Proof

• assume rank $[i] \in \{k+1,\ldots,2^k\}$