Claim

$$\#(i \to j\colon \log^*(\text{rank}[i]) < \log^*(\text{rank}[j]))$$
$$\leq O(m \log^* n)$$

Proof

There are at most $\log^* n$ different values for $\log^*(\text{rank})$. □