Cylindrical coordinate system

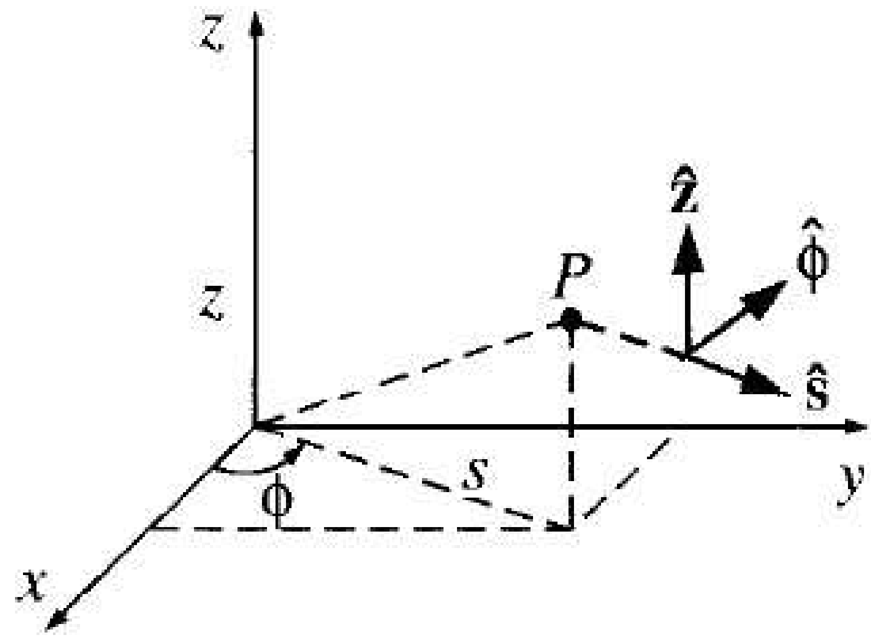
s

Cylindrical Coordinates

$$0 \le s \le \infty$$

$$0 \le \varphi \le 2\pi$$

$$-\infty \le z \le \infty$$



s

φ

z

$$x = s\cos\phi, \qquad y = s\sin\phi, \qquad z = z$$

$$s = \sqrt{x^2 + y^2} \qquad \varphi = \tan^{-1}\left(\frac{y}{x}\right) \qquad z = z$$

Unit Vectors

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

$$\hat{s} = \frac{\frac{\partial \vec{r}}{\partial s}}{\left|\frac{\partial \vec{r}}{\partial s}\right|} \qquad \hat{\varphi} = \frac{\frac{\partial \vec{r}}{\partial \varphi}}{\left|\frac{\partial \vec{r}}{\partial \varphi}\right|} \qquad \hat{z} = \frac{\frac{\partial \vec{r}}{\partial z}}{\left|\frac{\partial \vec{r}}{\partial z}\right|}$$

$$\vec{r} = s\cos\varphi\,\hat{i} + s\sin\varphi\,\hat{j} + z\hat{k}$$

$$\hat{s} = \cos\varphi\,\hat{i} + \sin\varphi\,\hat{j}$$

$$\hat{\varphi} = -\sin\varphi\,\hat{i} + \cos\varphi\,\hat{j}$$

$$\hat{z} = \hat{k}$$

Sketch the vector functions on XY plane

$$\vec{V} = s\hat{s}$$

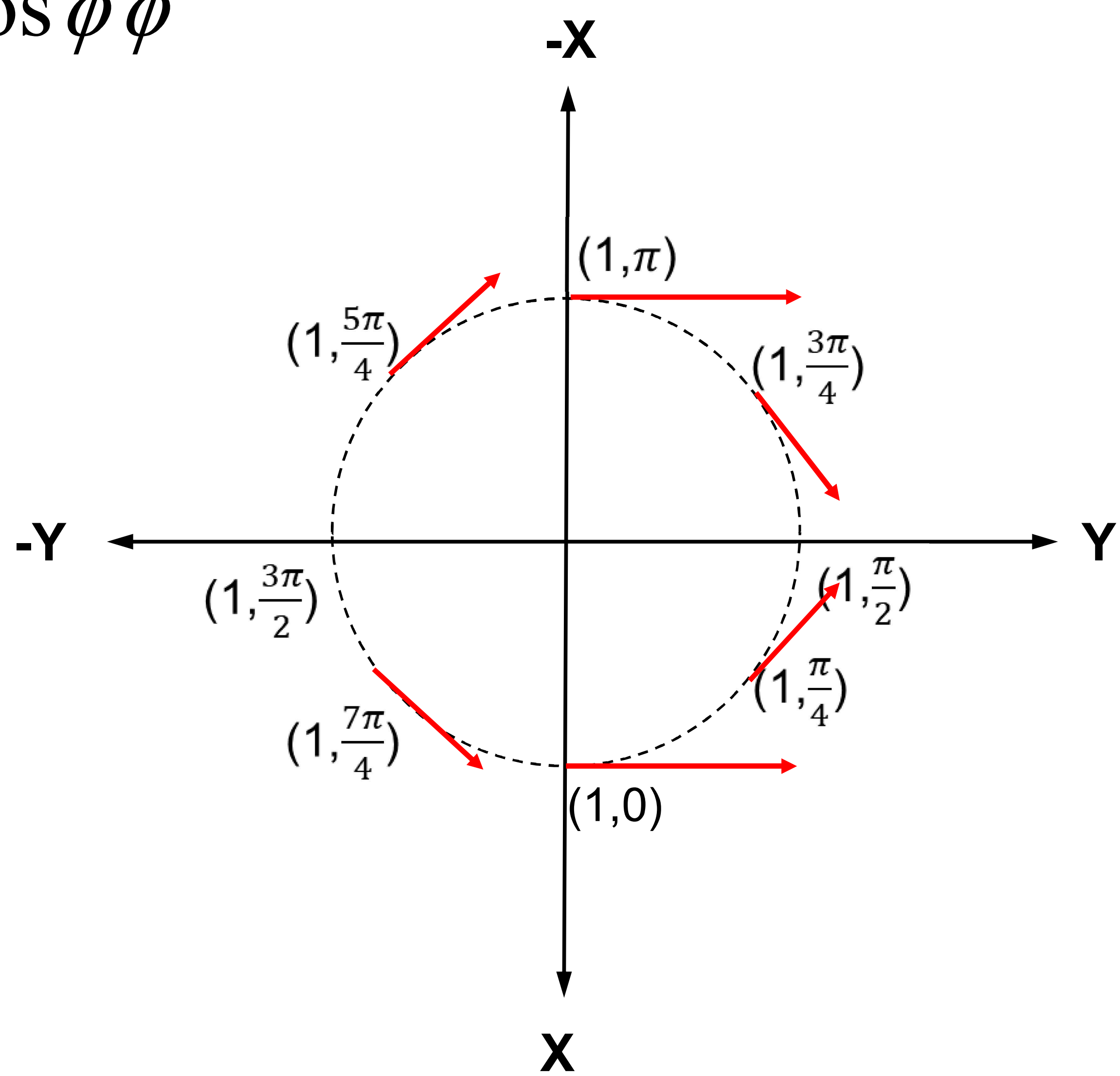
$$\vec{V} = s\hat{\phi}$$

$$\vec{V} = \hat{\phi} + \hat{s}$$

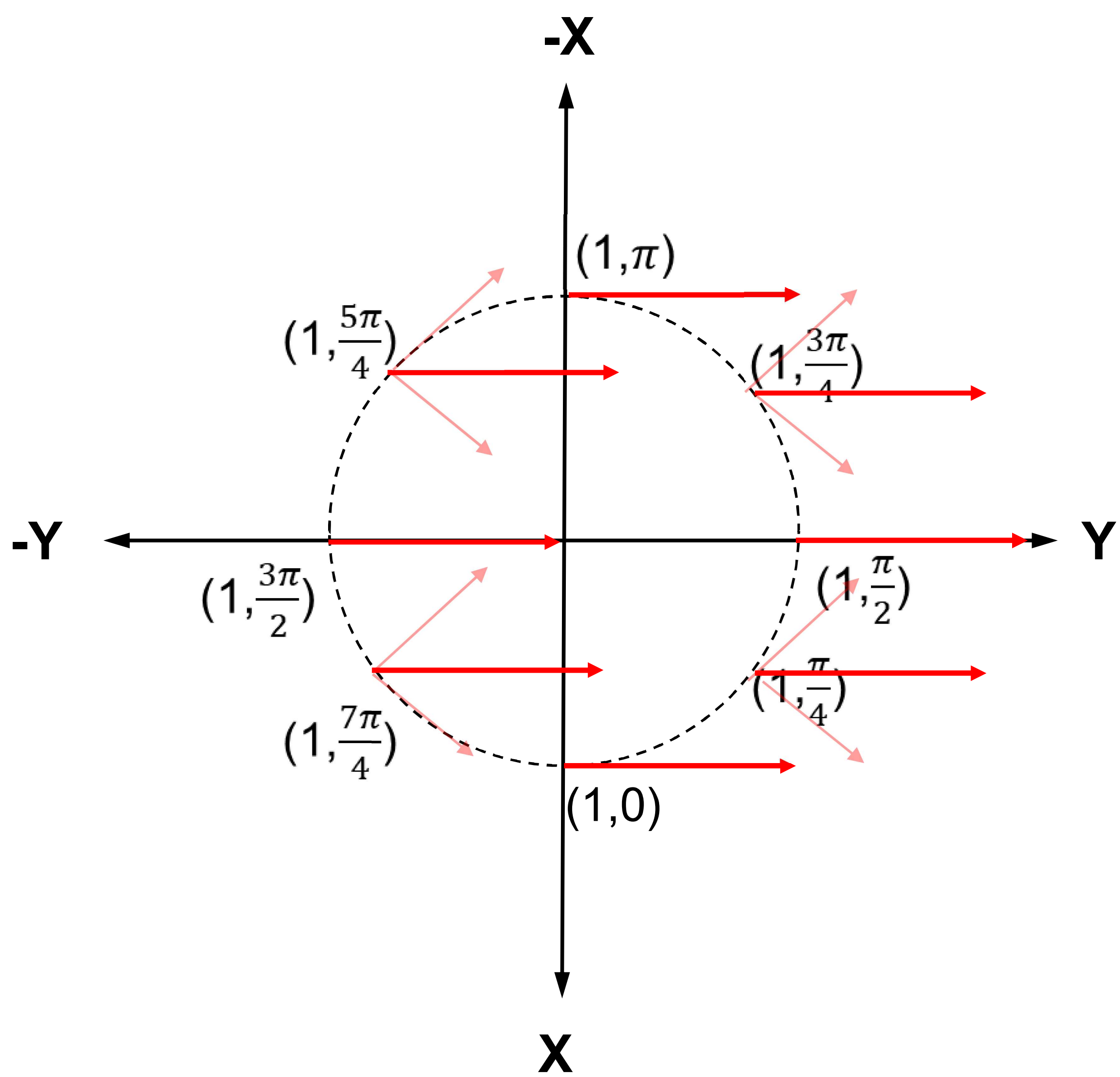
$$\vec{V} = \cos\phi\,\hat{\phi}$$

$$\vec{V} = \cos\phi\,\hat{\phi} + \sin\phi\,\hat{s}$$

$$\vec{V} = \cos\phi\,\hat{\phi}$$



$$\vec{V} = \cos\phi\,\hat{\phi} + \sin\phi\,\hat{s}$$



$$\vec{A} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$$\vec{A} = a_s \hat{s} + a_\varphi \hat{\varphi} + a_z \hat{z}$$

$$a_x = a_s \cos\phi - a_\phi \sin\phi$$
$$a_y = a_s \sin\phi + a_\phi \cos\phi$$
$$a_z = a_z$$

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_s \\ a_\varphi \\ a_z \end{bmatrix}$$

$$a_s = a_x \cos\phi + a_y \sin\phi$$
$$a_\phi = -a_x \sin\phi + a_y \cos\phi$$
$$a_z = a_z$$

$$\begin{bmatrix} a_s \\ a_\phi \\ a_z \end{bmatrix} = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$$

Infinitesimal vector

$$dl_s = ds, \qquad dl_\phi = s\,d\phi, \qquad dl_z = dz$$

$$d\mathbf{l} = ds\,\hat{\mathbf{s}} + s\,d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}$$