

# Basic Data Structures: Trees

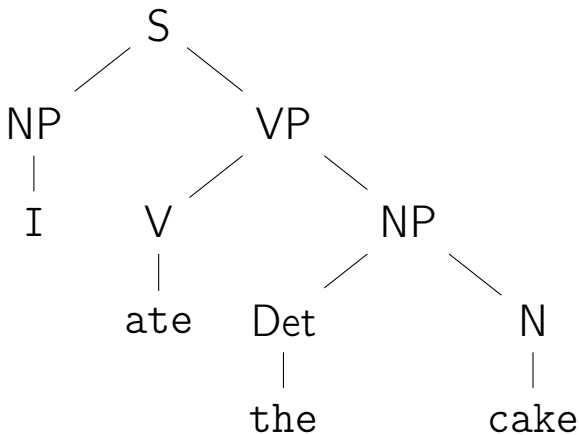
Neil Rhodes

Department of Computer Science and Engineering  
University of California, San Diego

Data Structures  
Data Structures and Algorithms

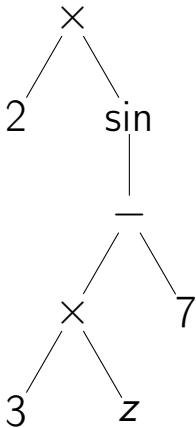
# Syntax Tree for a Sentence

I ate the cake

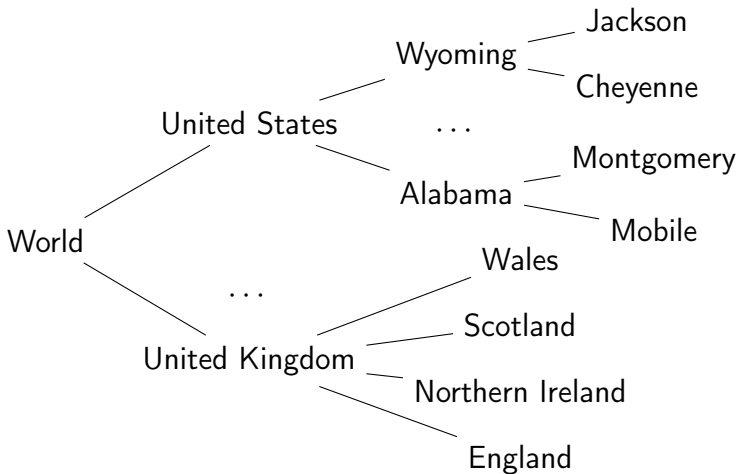


# Syntax tree for an Expression

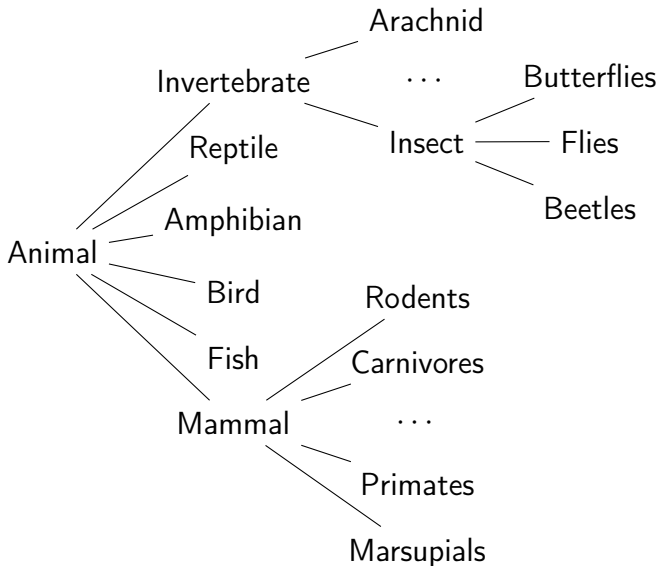
$$2 \sin(3z - 7)$$



# Geography Hierarchy



# Animal Kingdom (partial)

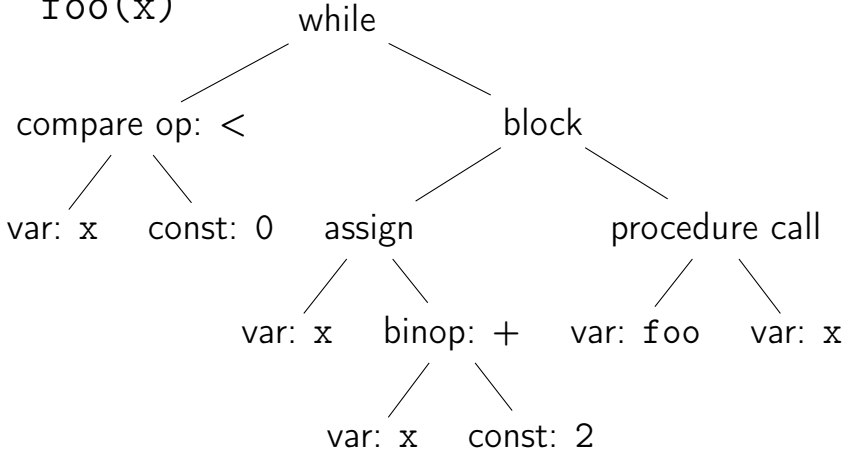


# Abstract Syntax Tree for Code

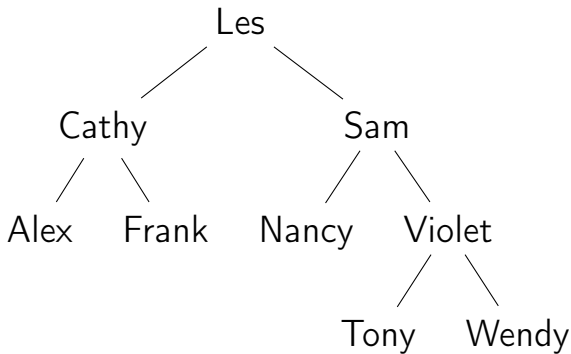
```
while x < 0:
```

```
    x = x + 2
```

```
    foo(x)
```



# Binary Search Tree



## Definition

A **Tree** is:

- empty, or
- a node with:
  - a key, and
  - a list of child trees.



# Simple Tree

Empty tree:

Tree with one node:

Fred

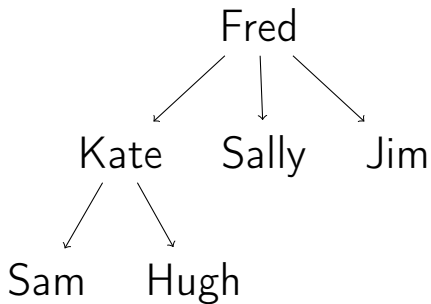
Tree with two nodes:

Fred

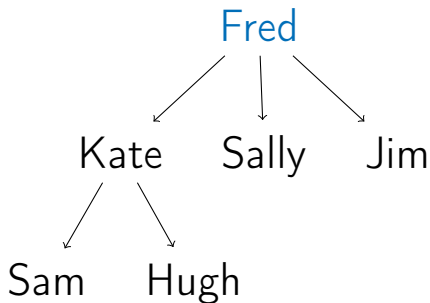
|

Sally

# Terminology

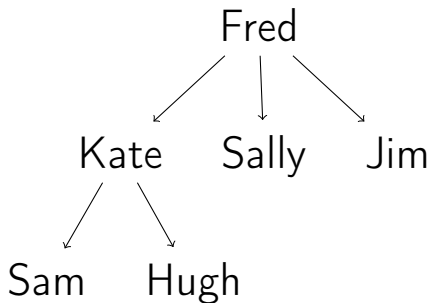


# Terminology



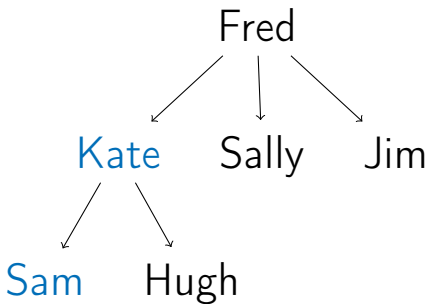
*Root:*  
top node in the tree

# Terminology



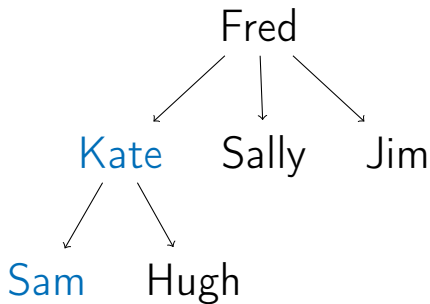
*A child* has a line down directly  
from a *parent*

# Terminology



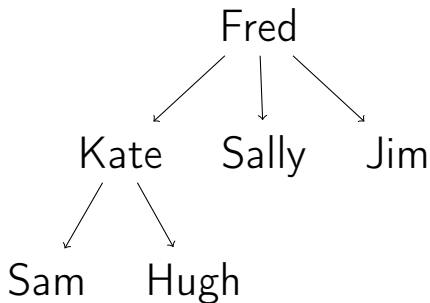
Kate is a *parent* of Sam

# Terminology



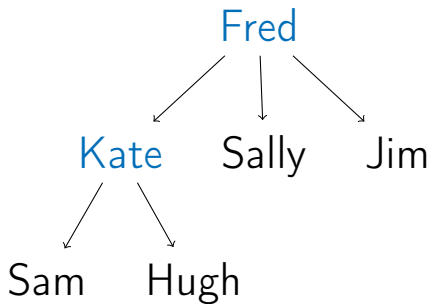
Sam is a *child* of Kate

# Terminology



***Ancestor:***  
parent, or parent of parent, etc.

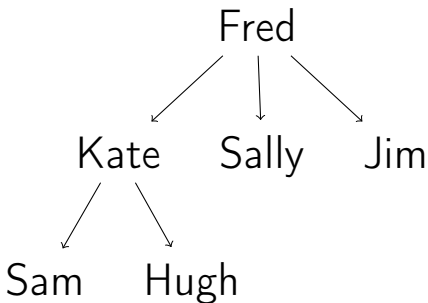
# Terminology



***Ancestors of Sam***

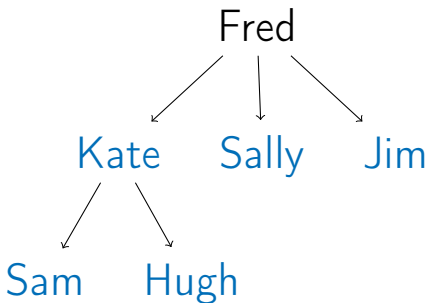


# Terminology



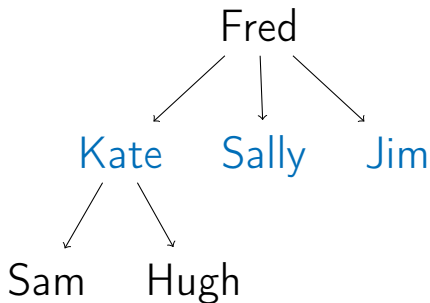
*Descendant:*  
child, or child of child, etc.

# Terminology



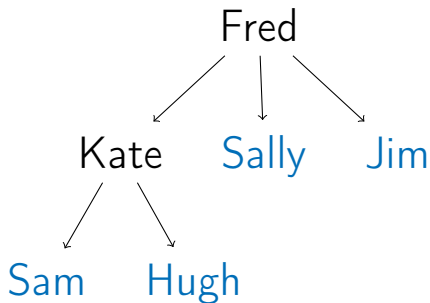
*Descendants of Fred*

# Terminology



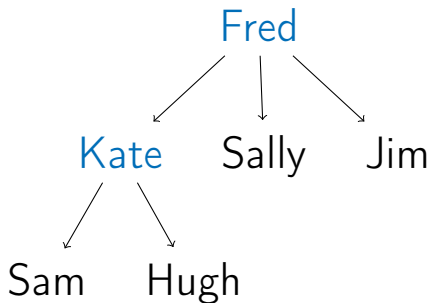
***Sibling:***  
sharing the same parent

# Terminology



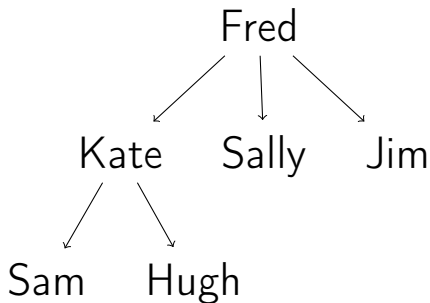
*Leaf:*  
node with no children

# Terminology



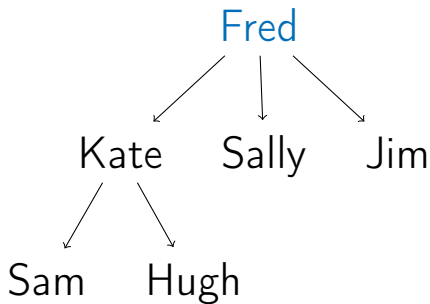
**Interior node  
(non-leaf)**

# Terminology



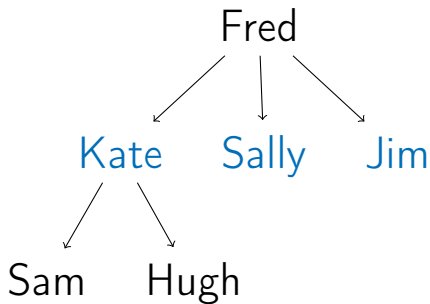
*Level:*  $1 + \text{num edges between root and node}$

# Terminology



**Level 1**

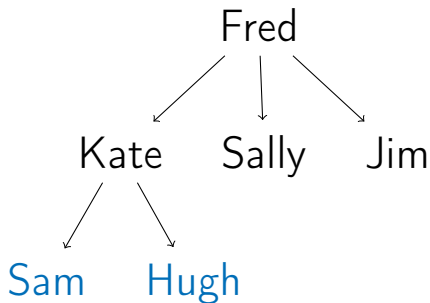
# Terminology



Level 2

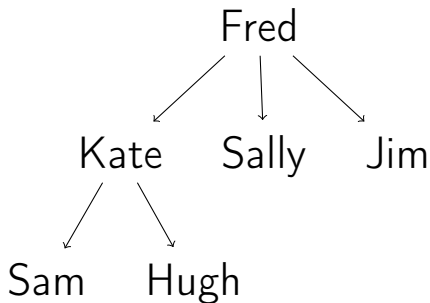


# Terminology



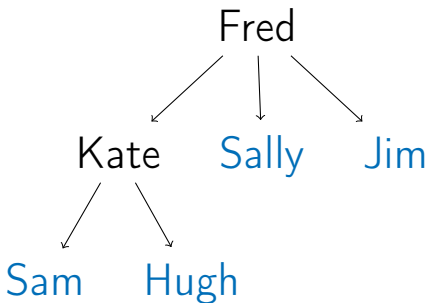
**Level 3**

# Terminology



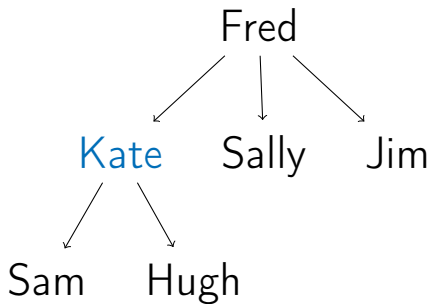
*Height:* maximum depth of subtree  
node and farthest leaf

# Terminology



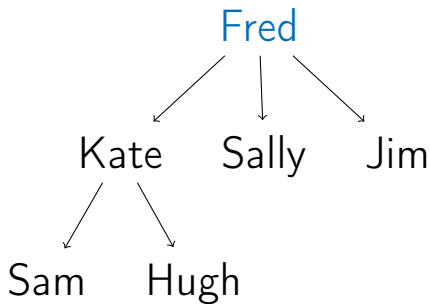
**Height 1**

# Terminology



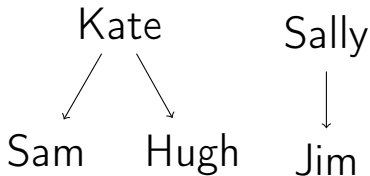
**Height 2**

# Terminology



**Height 3**

# Terminology



***Forest:***  
collection of trees

Node contains:

- key
- children: list of children nodes
- (optional) parent

For binary tree, node contains:

- key
- left
- right
- (optional) parent



## Height(*tree*)

```
if tree = nil:  
    return 0  
return 1 + Max(Height(tree.left),  
               Height(tree.right))
```

Size(*tree*)

```
if tree = nil  
    return 0  
return 1 + Size(tree.left) +  
           Size(tree.right)
```

# Walking a Tree

Often we want to visit the nodes of a tree in a particular order.

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For example, print the nodes of the tree.

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- Depth-first: We completely traverse one sub-tree before exploring a sibling sub-tree.

# Walking a Tree

Often we want to visit the nodes of a tree in a particular order.

For example, print the nodes of the tree.

- Depth-first: We completely traverse one sub-tree before exploring a sibling sub-tree.
- Breadth-first: We traverse all nodes at one level before progressing to the next level.

# Depth-first

InOrderTraversal(*tree*)

```
if tree = nil:
```

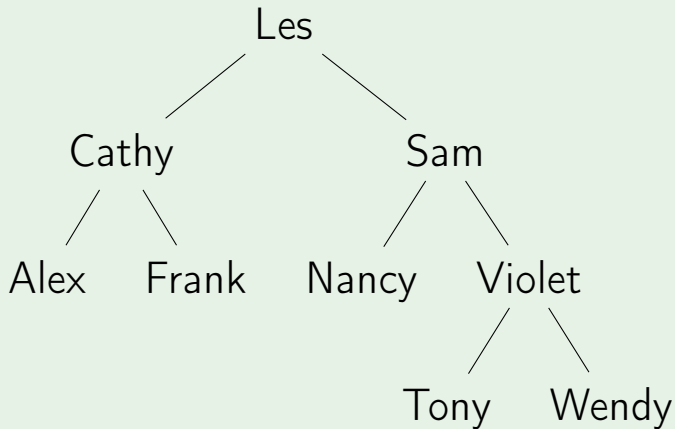
```
    return
```

```
InOrderTraversal(tree.left)
```

```
Print(tree.key)
```

```
InOrderTraversal(tree.right)
```

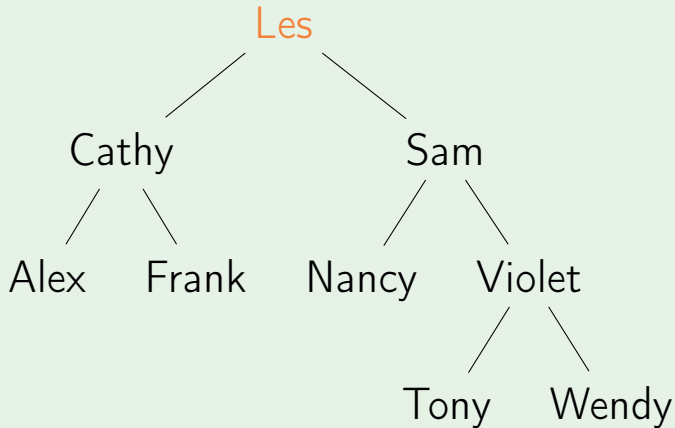
# InOrderTraversal



**Output:**



# InOrderTraversal



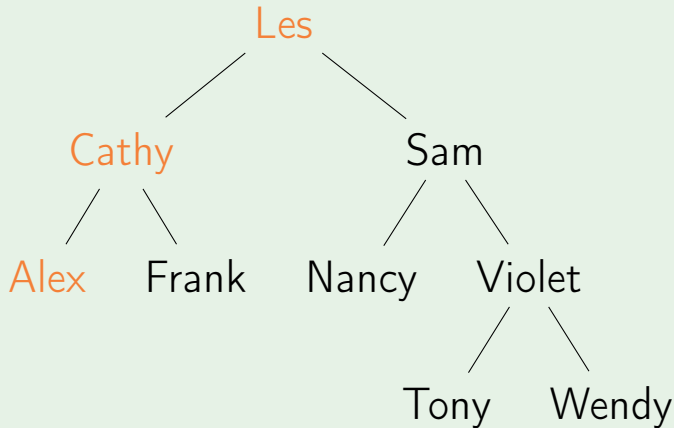
**Output:**

# InOrderTraversal



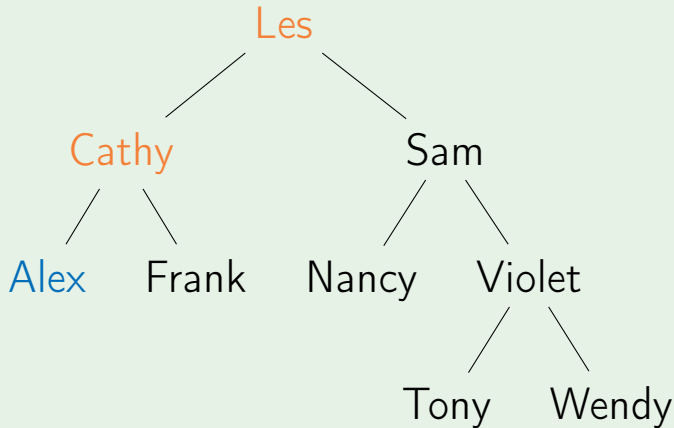
**Output:**

# InOrderTraversal



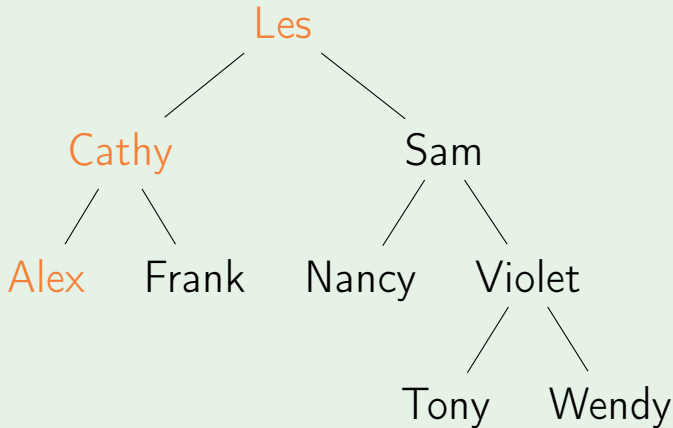
**Output:**

# InOrderTraversal



**Output:** Alex

# InOrderTraversal



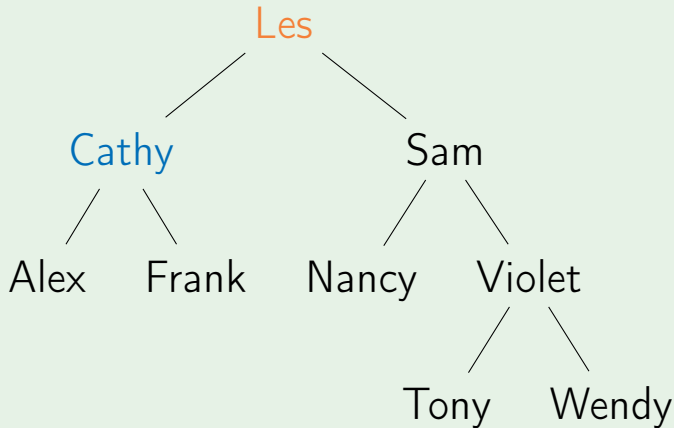
**Output:** Alex

# InOrderTraversal



**Output:** Alex

# InOrderTraversal



**Output:** Alex Cathy

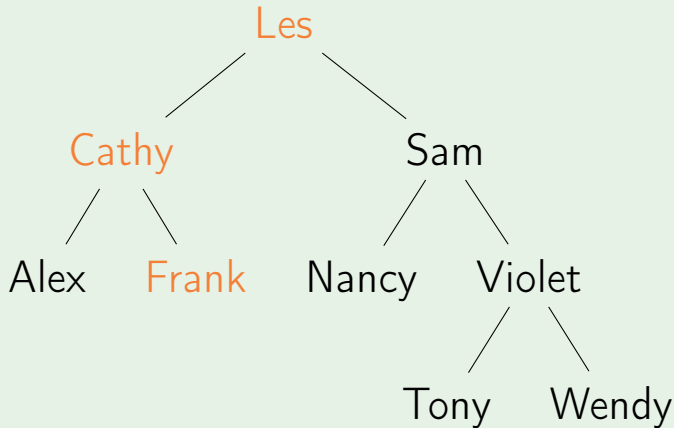
# InOrderTraversal



**Output:** Alex Cathy

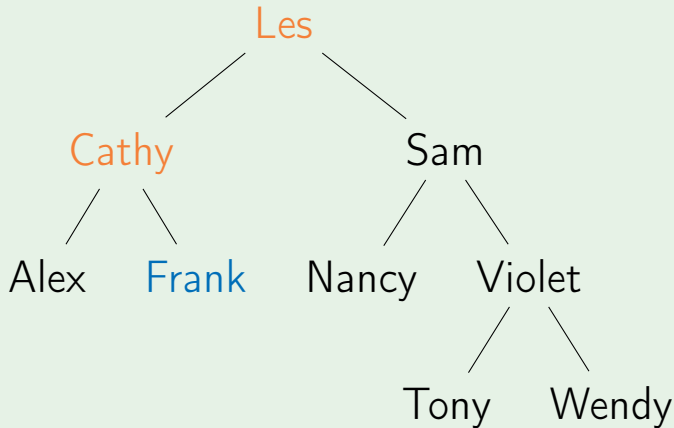


# InOrderTraversal



**Output:** Alex Cathy

# InOrderTraversal



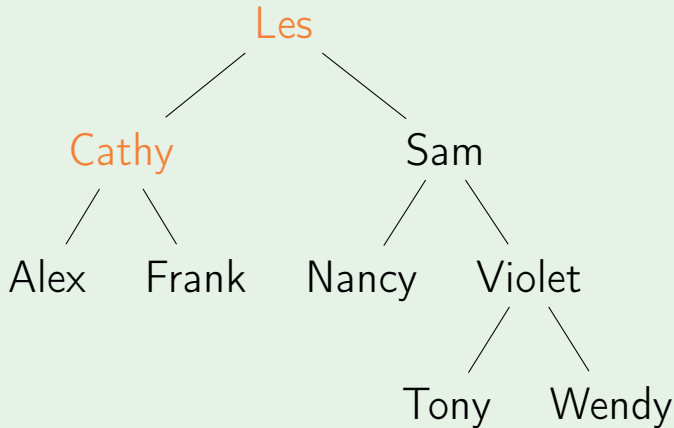
**Output:** Alex Cathy Frank

# InOrderTraversal



**Output:** Alex Cathy Frank

# InOrderTraversal



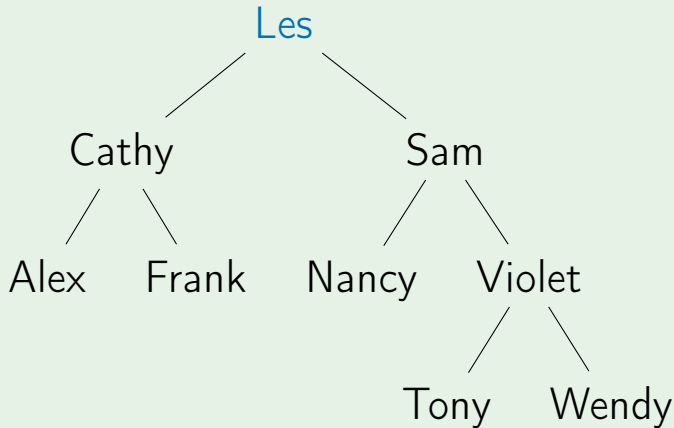
**Output:** Alex Cathy Frank

# InOrderTraversal



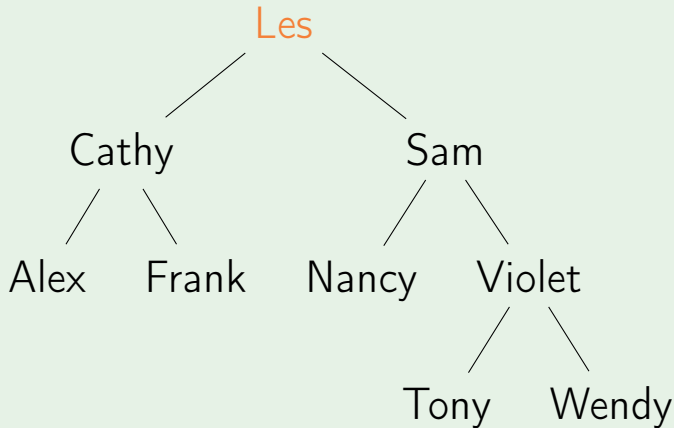
**Output:** Alex Cathy Frank

# InOrderTraversal



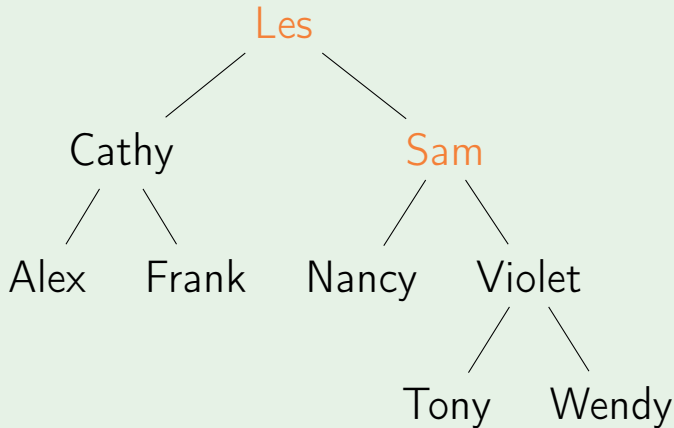
**Output:** Alex Cathy Frank Les

# InOrderTraversal



**Output:** Alex Cathy Frank Les

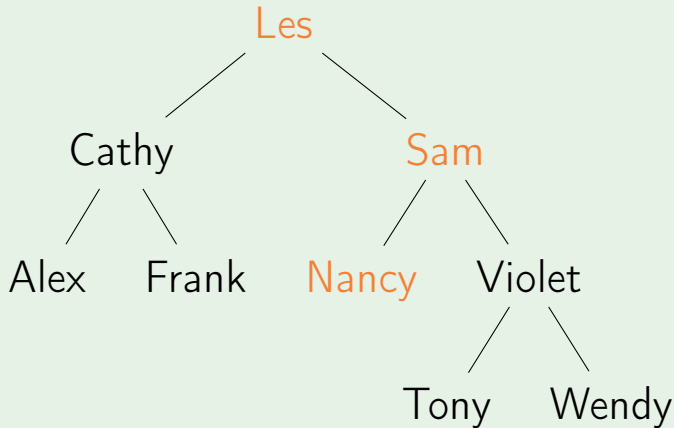
# InOrderTraversal



**Output:** Alex Cathy Frank Les



# InOrderTraversal



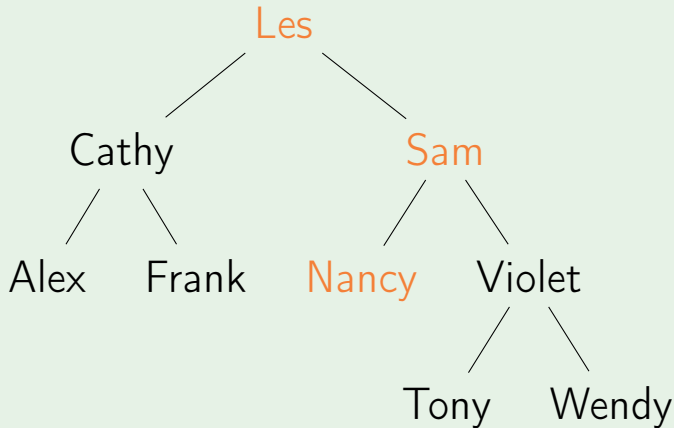
**Output:** Alex Cathy Frank Les

# InOrderTraversal



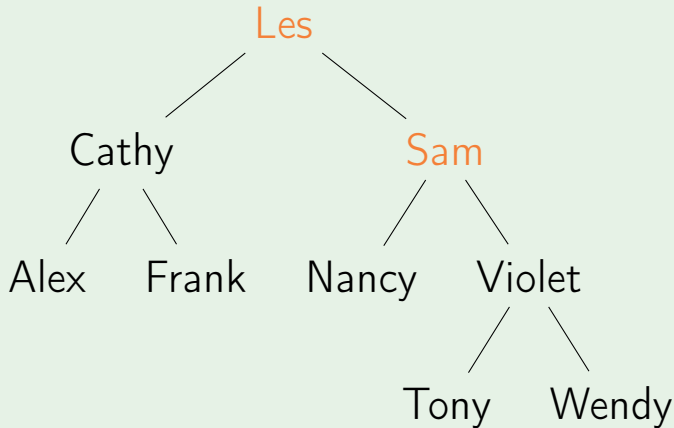
**Output:** Alex Cathy Frank Les Nancy

# InOrderTraversal



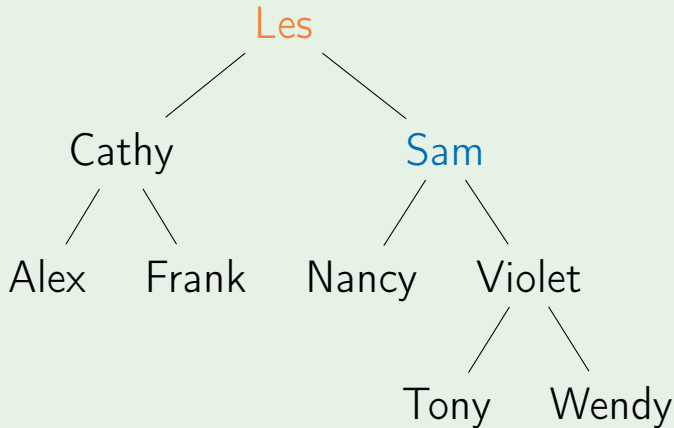
**Output:** Alex Cathy Frank Les Nancy

# InOrderTraversal



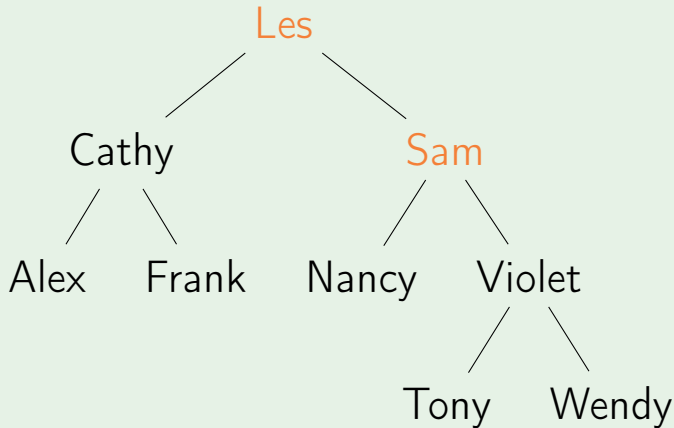
**Output:** Alex Cathy Frank Les Nancy

# InOrderTraversal



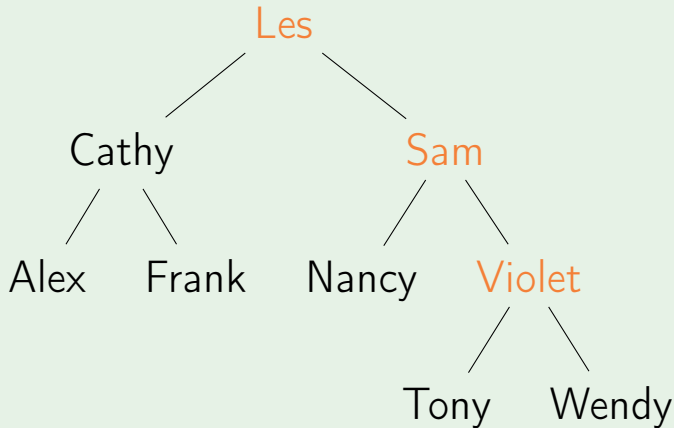
**Output:** Alex Cathy Frank Les Nancy Sam

# InOrderTraversal



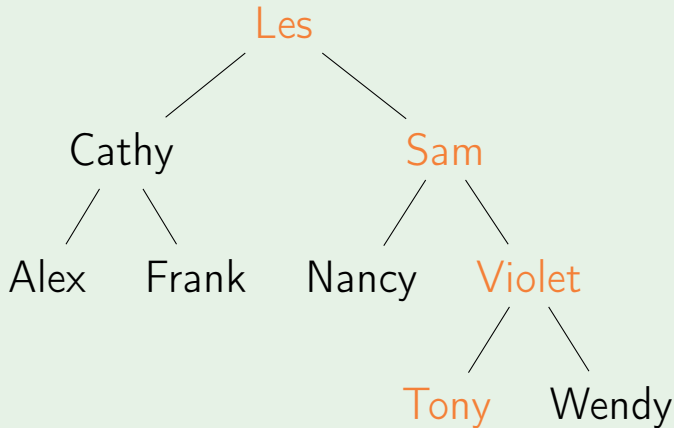
**Output:** Alex Cathy Frank Les Nancy Sam

# InOrderTraversal



**Output:** Alex Cathy Frank Les Nancy Sam

# InOrderTraversal



**Output:** Alex Cathy Frank Les Nancy Sam

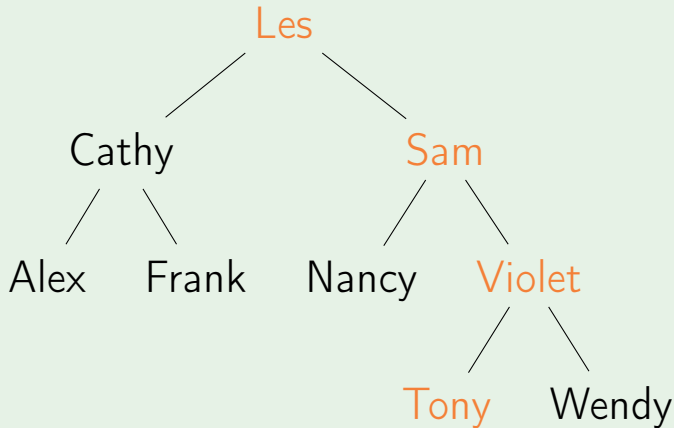


# InOrderTraversal



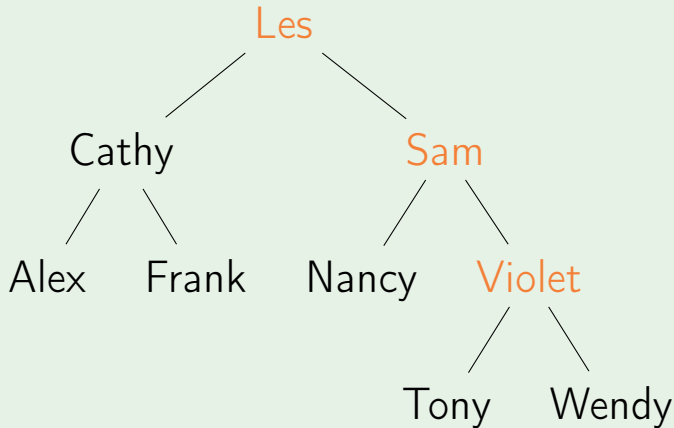
**Output:** Alex Cathy Frank Les Nancy Sam  
Tony

# InOrderTraversal



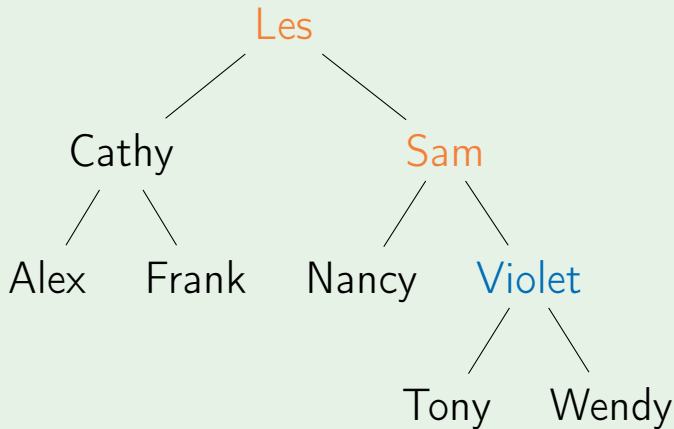
**Output:** Alex Cathy Frank Les Nancy Sam  
Tony

# InOrderTraversal



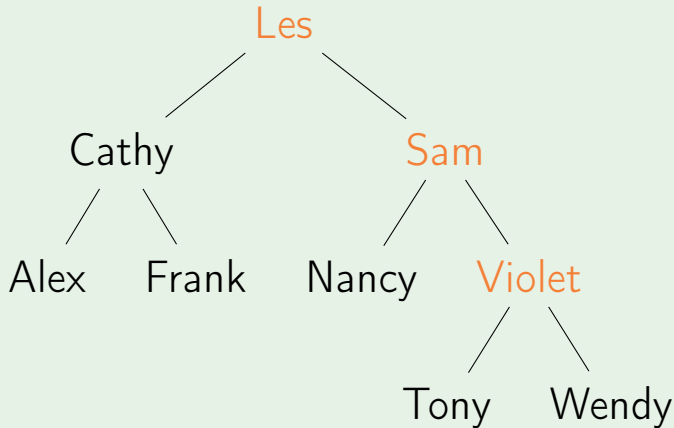
**Output:** Alex Cathy Frank Les Nancy Sam  
Tony

# InOrderTraversal



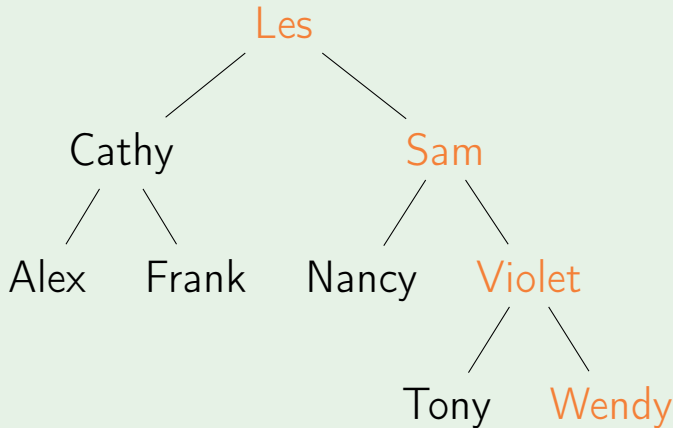
**Output:** Alex Cathy Frank Les Nancy Sam  
Tony Violet

# InOrderTraversal



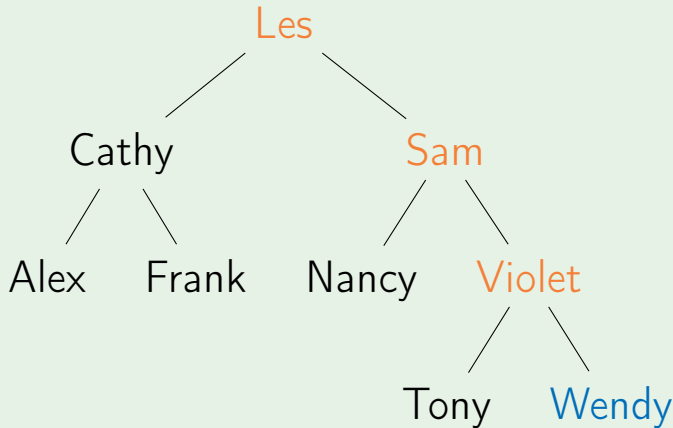
**Output:** Alex Cathy Frank Les Nancy Sam  
Tony Violet

# InOrderTraversal



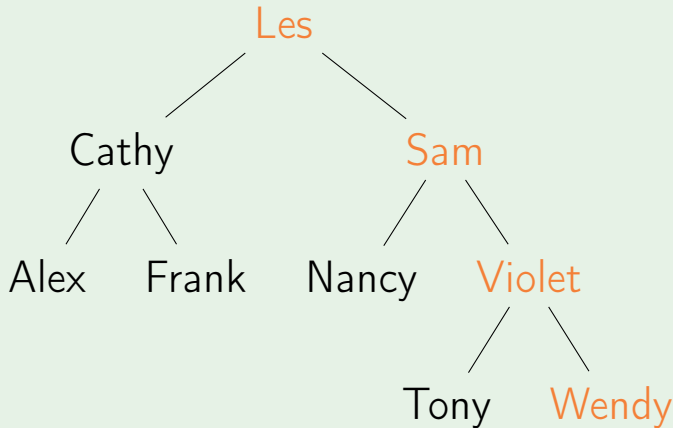
**Output:** Alex Cathy Frank Les Nancy Sam  
Tony Violet

# InOrderTraversal



**Output:** Alex Cathy Frank Les Nancy Sam  
Tony Violet Wendy

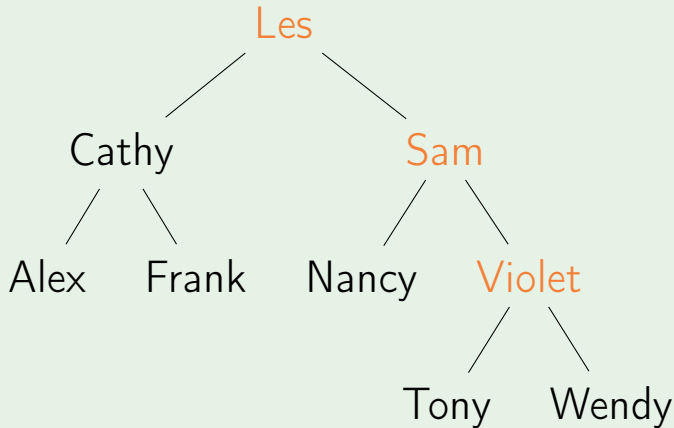
# InOrderTraversal



**Output:** Alex Cathy Frank Les Nancy Sam  
Tony Violet Wendy

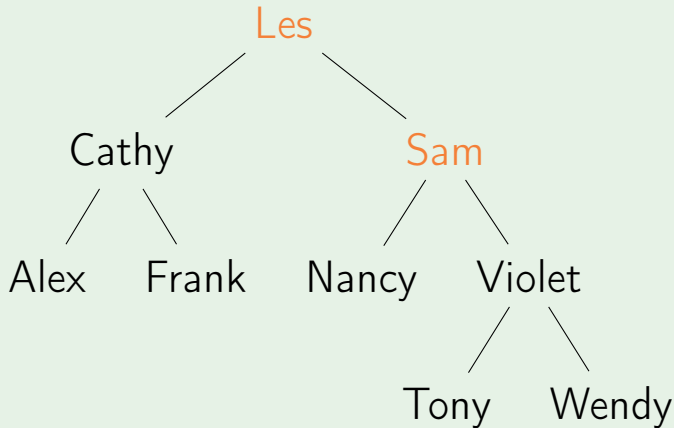


# InOrderTraversal



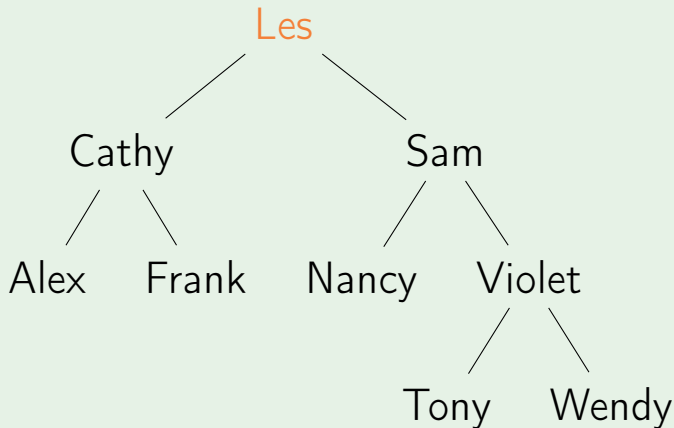
**Output:** Alex Cathy Frank Les Nancy Sam  
Tony Violet Wendy

# InOrderTraversal



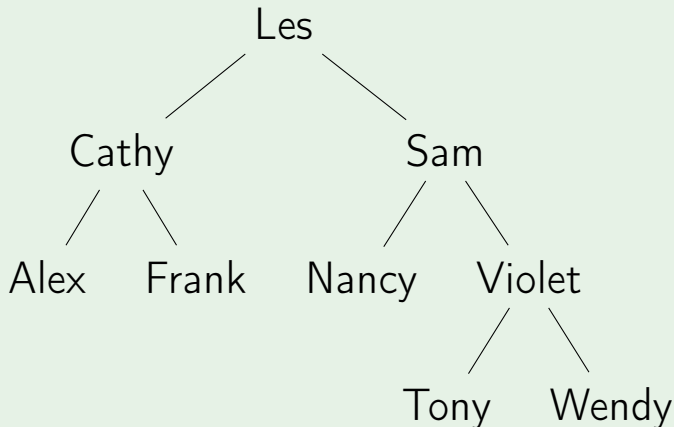
**Output:** Alex Cathy Frank Les Nancy Sam  
Tony Violet Wendy

# InOrderTraversal



**Output:** Alex Cathy Frank Les Nancy Sam  
Tony Violet Wendy

# InOrderTraversal



**Output:** Alex Cathy Frank Les Nancy Sam  
Tony Violet Wendy

# Depth-first

```
PreOrderTraversal(tree)
```

```
if tree = nil:
```

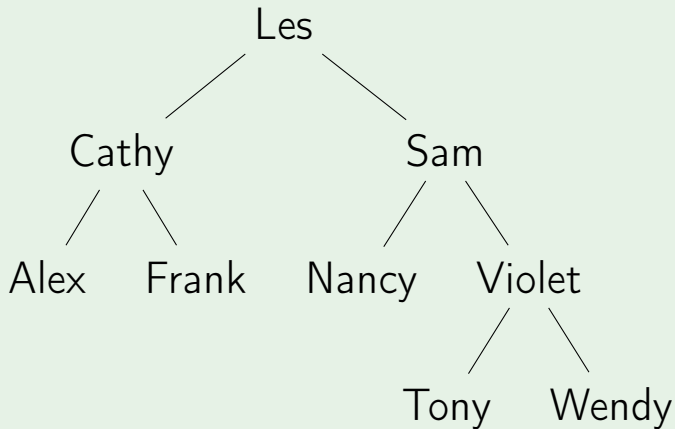
```
    return
```

```
Print(tree.key)
```

```
PreOrderTraversal(tree.left)
```

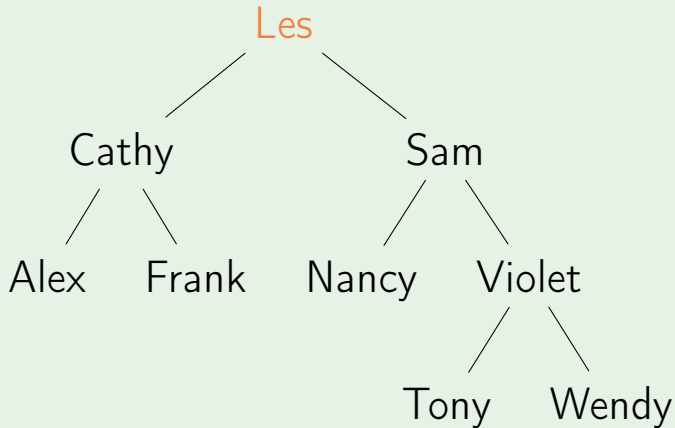
```
PreOrderTraversal(tree.right)
```

# PreOrderTraversal



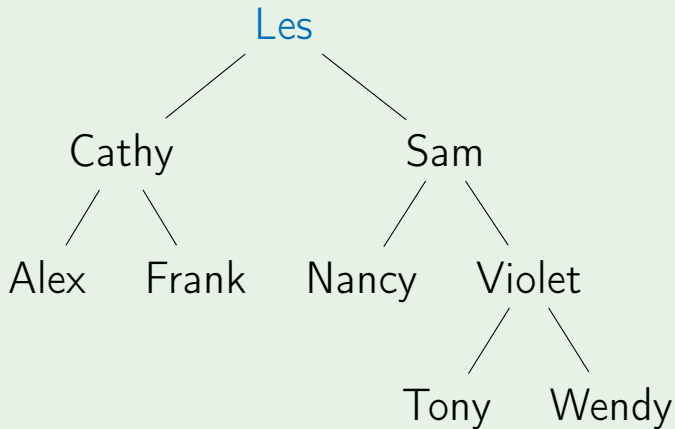
**Output:**

# PreOrderTraversal



**Output:**

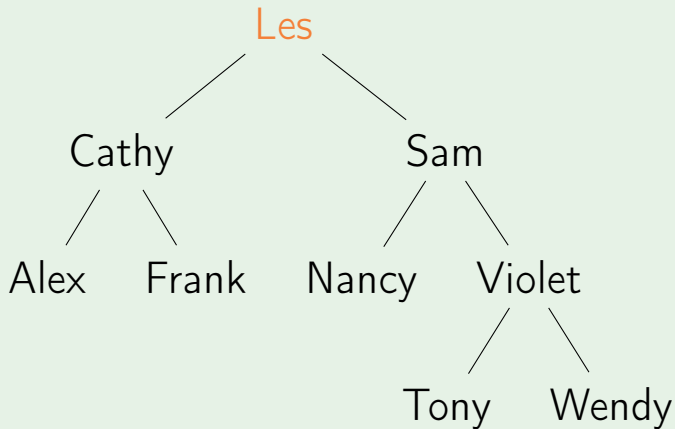
# PreOrderTraversal



**Output:** Les



# PreOrderTraversal



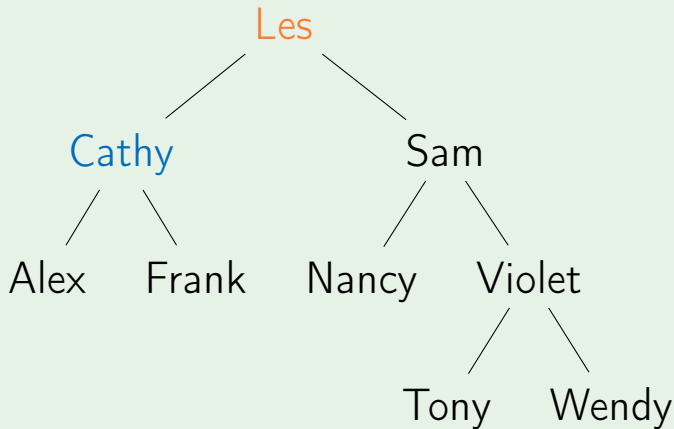
**Output:** Les

# PreOrderTraversal



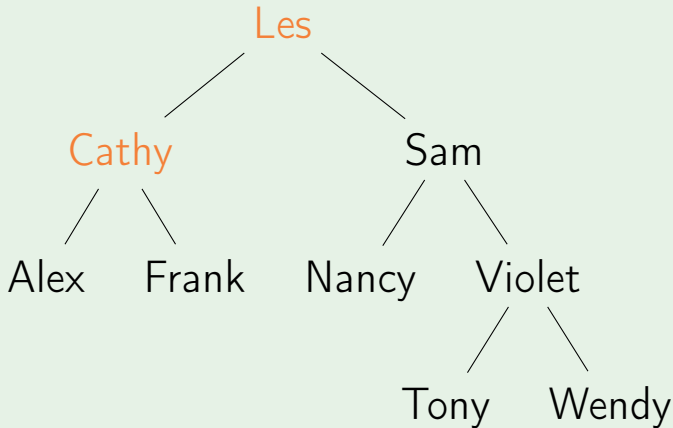
**Output:** Les

# PreOrderTraversal



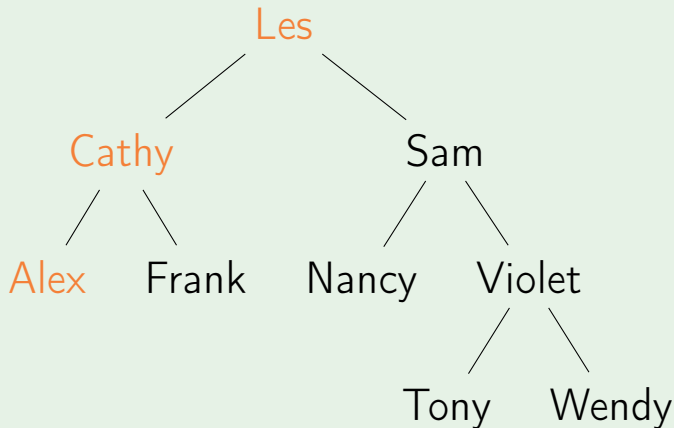
**Output:** Les Cathy

# PreOrderTraversal



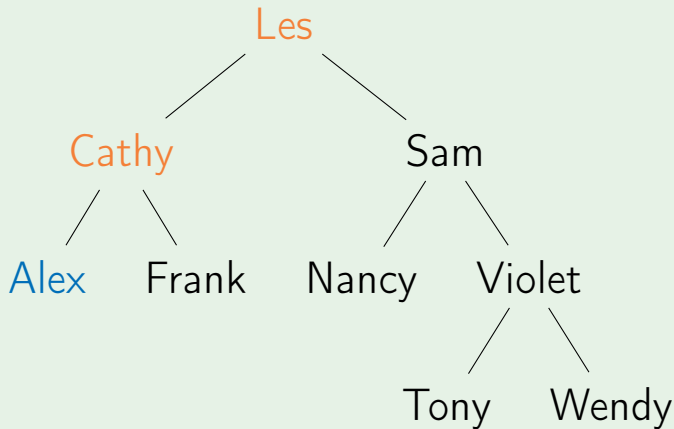
**Output:** Les Cathy

# PreOrderTraversal



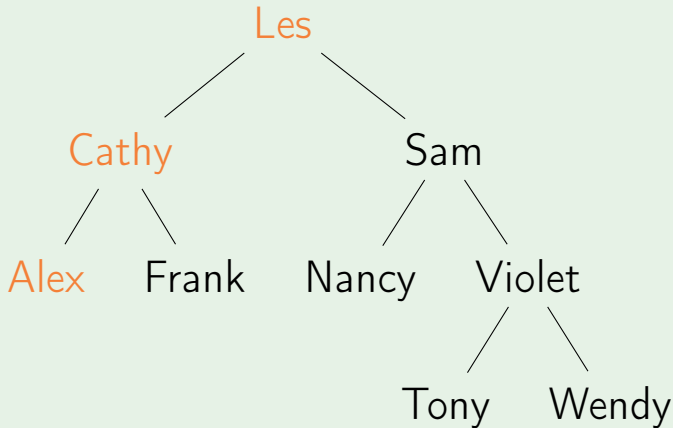
**Output:** Les Cathy

# PreOrderTraversal



**Output:** Les Cathy Alex

# PreOrderTraversal



**Output:** Les Cathy Alex

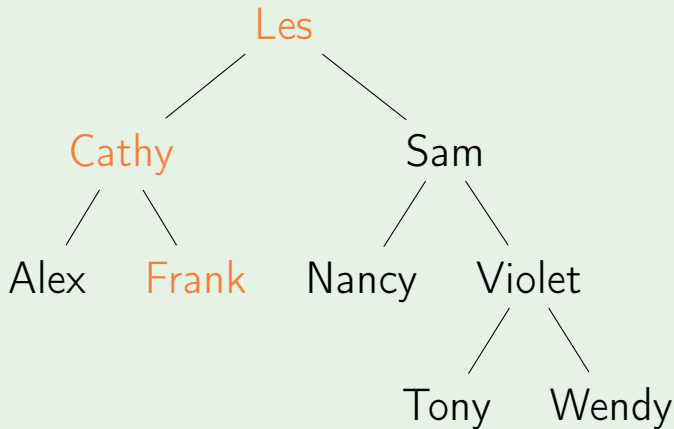
# PreOrderTraversal



**Output:** Les Cathy Alex

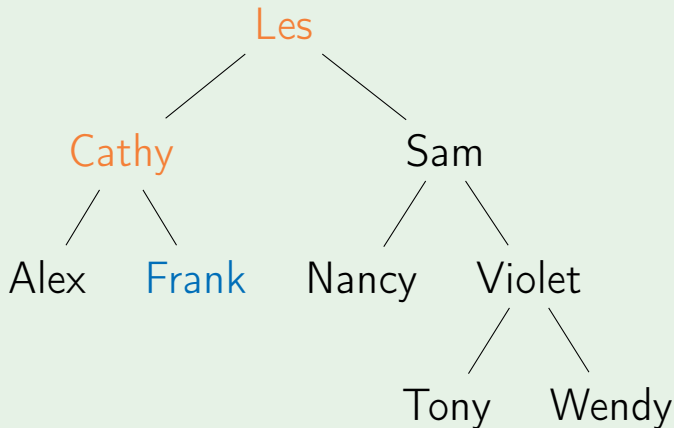


# PreOrderTraversal



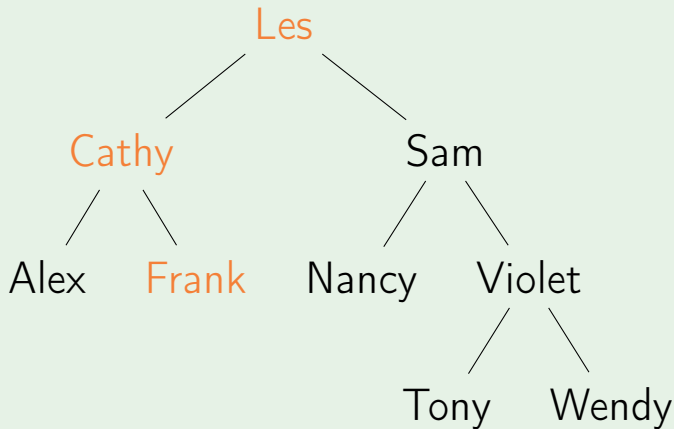
**Output:** Les Cathy Alex

# PreOrderTraversal



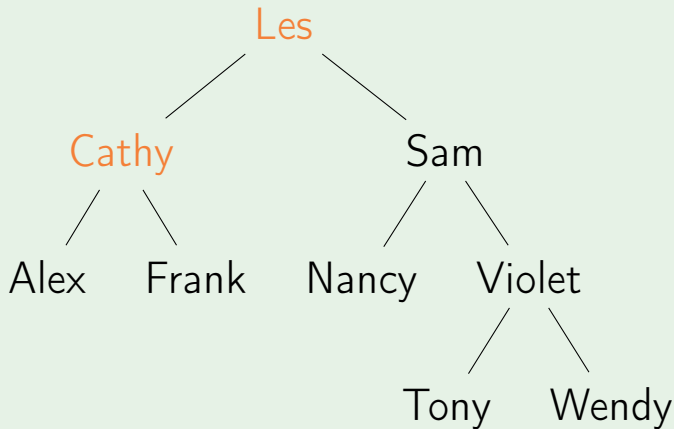
**Output:** Les Cathy Alex Frank

# PreOrderTraversal



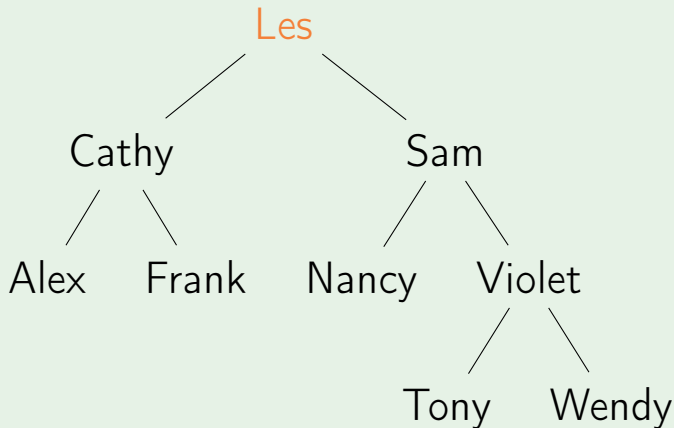
**Output:** Les Cathy Alex Frank

# PreOrderTraversal



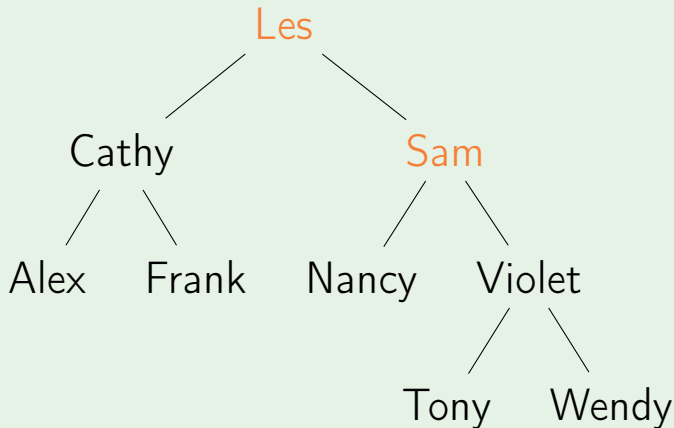
**Output:** Les Cathy Alex Frank

# PreOrderTraversal



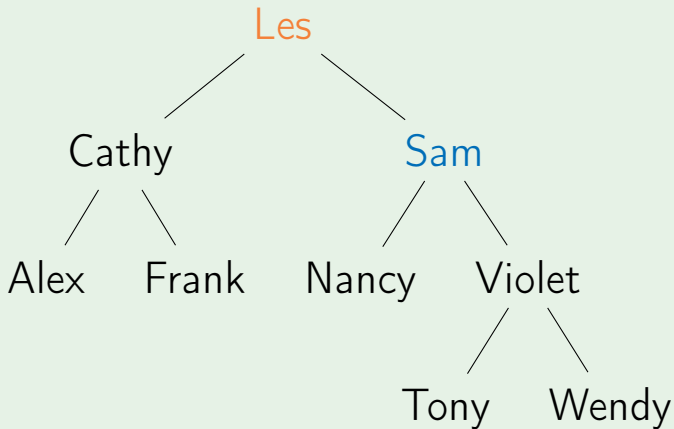
**Output:** Les Cathy Alex Frank

# PreOrderTraversal



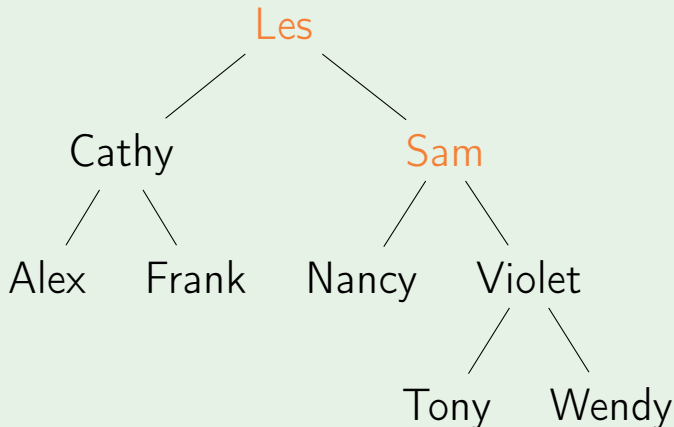
**Output:** Les Cathy Alex Frank

# PreOrderTraversal



**Output:** Les Cathy Alex Frank Sam

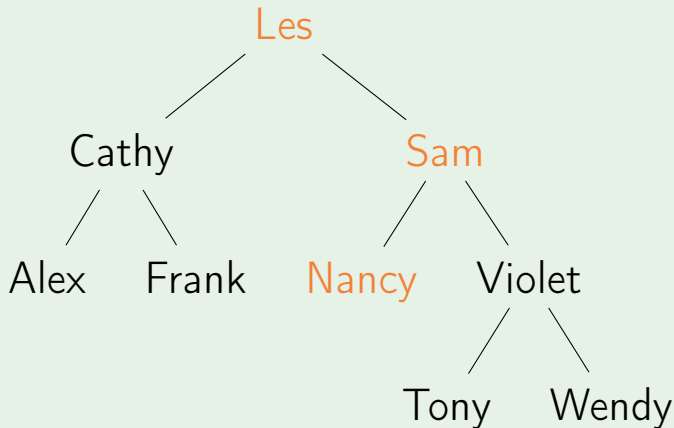
# PreOrderTraversal



**Output:** Les Cathy Alex Frank Sam



# PreOrderTraversal



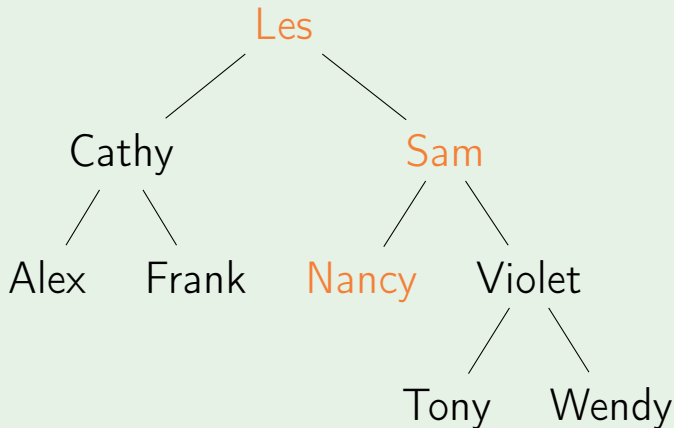
**Output:** Les Cathy Alex Frank Sam

# PreOrderTraversal



**Output:** Les Cathy Alex Frank Sam Nancy

# PreOrderTraversal



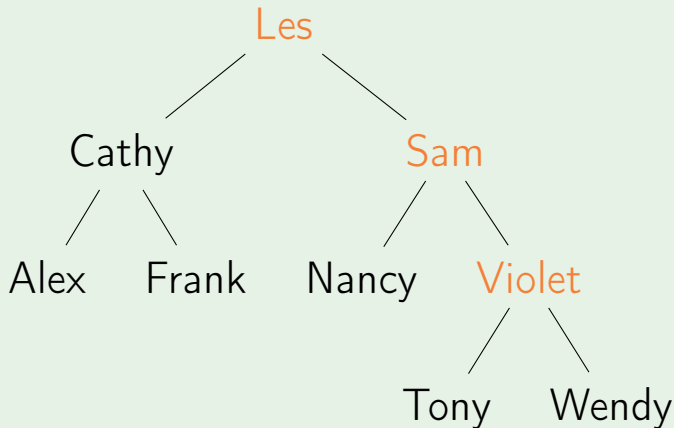
**Output:** Les Cathy Alex Frank Sam Nancy

# PreOrderTraversal



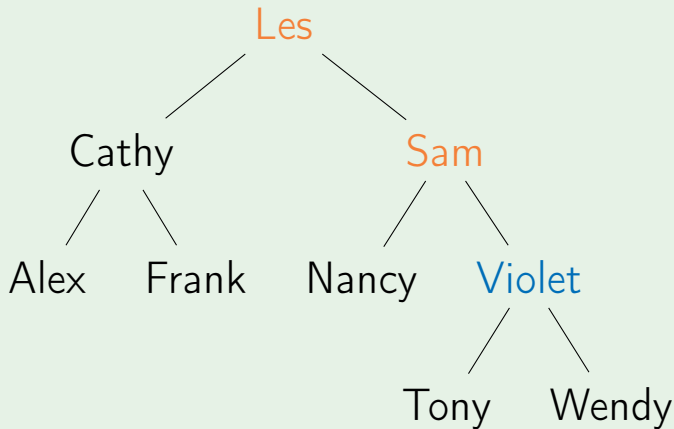
**Output:** Les Cathy Alex Frank Sam Nancy

# PreOrderTraversal



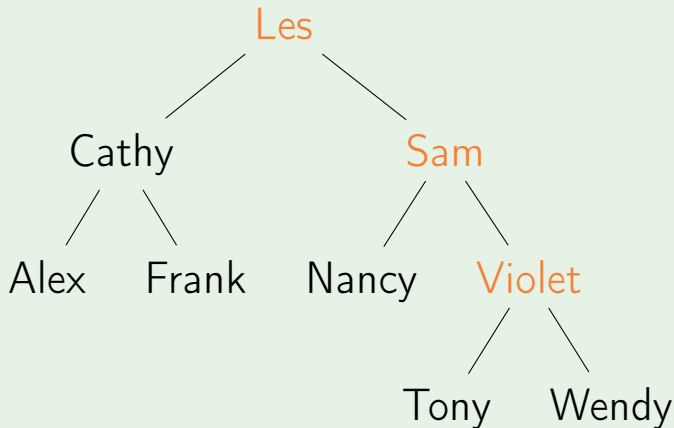
**Output:** Les Cathy Alex Frank Sam Nancy

# PreOrderTraversal



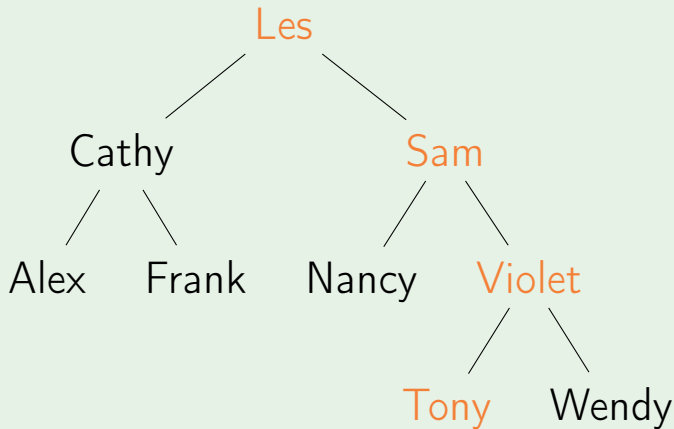
**Output:** Les Cathy Alex Frank Sam Nancy  
Violet

# PreOrderTraversal



**Output:** Les Cathy Alex Frank Sam Nancy  
Violet

# PreOrderTraversal



**Output:** Les Cathy Alex Frank Sam Nancy  
Violet

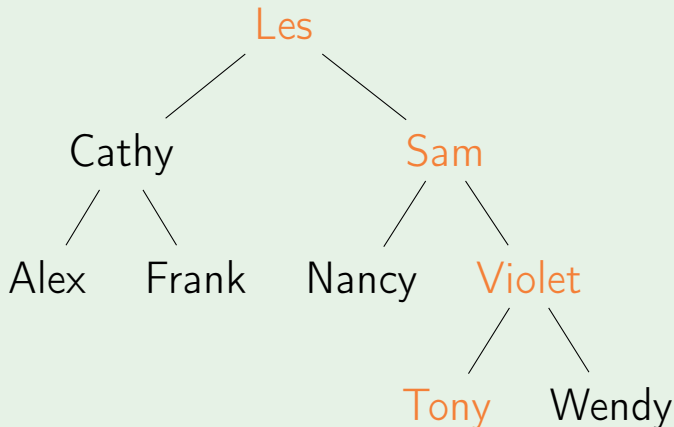


# PreOrderTraversal



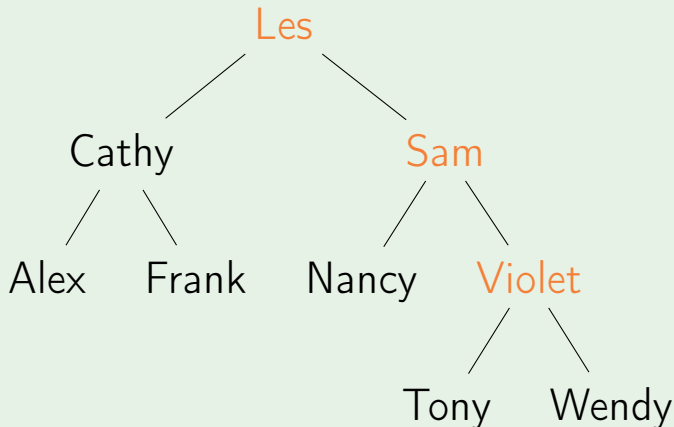
**Output:** Les Cathy Alex Frank Sam Nancy  
Violet Tony

# PreOrderTraversal



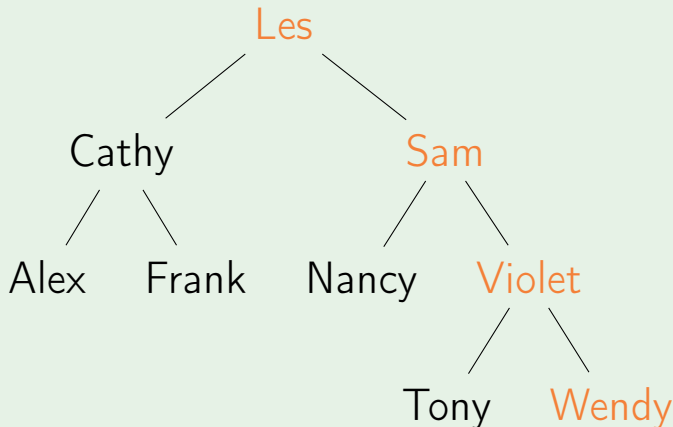
**Output:** Les Cathy Alex Frank Sam Nancy  
Violet Tony

# PreOrderTraversal



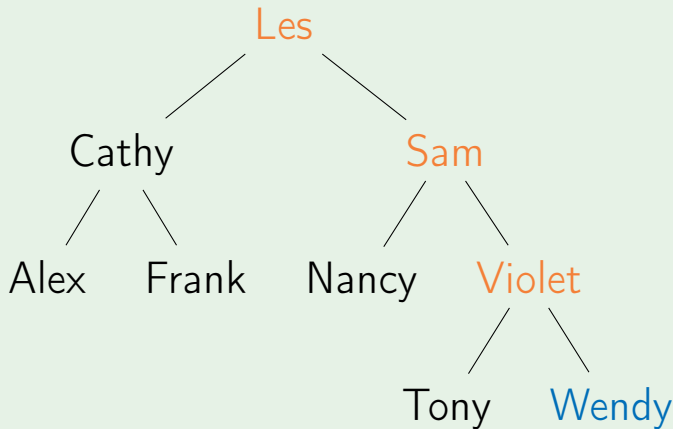
**Output:** Les Cathy Alex Frank Sam Nancy  
Violet Tony

# PreOrderTraversal



**Output:** Les Cathy Alex Frank Sam Nancy  
Violet Tony

# PreOrderTraversal



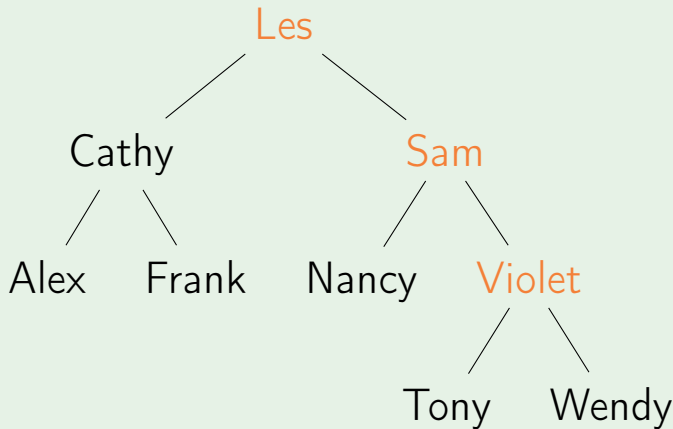
**Output:** Les Cathy Alex Frank Sam Nancy  
Violet Tony Wendy

# PreOrderTraversal



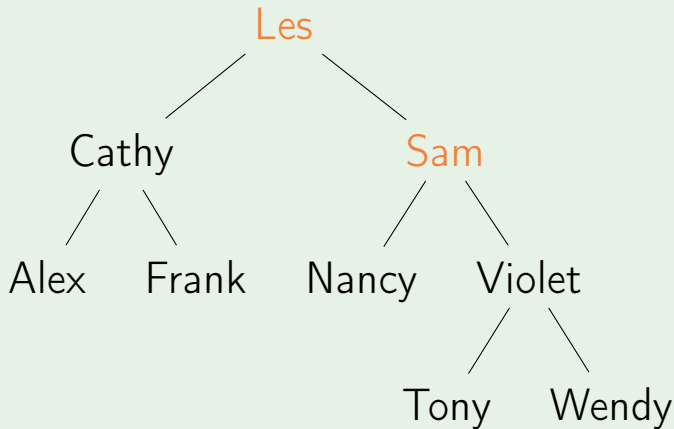
**Output:** Les Cathy Alex Frank Sam Nancy  
Violet Tony Wendy

# PreOrderTraversal



**Output:** Les Cathy Alex Frank Sam Nancy  
Violet Tony Wendy

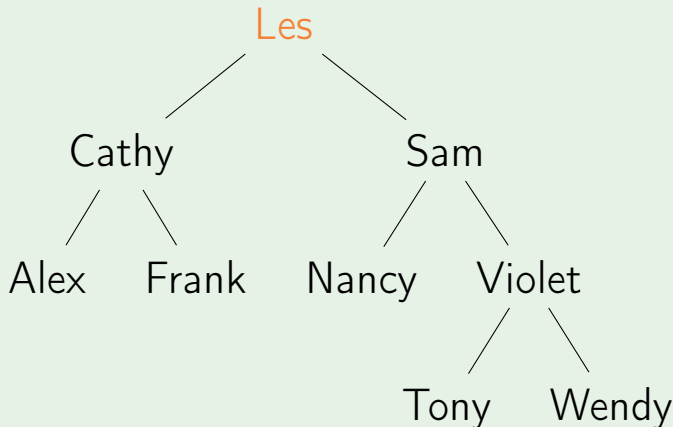
# PreOrderTraversal



**Output:** Les Cathy Alex Frank Sam Nancy  
Violet Tony Wendy

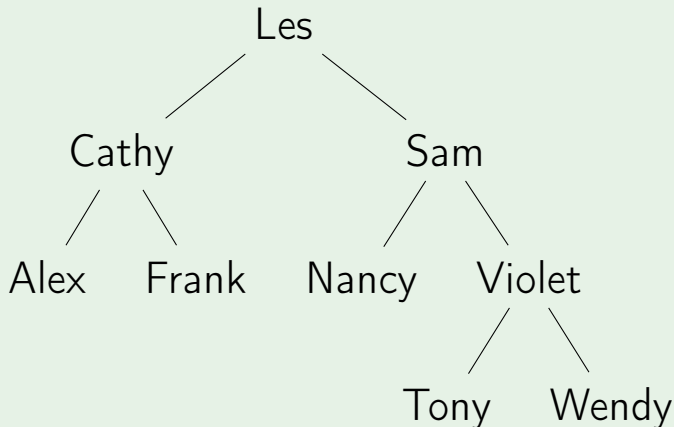


# PreOrderTraversal



**Output:** Les Cathy Alex Frank Sam Nancy  
Violet Tony Wendy

# PreOrderTraversal



**Output:** Les Cathy Alex Frank Sam Nancy  
Violet Tony Wendy

# Depth-first

PostOrderTraversal(*tree*)

```
if tree = nil:
```

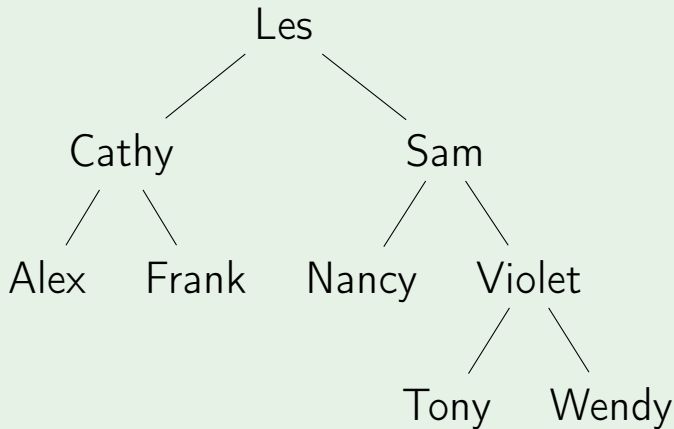
```
    return
```

```
PostOrderTraversal(tree.left)
```

```
PostOrderTraversal(tree.right)
```

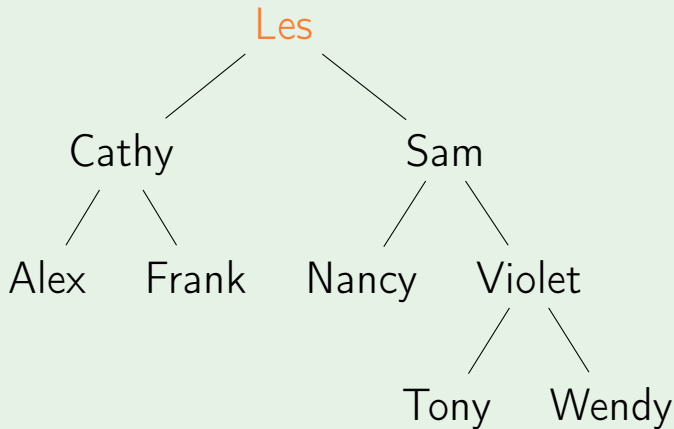
```
Print(tree.key)
```

# PostOrderTraversal



**Output:**

# PostOrderTraversal



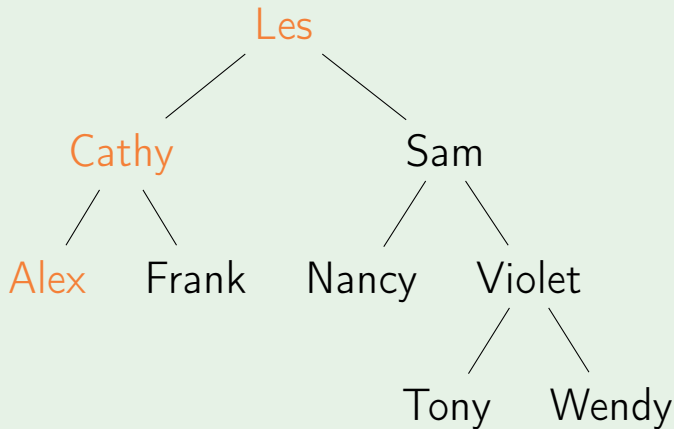
**Output:**

# PostOrderTraversal



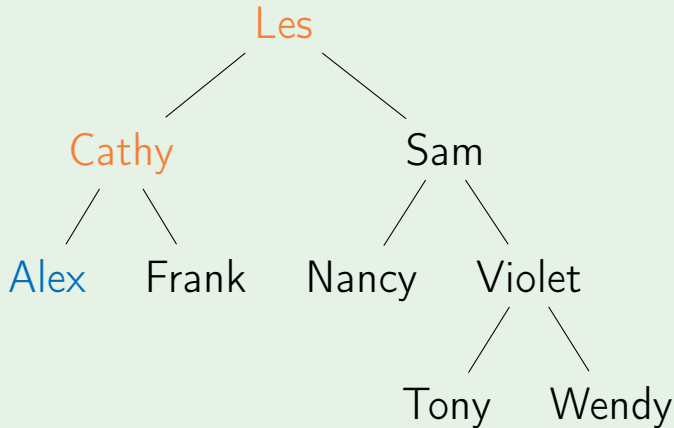
**Output:**

# PostOrderTraversal



**Output:**

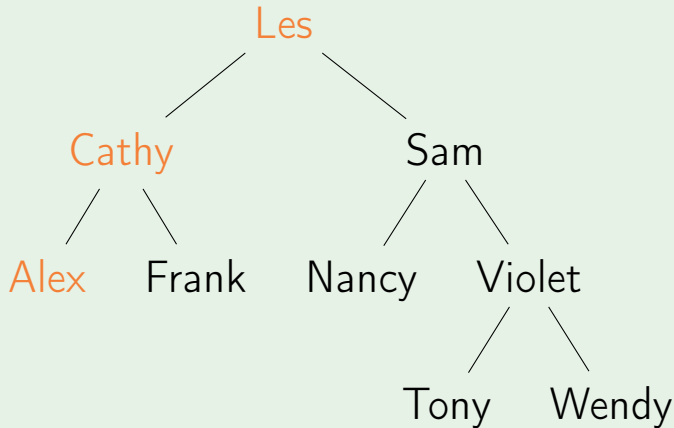
# PostOrderTraversal



**Output:** Alex



# PostOrderTraversal



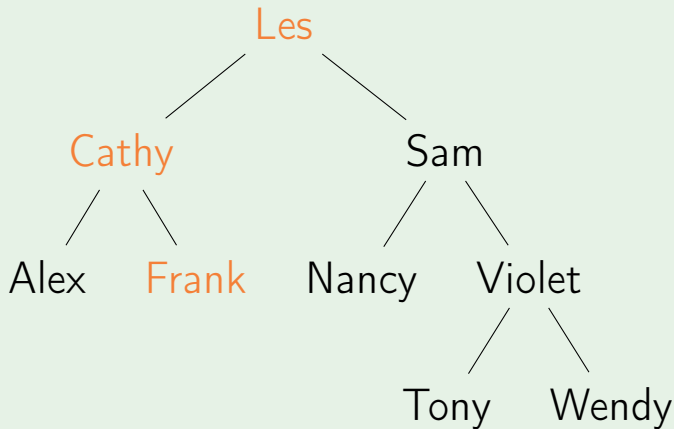
**Output:** Alex

# PostOrderTraversal



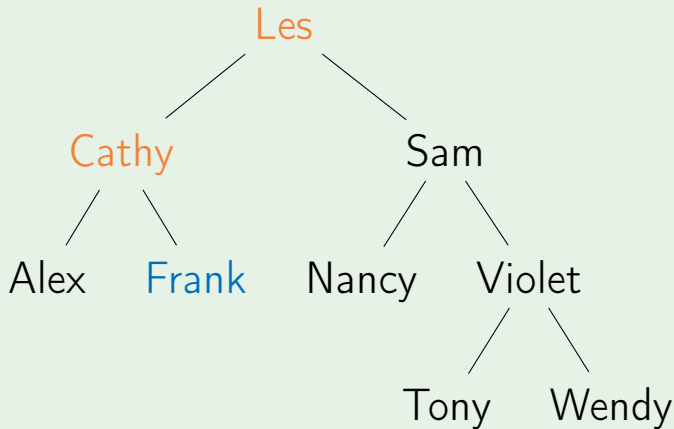
**Output:** Alex

# PostOrderTraversal



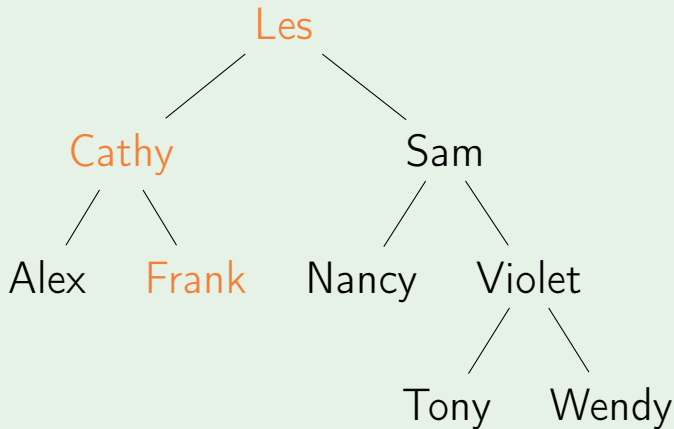
**Output:** Alex

# PostOrderTraversal



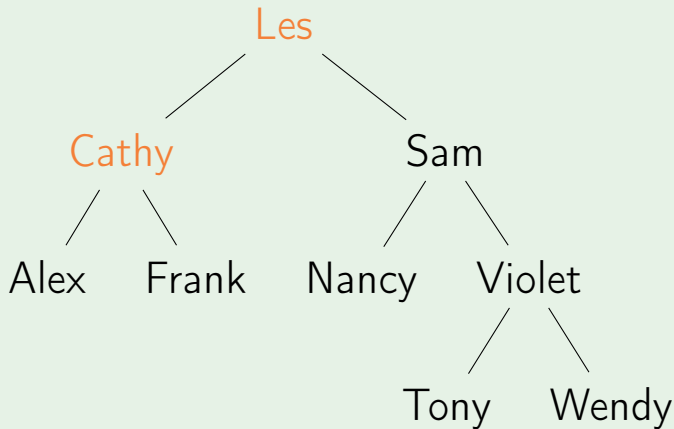
**Output:** Alex Frank

# PostOrderTraversal



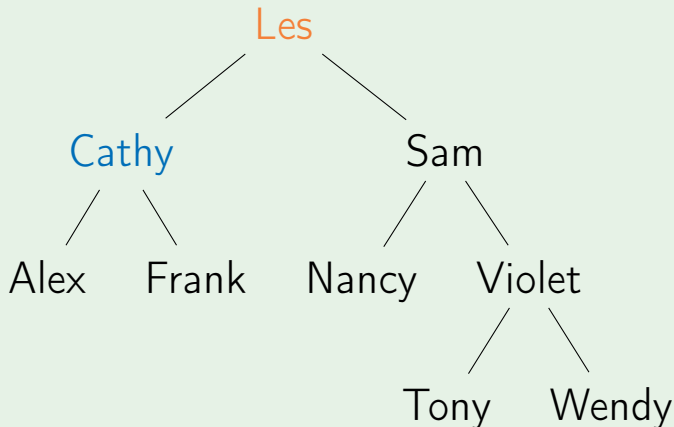
**Output:** Alex Frank

# PostOrderTraversal



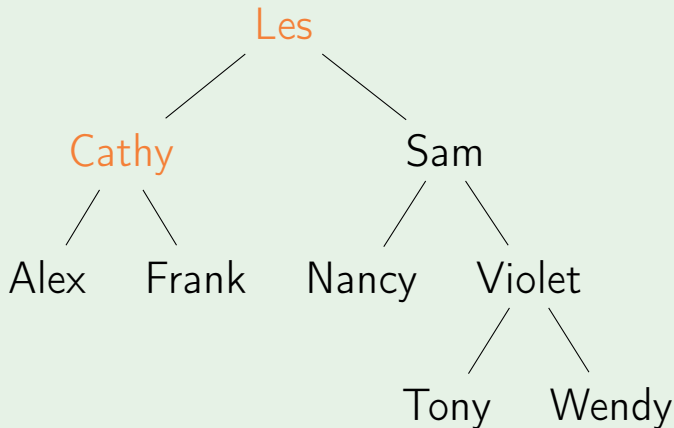
**Output:** Alex Frank

# PostOrderTraversal



**Output:** Alex Frank Cathy

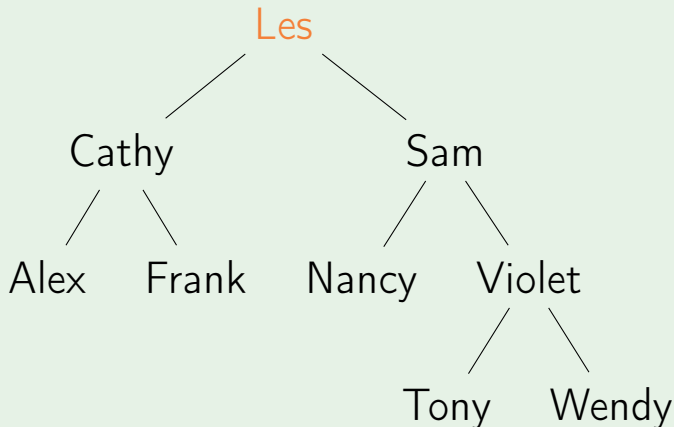
# PostOrderTraversal



**Output:** Alex Frank Cathy

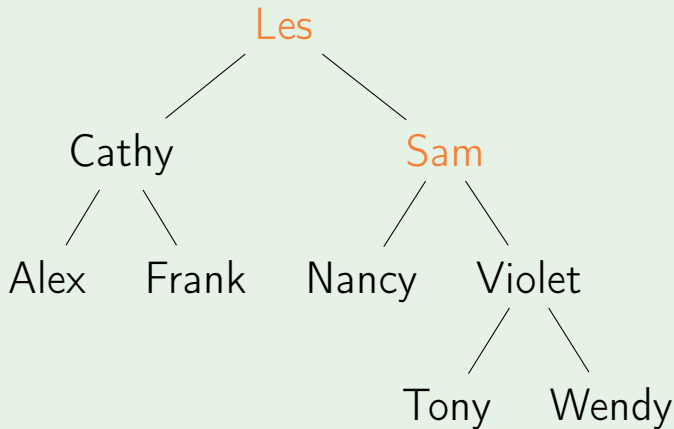


# PostOrderTraversal



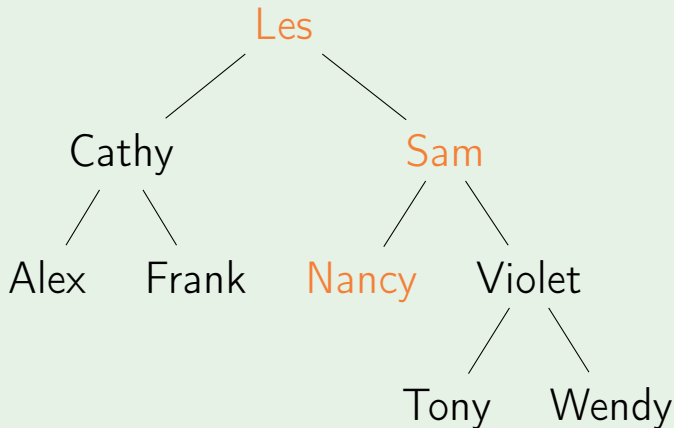
**Output:** Alex Frank Cathy

# PostOrderTraversal



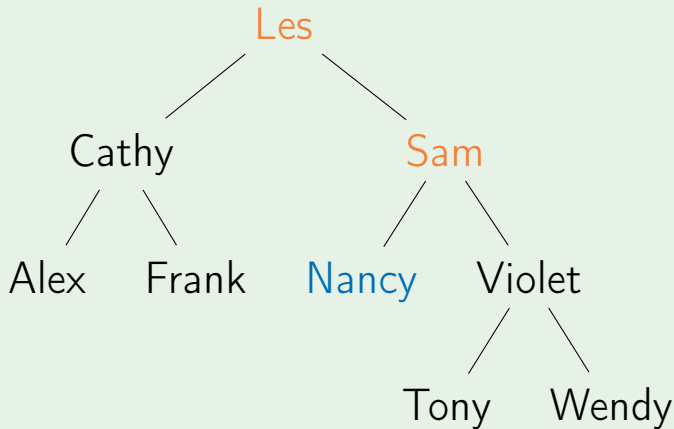
**Output:** Alex Frank Cathy

# PostOrderTraversal



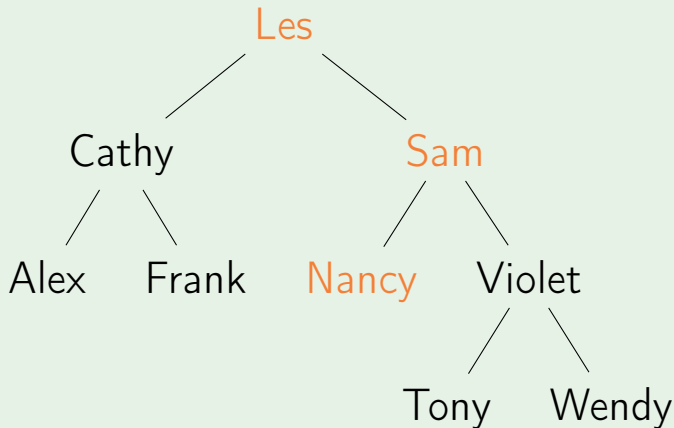
**Output:** Alex Frank Cathy

# PostOrderTraversal



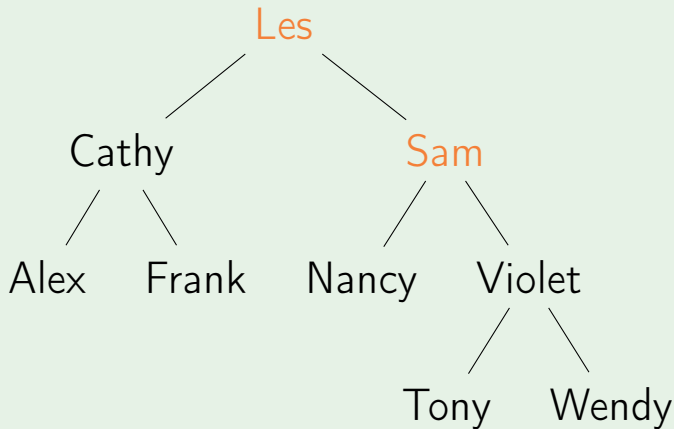
**Output:** Alex Frank Cathy Nancy

# PostOrderTraversal



**Output:** Alex Frank Cathy Nancy

# PostOrderTraversal



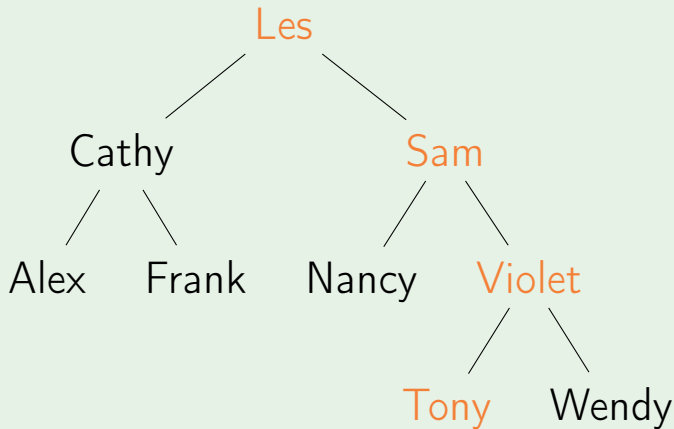
**Output:** Alex Frank Cathy Nancy

# PostOrderTraversal



**Output:** Alex Frank Cathy Nancy

# PostOrderTraversal



**Output:** Alex Frank Cathy Nancy

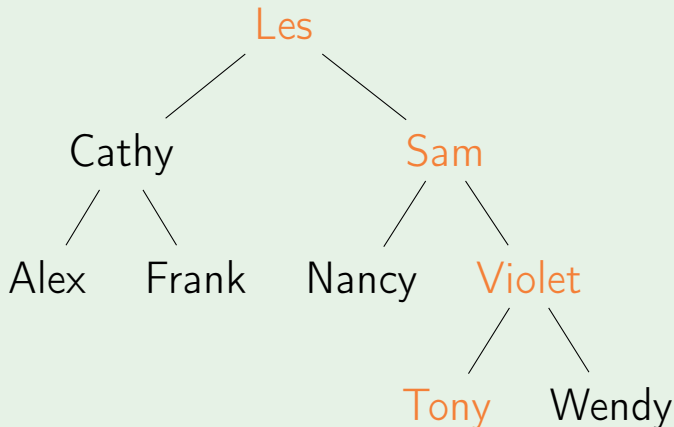


# PostOrderTraversal



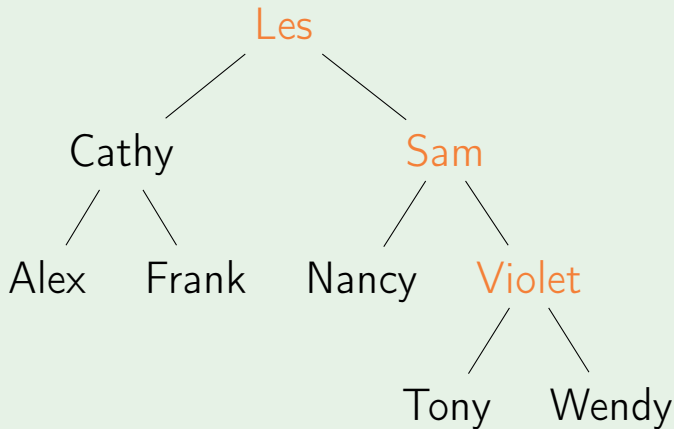
**Output:** Alex Frank Cathy Nancy Tony

# PostOrderTraversal



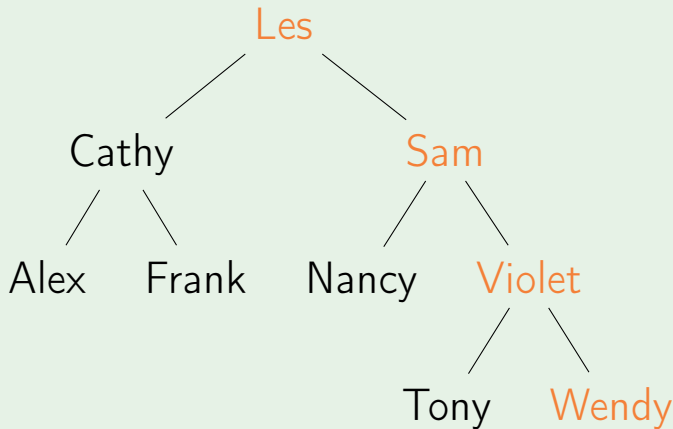
**Output:** Alex Frank Cathy Nancy Tony

# PostOrderTraversal



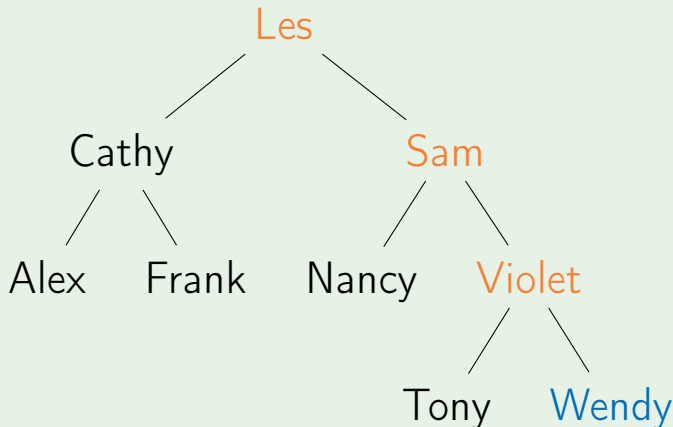
**Output:** Alex Frank Cathy Nancy Tony

# PostOrderTraversal



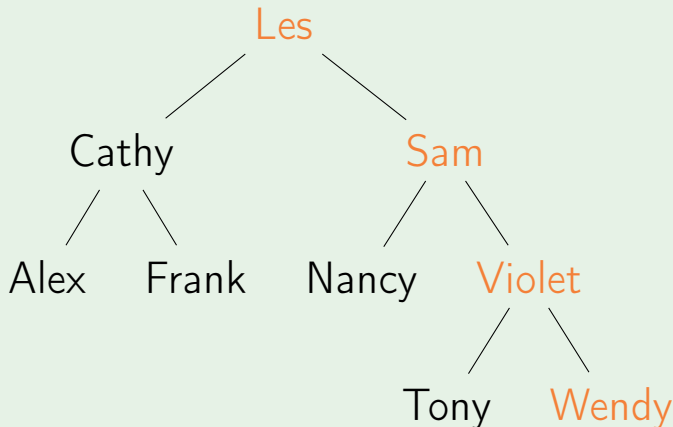
**Output:** Alex Frank Cathy Nancy Tony

# PostOrderTraversal



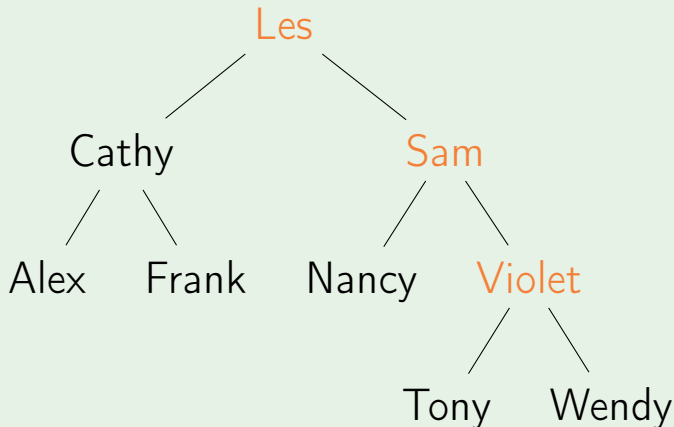
**Output:** Alex Frank Cathy Nancy Tony  
Wendy

# PostOrderTraversal



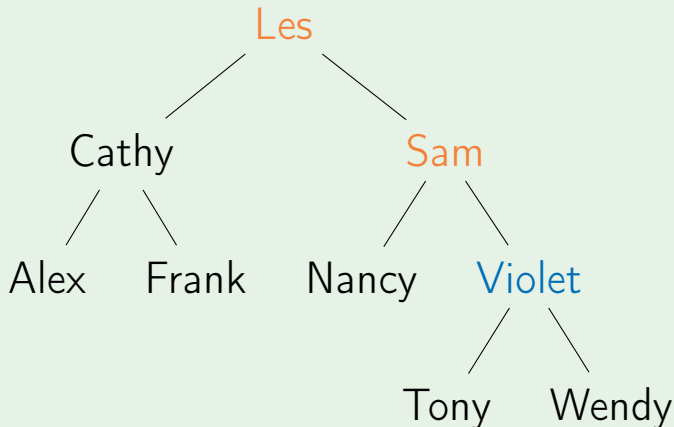
**Output:** Alex Frank Cathy Nancy Tony  
Wendy

# PostOrderTraversal



**Output:** Alex Frank Cathy Nancy Tony  
Wendy

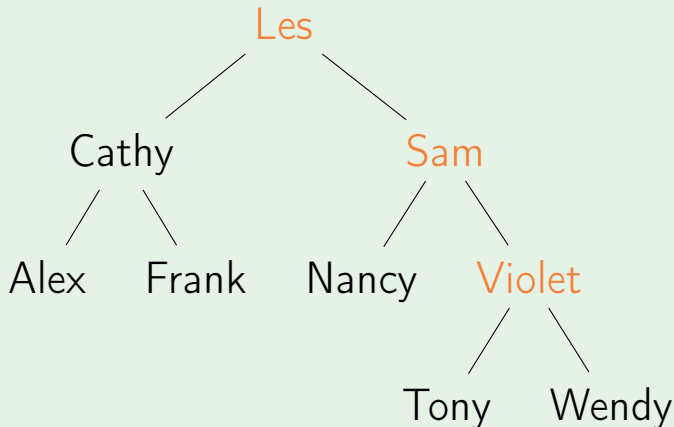
# PostOrderTraversal



**Output:** Alex Frank Cathy Nancy Tony  
Wendy Violet

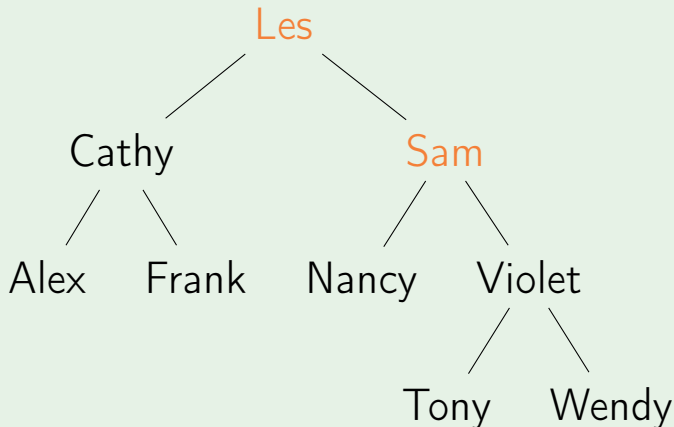


# PostOrderTraversal



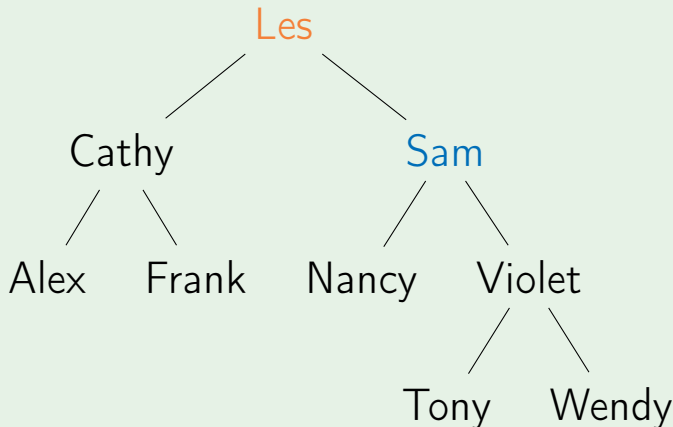
**Output:** Alex Frank Cathy Nancy Tony  
Wendy Violet

# PostOrderTraversal



**Output:** Alex Frank Cathy Nancy Tony  
Wendy Violet

# PostOrderTraversal



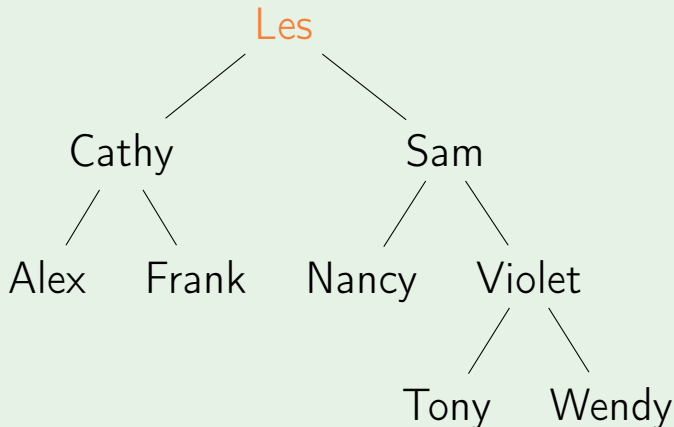
**Output:** Alex Frank Cathy Nancy Tony  
Wendy Violet Sam

# PostOrderTraversal



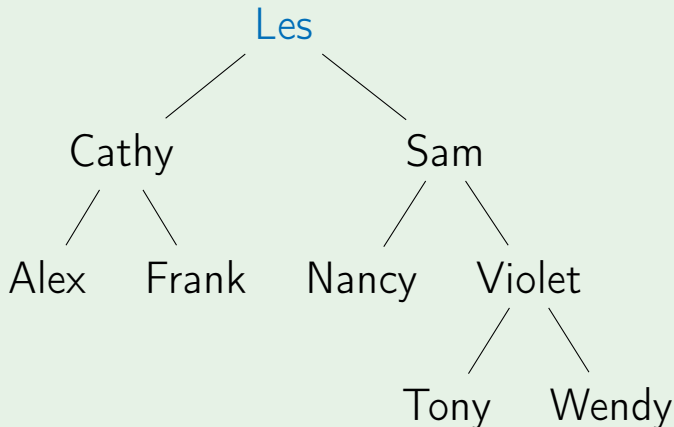
**Output:** Alex Frank Cathy Nancy Tony  
Wendy Violet Sam

# PostOrderTraversal



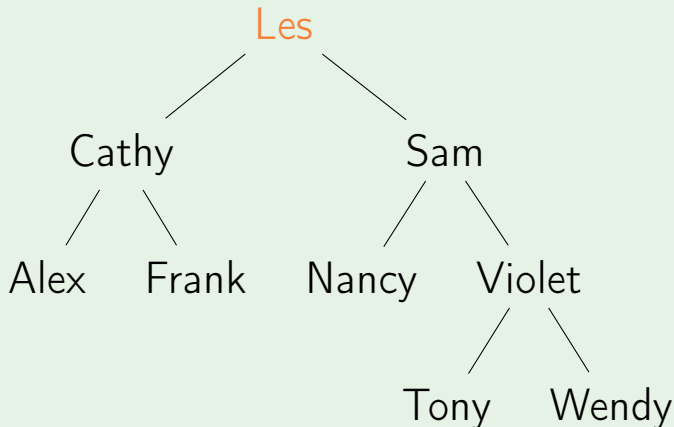
**Output:** Alex Frank Cathy Nancy Tony  
Wendy Violet Sam

# PostOrderTraversal



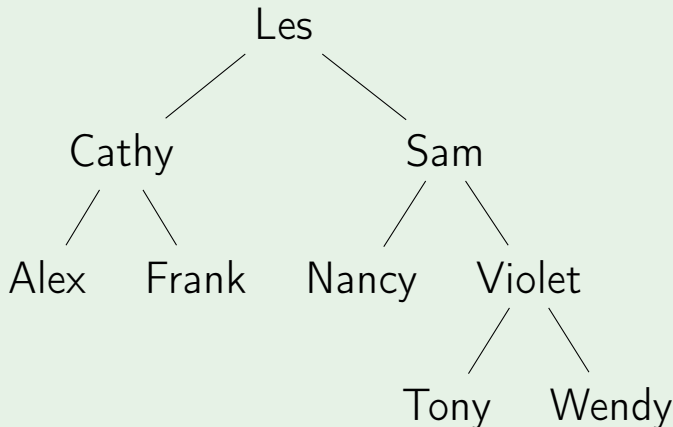
**Output:** Alex Frank Cathy Nancy Tony  
Wendy Violet Sam Les

# PostOrderTraversal



**Output:** Alex Frank Cathy Nancy Tony  
Wendy Violet Sam Les

# PostOrderTraversal



**Output:** Alex Frank Cathy Nancy Tony  
Wendy Violet Sam Les



# Breadth-first

## LevelTraversal(*tree*)

```
if tree = nil:    return
```

```
    Queue q
```

```
    q.Enqueue(tree)
```

# Breadth-first

## LevelTraversal(*tree*)

```
if tree = nil:    return  
  
Queue q  
q.Enqueue(tree)  
while not q.Empty():  
    node  $\leftarrow$  q.Dequeue()
```

# Breadth-first

## LevelTraversal(*tree*)

```
if tree = nil:    return

Queue q
q.Enqueue(tree)
while not q.Empty():
    node ← q.Dequeue()
    Print(node)
```

# Breadth-first

## LevelTraversal(*tree*)

```
if tree = nil:    return

Queue q
q.Enqueue(tree)
while not q.Empty():
    node  $\leftarrow$  q.Dequeue()
    Print(node)
    if node.left  $\neq$  nil:
        q.Enqueue(node.left)
```

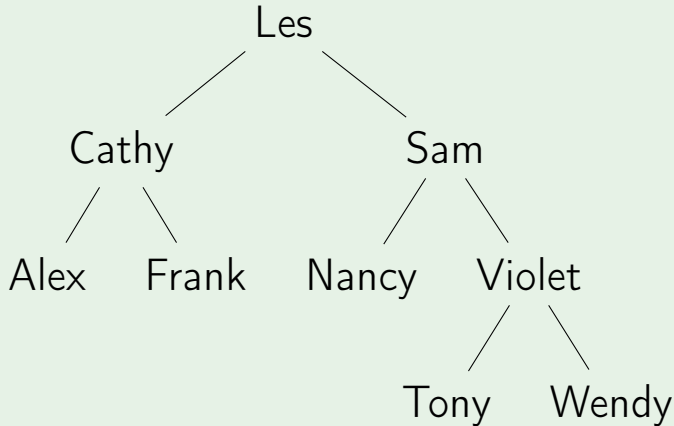
# Breadth-first

## LevelTraversal(*tree*)

```
if tree = nil:    return

Queue q
q.Enqueue(tree)
while not q.Empty():
    node  $\leftarrow$  q.Dequeue()
    Print(node)
    if node.left  $\neq$  nil:
        q.Enqueue(node.left)
    if node.right  $\neq$  nil:
        q.Enqueue(node.right)
```

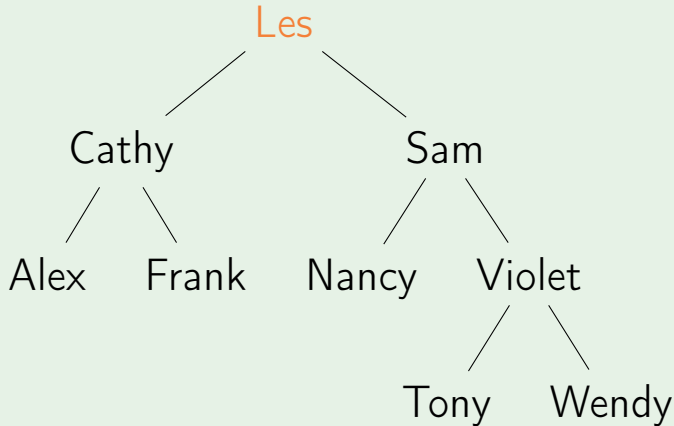
# LevelTraversal



**Output:**

**Queue:** Les

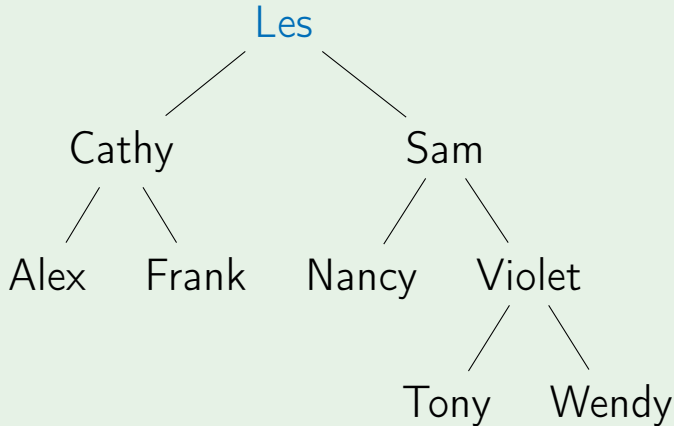
# LevelTraversal



**Output:**

**Queue:**

# LevelTraversal

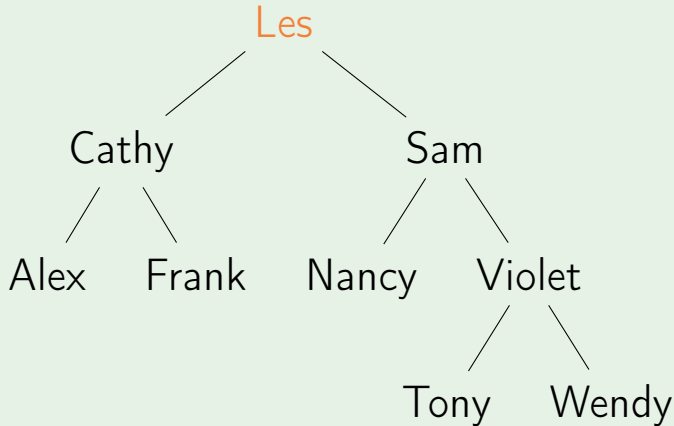


**Output:** Les

**Queue:**



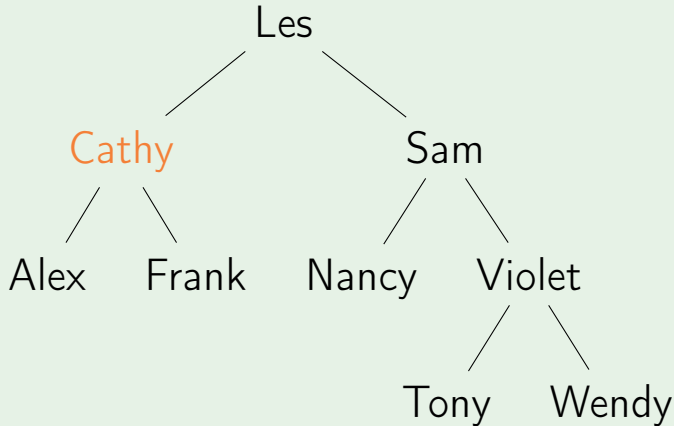
# LevelTraversal



**Output:** Les

**Queue:** Cathy, Sam

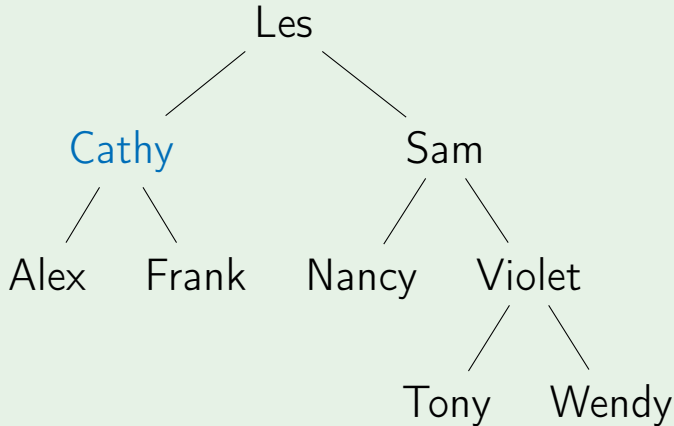
# LevelTraversal



**Output:** Les

**Queue:** Sam

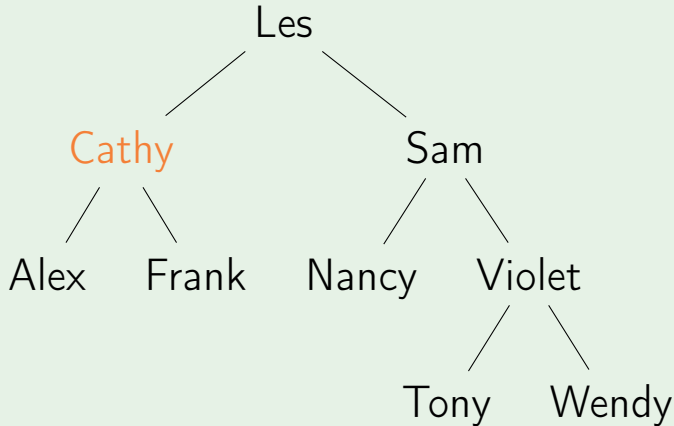
# LevelTraversal



**Output:** Les Cathy

**Queue:** Sam

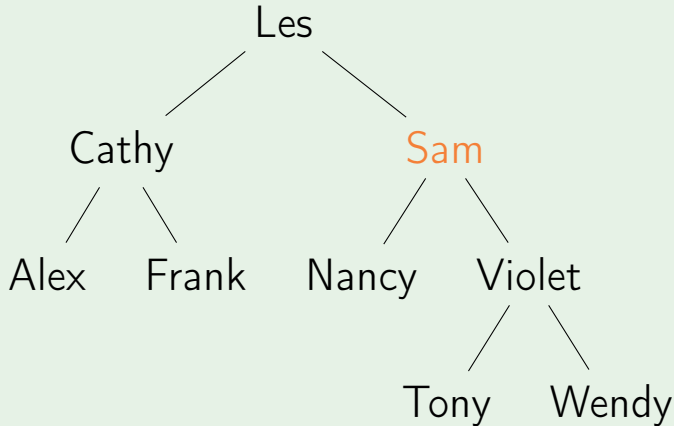
# LevelTraversal



**Output:** Les Cathy

**Queue:** Sam, Alex, Frank

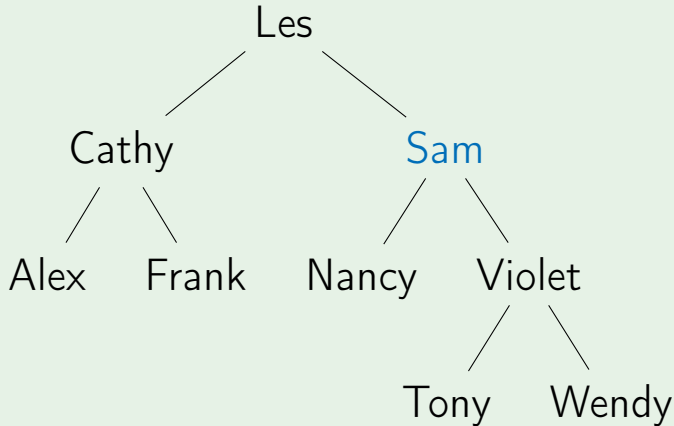
# LevelTraversal



**Output:** Les Cathy

**Queue:** Alex, Frank

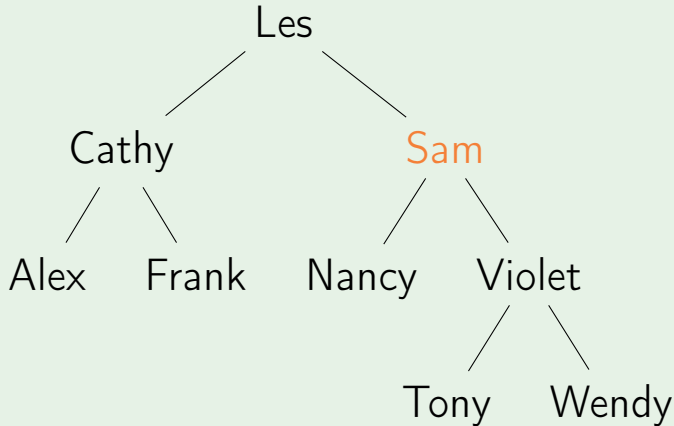
# LevelTraversal



**Output:** Les Cathy Sam

**Queue:** Alex, Frank

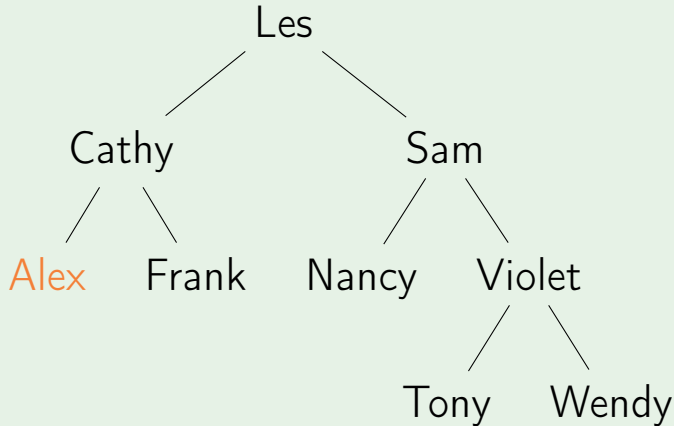
# LevelTraversal



**Output:** Les Cathy Sam

**Queue:** Alex, Frank, Nancy, Violet

# LevelTraversal

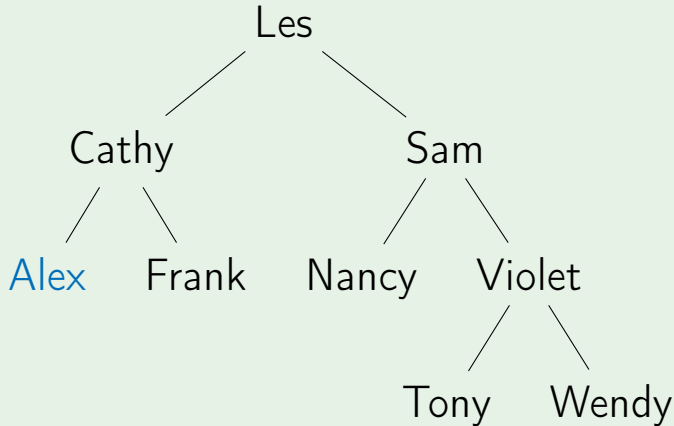


**Output:** Les Cathy Sam

**Queue:** Frank, Nancy, Violet



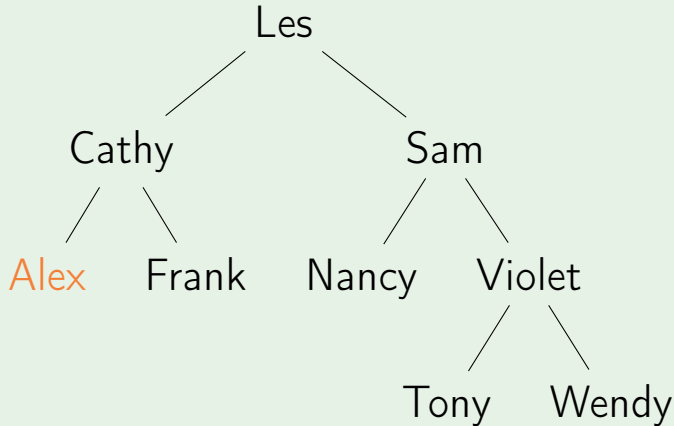
# LevelTraversal



**Output:** Les Cathy Sam Alex

**Queue:** Frank, Nancy, Violet

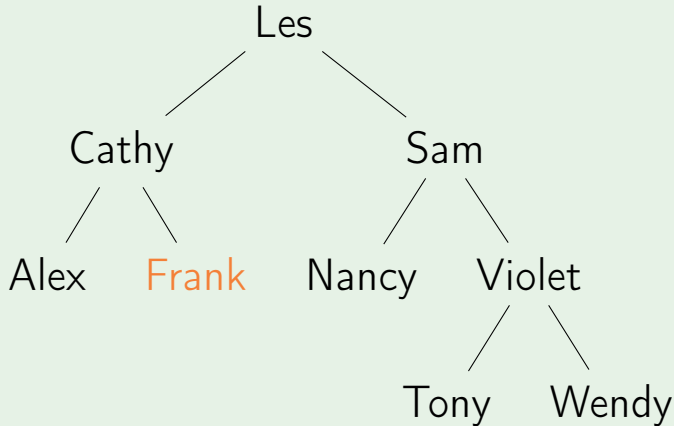
# LevelTraversal



**Output:** Les Cathy Sam Alex

**Queue:** Frank, Nancy, Violet

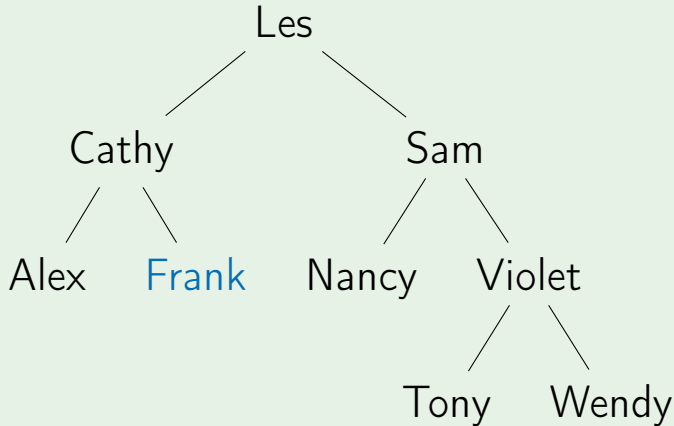
# LevelTraversal



**Output:** Les Cathy Sam Alex

**Queue:** Nancy, Violet

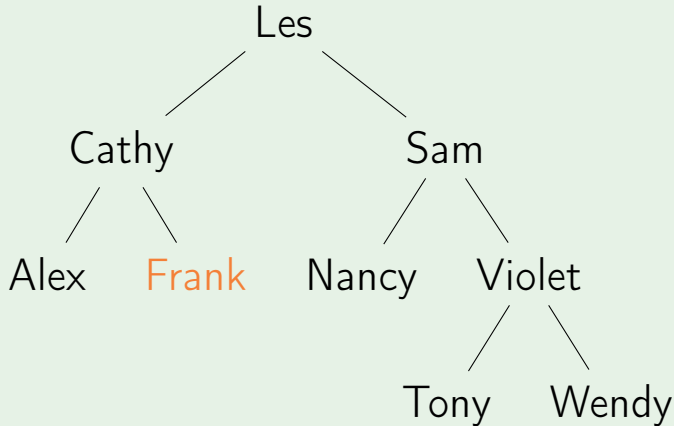
# LevelTraversal



**Output:** Les Cathy Sam Alex Frank

**Queue:** Nancy, Violet

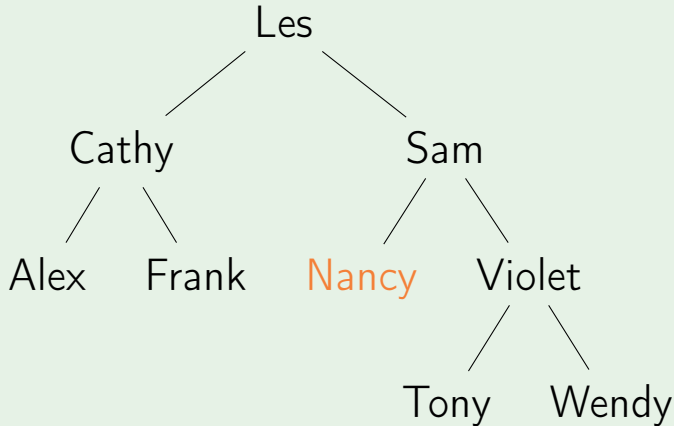
# LevelTraversal



**Output:** Les Cathy Sam Alex Frank

**Queue:** Nancy, Violet

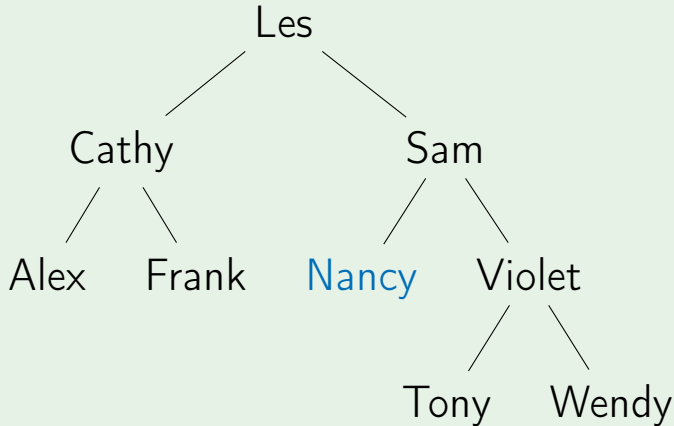
# LevelTraversal



**Output:** Les Cathy Sam Alex Frank

**Queue:** Violet

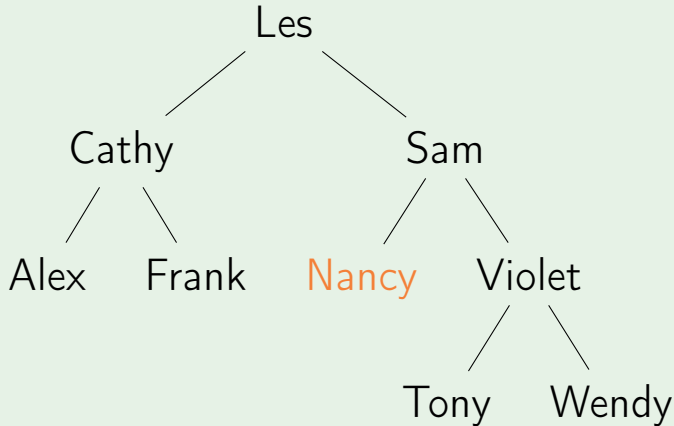
# LevelTraversal



**Output:** Les Cathy Sam Alex Frank Nancy

**Queue:** Violet

# LevelTraversal

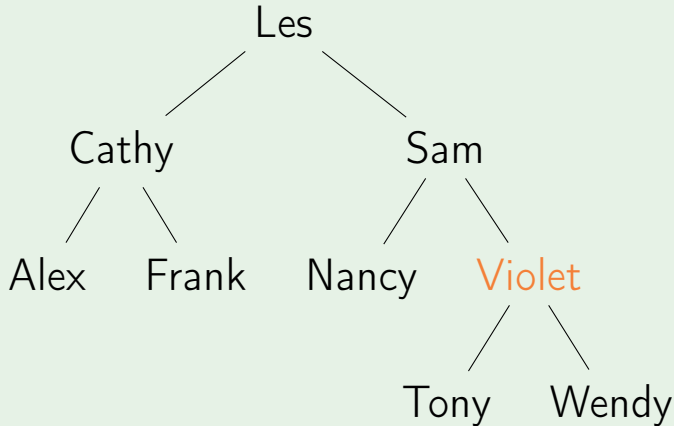


**Output:** Les Cathy Sam Alex Frank Nancy

**Queue:** Violet



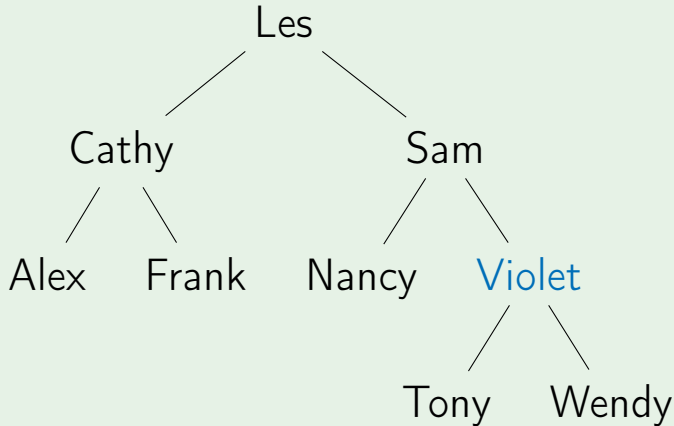
# LevelTraversal



**Output:** Les Cathy Sam Alex Frank Nancy

**Queue:**

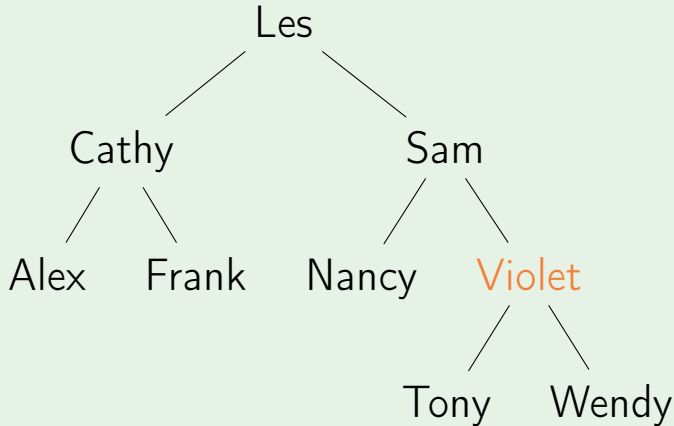
# LevelTraversal



**Output:** Les Cathy Sam Alex Frank Nancy  
Violet

**Queue:**

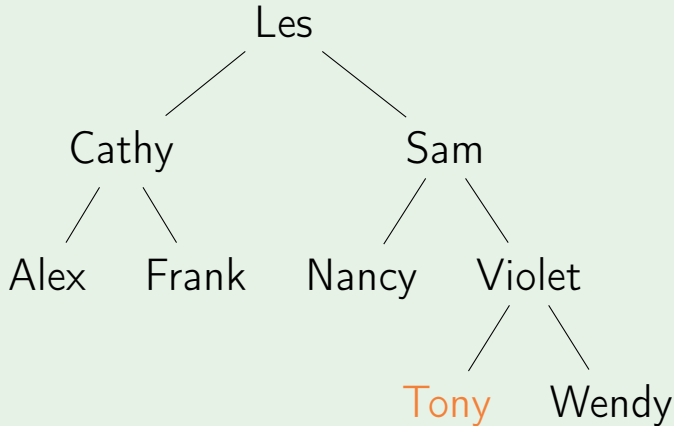
# LevelTraversal



**Output:** Les Cathy Sam Alex Frank Nancy  
Violet

**Queue:** Tony Wendy

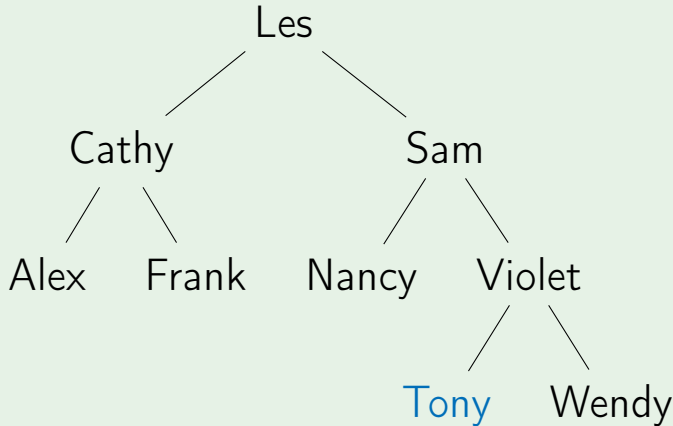
# LevelTraversal



**Output:** Les Cathy Sam Alex Frank Nancy  
Violet

**Queue:** Wendy

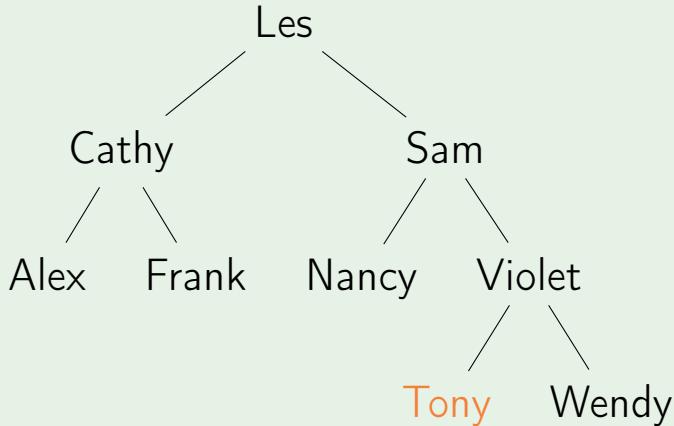
# LevelTraversal



**Output:** Les Cathy Sam Alex Frank Nancy  
Violet Tony

**Queue:** Wendy

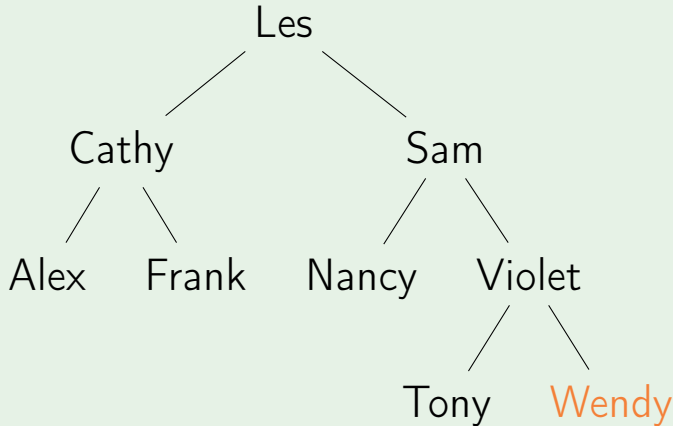
# LevelTraversal



**Output:** Les Cathy Sam Alex Frank Nancy  
Violet Tony

**Queue:** Wendy

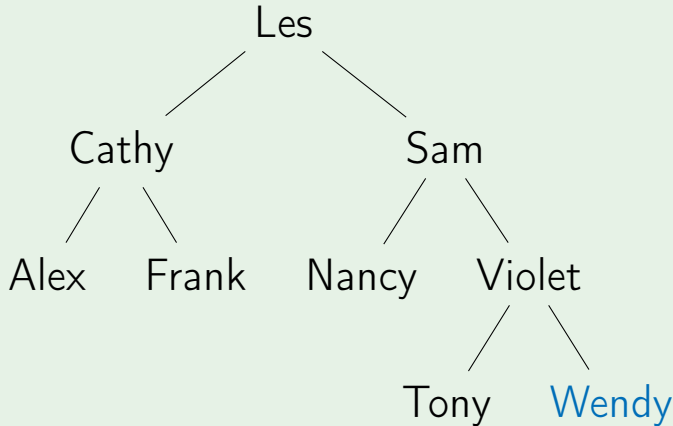
# LevelTraversal



**Output:** Les Cathy Sam Alex Frank Nancy  
Violet Tony

**Queue:**

# LevelTraversal

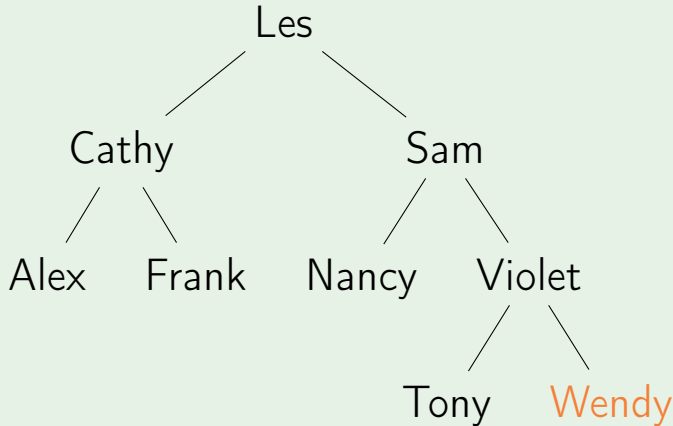


**Output:** Les Cathy Sam Alex Frank Nancy  
Violet Tony Wendy

**Queue:**



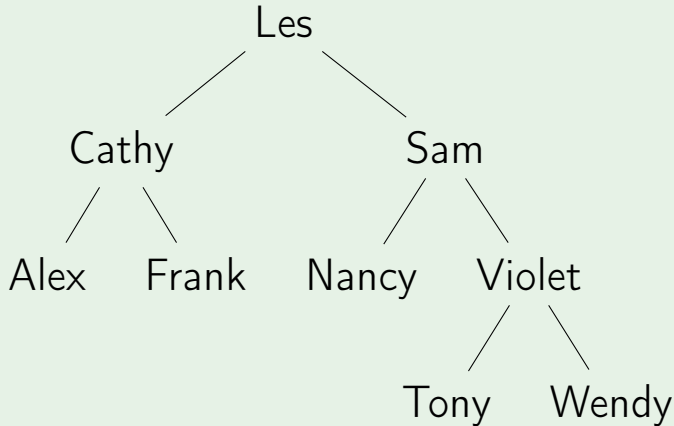
# LevelTraversal



**Output:** Les Cathy Sam Alex Frank Nancy  
Violet Tony Wendy

**Queue:**

# LevelTraversal



**Output:** Les Cathy Sam Alex Frank Nancy  
Violet Tony Wendy

**Queue:**

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- Trees have a key and children.
- Tree walks: DFS (pre-order, in-order, post-order) and BFS.
- When working with a tree, recursive algorithms are common.
- In Computer Science, trees grow down!

# For Tree-traversal quiz

