**The LNM Institute of Information Technology**
**Jaipur, Rajsthan**

**MATH-I** ■ Assignment #9

(Real-valued Functions of Several Variables: Differentiability and partial derivatives)

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Q1. Examine the following functions for differentiability at the point $(0,0)$ where $f(0,0) = 0$ and $f(x,y)$ for $(x,y) \neq (0,0)$ is given by
$(a)$ $\frac{-3x}{\sqrt{x^2+y^2}}$, $(b)$ $\frac{5x}{x^2+x+y^2}$, $(c)$ $x^2y(\frac{x^2-y^2}{x^2+y^2})$, $(d)$ $\frac{3x^2y}{x^2+2y^2}$ $(e)$ $x^2 \sin\frac{1}{x} + y^2 sin\frac{1}{y}$.

Q2. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} 1, & \text{if } x = 0 \text{ or if } y = 0 \\ 0, & \text{otherwise.} \end{cases}$$

Show that the function satisfy the following:
$(a)$ $f(x,y)$ is not continuous at $(0,0)$,
$(b)$ the partial derivatives exist at $(0,0)$.

Q3. Let

$$f(x,y) = \begin{cases} xy\left(\frac{x^2-y^2}{x^2+y^2}\right), & \text{if } \quad (x,y) \neq (0,0) \\ 0, & \text{if } \quad (x,y) = (0,0). \end{cases}$$

Prove that
$(a)$ $f_x(0,y) = -y$ and $f_y(x,0) = x$ for all $x$ and $y$,
$(b)$ $f_{xy}(0,0) = 1$ and $f_{yx}(0,0) = -1$ and
$(c)$ $f(x,y)$ is differentiable at $(0,0)$.

Q4. Let

$$f(x,y) = \begin{cases} xy\left(\frac{2x^2-3y^2}{x^2+y^2}\right), & \text{if } \quad (x,y) \neq (0,0) \\ 0, & \text{if } \quad (x,y) = (0,0). \end{cases}$$

Prove that $f_{xy}(0,0) \neq f_{yx}(0,0)$, but $f(x,y)$ is differentiable at $(0,0)$.

Q5. Consider the function $f : \mathbb{R}^2 \to \mathbb{R}$ defined by

$$f(x,y) = \begin{cases} \frac{y^3}{x^2+y^2}, & \text{if } (x,y) \neq (0,0) \\ 0, & \text{otherwise.} \end{cases}$$

Show that $f$ is continuous at $(0,0)$, it has partial derivatives at $(0,0)$ but not differentiable at $(0,0)$.