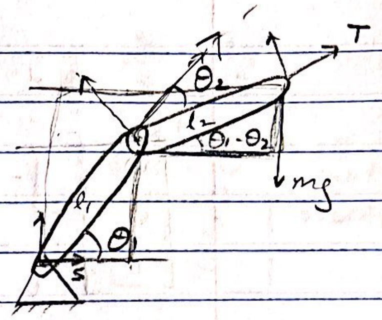
SAGAR SACHDEV

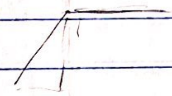
ROBOT DYNAMICS HOMEWORK-4



1.1

$$\cancel{\begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & l_1\cos(\theta_1) \\ \sin(\theta_1) & \cos(\theta_1) & l_1\sin(\theta_1) \\ 0 & 0 & 1 \end{bmatrix}}$$

$$\begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & l_1\cos(\theta_1) \\ \sin(\theta_1) & \cos(\theta_1) & 0 & l_1\sin(\theta_1) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = g_{sl_1 l_2}$$



$$\begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & l_2\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & l_2\sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = g_{l_2 t}$$

$$g_{st} = g_{l_1 l_2}\, g_{l_2 t}$$

$$\text{Force} = \begin{bmatrix} 0 \\ -mg \\ 0 \end{bmatrix} \qquad -mg\,(l_2\cos(\theta_1-\theta_2) + l_1\cos(\theta_1)),$$

rotation about z

$$\text{Torque} = \begin{bmatrix} 0 \\ 0 \\ -mg(l_2\cos(\theta_1-\theta_2) + l_1\cos(\theta_1)) \end{bmatrix}$$

$$\text{Rotation about } z \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \omega$$

$$P = \begin{bmatrix} -l_2 \\ 0 \\ 0 \end{bmatrix} : -\omega \times P = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} -l_2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -(l_2) \\ 0 \end{bmatrix}$$

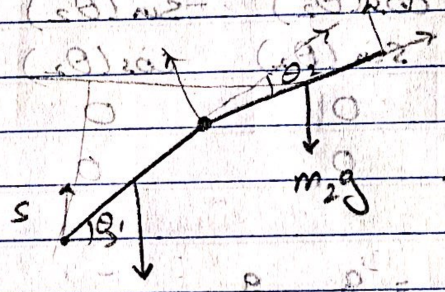
$$\text{Pt. 2}: \begin{bmatrix} -l_2 - l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix} \times \qquad = \begin{bmatrix} 0 \\ l_2 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$l_1 \times m_2$

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} -l_2 - l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -l_1 s_1 \\ -(-l_2 - l_1 c_1) \\ 0 \end{bmatrix}$$

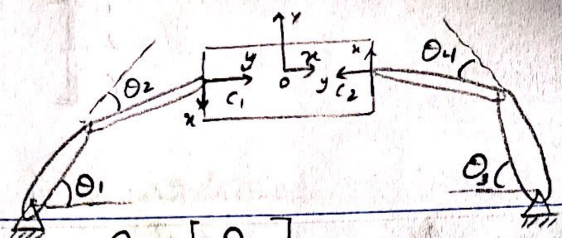
$$\Rightarrow \begin{bmatrix} -l_1 s_1 \\ -l_2 - l_1 c_1 \\ 0 \end{bmatrix}$$



$$-M_1 g\left(\frac{l_1}{2}\cos(\theta_1)\right) \text{ —— } (1)$$

$$\cancel{M_2 g\left(\frac{l_2}{2}\cos(\theta_2+\theta_1)\right)} \text{ —— } (2)$$

$$-M_2 g\left(\frac{L_2}{2}\cos(\theta_1+\theta_2) + L_1\cos(\theta_1)\right) \text{ —— } (2)$$



$$O = \begin{bmatrix} x_0 \\ y_0 \\ \phi_0 \end{bmatrix} \qquad \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}$$

$$R = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$\begin{matrix} \cos(90) & -\sin 90 \\ \sin 90 & \cos 90 \end{matrix}$$

2.1 $$G = \begin{bmatrix} Ad^T_{g_{oc_1}^{-1}} B_{c_1} & Ad^T_{g_{oc_2}^{-1}} B_{c_2} \end{bmatrix}$$

$$\Rightarrow g_{oc_1} = \begin{bmatrix} 0 & +1 & 0 & -lw/2 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad g_{oc_2} = \begin{bmatrix} 0 & -1 & 0 & lw/2 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B_{c_1} = B_{c_2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$G_{3D} = \begin{bmatrix} 0 & 1 & 0 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{lw}{2} & 0 & 0 & \frac{lw}{2} & 0 & 0 \end{bmatrix} ; \quad G_{2D} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ \frac{lw}{2} & 0 & \frac{lw}{2} & 0 \end{bmatrix}$$

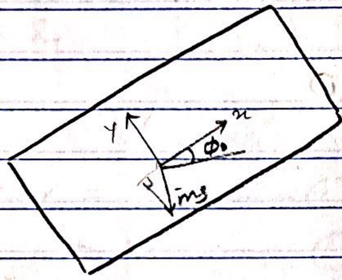
$$FC \Rightarrow U(f_{c_1}) = \begin{bmatrix} f_{c_1 y} \\ \mu f_{c_1 y} - \sqrt{f_{c_1 x}^2 + f_{c_1 z}^2} \end{bmatrix} \geq 0$$

$$U(f_{c_2}) = \begin{bmatrix} f_{c_2 y} \\ \mu f_{c_2 y} - \sqrt{f_{c_2 x}^2 + f_{c_2 z}^2} \end{bmatrix} \geq 0$$

$$FC = FC_1 \times FC_2$$

2.2 Since two columns are linearly dependent, the matrix has a rank of 3, which is maximum for a planar problem as a result, it is a force-closure grasp.

2.3



$$F = \begin{bmatrix} mg\sin(\phi_0) \\ mg\cos(\phi_0) \\ 0 \end{bmatrix}$$

$$F_e = G f_c$$

$$G \begin{bmatrix} f_{c_1 x} \\ f_{c_1 y} \\ f_{c_2 x} \\ f_{c_2 y} \end{bmatrix} = \begin{bmatrix} mg\sin(\phi_0) \\ mg\cos(\phi_0) \\ 0 \end{bmatrix}$$

$\sum F_y = 0$:

$$\begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ \frac{lw}{2} & 0 & \frac{lw}{2} & 0 \end{bmatrix} \begin{bmatrix} f_{c_1 x} \\ f_{c_1 y} \\ f_{c_2 x} \\ f_{c_2 y} \end{bmatrix} =$$

$$f_{c_1 y} - f_{c_2 y} = +mg\sin(\phi_0) \quad \text{—— } 1$$

$$-f_{c_1 x} + f_{c_2 x} = mg\cos(\phi_0) \quad \text{—— } 2$$

$$\frac{lw}{2} f_{c_1 x} + \frac{lw}{2} f_{c_2 x} = 0$$

$$f_{c_1 x} = -f_{c_2 x} \quad \text{—— } 3$$

$$2 f_{c_2 x} = mg\cos(\phi_0)$$

$$f_{c_2 x} = \frac{mg\cos(\phi_0)}{2}, \quad f_{c_1 x} = \frac{mg\cos(\phi_0)}{2}$$

Scanned with CamScanner

$$\mu f_{cy_1} - \sqrt{f_{c_1x}^2} \geq 0$$

$$\mu f_{cy_1} - \sqrt{\left(\frac{mg\cos(\Phi_0)}{2}\right)^2} \geq 0$$

$$\Rightarrow \mu f_{cy_1} - \frac{mg\cos(\Phi_0)}{2} \geq 0$$

$$\Rightarrow \mu f_{cy_1} \geq \frac{mg\cos(\Phi_0)}{2}$$

$$\Rightarrow f_{cy_1} \geq \frac{mg\cos(\Phi_0)}{2\mu}$$

$$\Rightarrow f_{cy_1} \geq \frac{|f_{c_1x}|}{\mu}$$

$$\frac{mg\cos(\Phi_0)}{2\mu} - f_{c_2y} = mg\sin(\Phi_0)$$

$$f_{c_2y} \geq \frac{|mg\cos(\Phi_0)|}{2\mu} - mg\sin(\Phi_0)$$

2-4

$$\begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ \frac{lw}{2} & 0 & \frac{lw}{2} & 0 \end{bmatrix} \begin{bmatrix} f_{c_1x} \\ f_{c_1y} \\ f_{c_2x} \\ f_{c_2y} \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} \lambda = 0$$

$\therefore f_{c_1y}$ and f_{c_2y} are internal forces

2.4

$$\begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ \frac{lw}{2} & 0 & \frac{lw}{2} & 0 \end{bmatrix} \begin{bmatrix} f_{c_1 x} \\ f_{c_1 y} \\ f_{c_2 x} \\ f_{c_2 y} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$f_{c_1 y} - f_{c_2 y} = 0 \quad \text{——} \quad (1)$$

$$-f_{c_1 x} + f_{c_2 x} = 0 \quad \text{——} \quad (2)$$

$$\frac{lw}{2} f_{c_1 x} + \frac{lw}{2} f_{c_2 x} = 0 \quad \text{——} \quad (3)$$

$$f_{c_1 y} = f_{c_2 y}$$

$$\left.\begin{aligned} f_{c_1 x} &= f_{c_2 x} \\ f_{c_1 x} &= -f_{c_2 x} \end{aligned}\right\} \text{doesn't make sense unless } f_{c_1 x} = f_{c_2 x} = 0$$

$$\therefore \quad \cancel{\begin{bmatrix} f_{c_1 x} \\ f_{c_1 y} \\ f_{c_2 x} \\ f_{c_2 y} \end{bmatrix}} \Rightarrow \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} x$$

$\Rightarrow f_{c_1 y}$ and $f_{c_2 y}$ are internal forces

2.5

$$f_{c_{2y}} \geq \frac{|mg\cos(\phi_0)|}{2\mu} - mg\sin(\phi_0)$$

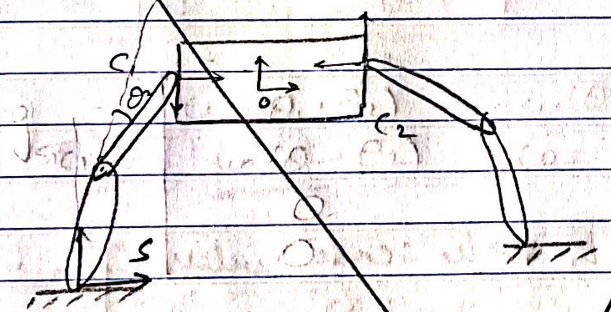
↳ This is (−)ve because its direction is opp.

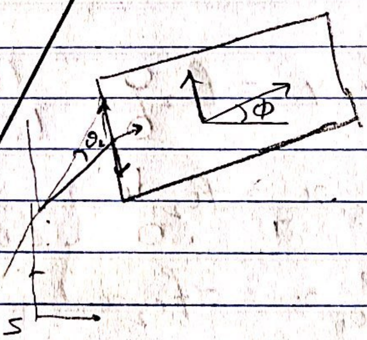
The above value will be minimum when $\phi_0 = 90°$

$\therefore \; f_{c_{2y}} \geq mg$

2.6

$$J_h(\theta, x_0) = \begin{bmatrix} \overbrace{B_{c_1}^T Ad^{-1}_{g_{s_1c_1}} J^s_{s_1f_1}(\theta_{f_1})}^{4\times 2} & 0 \\ 0 & \overbrace{B_{c_n}^T Ad^{-1}_{g_{s_nc_n}}(\theta_{f_n})}^{4\times 2} \end{bmatrix}_{8\times 4}$$

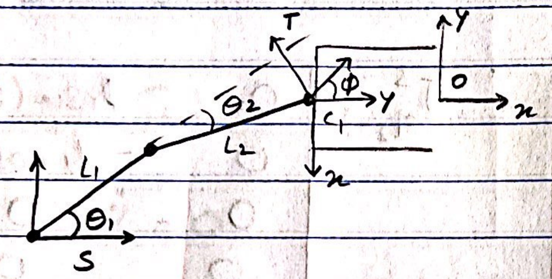




$$B_{c_1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad g_{s_1} = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 & l_1 c_1 \\ \sin(\theta_1) & \cos(\theta_1) & 0 & l_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2.6

$$G = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -1 & 0 & 1 & 0 \\ \frac{lw}{2} & 0 & \frac{lw}{2} & 0 \end{bmatrix}_{2D} \qquad B_{c_1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}_{2D}$$



$$J^s_{st} = \begin{bmatrix} 0 & l_1 \sin(\theta_1) \\ 0 & -l_1 \cos(\theta_1) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

$g_{sc_1} = g_{sp_1} g_{po} g_{oc_1}$

$=$

$$B_{c_i} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$J^{s}_{st_1} = \begin{bmatrix} 0 & l_1 \sin(\theta_1) \\ 0 & -l_1 \cos(\theta_1) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \qquad J^{s}_{st_2} = \begin{bmatrix} 0 & l_1 \sin(\theta_3) \\ 0 & -l_1 \cos(\theta_3) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

$$g_{sc_1} = g_{st_1} \, g_{fc}$$

$$g_{st_1} = \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 & l_2\cos(\theta_1+\theta_2) + l_1\cos\theta_1 \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 & l_2\sin(\theta_1+\theta_2) + l_1\sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g_{fc_1} = \begin{bmatrix} \cos(\theta_1+\theta_2+90-\varphi) & -\sin(\theta_1+\theta_2+90-\varphi) & 0 & 0 \\ \sin(\theta_1+\theta_2+90-\varphi) & \cos(\theta_1+\theta_2+90-\varphi) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

\- $(g_{sc_1}, g_{sc_2}, J_h$ all computed in MATLAB)

↳ Ans. in Matlab script.

$$\cancel{J_h = l_2 \cos(\theta_1 - \varphi + \theta_2) + l_1 \cos(\theta_1 - \varphi + 2\theta_2) \quad \begin{matrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{matrix}}$$

2.7 Null (J_h) = Internal motions

$$J_h \dot{\theta} = 0$$

$$\dot{\theta} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

∴ No internal motions

2.8 Since the internal motions for 2.7 are 0, the two fingers are losing a DOF. The null space of J_h is therefore corresponding to the DOF that is lost, which in turn corresponds to the internal motions