

Hashing: Introduction

Michael Levin

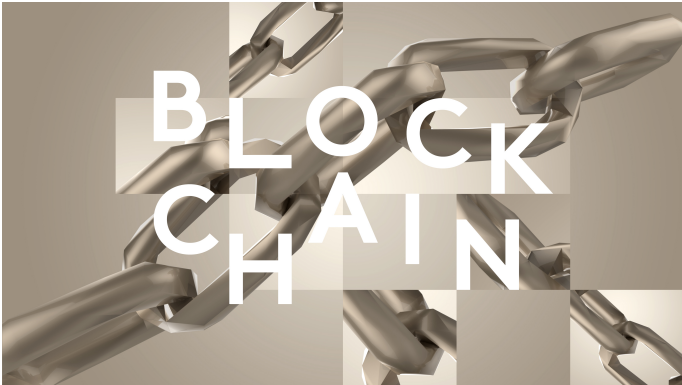
Department of Computer Science and Engineering
University of California, San Diego

Data Structures Fundamentals
Algorithms and Data Structures

Outline

- 1 Applications
- 2 Phone Book
- 3 International Phone Numbers
- 4 Hash Functions
- 5 Chaining
- 6 Chaining Implementation and Analysis
- 7 Hash Tables

Blockchain



Programming Languages



C#



C++



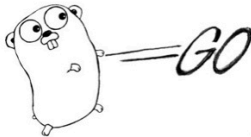
Objective-C



python



Perl



THE

C

PROGRAMMING
LANGUAGE



Programming Languages



C#



C++



Objective-C



python



Perl

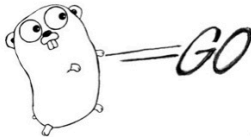
dict

THE

C



Visual Basic



JavaScript

PROGRAMMING
LANGUAGE

Programming Languages



C#



Objective-C

C++

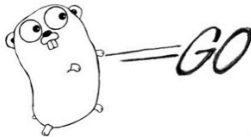


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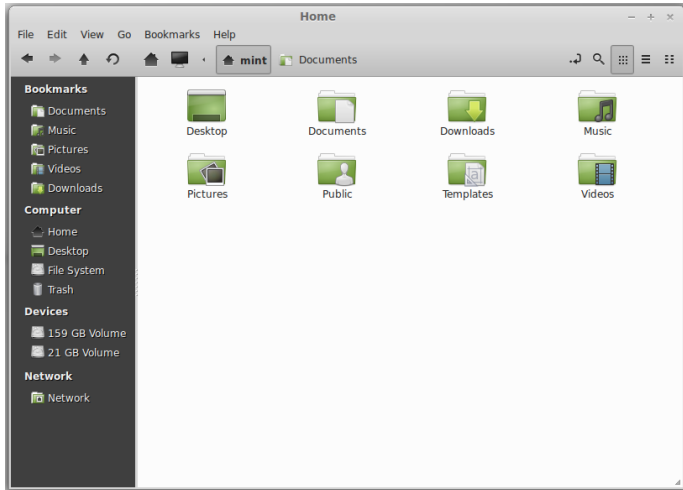
PROGRAMMING
LANGUAGE




Programming Languages

Keywords: `for`, `if`, `while`, `int`, ...

File Systems




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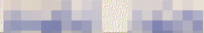
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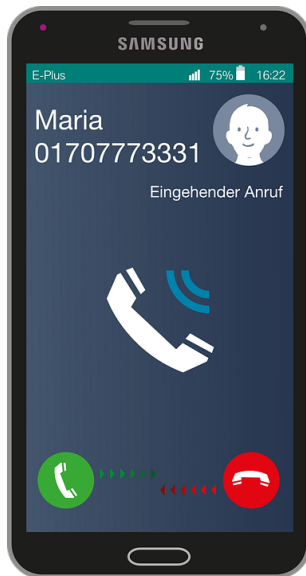
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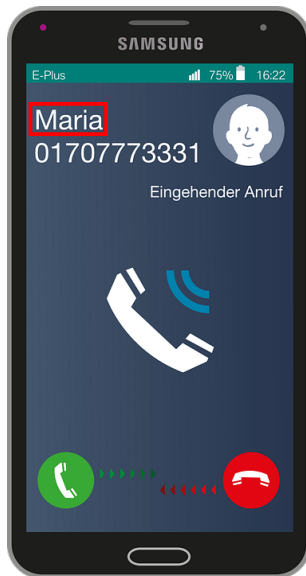
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- 2 **Phone Book**
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- 5 Chaining
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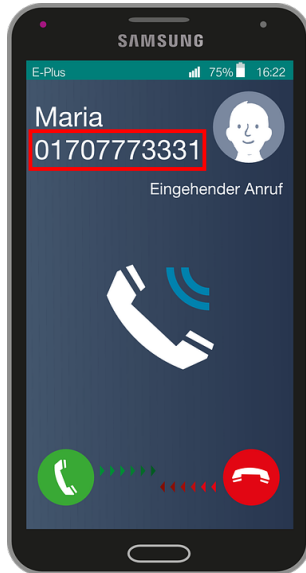
Who's Calling?



Who's Calling?



Who's Calling?



Phone Book

Phone number	Name
01707773331	Maria
239-17-17	Sasha
575-75-75	Helen

Phone to Name

We are going to focus on retrieving name by phone number for now

Local Phone Numbers

- Like 123-23-23

Local Phone Numbers

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- Typically up to 7 digits

Local Phone Numbers

- Like 123-23-23
- Typically up to 7 digits
- Sufficient for $10^7 = 10\,000\,000$ phone numbers

Convert Phone Number to Integer

Examples

123-23-23 \rightarrow 1 232 323

049 12 12 \rightarrow 491 212

5757575 \rightarrow 5 757 575

Direct Addressing

10^7 rows

Phone number	Name
00000000	
...	
2391717	Sasha
...	
5757575	Helen
...	
99999999	

Direct Addressing

- Store phone book as array of size 10^7
- Names are values of the array
- To retrieve name by phone number, convert phone number to integer first
- Use the resulting integer as index in the array of names

GetName(phoneNumber)

```
index  $\leftarrow$  ConvertToInt(phoneNumber)  
return phoneBookArray[index]
```

SetName(phoneNumber, name)

```
index  $\leftarrow$  ConvertToInt(phoneNumber)  
phoneBookArray[index]  $\leftarrow$  name
```

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Asymptotics

For a phone book with n contacts,

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For a phone book with n contacts,

- Retrieve name by phone number in $O(1)$
- Set name for a phone number in $O(1)$
- Memory consumption is $O(|U|)$, where U is the set of all possible phone numbers

Conclusion

- Local phone numbers are up to 7 digits long
- Can store them in an array of size 10^7
- This scheme is called **direct addressing**
- It is the simplest form of hashing

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+594 700 123 233 455

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- Using direct addressing requires array of size 10^{15} , which would take 7PB (7 petabytes) to store one phone book (1PB = 1024TB, 1TB = 1024GB)

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+594 700 123 233 455
- Using direct addressing requires array of size 10^{15} , which would take 7PB (7 petabytes) to store one phone book (1PB = 1024TB, 1TB = 1024GB)
- Your phone memory is probably at most 256GB, so you would need 28762 phones to store your phone book :)

Idea

- Direct addressing requires too much memory

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- Array is huge because it has a cell for every possible phone number

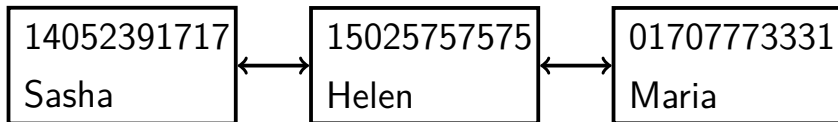
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- Let's store only the known phone numbers

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- Array is huge because it has a cell for every possible phone number
- Let's store only the known phone numbers
- Put pairs (Phone number, Name) into a doubly-linked list

Idea



Operations

- To add a contact, just insert new pair (Phone number, Name) into the list in $O(1)$

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- To retrieve name by phone number, search through the list...
- ...in $O(n)$, where n is the total number of contacts
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Idea 2

- Retrieving a name by phone number is slow, because we need to look through the whole list

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- Retrieving a name by phone number is slow, because we need to look through the whole list
- Let's put the pairs (Phone number, Name) in a dynamic array sorted by phone number!

Idea 2

01707773331	Maria
14052391717	Sasha
15025757575	Helen

Operations

- Retrieve name by phone number using binary search in $O(\log n)$

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- To insert a new contact, find appropriate position in $O(\log n)$, then insert in...
- ... $O(n)$, because we need to first move part of the array 1 position to the right
- Too slow again

Conclusion

- International numbers can be up to 15 digits long
- Direct addressing requires 7 petabytes of memory
- Simple list-based and array-based approaches are too slow
- Next videos — solution using hashing

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Encoding Phone Numbers

- Encode international phone numbers with small numbers

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- Encode international phone numbers with small numbers
- E.g. numbers from 0 to 999
- Different codes for the phone numbers in the phone book

Hash Function

Definition

For any set of objects S and any integer $m > 0$, a function $h : S \rightarrow \{0, 1, \dots, m - 1\}$ is called a **hash function**.

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m is called the **cardinality** of hash function h .

Desirable Properties

- Hash function should be fast to compute

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- Hash function should be fast to compute
- Different values for different objects
- Direct addressing with $O(m)$ memory
- Want small cardinality m
- Impossible to have all different values if number of objects $|S|$ is more than m (by pigeonhole principle)

Collisions

Definition

When $h(o_1) = h(o_2)$ and $o_1 \neq o_2$, this is a collision.

Desirable Properties

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- Hash function should be fast to compute
- ~~Different values for different objects~~
Small probability of collision
- Small enough cardinality m

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Map

Store mapping from objects to other objects:

- Filename \rightarrow location of the file
- Phone number \rightarrow name
- Name \rightarrow phone number

Map

Definition

Map from set S of objects to set V of values is a data structure with methods `HasKey(object)`, `Get(object)`, `Set(object, value)`, where $\text{object} \in S, \text{value} \in V$.

Map

Definition

In a Map from S to V , objects from S are usually called **keys** of the Map. Objects from V are called **values** of the Map.

Chaining for Phone Book

0
1
2
3
4
5
6
7

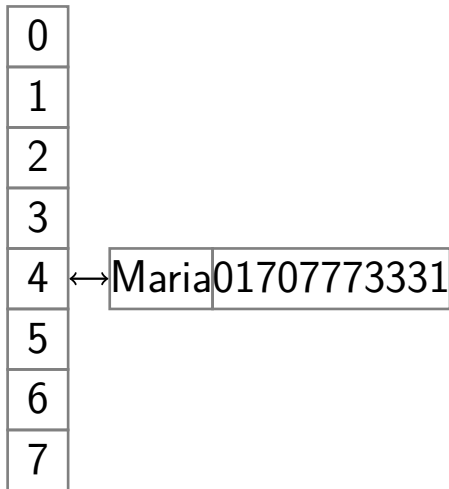
Chaining for Phone Book

$$h(01707773331) = 4$$

0
1
2
3
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7

Chaining for Phone Book

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Chaining for Phone Book

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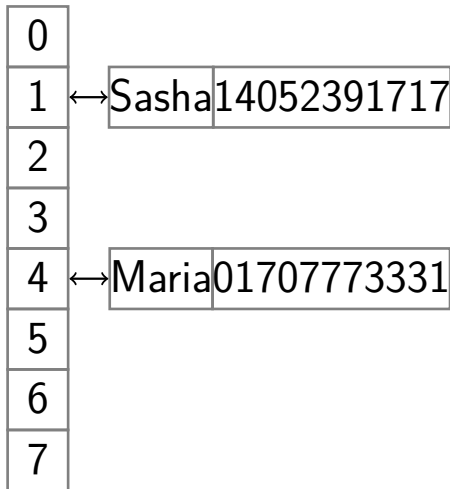
$$h(14052391717) = 1$$

0	
1	
2	
3	
4	← Maria01707773331
5	
6	
7	

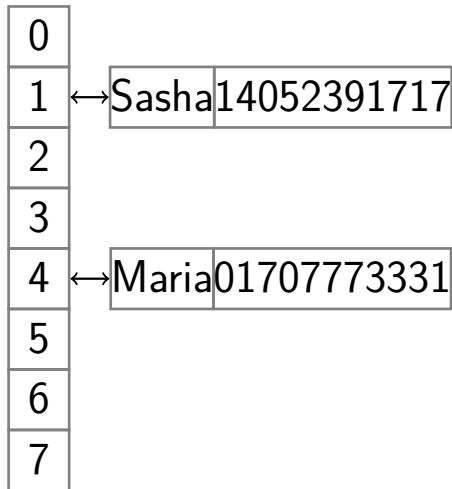
Chaining for Phone Book

$$h(01707773331) = 4$$

$$h(14052391717) = 1$$



Chaining for Phone Book



$$h(01707773331) = 4$$

$$h(14052391717) = 1$$

$$h(15025757575) = 4$$

Chaining for Phone Book

0
1 ↔ Sasha14052391717
2
3
4 ↔ Maria01707773331 ↔ Helen15025757575
5
6
7

$$h(01707773331) = 4$$

$$h(14052391717) = 1$$

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Chaining for Phone Book

- Select hash function h of cardinality m

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- Create array `Chains` of size m

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- Each element of `Chains` is a doubly-linked list of pairs (name, phoneNumber), called *chain*

Chaining for Phone Book

- Select hash function h of cardinality m
- Create array `Chains` of size m
- Each element of `Chains` is a doubly-linked list of pairs $(\text{name}, \text{phoneNumber})$, called *chain*
- Pair $(\text{name}, \text{phoneNumber})$ goes into chain at position $h(\text{ConvertToInt}(\text{phoneNumber}))$ in the array `Chains`

Chaining for Phone Book

- To look up name by phone number, go to the chain corresponding to phone number and look through all pairs

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- To look up name by phone number, go to the chain corresponding to phone number and look through all pairs
- To add a contact, create a pair (name, phoneNumber) and insert it into the corresponding chain
- To remove a contact, go to the corresponding chain, find the pair (name, phoneNumber) and remove it from the chain

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Implementation

Chains — array of chains

Each chain is a list of pairs (object, value)

HasKey(object)

```
chain ← Chains[hash(object)]  
for (key, value) in chain:  
    if key == object:  
        return true  
return false
```

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Get(object)

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Implementation

Set(object, value)

```
chain ← Chains[hash(object)]  
for pair in chain:  
    if pair.key == object:  
        pair.value ← value  
        return  
chain.Append((object, value))
```

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Asymptotics

Lemma

Let c be the length of the longest chain in `Chains`. Then the running time of `HasKey`, `Get`, `Set` is $\Theta(c + 1)$.

Asymptotics

Proof

- If the chain corresponding to the object is non-empty, but the object is not found in the chain, we will scan all c items — $\Theta(c) = \Theta(c + 1)$

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- If the chain corresponding to the object is non-empty, but the object is not found in the chain, we will scan all c items — $\Theta(c) = \Theta(c + 1)$
- If $c = 0$, we still need $O(1)$ time, thus the need for “+1” □

Asymptotics

Lemma

Let n be the number of different objects currently in the map and m be the cardinality of the hash function. Then the memory consumption for chaining is $\Theta(n + m)$.

Asymptotics

Proof

- $\Theta(n)$ to store n pairs (object, value)

Asymptotics

Proof

- $\Theta(n)$ to store n pairs (object, value)
- $\Theta(m)$ for array Chains of size m ☐

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Set

Definition

Set is a data structure with methods
`Add(object)`, `Remove(object)`,
`Find(object)`.

Set

Examples

- Students on campus

Set

Examples

- Students on campus
- Phone numbers of contacts

Set

Examples

- Students on campus
- Phone numbers of contacts
- Keywords in a programming language

Implementing Set

Two ways to implement a set using chaining:

- Set is equivalent to map from S to $V = \{true\}$

Implementing Set

Two ways to implement a set using chaining:

- Set is equivalent to map from S to $V = \{true\}$
- Store just objects instead of pairs (object, value) in the chains

Implementation

Find(object)

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Add(object)

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chain ← Chains[hash(object)]  
for key in chain:  
    if key == object:  
        return  
chain.Append(object)
```

Implementation

Add(object)

```
chain ← Chains[hash(object)]  
for key in chain:  
    if key == object:  
        return  
chain.Append(object)
```

Implementation

Remove(object)

```
if not Find(object):  
    return  
chain ← Chains[hash(object)]  
chain.Erase(object)
```

Implementation

```
Remove(object)
```

```
if not Find(object):
```

```
    return
```

```
chain ← Chains[hash(object)]
```

```
chain.Erase(object)
```


Implementation

Remove(object)

```
if not Find(object):
```

```
    return
```

```
chain ← Chains[hash(object)]
```

```
chain.Erase(object)
```

Implementation

Remove(object)

```
if not Find(object):  
    return
```

```
chain ← Chains[hash(object)]  
chain.Erase(object)
```

Implementation

Remove(object)

```
if not Find(object):  
    return
```

```
chain ← Chains[hash(object)]  
chain.Erase(object)
```

Hash Table

Definition

An implementation of a Set or a Map using hashing is called a hash table.

Programming Languages

Set:

- `unordered_set` in C++
- `HashSet` in Java
- `set` in Python

Map:

- `unordered_map` in C++
- `HashMap` in Java
- `dict` in Python

Conclusion

- Chaining is a technique to implement a hash table

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- Number of objects n , hash function cardinality m , longest chain length c
- Memory consumption is $\Theta(n + m)$
- Operations work in time $\Theta(c + 1)$
- How to make both m and c small?