Classical Physics (mechanics)

Introduction to physics (mechanics) - Kleppener Kolenkow

$$v = \frac{d\vec{r}}{dt} = \dot{r}\hat{r} + r\frac{d\hat{r}}{dt}$$

$$\boxed{\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}}$$

$v_r = \dot{r}$ = radial vel.

$v_\theta = r\dot{\theta}$ = tangential vel.

$= r\omega$

$\omega = \dot{\theta}$ = angular vel.

$$\boxed{\frac{d\hat{\theta}}{dt} = -\dot{\theta}\hat{r}}$$

$$\hat{r} = \cancel{\hat{x}}\ \hat{x}\cos\theta + \hat{y}\sin\theta$$
$$\hat{\theta} = -\hat{x}\sin\theta + \hat{y}\cos\theta$$

Vel. & acc^n in plane polar Coordinates

$$\vec{r} = r\hat{r}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$\left(\frac{d\hat{r}}{dt} = \dot{\theta}\hat{\theta}\right)$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \ddot{r}\hat{r} + \dot{r}\frac{d\hat{r}}{dt} + \dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} + r\dot{\theta}\frac{d\hat{\theta}}{dt}$$

$$= \ddot{r}\hat{r} + \dot{r}\dot{\theta}\hat{\theta} + \dot{r}\dot{\theta}\hat{\theta} + r\ddot{\theta}\hat{\theta} - r\dot{\theta}^2\hat{r}$$

$$= \underbrace{(\ddot{r} - r\dot{\theta}^2)}_{a_r}\hat{r} + \underbrace{(2\dot{r}\dot{\theta} + r\ddot{\theta})}_{a_\theta}\hat{\theta}$$

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$\dot{\theta} = \omega$

$a_r \begin{cases} \ddot{r} = \text{linear acc}^n \text{ in radial direc}^n \\ r\dot{\theta}^2 = \text{Centripetal. acc}^n \end{cases}$

$a_\theta \begin{cases} 2\dot{r}\dot{\theta} = \text{Coriolis acc}^n \text{ in tengential. direc}^n \\ r\ddot{\theta} = \text{linear acc}^n \text{ in tengential direc}^n. \end{cases}$

Q. Particle moving in a plane with Cons. radial vel. 4 m/s & Cons. angular vel. 2 rad/s. Find mag. of (i) v (2) a at 3 m from origin.

$v = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta} = 4\hat{r} + 6\hat{\theta}.$

$v = \sqrt{16 + (2\times3)^2} = \sqrt{16+36} = \sqrt{52}$ m/s

$\vec{a} = (\cancel{\ddot{r}} - r\dot{\theta}^2)\hat{r} + (2\dot{r}\dot{\theta} + \cancel{r\ddot{\theta}})\hat{\theta}$

$\vec{a} = -12\hat{r} + 16\hat{\theta}$

$a = 20 \text{ m/s}^2$

Q. A bead moves with uniform radial speed u m/s on spoke of wheel rotating with uniform ~~ag~~ ang. vel. ω rad s⁻¹ s.t. at $t=0$ bead is at origin & spoke is on x-axis. Find vel. at $t=T$ in polar coordinate.

$v = u\hat{r} + r\omega\hat{\theta}$

$= u\hat{r} + uT\omega\hat{\theta}$

Newton's Law :-

(i) if $\vec{F}_{ext} = 0 \Rightarrow \vec{p}$ and $\vec{v}$ is Cons.

(ii) $\vec{F}_{ext} = \frac{d\vec{p}}{dt} = m\vec{a}$

$\vec{F}_{AB} = -\vec{F}_{BA}$

First law define an inertial FOR = FOR at rest or Const. vel. w.r.t body being observed

'It is always possible to find an IFOR for an isolated body
↓
no net ext. force

Second Law :-

$$\vec{F}_{ext} = \frac{d\vec{p}}{dt} \rightarrow \text{general}$$

$$= m\vec{a}$$

Assumption - (i) Point particle
(ii) Mass is Cons.
(iii) non relativistic Posiⁿ $v << c$

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

3rd law -

$\vec{F}_{AB} = -\vec{F}_{BA}$ verifies the state of isolation of a body.

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Free body diagram :-

1. Separate the bodies & treat them as point
   & abz.
(2) Draw forces act acting on body

$$\vec{v} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

There is no inertial FOR on earth
& actual earth is also NIFOR

To Calculate $a_r = r\dot{\theta}^2$ for earth

$$r\dot{\theta}^2 = R\omega^2$$

$$R = 6400 \text{ km}$$

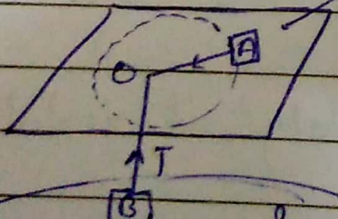
$$\omega = \frac{2\pi}{T} = \frac{2\pi}{24 \times 3600}$$

$$a_r = \frac{6400 \times 10^3 \times (4\pi)^2}{(24 \times 3600)^2} \simeq 0.034 \ m/s^2$$

$$\frac{g}{a_r} \simeq 300$$

eg.

frictionless.



$T = m_A r_0 \omega^2$ X

$m_B g - T = m_B a_B$

at $t=0$ & $V_B = 0$
and A is moving with uniform Circular motion
with $r = r_0$ & angular vel. ω.
if B is release at $t=0$
than what is acc?

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$L = r + z$

$\frac{dL}{dt} = \frac{dr}{dt}$

$\ddot{r} = -\ddot{z} = -a_B$

$T = m_A(r\omega^2 - \ddot{r})$

$w_B - m_A r\dot{\theta}^2 + m_A\ddot{r} = m_B a_B$

at $t = 0$ $\quad w_B - m_A r_0 \omega_0^2 - m_A a_B = m_B a_B$

Assignment - 1

$v = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$

$a = \underbrace{(\ddot{r} - r\dot{\theta}^2)}_{a_r}\hat{r} + \underbrace{(2\dot{r}\dot{\theta} + r\ddot{\theta})}_{a_\theta}\hat{\theta}$

(3) $N = 80$ rev/min $\rightarrow \dot{\theta} = 2\pi \times \frac{80}{60}$ rad/s

$\dot{N} = -280$ rev/min/s; $\ddot{\theta} = -2\pi \times \frac{280}{60}$ rad/s^2

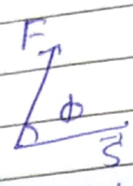
$r = 200$ mm

$\dot{r} = -300$ mm/s

$\ddot{r} = 0$

Work & Energy

+ve & -ve work
work energy Th.

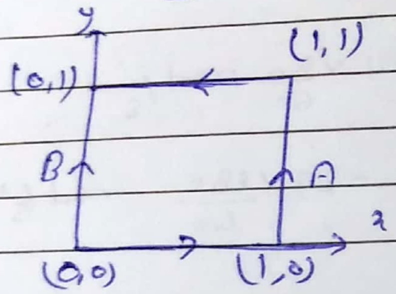


$W = \vec{F}\cdot\vec{S} = FS\cos\phi$

$0 \le \phi \le 90^\circ$, $270 \le \phi < 360^\circ$ → +ve work

$90 < \phi < 270$ -ve work

Q. $\vec{F} = A(xy\,\hat{i} + y^2\hat{j})$ find whether C/NC by calculating W_{ba} for 2 path



Path (A) · (0,0) → (1,0) → (1,1) → (0,1)

path (B) (0,0) → (0,1)

Sol. $\int_A \vec{F}\cdot d\vec{r} = \int_{x=0}^{1} xy\,dx + \int_{y=0}^{1} y^2 dy + \int_{x=1}^{0} xy\,dx$

$= \cancel{y\left(\frac{1}{2}\right)} + \frac{1}{3} + \left(-\frac{1}{2}\right)$

$0 + \frac{1}{3} - \frac{1}{2} = -\frac{1}{6}$

For Path B

$\int_B \vec{F}\cdot d\vec{r} = \int_{y=0}^{1} y^2 dy = \frac{1}{3}$

$W^A_{ba} \ne W^B_{ba}$ → Non Conservative

Assignment - 4

(1) (a) $M\frac{dv}{dt} = \vec{F}_{ext} + \vec{u}_{rel}\frac{dM}{dt}$

$M\frac{dv}{dt}\hat{j} = -Mg\,\hat{j} + (-v_0)\hat{j}\,\frac{dM}{dt}$

$\frac{dv}{dt} = -\frac{Mg}{M} - \frac{v_0}{M}\frac{dM}{dt}$

$a = -g - \frac{v_0}{M}(-u) = -g + \frac{v_0 u}{M}$

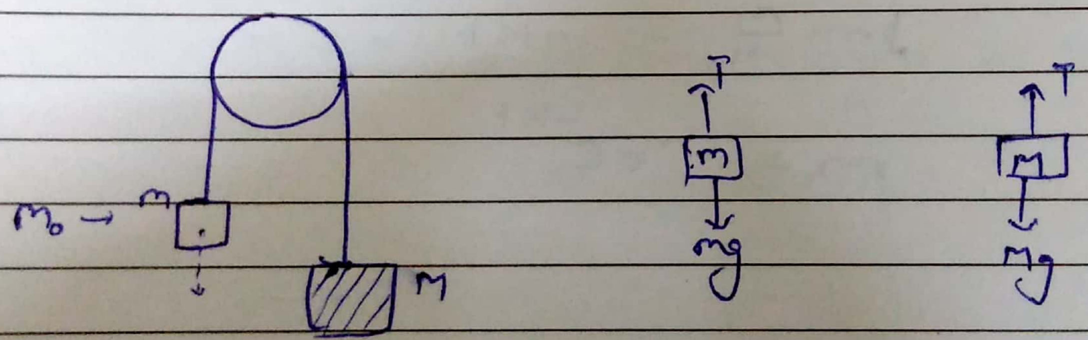
(b) $\frac{dv}{dt} = -g - \frac{v_0}{M}\frac{dM}{dt}$

$\frac{dM}{dt} = -\left(\frac{dv}{dt} + g\right)\frac{M}{v_0} = -(0.5g+g)\frac{M_0}{v_0}$

$= \frac{-1.5 \times 10 \times 1000 \times 1000}{2000}$

$= 7500\ kg/s$

(2)



$F_{ext} + u\frac{dm}{dt} = M\frac{dv}{dt}$

$(T-mg)\hat{j} + (-u)\left(\frac{dm}{dt}\right)\hat{j} = ma\,\hat{j}$

$(T-mg) + (-u)(-Km) = ma$

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for larger mass

$$Mg - T = Ma$$
$$T = M(g-a)$$

$$mg - ma - mg) + \mu K m = ma$$

$$a = \frac{M(g-a)}{m} - g + \mu K$$

$$a\left(1 + \frac{M}{m}\right) = \frac{Mg}{m} - g + \mu K$$

$$a = \frac{Mg - mg + m\mu K}{(M+m)}$$

$$\frac{dm}{dt} = -Km$$

$$\int_{m_0}^{m} \frac{dm}{m} = -\int_{0}^{t} K\, dt$$

$$\ln \frac{m}{m_0} = -Kt$$

$$m = m_0 e^{-Kt}$$

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Q.3. $\cancel{\vec{F}_{ext}}^{\,0} + \vec{u}_{rel}\frac{dM}{dt} = M\frac{dv}{dt}$

Scalar $-u_{rel}\frac{dm}{dt} = M\frac{dv}{dt}$

$u_{rel} = K\sqrt{M}$

$-K\sqrt{M}\frac{dM}{dt} = M\frac{dv}{dt}$

$-K\frac{dM}{dt} = \sqrt{M}\frac{dv}{dt}$

$-K\int_{m_i}^{m}\frac{dM}{M} = \int_0^v dv$

$-2K\left[\sqrt{M}\right]_{m_i}^{m_f} = \Delta v$

$-2K\left(\sqrt{3600}-\sqrt{6400}\right) = 2$

$K = 1/20$

$-2\times 1/20\left[\sqrt{900}-\sqrt{6400}\right. = v$

$v = 5\ m/s$

Q(4)

(a) variable mass app.

$$F_{ext} + u_{rel}\frac{dm}{dt} = M\frac{dv}{dt}$$

$$Mg(-\hat{j}) + v(\hat{j})\frac{dm}{dt} = M\frac{dv}{dt}(-\hat{j})$$

$u_{rel} = v(\hat{j})$: relative vel. of Smaller mass w.r.t bigger mass

$$-Mg + v\frac{dm}{dt} = -M\frac{dv}{dt}$$

$$v\frac{dm}{dt} + M\frac{dv}{dt} = Mg$$

$$v(k\cancel{M}v) + \cancel{M}\frac{dv}{dt} = \cancel{M}g$$

$$\frac{dv}{dt} = g - kv^2$$

$v \uparrow$ing $\frac{dv}{dt}$ $\downarrow$ing

at a time $\frac{dv}{dt} = 0$

$$g = kv^2$$

$$v = \sqrt{\frac{g}{k}}$$ ← terminal vel.

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(6) (a) Force = Thrust = $\left| u_{rel} \cdot \frac{dm}{dt} \right| \times A$

$= 5 \times 10^{-3} \times 10^{-3} \frac{kg}{cm^2 \cdot s} \times 500\ cm^2$

$= 2.5 \times 10^{-3}\ N.$

(b) $U_{rel} = 5 + 2 = 7\ m/s$

$\frac{dm}{dt} = ?$ when U_{rel} is $5\ m/s$ $\Rightarrow \frac{dm}{dt}$ is $10^{-3} \frac{g}{cm^2 s}$

is $7\ m/s$ $\frac{dm}{dt} = \frac{7}{5} \times 10^{-3} \frac{g}{cm^2}$

$F = 7 \times \frac{7}{5} \times 10^{-3} \times 10^{-3} \times 500$

$= 4.9 \times 10^{-3}\ N$

(7) $F_{ext} + u_{rel} \frac{dm}{dt} = m \frac{dv}{dt}$

$(mg + mbv)(-\hat{j}) + u(-\hat{j}) \frac{dm}{dt} = m \frac{dv}{dt} (\hat{j})$

$-(mg + mbv) - u(-rm) = m \frac{dv}{dt}$

$\frac{dv}{dt} + bv = ru - g$

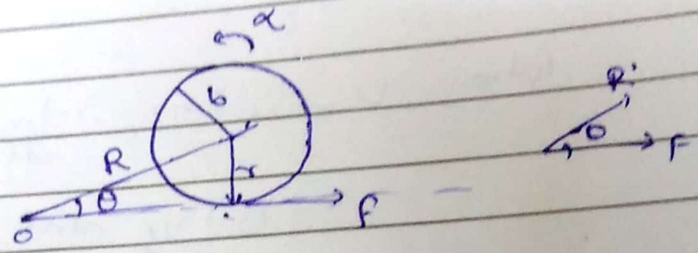
Rotational motion

Torque for rotational + translational motion

$$L = I_0\omega + (\vec{R}\times m\vec{v})_z$$

$$\tau = \frac{dL}{dt} = I_0\alpha + (\vec{R}\times\vec{F})_z$$

eg



Rotation torque $\quad I_0\alpha = \vec{r}\times F = bF$

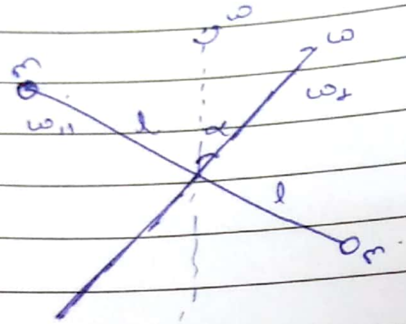
$$\alpha = \frac{bF}{I_0} = \frac{2F}{mb}$$

$$\vec{R}\times\vec{F} = -RF\sin\theta = -bF$$

Net torque = 0

Rotation with non-fixed axis :-

Rotating Skew Rod :- rod of length 2l bin ing (a) massless fixed ω about 2 Point masses m each direction $\vec{\omega}$. find $\vec{L}$



$$L = \vec{r}_i \times m_i \vec{v}_i$$

$$\vec{v} = \vec{\omega} \times \vec{r}$$

$$= \underbrace{\vec{\omega}_{\parallel} \times \vec{r}}_{0} + \underbrace{\vec{\omega}_{\perp} \times \vec{r}}_{\omega_\perp l}$$

$$L = 2l\, m\, \omega_\perp l$$

$$= 2ml^2\omega_\perp$$

$$= 2ml^2\omega\cos\alpha$$

$$\vec{L} = \sum \vec{r}_j \times m_j \vec{v}_j = \sum \vec{r}_j \times m_j (\vec{\omega} \times \vec{r}_j)$$

$$\vec{r} \times (\vec{\omega} \times \vec{r})$$

$$\vec{\omega} \times \vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \omega_x & \omega_y & \omega_z \\ x & y & z \end{vmatrix}$$

$$= \underbrace{(z\omega_y - y\omega_z)}_{P}\hat{i} + \overbrace{(x\omega_z - z\omega_x)}^{Q}\hat{j} + \underbrace{(y\omega_x - x\omega_y)}_{R}\hat{k}$$

$$\vec{r} \times (\vec{\omega} \times \vec{r}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & z \\ P & Q & R \end{vmatrix}$$

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Simple Harmonic motion

$F_x = -kx$

$$\boxed{\frac{d^2x}{dt^2} + \omega_0^2 x = 0}$$

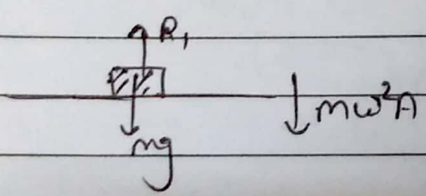
$\omega_0^2 = \frac{k}{m}$

$\omega_0 = \sqrt{\frac{k}{m}}$

$A\sin(\omega_0 t+\phi)$ $\quad$ $A\cos(\omega_0 t+\phi)$

Disp	$x = A\cos(\omega t+\phi)$	
vel	$v = -A\omega\sin(\omega t+\phi)$	$\rightarrow v = \pm\,\omega\sqrt{A^2-x^2}$
acc^n	$a = -A\omega^2\cos(\omega t+\phi)$	$\rightarrow a = -\omega^2 x$
K.E	$K.E = \frac{1}{2}mv^2$	$KE = \frac{1}{2}m\omega^2(A^2-x^2)$
P.E	$P.E = \frac{1}{2}kx^2$ $= \frac{1}{2}(m\omega_0^2)(A\cos(\omega t+\phi))^2$	$PE = \frac{1}{2}m\omega^2 x^2$
total E	$T.E = \frac{1}{2}m\omega^2A^2$	$E = \frac{1}{2}m\omega^2A^2$

Q



$mg - R_1 = m\omega^2 A$

when Contact loose

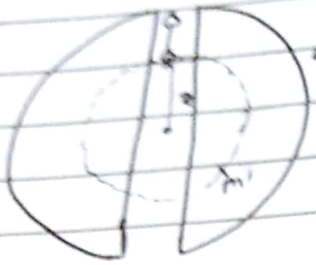
$R_1 = 0$

$\omega^2 = \frac{g}{A}$

$\omega = \sqrt{\frac{g}{A}}$

$$\boxed{f = \frac{1}{2\pi}\sqrt{\frac{g}{A}}}$$

Q



$$F = \frac{GmM'}{x^2}$$

$$F = -\frac{GmMx^{\cancel{3}}}{R^3 \cancel{x^2}}$$

$$F = -\left(\frac{GmM}{R^3}\right) x$$

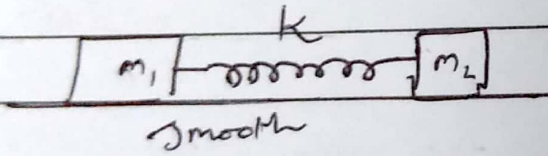
$$M' = \frac{M}{\cancel{\frac{4}{3}\pi} R^3} \cancel{\frac{4}{3}\pi} x^3$$

$$= \frac{Mx^3}{R^3}$$

$$\cancel{m}a = -\underbrace{\left(\frac{GM\cancel{m}}{R^3}\right)}_{\omega_0^2} x$$

$$\omega_0 = \sqrt{\frac{GM}{R^3}}$$

$$T = 2\pi\sqrt{\frac{R^3}{GM}}$$



$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$T = 2\pi\sqrt{\frac{\mu}{k}}$$

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Damped Harmonic Oscillator.

eq^n for undamped Simple Harmonic oscillator.

$$F = -kx \Rightarrow m\frac{d^2x}{dt^2} = -kx$$

here we add one more term that is damping force proportional to velocity $= -bv$

So eq^n becomes

damping term

$$m\frac{d^2x}{dt^2} = -kx - b\frac{dx}{dt}$$

or $\ddot{x} + \frac{k}{m}x + \frac{b}{m}\dot{x} = 0$

$$\boxed{\ddot{x} + \frac{b}{m}\dot{x} + \frac{k}{m}x = 0}$$

γ (damping term) $\qquad \omega_0^2 \leftarrow$ oscillatory term

$$\boxed{\begin{matrix}\gamma = b/m \\ \omega_0^2 = k/m\end{matrix}}$$

to solve this differential eq^n

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0 \quad \text{---} \ (1)$$

we substitute $x = e^{\alpha t}$ in above eq (1)

$\alpha^2 e^{\alpha t} + r\alpha e^{\alpha t} + \omega_0^2 e^{\alpha t} = 0$

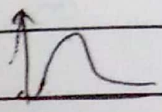
$\alpha^2 + r\alpha + \omega_0^2 = 0$

$$\boxed{\alpha_{1,2} = -\frac{r}{2} \pm \sqrt{\frac{r^2}{4} - \omega_0^2}}$$

general sol^n $\left(x = A e^{\alpha_1 t} + B e^{\alpha_2 t}\right)$

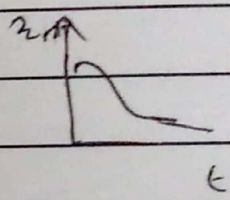
the term $\sqrt{\frac{r^2}{4} - \omega_0^2}$ is very important

~~Possibilites~~ $(>, =, <) \; 0$



if $\frac{r^2}{4} > \omega_0^2$ heavily damped oscillation (over damped)

$x(t) = c_1 e^{-r_1 t} + c_2 e^{-r_2 t}$



if $\frac{r^2}{4} - \omega_0^2 = 0$ critical damping

$$\left[x(t) = c_1 e^{-r_c t} + c_2 t e^{-r_c t}\right]$$

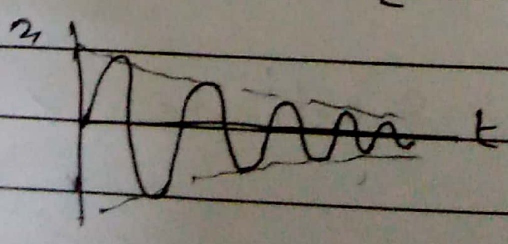
if $\frac{r^2}{4} - \omega_0^2 < 0$ oscillation (ω_0) dominates damping (r)

lightly damped oscilla^n (under damped osci^n)

$x(t) = e^{-rt}\left(c_1 e^{i\omega_1 t} + c_2 e^{-i\omega_1 t}\right)$

or

$x(t) = A e^{-rt} \cos(\omega_1 t + \phi)$



Lightly Damped / underdamped oscillator

$\omega_0^2 > \frac{r^2}{4}$

$\alpha_{1,2} = -\frac{r}{2} \pm i\sqrt{\omega_0^2 - \frac{r^2}{4}}$ ← Say ω_1

$= -\frac{r}{2} \pm i\omega_1$

$x(t) = c_1 e^{(-\frac{r}{2}+i\omega_1)t} + c_2 e^{(-\frac{r}{2}-i\omega_1)t}$

$= e^{-\frac{r}{2}t}\left(c_1 e^{i\omega_1 t} + c_2 e^{-i\omega_1 t}\right)$

$$\boxed{x(t) = Ae^{-\frac{r}{2}t}[\cos(\omega_1 t + \phi)]}$$

$\omega_1^2 = \omega_0^2 - \frac{r^2}{4}$

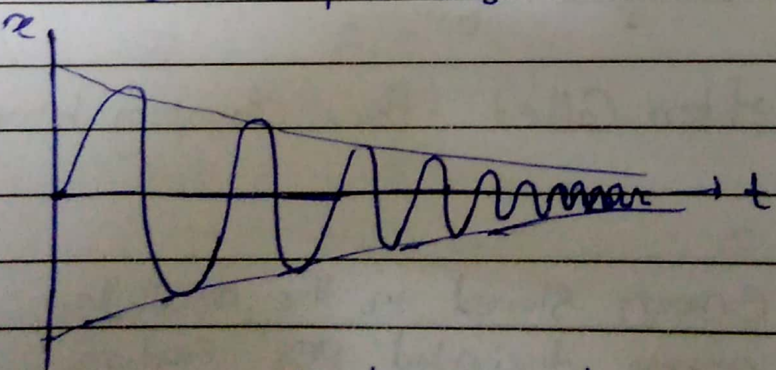
OR

$x(t) = Ae^{-\frac{r}{2}t}[A\cos\omega_1 t + B\sin\omega_1 t]$

ω_1 and ω_0

Since $\omega_1^2 = \omega_0^2 - \frac{r^2}{4}$ $\omega_0^2 = \frac{k}{m}$, $r = \frac{b}{m}$

So $\omega_1 < \omega_0$



So displacement is given by

$x(t) = Ae^{-r/2\,t} \cdot \cos(\omega_1 t + \phi)$

$A(t) = Ae^{-r/2\,t}$ Amplitude

~~A(t) =~~

$$A(t) = A e^{-bt/2m} \cos(\omega_1 t + \phi) \qquad \tau = \frac{2m}{b}$$

$$= A e^{t/\tau} \cos(\omega_1 t + \phi)$$

Energy, E(t) —

$$E(t) = K(t) + U(t)$$

$$= \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

from above express^n one can find v & treating damping to be small $\omega_1^2 \approx \omega_0^2$

$$E(t) = \frac{1}{2}kA^2 e^{-\gamma t} = E_0 e^{-\gamma t}$$

$$E_0 = \frac{1}{2}kA^2, \text{ energy at } t=0$$

decay can be characterized by the time τ required for the energy to drop to $\frac{1}{e}$ of its initial value.

$$E(\tau) = \cancel{E_0} e^{-\gamma\tau} = e^{-1} E_0$$

$$\tau = \frac{1}{\gamma} = \frac{m}{b}$$

τ is often called the damping time

Quality factor —

$$Q = \frac{\text{energy stored in the oscillator}}{\text{energy dissipated per radian}} = \frac{E}{\gamma E/\omega_1} = \frac{\omega_1}{\gamma} \simeq \frac{\omega_0}{\gamma}$$

Energy dissipated $\frac{dE}{dt} = -\gamma E_0 \gamma^{-\gamma t} = -\gamma E$

~~Energy dissipated~~ in time t $= \frac{dE}{dt}(\delta t) = -\gamma E \frac{1}{\omega_1}$

$$\left(\frac{T}{2\pi}\right) = \frac{1}{\omega_1}$$

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for lightly damped $Q >> 1$
heavily damped $\Rightarrow$ low Q
undamped oscillator has infinite Q

Q A musician tuning fork rings at A above middle C 440 Hz. A sound level meter indicates that the sound intensity decrease by a factor of 5 in 4s. What is the Q of tuning fork?

Soln

$$5 = \frac{E_0 e^0}{E_0 e^{-4\gamma}} = e^{4\gamma}$$

$$\ln 5 = 4\gamma$$
$$\gamma = \frac{\ln 5}{4}$$

$$\omega_0 = 2\pi f = 2\pi(440)$$

$$Q = \frac{\omega_0}{\gamma} = \frac{440 \times 4 \times 2\pi}{\ln 5} \simeq 700$$

Q $T = 1.2s$, amplitude of oscillation decreased by a factor of 2 after 3 period. What is the estimated Q of this system.

$$2 = \frac{A_0 e^0}{A_0 e^{-\frac{\gamma}{2}(3.6)}}$$

$$t = 3(1.2) = 3.6$$

$$\ln 2 = 1.8\gamma$$
$$\gamma = 0.39 \text{ sec}^{-1}$$

$$Q = \frac{\omega_0}{\gamma} = \frac{2\pi/T}{0.39} = \frac{2\pi/1.2}{0.39} = 13$$

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Page Note Book

Q An under damped harmonic oscillator has $K = 2\,N/m$ $m = 1\,kg$ and $b = 0.1\,kg/s$. How many oscillation does the system make before the amplitude decreases to $\frac{1}{e}$ out its initial value.

Sol We need to find out total angle $(\omega_1 t)$

$$A(t) = A e^{-\frac{\gamma}{2}t} = \frac{1}{e} = e^{-\frac{\gamma}{2}t}$$

$$\boxed{\gamma = \frac{b}{m} = 0.1/s}$$

$$t = \frac{2}{\gamma} = 20 \text{ sec.}$$

$$\omega_1 t = \sqrt{\omega_0^2 - \frac{\gamma^2}{4}}\; t = \sqrt{4 - \frac{0.01}{4}} \times 20 = 28.26 \text{ rad}$$

$\omega_0^2 = \frac{k}{m}$
$= 2$

numbers out oscillatⁿ $= \frac{28.26}{2\pi} = 4.5$ (Ans)

Variable mass

thrust of the rocket

$$F_{ext} + \vec{u}_{rel}\frac{dM}{dt} = M\frac{dv}{dt}$$

$u_{rel} = \vec{U} - \vec{V} =$ velocity of the impacting material relative to the object. velocity of the smaller mass w.r.t. to bigger mass.

rocket eq^n in free space $\boxed{\Delta V = u_{rel} \ln \frac{m_0}{m_1}}$

if gravitational field, $\boxed{V_y = -gt + u_{rel} \ln \frac{m_0}{m_f}}$

Work Power & Energy

(1) work done by conservative force is equal to change in potential energy.

$$W_C = -\Delta U = -(U_f - U_i) = U_i - U_f$$

(2) $W_{all} = \Delta K = K_f - K_i$

(3) $W_{nc} + W_{ext} = \Delta E = E_f - E_i$
$= (K_f + U_f) - (K_i + U_i)$

Rigid Body Dynamics

$$\boxed{v = \vec{\omega} \times \vec{r}}$$

$$\vec{a} = \underbrace{\vec{\alpha} r}_{\text{tangential}} - \underbrace{\omega^2 r}_{\text{radial}}$$

angular momentum :- $\boxed{\vec{L}_o = \vec{r} \times \vec{p}} = \vec{r} \times m\vec{v}$

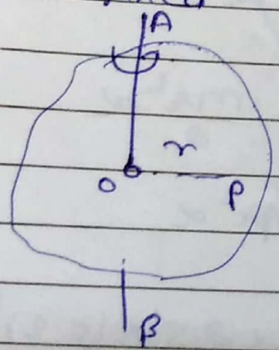
$= m(\vec{r} \times \vec{v})$

$= mvr\sin\theta$

$= mvr_1$

angular momentum is not an intrinsic property of a moving particle because it will change from pt to pt.

angular momentum of a rigid body rotating about a fixed axis :-



$L = \Sigma m_i r_i v_i \qquad v_i = r_i \omega$

$L = \Sigma m_i r_i^2 \omega = \omega \Sigma m r_i^2 = I\omega$

Conservation of angular momentum :-

$L = \vec{r} \times \vec{p}$ (angular m)

$$\frac{dL}{dt} = \vec{r} \times \frac{dP}{dt} + \frac{d\vec{r}}{dt} \times \vec{p} = \underbrace{\vec{r} \times \vec{F}}_{\tau} + \underbrace{\vec{v} \times \vec{p}}_{0}$$

$$\cancel{dt} \quad \frac{dL}{dt} = \vec{r} \times \vec{F} = \vec{\tau}$$

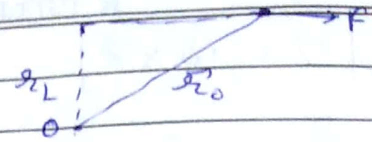
if $\tau_{ext} = 0 \qquad \frac{d\vec{L}}{dt} = 0 \qquad L = \text{Cons}$

$I\omega = \text{Cons}$

$(I_1\omega_1 = I_2\omega_2)$

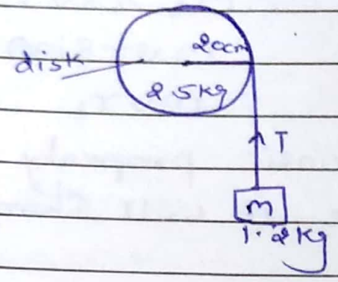
g. 4

Torque :- $\left[\vec{\tau}_o = \vec{r}_o \times \vec{F}\right]$



$\left[\tau = I\alpha\right]$

Q $m = 2.5\,kg, \quad r = 20\,cm$



$mg - T = ma \rightarrow mg - T = mr\alpha$

$a = r\alpha$

$\tau = I\alpha$

$\tau = \frac{mr^2}{2}\alpha$

$\cancel{T \times r = \frac{2.5 \times (0.2)^2 \alpha}{2}}$

$(\cancel{mg - mr\alpha})\ Tr = \frac{mr^2 \alpha}{2}$

$(mg - mr\alpha) r = \frac{mr^2}{2}\alpha$

$\cancel{m}gr = \frac{3}{2}\cancel{m}r^2\alpha$

$1.2 \times 10 \times 0.2 = \frac{3}{2} \times 2.5 \times (0.2)^{\cancel{2}\,1} \alpha$

$\frac{\cancel{12}^{\,4} \times 2}{3 \times 2.5 \times 0.2}$

$TR = I\alpha$

$T = \frac{MR}{2}\alpha$

$mg - T = ma_y$

$mg - \frac{MR\alpha}{2} = mR\alpha$

$mg = \left(\frac{M}{2} + m\right) a$

$a = \frac{1.2 \times 10}{\left(\frac{2.5}{2} + 1.2\right)}$

$= \cancel{12} \quad \frac{24}{4.9}$

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Central force Motion

$$\vec{F} = f(r)\,\hat{r} = m\ddot{\vec{r}}$$

if $f(r) < 0$ ← attractive towards o
if $f(r) > 0$ ← repulsive from o

Properties of a particle moving under the influence of Central force :-

(i) Path of Particle must be plane Curve i.e it must lie on a plane

(ii) A.M. of Particle is Conserved (Cons. with time)
Both A.M. & energy are Cons. for motion.

(iii) The particle should follow law of Areas.

Potential energy :-

$$F(r) = -\frac{du}{dr}$$

$$W = \int_{r_1}^{r_2} \vec{F}\cdot d\vec{r} = \int_{r_1}^{r_2} f(r)\,\hat{r}\cdot dr = \int_{r_1}^{r_2} f\,dr = U(r_1) - U(r_2)$$

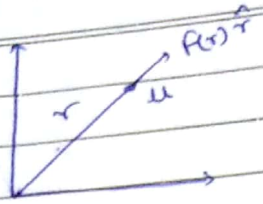
$$\boxed{\vec{\nabla}\times\vec{F} = 0}$$

Uniform Circular Motion :-

every Central force Can produce Circular Motion Provided that initial radius r and Speed v Satisfy the eq^n for the Centripetal Force

$$\boxed{\frac{mv^2}{r} = f(r)}$$

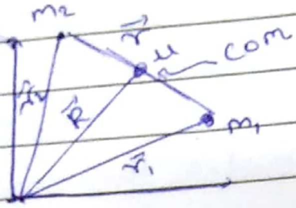
$$\left[\mu = \frac{m_1 m_2}{m_1 + m_2}\right]$$



$$\left[\mu \ddot{\vec{r}} = f(r)\,\hat{r}\right]$$

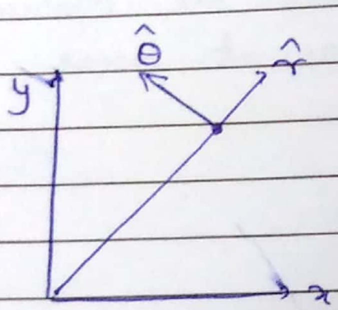
$$\vec{r} = \vec{r}_1 - \vec{r}_2 \quad \text{—(i)}$$

$$R = \frac{m_1 \vec{r}_1 + m_2 \vec{r}_2}{m_1 + m_2} \quad \text{—(ii)}$$



on solving

$$\left[\begin{aligned} \vec{r}_1 &= \vec{R} + \left(\frac{m_2}{m_1 + m_2}\right)\vec{r} \\ \vec{r}_2 &= \vec{R} - \left(\frac{m_1}{m_1 + m_2}\right)\vec{r} \end{aligned}\right]$$



$$P = r\hat{r}$$

$$v = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

$$a = (\ddot{r} - r\dot{\theta}^2)\,\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{\theta}$$

$$\frac{d\hat{r}}{dt} = \dot{\theta}\hat{\theta}$$

$$\hat{r} = i\cos\theta + j\sin\theta$$

$$\hat{\theta} = -i\sin\theta + j\cos\theta$$

$$\frac{d\hat{\theta}}{dt} = -\dot{\theta}\hat{r}$$

apply for two body problem

$$\mu \vec{a} = f(r)\,\hat{r}$$

$$\mu\left[(\ddot{r} - r\dot{\theta}^2)\,\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\,\hat{\theta}\right] = f(r)\,\hat{r}$$

equating $\hat{r}$ and $\hat{\theta}$ Component

$$\left.\begin{aligned} \mu(\ddot{r} - r\dot{\theta}^2) &= f(r) \\ \mu(r\ddot{\theta} + 2\dot{r}\dot{\theta}) &= 0 \end{aligned}\right] \Rightarrow$$ equation of motion in Central force field.

Date

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$$\left[r^2\dot\theta = \text{Constant} = h\right]$$

$$\tau = \vec{r}\times\vec{F} = r\hat{r}\times f(r)\hat{r} = 0$$

$$\frac{dL}{dt} = 0 \Rightarrow L = \text{Cons.}$$

$$L = \vec{r}\times\vec{p} = \vec{r}\times\mu\vec{v}$$

$$= r\hat{r}\times\mu(\dot{r}\hat{r} + r\dot\theta\hat\theta)$$

$$= \mu r\dot{r}\,\underbrace{\hat{r}\times\hat{r}}_{0} + \mu r^2\dot\theta\,\hat{r}\times\hat\theta$$

$$= \mu r^2\dot\theta\,\hat{k}$$

$$\boxed{|\vec{L}| = l = \mu r^2\dot\theta} \leftarrow \text{Cons. of motion}$$

energy :-

$$E = \frac{1}{2}\mu v^2 + U(r) = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot\theta^2) + U(r)$$

$$\dot\theta = \frac{l}{\mu r^2}$$

$$E = \frac{1}{2}\mu\dot{r}^2 + \underbrace{\frac{1}{2}\frac{l^2}{\mu r^2} + U(r)}_{U_{eff}(r)}$$

$$\boxed{E = \frac{1}{2}\mu\dot{r}^2 + U_{eff}(r)}$$

$$\boxed{\frac{dr}{dt} = \sqrt{\frac{2}{\mu}(E - U_{eff})}}$$

$$\dot\theta = \frac{l}{\mu r^2}$$

$$\int d\theta = \int\frac{l}{\mu r^2}\,dt$$

law of Areas :-

$$\boxed{\dot{A} = \frac{1}{2}r^2\dot\theta\,\hat{k} = \frac{l}{2\mu}\hat{k}}$$

$$r_0 = \frac{l^2}{\mu C}$$

$$\epsilon = \sqrt{1 + \frac{2El^2}{\mu C^2}}$$

$$\boxed{r = \frac{r_0}{1 - \epsilon \cos\theta}}$$

$$r_{min} = \frac{r_0}{1-\epsilon}$$

$$r_{max} = \frac{r_0}{1+\epsilon}$$

$$\frac{r_{max}}{r_{min}} = \frac{1+\epsilon}{1-\epsilon}$$

$$A = r_{min} + r_{max}$$

$$A = r_0\left(\frac{1}{1+\epsilon_0} + \frac{1}{1-\epsilon_0}\right) = \frac{2r_0}{1-\epsilon^2}$$

$\cancel{A}$

length of major axis

$$\boxed{A = \frac{C}{-E}}$$

C ← Umm

E ← total energy