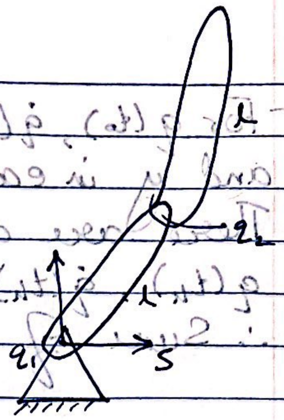
SAGAR SACHDEV

# HOMEWORK-12




**1.1** $M_1 = 1$ $\quad \min \int \tau^T \tau \, dt$

$I_1 = \frac{1}{12}$

$$q(t_0) = \begin{bmatrix} -\pi/2 \\ 0 \end{bmatrix}$$

$$\dot{q}(t_0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Constraints $= \overset{2}{q(t_0)}, \overset{2}{\dot{q}(t_0)}, \overset{2}{q(t_f)}, \overset{2}{\dot{q}(t_f)},$
$\dot{q}(t_{k+1}), q(t_{k+1}), q(t_k), \dot{q}(t_k)$ $\quad (75,\ 2)$

$$q(t_f) = \begin{bmatrix} \pi/2 \\ 0 \end{bmatrix}$$

Decision Vars $= q(t_k), \dot{q}(t_k), \tau(t_k)$

$$\dot{q}(t_f) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

No. of constraints $= 8 + 75(4)$
$= \boxed{308}$

$t = 1.5s$

$t_s = 20ms$

No. of decision variables $= 6 \times 76$
$= \boxed{456}$

$M\ddot{q} + C\dot{q} + N + A^T\lambda = \tau \quad (\lambda = 0)$

range of $k = 1 \to 76$

6 decision vars × 76 timesteps
$(q(t_k), \dot{q}(t_k), \tau(t_k))$

$$\min \sum_{k=0}^{N-1} \underbrace{(t_{k+1} - t_k)}_{t_s} \left( \frac{\tau^T(t_k)\tau(t_k) + \tau^T(t_{k+1})\tau(t_{k+1})}{2} \right)$$

w.r.t $q(t_k), \dot{q}(t_k), \tau(t_k)$

$$\text{s.t: } \dot{q}(t_{k+1}) = \dot{q}(t_k) + t_s \left( \frac{\ddot{q}(t_{k+1}) + \ddot{q}(t_k)}{2} \right)$$

$$q(t_{k+1}) = q(t_k) + t_s \left( \frac{\dot{q}(t_{k+1}) + \dot{q}(t_k)}{2} \right)$$

For $q(t_0)$, $\dot{q}(t_0)$, $q(t_f)$, $\dot{q}(t_f)$ there are 2 variables, one in $x$ and $y$ in each. Therefore there are $2+2+2+2$ constraints. There are also 75 timesteps for each of $q(t_{k+1})$, $\dot{q}(t_{k+1})$, $q(t_k)$, $\dot{q}(t_k)$. Therefore there are $75(4) = 300$ constraints.

$\therefore$ Sum of constraints $= 300 + 8 = \boxed{308}$ (in $x$ & $y$)

There are 2 decision variables each in $q(t_k)$, $\dot{q}(t_k)$, $\tau(t_k)$. Therefore there are 6 decision variables. For each of the above there are $75+1$ timesteps.

$\therefore$ Total no. of decision variables $= 76 \times 6 = \boxed{456}$

1.3 In terms of the trajectory, the ~~graph~~ trajectory in 1.3 is slightly wider than the trajectory in 1.2. This is because in 1.3, the ball is ~~being~~ trying to go to the center because it ~~has~~ is the position with the least torque however it cannot go to that position due to the constraint. ~~In terms~~ This can also be seen in the cost in the way that cost in 1.2 is 79.0438 and cost in 1.3 is 119.7581

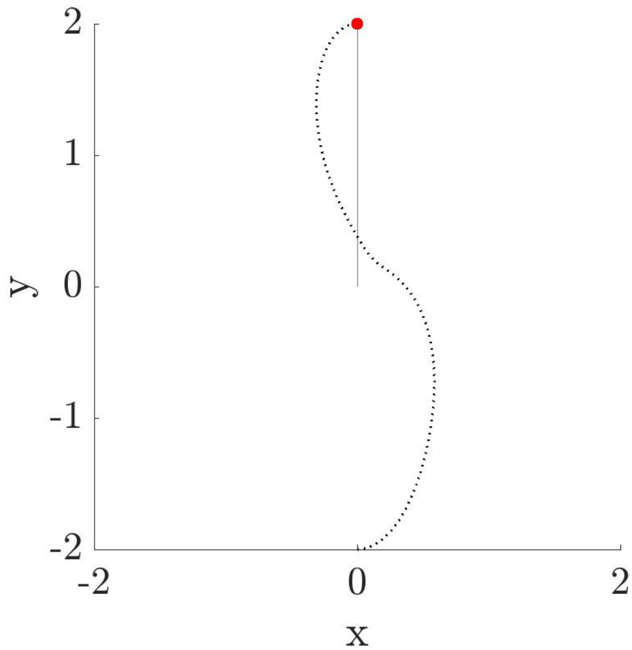



Fig from 1.2

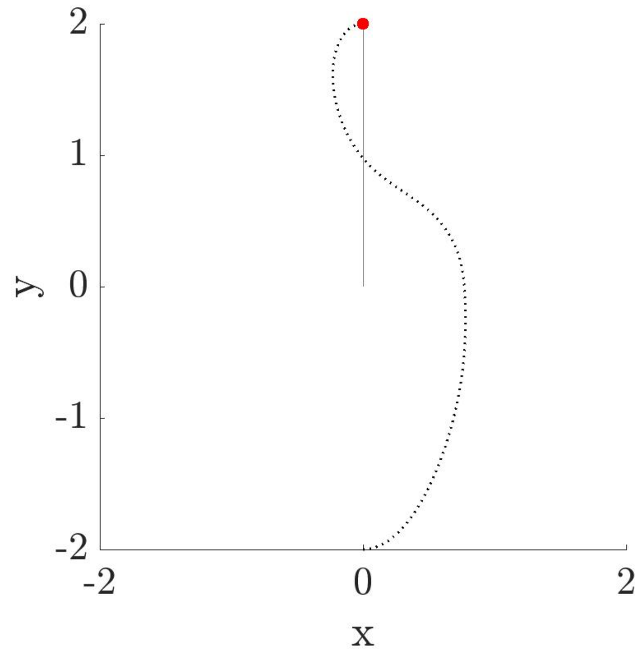



Fig. from 1.3