

Ch-3 Operational Amplifiers

The OP-AMP -

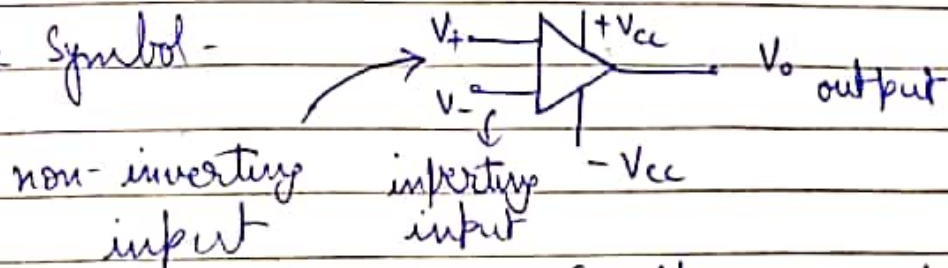
- 1) The OPAMP is essentially speaking, a voltage amplifier with very high gain.
- 2) Essentially, the OPAMP contains a large number of individual amplifier stages, interconnected.
- 3) The OPAMP circuitry contains a large number of Bipolar Junction Transistors (BJT's), Field effect transistors (FET's), resistors etc.
- 4) The OPAMP circuitry occupies very little space and is hence suited to being made available in the Integrated Circuit (IC) form.
- 5) The OPAMP Circuitry, for the sake of ease of handling, is contained in a large multi-pin package.

Characteristics of an Ideal OP-AMP -

- 1) Infinitely - large open loop gain.
- 2) Infinitely - large Input Impedance
- 3) Zero Output Impedance
- 4) Zero Input Current.

The OPAMP is capable of producing operations like addition, subtraction, multiplication, integration, differentiation etc. For each application, special circuitry needs to be designed.

The Symbol -



$$V_d = V_+ - V_- \quad \text{--- (1) (Difference input)}$$

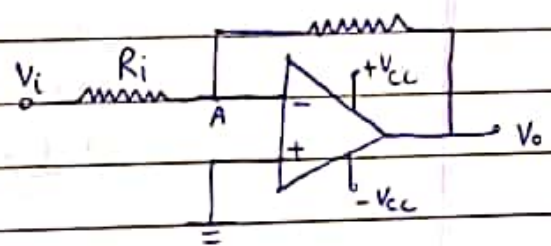
$$V_o = A[V_d] = A[V_+ - V_-]$$

A = open loop gain (Between $10^5 \pm 10^6$)

g- Under ideal conditions,
OPAMP's input $I = 0$

Also because A is large,

$$V_d \approx 0$$



In this case, since we gave grounded + terminal

$$V_+ = 0 \quad \text{--- (2)}$$

$$V_- \approx 0 \quad \text{--- (3) Virtual ground concept}$$

KCL at (A)

$$\frac{V_i}{R_i} + \frac{V_o}{R_f} = 0 \quad \text{--- (5)}$$

The actual gain

$$A_v \triangleq \frac{V_o}{V_i} \quad \text{--- (6)}$$

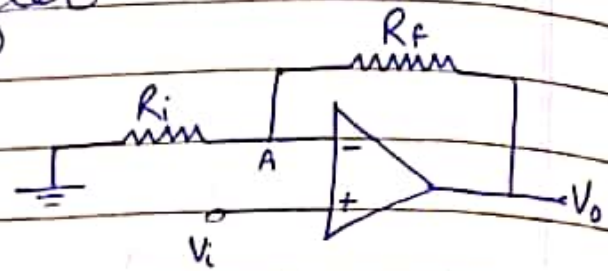
$$= -\frac{R_f}{R_i} \quad \text{--- (7)}$$

This is an example of inverting amplifier
As output is negative to that of input

Example 2 - Non-inverting Amplifier

$$V_F = V_i \quad \text{--- (8)}$$

$$V_i \approx V_i \quad \text{--- (9)}$$



KCL at (A)

$$\frac{V_i}{R_i} = - \left(\frac{V_i - V_o}{R_f} \right) \quad \text{--- (10)}$$

$$\begin{aligned} \therefore A_v &\triangleq \frac{V_o}{V_i} \\ &= 1 + \frac{R_f}{R_i} \quad \text{--- (12)} \\ &= 1 \quad (\text{if } R_f = 0) \end{aligned}$$

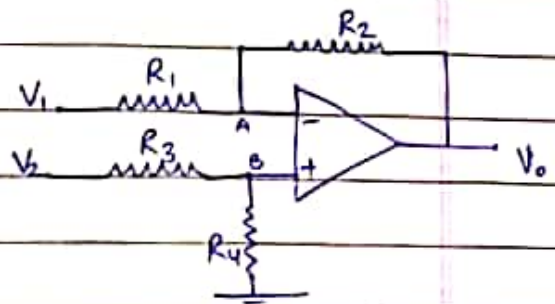
- Q. Design a unity gain amplifier for
- (a) Inverting application
 - (b) non-inverting application

Ans. (a) $|A_v| = 1$
 $A_v = - \frac{R_f}{R_i}$

(b) $R_f = 0$

The Difference Amplifier

Because of the "virtual" short-circuit between + and - inputs of the OPAMP we write

$$V_A \approx V_B \quad - (1)$$


KCL at A

$$\frac{V_1 - V_A}{R_1} = \frac{V_A - V_O}{R_2} \quad - (2)$$

KCL at B

$$\frac{V_2 - V_B}{R_3} = \frac{V_B}{R_4} \quad - (3)$$

By ①, ② & ③

$$V_O = -\frac{R_2}{R_1} V_1 + \left(\frac{R_4}{R_3 + R_4} \right) \left(\frac{R_1 + R_2}{R_1} \right) V_2 \quad - (4)$$

It is not easy to work out the gain of the circuit based on eq ④

Let us rewrite ④ as

$$V_O = -\alpha V_1 + \beta V_2 \quad - (5)$$

where $\alpha = \frac{R_2}{R_1} \quad - (6)$

and $\beta = \left(\frac{R_4}{R_3 + R_4} \right) \left(\frac{R_1 + R_2}{R_1} \right) \quad - (7)$

Special Case : If $\alpha = \beta$

$$V_O = \alpha (V_2 - V_1) \quad - (8)$$

We define the operation of the circuit in terms of the following two inputs -

- Differential Input** - When V_1 and V_2 are simultaneously present and have 180° phase difference between them.
- Common Mode Input** - When V_1 and V_2 are simultaneously present and have zero phase shift between them.

The differential input is defined as

$$V_d = V_2 - V_1 \quad \text{--- (9)}$$

and common mode input is defined as

$$V_{cm} = \frac{V_2 + V_1}{2} \quad \text{--- (10)}$$

$$V_2 = \frac{V_d + 2V_{cm}}{2} \quad \text{--- (11)}$$

$$V_1 = \frac{2V_{cm} - V_d}{2} \quad \text{--- (12)}$$

Using (10) & (11) in (5) we get

$$V_o = A_d V_d + A_{cm} V_{cm}$$

Differential Gain

Common - Mode Gain

where

$$A_d = \frac{1}{2} \left[\frac{R_2}{R_1} + \left(\frac{R_4}{R_3 + R_4} \right) \left(\frac{R_1 + R_2}{R_2} \right) \right] \quad \text{--- (14)} = \frac{1}{2} (\alpha + \beta)$$

$$A_{cm} = \left(\frac{R_4}{R_3 + R_4} \right) \left(\frac{R_1 + R_2}{R_1} \right) - \frac{R_2}{R_1} \quad \text{--- (15)} = \beta - \alpha$$

Common Mode Rejection Ratio (CMRR)

$$CMRR \triangleq \left| \frac{A_d}{A_{cm}} \right| \quad - (16)$$

In dB terms,

$$CMRR(dB) \triangleq 20 \log \left| \frac{A_d}{A_{cm}} \right| \quad - (17)$$

Prob 1 - Define an OP-Amp.

- Q. Design an OP-Amp based Difference amplifier with the output voltage $V_o = V_2 - 2V_1$ where V_1 & V_2 are the input voltage.

Ans. $\alpha = \frac{R_2}{R_1} = 2$ $\beta = 1 \Rightarrow \frac{R_4}{R_3} = \frac{1}{2}$

Choose $R_1 = 1k\Omega = R_4$

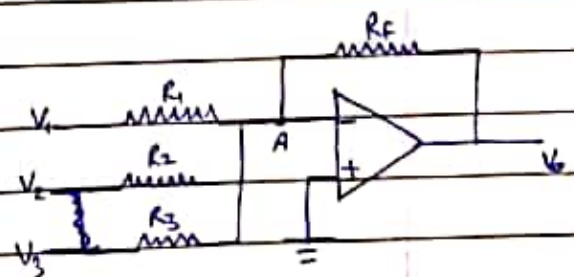
Summing Amplifier

$$V_A = 0 \quad - (1)$$

KCL

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} = -\frac{V_o}{R_f}$$

$$\Rightarrow V_o = - \left[\frac{R_f}{R_1} V_1 + \frac{R_f}{R_2} V_2 + \frac{R_f}{R_3} V_3 \right] \quad - (2)$$



OPAMP - Based Integrator

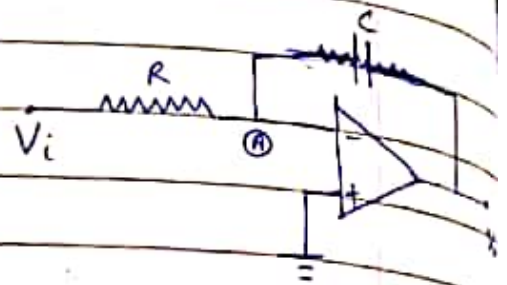
KCL

$$V_A \approx 0 \text{ --- (1)}$$

$$\frac{V_i - 0}{R} = C \frac{d[0 - V_o]}{dt} = -C \frac{dV_o}{dt}$$

$$\therefore V_i = -RC \frac{dV_o}{dt}$$

$$\Rightarrow V_o = \frac{-1}{RC} \int V_i dt \text{ --- (2)}$$



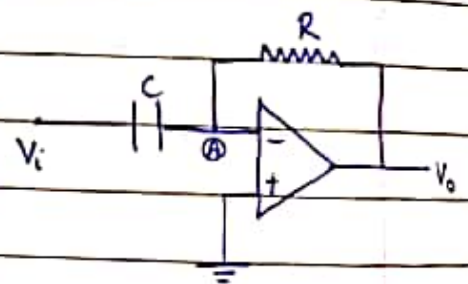
OPAMP - Based Differentiator

$$V_A \approx 0 \text{ --- (1)}$$

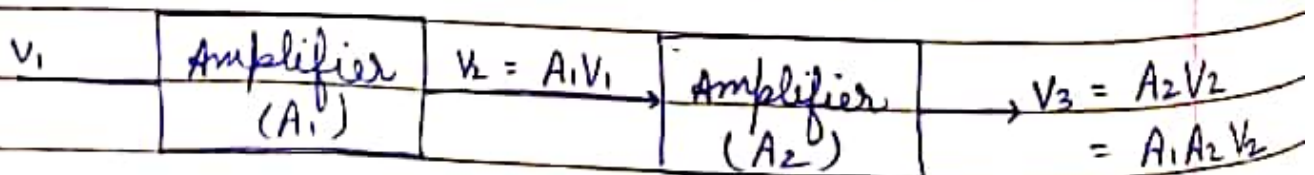
KCL

$$C \frac{dV_i}{dt} = \frac{0 - V_o}{R} = -\frac{V_o}{R}$$

$$\therefore V_o = -RC \frac{dV_i}{dt} \text{ --- (2)}$$



Cascading of two (or more) circuits



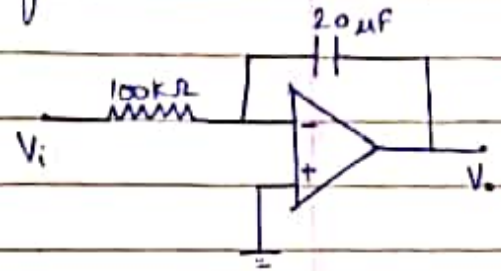
for N cascaded circuits $V_o = V_N = A_1 A_2 A_3 \dots A_N V_i$

8. For the circuit shown below, at $t=0$, $V_{in} = D.C$ of 2.5 mV is applied. Find V_o for $t > 0$.

Ans. $RC = 10^5 \times 20 \times 10^{-6} = 2 \text{ seconds}$

$$V_o = -\frac{1}{2} \int (2.5 \times 10^{-3}) dt$$

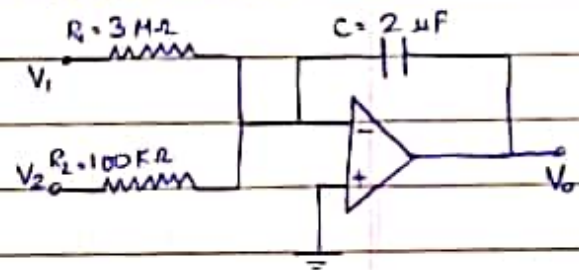
$$= -1.25 \times 10^{-3} t \text{ Volts}$$



9. Determine V_o in the circuit shown below.

Assume that $V_1 = 10 \cos 2t \text{ mV}$

$V_2 = 0.5t \text{ mV}$



Ans. $R_1 C = 6 \text{ seconds}$

$$R_2 C = 0.2 \text{ second}$$

$$V_o = -\frac{1}{R_1 C} \int V_1 dt - \frac{1}{R_2 C} \int V_2 dt$$

$$= -\frac{1}{6} [5 \sin 2t] - \frac{1}{0.2} [0.25 t^2]$$

$$= -0.833 \sin 2t - 1.25 t^2 \text{ mV}$$

10. Sketch the output voltage with Time for the following circuit. Assume $V_o = 0$ at $t=0$.

Ans. $0 \leq t \leq 2$ $V_i = 2t$

$$V_i = a + bt$$

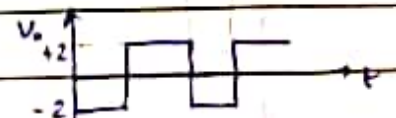
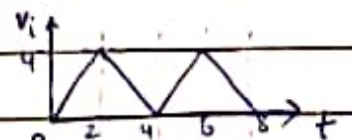
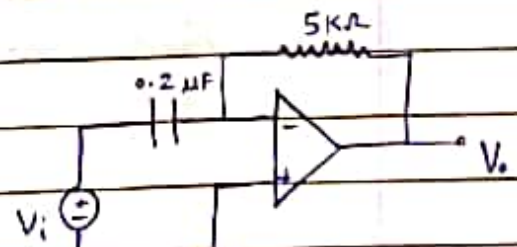
At $t=2$, $V=4 \Rightarrow 4 = a + 2b$

At $t=4$, $V=0 \Rightarrow 0 = a + 4b$

$$4 = -2b \Rightarrow b = -2$$

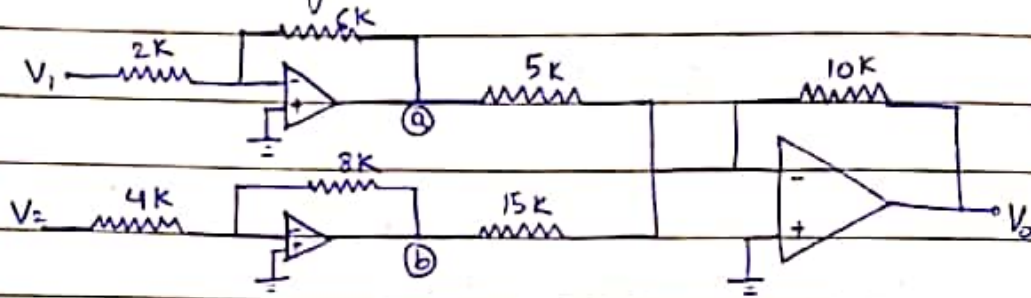
$$a = 8$$

$$V_i = 8 - 2t$$



$$\begin{aligned}
 V_o &= -5 \times 10^3 \times 0.2 \times 10^{-6} \times -2 \\
 &= -10^{-3} \times -2 \\
 &= 2 \times 10^{-3}
 \end{aligned}$$

Q. Calculate V_o for the circuits shown below.



Ans. $V_a = -3V_1$, $V_b = -2V_2$

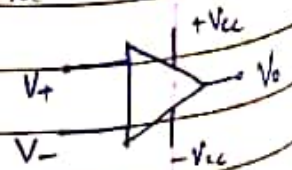
$$\begin{aligned}
 V_o &= \frac{-10}{5} V_a - \frac{10}{15} V_b = \frac{30}{5} V_1 + \frac{20}{15} V_2 \\
 &= \frac{1}{5} (30V_1 + 20V_2)
 \end{aligned}$$

Voltage Transfer Characteristic (VTC) of an OP-AMP

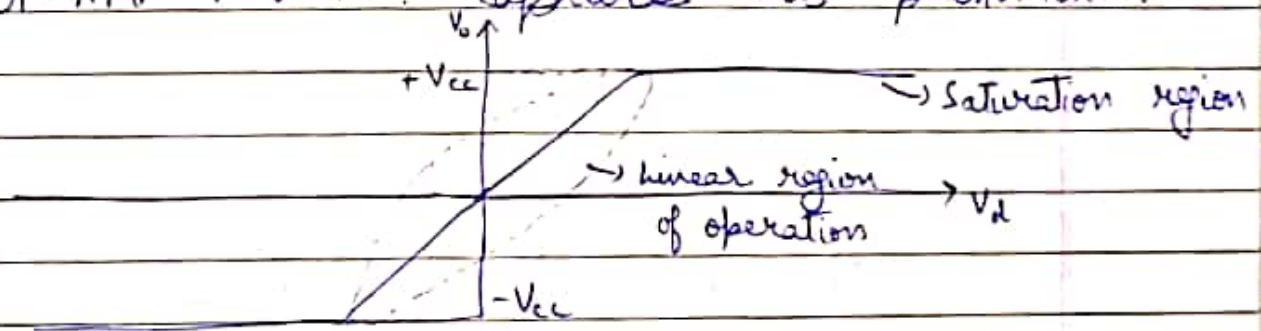
When an OP-AMP operates in open loop (i.e. nothing connected between output and inputs), the output voltage is given by $V_o = AV_d$ — (1)

When the difference input, V_d is written as $V_d = V_+ - V_-$ — (2)

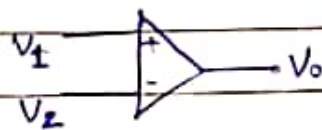
and A is known as the open-loop gain. A typically lies between 10^5 and 10^6 .



Since A is quite large, even an extremely small V_i can produce large V_o if we strictly go by eq. (1). In reality, however, the moment the magnitude of V_o exceeds that of V_{cc} , the V_o value settles to a constant value $\approx +V_{cc}$ or $-V_{cc}$, because of saturation of the semiconductor devices being used in the various amplifiers stages inside the OP-AMP. V.T.C. captures this phenomenon.



Comparator - Compares the input voltages (or two input currents) and produces a digital output whose value depends on which of the inputs has a larger magnitude.



If $V_1 > V_2$, $V_0 = +V_{cc}$

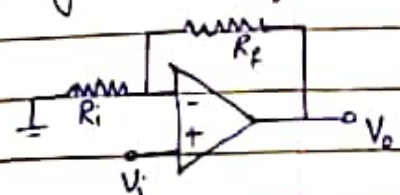
If $V_1 < V_2$, $V_0 = -V_{cc}$

Voltage Follower (also known as Buffer Amplifier) is nothing but a non-inverting amplifier with unity gain.

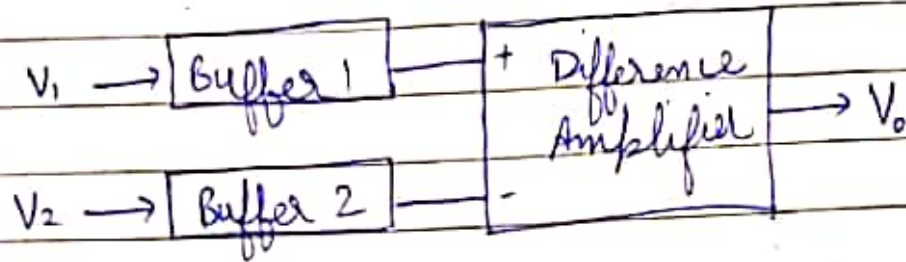
$$A_v = 1 + \frac{R_f}{R_i}$$

$$= 1 \text{ if } R_f = 0$$

Output and input are quite isolated.



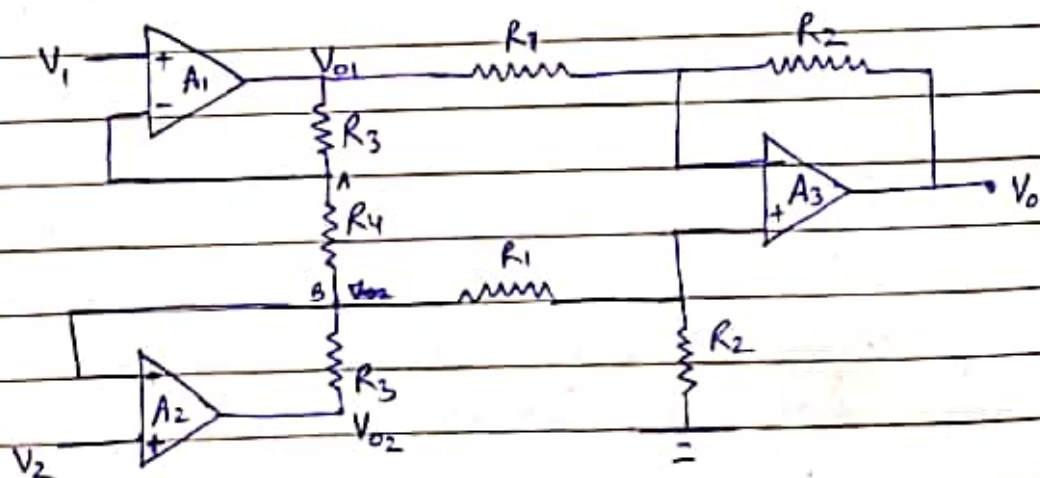
Instrumentation Amplifier



Advantages -

1. High CMRR is possible
2. High Input Impedance
3. Good Sensitivity to small inputs

Detailed Circuit



$$V_o = A_d (V_{o2} - V_{o1}) + A_{cm} \left(\frac{V_{o2} + V_{o1}}{2} \right) \quad \text{--- (1)}$$

If $R_3' = R_1$ and $R_4' = R_2$

$$A_d = \frac{1}{2} \left[\frac{R_2}{R_1} + \frac{R_4'}{R_3' + R_4'} \cdot \frac{R_1 + R_2}{R_1} \right] = \frac{R_2}{R_1}$$

$$A_{cm} = \frac{R_4'}{R_3' + R_4'} \cdot \frac{R_1 + R_2}{R_1} - \frac{R_2}{R_1} = 0$$

$$\Rightarrow V_o = \frac{R_2}{R_1} (V_{o2} - V_{o1}) \quad \text{--- (2)}$$

$$I = \frac{V_{01} - V_{02}}{2R_3 + R_4} \quad \text{--- (3)}$$

$$V_A = V_1 \quad \text{--- (4)}$$

$$V_B = V_2 \quad \text{--- (5)}$$

$$I = \frac{V_A - V_B}{R_4} = \frac{V_1 - V_2}{R_4} \quad \text{--- (6)}$$

We should get

$$V_0 = \frac{R_2}{R_1} \left(\frac{2R_3 + R_4}{R_4} \right) (V_2 - V_1) \quad \text{--- (7)}$$

Special Case -

Let $R_1 = R_2 = R_3 = R$ and let $R_4 = R_G$ (variable)

$$\text{Then } V_0 = \frac{2R + R_G}{R_G} (V_2 - V_1)$$

$$= \left[1 + \frac{2R}{R_G} \right] (V_2 - V_1) \quad \text{--- (8)}$$

Q. For the I.A already considered, $R_1 = R_2 = R_3 = 10 \text{ k}\Omega$,

$V_1 = 2.011 \text{ V}$, $V_2 = 2.017 \text{ V}$ and $R_G = 500 \Omega$.

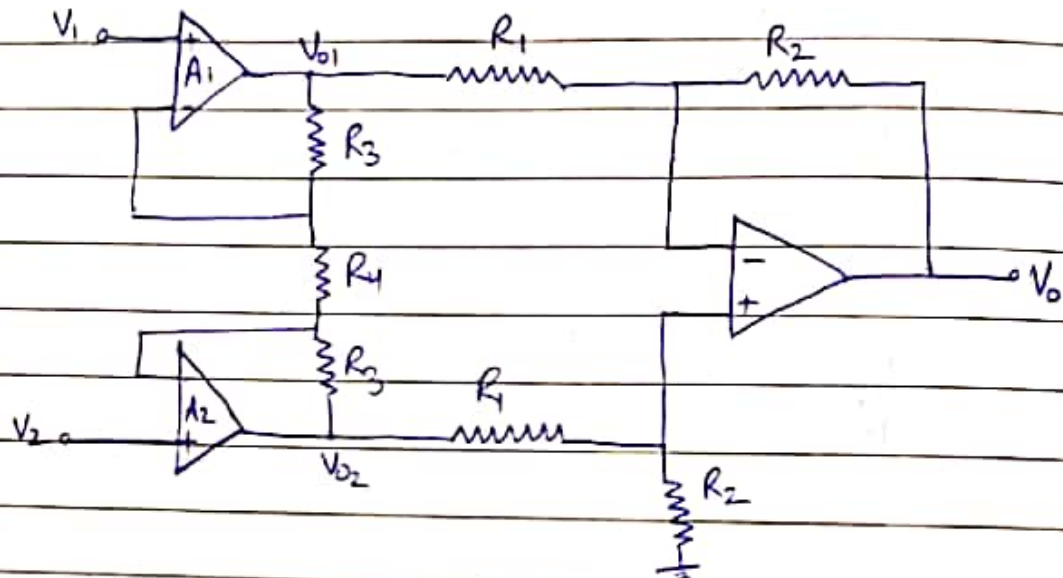
Calculate V_0 .

$$\text{Ans. Gain} = 1 + \frac{2R}{R_G} = 1 + \frac{2 \times 10^4 \times 10^3}{500} = 41$$

$$V_2 - V_1 = 6 \text{ mV}$$

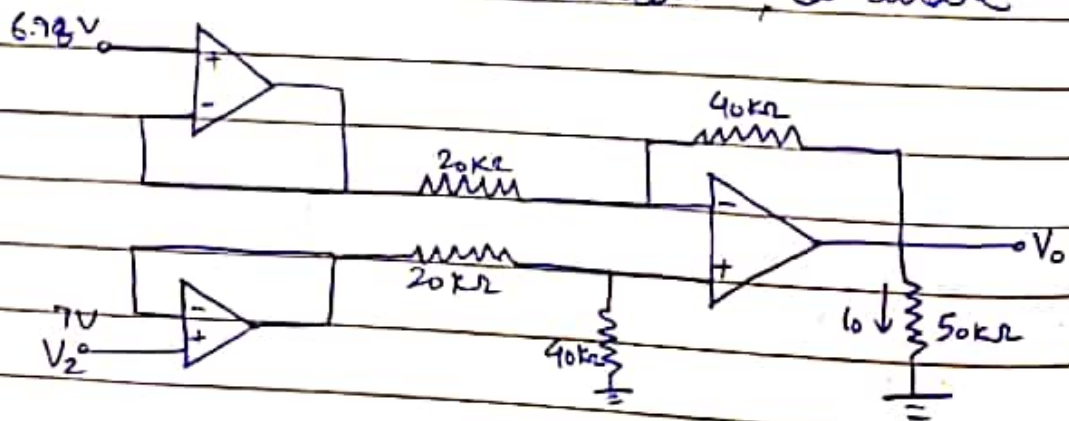
$$V_0 = 41 \times 6 \times 10^{-3}$$

Instrumentation Amplifier Circuit



$$V_0 = \frac{R_2}{R_1} \left(1 + \frac{2R_3}{R_4} \right) (V_2 - V_1)$$

Q. In the circuit shown below, calculate V_0 is



Ans.

$R_1 = 20k\Omega$	$R_2 = 40k\Omega$	$R_3 = 0$	$R_4 = \infty$
$V_1 = 6.98V$	$V_2 = 7V$		

$\therefore V_0 = 40mV$
 $I_D = \frac{40 \times 10^{-3}}{50 \times 10^3} A$
 $= 0.8 \mu A$

$$\begin{aligned} \text{Inductor: } & \vec{I} \text{ mm} \quad V = j\omega L I \\ & \leftarrow \vec{V} \rightarrow \quad Z = j\omega L \\ & \quad \quad \quad Z = jX_L \end{aligned}$$

$$\begin{aligned} \text{Capacitor: } & \vec{I} \text{ mm} \quad V = Z I \\ & \quad \quad \quad Z = \frac{-j}{\omega C} = \frac{1}{j\omega C} \end{aligned}$$

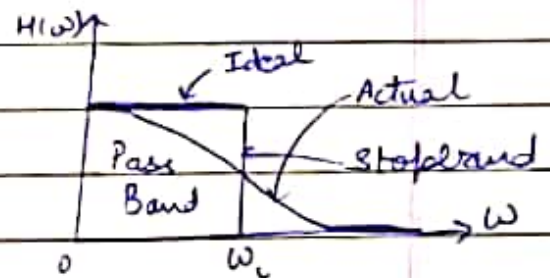
Low Pass Filter

$$V_i(\omega) \rightarrow \boxed{\quad} \rightarrow V_o(\omega)$$

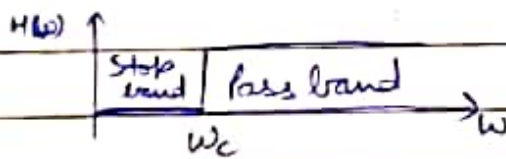
Transfer Function

$$H(\omega) \triangleq \frac{V_o(\omega)}{V_i(\omega)} \quad \text{--- (1)}$$

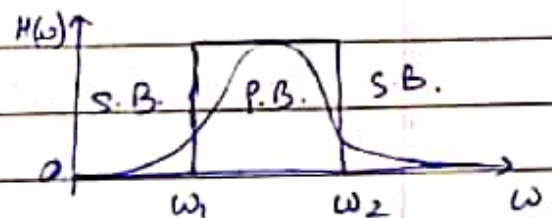
ω_c = Cutoff angular freq.



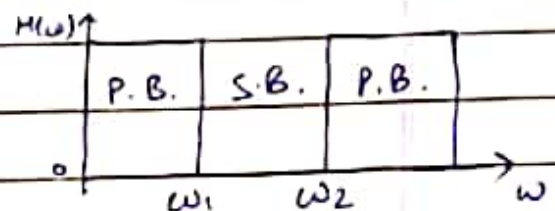
High Pass Filter (HPF)



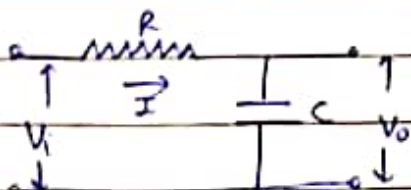
Band Pass Filter (BPF)



Band Stop Filter (BSF)



Ex-1.



L.P.F.

$$I = \frac{V_i}{Z_R + Z_C} = \frac{V_i}{R - \frac{j}{\omega C}} = \frac{V_i}{R + \frac{j}{\omega C}} \quad \text{--- (2)}$$

$$V_o = Z_C I = \frac{1}{j\omega C} \times \frac{V_i}{R + \frac{j}{\omega C}}$$

$$V_o = \frac{V_i}{1+j\omega RC} \quad - (2)$$

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)} = \frac{1}{1+j\omega RC} \quad - (3)$$

$$|H(\omega)| = \frac{1}{\sqrt{1+\omega^2 R^2 C^2}} \quad - (4)$$

$$\text{At } \omega = \omega_c, |H| = \frac{1}{\sqrt{2}} \quad - (5)$$

$$\therefore \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1+\omega_c^2 R^2 C^2}}$$

$$\therefore 2 = 1 + \omega_c^2 R^2 C^2$$

$$\therefore \omega_c = \frac{1}{\sqrt{RC}} \quad - (6)$$

$$\therefore f_c = \frac{1}{2\pi\sqrt{RC}} \quad - (7)$$

Q. Using a $1\mu F$ capacitor design RC LPF for
 @ $f_c = 1 \text{ MHz}$ (b) $f_c = 1 \text{ KHz}$ (c) $f_c = 100 \text{ MHz}$

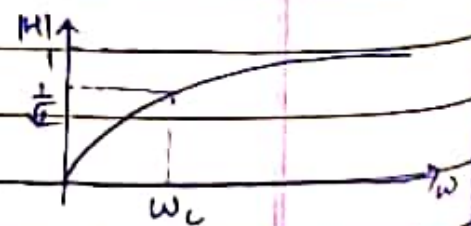
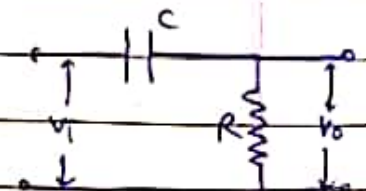
Ans. a. $1 \times 10^6 = \frac{1}{2\pi R \times 1 \times 10^{-6}}, R = \frac{1}{2\pi}$

Q. Find $H(\omega)$?

Ans. $I = \frac{V_i}{R + 1/j\omega C}$

$$V_o = IR$$

$$H(\omega) = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC}$$



Filters examples 1 & 2 were of "RC Filter" category. They can also be termed "Passive" filters because to DC input is needed. They can also be termed "First-order" filters because the mathematical expression for $H(\omega)$ contains only ω terms (now 2 terms, now 3 terms, etc.)

Example-3. Passive First-Order RL Filters (LPF)

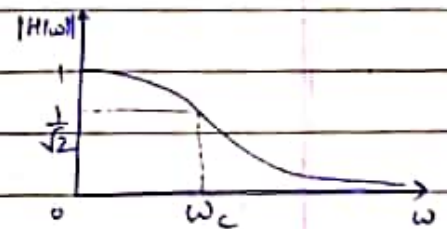
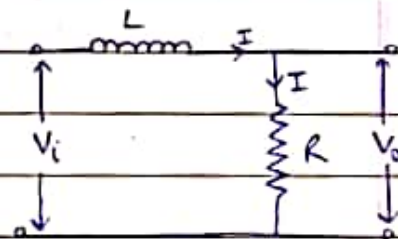
$$I = \frac{V_i}{R + j\omega L} \quad - (1)$$

$$V_o = RI = \frac{RV_i}{R + j\omega L} \quad - (2)$$

$$H(\omega) \triangleq \frac{V_o(\omega)}{V_i(\omega)} = \frac{R}{R + j\omega L}$$

$$\therefore H(\omega) = \frac{1}{1 + j\omega \frac{L}{R}} \quad - (3)$$

$$\therefore |H(\omega)| = \frac{1}{\sqrt{1 + \frac{\omega^2 L^2}{R^2}}} \quad - (4)$$



At cut off,

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + \frac{\omega_c^2 L^2}{R^2}}}$$

$$\therefore 2 = 1 + \frac{\omega_c^2 L^2}{R^2}$$

$$\Rightarrow \omega_c = \frac{R}{L} \quad - (5)$$

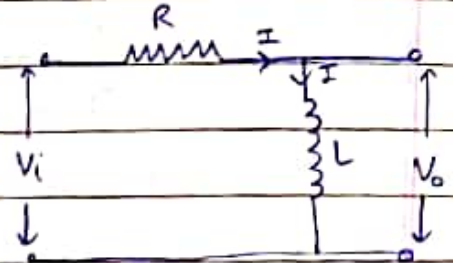
$$\therefore f_c = \frac{R}{2\pi L} \quad - (6)$$

Example-4. Passive First-Order Filter (HFF)

$$I = \frac{V_i}{R + j\omega L} \quad - (1)$$

$$V_o = j\omega L I$$

$$= \frac{j\omega L V_i}{R + j\omega L} \quad - (2)$$



$$\therefore H(\omega) \triangleq \frac{V_o(\omega)}{V_i(\omega)} = \frac{j\omega L}{R + j\omega L} \quad - (3)$$

$$\therefore |H(\omega)| = \frac{\omega L}{\sqrt{R^2 + \omega^2 L^2}} = \frac{1}{\sqrt{1 + \left(\frac{R}{\omega L}\right)^2}} \quad - (4)$$

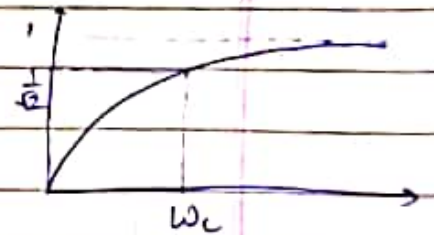
At ω_{cutoff}

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + \frac{R^2}{\omega_c^2 L^2}}}$$

$$\therefore 2 = 1 + \frac{R^2}{\omega_c^2 L^2}$$

$$\Rightarrow \omega_c = \frac{R}{L} \quad - (5)$$

$$\therefore f_c = \frac{R}{2\pi L} \quad - (6)$$



- Q. For R-L LPF, if $L = 3 \mu H$, what R is need to obtain $f_c = 350 \text{ kHz}$?

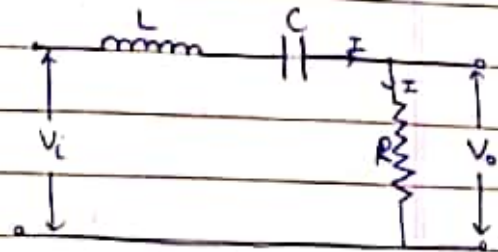
Ans. ~~13.19 ohm~~ 6.6Ω

Example-5. Second-order Passive Filter

$$I = \frac{V_i}{R + j\omega L - \frac{j}{\omega C}} \quad \text{--- (1)}$$

$$V_o = RI \quad \text{--- (2)}$$

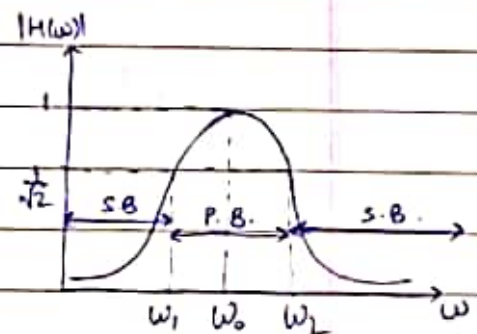
$$V_o = \frac{RV_i}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$$



$$\therefore H(\omega) \triangleq \frac{V_o(\omega)}{V_i(\omega)} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)} \quad \text{--- (2)}$$

$$= \frac{1}{1 + j\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)} \quad \text{--- (3)}$$

$$|H(\omega)| = \frac{1}{\sqrt{1 + \left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)^2}} \quad \text{--- (4)}$$



ω_0 = Center frequency
 = Resonance frequency
 = Design frequency

At cutoff

$$\frac{1}{\sqrt{2}} = \frac{1}{\sqrt{1 + \left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)^2}} \quad \text{--- (5)}$$

At resonance, $\omega = \omega_0$ and $|H(\omega)| = 1$

$$\therefore \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \quad \text{--- (5)}, \quad f_0 = \frac{1}{2\pi\sqrt{LC}} \quad \text{--- (6)}$$

$$2 = 1 + \frac{\left(\omega L - \frac{1}{\omega C}\right)^2}{R^2} \Rightarrow \left(\omega L - \frac{1}{\omega C}\right)^2 = R^2$$

$$\omega^2 L^2 + \frac{1}{\omega^2 C^2} - \frac{2L}{C} = R^2 \quad \text{--- (7)}$$

This is a fourth order equation

The two non-negative solutions to eq. (7) have been shown to be

$$\omega_1 = \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \quad \text{--- (8)}$$

$$\text{and } \omega_2 = \frac{+R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \quad \text{--- (9)}$$

Bandwidth is that range of ω (& f) over which the filter's performance is considered to be good enough.

$$\therefore \text{Bandwidth } B \triangleq \omega_2 - \omega_1 \quad \text{--- (10)}$$

$$= \frac{R}{L} \quad \text{--- (11)}$$

Relative Bandwidth

$$R.B.W. \triangleq \frac{\omega_2 - \omega_1}{\omega_0} \quad \text{--- (12)}$$

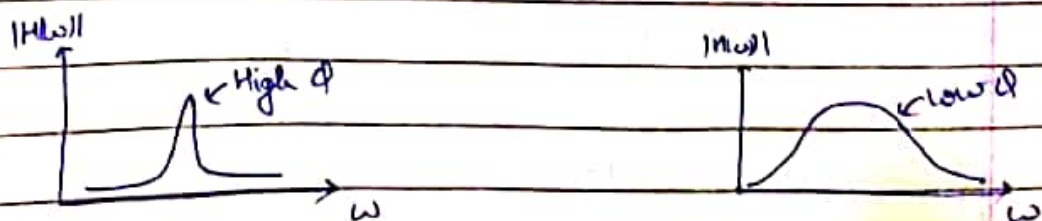
Quality Factor

$$Q = \frac{\omega_0 L}{R} \quad \text{--- (14)}$$

$$Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{\omega_0}{B.W.} \quad \text{--- (15)}$$

Percentage Bandwidth

$$P.B.W. = 100 \times \frac{\omega_2 - \omega_1}{\omega_0} \quad \text{--- (13)}$$



Q. For the RLC BPF, $L = 3\mu\text{H}$, $C = 3\mu\text{F}$. Calculate f_0 . What R value is needed to obtain $Q = 50$?

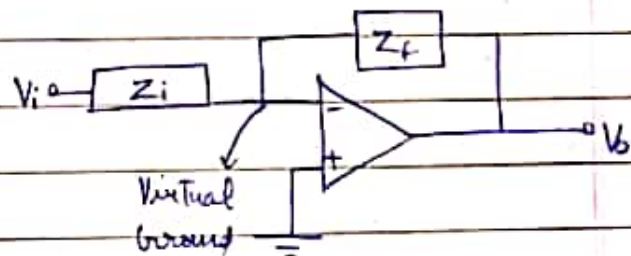
Ans $f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{10^6}{2\pi \times 3} \text{ Hz}$

$$R = \frac{\omega_0 L}{Q} = \frac{1}{50} \Omega$$

Active Filters Using op-AMP's

① Inverting Mode :-

$$\frac{V_i}{Z_i} = -\frac{V_o}{Z_f} \quad \text{--- (1)}$$



$$\begin{aligned} \therefore H(\omega) &\triangleq \frac{V_o(\omega)}{V_i(\omega)} \\ &= -\frac{Z_f}{Z_i} \quad \text{--- (2)} \end{aligned}$$

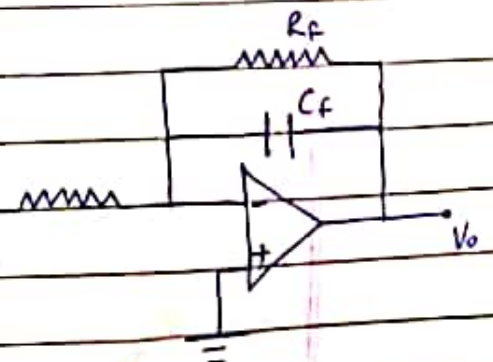
Ex-1. Active LPF

$$\frac{1}{Z_f} = \frac{1}{R_f} + \frac{1}{[1/j\omega C_f]}$$

$$Z_f = R_f \times \frac{1}{j\omega C_f}$$

$$R_f + \frac{1}{j\omega C_f}$$

$$= \frac{j\omega C_f R_f}{1 + j\omega C_f R_f} \quad \text{--- (3)}$$



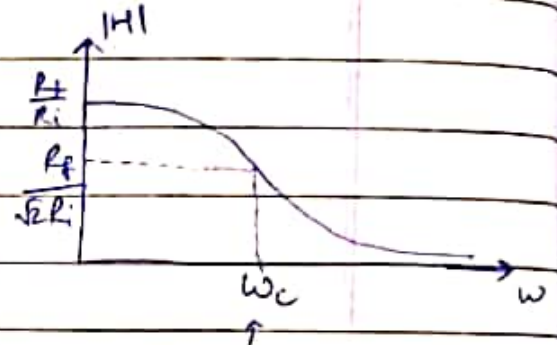
$$Z_i = R_i \quad - (4)$$

$$H(\omega) = -\frac{R_f}{R_i} \left[\frac{1}{1+j\omega R_f C_f} \right] \quad - (5)$$

$$|H(\omega)| = \left[\frac{R_f}{R_i} \right] \left[\frac{1}{\sqrt{1+\omega^2 R_f^2 C_f^2}} \right] \quad - (6)$$

$$\omega_c = \frac{1}{R_f C_f} \quad - (7)$$

$$f_c = \frac{1}{2\pi R_f C_f} \quad - (8)$$



Cutoff Angular Freq
(Carrier Angular Freq)

Example-2. Active HPF

$$H(\omega) \triangleq \frac{V_o(\omega)}{V_i(\omega)}$$

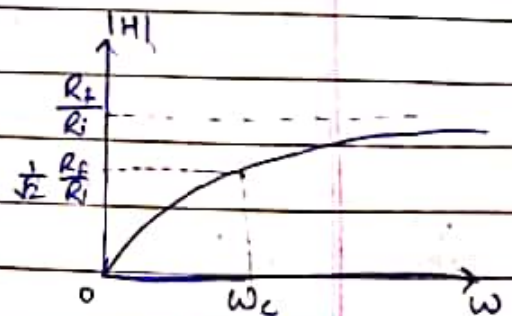
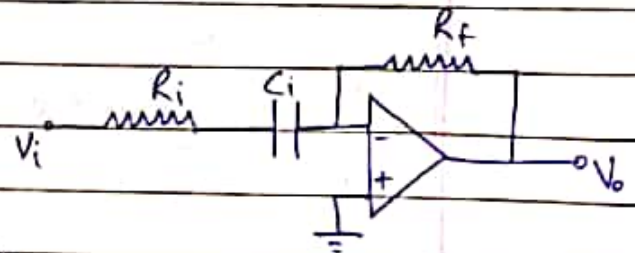
$$= -\left(\frac{R_f}{R_i} \right) \left(\frac{j\omega C_i R_i}{1+j\omega C_i R_i} \right) \quad - (1)$$

D.C. Gain

$$|H(\omega)| = \frac{R_f}{R_i} \frac{\omega C_i R_i}{\sqrt{1+\omega^2 C_i^2 R_i^2}}$$

$$= \frac{R_f}{R_i} \left[\frac{1}{\sqrt{1+\frac{1}{\omega^2 C_i^2 R_i^2}}} \right] \quad - (2)$$

$$\omega_c = \frac{1}{R_i C_i} \quad - (3)$$



Q. Design an inverting LPF (active) with a D.C Gain = 4 and Carrier Freq = 500Hz \wedge $C_f = 0.1 \mu F$

Ans. $\omega_c = \frac{1}{R_f C_f} = 2\pi 500 = \frac{1}{R_f \cdot 0.1 \times 10^{-7}}$

$R_f = \frac{1}{2\pi \times 500 \times 10^{-7}} = 3.18 \text{ k}\Omega$

$R_i = \frac{R_f}{4} = \frac{3.18}{4} \text{ k}\Omega$

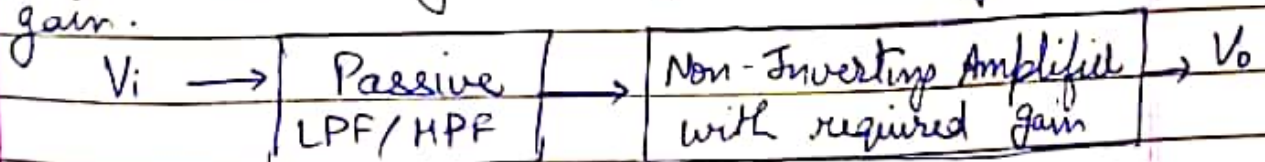
1. Design an inverting active HPF with a gain of 5 and $f_c = 2 \text{ kHz}$. Use $0.1 \mu\text{F}$ capacitor.

Ans. $f_c = \frac{1}{2\pi R_i C_f} \Rightarrow R_i = \frac{1}{2\pi \times 2000 \times 0.1 \times 10^{-6}} = 0.796 \text{ k}\Omega$

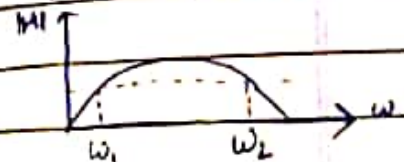
$R_f = 5R_i = 0.398 \text{ k}\Omega$

Active Non-Inverting Filters

- 1) Cascade a known inverting design with an inverting amplifier having unity Gain.
- 2) Use a passive design and Cascade it with a non-inverting amplifier having the designed gain.

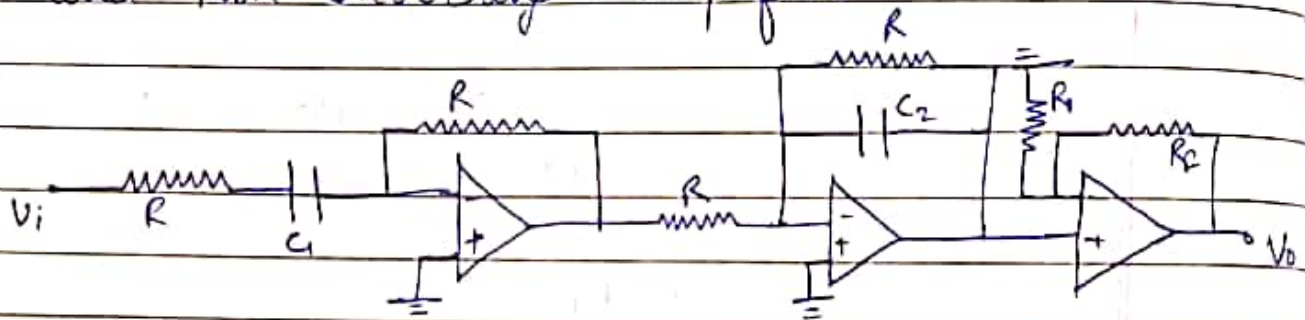


Band Pass Filters (BPF) can be obtained by cascading a suitably designed HPF with a suitably designed LPF.



Band Stop Filters can also be designed using LPF & HPF combinations

Active BPF using the cascade of HPF, LPF, and Non-Inverting Amplifier



$$H(\omega) = \left[\frac{-j\omega RC_1}{1 + j\omega RC_1} \right] \cdot \left[\frac{-1}{1 + j\omega RC_2} \right] \cdot \left[1 + \frac{R_f}{R_i} \right] \quad \text{--- (1)}$$

$$|H(\omega)| = \left(1 + \frac{R_f}{R_i} \right) \left[\frac{\omega RC_1}{\sqrt{1 + \omega^2 R^2 C_1^2} \sqrt{1 + \omega^2 R^2 C_2^2}} \right] \quad \text{--- (2)}$$

$$= \frac{1 + R_f/R_i}{\sqrt{1 + \frac{1}{\omega^2 R^2 C_1^2}} \sqrt{1 + \omega^2 R^2 C_2^2}} \quad \text{--- (3)}$$

$$\omega_1 \triangleq \frac{1}{RC_1} \quad \text{--- (4)}$$

$$\omega_2 \triangleq \frac{1}{RC_2} \quad \text{--- (5)}$$

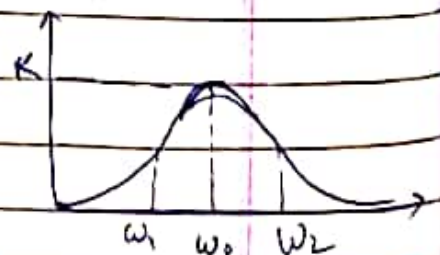
For max $|H(\omega)|$

$$\frac{d|H(\omega)|}{d\omega} = 0 \quad \text{--- (6)}$$

$$\frac{d}{d\omega} \left[\frac{1}{\sqrt{f_1 + f_2}} \right] = 0 \quad \text{--- (7)}$$

$$\text{When } f_1 \triangleq 1 + \frac{1}{\omega^2 R^2 C_1^2} \quad \text{--- (8)}$$

$$f_2 \triangleq 1 + \omega^2 R^2 C_2^2 \quad \text{--- (9)}$$



K = Passband gain

ω_0 = Center Angular Frequency

(Resonance Angular Frequency)

Yield

$$\frac{d}{dw} [(f_1 f_2)^{1/2}] = 0$$

$$\Rightarrow \frac{-1}{2} (f_1 f_2)^{-3/2} \frac{d(f_1 f_2)}{dw} = 0$$

Since $f_1 f_2 \neq 0$ we get

$$\frac{d(f_1 f_2)}{dw} = 0 \quad \text{--- (10)}$$

$$f_1 \frac{df_2}{dw} + f_2 \frac{df_1}{dw} = 0 \quad \text{--- (11)}$$

$$\frac{df_1}{dw} = \frac{-2}{RC_1 \omega^2}$$

$$\omega_0 = \frac{1}{R \sqrt{C_1 C_2}} \quad \text{--- (14)}$$

$$= \frac{1}{\sqrt{R C_1} \sqrt{R C_2}} = \sqrt{\omega_1 \omega_2} \quad \text{--- (15)}$$

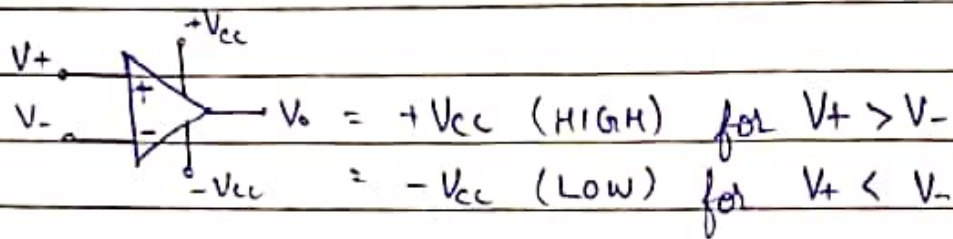
We define $f_1 = \frac{\omega_1}{2\pi}$

Passband Gain

$$|H(\omega)| = \frac{1 + R_1/R_2}{\sqrt{1 + \frac{1}{\omega^2 R^2 C_1^2}} \sqrt{1 + \omega^2 R^2 C_2^2}}$$

$$K = 1 + R_1/R_2$$

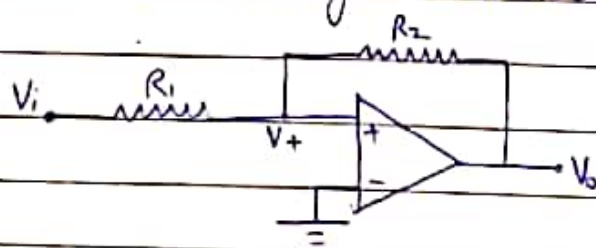
Comparator circuit using an OP-AMP in open loop.



Disadvantage - Even for a small change in input (caused by say random noise signals), the output can switch from one state to another in quite an unpredictable fashion (random behaviour).

Schmitt Trigger: is a comparator circuit which uses ~~possible~~ positive feedback to make the output more predictable.

Non-Inverting Schmitt Trigger:



we deliberately make V_+ high enough or low enough

KCL

$$\frac{V_i - V_+}{R_1} + \frac{V_0 - V_+}{R_2} = 0 \quad \text{--- (1)}$$

$$V_+ \left[\frac{1}{R_1} + \frac{1}{R_2} \right] = \frac{V_i}{R_1} + \frac{V_0}{R_2}$$

$$V_+ \left[\frac{R_1 + R_2}{R_1 R_2} \right] = \frac{V_i}{R_1} + \frac{V_0}{R_2}$$

$$V_+ = \left[\frac{R_2}{R_1 + R_2} \right] V_i + \left[\frac{R_1}{R_1 + R_2} \right] V_o \quad \text{--- (2)}$$

- ④ Let us assume that, in the beginning, output is HIGH that is $V_o = +V_{cc}$. Then eq. (2) becomes

$$V_+ = \left[\frac{R_2}{R_1 + R_2} \right] V_i + \left[\frac{R_1}{R_1 + R_2} \right] V_{cc} \quad \text{--- (3)}$$

The output will always switch to the other state when V_+ crosses 0 in either direction. For $V_+ = 0$ eq. (3) yields

$$V_i = -\frac{R_1}{R_2} V_{cc} \quad \text{--- (4)}$$

We define a Threshold voltage V_{th} by

$$V_{th} = \frac{R_1}{R_2} V_{cc} \quad \text{--- (5)}$$

so that eq. (4) can be written as $V_i = -V_{th}$ --- (6)

- ⑤ If V_i falls below $-V_{th}$, the output will switch to low, i.e. $V_o = -V_{cc}$.

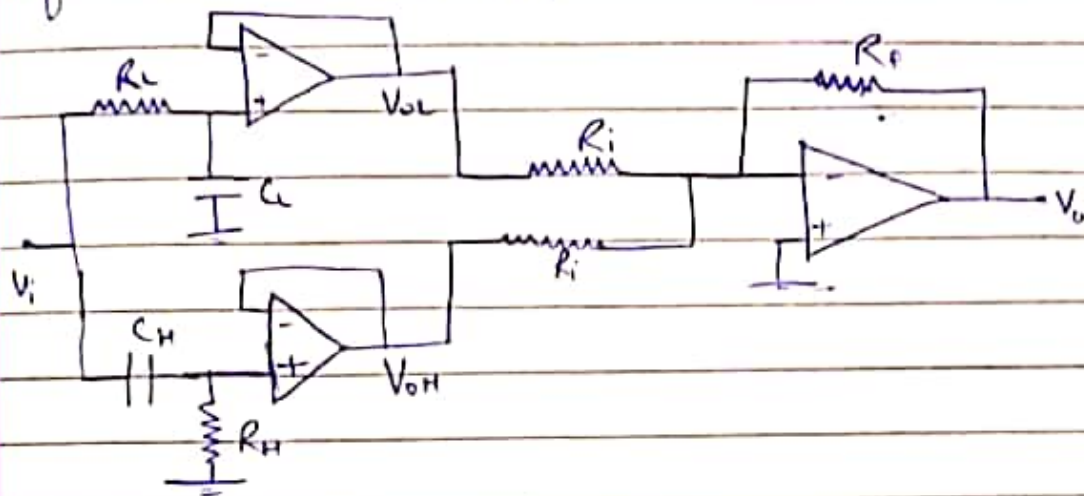
Then eq. (2) will yield

$$V_+ = \left(\frac{R_2}{R_1 + R_2} \right) V_i - \left(\frac{R_1}{R_1 + R_2} \right) V_{cc} \quad \text{--- (7)}$$

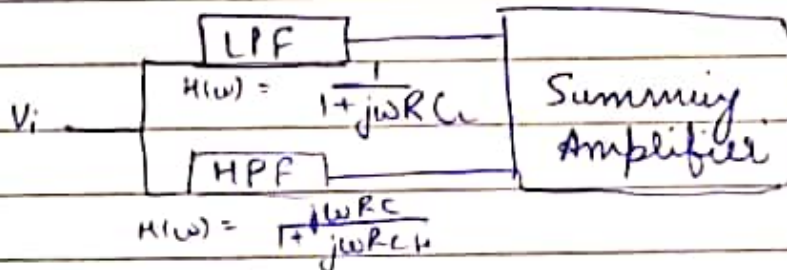
Hence the output will now switch at $V_{in} = +V_{th}$ --- (8)

- ⑥ This sequence can continue

Q. Determine the transfer function and nature of the Active filter



Ans.



$$V_{OL} = \frac{V_i}{1 + j\omega R_C C} \quad - (1)$$

$$V_{OH} = \frac{j\omega R_H C_H V_i}{1 + j\omega R_H C_H} \quad - (2)$$

$$V_o = \frac{-R_F}{R_i} \left[\frac{1}{1 + j\omega R_C C} + \frac{j\omega R_H C_H}{1 + j\omega R_H C_H} \right] V_i \quad - (3)$$

$$H(\omega) = \frac{V_o(\omega)}{V_i(\omega)}$$

$$= \frac{-R_F}{R_i} \left[\frac{1}{1 + j\omega R_C C} + \frac{1}{1 - \frac{j}{\omega R_H C_H}} \right] \quad - (4)$$