SAGAR SACHDEV

HW-9

1.1 $J = \{ \{c_1, c_2\}, \{c_1\}, \{c_2\}, \{\} \}$

1.2 $D_{c_1} = \{ (q, \dot{q}) \in TQ : a_{c_1}(q) = 0,\ A_{c_1}\dot{q} = 0,\ a_{c_2}(q) \geq 0 \}$

$D_{c_2} = \{ (q, \dot{q}) \in TQ : a_{c_2}(q) = 0,\ A_{c_2}\dot{q} = 0,\ a_{c_1}(q) \geq 0 \}$

$D_{c_1,c_2} = \{ (q, \dot{q}) \in TQ : a_{c_1}(q) = 0,\ A_{c_1}\dot{q} = 0,\ a_{c_2}(q) = 0,\ A_{c_2}\dot{q} = 0 \}$

$D_{\{\}} = \{ (q, \dot{q}) \in TQ : a_{c_2}(q) > 0,\ a_{c_1}(q) > 0 \}$

1.3 $F_{c_1} : M\ddot{q} + C\dot{q} + N + A_{c_1}^T \lambda_{c_1} = \Upsilon$

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & \frac{1}{3}m(w^2+h^2) \end{bmatrix} = M$$

$$C_{ij} = \left\{ \frac{1}{2}\sum_{k=1} \left(\frac{\partial M_{ij}}{\partial q_k} + \frac{\partial M_{ik}}{\partial q_j} - \frac{\partial M_{kj}}{\partial q_i} \right) \dot{q}_k \right.$$

$\because M$ is not dependent on q, $c = 0$

$V =$

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1.3 $F_{\xi\zeta}$: $M\ddot{q} + C\dot{q} + N + A^T_{\xi\zeta}\lambda_{\xi\zeta} = \Upsilon$

$$M = \begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & \frac{1}{3}m(\omega^2+h^2) \end{bmatrix} ; \quad C_{ij} = \frac{1}{2}\sum_{k=1}^{n}\left(\frac{\partial M_{ij}}{\partial q_k} + \frac{\partial M_{ik}}{\partial q_j} - \frac{\partial M_{kj}}{\partial q_i}\right)\dot{q}_k$$

Since M is not dependent on q, $C = 0$

$V = mgy$

$$N = \begin{bmatrix} \frac{\partial V}{\partial x} \\ \frac{\partial V}{\partial y} \\ \frac{\partial V}{\partial \theta} \end{bmatrix} = \begin{bmatrix} 0 \\ mg \\ 0 \end{bmatrix}$$

$$\cancel{A_{\xi\zeta} = \begin{bmatrix} 0 & 1 & h\sin(\theta) - \omega\cos(\theta) \\ 0 & 1 & \omega\cos(\theta) + h\sin(\theta) \end{bmatrix}}$$

$$A_{\xi\zeta} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Upsilon = \begin{bmatrix} 0 \\ 0 \\ \end{bmatrix}$$

$F_{c_1} = M\ddot{q} + C\dot{q} + N + A^T_{c_1}\lambda_{c_1} = \Upsilon$

M, C, N and Υ stay the same

$$a(q) = \begin{bmatrix} y - h\cos(\theta) - \omega\sin(\theta) \\ 0 \end{bmatrix}$$

$$A_{c_1} = \begin{bmatrix} 0 & 1 & h\sin(\theta) - \omega\cos(\theta) \\ 0 & 0 & 0 \end{bmatrix}$$

$F_{c_2} = M\ddot{q} + C\dot{q} + N + A^T_{c_2}\lambda_{c_2} = \Upsilon$

$$a(q) = \begin{bmatrix} 0 \\ y - h\cos(\theta) + \omega\sin(\theta) \end{bmatrix}$$

M, C, N and Υ stay the same

$$A_{c_2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & \cancel{h\cos(\theta) + \omega\sin(\theta)} \end{bmatrix}$$

$h\sin(\theta) + \omega\cos(\theta)$

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$\cancel{F_{c_1c_2} = M\ddot{q} + C\dot{q} + N + A^T_{c_1c_2}\lambda}$

$$F_{c_1c_2} = M\ddot{q} + C\dot{q} + N + A^T_{c_1}\lambda_{c_1} + A^T_{c_2}\lambda_{c_2} = \Upsilon$$

Everything stays the same.

1.4

$$\Gamma = \{(c_1, c_{12}), (c_2, c_{12}), (c_{12}, c_1), (c_{12}, c_2)$$
$$(c_1, c_2), (c_2, c_1)$$
$$(c_{\{\}}, c_1), (c_{\{\}}, c_2), (c_{\{\}}, c_{12})$$
$$(c_{12}, c_{\{\}}), (c_1, c_{\{\}}), (c_2, c_{\{\}})$$

$$\tilde{\Gamma} = \{(c_1, c_2), (c_2, c_1),$$
$$(c_{\{\}}, c_1), (c_{\{\}}, c_2), (c_{\{\}}, c_{12})\}$$

1.5

$$G_{\{c_1, c_{12}\}} = \{(q,\dot{q}) \in D_{c_1} : a_{c_{12}}(q) = 0,\ A_{c_{12}}\dot{q} < 0\}$$

$$G_{\{c_2, c_1\}} = \{(q,\dot{q}) \in D_{c_2} : a_{c_1}(q) = 0,\ A_{c_1}\dot{q} < 0\}$$

$$G_{\{\{\}, c_1\}} = \{(q,\dot{q}) \in D_{\{\}} : a_{c_1}(q) = 0,\ A_{c_1}\dot{q} < 0\}$$

$$G_{\{\{\}, c_2\}} = \{(q,\dot{q}) \in D_{\{\}} : a_{c_2}(q) = 0,\ A_{c_2}\dot{q} < 0\}$$

$$G_{\{\{\}, c_{12}\}} = \{(q,\dot{q}) \in D_{\{\}} : a_{c_{12}}(q) = 0,\ A_{c_{12}}\dot{q} \neq 0\}$$

1.6

$$R_{\{c_1, c_2\}}(q, \dot{q}^-) = \{(q, \dot{q}^+) \in D_{c_2}$$

$$R_{\{c_2, c_1\}}(q, \dot{q}^-) = (q, \dot{q}^+) \in D_{c_1}$$

$$R_{\{\{\}, c_{12}\}}(q, \dot{q}^-) = (q, \dot{q}^+ \in D_{c_{12}})$$

$$R_{\{\{\}, c_1\}}(q, \dot{q}^-) = (q, \dot{q}^+) \in D_{c_1}$$

$$R_{\{\{\}, c_2\}}(q, \dot{q}^-) = (q, \dot{q}^+) \in D_{c_2}$$

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1.7

$h > w$

$\{1\}$ to $\{1,2\}$ and $\{2\}$ to $\{1,2\}$ are not achievable since it is a plastic impact for each mode, it would keep oscillating back and forth.