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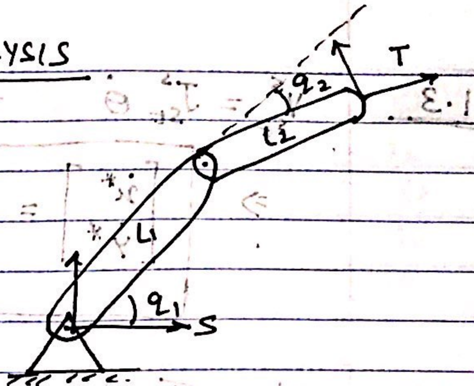
HOMEWORK-3 : ROBOT DYNAMICS & ANALYSIS



$$g_{st}(q) = \begin{bmatrix} \cancel{\cos(q_1+q_2)} & \cancel{-\sin(q_1+q_2)} \\ \cancel{\sin(q_1+q_2)} & \\ \cancel{0} & \\ \cancel{0} & \end{bmatrix}$$

$$g_{st}(q) = \begin{bmatrix} \cos(q_1+q_2) & -\sin(q_1+q_2) & 0 & l_1\cos(q_1) + l_2\cos(q_1+q_2) \\ \sin(q_1+q_2) & \cos(q_1+q_2) & 0 & l_1\sin(q_1) + l_2\sin(q_1+q_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{f(q) = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_1\cos(q_1) + l_2\cos(q_1+q_2) \\ l_1\sin(q_1) + l_2\sin(q_1+q_2) \end{bmatrix}}$$

1.2 Analytic Jacobian:

$$\Rightarrow f(q) = \begin{bmatrix} l_1\cos(q_1) + l_2(\cos(q_1)\cos(q_2) - \sin(q_1)\sin(q_2)) \\ l_1\sin(q_1) + l_2(\sin(q_1)\cos(q_2) + \cos(q_1)\sin(q_2)) \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} -l_1\sin(q_1) + (-l_2\sin(q_1+q_2)) & -l_2\sin(q_1+q_2) \\ l_1\cos(q_1) + l_2\cos(q_1+q_2) & l_2\cos(q_1+q_2) \end{bmatrix}$$

$$\Rightarrow J^a = \begin{bmatrix} -l_1\sin(q_1) - l_2\sin(q_1+q_2) & -l_2\sin(q_1+q_2) \\ l_1\cos(q_1) + l_2\cos(q_1+q_2) & l_2\cos(q_1+q_2) \end{bmatrix}$$

1.3 $V^s_{st} = J^s_{st}\dot{\theta} \Rightarrow V^b_{st} = J^s_{st}\dot{q}$

$$\begin{bmatrix} \dot{x}^* \\ \dot{y}^* \end{bmatrix} = J^a \cdot \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} = J^{a^{-1}} \begin{bmatrix} \dot{x}^* \\ \dot{y}^* \end{bmatrix}$$

$\Rightarrow$ To take determinant of J^a,

$$J^a = \begin{bmatrix} -l\sin(q_1) - l_2\sin(q_1+q_2) & -l_2\sin(q_1+q_2) \\ l\cos(q_1) + l_2\cos(q_1+q_2) & l_2\cos(q_1+q_2) \end{bmatrix}$$

using mathematica,

$$J^{a^{-1}} = \frac{1}{l_1 l_2 \sin(q_2)} \begin{bmatrix} l_2\cos(q_1+q_2) & l_2\sin(q_1+q_2) \\ -l_1\cos(q_1) + l_2\cos(q_1+q_2) & -l_1\sin(q_1) - l_2\sin(q_1+q_2) \end{bmatrix}$$

where $\det J^{a^{-1}} = \dfrac{1}{l_1 l_2 \sin(q_2)}$

$l_1 l_2 \sin(q_2) = 0$ only at 0 and π

1.4 The analytic Jacobian has a singularity at $q_2 = 0$ or π which is when J loses rank. When a singularity occurs a degree of freedom is lost and motion cannot occur.

For linear independence, J^a must be invertible and $q_2 \neq 0$ or $n\pi$

1.5

$$\xi_1 = \begin{bmatrix} -\begin{bmatrix}0\\0\\1\end{bmatrix} \times \begin{bmatrix}0\\0\\0\end{bmatrix} \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix}0\\0\\0\\0\\0\\1\end{bmatrix}$$

$$\xi_2 = \begin{bmatrix} -\begin{bmatrix}0\\0\\1\end{bmatrix} \times \begin{bmatrix}l_1\cos(q_1)\\ l_1\sin(q_1)\\ 0\end{bmatrix} \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix}l_1\sin(q_1)\\ -l_1\cos(q_1)\\ 0\\0\\0\\1\end{bmatrix}$$

$$\therefore J^s_{st} = \begin{bmatrix} 0 & l_1\sin(q_1) \\ 0 & -l_1\cos(q_1) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} ; \quad J^b_{st} = \begin{bmatrix} l_1\sin(q_2) & 0 \\ (l_1+l_2)\cos q_2 & l_2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix}$$

$$J^b_{st}\cdot\dot{q} = \begin{bmatrix} l_1\sin(q_2) & 0 \\ (l_1+l_2)\cos(q_2) & l_2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix}$$

If ~~l_2 q_2 = 0 or~~ π,

If $q_2 = 0$ or π

$$\begin{bmatrix} 0 & 0 \\ l_1 \pm l_2 & l_2 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}$$

$J^b_{st}\,\dot{q}$ will always yield the velocity in the x-direction to be 0.

Hence it is non-singular since the rank is always 2

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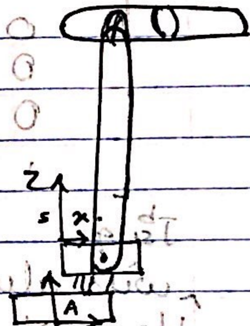
2.1 $g_{st}(0) = \begin{bmatrix} 1 & 0 & 0 & 1407 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1855 \\ 0 & 0 & 0 & 1 \end{bmatrix}$



2.2 $g_{SL1} = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 680 \\ 0 & 0 & 0 & 1 \end{bmatrix}$; $g_{L4L5} = \begin{bmatrix} c_5 & 0 & s_5 & 887 \\ 0 & 1 & 0 & 0 \\ -s_5 & 0 & c_5 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$g_{L1L2} = \begin{bmatrix} c_2 & 0 & s_2 & 320 \\ 0 & 1 & 0 & 0 \\ -s_2 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$; $g_{5T} = \begin{bmatrix} 1 & 0 & 0 & 200 \\ 0 & c_6 & -s_6 & 0 \\ 0 & s_6 & c_6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$g_{L2L3} = \begin{bmatrix} c_3 & 0 & s_3 & 0 \\ 0 & 1 & 0 & 0 \\ -s_3 & 0 & c_3 & 975 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

All these are taken using s-frame from diagram

$g_{L3L4} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_4 & -s_4 & 0 \\ 0 & s_4 & c_4 & 200 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

2.3

$$\xi_1 = \begin{bmatrix} -\omega_1 \times p_1 \\ \omega_1 \end{bmatrix} \Rightarrow -\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 680 \end{bmatrix} = V_1$$

$$\xi_2 = \begin{bmatrix} -\omega_2 \times p_2 \\ \omega_2 \end{bmatrix} \Rightarrow -\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 320 \\ 0 \\ 0 \end{bmatrix} = V_2$$

$$\xi_3 = \begin{bmatrix} -\omega_3 \times p_3 \\ \omega_3 \end{bmatrix} \Rightarrow -\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 975 \end{bmatrix} = V_3$$

$$\xi_4 = \begin{bmatrix} -\omega_4 \times p_4 \\ \omega_4 \end{bmatrix} \Rightarrow -\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 200 \\ 0 \\ 0 \end{bmatrix} = V_4$$

$$\xi_5 = \begin{bmatrix} -\omega_5 \times p_5 \\ \omega_5 \end{bmatrix} \Rightarrow -\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 200 \end{bmatrix} = V_5$$

$$\xi_6 = \begin{bmatrix} -\omega_6 \times p_6 \\ \omega_6 \end{bmatrix} \Rightarrow -\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \times \begin{bmatrix} 887 \\ 0 \\ 0 \end{bmatrix} = V_6$$

2.4 $\| g_{st}(\theta) - g^* \|$

$cost(q) = sum(abs(g_{st}(q) - g^*_{st}), all)$

if $cost(q) < 0.1$

stop.

$$g^* = \begin{bmatrix} 1 & 0 & 0 & 1407 \\ 0 & 1 & 0 & 100 \\ 0 & 0 & 1 & 1855 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\cancel{g_{st}(\theta) =}$