SAGAR SACHDEV

ROBOT DYNAMICS AND ANALYSIS H.W 10

$\mu = 1$

$g = 9.8$

$a_1(x,y) = y$

$a_2(x,y) = x+y+1$

$a_3(x,y) = (x-2)^2 + (y-1)^2 - 2$

$J = \{\{1\}, \{2\}, \{3\}, \{\}, \{1,3\}\}$

$\Gamma = \{(\{\},\{1\}), (\{1\},\{\}), (\{\},\{2\}), (\{2\},\{\}),$
$(\{\},\{3\}), (\{3\},\{\}), (\{\},\{1,3\}), (\{1,3\},\{\}),$
$(\{1\},\{2\}), (\{2\},\{1\}), (\{1\},\{3\}), (\{3\},\{1\})$
$(\{1\},\{1,3\}), (\{1,3\},\{1\}), (\ldots)$

$\tilde{\Gamma} = \{(\{\},\{1\}), (\{\},\{2\}), (\{\},\{3\}), (\{1\},\{2\}),$
$(\{2\},\{1\}), (\{3\},\{\}), (\{1\},\{1,3\})$

$D \Rightarrow D_{\{\}} = \{(q,\dot{q}) \in TQ : a_1(q) \geq 0,\ a_2(q) \geq 0,\ a_3(q) \geq 0\}$

$D_{\{1\}} = \{(q,\dot{q}) \in TQ : a_2(q) \geq 0,\ a_3(q) \geq 0,\ A_1\dot{q} = 0\}$

$D_{\{2\}} = \{(q,\dot{q}) \in TQ : a_1(q) \geq 0,\ a_3(q) \geq 0,\ A_2\dot{q} = 0\}$

$D_{\{3\}} = \{(q,\dot{q}) \in TQ : a_1(q) \geq 0,\ a_2(q) \geq 0,\ A_3\dot{q} = 0\}$

$D_{\{1,3\}} = \{(q,\dot{q}) \in TQ : a_2(q) \geq 0,\ a_1(q) = 0,\ a_3(q) = 0,$
$A_1\dot{q} = 0,\ A_3\dot{q} = 0\}$

Ex: $A_1 = \begin{bmatrix} 0 & 1 \end{bmatrix}$ $\quad x^2 + 4 = 4x + y^2 + 1 - 2y - 2$

$A_2 = \begin{bmatrix} 1 & 1 \end{bmatrix}$ $\qquad A_{1,3} = \begin{bmatrix} 0 & \\ 2x-4 & 2y-2 \end{bmatrix}$

$A_3 = \begin{bmatrix} 2x-4 & 2y-2 \end{bmatrix}$

$M = \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}$ $\quad C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\quad V = mgy$

$N = \begin{bmatrix} 0 \\ mg \end{bmatrix}$

$\Upsilon = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$F_{\{\}} \Rightarrow M\ddot{q} + C\dot{q} + N + \cancel{A^T\lambda} = \Upsilon$

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ mg \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$F_{\{1\}} \Rightarrow M\ddot{q} + C\dot{q} + N + A_1^T\lambda = \Upsilon$

$$\Rightarrow \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ mg \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\lambda = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$F_{\{2\}} \Rightarrow \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ mg \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix}\lambda = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$F_{\{3\}} \Rightarrow \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ mg \end{bmatrix} + \begin{bmatrix} 2x-4 \\ 2y-2 \end{bmatrix}\lambda = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$F_{\{1,3\}} \Rightarrow \begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ mg \end{bmatrix} + \begin{bmatrix} 0 & 2x-4 \\ 1 & 2y-2 \end{bmatrix}\lambda = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Scanned with CamScanner

$$G_{\{\},\{1\}} = \{q, \dot{q} \in D_{\{\}} : a_1(q) = 0,\ A_1\dot{q} < 0\}$$

$$G_{\{\},\{2\}} = \{q, \dot{q} \in D_{\{\}} : a_2(q) = 0,\ A_2\dot{q} < 0\}$$

$$G_{\{\},\{3\}} = \{\dot{q}, \dot{q} \in D_{\{\}} : a_3(q) = 0,\ A_3\dot{q} < 0\}$$

$$G_{\{1\},\{2\}} = \{q, \dot{q} \in D_1 : a_2(q) = 0,\ A_2\dot{q} < 0\}$$

$$G_{\{2\},\{1\}} = \{q, \dot{q} \in D_2 : a_1(q) = 0,\ A_1\dot{q} < 0\}$$

$$G_{\{3\},\{\}} = \{q, \dot{q} \in D_3 : u_3(\lambda) < 0\}$$

$$G_{\{1\},\{1,3\}} = \{q, \dot{q} \in D_1 : a_3(q) = 0,\ A_1\dot{q} < 0,\ A_3\dot{q} < 0\}$$

$$\begin{cases}
R_{\{\},\{1\}}(q, \dot{q}^-) = (q, \dot{q}^+) \in D_1 \\
R_{\{\},\{2\}}(q, \dot{q}^-) = (q, \dot{q}^+) \in D_2 \\
R_{\{\},\{3\}}(q, \dot{q}^-) = (q, \dot{q}^+) \in D_3 \\
R_{\{1\},\{2\}}(q, \dot{q}^-) = (q, \dot{q}^+) \in D_2 \\
R_{\{2\},\{1\}}(q, \dot{q}^-) = (q, \dot{q}^+) \in D_1 \\
R_{\{3\},\{\}}(q, \dot{q}^-) = (q, \dot{q}^+) \in D_{\{\}} \\
R_{\{1\},\{1,3\}}(q, \dot{q}^-) = (q, \dot{q}^+) \in D_{\{1,3\}}
\end{cases}
,\quad \dot{q}^+ = \dot{q}^- - A_J^{*T}(q) A_J(q)\dot{q}^-$$

1.3

For (0,5)

```
te =

    1.0102


contactMode =

     []
```

For (-1.5,5)

```
te =

    0.9583


contactMode =

     []


te =

    1.0595


contactMode =

     2


te =

    1.4447


contactMode =

     1
```

For (1.5,5)

```
te =

    0.7392


contactMode =

     3


te =

    0.8920


contactMode =

     []


te =

    1.3706


contactMode =

     1


te =

    1.7155


contactMode =

     2


te =

    2.2402


contactMode =

     1

~~
```

For (1,5)

```
te =

    0.7825


contactMode =

     []


te =

    1.1404


contactMode =

     []


te =

    1.3041


contactMode =

     1


te =

    2.0866


contactMode =

     2


te =

    3.1298


contactMode =

     1
```