Introduction to Robotics

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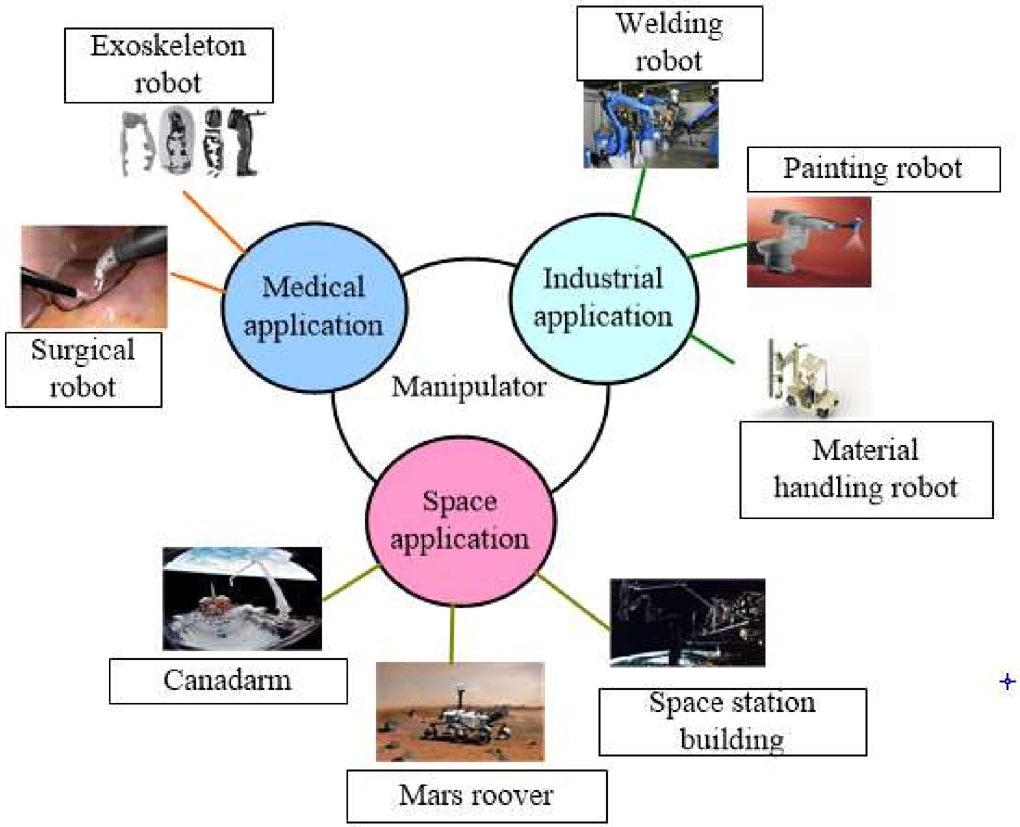


Pick and Place - Revel Robotic Manipulator.mp4

Introduction

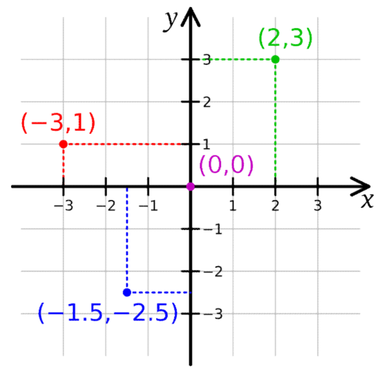
- Need of representing position and orientation???

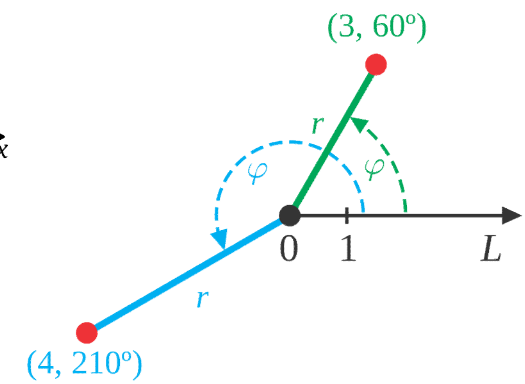
- Parts and Tools are moving around space by some sort of mechanism.

- Using our knowledge on coordinate systems and conventions , we can manipulate mathematical quantities representing position and orientation.

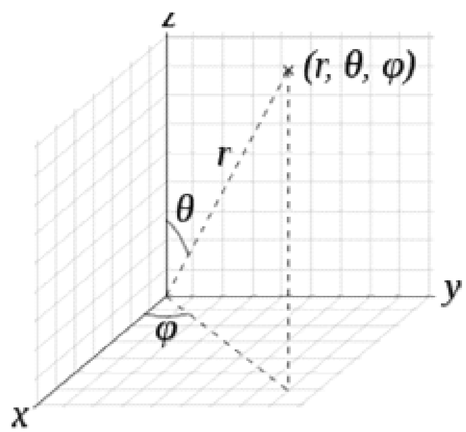
  - **Universe co-ordinate system**

- **Cartesian Coordinate**: coordinate system that specifies each point uniquely in a plane by a set of numerical **coordinates**
- **Polar Coordinate**: Two-dimensional coordinate system in which each point on a plane is determined by a distance from a reference point and an angle from a reference direction.

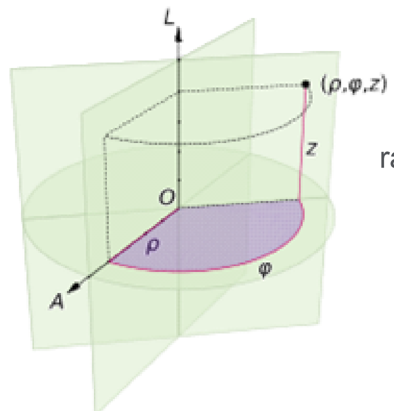




Radial distance r, polar angle θ (theta), and azimuthal angle φ (phi)



Spherical coordinates



Cylindrical coordinates

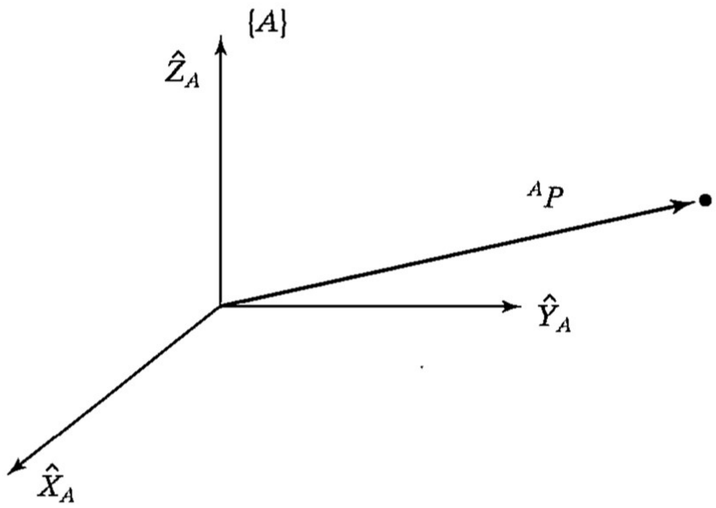
radial distance ρ , angular coordinate φ , and height z .

Position:

- we can locate any point in the universe with a 3 x 1 position vector.

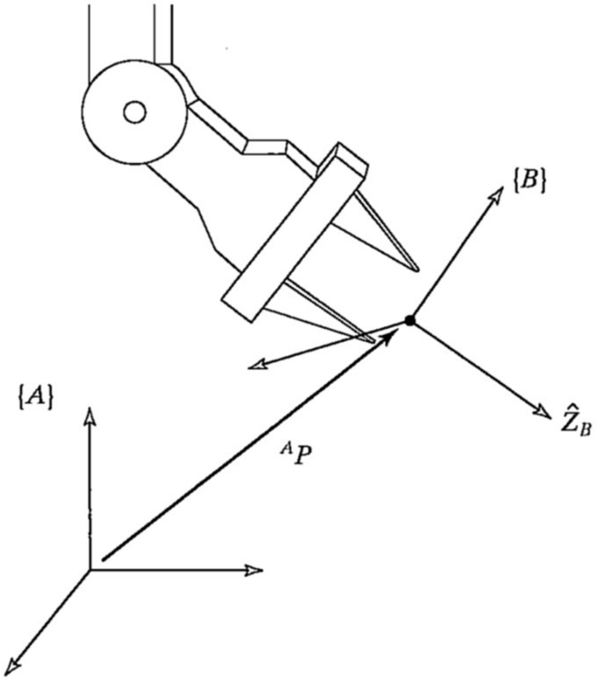
$^{A}p \longrightarrow$ Components of ^{A}P have numerical values that indicate distances along the axes of {A}

$$^{A}P = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}$$

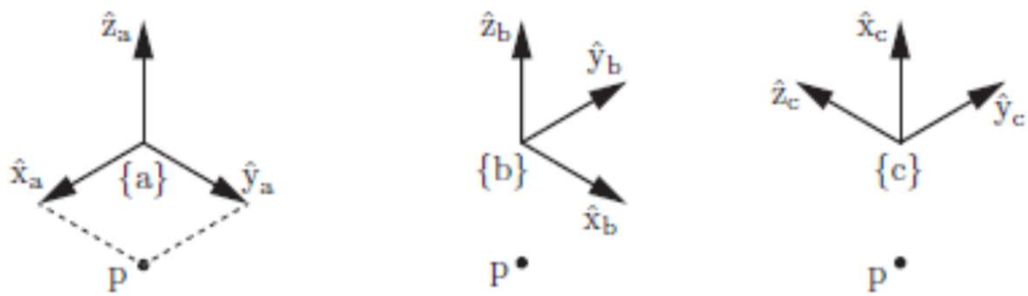


Orientation:

- To describe the orientation of a body, we will attach a coordinate system to the body and then give a description of this coordinate system relative to the reference system



Description of {B} relative to (A) gives the Orientation of the body.



- Positions of points are described with vectors
- Orientations of bodies are described with an attached coordinate system

$$\{B\} = \hat{X}_B \quad \hat{Y}_B \quad \hat{Z}_B$$

Unit Vector giving principle direction of coordinated system {B}

Principle direction of coordinated system {B} with respect to {A} $= \quad {}^{A}\hat{X}_B \quad {}^{A}\hat{Y}_B \quad {}^{A}\hat{Z}_B$

When we stack these three unit vectors together as the columns of a 3 x 3 matrix

Describes Orientation of {B} W.R.T {A}

Rotation matrix

$$^{A}_{B}R = \left[\, {}^{A}\hat{X}_B \quad {}^{A}\hat{Y}_B \quad {}^{A}\hat{Z}_B \,\right] = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}.$$

$$
{}^{A}_{B}R = \left[\ {}^{A}\hat{X}_B \quad {}^{A}\hat{Y}_B \quad {}^{A}\hat{Z}_B\ \right] = \begin{bmatrix} \hat{X}_B \cdot \hat{X}_A & \hat{Y}_B \cdot \hat{X}_A & \hat{Z}_B \cdot \hat{X}_A \\ \hat{X}_B \cdot \hat{Y}_A & \hat{Y}_B \cdot \hat{Y}_A & \hat{Z}_B \cdot \hat{Y}_A \\ \hat{X}_B \cdot \hat{Z}_A & \hat{Y}_B \cdot \hat{Z}_A & \hat{Z}_B \cdot \hat{Z}_A \end{bmatrix}.
$$

Dot product of two unit vectors yields the cosine of the angle between them
Components of rotation matrices are often referred to as Direction cosines.

$$
{}^{A}_{B}R = \left[\ {}^{A}\hat{X}_B \quad {}^{A}\hat{Y}_B \quad {}^{A}\hat{Z}_B\ \right] = \begin{bmatrix} {}^{B}\hat{X}_A^T \\ {}^{B}\hat{Y}_A^T \\ {}^{B}\hat{Z}_A^T \end{bmatrix}.
$$

$$
{}^{B}_{A}R = {}^{A}_{B}R^T.
$$

$$
{}^{A}_{B}R = {}^{B}_{A}R^{-1} = {}^{B}_{A}R^T.
$$

$$
{}^{A}_{B}R^T\, {}^{A}_{B}R = \begin{bmatrix} {}^{A}\hat{X}_B^T \\ {}^{A}\hat{Y}_B^T \\ {}^{A}\hat{Z}_B^T \end{bmatrix} \left[\ {}^{A}\hat{X}_B \quad {}^{A}\hat{Y}_B \quad {}^{A}\hat{Z}_B\ \right] = I_3,
$$