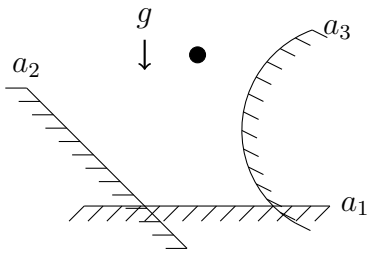
Homework 10: Hybrid Systems Simulation

24-760 Robot Dynamics & Analysis
Fall 2021

Name: _____

Problem 1) Falling Ball



Consider a point particle that can make plastic frictionless impact with several constraints. Assume the particle is mass 1 and gravity is 9.8. Let the constraints be $a_1(x, y) = y$, $a_2(x, y) = x + y + 1$, and $a_3(x, y) = (x - 2)^2 + (y - 1)^2 - 2$.

1.1) What is the hybrid dynamical system for this problem? That is, what are all of the components of $\mathcal{H} = (\mathcal{J}, \Gamma, \mathcal{D}, \mathcal{F}, \mathcal{G}, \mathcal{R})$? Consider both impact (IV complementarity) and liftoff (FA complementarity) transitions. You may limit the hybrid system to only the feasible transitions ($\tilde{\Gamma}$ instead of Γ). For simplicity, assume the particle does not impact multiple constraints at once from the unconstrained mode.

1.2) Simulate the system in Matlab using `ode45` and an event function. The `odefun` should capture the continuous dynamics $\mathcal{F}$, while the event function detects the guard conditions $\mathcal{G}$. Apply the reset function outside of the `ode45` execution. You may want to make separate Matlab functions to calculate $a, A, \dot{A}, \mathcal{F}, \mathcal{R}$, the block matrix inverse, etc. To solve the complementarity problems, CP_{IV} and CP_{FA}, you do not need to use a computationally efficient algorithm, simply check the complementarity conditions for all possible modes (modes in the local scope, $\mathcal{I}$) and return the (hopefully unique) mode that satisfies the constraints. Here are two pages documenting these Matlab features:
https://www.mathworks.com/help/matlab/ref/ode45.html
https://www.mathworks.com/help/matlab/math/ode-event-location.html
Hint: Start with just a single constraint a_1, and then add in a_2 and a_3. If your simulation is missing events, you may want to try using the `MaxStep` option.

1.3) Run four simulations starting at $(0, 5)$, $(-1.5, 5)$, $(1.5, 5)$, and $(1, 5)$ with zero velocity. Run each simulation for 5 seconds. What contact mode transitions occur and at what times?