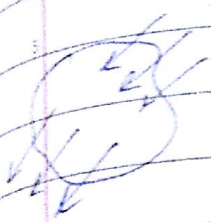
MAGNETOSTATICS

* Field is generated due to a steady current
* steady current :

$\boxed{\nabla \cdot \vec{J} = 0}$ ⇒ No. of charges arriving at a place = No. of charges leaving that place (No accumulation / No depletion)



Total current crossing surface : $\oiint \vec{J} \cdot d\vec{a}$

$$\oiint \vec{J} \cdot d\vec{a} = -\frac{\partial}{\partial t} \iiint \rho \, d\tau \quad : \text{change of volume charge density}$$

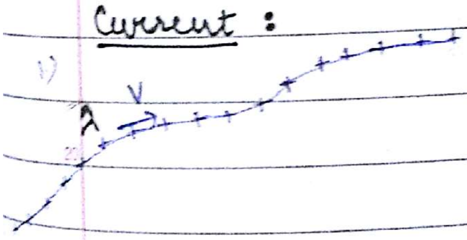
going out, charge density ↓es inside region

$$\Rightarrow \iiint (\vec{\nabla} \cdot \vec{J}) \, d\tau + \iiint \frac{\partial \rho}{\partial t} \, d\tau = 0$$

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

→ 4 steady current, $\nabla \cdot \vec{J} = 0$

Current :

1) 

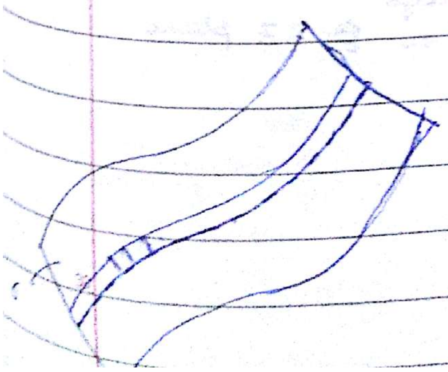
line current :

$$\vec{I} = \frac{dQ}{dt} = \lambda \vec{v} \quad (\text{in free space})$$

$$\text{length} = v\Delta t = \Delta l$$

$$I = \frac{\Delta Q}{\Delta t} = \frac{\lambda \Delta l}{\Delta t} = \frac{\lambda v \Delta t}{\Delta t} = \lambda v$$

2) Surface current :



surface current : current per unit length ⊥r to it

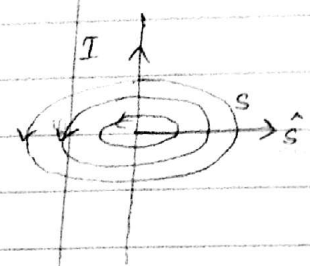
$$\vec{K} = \sigma \vec{v}$$

* When in free space, charges can move in any
I is vector

3.) Volume current

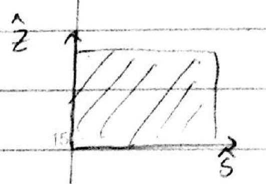
Volume current $\vec{J} = \rho \vec{v}$
current per unit area
$\perp^r$ to it

Biot - Savart Law

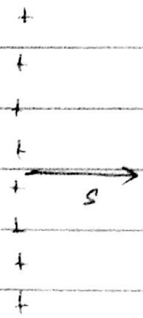


① $|\vec{B}| \propto I$, ③ $\vec{B} \perp^r$ to zs plane

② $|\vec{B}| \propto \frac{1}{s}$



Earlier →



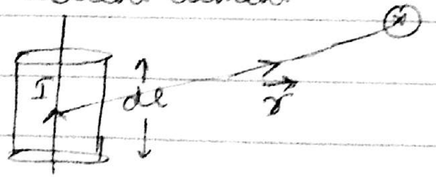
$\bar{E} = \frac{\lambda}{2\pi\epsilon_0 s} \Rightarrow \bar{E} \propto \frac{1}{s}$

⇒

Similar to point charges above

dl I : current element

current element



always $\perp^r$ to ~~z~~ SZ plane

$$d\vec{B} = C \frac{(I d\vec{l} \times \hat{r})}{r^2}$$

$$\boxed{d\vec{B} = \frac{\mu_0}{4\pi} \frac{(I\, d\vec{l} \times \hat{r})}{r^2}}$$

* $I\,\vec{dl}$ is not smallest source of magnetostatic field while single point charge is smallest source of electrostatic field

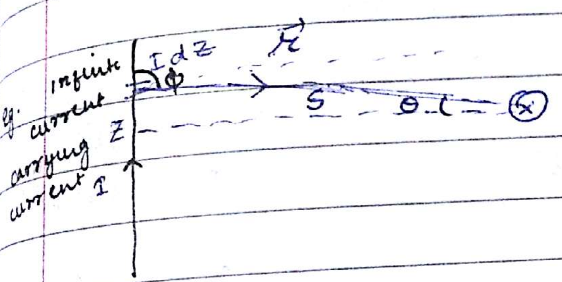
for continuous charge system :

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{I}\times\hat{r}}{r^2}\, dl' \quad : \text{ line charge distribution}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{K}\times\hat{r}}{r^2}\, da' \quad : \text{ surface " "}$$

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{\vec{J}\times\hat{r}}{r^2}\, d\tau' \quad : \text{ volume " "}$$

eg. infinite current carrying current I



{ only when it is of infinite length, we can say that current is steady }

$$\int d\vec{B} = \int \frac{\mu_0}{4\pi}\frac{I\,dz \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi}\int \frac{I\,dz\,\sin\phi}{r^2}$$

$$= \frac{\mu_0}{4\pi}\int \frac{I\,dz\,\sin(90^\circ+\theta)}{r^2} = \frac{\mu_0}{4\pi}\int \frac{I\,dz\,\cos\theta}{r^2}$$

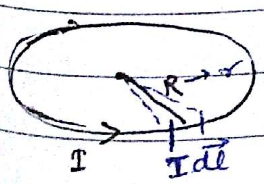
$$= \frac{\mu_0}{4\pi}\int \frac{I\,dz\,\cos\theta}{z^2+r^2} = \frac{\mu_0 I}{4\pi}\int \frac{s\;dz}{(z^2+s^2)^{3/2}} \qquad \begin{aligned} z &= s\tan\theta \\ dz &= s\sec^2\theta\, d\theta \end{aligned}$$

$$= \int \frac{\cancel{s}\,(\cancel{s}\sec^2\theta\, d\theta)}{s^{\cancel{3}}\sec^3\theta} = \int \frac{1}{s}\cos\theta\, d\theta$$

$$= \frac{\mu_0 I}{4\pi s}\Big[\sin\theta\Big]_{-\pi/2}^{\pi/2} = \frac{\mu_0 I}{4\pi s}\left[\frac{2\cancel{z}}{\sqrt{s^2+z^2}}\right]_{\lim_{z\to\infty}} = \boxed{\frac{\mu_0 I}{2\pi s}\hat{\phi}}$$

eg. In a loop :

$$d\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\,d\vec{l}\times\hat{r}}{r^2} \qquad [\text{here, angle b/w } \hat{r} \text{ \& } d\vec{l}' = 90^\circ]$$



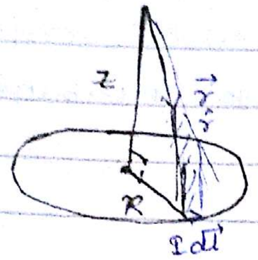
$$= \frac{\mu_0}{4\pi}\int \frac{I\,dl}{r^2}$$

$$= \frac{\mu_0 I}{4\pi r^2}\int_0^{2\pi r} dl$$

r → Radius

$$= \frac{\mu_0 I}{4\pi R^2}(2\pi R) = \frac{\mu_0 I}{2r}\hat{z}$$

Ques



magnetic field will always be $\perp^r$ to the plane

$$\vec{B} = \int d\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\,dl}{r^2}$$

$$= \frac{\mu_0 I}{4\pi}\int_0^{2\pi R}\frac{dl}{R^2+z^2} = \frac{\mu_0 I\,(2R)}{2(R^2+z^2)}\,\hat{r}$$

Lorentz force :

$$\vec{F}_L = q(\vec{V}\times\vec{B})$$
$$= I(\vec{dl}\times\vec{B}') = (I\vec{dl})\times\vec{B}$$

$\vec{F}_e = q\vec{E}$

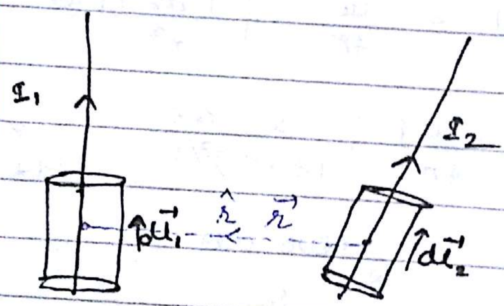
$\vec{F}_L = (I\vec{dl})\times\vec{B}$

Properties :-

→ Work done by Lorentz force :

$$\vec{F}_L\cdot\vec{dl} = I(\vec{dl}\times\vec{B})\cdot\vec{dl} = \underline{0}$$

•)



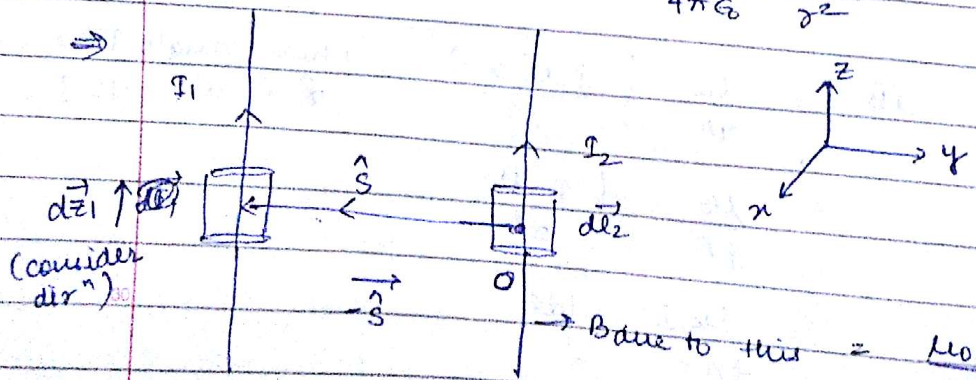
$B_{at\,I}$

$$F\,(\text{at } I_1 \text{ due to } I_2) = (I_1\,dl_1)\times\frac{\mu_0}{4\pi}\left(\frac{I_2\,dl_2\times\hat{r}}{r^2}\right)$$

$$\rightarrow F_L = \frac{\mu_0}{4\pi}\,\frac{(I_1\,dl_1)\times(I_2\,\vec{dl}_2)\times\hat{r}}{r^2}$$

similar to $\vec{F}_e = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$

⇒



$$\rightarrow B \text{ due to this} = \frac{\mu_0 I_2}{2\pi s}\hat{\phi}$$

$$F_L = (I_1\,d\vec{z}_1)\times\left(\frac{\mu_0 I_2}{2\pi s}\right)\hat{\phi} = \frac{\mu_0 I_1 I_2\,dz}{2\pi s}(-\hat{s})$$

- Force b/w the two wires will be attractive type of force

Electrostatics

Magnetostatics

- $d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq}{r^2}\hat{r}$ | $d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{l}\times\hat{r}}{r^2}$

- $\vec{F_e} = q\vec{E}$ | $\vec{F_L} = (I\,d\vec{l}\,)\times\vec{B}$

- $E = \frac{1}{4\pi\epsilon_0}\iiint \frac{\rho\,d\tau'}{r^2}\hat{r}$ | $\vec{B} = \frac{\mu_0}{4\pi}\iiint \frac{\vec{J}\times\hat{r}}{r^2}\,d\tau'$

- $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}$ | $\vec{\nabla}\cdot\vec{B} = 0$

- $E = -\nabla V$

- $\vec{\nabla}\times\vec{E} = 0$ | $\vec{\nabla}\times\vec{B} = \mu_0\vec{J}$

⇒ calc^n of $\vec{\nabla}\cdot B$

$$B(r) = \frac{\mu_0}{4\pi}\iiint \frac{J(r')\times r'}{r^2}\,d\tau'$$

$\left\{ \nabla\cdot(\vec{A}\times\vec{B}) = \vec{B}\cdot(\vec{\nabla}\times\vec{A}) - \vec{A}\cdot(\vec{\nabla}\times\vec{B}) \right\}$

0 (it is in terms of r' while $\vec{\nabla}$ is wrt r)

$$\vec{\nabla}\cdot B(r) = \frac{\mu_0}{4\pi}\left[\iiint \frac{r'}{r^2}\cdot\left(\vec{\nabla}\times J(r')\right)d\tau' - \iiint J(r')\cdot\left(\vec{\nabla}\times\frac{r'}{r^2}\right)d\tau'\right]$$

Electrostatic vector field (conservative vector field)

$\boxed{\vec{\nabla}\times \text{conservative vector field} = 0}$

$\boxed{\vec{\nabla}\cdot\vec{B} \equiv 0}$ ⇒ change of flux per unit volume = 0

⇒ $\oiint \vec{B}\cdot d\vec{a} \equiv 0$ $\left\{ \oiint \vec{E}\cdot d\vec{a} = \frac{q_{encl}}{\epsilon_0} \right\}$

→ ~~when~~ $\oiint \vec{E}\cdot d\vec{a} = 0$ when $\vec{E} = 0$ or $\vec{E} \neq 0$ ~~but~~ & $q_{encl} = 0$ (equal & opp. charges)

Similarly, here, $\oiint \vec{B}\cdot d\vec{a} = 0$ ⇒ two types of sources with opp ~~diff~~ types, they always coexist

Imp. ★★

* Magnetic monopole does not exist.

Date 18 / /

⇒ calc^n of $\bar{\nabla} \times \bar{B}$

$$\bar{B}(r) = \frac{\mu_0}{4\pi} \frac{I\, d\bar{l} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi} \iiint \frac{J(r') \times \hat{\mathscr{r}}}{\mathscr{r}^2} d\tau'$$

Using $\nabla \times (\bar{A} \times \bar{B}) = (\bar{B}\cdot\bar{\nabla})\bar{A} - (\bar{A}\cdot\bar{\nabla})\bar{B} + \bar{A}(\bar{\nabla}\cdot\bar{B}) - \bar{B}(\bar{\nabla}\cdot$

$$\Rightarrow \bar{\nabla} \times \bar{B}(r) = \bar{\nabla} \times \left\{ \quad \right\}$$

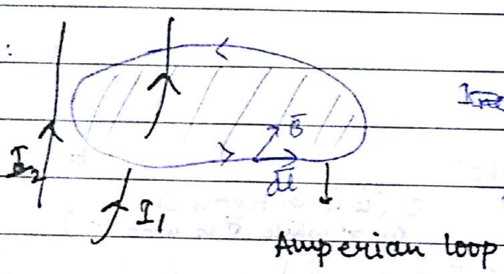
$\Rightarrow$ $\boxed{\bar{\nabla} \times \bar{B} = \mu_0 J(r)}$ ⇒ magnetic field is non-conservative vector field.

using stoke's theorem

$$\iint (\bar{\nabla} \times \bar{B})\, d\bar{a} = \mu_0 \iint J(r)\, da$$

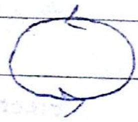
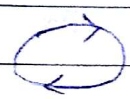
$\boxed{\oint \bar{B}\cdot d\bar{l} = \mu_0 I_{encl}}$ → Ampere's law (Integral form)

Interpretation :



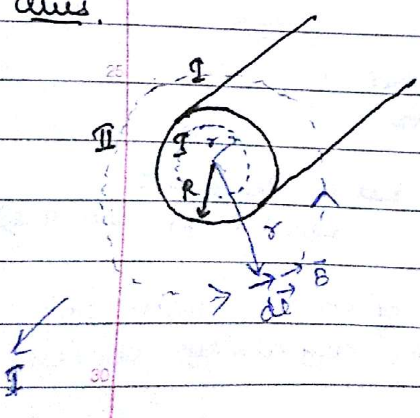
~~Ire~~ I_{encl} : penetrates the ~~closed~~ open surface

$I_{encl} = I_1$ (not I_2)

 : upwards : +ive I  : -ive I

* ~~Ap~~ Ampere's law is analog to Gauss's law.

Ques.



Find B_I and B_{II}

I : current flowing in the ~~wire~~ wire.

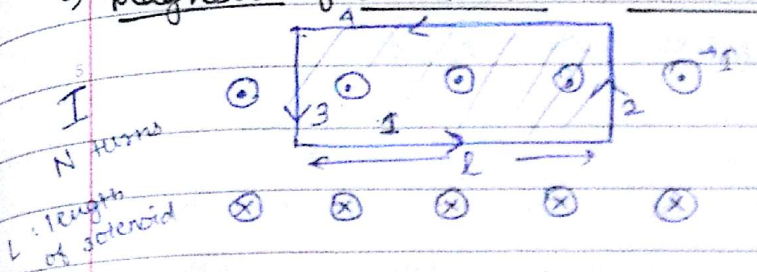
$$\text{II} : \oint B_{II}\cdot d\vec{l} = B_{II}(2\pi r) = \mu_0 I_{encl}$$

$$\Rightarrow \boxed{|\vec{B_{II}}| = \frac{\mu_0 I}{2\pi r}} \text{ Ans.}$$

$$\text{I} : \oint \vec{B_I}\cdot d\vec{l} = \mu_0 I_{encl} \qquad B_I(2\pi r) = \mu_0 \frac{I}{\pi R^2} \times \pi r^2$$

$$\boxed{|\vec{B_T}| = \frac{\mu_0 I r}{2R^2}}$$ Ans.

→ <u>Magnetic field inside a solenoid</u> :



$n = \frac{N}{L}$: no of turns per unit length

* Always takes $\vec{B} \parallel d\vec{l} \Rightarrow$ Ampere's loop.

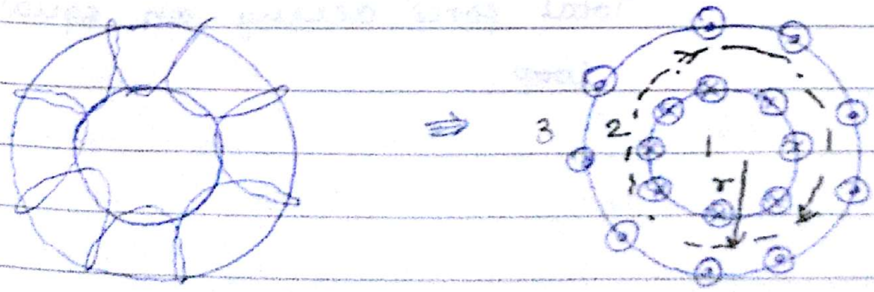
For 4^{th} Region (outside loop), magnetic field = 0

For 2nd ($\perp^r$ $d\vec{l}$ & $\vec{B}$) → $\vec{B} \cdot d\vec{l} = 0$

For 3^{rd} : $B\,l = \mu_0 \left[I \times \frac{N}{L} \times l \right]$

$$\rightarrow \boxed{|\vec{B}| = \frac{\mu_0 N I}{L} = \mu_0 n I}$$

→ <u>Toroid</u> :



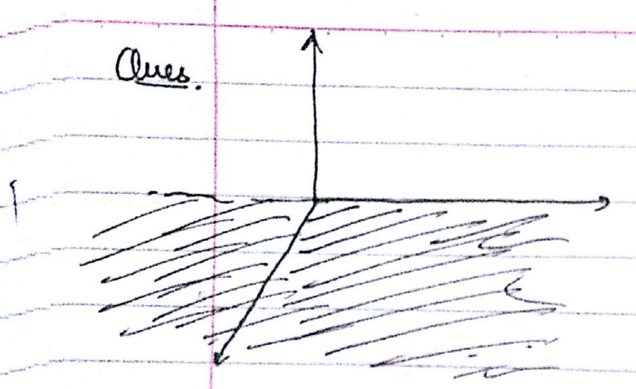
$\vec{B_1} = 0$ [$I_{encl} = 0$]

$\vec{B_2}(2\pi r) = \mu_0 I_{encl} = \mu_0 (IN)$

$$\boxed{B_2 = \frac{\mu_0 N I}{2\pi r}}$$

$\vec{B_3} = 0$ [$I_{encl} = I^+ + I^- = 0$]

Ques.

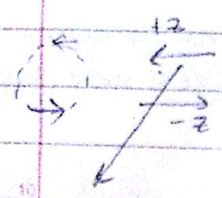


$\vec{K} = c\hat{x}$ $\quad \vec{B} = ?$

$\Downarrow$

surface current

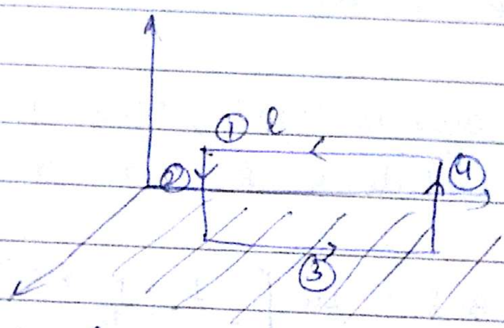
$\bar{B}$: it follows dir^n of $\hat{\phi}$



B/w all lines, $\bar{B}$ will be 0 as they will cancel each other.

$\bar{B}$ will only be in $\leftarrow \hat{y}$

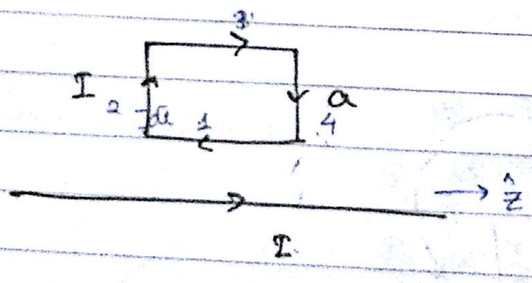
Amperean loop :



$$\bar{B}(2l) = \mu_0 (KL) = \mu_0 K L$$

$$\boxed{\bar{B} = \frac{\mu_0 K}{2}}$$

Ques.



Total force acting on square loop.

$$\bar{F}_B = q(\bar{V} \times \bar{B}) = I\,d\bar{l} \times \bar{B}$$

$$B = \frac{\mu_0 I}{2\pi s}\hat{\phi}$$

$$\bar{F}_{B_1} = I a (-\hat{z}) \times \frac{\mu_0 I}{2\pi s}(\hat{\phi})$$

$$= \frac{\mu_0 I^2 a}{2\pi s}\hat{s}$$

$F_2 = 0$ [illegible] F_{3} [illegible]

$\vec{F}_3 = \int I \, dl \, \frac{\mu_0 I}{4\pi a} = \frac{\mu_0 I^2}{2\pi} \int [illegible] \frac{dy}{y} \, [illegible]$

$= \frac{\mu_0 I^2}{2\pi} \ln \left| [illegible] \right| \, [illegible]$

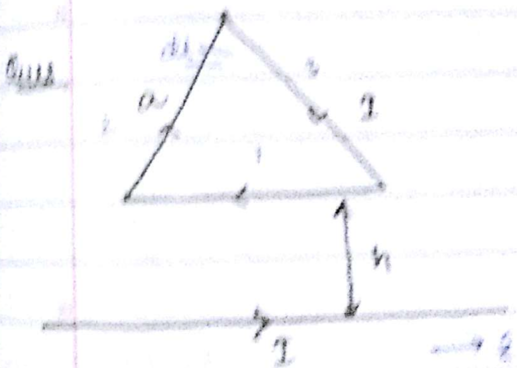
$\vec{F}_{41} = \int [illegible] \frac{\mu_0 I}{2\pi} \left([illegible] \right) = \int \frac{\mu_0 I^2}{2\pi} \frac{dy}{y} \left([illegible] \right)$

$= \frac{\mu_0 I^2}{2\pi} \ln \left| [illegible] \right| \left([illegible] \right)$

$\vec{F}_3 + \vec{F}_{41} = 0$

$\vec{F}_4 = \int I \, (\ldots)$

Ques



$d\vec{l}_2 = dx\hat{x} + dz\hat{z}$

$d\vec{l}_4 = -dx\hat{x} + dz\hat{z}$

we will see that $\hat{x}$ components in both will cancel each other

19-4-19 Electrostatics : $\vec{\nabla} \times \vec{E} = 0$

also, we know that $\vec{\nabla} \times \vec{\nabla}\varphi = 0$

$\Rightarrow \vec{E} = -\nabla V$ ↳ scalar potential

Magnetostatics :

$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J} \Rightarrow$ non-conservative

Hence, $\vec{B}$ can't be written in form of scalar potential

But we know, $\vec{\nabla} \cdot \vec{B} = 0$

$\nabla \cdot (\vec{\nabla} \times \vec{A}) = 0$

also,

$\Rightarrow \vec{B} = \nabla \times \vec{A}$ ↳ magnetic vector potential

* Here, $\bar{A}$ doesn't have any physical meaning
But it is imp. to study as it ~~p~~ is analogous to V in Elec

Electrostatics	Magnetostatics
$\bar{E} = -\nabla V$	$\bar{B} = \bar{\nabla} \times \bar{A}$
$V \longrightarrow V + C$	$\bar{A} \longrightarrow \bar{A} + \bar{\nabla}\psi$ (scalar funcⁿ)
$\bar{E} = -\nabla V$	$\Rightarrow \bar{B} = \bar{\nabla} \times \bar{A}$

⇒ In calculating $\nabla \bar{A}$, we add $\bar{A} \rightarrow \bar{A} + \bar{\nabla}\psi$

$$\bar{\nabla} \cdot (\bar{A} + \bar{\nabla}\psi) = 0$$

$$\bar{\nabla} \cdot \bar{A} + \nabla^2 \psi = 0$$

$$\cancel{\bar{\nabla}\cdot\bar{A}} \quad \boxed{\nabla^2 \psi = -\bar{\nabla} \cdot \bar{A}}$$

Hence, $\boxed{\bar{\nabla} \cdot \bar{A} = 0}$ $\boxed{\bar{\nabla} \times \bar{A} = \bar{B}}$

Ampere's law :

$$\bar{\nabla} \times \bar{B} = \mu_0 \vec{J}$$

$$\bar{\nabla} \times (\bar{\nabla} \times \bar{A}) = \mu_0 \vec{J}$$

$$\Rightarrow \bar{\nabla}(\underbrace{\bar{\nabla} \cdot \bar{A}}_{0}) - \nabla^2 A = \mu_0 \bar{J}$$

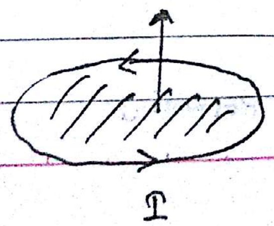
$$\Rightarrow \boxed{\nabla^2 A = -\mu_0 \vec{J}} , \quad \nabla^2 V = -\frac{\rho}{\epsilon_0} \text{ (In electrostatics)}$$

solⁿ of V : $\frac{1}{4\pi\epsilon_0} \iiint \frac{\rho \, d\tau'}{r}$

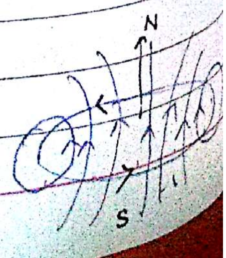
solⁿ of A : $A = \frac{\mu_0}{4\pi} \iiint \frac{\bar{J} \, d\tau'}{r}$

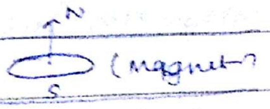
→ We earlier used multipole expⁿ ~~Eq~~ to find V due to charge system. Similarly, it can be used in magnetostatics.

Eq.



$\vec{A}$ $\quad \vec{\mu}_m = \vec{A} I$
(magnetic dipole moment)





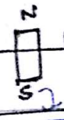
N S

$S \longrightarrow N$ (it will allign itself in dir^n of Magnetic field)

$$V = \frac{1}{4\pi\epsilon_0} \frac{\vec{P} \cdot \hat{r}}{r^2}$$

$$\vec{A} \text{ (here)} = \frac{\mu_0}{4\pi} \frac{\vec{\mu}_m \times \hat{r}}{r^2}$$

Ques

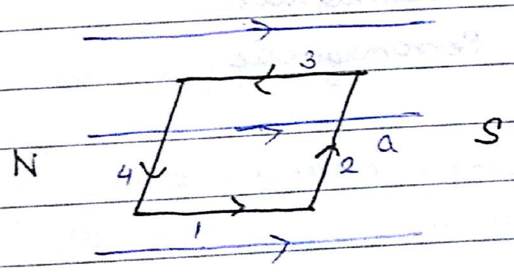


Bar magnet (Ferromagnetic material)

N S

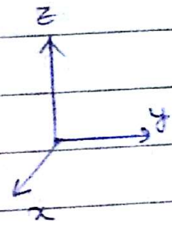
$\boxed{S \longrightarrow N}$ gets alligned (Why?)

→



$$\vec{F} = (I d\vec{l} \times \vec{B})$$

$$\vec{F_1} = \vec{F_3} = 0$$

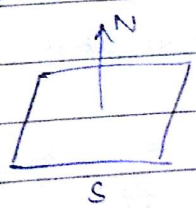


$$\vec{F_2} = \cancel{I\,dl}\; I a(-\hat{i}) \times B(\hat{j}) = -IaB\,\hat{k}$$

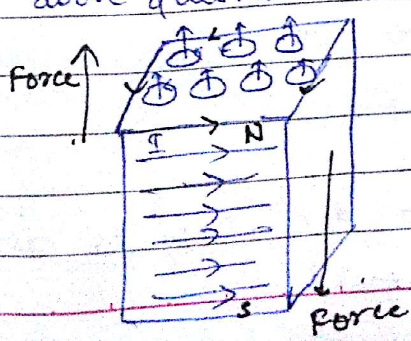
$$\vec{F_4} = IaB\,\hat{k}$$

It experiences a torque

It will rotate s.t. its top is in the dir^n of S



→ In above question (Bar Magnet)



all atoms show same magnetic dipole moment. Hence, some current exists due to motion of $e^{\ominus}$

Net current $= 0$

But for boundary, current exists (Bound Current)

Due to the force, it will experience torque which will result ~~show~~ its allignment in dir^n of magnetic field.

Magnetic field inside a Magnetic Material

Dielectric Material — Magnetic Material

$\vec{E}_{net} (< \vec{E}_{ext})$

$|\vec{B}_{net}| < |\vec{B}_{ext}|$

$|\vec{B}_{net}| > |\vec{B}_{ext}|$

$|\vec{B}_{net}| >> |\vec{B}_{ext}|$

$\xrightarrow{}$ E_{ext} $\qquad\qquad$ $\xrightarrow{}$ B_{ext}

3 cases possible :

1.) $|\vec{B}_{net}| < |\vec{B}_{ext}|$ $\longrightarrow$ **Diamagnetic**

2.) $|\vec{B}_{net}| > |\vec{B}_{ext}|$ $\longrightarrow$ **Paramagnetic**

3.) $|\vec{B}_{net}| >> |\vec{B}_{ext}|$ $\longrightarrow$ **Ferromagnetic**

1.) & 2.) : some field is generated inside the solid
some I must be responsible for it.

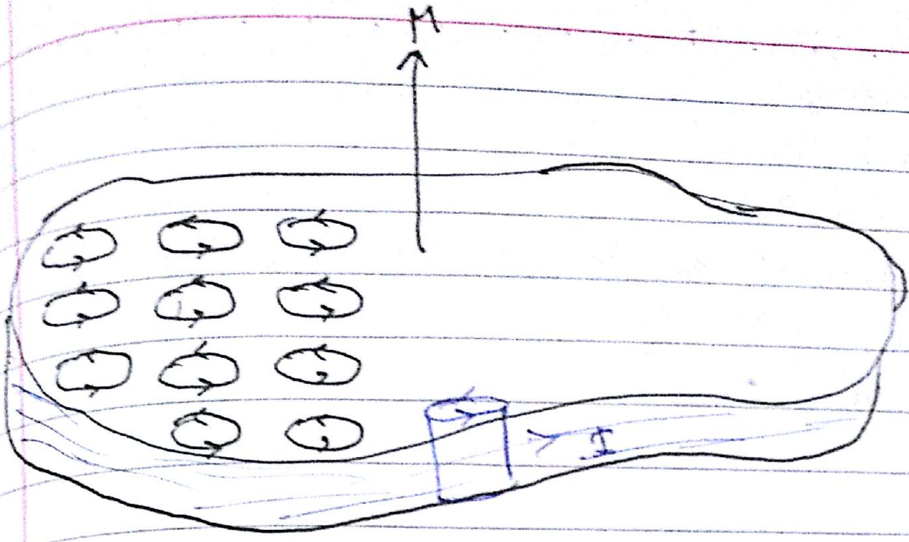
The magnetic material gets magnetised (Magnetic dipole (Molecular/atomic) gets generated) after applying $\vec{B}_{ext}$.

$\vec{M}$ $\leftarrow$ Magnetization vector

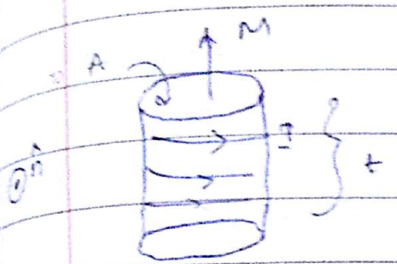
$$\vec{M} = \frac{\sum \vec{\mu}_m}{\Delta V} \quad : \text{Magnetic dipole moment per unit volume}$$

Dimension : $\dfrac{\overset{\text{area}}{A} I}{\Delta V} = \dfrac{L^2 A}{L^3} \Rightarrow \dfrac{I}{L}$; Surface current density $\Rightarrow K$

Magnetic dipole : get alligned in dir^n / opp to dir^n of magnetic field



No I_{net} inside.
Only I exist on boundary (although no motion of e^-)



$|\vec{\mu}_m| = |\vec{M}|\, A t \Rightarrow |\vec{M}| = \frac{I}{t}$

$|\vec{\mu}_m| = A\, I$

$|\vec{M}| = K$

$\boxed{\vec{M} \times \hat{n} = K_b}$: Bound surface current density

$\Rightarrow$ I $\perp^r$ to M & A

* If magnetisation is non-uniform,

less magnetisation $\Rightarrow$ less I

More magnetisation $\Rightarrow$ more I

$\Rightarrow I_{net}$ inside $\neq 0$

* Bound volume ~~Charge~~ Current Density

$$\boxed{\vec{J}_b = \vec{\nabla} \times \vec{M}}$$

We can find $\vec{B}$ using Ampere's law.

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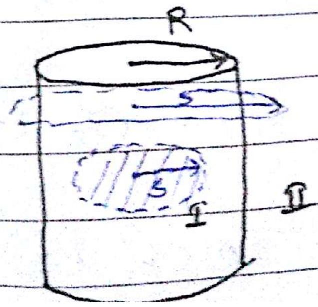
Ex.



$\vec{M} = c s^2 \hat{\phi}$, c : const.

$\vec{B}_I = ?$ $\vec{B}_{II} = ?$

⇒ There is no free I.

∴ It has symmetry,
Using Ampere's law,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$$

$\vec{B}_1$: same direct^n as $\vec{M}$ ⇒ Ampere's loop : $\hat{\phi}$

↳ no surface (No K_b)

$$B(2\pi s) = \mu_0 \left[I_{encl} \right]$$

$$J_B = \vec{\nabla} \times \vec{M}$$

$$= \vec{\nabla} \times (cs^2 \hat{\phi})$$

$$= \frac{1}{s}\left[\frac{\partial}{\partial s}(sM_\phi)\right]\hat{z} = \frac{1}{s}\frac{\partial}{\partial s}(cs^3) = 3cs\,\hat{z}$$

Current per unit area $\perp^r$ to it.

~~I_{Rec}~~

$$I_{encl} = \iint J_b \cdot d\bar{a}$$

$$= \int_0^s\int_0^{2\pi} 3cs\,\cancel{\hat{z}} \cdot s\,d\phi\,ds$$

$$(2\pi s)\,\bar{B}_1 = \frac{\cancel{3}c\,s^3}{\cancel{3}} \cdot 2\pi = \mu_0(2cs^3\pi)$$

$$\Rightarrow \bar{B}_1 = \mu_0 s^2 c\,\hat{\phi}$$

~~$\oint B_2$~~ $\cancel{\bar{B}_2 (2\pi s) = \int_0^R\int_0^{2\pi} 3cs\, s\,d\phi\,ds}$ $\ominus$

~~$\Rightarrow \bar{B}_2 = \mu_0$~~

Outside, we have both K_b and J_b.

$$\vec{K}_b = cs^2\hat{\phi} \times \hat{s}\Big|_{s=R} = -cs^2\hat{z}\Big|_{s=R} = cR^2(-\hat{z})$$

~~$I_{encl} = \iint_0^{2\pi} cR^2$~~ $\quad \vec{J}_b = 3cs\,\hat{z}$

~~Total current =~~ $\oint$

surface I due to $\vec{K}_b$ = $cR^2\,(2\pi R) = 2\pi cR^3\,(-\hat{z})$

$$\text{Total volume } I = \int_0^R\int \bar{J}_B \cdot d\bar{a} = 2\pi cR^3\,(\hat{z})$$

⇒ total ~~B~~ $\bar{I} = 0$ ⇒ $\bar{B}_{Total} = 0$ (outside)

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$\Rightarrow$ $\boxed{\bar{B}_2 = 0}$

$$\oint \bar{B}\cdot d\bar{l} = \mu_0 I_{encl}$$ → includes both bounded and free current.

$$\bar{\nabla} \times \bar{B} = \mu_0 \bar{J}$$
$$= \mu_0 (\bar{J}_{bound} + \bar{J}_{free})$$

$$\bar{\nabla} \times \frac{\bar{B}}{\mu_0} = \bar{\nabla} \times \bar{M} + \bar{J}_{free}$$

$$\bar{\nabla} \times \underbrace{\left(\frac{\bar{B}}{\mu_0} - \bar{M}\right)}_{\bar{H}} = \bar{J}_{free}$$

$$\boxed{\bar{\nabla} \times \bar{H} = \bar{J}_{free}}$$

Dimension of $\bar{H}$: $\bar{B} = \mu_0 \bar{H} + \mu_0 \bar{M}$
⇓
dimension of $\bar{M}$ → it is also some kind of surface ?

$$\bar{B} = \mu_0 \bar{H} + \mu_0 \bar{M}$$

$\mu_0 \bar{H}$ ↑ contribution due to free current

$\mu_0 \bar{M}$ ↑ contribution due to bound current

→ When I is passed in solenoid, $\bar{B}$ is developed due to $\bar{H}$.
Now, when iron (ferromagnetic) is placed inside it, $\bar{B}$ due to $\bar{M}$ also gets added

$$\oint \bar{H}\cdot d\bar{l} = I_{free}$$

$\bar{H}$: dir^n of $\bar{B}$

Here, $H(2\pi s) = 0$ $[I_{free} = 0]$

$\Rightarrow H_\phi = 0$

$\Rightarrow \bar{B}_1 = \mu_0 \bar{M}$

$\Rightarrow$ $\boxed{\vec{B}_1 = \mu_0 (c s^2 \hat{\phi})}$

$\bar{B}_2 = 0$

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$$\boxed{\vec{M} = \chi \vec{H}}$$

χ → Magnetic susceptibility (dimensionless quantity)

diamagnetic : χ $(10^{-4} - 10^{-6})$ [-ive]

paramagnetic : " (+ive)

ferromagnetic (valid only for low field) $(10^{4} - 10^{12})$

χ : how easily you can magnetise ~~an~~ object due to free I.

$$\vec{B} = \mu_0 \bar{H} + \mu_0 \bar{M}$$

$$= \mu_0 [1 + \chi] \bar{H}$$

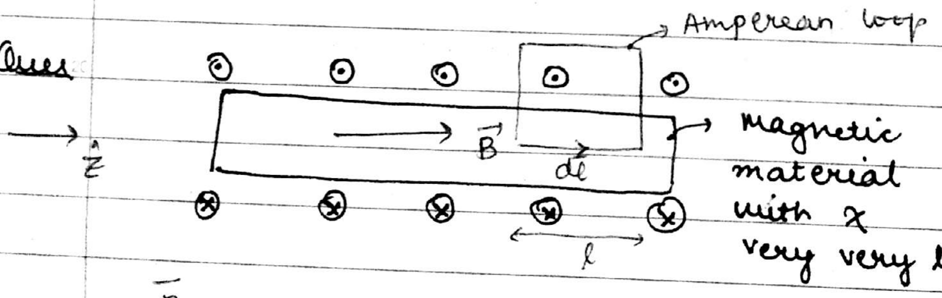
$$\boxed{\bar{B} = \mu \bar{H}}$$

μ → Permeability

$$\frac{\mu}{\mu_0} = 1 + \chi = \mu_r \leftarrow \text{Relative permeability}$$

$$\Rightarrow \boxed{\vec{B} = \mu_0 \mu_r \bar{H}}$$

Ques



No. of turns = N
current : I
length : L

$\bar{B}_{outside} = 0$

$$\oint \bar{H} \cdot d\bar{l} = \cancel{\mu_0}\, I_{free} \qquad \frac{N\,l\,I}{L}$$

$$\bar{H}(l) = \cancel{\mu_0}\frac{NI}{L}(l) \Rightarrow \bar{H} = \frac{NI}{L}$$

$$\bar{B} = \mu \bar{H}$$

$$\bar{B} = \mu_0 (1 + \chi) \bar{H}$$

$$\Rightarrow \bar{B} = \mu_0 (1 + \chi) \frac{NI}{L}$$ (very high just because χ is very high)

$$= \frac{\mu_0 N}{L} I + \frac{\mu_0 N}{L} \chi I = \mu_0 \bar{H} + \mu_0 \bar{M}$$

Bound Current = ??

$\cancel{\bar{K}_b = \bar{M}\times\hat{n}}$ $\quad \bar{M} = \chi \bar{H}$

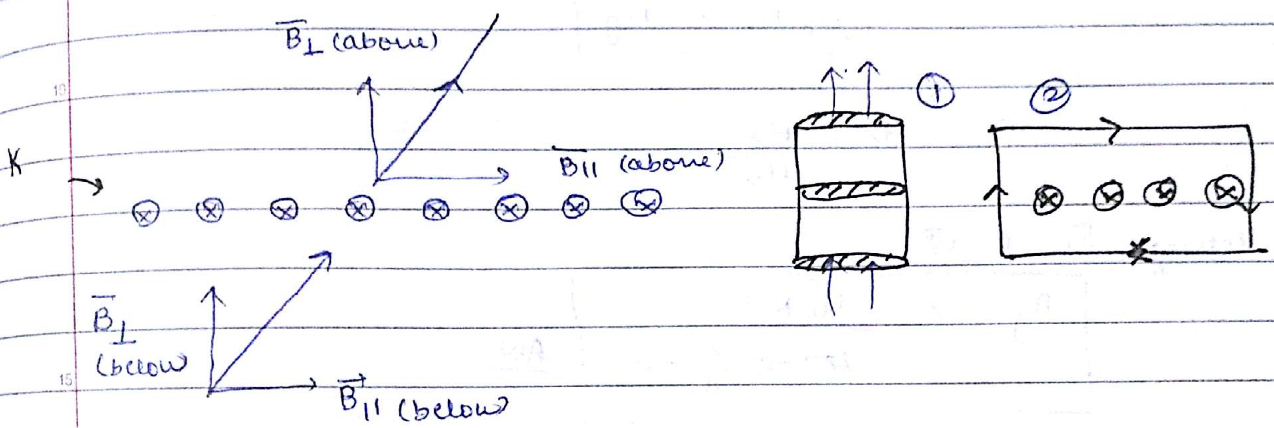
$$\bar{M} = \chi \frac{N}{L} I$$

$$\bar{J}_b = 0 \quad (\text{const.})$$

$$\bar{K}_b = \bar{M}\times\hat{n} = \chi \frac{N I}{L}(\hat{z}) \times \hat{s} = \frac{\chi N I}{L}\hat{\phi}$$

(similar to current shown)

Boundary Condition of Magnetic Field :



Two cond^{n}s that $\bar{B}$ follow:

1.) Ampere's law

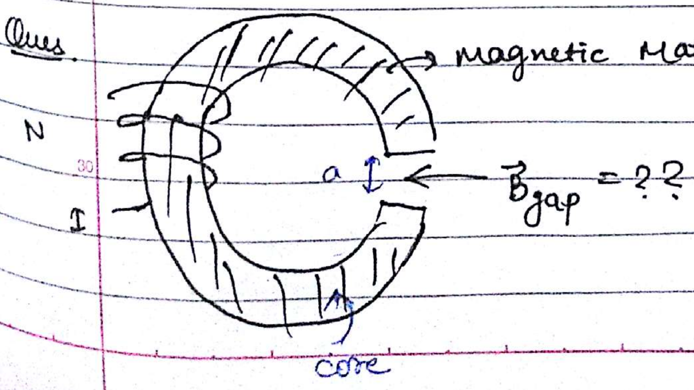
2.) $\bar{\nabla}\cdot\bar{B} = 0$

$$\oiint \bar{B}\cdot d\bar{a} = 0 \quad , \quad \oint \bar{B}\cdot d\bar{l} = \mu_0 I_{encl}$$

① → $B_{\perp r}(\text{above}) = B_{\perp r}(\text{below})$ $\quad$ [above − below = 0]

② $\bar{B}_{\parallel}(\text{above}) - \bar{B}_{\parallel}(\text{below}) = \mu_0 K$

Ques.



Applying ampere's law

$$\oint \bar{H}\cdot d\bar{l} = I_{free} = NI$$

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$$H_c\,(2\pi r - a) + H_{gap}\,a = NI \quad \text{—— (1)}$$

↓
core

$$B_c = \mu_0\mu_r \bar{H} \quad , \quad B_g = \mu_0 H_g$$

B (⊥r to surface)

$B_\perp$: continuous

$$\Rightarrow |\bar{B}_c| = |\bar{B}_g|$$

$$\Rightarrow \mu_0\mu_r|\bar{H}| = \mu_0|\bar{H}_g|$$

$$\Rightarrow \boxed{\mu_r H_c = H_g} \quad \text{—— (2)}$$

$$\Rightarrow H_c = \frac{H_g}{\mu_r}$$

Using (1) & (2),

$$\boxed{B_g = \frac{\mu_0 N I}{(2\pi r - a)/\mu_r + a}}$$

Ans.