Proof

- assume $\texttt{rank}[i] \in \{k+1, \ldots, 2^k\}$
- the number of nodes with rank lying in this interval is at most

$$\frac{n}{2^{k+1}} + \frac{n}{2^{k+2}} + \cdots \leq \frac{n}{2^k}$$

- after a call to $\texttt{Find}(i)$, the node i is adopted by a new parent of strictly larger rank
- after at most 2^k calls to $\texttt{Find}(i)$, the parent of i will have rank from a different interval