V(t) = V(\infty) + [V(0) - V(\infty)]e^{-t/\tau}$

$V(\infty) = 30V$

$\tau = RC$

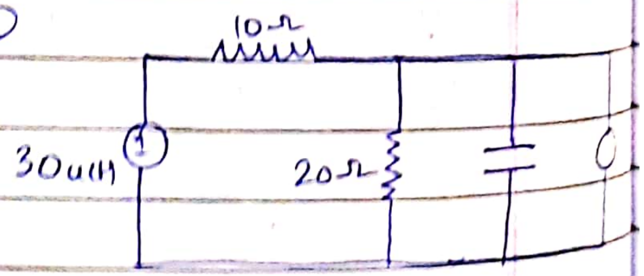
$= \frac{1}{2} \times 10^3 \times 4 \times 10^3 = 2$

$V(t) = 30 - 15e^{-t/2}$

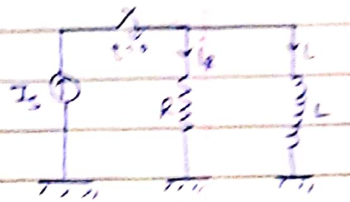
$V(1) = 30 - 15e^{-1/2} = 20.9V$

$V(4) = 30 - 15e^{-2} = 27.9V$

Q. Calculate i & V for $t > 0$



Step Response of RLC



Let $I_L(0^-) = I_L(0^+) = I_0$

$$V(t) = V(\infty) + [V(0) - V(\infty)]\, e^{-t/\tau}$$
$$= V_0 e^{-t/\tau} + V_s(1 - e^{-t/\tau})$$

for $t > 0$

$$I_s u(t) = i_R + i$$
$$= \frac{V_L}{R} + i$$
$$= \frac{L}{R}\frac{di}{dt} + i$$

$$\frac{L}{R}\frac{di}{dt} = I_s u(t) - i$$

$$\frac{di}{I_s u(t) - i} = \frac{R}{L} dt$$

$$\frac{di}{i - I_s u(t)} = -\frac{R}{L} dt$$

$$\int_{I_0}^{i(t)} \frac{di}{i - I_s u(t)} = -\frac{R}{L}\int_0^t dt$$

$$\int_{I_0}^{i(t)} \frac{di}{i - I_0} = -\frac{R}{L}\int_0^t dt$$

$$[\ln(i - I_0)]_{I_0}^{i(t)} = -\frac{R}{L}[t]_0^t$$

$$\ln \frac{i(t) - I_s}{I_0 - I_s} = -\frac{R}{L} t$$

$$\frac{i(t) - I_s}{I_0 - I_s} = e^{-R/L\, t}$$

$$i(t) = I_s + (I_0 - I_s)\, e^{-t/\tau}$$
$$= \frac{V_s}{R} + \left(I_0 - \frac{V_s}{R}\right) e^{-t/\tau}$$

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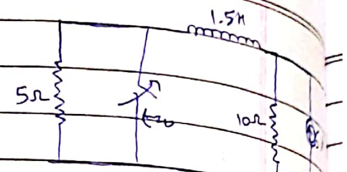
Q. 2 Find $i(t)$ $t>0$, I_0, $I(\infty)$,

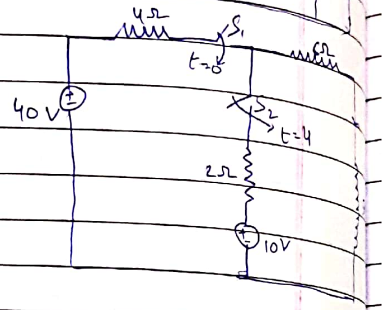
R_{th}, τ, $i(t)$?



Q - 3. Circuit S_1 closed at $t=0$

S_2 closed at $t = 4$ sec.

Find $i(t)$, $i(2)$, $i(5)$



Ans - 3. $0 \le t \le 4$

$i_0 = 0$

Only S_1 is closed

$i(t) = i(4) + (i_0 - i(4))e^{-t/\tau}$

$= 4 + [0 - 4] e^{-t/\tau}$

$= 4(1 - e^{-2t})$

$4 \le t < \infty$

$= i(t) = 2.72 + 1.27 e^{-1.46(t-4)}$

$i(2) = 4(1 - e^{-4})$

Initial and final values (transient and steady state)

$t = 0^- / 0^+$

$R \rightarrow \frac{dV_R}{dt} \rightarrow V_R \rightarrow KVL, KCL$, $\frac{dV_R}{dt}$, $\left.\frac{dV_R}{dt}\right|_{t=0}$

$L \rightarrow i(0^-) = i(0^+) = I_0$ (nature)

$V_L = L\frac{di}{dt}$ $\quad \frac{di}{dt}(0^+) \rightarrow$ loop $\rightarrow V_L = \frac{di}{dt} = \frac{V_L}{L}$

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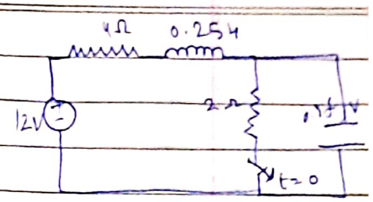

Q. i) $i(0^+)$, $V(0^+)$

ii) $\frac{di(0^+)}{dt}$, $\frac{dV(0^+)}{dt}$

iii) V_∞, I_∞

Ans. i) $t<0$

$i(0^+) = \frac{12}{6} = 2A$

$V(0^+) = 2 \times i = 4A$

ii) $V_L = L\frac{di}{dt}$

$\frac{di}{dt} = \frac{V_L}{L}$

$12 = V_{4} + V_L + V_C$

$12 = V_{4\Omega}(0^+) + V_L + V_C(0^+)$

$12 = i \times 4 + V_L(0^+) + 4$

$12 = 4\Omega\, i(0^+) + V_L(0^+) + 4$

$12 = 4 \times 2 + V_L(0^+) + 4$

$V_L(0^+) = 12 - 8 - 4 = 0$

$\frac{di(0^+)}{dt} = \frac{V_L}{L} = \frac{0}{0.25} = 0A$

$i = \frac{C\,dv}{dt}$, $\frac{dv}{dt} = \frac{i}{C}$

$\frac{dV(0^+)}{dt} = \frac{i(0^+)}{C}$

$= \frac{2}{0.1}$

$= 20V/sec$

$i(\infty) = 0$ $V(\infty) = 12V$

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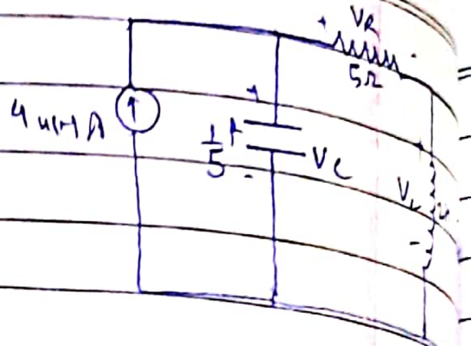
Q. Calculate

i) $i_L(0^+)$, $V_L(0^+)$, $V_R(0^+)$

ii) $\frac{di_L(0^+)}{dt}$, $\frac{di_L}{dt}(0^+)$, $\frac{dV_R(0^+)}{dt}$

iii) $i_L(\infty)$, $V_L(\infty)$, $V_2(\infty)$



Ans. i) $i(0^+) = -6A$

$V_L(0^+) = 0A$

$V_R(0^+) = 0$

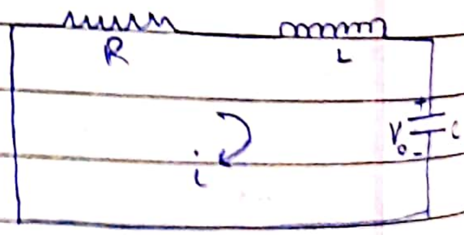
ii) $V_L = L\frac{di}{dt}$, $\frac{di}{dt} = \frac{V_L}{L} = \frac{0}{0} = 0$

$V_C = V_R + V_L$

$V_L = V_C - V_R$

$= 0 - (5 \times 25 \times 2.$

Source free RLC



Let $i(0^-) = i(0^+) = I_0$

$v(0^-) = v(0^+) = V_0$

Using KVL

$$V_R + V_L + V_C = 0$$

$$iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0$$

$$R\frac{di}{dt} + L\frac{d^2i}{dt^2} + \frac{i}{C} = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC} = 0$$

$$i = Ae^{st}$$

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$$\frac{d^2}{dt^2}(Ae^{st}) + \frac{R}{L}\frac{d}{dt}(Ae^{st}) + \frac{Ae^{st}}{LC} = 0$$

$$As^2e^{st} + \frac{R}{L}sAe^{st} + \frac{Ae^{st}}{LC} = 0$$

$$Ae^{st}\left(s^2 + \frac{R}{L}s + \frac{1}{LC}\right) = 0$$

$$i = Ae^{st} \neq 0$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$$b = \frac{R}{L}, \quad c = \frac{1}{LC}, \quad a = 1$$

$$S_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-\frac{R}{L} \pm \sqrt{\left(\frac{R}{L}\right)^2 - \frac{4}{LC}}}{2} = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

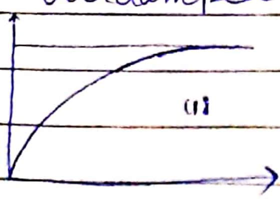
$$S_{1,2} = \frac{-R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$$

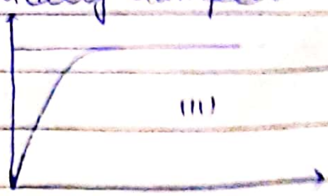
Assumme $\alpha = \frac{R}{2L}$ & $\omega_0 = \frac{1}{\sqrt{LC}}$

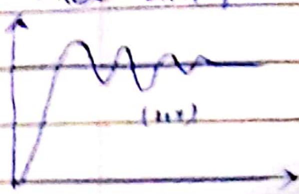
$\alpha \rightarrow$ damping factor

ω_0 = indamped natural frequency / resonanant freq

$\alpha > \omega_0$	$\alpha = \omega_0$	$\alpha < \omega_0$
$S_{1,2}$: ~~discrete~~ distinct real	$S_{1,2}$ = real and equal	complex & comp. pair
$i(t) = A_1e^{s_1t} + A_2e^{s_2t}$	$i(t) = (A_1 + A_2)e^{-\alpha t}$	$i = e^{-\alpha t}(\cos\omega_d t + \sin\omega_d t)$
		$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$
Overdamped	Critically damped	Under damped







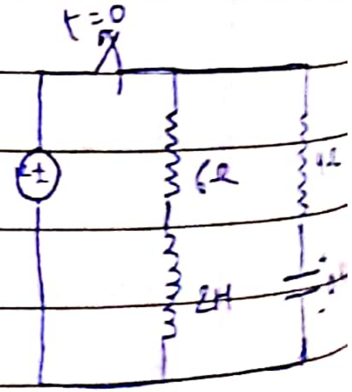
2. Calculate α & ω_0

3. Compare α & ω_0 to write $i(t)$

4. Calculate $S_{1,2} \rightarrow$ roots of characteristic equation

5. Calculate constant A_1, A_2 using Initial Conditions & derivative

Q. Cal. $\frac{di}{dt}$



Ans.

$$V_R + V_L = V_4 + V_C$$

$$V_R(0^+) + L\frac{di}{dt} = V_4(0^+) + V_C(0^+)$$

$$6 \times 2 + 2 \times \frac{di}{dt} = 4 \times (-2) + 12$$

$$\frac{di}{dt}(0^-) = -4 \text{ P/sec.}$$

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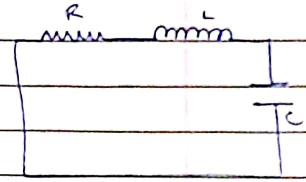
Source free RLC Circuit

1) Series RLC



$$\alpha = \frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

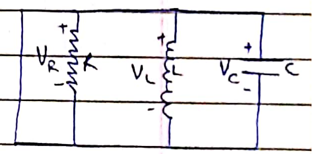
$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

$$i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \rightarrow \text{over damped}$$

$$= (A_1 + A_2) e^{-\alpha t} \rightarrow \text{critically damped}$$

$$= e^{-\alpha t} (A_1 \cos \omega_d t + A_2 \sin \omega_d t) \rightarrow \text{Under damped}$$

2) Parallel RLC



$$i_R + i_L + i_C = 0$$

$$\frac{V_R}{R} + \frac{1}{L}\int V dt + C\frac{dv}{dt} = 0$$

$$\frac{V}{R} + \frac{1}{L}\int V dt + C\frac{dv}{dt} = 0$$

$$\frac{1}{R}\frac{dv}{dt} + \frac{1}{L} V + C\frac{d^2v}{dt^2} = 0$$

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{v}{LC} = 0$$

$$V = Ae^{st}$$

$$\frac{d^2}{dt^2}(Ae^{st}) + \frac{1}{RC}\frac{d}{dt}(Ae^{st}) + \frac{Ae^{st}}{LC} = 0$$

$$As^2 e^{st} + \frac{1}{RC} Ase^{st} + \frac{Ae^{st}}{LC} = 0$$

$$Ae^{st}\left(s^2 + \frac{s}{RC} + \frac{1}{LC}\right) = 0$$

Since $Ae^{st} \neq 0$

$$s^2 + \frac{s}{RC} + \frac{1}{LC} = 0$$

Let $\alpha = \frac{1}{2RC}$, $\omega_0 = \frac{1}{\sqrt{LC}}$

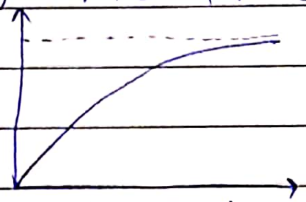
$= \frac{1}{2\tau}$

$S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

$\alpha > \omega_0$

distinct

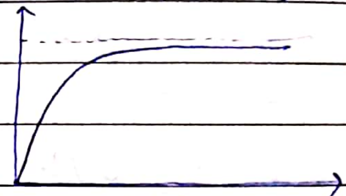
$V(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$



Overdamped

$\alpha = \omega_0$

$V(t) = (A_1 + A_2) e^{-\alpha t}$



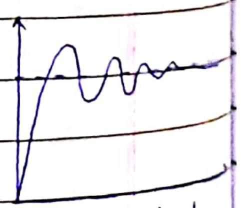
Critically damped

$\alpha < \omega_0$

Complex Conjugate

$V(t) = e^{-\alpha t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t)$

$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$



Under damped

Steps -

1) $t < 0$ - Cal. I_0, V_0

2) $t = 0$ - $\frac{dI}{dt}$, $\frac{dV}{dt}$

3) $t > 0$ - ① cal. α, ω_0

② $V(t)$

③ A_1, A_2

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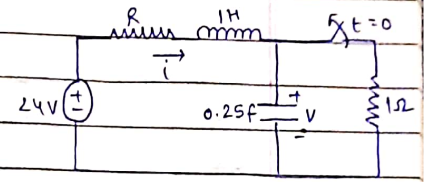
Q. For $t > 0$, calculate $i(t)$

$R = 5\Omega$, $R = 4\Omega$, $R = 1\Omega$



Ans. $i(0^-) = i(0^+) = I_0 = 4A$

$V(0^-) = V(0^+) = V_0 = 4V$

$\frac{di}{dt}(t=0) \rightarrow$ $24 = V_R + V_L + V_C$

$24 = iR + V_L + 4$

$V_L(0) = 0$

$t > 0$, $\alpha = \frac{R}{2L} = 2.5$

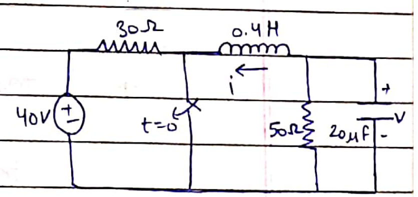
$\omega = \frac{1}{\sqrt{LC}} = 2$

$\alpha > \omega_0$ Overdamped

$R = 4\Omega \rightarrow$ Critically $\alpha = \omega_0$

$R = 1\Omega \rightarrow \alpha < \omega_0$ Under damped

Q. Cal. $V(t)$ for $t > 0$



Ans. $i(0) = -0.5A$

$V(0) = 25V$

$i + i_R + i_C = 0$

$-0.5 + \frac{V_R}{50} + \frac{Cdv}{dt} = 0 \rightarrow \frac{dv}{dt} = 0$

$t > 0$,

$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 50 \times 20 \times 10^{-6}} = 500$

$\omega = 354$

$\alpha > \omega_0 \rightarrow$ overdamped

$S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

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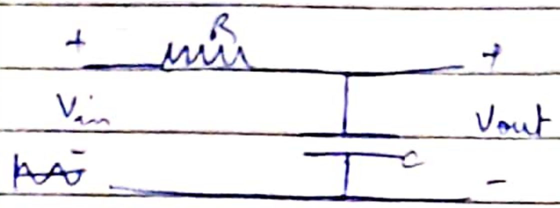
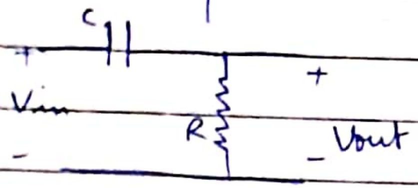


$$V_{out} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} V_{in} = \frac{1}{1+j\omega RC}$$

$$\frac{V_o}{V_{in}} = \frac{1}{1+j\omega RC} = \frac{1}{1+sRC} \qquad s = j\omega$$

$$H\,(\text{gain}) = \left|\frac{V_o}{V_{in}}\right| = \frac{1}{\sqrt{1+(\omega RC)^2}}$$

$$\phi = -\tan^{-1}(\omega RC)$$



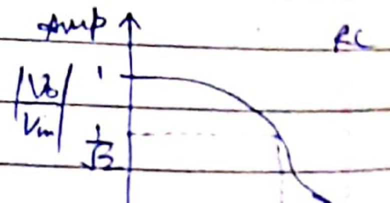
$$V_{out} = \frac{R}{R+\frac{1}{j\omega C}} V_{in} = \frac{1}{1+\frac{1}{j\omega RC}} V_{in}$$

$$V_{out} = \frac{1}{1+\frac{1}{sRC}} V_{in}$$

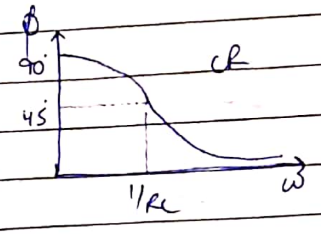
$$\frac{V_o}{V_{in}} = \frac{1}{1-j/\omega RC} \Rightarrow \left|\frac{V_o}{V_{in}}\right| = \frac{1}{\sqrt{1+\left(\frac{1}{\omega RC}\right)^2}}$$

$$\phi = -\tan^{-1}\left(\frac{-1}{\omega RC}\right)$$

$$= \tan^{-1}\left(\frac{1}{\omega RC}\right)$$



Laplace Transform



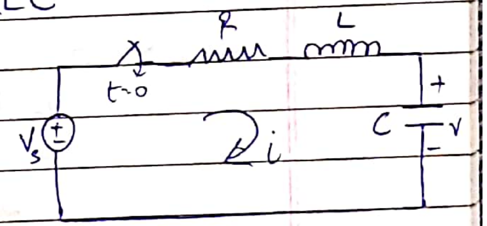
$P_{out} = \frac{P_{in}}{2}$ $\qquad$ $dB = 10 \log_{10}\left(\frac{P_o}{P_{in}}\right)$

$V_{out}^2 = \frac{V_{in}^2}{2}$ $\qquad$ $= 10 \log_{10}\left(\frac{V_o}{V_{in}}\right)^2$

$V_{out} = \frac{V_{in}}{\sqrt{2}}$ $\qquad$ $= 20 \log_{10}\left(\frac{V_o}{V_{in}}\right)$

$\frac{V_{out}}{V_{in}} = \frac{1}{\sqrt{2}}$ $\qquad$ $LT = \frac{(s+3)}{(s+4)} = \frac{(3+j\omega) \rightarrow +20dB\ sec}{(4+j\omega) \rightarrow -20dB\ sec}$

Step Response of Series RLC



KVL $\rightarrow$

$V_R + V_L + V_C = V_s$

$iR + L\frac{di}{dt} + V_C = V_s$

$RC\frac{dv}{dt} + LC\frac{d^2v}{dt^2} + V = V_s$

$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{V}{LC} = \frac{V_s}{LC}$

Non Homogeneous $\rightarrow RHS \neq 0$

Total Solution $= \underset{LHS}{C.F.} + \underset{RHS}{P.I.}$

Equality $RHS = 0$ $\qquad$ $\frac{RHS}{C.F.}$

$$\frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v}{LC} = \frac{V_s}{LC}$$

$$C.F. \Rightarrow \frac{d^2v}{dt^2} + \frac{R}{L}\frac{dv}{dt} + \frac{v}{LC} = 0 \rightarrow \left(D^2 + \frac{RD}{L} \ldots\right.$$

$$i(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$$

$$v(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$$

$$P.I. = \frac{\frac{V_s}{LC}}{\left(D^2 + \frac{RD}{L} + \frac{1}{LC}\right)} = \frac{V_s}{(1 + RCD + LCD^2)}$$

$$= \frac{V_s}{LC}(1 + RCD + LCD^2)^{-1}$$

$$= V_s(1 - RCD + LCD^2 + \ldots)$$

$$= V_s - 0 - 0$$

$$= V_s$$

$$V(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t} + V_s$$

$$= \underbrace{V_t(t)}_{\text{Transient}} + \underbrace{V_s(t)}_{\text{Steady State}}$$

V_t

Over damped $\alpha < \omega_0$

Critically $\alpha = \omega_0$

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$$i_R + i_L + i_C = I_s$$

$$\frac{V}{R} + i_L + C\frac{dv}{dt} = I_s$$

$$\frac{L}{R}\frac{di}{dt} + i + C\frac{d}{dt}\left(L\frac{di}{dt}\right) = I_s$$

$$LC\frac{d^2i}{dt^2} + \frac{L}{R}\frac{di}{dt} + i = I_s$$

$$\frac{d^2i}{dt^2} + \frac{1}{RC}\frac{di}{dt} + \frac{i}{LC} = \frac{I_s}{LC}$$

General sol. = C.F. + P.I.

$$i(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t} + I_s$$

Series RLC

$$\alpha = \frac{R}{2L} \qquad \omega_0 = \frac{1}{\sqrt{LC}} \qquad \omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

Source free

$$i(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$$

$\alpha > \omega_0$: $i(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t}$

$\alpha = \omega_0$: $i(t) = (A_1 + A_2 t)e^{-\alpha t}$

$\alpha < \omega_0$: $i(t) = e^{-\alpha t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t)$

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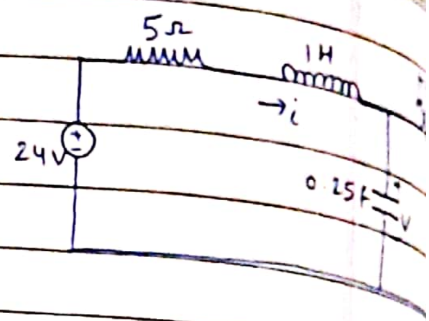
8 Calculate $V(t)$ $I(t)$ for $t>0$



Ans. $I_0 = 4A$

$V_0 = 4V$

$V(t) = A_1 e^{S_1 t} + A_2 e^{S_2 t} + 24$

$\alpha = 2.5$ $\quad \omega_0 = 2 \quad \alpha > \omega_0$

$S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$

$= -1, -4$

$\frac{dv}{dt}(t=0) = 16$ $\qquad i(0) = C\frac{dv}{dt}$

$V(t) = A_1 + A_2 + 24 = 4$

$A_1 + A_2 = -20$ — ①

$\frac{dV(t)}{dt} = -A_1 e^{-t} - 4A_2 e^{-4t}$

$16 = -A_1 - 4A_2$ — ②

$A_1 = \frac{-64}{3}$ $\qquad A_2 = \frac{4}{3}$

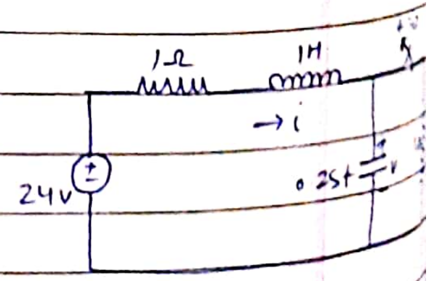
$V(t) = -\frac{64}{3}e^{-t} + \frac{4}{3}e^{-4t} + 24$

$i(t) = C\frac{d}{dt}V(t)$

$= \frac{1}{4}\left(\frac{64}{3}e^{-t} - \frac{16}{3}e^{-4t} + 0\right)$

$= \frac{16}{3}e^{-t} - \frac{4}{3}e^{-4t}$ A

2. Calculate $V(t)$, $I(t)$ for $t>0$



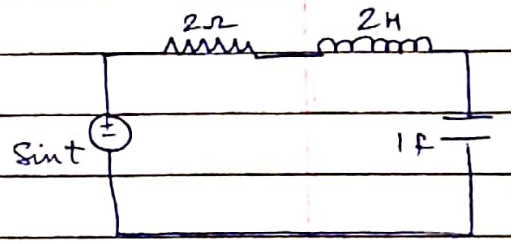
Ans. $I_0 = 12A$

$V_0 = 12V$

$$V(t) = e^{-\alpha t}(A_1 \cos \omega_d t + A_2 \sin \omega_d t) + 24$$
$$= e^{-0.5t}(A_1 \cos(1.93)t + A_2 \sin(1.93)t)$$

$\left.\begin{array}{l}\alpha = 0.5 \\ \omega_0 = 2\end{array}\right\}$ $\alpha < \omega_0$.

Q. Find expression in 2^{nd} derivative.



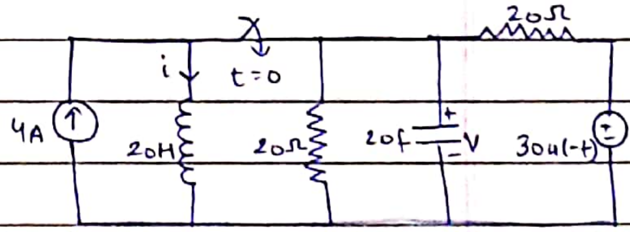
Ans. $V_R + V_L + V_C = \sin t$

$$iR + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = \sin t$$

$$R\frac{di}{dt} + L\frac{d^2 i}{dt^2} + \frac{i}{C} = \cos t$$

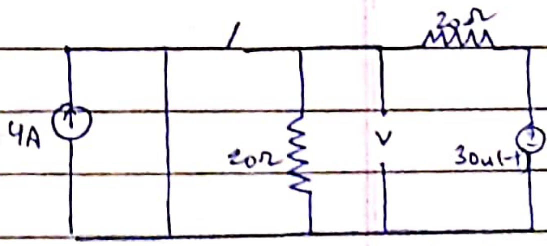
$$\Rightarrow 2\frac{d^2 i}{dt^2} + 2\frac{di}{dt} + i = \cos t$$

Q. Calculate $i(0)$, $V(0)$.



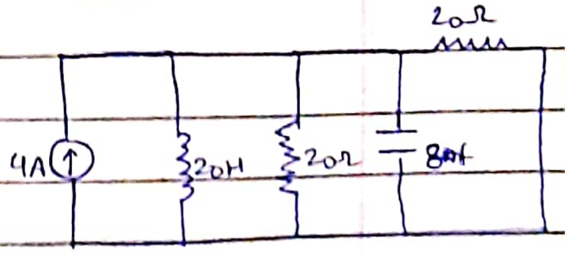
Ans. $u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$

$u(-t) = \begin{cases} 0, & t > 0 \\ 1, & t \leq 0 \end{cases}$



$t < 0$, $i(0) = 4A$

$V(0) = 15V$



$t = 0$, $V = L\frac{di}{dt}$

$$\frac{di}{dt}(t=0) = \frac{V_L(0)}{L} = \frac{15}{20} = \frac{3}{4}$$

$$\alpha = \frac{1}{2RC} = \frac{1}{2 \times 10 \times 8 \times 10^{-3}} = 6.25$$

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$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20 \times 8 \times 10^{-3}}} = 2.5$$

$\alpha > \omega \rightarrow$ overdamped

$$S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$$= -11.928 \quad , -0.5218$$

$$i(t) = A_1 e^{-11.928t} + A_2 e^{-0.5218t} + 4$$

$$i(0) = A_1 + A_2 + 4$$

$$4 = A_1 + A_2 + 4$$

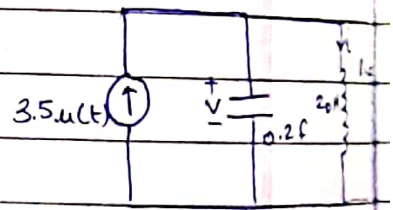
$$A_1 + A_2 = 0 \quad — ①$$

$$\frac{di(t)}{dt} = -11.928 A_1 e^{-11.928t} + (-0.5218) A_2 e^{-0.5218t}$$

$$\frac{3}{4} = -11.928 A_1 - 0.5218 A_2 \quad — ②$$

$A_1 = 0.2055$ $\qquad$ $A_2 = -0.0655$

Q. Calculate $i(t)$ & $V(t)$ for $t > 0$



Ans. $\alpha = \frac{1}{2RC} = 0$

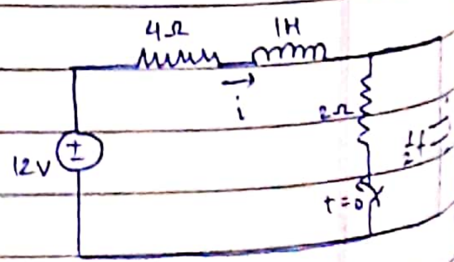
$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{20 \times 0.2}} = \frac{1}{2}$$

$\omega > \alpha \rightarrow$ underdamped

$i(t) = 3.5(1 - \cos 0.5t)$ A

$V(t) = 1.75 \sin 0.5t$

Q. Find $i(t)$ & $V(t)$ for $t > 0$



Ans. $i(0) = 0$

$V(0) = 12$

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Step 1 - Calculate initial conditions.

Step 2 - Calculate $x(t)$ by turning off independent source. i.e. $\frac{d^2v}{dt^2}$ or $\frac{d^2 i}{dt^2}$ $\to$ KCL, $\to$ KVL

Step 3 - Calculate $x(\infty)$

Step 4 - Total response $= x_t(t) + x(\infty)$

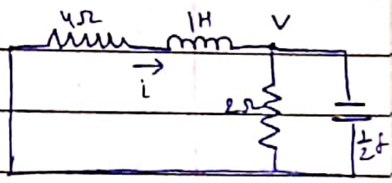
Step 5 - Constants using initial conditions

Ans. (2) $\frac{di(t=0)}{dt} = \frac{V_L}{L} = \frac{12 - 4i(0) - V}{L} = \frac{12 - 0 - 12}{L} = 0$

$\frac{dV(t=0)}{dt} = \frac{i_c}{C} = \frac{-6}{1/2} = -12\,V/sec.$

$-i + \frac{V}{2} + i_c = 0 \Rightarrow i_c = i - \frac{V}{2} = 0 - \frac{12}{2} = -6$

(3) KCL $\to$ $-i + \frac{V}{2} + i_c = 0$



$i = \frac{V}{2} + \frac{c\,dV}{dt}$

KVL $\to$ $-4i - V_L - V_c = 0$

$-4i - L\frac{di}{dt} - V = 0$

$-4\left(\frac{V}{2} + \frac{1}{2}\frac{dv}{dt}\right) - \frac{d}{dt}\left(\frac{V}{2} + \frac{1}{2}\frac{dv}{dt}\right) - V = 0$

$\Rightarrow -2V - \frac{2dv}{dt} - \frac{1}{2}\frac{dv}{dt} - \frac{1}{2}\frac{d^2v}{dt^2} - V = 0$

$\Rightarrow \frac{d^2V}{dt^2} + \frac{5dv}{dt} + 6V = 0 \to V_t$

$S_{1,2} = -2, -3$

$V_{ss}(\infty) = 4V$

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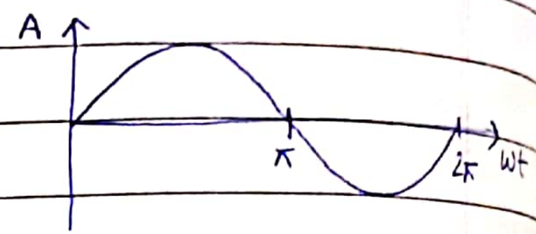
$$V(t) = V_t(t) + V_{ss}(t)$$
$$= A_1 e^{-2t} + A_2 e^{-3t} + 4$$

Sinusoidal and Phasers



$f(t) = \sin t$

$T\omega = 2\pi$

$f(t+T) = \sin\omega(t+T)$
$= \sin(\omega t + \omega T)$
$= \sin\omega t$

Cycle → 1 wave

T → Time period

f → cycle/sec

Phase is the anticlockwise rotating vector

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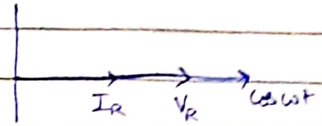
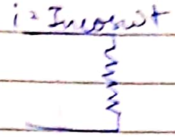
1) Across R

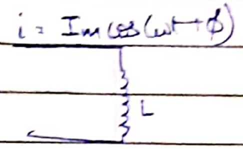
$i = I_m \cos\omega t$

$V = I_m R \cos\omega t$





2) Across L



$i = I_m \cos(\omega t + \phi)$

$V = L\, I_m\,(-\omega \sin(\omega t + \phi))$

$= -\omega L\, I_m \sin(\omega t + \phi)$

$= -\omega L\, I_m \cos(\omega t + \phi - 90^\circ)$

$= -\omega L\, I_m \angle \phi - 90^\circ$

$= -\omega L\, I_m \angle \phi\, e^{-i\pi/2}$

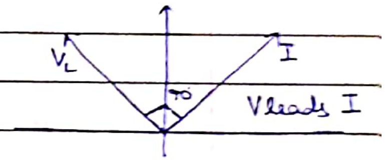
$= -\omega L\, I_m \cos(\omega t + \phi)$

$V = (j\omega L)\, I_m \cos(\omega t + \phi)$

$j\omega L = 0 + j\omega L$

$j\omega L = \omega L$

$\theta = \pi/2$



Phasor

Representation $\rightarrow$ Rectangular $\rightarrow \alpha + j\beta$

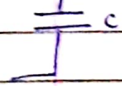
$\rightarrow$ Polar $\rightarrow r\angle\phi$ where $r = \sqrt{\alpha^2 + \beta^2}$

$\phi = \tan^{-1}\left(\frac{\beta}{\alpha}\right)$

$\rightarrow$ Exponent form $\rightarrow re^{j\theta}$, $e^{j\theta} = \cos\theta + j\sin\theta$

$j^2 = -1$, $j = \sqrt{-1}$

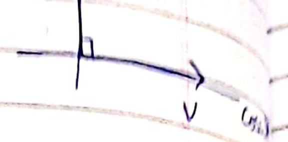
3) Across C



$i = C\frac{dv}{dt}$

$= \omega C V_m \angle 90^\circ$

Current leads Voltage



Impedence (z)

1) Resistor

$i = I_m \cos\omega t$ $\quad\quad V = I_m R \cos\omega t$

$z = \frac{V}{I} = R\ \Omega$

2) Inductor

$i = I_m \cos\omega t$

$= I_m \angle 0^\circ$

$V = \omega L I_m e^{j90^\circ}$

$= \omega L I_m (0 + j + 1)$

$= j\omega L I_m \angle 0^\circ$

$Z = \frac{V}{I} = j\omega L\ \Omega$

$|z| = \omega L$

3) Capacitor

$V = V_m \cos\omega t$

$i = \omega C V_m e^{j90^\circ}$

$= \omega C V_m (0 + j)$

$= j\omega C V_m$

$Z = \frac{V}{I} = \frac{1}{j\omega C}$

$|z| = -\frac{1}{\omega C}\ \Omega$

→ For addition, subtraction use rectangular form

→ For multiplication, division use polar form

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$V = V_{m_1}\cos(\omega t+\phi_1) + V_{m_2}\cos(\omega t+\theta_2) + \ldots V_{m_n}\cos(\omega t+\theta_n)$

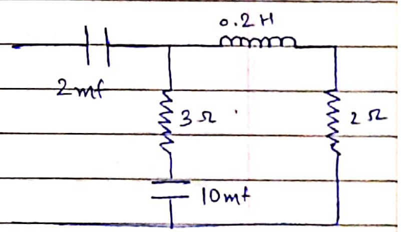
$= V_{m_1} e^{j(\omega t+\phi_1)} + V_{m_2} e^{j(\omega t+\theta_2)} + \ldots + V_{m_n} e^{j(\omega t+\theta_n)}$

Q. Calculate $i(t)$, $v(t)$

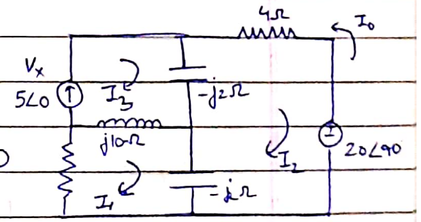
Ans. $i = \frac{V}{2} = \frac{4\cos 5t}{2+j5} = \frac{4\angle 0}{\sqrt{29}\angle\tan^{-1}\left(\frac{5}{2}\right)} = \frac{4}{5.4}\angle -\tan^{-1}\left(\frac{2}{5}\right)$

Q. $\omega = 50$ rad/sec.

Calculate total Impedence



① Mesh



$M_1 \rightarrow$

$-8I_1 - j10(I_1 - I_3) - (j2)(I_1 - I_2) = 0$

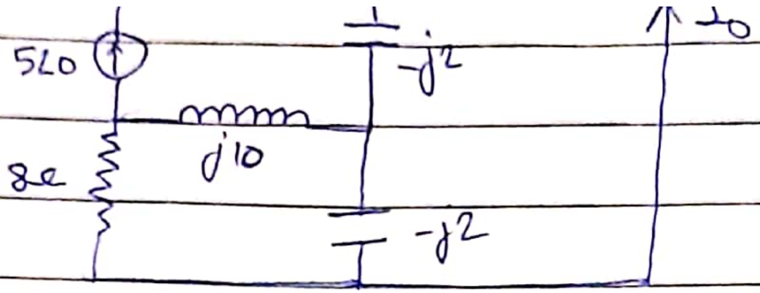
$M_2 \rightarrow$

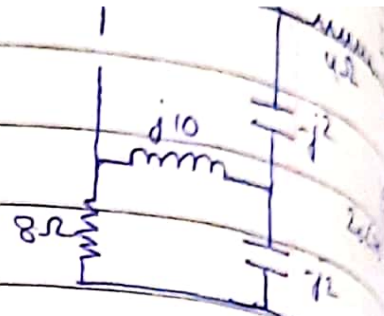
$-(-j2)(I_2 - I_1) - (-j2)(I_2 - I_3) - 4I_2 - 20\angle 90 = 0$

$M_3 \rightarrow$

$V_x - (-j2)(I_3 - I_2) - j10(I_3 - I_1) = 0$ $\qquad I_3 = 5\angle 0$

$I_0 = 6.12\angle 144.28^\circ = -I_2$





$I_o = I_o' + I_o''$

$I_o' = -2.35 + j2.353$

$= 2.353\angle -45$

$I_o'' = -2.647 + j1.76$

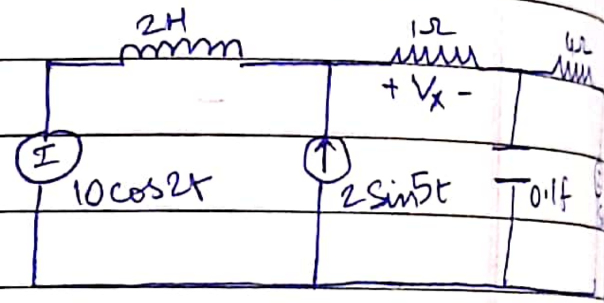
$= 2.87\angle -23.95^\circ$

$I_o = (-2.353 + j2.353) + (-2.647 + j1.176)$

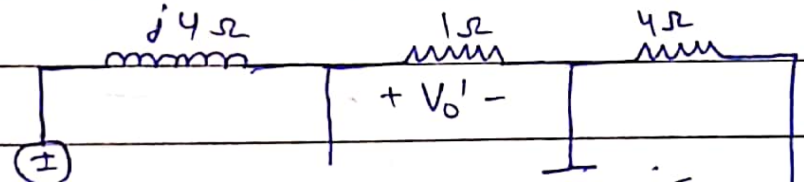
$= -5 + j3.529$

$= 6.12\angle 144.78^\circ$

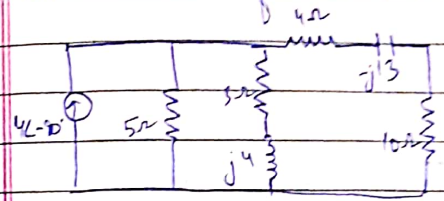
(2) Superposition

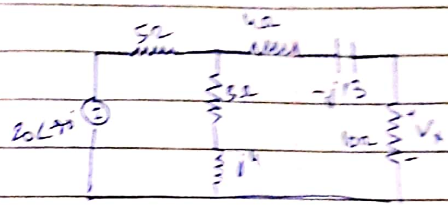


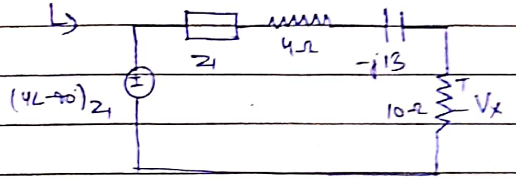
$\omega = 2 \rightarrow$



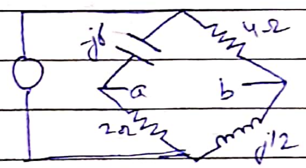
③ Source Transformation

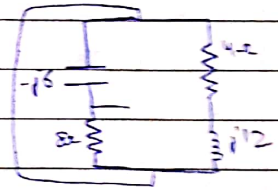






3.





Q.