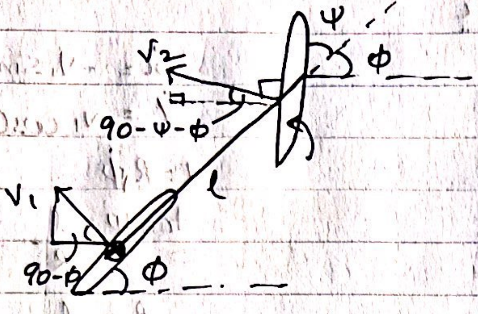
SAGAR SACHDEV

ROBOT DYNAMICS HOMEWORK - 5

2)

1.1



$$\Rightarrow \begin{bmatrix} -V_2\cos(90-\psi-\phi) \\ V_2\sin(90-\psi-\phi) \end{bmatrix} \Rightarrow \begin{bmatrix} -V_2\cos(90-(\psi+\phi)) \\ V_2\sin(90-(\psi+\phi)) \end{bmatrix}$$

$\Rightarrow \dot{x} = -V_2\sin(\psi+\phi)$

$\Rightarrow \dot{y} = (V_2)\cos(\psi+\phi)$

~~$-V_2\sin(\psi+\phi)$~~ $-V_1\sin(\phi) = \dot{x}$

$V_1\cos(\phi) = \dot{y}$

$$\begin{bmatrix} 0 & -\sin(\phi) & \cos(\phi) & 0 \\ 0 & -\sin(\psi+\phi) & +\cos(\psi+\phi) & +l\cos(\psi) \end{bmatrix}\begin{bmatrix} \dot{\psi} \\ \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

3) $\Rightarrow A(q)$ $\dot{q}$ = 0

1.2 $V = R\dot{\phi}$

$\tan\psi = \frac{l}{R}$

$\Rightarrow R = \frac{l}{\tan\psi}$

$\dot{\phi} = \frac{V}{\left(\frac{l}{\tan\psi}\right)}$

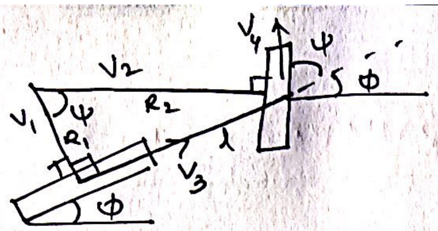
$\Rightarrow \frac{V\tan\psi}{l} = \dot{\phi}$

$$\therefore \dot{q} \Rightarrow \begin{bmatrix} 0 & 1 \\ \cancel{\cos(\phi)} & 0 \\ \sin(\phi) & 0 \\ \frac{\tan\psi}{l} & 0 \end{bmatrix}\begin{bmatrix} V \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} \dot{\psi} \\ \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix}$$

$\Rightarrow$ Rear wheel

$\dot{x} = -V_2\sin(\psi+\phi)$

$\dot{y} =$



1.2 For Rear wheel

$$\dot{x} = V_3 \cos(\phi) \qquad \tan\psi = \frac{L}{R_1}$$
$$\dot{y} = V_3 \sin(\phi)$$

$$\begin{bmatrix} \dot{\psi} \\ \dot{x} \\ \dot{y} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \cos(\phi) & 0 \\ \sin(\phi) & 0 \\ \frac{\tan(\psi)}{L} & 0 \end{bmatrix} u \qquad V = R_1 \dot{\theta} \qquad R_1 = \frac{L}{\tan\psi}$$

$$\Rightarrow \dot{\theta} = \frac{V}{R_1}$$
$$\dot{\theta} = \frac{V \tan\psi}{L}$$

$$\Rightarrow \begin{bmatrix} \dot{\psi} \\ \dot{x} \\ \dot{y} \\ \phi \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \cos(\phi) & 0 \\ \sin(\phi) & 0 \\ \frac{\tan(\psi)}{L} & 0 \end{bmatrix} \begin{bmatrix} V \\ \dot{\psi} \end{bmatrix}$$

$$\dot{q} = \underbrace{}_{H(q)} \underbrace{}_{u}$$

1.3 $A\dot{q} = 0$

$\Rightarrow AH(q)\,u = 0$

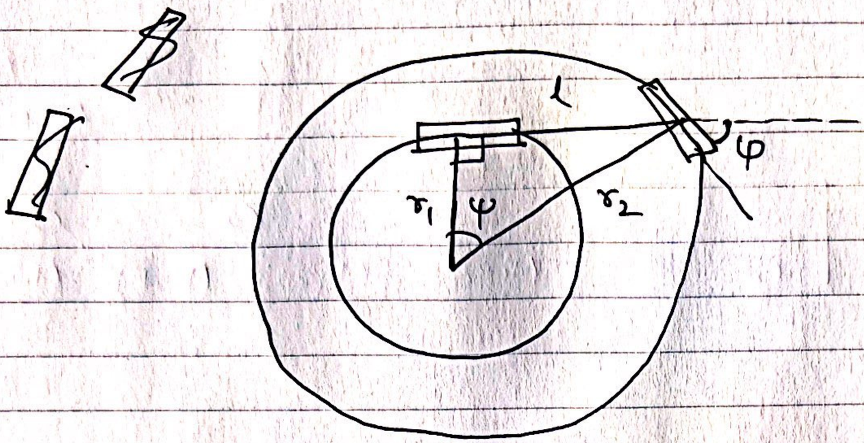
$$\therefore \begin{bmatrix} 0 & -\sin(\phi) & \cos(\phi) & 0 \\ 0 & -\sin(\phi+\psi) & \cos(\phi+\psi) & l\cos(\psi) \end{bmatrix} \begin{bmatrix} 0 & 1 \\ \cos(\phi) & 0 \\ \sin(\phi) & 0 \\ \frac{\tan(\psi)}{L} & 0 \end{bmatrix} u$$

$$\Rightarrow \begin{bmatrix} -\cancel{\sin(\phi)\cos(\phi)} + \cancel{\sin(\phi)\cos(\phi)} + 0 & 0 \\ -\sin(\phi+\psi)\cos(\phi) + \sin(\phi)\cos(\phi+\psi) + \sin\psi & 0 \end{bmatrix} u$$

Using MATLAB for simplification

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} u$$

1.4



Rear wheel : $\frac{L}{r_1} = \tan(\psi)$

$$\boxed{r_1 = \frac{L}{\tan\psi}}$$