The Fundamental Theorem for Gradients

The integral of a derivative over a region is equal to the value of the function at the boundary

$$\boxed{\int_{\mathbf{a}\atop\mathcal{P}}^{\mathbf{b}} (\nabla T) \cdot d\mathbf{l} = T(\mathbf{b}) - T(\mathbf{a}).}$$

$$dT = (\nabla T) \cdot d\mathbf{l}_1$$

Difference of function’s value at b and a

Corollary 1: $\int_{\mathbf{a}}^{\mathbf{b}} (\nabla T) \cdot d\mathbf{l}$ is independent of path taken from **a** to **b**.

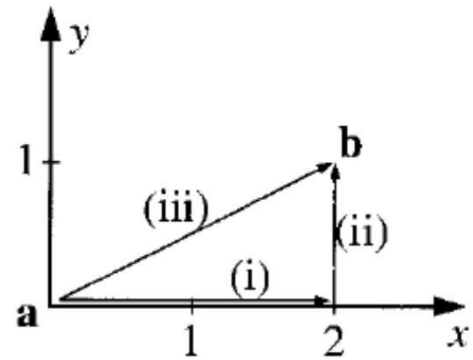
Corollary 2: $\oint (\nabla T) \cdot d\mathbf{l} = 0$, since the beginning and end points are identical, and hence $T(\mathbf{b}) - T(\mathbf{a}) = 0$.

$$\vec{\nabla} \times \vec{\nabla} \boldsymbol{T} = 0$$

$$\vec{\nabla} \times \vec{\boldsymbol{F}} = 0$$

F conservative field

Let $T = xy^2$, and take point **a** to be the origin $(0, 0, 0)$ and **b** the point $(2, 1, 0)$. Check the fundamental theorem for gradients.



The Fundamental Theorem for Divergences

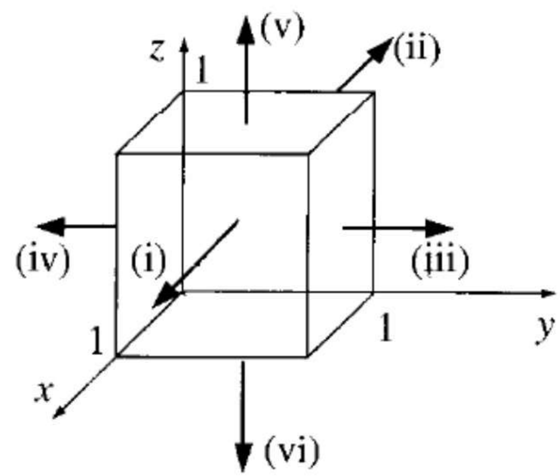
$$\boxed{\int_{\mathcal{V}} (\nabla \cdot \mathbf{v})\, d\tau = \oint_{\mathcal{S}} \mathbf{v} \cdot d\mathbf{a}.}$$

The integral of a derivative over a region is equal to the value of the function at the boundary

$$\nabla \cdot \vec{V} = \text{outflow} - \text{inflow} \longrightarrow \begin{array}{ll} \text{+ve} & \text{(source)} \\ \text{-ve} & \text{(sink)} \end{array}$$

$$\int \text{(faucets within the volume)} = \oint \text{(flow out through the surface)}$$

Check the divergence theorem using the function $\mathbf{v} = y^2\,\hat{\mathbf{x}} + (2xy + z^2)\,\hat{\mathbf{y}} + (2yz)\,\hat{\mathbf{z}}$



The Fundamental Theorem for Curls Stokes' theorem

$$\boxed{\int_{\mathcal{S}} (\nabla \times \mathbf{v}) \cdot d\mathbf{a} = \oint_{\mathcal{P}} \mathbf{v} \cdot d\mathbf{l}.}$$

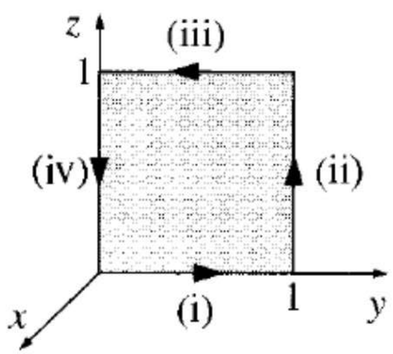
The integral of a derivative over a region is equal to the value of the function at the boundary

rotational force field / non-conservative force field

Corollary 1: $\int (\nabla \times \mathbf{v}) \cdot d\mathbf{a}$ depends only on the boundary line, not on the particular surface used.

Corollary 2: $\oint (\nabla \times \mathbf{v}) \cdot d\mathbf{a} = 0$ for any closed surface.

Suppose $\mathbf{v} = (2xz + 3y^2)\hat{\mathbf{y}} + (4yz^2)\hat{\mathbf{z}}$. Check Stokes' theorem for the square surface



Summary of vector Calculus

- **Concept of Vector Field**
- **Gradient [converts scalar field to a vector field]**
- **Divergence [A measure of change of flux per unit volume]**
- **Curl [measure of rotational nature of a vector field]**
- **Line integration**
- **Surface integration**
- **Volume integration**
- **Fundamental theorem for Gradients**
- **Fundamental theorem for Divergences**
- **Fundamental theorem for Curls**