The LNM Institute of Information Technology
Jaipur, Rajsthan

MATH-I ■ Assignment #4

(Infinite Series)

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1. Find the values of p for which the following series are convergent or divergent:
   $(a)\ \sum_{n=1}^{\infty} \frac{1}{n^p}$, $\qquad (b)\ \sum_{n=1}^{\infty} \frac{1}{n((logn)^p)}$

2. Test the series $\sum_{n\geq 1} \tan^{-1}(e^{-n})$ and the series $\sum_{n\geq 1} \left(1-\frac{1}{n}\right)^{n^2}$ for convergence.

3. Show that if $\sum_{n=1}^{\infty}|a_n|$ converges then $\sum_{n=1}^{\infty}a_n$ converges. Give an example to show that the converse need not be true.

4. In each of the following cases, discuss the convergence/divergence of the series $\sum_{n\geq 1} a_n$, where a_n equals:
   $(a)\ 2^{-n-(-1)^n}$, $\qquad (b)\ \left(1+\frac{1}{n}\right)^{n(n+1)}$.

5. Let $\{a_n\}$ be a decreasing sequence, $a_n \geq 0$ and $\lim_{n\to\infty} a_n = 0$. For each $n \in \mathbb{N}$, let $b_n = \frac{a_1 + a_2 + \ldots + a_n}{n}$. Show that $\sum_{n\geq 1}(-1)^n b_n$ converges.

6. Determine the values of x for which the following series converges:
   $(a)\ \sum_{n\geq 1} \frac{(x-1)^{2n}}{n^2 3^n}$, $\qquad (b)\ \sum_{n\geq 1} \frac{n^3}{3^n} x^n$, $\qquad (c)\ \sum_{n\geq 1} \frac{(2n)!}{(2^n n!)^2} \frac{x^{2n+1}}{2n+1}$.