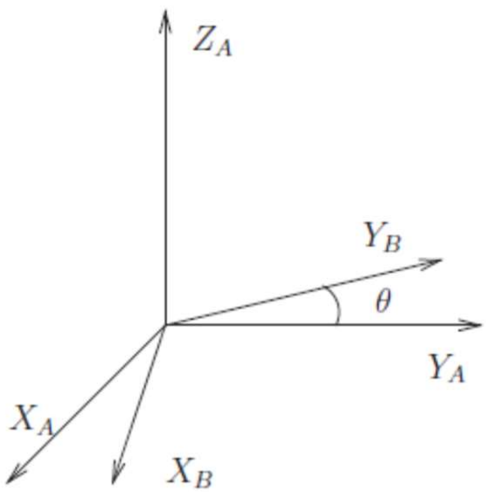
# Introduction to Robotics

*Soumya S*

# Orientation-Introduction

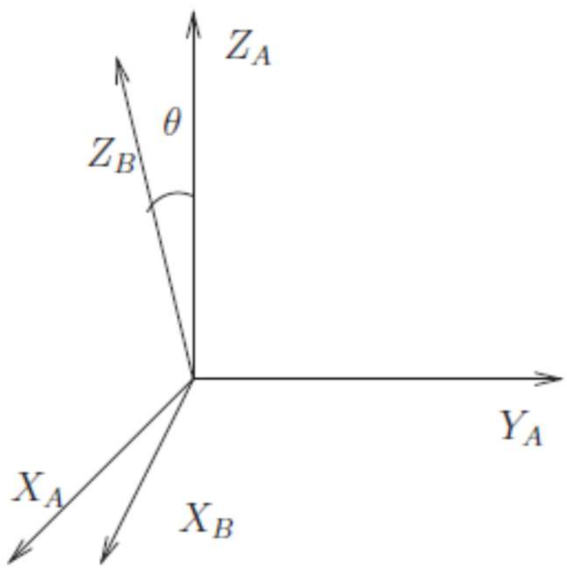
# Rotation On Z Axis




$$\begin{array}{c} \\ X_A \\ Y_A \\ Z_A \end{array} \begin{array}{c} \begin{array}{ccc} X_B & Y_B & Z_B \end{array} \\ \begin{bmatrix} Cos\theta & -Sin\theta & 0 \\ Sin\theta & Cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$X_A \,.\, X_B = a.a\ Cos\theta$$
$$X_A \,.\, Y_B = a.b\ Cos(90+\theta) = -sin\theta$$
$$Y_A \,.\, X_B = a.b\ Cos(90-\theta) = Sin\theta$$
$$Y_A \,.\, Y_B = b.b\ Cos\theta$$

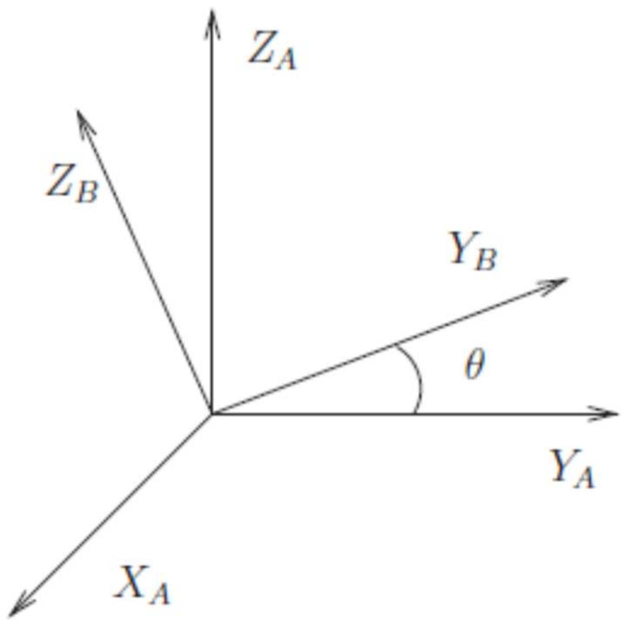
# Rotation On  Y Axis




$$
\begin{array}{c c}
 & \begin{array}{ccc} X_B & Y_B & Z_B \end{array} \\
\begin{array}{c} X_A \\ Y_A \\ Z_A \end{array} & \begin{bmatrix} Cos\theta & 0 & Sin\theta \\ 0 & 1 & 0 \\ -Sin\theta & 0 & Cos\theta \end{bmatrix}
\end{array}
$$

$$X_A\,.X_B = a.a\;Cos\theta$$
$$X_A\,.Z_B = a.b\;Cos(90-\theta) = sin\theta$$
$$Z_A\,.X_B = a.b\;Cos(90+\theta) - Sin\theta$$
$$Z_A\,.Z_B = b.b\;Cos\theta$$

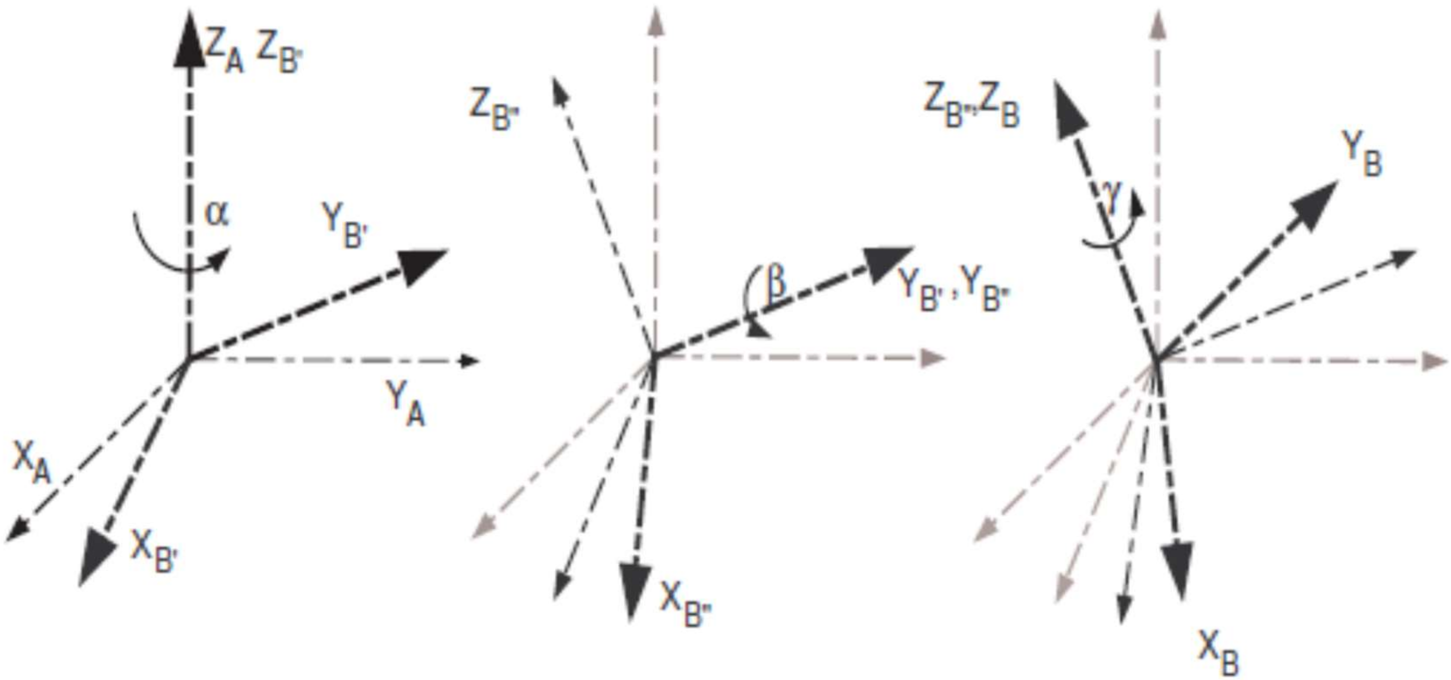
## Rotation On  X Axis




$$
\begin{array}{cc}
 & \begin{array}{ccc} X_B & Y_B & Z_B \end{array} \\
\begin{array}{c} X_A \\ Y_A \\ Z_A \end{array} & \begin{bmatrix} 1 & 0 & 0 \\ 0 & Cos\theta & -Sin\theta \\ 0 & Sin\theta & Cos\theta \end{bmatrix}
\end{array}
$$

$$X_A \,.\, X_B = 1$$
$$Y_A \,.\, Y_B = a.b\; Cos(\theta) = Cos(\theta)$$
$$Y_A \,.\, Z_B = a.b\; Cos(90 + \theta) = -Sin(\theta)$$
$$Z_A \,.\, Y_B = a.b\; Cos(90 - \theta) = \sin(\theta)$$

## Z-Y-Z Euler Axis



$$ {}^{A}_{B}R = {}^{A}_{B'}R \; {}^{B'}_{B''}R \; {}^{B''}_{B}R $$

$$ \begin{aligned} {}^{A}_{B'}R &= R(\hat{Z}_A, \alpha) = R_Z(\alpha) \\ {}^{B'}_{B''}R &= R(Y_{B'}, \beta) = R_Y(\beta) \\ {}^{B''}_{B}R &= R(Z_{B''}, \gamma) = R_Z(\gamma) \end{aligned} $$

$$ {}^{A}_{B}R = R_Z(\alpha)\, R_Y(\beta)\, R_Z(\gamma) $$

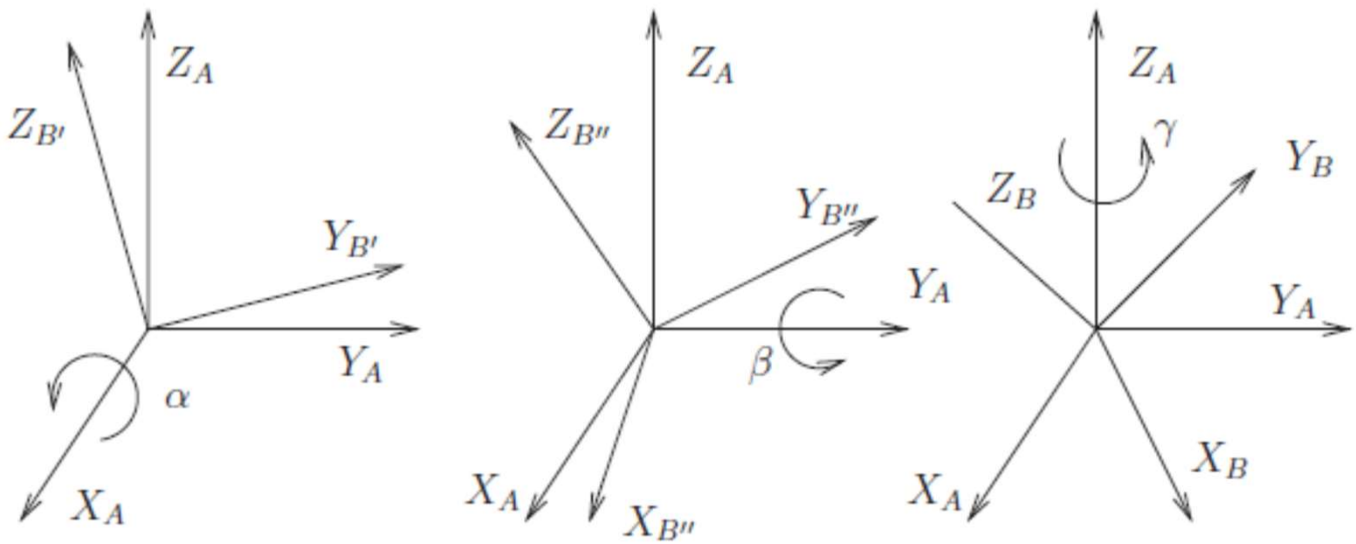
$$ = \begin{pmatrix} c\alpha c\beta c\gamma - s\alpha s\gamma & -c\alpha c\beta s\gamma - s\alpha c\gamma & c\alpha s\beta \\ s\alpha c\beta c\gamma + c\alpha s\gamma & -s\alpha c\beta s\gamma + c\alpha c\gamma & s\alpha s\beta \\ -s\beta c\gamma & s\beta s\gamma & c\beta \end{pmatrix} $$

$$
{}^{A}_{B}R \;=\; R_Z(\alpha)\, R_Y(\beta)\, R_Z(\gamma)
$$

$$
= \begin{pmatrix} c\alpha c\beta c\gamma - s\alpha s\gamma & -c\alpha c\beta s\gamma - s\alpha c\gamma & c\alpha s\beta \\ s\alpha c\beta c\gamma + c\alpha s\gamma & -s\alpha c\beta s\gamma + c\alpha c\gamma & s\alpha s\beta \\ -s\beta c\gamma & s\beta s\gamma & c\beta \end{pmatrix}
$$

Matrix multiplication of rotation matrices is not commutative

# X-Y-Z Fixed Angles



$$
\begin{aligned}
{}^{A}_{B'}R &= R(\hat{X}_A, \alpha) = R(X_{B'}, \alpha) = R_X(\alpha) \\
{}^{B'}_{B''}R &= R(Y_A, \beta) \neq R_Y(\beta) \\
{}^{B''}_{B}R &= R(\hat{Z}_A, \gamma) \neq R_Z(\gamma)
\end{aligned}
$$

$$
\begin{aligned}
{}^{A}X_{B''} &= R_Y(\beta)\,{}^{A}X_{B'} \\
{}^{A}Y_{B''} &= R_Y(\beta)\,{}^{A}Y_{B'} \\
{}^{A}Y_{B''} &= R_Y(\beta)\,{}^{A}Y_{B'}
\end{aligned}
$$

$$
{}^{B'}_{B''}R = R_X^T(\alpha)\ R_Y(\beta)\ R_X(\alpha)
$$

$$
{}^{A}_{B}R = R_Z(\gamma)\ R_Y(\beta)\ R_X(\alpha)
$$