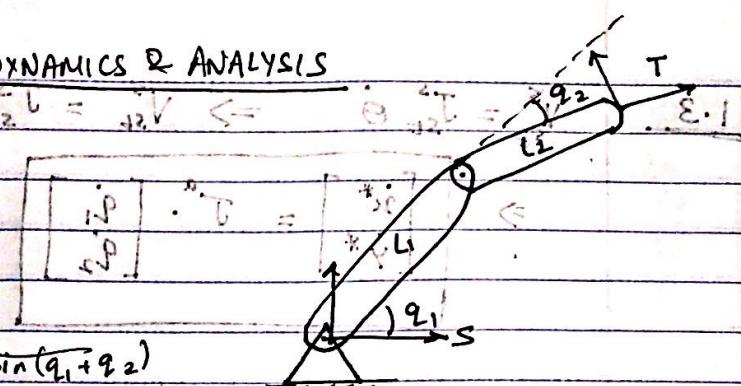


HOMWORK - 3 : ROBOT DYNAMICS & ANALYSIS

$$g(q) = \begin{bmatrix} \cos(q_1+q_2) & -\sin(q_1+q_2) \\ \sin(q_1+q_2) & \cos(q_1+q_2) \end{bmatrix}$$

Ques. Show that it moves

$$g_{st}(q) = \begin{bmatrix} \cos(q_1+q_2) & -\sin(q_1+q_2) & 0 & l_1 \cos(q_1) + l_2 \cos(q_1+q_2) \\ \sin(q_1+q_2) & \cos(q_1+q_2) & 0 & l_1 \sin(q_1) + l_2 \sin(q_1+q_2) \\ 0 & 0 & (0+0^T) \text{ t.e.} & 0 \end{bmatrix}$$

$$f(q) = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_1 \cos(q_1) + l_2 \cos(q_1+q_2) \\ l_1 \sin(q_1) + l_2 \sin(q_1+q_2) \end{bmatrix} \quad \therefore \quad \text{Ques.}$$

1.2 Analytic Jacobian:

$$\Rightarrow f(q) = \begin{bmatrix} l_1 \cos(q_1) + l_2 (\cos(q_1) \cos(q_2) - \sin(q_1) \sin(q_2)) \\ l_1 \sin(q_1) + l_2 (\sin(q_1) \cos(q_2) + \cos(q_1) \sin(q_2)) \end{bmatrix}$$

$$\Rightarrow J^a = \begin{bmatrix} -l_1 \sin(q_1) + (-l_2 \sin(q_1+q_2)) & -l_2 \sin(q_1+q_2) \\ l_1 \cos(q_1) + l_2 \cos(q_1+q_2) & l_2 \cos(q_1+q_2) \end{bmatrix}$$

$$\underline{1.3} \quad V_{st}^s = J_{st}^s \dot{\theta} \Rightarrow V_{st}^b = J_{st}^s q \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \underline{s} \quad \underline{2.1}$$

$$\begin{bmatrix} \dot{x}^* \\ \dot{y}^* \end{bmatrix} = J^a \cdot \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \underline{0}$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = J^{a^{-1}} \begin{bmatrix} \dot{x}^* \\ \dot{y}^* \end{bmatrix}$$

$$\Rightarrow \text{To take determinant} = \begin{vmatrix} (J^a)_{11} & (J^a)_{12} \\ (J^a)_{21} & (J^a)_{22} \end{vmatrix} \times \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \underline{s}$$

$$J^a = \begin{bmatrix} -l_1 \sin(q_1) - l_2 \sin(q_1 + q_2) & 0 & -l_2 \sin(q_1 + q_2) \\ l_1 \cos(q_1) + l_2 \cos(q_1 + q_2) & 0 & l_2 \cos(q_1 + q_2) \end{bmatrix}$$

using mathematica,

$$\begin{bmatrix} J^{a^{-1}}_{11} & J^{a^{-1}}_{12} \\ J^{a^{-1}}_{21} & J^{a^{-1}}_{22} \end{bmatrix} = \frac{1}{l_1 l_2 \sin(q_2)} \begin{bmatrix} l_2 \cos(q_1 + q_2) & l_2 \sin(q_1 + q_2) \\ -l_1 \cos(q_1) + l_2 \cos(q_1 + q_2) & -l_1 \sin(q_1) - l_2 \sin(q_1 + q_2) \end{bmatrix}$$

where $\det J^{a^{-1}} = \frac{1}{l_1 l_2 \sin(q_2)}$

$$l_1 l_2 \sin(q_2) = 0 \quad \text{only at } 0 \text{ and } \pi$$

1.4 The analytic Jacobian has a singularity at $q_2 = 0$ or π which is when J loses rank. When a singularity occurs a degree of freedom is lost and motion cannot occur.

For linear independence, J^a must be invertible and $q_2 \neq 0$ or π .

1.5

$$\begin{aligned} \vec{\gamma}_1 &= - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \vec{\gamma}_1^T = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \quad \vec{V} \leftarrow \dot{\theta} \vec{\gamma}_1^T = \dot{\theta} \vec{V} \quad \underline{S.1} \\ &\quad \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} * & * & * \\ * & * & * \end{bmatrix} \end{aligned}$$

$$\begin{aligned} \vec{\gamma}_2 &= - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} l_1 \cos(q_1) \\ l_1 \sin(q_1) \\ 0 \end{bmatrix} = \begin{bmatrix} l_1 \sin(q_1) \\ 0 \\ 0 \end{bmatrix} \quad \text{dotted at } \vec{\gamma}_2^T = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix} \\ &\quad \begin{bmatrix} (s_p + p) \sin q_1 & 0 & (s_p + p) \cos q_1 \\ 0 & (s_p + p) \cos q_1 & -(s_p + p) \sin q_1 \\ 1 & 0 & 0 \end{bmatrix} = \vec{\gamma}_2^T \quad \text{will do} \end{aligned}$$

$$\begin{aligned} \therefore J_{st}^s &= \begin{bmatrix} 0 \sin q_1 \cos(q_1) p + (s_p + p) \sin q_1 \\ 0 \sin q_1 \sin(q_1) p + (s_p + p) \cos q_1 \\ 0 \end{bmatrix} \quad J_{st}^b = \begin{bmatrix} l_1 \sin(q_2) \\ (l_1 + l_2) \cos q_2 \\ l_2 \end{bmatrix} \\ &\quad \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \vec{\gamma}_2^T \quad \text{will do} \\ &\quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & (s_p + p) \sin q_1 \\ 1 & 1 & 1 \end{bmatrix} \end{aligned}$$

π bar 0 \rightarrow who $\theta = (s_p + p) \sin q_1$.

$$J_{st}^b \cdot \dot{q} = \begin{bmatrix} l_1 \sin(q_2) \\ 0 \\ 0 \end{bmatrix}$$

π bar 0 \Rightarrow π bar $(l_1 + l_2) \cos(q_2)$ when $\vec{\gamma}_2^T = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$ will do $\underline{H.1}$

when π bar 0 \Rightarrow π bar 0 when $\vec{\gamma}_2^T = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix}$ is done

now π bar 0 \Rightarrow π bar 0

now determine π bar $\vec{\gamma}_2^T$ whether π bar 0 \Rightarrow

~~π bar 0 \Rightarrow π , π bar 0 \neq π~~

If $q_2 = 0$ or π

$$\begin{array}{|c|c|} \hline \cancel{0} & 0 \\ \hline l_1 \pm l_2 & l_2 \\ \hline 0 & 0 \\ \hline 0 & 0 \\ \hline 0 & 0 \\ \hline 1 & 1 \\ \hline \end{array} \quad \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$J_{st}^b \dot{q}$

will always yield the velocity in the x -direction to be 0.
Hence it is non-singular since the rank is always 2

2.1 $g_{st} (0) = \begin{bmatrix} 1 & 0 & 0 & 1407 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1855 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ $\pi \times 0 = \rho$ ✓

$$\begin{array}{c|c} \text{Row 1} & \text{Row 2} \\ \text{Row 2} & \text{Row 3} \\ \text{Row 3} & \text{Row 4} \\ \text{Row 4} & \text{Row 1} \end{array}$$

Ondal maitineb-e illi a filoohi ill bain 2000
aynabi a dwo illi kina salmiz-han 0.5 m length

2.2 $g_{SL1} = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 680 \\ 0 & 0 & 0 & 1 \end{bmatrix}; g_{LU45} = \begin{bmatrix} c_5 & 0 & s_5 & 887 \\ 0 & 1 & 0 & 0 \\ -s_5 & 0 & c_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$g_{L1L2} = \begin{bmatrix} c_2 & 0 & s_2 & 320 \\ 0 & 1 & 0 & 0 \\ -s_2 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; g_{ST} = \begin{bmatrix} 1 & 0 & 0 & 200 \\ 0 & c_6 & -s_6 & 0 \\ 0 & s_6 & c_6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$g_{L2L3} = \begin{bmatrix} c_3 & 0 & s_3 & 0 \\ 0 & 1 & 0 & 0 \\ -s_3 & 0 & c_3 & 975 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

All these are taken using s-frame from diagram

$g_{L3L4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_4 & -s_4 & 0 \\ 0 & s_4 & c_4 & 200 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$\zeta_1 = \begin{bmatrix} -\omega_1 \times p_1 \\ \omega_1 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 680 \end{bmatrix} = v_1 \quad \text{(*p) - (e)}$$

$$\zeta_2 = \begin{bmatrix} -\omega_2 \times p_2 \\ \omega_2 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 320 \\ 0 \\ 201 \end{bmatrix} = v_2 \quad \text{(*p)}$$

$$\zeta_3 = \begin{bmatrix} -\omega_3 \times p_3 \\ \omega_3 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 975 \end{bmatrix} = v_3 \quad \text{(*p)}$$

$$\zeta_4 = \begin{bmatrix} -\omega_4 \times p_4 \\ \omega_4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 200 \\ 0 \\ 0 \end{bmatrix} = v_4$$

$$\zeta_5 = \begin{bmatrix} -\omega_5 \times p_5 \\ \omega_5 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 200 \end{bmatrix} = v_5$$

$$\zeta_6 = \begin{bmatrix} -\omega_6 \times p_6 \\ \omega_6 \end{bmatrix} \Rightarrow \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 887 \\ 0 \\ 0 \end{bmatrix} = v_6$$

$$2.4 \quad ||g(\theta) - g^*|| \quad \text{cost}(g_s) = \sum_{\text{all}} (\text{abs.} (g_{st}(q_s) - g_{st}^*)^2)$$

if $\text{cost}(g_s) < 0.1$

stop.

$$g^* = \begin{bmatrix} 1 & 0 & 0 & 1407 \\ 0 & 1 & 0 & 100 \\ 0 & 0 & 1 & 1855 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g_{st}(\theta) \approx \begin{bmatrix} 0 & 0 \\ 0 & 250 \\ 0 & 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \left(\begin{bmatrix} 29 \times 2w \\ 2w \end{bmatrix} \right) = \begin{bmatrix} 5 \\ 2w \end{bmatrix}$$

$$g_{st}(\theta) \approx \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \left(\begin{bmatrix} 29 \times 2w \\ 2w \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 5 \\ 2w \\ 0 \end{bmatrix}$$

$$g_{st}(\theta) \approx \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \left(\begin{bmatrix} 29 \times 2w \\ 2w \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 5 \\ 2w \\ 0 \end{bmatrix}$$

$$g_{st}(\theta) \approx \begin{bmatrix} 188 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} - \left(\begin{bmatrix} 29 \times 2w \\ 2w \\ 0 \end{bmatrix} \right) = \begin{bmatrix} 5 \\ 2w \\ 0 \end{bmatrix}$$