Examples of significant figures are: 3.1415 (five significant figures); 9 (one significant figure); 0.00021 (two significant figures); .000210 (three significant figures). Leading zeros don't count, but trailing zeros do.

A useful rule-of-thumb is that the number of significant figures retained in the result of a calculation should equal the smallest number of significant figures of any number in the calculation. For instance, if the acceleration of gravity in a calculation is taken to be 9.8 m/s^2, the result of the calculation should be quoted to no more than two significant figures.

Experimental uncertainty can be expressed in several ways, for example as 72.53±0.20 or concisely as 72.53(20). Another way is the *parts-per notation* based on fractional error. The fractional error in our example is $0.20/72.53 = 2.8\times10^{-3}$, and the error can be stated as 2.8 parts per thousand, alternatively as 2.8 parts in 10^3.

Problems

*For problems marked *, refer to page 519 for a hint, clue, or answer.*

1.1 *Vector algebra 1**
Given two vectors $\mathbf{A} = (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + 7\hat{\mathbf{k}})$ and $\mathbf{B} = (5\hat{\mathbf{i}} + \hat{\mathbf{j}} + 2\hat{\mathbf{k}})$ find: (*a*) $\mathbf{A}+\mathbf{B}$; (*b*) $\mathbf{A}-\mathbf{B}$; (*c*) $\mathbf{A}\cdot\mathbf{B}$; (*d*) $\mathbf{A}\times\mathbf{B}$.

1.2 *Vector algebra 2**
Given two vectors $\mathbf{A} = (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 5\hat{\mathbf{k}})$ and $\mathbf{B} = (6\hat{\mathbf{i}} - 7\hat{\mathbf{j}} + 4\hat{\mathbf{k}})$ find: (*a*) $\mathbf{A}^2$; (*b*) $\mathbf{B}^2$; (*c*) $(\mathbf{A}\cdot\mathbf{B})^2$.

1.3 *Cosine and sine by vector algebra**
Find the cosine and the sine of the angle between $\mathbf{A} = (3\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$ and $\mathbf{B} = (-2\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}})$.

1.4 *Direction cosines*
The direction cosines of a vector are the cosines of the angles it makes with the coordinate axes. The cosines of the angles between the vector and the x, y, and z axes are usually called, in turn, α, β, and γ. Prove that $\alpha^2 + \beta^2 + \gamma^2 = 1$, using either geometry or vector algebra.

1.5 *Perpendicular vectors*
Show that if $|\mathbf{A}-\mathbf{B}| = |\mathbf{A}+\mathbf{B}|$, then $\mathbf{A}$ and $\mathbf{B}$ are perpendicular.

1.6 *Diagonals of a parallelogram*
Show that the diagonals of an equilateral parallelogram are perpendicular.

1.7 *Law of sines**
Prove the law of sines using the cross product. It should only take a couple of lines.

1.8 *Vector proof of a trigonometric identity*
Let $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ be unit vectors in the x–y plane making angles θ and ϕ with the x axis, respectively. Show that $\hat{\mathbf{a}} = \cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}}$, $\hat{\mathbf{b}} = \cos\phi\hat{\mathbf{i}} + \sin\phi\hat{\mathbf{j}}$, and using vector algebra prove that

$$\cos(\theta-\phi) = \cos\theta\cos\phi + \sin\theta\sin\phi.$$

1.9 *Perpendicular unit vector**
Find a unit vector perpendicular to $\mathbf{A} = (\hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}})$ and $\mathbf{B} = (2\hat{\mathbf{i}} + \hat{\mathbf{j}} - 3\hat{\mathbf{k}})$.

1.10 *Perpendicular unit vectors**
Given vector $\mathbf{A} = 3\hat{\mathbf{i}} + 4\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$,
(*a*) find a unit vector $\hat{\mathbf{B}}$ that lies in the x–y plane and is perpendicular to $\mathbf{A}$.
(*b*) find a unit vector $\hat{\mathbf{C}}$ that is perpendicular to both $\mathbf{A}$ and $\hat{\mathbf{B}}$.
(*c*) Show that $\mathbf{A}$ is perpendicular to the plane defined by $\hat{\mathbf{B}}$ and $\hat{\mathbf{C}}$.

1.11 *Volume of a parallelepiped*
Show that the volume of a parallelepiped with edges $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ is given by $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})$.

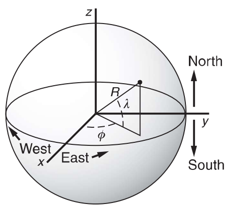
1.12 *Constructing a vector to a point*
Consider two points located at $\mathbf{r}_1$ and $\mathbf{r}_2$, separated by distance $r = |\mathbf{r}_1 - \mathbf{r}_2|$. Find a vector $\mathbf{A}$ from the origin to a point on the line between $\mathbf{r}_1$ and $\mathbf{r}_2$ at distance xr from the point at $\mathbf{r}_1$ where x is some number.

1.13 *Expressing one vector in terms of another*
Let $\mathbf{A}$ be an arbitrary vector and let $\hat{\mathbf{n}}$ be a unit vector in some fixed direction. Show that $\mathbf{A} = (\mathbf{A}\cdot\hat{\mathbf{n}})\hat{\mathbf{n}} + (\hat{\mathbf{n}}\times\mathbf{A})\times\hat{\mathbf{n}}$.

1.14 *Two points*
Consider two points located at $\mathbf{r}_1$ and $\mathbf{r}_2$, and separated by distance $r = |\mathbf{r}_1 - \mathbf{r}_2|$. Find a time-dependent vector $\mathbf{A}(t)$ from the origin that is at $\mathbf{r}_1$ at time t_1 and at $\mathbf{r}_2$ at time $t_2 = t_1 + T$. Assume that $\mathbf{A}(t)$ moves uniformly along the straight line between the two points.

1.15 *Great circle**
The shortest distance between two points on the Earth (considered to be a perfect sphere of radius R) is the distance along a great circle — the arc of a circle formed where a plane passing through the two points and the center of the Earth intersects the Earth's surface.

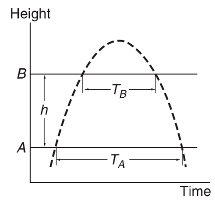
The position of a point on the Earth is specified by the point's longitude ϕ and latitude λ. Longitude is the angle between the meridian (a line from pole to pole) passing through the point and the "prime" meridian passing through Greenwich U.K. Longitude is taken to be positive to the east and negative to the west. Latitude



is the angle from the Equator along the point's meridian, taken positive to the north.

Let the vectors from the center of the Earth to the two points be $\mathbf{r}_1$ and $\mathbf{r}_2$. The cosine of the angle θ between them can be found from their dot product, so that the great circle distance between the points is $R\theta$.

Find an expression for θ in terms of the coordinates of the two points. Use a coordinate system with the x axis in the equatorial plane and passing through the prime meridian; let the z axis be on the polar axis, positive toward the north pole, as shown in the sketch.



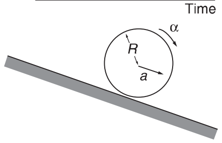
1.16 *Measuring g*

The acceleration of gravity can be measured by projecting a body upward and measuring the time that it takes to pass two given points in both directions.

Show that if the time the body takes to pass a horizontal line A in both directions is T_A, and the time to go by a second line B in both directions is T_B, then, assuming that the acceleration is constant, its magnitude is

$$g = \frac{8h}{T_A{}^2 - T_B{}^2},$$

where h is the height of line B above line A.



1.17 *Rolling drum*

A drum of radius R rolls down a slope without slipping. Its axis has acceleration a parallel to the slope. What is the drum's angular acceleration α?

1.18 *Elevator and falling marble**

At $t = 0$, an elevator departs from the ground with uniform speed. At time T_1 a child drops a marble through the floor. The marble falls with uniform acceleration $g = 9.8\ \mathrm{m/s^2}$, and hits the ground T_2 seconds later. Find the height of the elevator at time T_1.

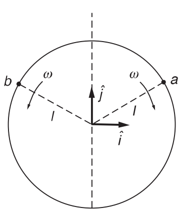
1.19 *Relative velocity**

By *relative velocity* we mean velocity with respect to a specified coordinate system. (The term velocity, alone, is understood to be relative to the observer's coordinate system.)

(*a*) A point is observed to have velocity $\mathbf{v}_A$ relative to coordinate system A. What is its velocity relative to coordinate system B, which is displaced from system A by distance $\mathbf{R}$? ($\mathbf{R}$ can change in time.)

(*b*) Particles a and b move in opposite directions around a circle with angular speed ω, as shown. At $t = 0$ they are both at the point $\mathbf{r} = l\hat{\mathbf{j}}$, where l is the radius of the circle.

Find the velocity of a relative to b.



1.20 *Sportscar*

A sportscar, Electro-Fiasco I, can accelerate uniformly to 100 km/h in 3.5 s. Its *maximum* braking rate cannot exceed $0.7g$. What is the minimum time required to go 1.0 km, assuming it begins and ends at rest?

1.21 *Particle with constant radial velocity**

A particle moves in a plane with constant radial velocity $\dot{r} = 4$ m/s, starting from the origin. The angular velocity is constant and has magnitude $\dot{\theta} = 2$ rad/s. When the particle is 3 m from the origin, find the magnitude of (*a*) the velocity and (*b*) the acceleration.

1.22 *Jerk*

The rate of change of acceleration is known as "jerk." Find the direction and magnitude of jerk for a particle moving in a circle of radius R at angular velocity ω. Draw a vector diagram showing the instantaneous position, velocity, acceleration, and jerk.

1.23 *Smooth elevator ride**

For a smooth ("low jerk") ride, an elevator is programmed to start from rest and accelerate according to

$$a(t) = (a_m/2)[1 - \cos(2\pi t/T)] \qquad 0 \le t \le T$$
$$a(t) = -(a_m/2[(1 - \cos(2\pi t/T)] \qquad T \le t \le 2T$$

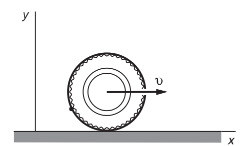
where a_m is the maximum acceleration and $2T$ is the total time for the trip.

(*a*) Draw sketches of $a(t)$ and the jerk as functions of time.

(*b*) What is the elevator's maximum speed?

(*c*) Find an approximate expression for the speed at short times near the start of the ride, $t \ll T$.

(*d*) What is the time required for a trip of distance D?



1.24 *Rolling tire*

A tire of radius R rolls in a straight line without slipping. Its center moves with constant speed V. A small pebble lodged in the tread of the tire touches the road at $t = 0$. Find the pebble's position, velocity, and acceleration as functions of time.

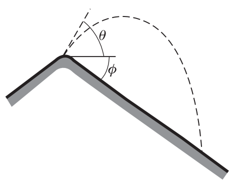
1.25 *Spiraling particle*

A particle moves outward along a spiral. Its trajectory is given by $r = A\theta$, where A is a constant. $A = (1/\pi)$ m/rad. θ increases in time according to $\theta = \alpha t^2/2$, where α is a constant.

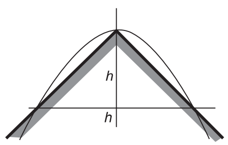
(*a*) Sketch the motion, and indicate the approximate velocity and acceleration at a few points.

(*b*) Show that the radial acceleration is zero when $\theta = 1/\sqrt{2}$ rad.

(*c*) At what angles do the radial and tangential accelerations have equal magnitude?



1.26 *Range on a hill**
An athlete stands at the peak of a hill that slopes downward uniformly at angle ϕ. At what angle θ from the horizontal should they throw a rock so that it has the greatest range?



1.27 *Peaked roof**
A peaked roof is symmetrical and subtends a right angle, as shown. Standing at a height of distance h below the peak, with what initial speed must a ball be thrown so that it just clears the peak and hits the other side of the roof at the same height?