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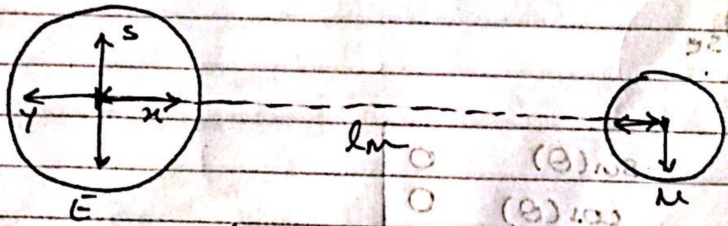
HOMEWORK-2: ROBOT DYNAMICS & ANALYSIS



$\theta = 270°$

1. Moon around earth = 1 revolution per 28 days
Moon around its axis = 1 revolution per 28 days

1.1 $$R_{se} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} = \begin{bmatrix} \cos(270) & -\sin(270) & 0 \\ \sin(270) & \cos(270) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$g_{se} = \begin{bmatrix} R_{se} & P_{se} \\ 0\ 0\ 0 & 1 \end{bmatrix}_{4\times4} \Rightarrow \begin{bmatrix} 0 & +1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$g_{se} = \begin{bmatrix} 0 & +1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1.2 $$q_e = \begin{bmatrix} 0 \\ r_e \\ 0 \end{bmatrix}$$

$$q_s = R_{se} \cdot q_e = \begin{bmatrix} 0 & +1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ r_e \\ 0 \end{bmatrix}$$

$$q_s = \begin{bmatrix} +r_e \\ 0 \\ 0 \end{bmatrix}$$

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1.3

$$V^b_{se} = \begin{bmatrix} R^T_{SE}\,\dot{P}_{SE} \\ (R^T_{SE}\,\dot{R}_{SE})^{\vee} \end{bmatrix} = \begin{bmatrix} v^b_{se} \\ w^b_{se} \end{bmatrix}$$

$$R_{SE} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_{SE}^T = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\dot{P}_{SE} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\dot{R}_{SE} = \begin{bmatrix} -\dot{\theta}\sin(\theta) & -\dot{\theta}\cos(\theta) & 0 \\ \dot{\theta}\cos(\theta) & -\dot{\theta}\sin(\theta) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$R^T_{SE}\,\dot{P}_{SE} = \begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ —— } (1)$$

$$\begin{bmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\sin(\theta) & -\cos(\theta) & 0 \\ \cos(\theta) & -\sin(\theta) & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{\theta} = R^T_{SE} \cdot \dot{R}_{SE}$$

$$\Rightarrow \begin{bmatrix} -\sin\theta\cancel{\cos\theta} + \sin(\theta)\cancel{\cos(\theta)} & -\cos^2(\theta) - \sin^2(\theta) & 0 \\ +\sin^2\theta + \cos^2\theta & \sin\theta\cos\theta - \sin\theta\cos\theta & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{\theta}$$

$$\Rightarrow \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \dot{\theta} \quad ; \quad \omega^b_{se} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \dot{\theta}$$

$$\therefore V^b_{SE} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \dot{\theta}$$

$$V^s_{SE} = \begin{bmatrix} -\dot{R}_{SE} R^T_{SE} P_{SE} + \dot{P}_{SE} \\ (\dot{R}_{SE} R^T_{SE})^\vee \end{bmatrix}$$

$$\Rightarrow -\dot{R}_{SE} R^T_{SE} P_{SE} + \dot{P}_{SE} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\dot{\theta} \begin{bmatrix} -\sin(\theta) & -\cos(\theta) & 0 \\ \cos(\theta) & -\sin(\theta) & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \hat{\omega}$$

$$\Rightarrow \dot{\theta} \begin{bmatrix} -\sin(\theta)\cos(\theta) + \sin(\theta)\cos(\theta) & -\sin^2(\theta) - \cos^2(\theta) & 0 \\ \cos^2(\theta) + \sin^2(\theta) & \sin(\theta)\cos(\theta) - \sin(\theta)\cos(\theta) & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \dot{\theta} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}; \quad V^s_{SE} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \dot{\theta}$$

$$\omega^s_{se} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \dot{\theta}$$

1.4

$\cancel{V_{qE} = V^s_{SE}\, q_E}$

$\cancel{= V_{qE} = V^s_{SE}\, q_E}$

$$\cancel{= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}}$$

$$V_{qE} = \underset{6\times1}{V^s_{SE}}\ \underset{3\times1}{q_E}$$

$\Rightarrow$

$$V_{q_s} = \hat{V}^s_{SE}\, q_E = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ r_e \\ 0 \\ 0 \end{bmatrix}$$

$$V_{q_s} = \begin{bmatrix} -r_e \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$V_{qe} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ r_e \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -r_e \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

1.5 $g_{sm} = R_{sm} q_m + P_{sm}$

$$\Rightarrow g_{sm} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & l_m \\ \sin(\theta) & \cos(\theta) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

when $\theta = \pi$

$$g_{sm} = \begin{bmatrix} -1 & 0 & 0 & l_m \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$g_{sm} = g_{se} \times g_{em}$

$\Rightarrow g_{em} = g_{se}^{-1} g_{sm}$

$\Rightarrow g_{se}^{-1} = R_{se} q_e + P_{se} \Rightarrow q_s$ $\qquad q_e = R_{se}^T q_s - R_{se}^T P_{se}$

$\Rightarrow q_s - P_{se} = R_{se} q_e$

$\Rightarrow g_{se}^{-1} = R_{se}^T q_s - R_{se}^T P_{se} \Rightarrow \begin{bmatrix} R_{SE}^T & -R_{SE}^T \cdot P_{SE} \\ 000 & 1 \end{bmatrix}$

$$g_{SE}^{-1} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$g_{em} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & l_m \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} =$$

$$g_{em} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & l_m \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

1.6 $\hat{V}_{ab}^{b} = g_{ab}^{-1}\,\dot{g}_{ab} = g_{sm}^{-1} \cdot \dot{g}_{sm}$

$$g_{sm} = \begin{bmatrix} \cos(\theta_{sm}) & -\sin\theta_{sm} & 0 & l_{sm}\cos(\phi) \\ \sin(\theta_{sm}) & \cos\theta_{sm} & 0 & l_{sm}\sin(\phi) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\dot{\theta}_{sm} = \dot{\phi}_{sm}$

$$\dot{g}_{sm} = \begin{bmatrix} -\theta_{sm}\sin\theta_{sm} & -\dot{\theta}_{sm}\cos(\theta_{sm}) & 0 & -\dot{\theta}_{sm}\, l_m \sin(\phi) \\ \dot{\theta}_{sm}\cos\theta_{sm} & -\dot{\theta}_{sm}\sin(\theta_{sm}) & 0 & \dot{\theta}_{sm}\, l_m \cos(\phi) \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Plug in $\theta_{sm} = \pi$

$$\Rightarrow \dot{\theta} \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & l_m \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \hat{V}_{sm}^{b}$$

1.7 $V^{s}_{sm} = Ad_{g_{sm}} = \begin{bmatrix} R_{sm} & \hat{P}_{sm} R_{sm} \\ 0 & R_{sm} \end{bmatrix}$

$$P_{sm} = \begin{bmatrix} l_m \\ 0 \\ 0 \end{bmatrix}$$

$$\hat{P}_{sm} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -l_m \\ 0 & l_m & 0 \end{bmatrix}$$

$$\hat{P}_{sm} R_{sm} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -l_m \\ 0 & l_m & 0 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -l_m \\ -l_m \sin\theta & l_m \cos\theta & 0 \end{bmatrix}$$

$$V^{s}_{sm} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 & 0 & 0 \\ -\sin(\theta) & \cos(\theta) & 0 & 0 & 0 & -l_m \\ 0 & 0 & 1 & -l_m \sin\theta & l_m \cos\theta & 0 \\ 0 & 0 & 0 & 0 & l_m & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & -\sin\theta & 0 \\ 0 & 0 & 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow V^{s}_{sm} = \begin{bmatrix} 0 \\ -l_m \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad V^{s}_{sm} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

the origin remains the same from everywhere.

1.8. V^s_{em}

We have V^s_{sm} and V^b_{sm} and V^s_{se} and V^b_{se}

$$V^s_{em} = V^s_{es} + Ad_{g_{es}} V^s_{sm}$$

$$\hat{V}^s_{sm} = \left[\begin{array}{c|c} \hat{\omega}^s_{sm} & v^s_{sm} \\ \hline 000 & 0 \end{array}\right]$$

$$\Rightarrow V^s_{es} = -V^b_{se}$$

$$\Rightarrow (g_{se})^{-1} = g_{es}$$

$$\Rightarrow V^s_{em} = -V^b_{se} + Ad_{(g_{se})^{-1}} V^s_{sm}$$

$$(g_{se})^{-1} = \begin{array}{cc} R_{se} & P_{se} \\ \left[\begin{array}{ccc|c} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] & \end{array}$$

$$\Rightarrow Ad_{g_{se}^{-1}} = \begin{bmatrix} R_{se} & 0 \\ 0_{3\times3} & R_{se} \end{bmatrix}$$

If unclear, pls check at the end. $\longrightarrow$

$$= \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & -\sin\theta & 0 \\ 0 & 0 & 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\dot{V}^s_{em} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1/28 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -1 + 1/28 \end{bmatrix}$$

$$\omega^s_{em} = -1 + \frac{1}{28} = \left| -\frac{27}{28} \right|^{-1}$$

$$\Rightarrow \boxed{\frac{28}{27}}$$

Unclear matrices!

1.8

$$\begin{bmatrix} \cos(\theta) & -\sin(\theta) & 0 & 0 & 0 & 0 \\ \sin(\theta) & \cos(\theta) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos(\theta) & -\sin(\theta) & 0 \\ 0 & 0 & 0 & \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\rightarrow \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$