

Claim

$$\begin{aligned} \#(i \rightarrow j: \log^*(\text{rank}[i]) < \log^*(\text{rank}[j])) \\ \leq O(m \log^* n) \end{aligned}$$