Mod 6 Numerical Methods

① To find root sol eq^n

① Bisection ⓑ Regulare falsi ⓒ Newton Raphson

② Solve system of lineare equation

ⓐ Gauss Jacobi ⓑ Gauss Seidel

Algebric Expressions

Expressions involving only polynomials

Trancendental Expression

That has fun^n other than polynomials

Eg $\sin x$, $\tan^{-1}(x)$ $\boxed{e^x + x^2 + \sin x}$

Bisectiom Method.

To find root of $f(x)$ OR To solve $f(x) = 0$

Let the root lie in interval (a, b)

$x_0 = \frac{a+b}{2} \longrightarrow (a, b)$

$x_1 = \frac{x_0 + b}{2} \longrightarrow (x_0, b)$

$x_2 = \frac{x_0 + x_1}{2} \longrightarrow (x_0, x_1)$

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Q1 Using Bisection Meth to sol $x^4 = 5x - 5$
Correct upto 2 decimal

→

Solve $x^4 - 5x - 5x = 0$

$f(x) = x^4 - 5x - 5$

$f(-1) = 1 > 0$

$f(0) = -5 < 0$

$f(-0.5) = -2.4375 < 0$

Root lies in $(-1, -0.5)$

$x_0 = \frac{-1 + (-0.5)}{2} = -0.75$

$f(x_0) = f(0.75) = -0.9335 < 0$

Root lies in $(-1, -0.75)$

$x_1 = \frac{-1 + (0.75)}{2} = -0.875$

$f(x_1) = f(0.875) = -0.0388 < 0$

Root lies in $(-1, -0.875)$

$x^2 = \frac{-1 + (-0.875)}{2} = -0.937$

$f(x_2) = f(0.937) = 0.4599 > 0$

Root lies in $(-0.9375, -0.875)$

$x^3 =$

$x^3 = \frac{-0.9375 + (-0.875)}{2} = -0.9062$

$f(x_3) = f(-0.9062) = 0.2053 > 0$

Root lies in $(-0.9062, -0.875)$

$x_4 = \frac{-0.9062 + (-0.875)}{2} = -0.8906$

$f(x_4) = f(-0.8906) - 0.08211 > 0$

Root lies in $(-0.8906, -0.875)$

$$x_S = \frac{-0.8906 + (-0.975)}{2} \approx -0.9328$$

$$f(x_S) = f(-0.9328) = \quad > 0$$

$$(-0.9328, -0.975)$$

$$x_0 = \frac{-0.9325 + (-0.975)}{2}$$

$$\approx 0.9789$$

Numerical methods

Bisection method

Step ① use ure calculator to find out an interval [a,b] with a,b having an opposite sign

Step ② Calculate the midpoints of the interval by using the formula $c = \frac{a+b}{2}$

Step ③ check the midpoints f(c) if f(c) = 0 the root lies at the point c

if f(c) has the same sign to that of f(a) then the new interval becomes [c, b]

if f(c) has the same sign to that of f(b) then the new interval becomes [a, c]

Step ④ repeat the step 2 & 3 until you're sufficiently close to zero or asper the nos of iterations told by them

Newton Raphson method

Step ① use your calculator to find the interval at which $f(x)$ has opposite sign

Step ② Always take initial approximation to be x_0 in the cases of newton Raphson

Step ③ calculate $f(x_n)$ & $f'(x_n)$

Step ③ Compute the approximation by using the formula

$$\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}}$$

Step ④ continue the approximation until $|x_{n+1} - x_n| < \text{error}$

oh & btw hope you only put out the value of $f(x_n)$ or $f'(x_n)$ & not the whole fucking thing

Regula falsi method

Step ① choose a & b in such an way that f(a) & f(a) are that of the opposite sign

Step ② Count the approximations using this formula

$$\boxed{x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)}}$$

Step ③ if $f(x_1)$ & $f(a)$ have the opposite sign then the interval lies in between (x_1, a) & we've to replace b by x_1 & the formula becomes something like

$$x_2 = \frac{a\,f(x_1) - x_1 \cdot f(a)}{f(x_1) - f(a)},$$

the same would be true if $f(x_1)$ & $f(b)$ have opposite sign then we'd have to replace $a \rightarrow x_1$

Step ④ repeat Step ③ until desired Accuracy've been achieved

Gauss jacobi Method

(*) only applicable for an system of linear equations

Step ① get an system of linear equations

$$\left.\begin{aligned} a_1x + b_1y + c_1z &= d_1 \\ a_2x + b_2y + c_2z &= d_2 \\ a_3x + b_3y + c_3z &= d_3 \end{aligned}\right\} = \text{(I)}$$

Step ② In each one of them you'd need to find which numerical is greater amongst themselves

$|a_1| > |b_1| + |c_1| \qquad |b_2| > |a_2| + |c_2| \qquad |c_3| > |a_3| + |b_3|$

Step ③ now the system of the equation can be written as

$$\left.\begin{aligned} x &= \frac{1}{a_1}[d_1 - b_1y - c_1z] \\ y &= \frac{1}{b_2}[d_2 - a_2y - c_2z] \\ z &= \frac{1}{c_3}[d_3 - b_3y - a_3z] \end{aligned}\right\} - \text{(II)}$$

Step ④ If x^0, y^0, z^0 are the initial values of x, y, z respectively then

$$x^1 = \frac{1}{a_1}[d_1 - b_1 y^0 - c_1 z^0]$$

$$y^1 = \frac{1}{b_2}[d_2 - a_2 y^0 - c_2 z^0] \quad \text{— III}$$

$$z^1 = \frac{1}{c_3}[d_3 - b_3 y^0 - a_3 z^0]$$

basically put the initial values in equation no. II

$$x^2 = \frac{1}{a_1}[d_1 - b_1 y^1 - c_1 z^1]$$

$$y^2 = \frac{1}{b_2}[d_2 - a_2 y^1 - c_2 z^1] \quad \text{— IV}$$

$$z^2 = \frac{1}{c_3}[d_3 - b_3 y^1 - a_3 z^1]$$

Step 5 Proceed till convergence is assured

(*) Gauss Siedl is an method just like gauss Jacobi the only real diffrence is that you just keep on updating the values of x, y, z in each of the step so

say let x^0, y^0, z^0 be the initial values, then

$$x^1 = \frac{1}{a_1}(z_1 - b_1 y^0 - c_1 z^0]$$

$$y^1 = \frac{1}{b_2}(z_2 - a_2 x^1 - c_2 z^0) \quad \ldots$$

so on you get it

$$z^1 = \frac{1}{c_3}\left[z_3 - a_3 x^1 - b_3 y^1\right]$$

upon completion the next step would be to substitute the values of x^1, y^1, z^1 in x^2, y^2, z^2

To Solve using the gauss Jacobi Method

(a) $10x - 5y - 2z = 3;\ 4x - 10y + 3z = -3;\ x + 6y + 10z = -3$

So the 3 equations are:-

$$\left.\begin{aligned} 10x - 5y - 2z &= 3 \\ 4x - 10y + 3z &= -3 \\ x + 6y + 10z &= -3 \end{aligned}\right\}$$

$|10| > |-5| + |-2|$, $|-10| > |4| + |3|$, $|10| > |1| + |6|$

now let the initial values of each be x^0, y^0, z^0

. the formulas - $x' = \frac{1}{a_1}[d_1 - b_1 y - c_1 z]$

$$y' = \frac{1}{b_2}[d_2 - a_2 x - c_2 z]$$

$$z' = \frac{1}{c_3}[d_3 - a_3 x - b_3 y]$$

upon substituting the values

$$x' : \frac{1}{10}[3 - (-5)(1) - (-2)(1)]$$

$$x' = \frac{1}{\cancel{10}}[\cancel{3 + 5 + 2}]^{1} \quad \boxed{x' = 1}$$

$$\left.\begin{aligned} 10x - 5y - 2z &= 3 \\ 4x - 10y + 3z &= -3 \\ x + 6y + 10z &= -3 \end{aligned}\right\}$$

$$y' = \frac{1}{10}[-3 - (4)(1) - 3(1)]$$

$$y' = \frac{1}{10}(-10) \qquad \boxed{y' = -1}$$

$$z' \cdot \frac{1}{10}(-3 - (1)(1) - 6(1)]$$

$$\boxed{z' = -1}$$

$$\therefore \boxed{x' = 1,\ y' = -1,\ z' = -1}$$

on doing TS Again

Matrices

1. If A is orthogonal, find a, b, c where $A = \frac{1}{9}\begin{bmatrix} -8 & 4 & a \\ 1 & 4 & b \\ 4 & 7 & c \end{bmatrix}$

we know that condition of orthogonality $= \boxed{AA^t = I}$

$$\frac{1}{81}\begin{bmatrix} -8 & 4 & a \\ 1 & 4 & b \\ 4 & 7 & c \end{bmatrix}\begin{bmatrix} -8 & 1 & 4 \\ 4 & 4 & 7 \\ a & b & c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \frac{1}{81}\begin{bmatrix} -64+16a^2 & & \\ & 1+16+b^2 & \\ & & 16+49+c^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$-64+16+a^2 = 81$, $1+16+b^2 = 81$, $16+49+c^2 = 81$

$\boxed{a = 1135}$ $\boxed{b = 8}$ $\boxed{c = 4}$

2. Show that $\begin{bmatrix} \cos\phi\cos\theta & \sin\phi & \cos\phi\sin\theta \\ -\sin\phi\cos\theta & \cos\phi & -\sin\phi\sin\theta \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$ is orthogonal and find its inverse

TPT is a orthogonal $\begin{bmatrix} \cos\phi\cos\theta & \sin\phi & \cos\phi\sin\theta \\ -\sin\phi\cos\theta & \cos\phi & -\sin\phi\sin\theta \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$

$$\begin{bmatrix} \cos\phi\cos\theta & \sin\phi & \cos\phi\sin\theta \\ -\sin\phi\cos\theta & \cos\phi & -\sin\phi\sin\theta \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

Condition for orthogonality $\boxed{A A^T = I}$

$$\begin{bmatrix} \cos\phi\cos\theta & \sin\phi & \cos\phi\sin\theta \\ -\sin\phi\cos\theta & \cos\phi & -\sin\phi\sin\theta \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} \cos\phi\cos\theta & -\sin\phi\cos\theta & -\sin\theta \\ \sin\phi & \cos\phi & 0 \\ \cos\phi\sin\theta & -\sin\phi\sin\theta & \cos\theta \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\cos^2\phi\cos^2\theta + \sin^2\phi + \cos^2\phi\sin^2\theta = 1 \quad . \quad (1)$$

$$+\sin^2\phi\cos^2\theta + \cos^2\phi + \sin^2\phi\sin^2\theta = 1 \quad (2)$$

$\boxed{\sin^2\theta + \cos^2\theta = 1}$. This condition is always true

$$\cos^2\phi\cos^2\theta + \sin^2\phi + \cos^2\phi\sin^2\theta = 1$$

$$\cos^2\phi(\cos^2\theta + \sin^2\theta) + \sin^2\phi = 1$$

$\boxed{\cos^2\phi + \sin^2\phi = 1}$. This condition is also always true

Eqn (2) $+\sin^2\phi\cos^2\theta + \cos^2\phi + \sin^2\phi\sin^2\theta = 1$

$$\sin^2\phi(\cos^2\theta + \sin^2\theta) + \cos^2\phi = 1$$

$\boxed{\sin^2\phi + \cos^2\phi = 1}$ Always true condition hence proved that this matrix is indeed orthogonal

Since we know this is an orthogonal matrix hence

$$\boxed{A^{-1} = A^T}$$

$$\begin{bmatrix} \cos\phi\cos\theta & -\sin\phi\cos\theta & -\sin\theta \\ \sin\phi & \cos\phi & 0 \\ \cos\phi\sin\theta & -\sin\phi\sin\theta & \cos\theta \end{bmatrix} = A^{-1}$$

3. Prove that $\begin{bmatrix} \frac{2+i}{3} & \frac{2i}{3} \\ \frac{2i}{3} & \frac{2-i}{3} \end{bmatrix}$ is unitary.

Anything can be called as unitary matrix if $\boxed{AA^\theta = I}$

& $\boxed{A^\theta = A^{-1}}$

How to find inverse of an 2x2 matrix instantly -

the matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\left(\frac{2+i}{3}\right)\left(\frac{2-i}{3}\right) - \left(\frac{2i}{3}\right)\left(\frac{2i}{3}\right) \Rightarrow \frac{4-i^2}{9} - \left(\frac{4i^2}{9}\right)$$

$$\Rightarrow \frac{4+1+1}{9} \Rightarrow \cancel{\frac{6}{9}}\ \frac{2}{3}$$

$$\frac{3}{2}\begin{bmatrix} \frac{2-i}{3} & -2i/3 \\ -2i/3 & \frac{2+i}{3} \end{bmatrix}$$

$$A A^{\theta} = I$$

This must be the Conjugate transpose

$$\begin{bmatrix} \frac{2+i}{3} & 2i/3 \\ 2i/3 & \frac{2-i}{3} \end{bmatrix} \begin{bmatrix} \frac{2-i}{3} & -2i/3 \\ -2i/3 & \frac{2+i}{3} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Instead of finding the inverse of an matrix u can also directly take the transpose for the said equation $\Rightarrow A A^{\theta} = I$, $A^{\theta} = A^{-1} = A^{T}$ (if orthogonal)

$$\left(\frac{2+i}{3}\right)\left(\frac{2-i}{3}\right) + \left(\frac{2i}{3}\right)\left(\frac{2i}{3}\right) = 1$$

$$\Rightarrow \frac{4+1}{9} + \frac{-4(-1)}{9} \Rightarrow \frac{9}{9} = 1$$

now similarly applying this to other parts we get

$A A^{T} = I$ hence its unitary

where A^{T} · Conjugate transpose

4. Show that the matrix $\begin{bmatrix} a+ib & -c+id \\ c+id & a-ib \end{bmatrix}$ is unitary if $a^2+b^2+c^2+d^2=1$

As noticed earlier $A A^T = I$ where A^T = Conjugate transpose

$$\begin{bmatrix} a+ib & -c+id \\ c+id & a-ib \end{bmatrix}\begin{bmatrix} a-ib & +c-id \\ -c-id & a+ib \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$(a+ib)(a-ib) + (-c+id)(c-id) = 1$

$\Rightarrow a^2 - i^2b^2 - \; -c^2 - \cancel{cid} + \cancel{cid} - i^2d^2$

$\boxed{a^2+b^2+c^2+d^2 = 1}$

& now for (2,2) 2nd row, 2nd column

we get $\Rightarrow (c+id)(c-id) + (a+ib)(a-ib) = 1$

$\Rightarrow c^2 - (i)^2d^2 + a^2 - (ib)^2 = 1$

$\boxed{a^2+b^2+c^2+d^2 = 1}$

hence Proved that this equation is unitary only when

$a^2+b^2+c^2+d^2 = 1$

5. Find the value of p for which the matrix $A = \begin{bmatrix} p & p & 2 \\ 2 & p & p \\ p & 2 & p \end{bmatrix}$ will have

(i) rank 1 (ii) rank 2 (iii) rank 3

In order to solve these bitchass kindof sums here's the step

Step ① Calculate the determinant value of A

Step ② find the values of p for which the value of determinant is equals to "0"

Step ③ Determine the rank for different values of P

(iii) Rank ③ 3x3 has rank 3 if its determinant is non zero

(ii) Rank 2 if det (A) = 0 but at least one 2x2 minor is non zero

(i) Rank 1 :- if all 2x2 minors are zero but atleast one 1x1 minor (element) is non zero

now Apply those Steps

$$\det(A) = \begin{array}{c} \begin{matrix} + & - & + \end{matrix} \\ \begin{bmatrix} p & p & 2 \\ 2 & p & p \\ p & 2 & p \end{bmatrix} \end{array}$$

$$\Rightarrow p(p^2 - 2p) - p(2p - p^2) + 2(4 - p^2)$$

$$\Rightarrow \cancel{p^3} - 2p^2 - 2p^2 + \cancel{p^3} + 8 - 2p^2$$

$$\Rightarrow 2p^3 - 6p^2 + 8$$

to factor the expression to find values for P where values of determinants are (0)

Step ② Determine the values of P for rank 3

for an matrix of rank ③ if $\det(A) \neq 0$

$2(P+1)(p-2)^2 \neq$ when $p \neq -1$ & $P \neq 2$

the values for rank ②

if $\det(A) = 0$ but there exists atleast 1 non zero

2x2 minor for the determinant is 0 for $P=-1$ & $P=2$

it should be -1

Step ④ for the values of rank ①

if all 2x2 minors are 0 This only occur when $P=2$

& first row is non zero

7. Reduce the following matrices to normal form and find their rank:

(a) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & 6 & 10 \\ 4 & 6 & 8 \\ 15 & 27 & 39 \end{bmatrix}$ (d) $\begin{bmatrix} -\frac{3}{4} & \frac{9}{5} & -\frac{1}{2} \\ \frac{30}{2} & -18 & 5 \\ \frac{57}{4} & -\frac{81}{5} & \frac{9}{2} \end{bmatrix}$

(a) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix}$ by using the formula

$$A = I_n A I_m$$

where $I_m \Rightarrow$ nos of rows & I_n = nos of columns

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & -3 & 4 \\ 3 & -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 - R_1$, $R_3 - 3R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -4 & 3 \\ 0 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -3 & 0 & 0 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_2 - C_1$. $C_3 - C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & -5 & 2 \\ 0 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & 0 & 0 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 / -2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1/-2 & +5/2 & 1 \\ 0 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1/2 & 0 \\ 3 & 0 & 0 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 \mid -2 & +5/2 & 1 \\ 0 & -5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1/2 & 1/2 & 0 \\ 3 & 0 & 0 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Vector calculus

(a) $f(x, y, z) = xy^2 + yz^3$ at the point (2,-1,1) in the direction of the vector $\hat{i} + 2\hat{j} + 2\hat{k}$

let $\phi = xy^2 + yz^3$

$$\nabla\phi = \hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z}$$

$$\nabla\phi = \hat{i}(y^2) + \hat{j}(2yx + z^3) + \hat{k}(3z^2y)$$

$$= \nabla\phi_{(2,-1,1)} = \hat{i}(-1)^2 + \hat{j}(2(-1)(2) + (1)^3 + \hat{k}(3(1)^2(-1)$$

$$= \nabla\phi_{(2,-1,1)} = \hat{i}(1) + \hat{j}(-3) + \hat{k}(2)$$

$$= \hat{i} - 3\hat{j} + 2\hat{k}$$

let $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$

$\vec{b} = \hat{i} + 2\hat{j} + 2\hat{k}$

$$\overline{AB} = \vec{b} - \vec{a} = (\hat{i} + 2\hat{j} + 2\hat{k}) - (-2\hat{i} - \hat{j} + \hat{k})$$

$$= \hat{i} + 2\hat{j} + 2\hat{k} + 2\hat{i} + \hat{j} - \hat{k}$$

$$= 3\hat{i} + 3\hat{j} + \hat{k}$$

the formula for the directional derivative

$$DD = \frac{\nabla\phi \, \overline{AB}}{|\overline{AB}|} \Rightarrow \frac{(\hat{i} - 3\hat{j} + 2\hat{k})(3\hat{i} + 3\hat{j} + \hat{k})}{\sqrt{3^2 + 3^2 + 1^2}}$$

$$\Rightarrow \frac{3 - 3 + 2}{\sqrt{9+9}} \Rightarrow \underline{\underline{\frac{2}{\sqrt{19}}}} = DD$$

(b) $\phi = x^2yz + 4xz^2$ at the point (1,-2,-1) in the direction of the vector $2\hat{i} - \hat{j} - 2\hat{k}$

firstly $\nabla\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$

$$\nabla\phi = (2xyz + 4z^2)\hat{i} + (x^2z)\hat{j} + (x^2y + 8xz)\hat{k}$$

$$\nabla\phi_{(1,-2,-1)} = (2(1)(-2)(-1) + 4(-1)^2)\hat{i} + (1)^2(-1)\hat{j} + ((1)^2(-2) + 8(1)(-1))\hat{k}$$

$$\nabla\phi_{(1,-2,-1)} = (-5+4)\hat{i} + (-\hat{j}) + (-2-8)\hat{k}$$

$$= (-1)\hat{i} + (-\hat{j}) - 10\hat{k}$$

$$\overline{AB} = \bar{b} - \bar{a}$$

$$\bar{a} = \hat{i} - 2\hat{j} - \hat{k}$$

$$= (2\hat{i} - \hat{j} - 2\hat{k}) - (\hat{i} - 2\hat{j} - \hat{k})$$

$$\bar{b} = 2\hat{i} - \hat{j} - 2\hat{k}$$

$$= \hat{i} + \hat{j} - \hat{k}$$

Directional derivative $= \dfrac{\nabla\phi \cdot \overline{AB}}{|\overline{AB}|}$

$$= \frac{\underbrace{((-1)\hat{i} + (-\hat{j}) - 10\hat{k})(\hat{i} + \hat{j} - \hat{k})}}{\sqrt{1^2 + 1^2 + (-1)^2}}$$

$$= \frac{-1 - 1 + 10}{\sqrt{3}} = \underline{\underline{\frac{8}{\sqrt{3}}}} = DD$$

(c) $\phi = 4xz^2 - 3x^2yz^2$ at (2,-1,2) along $\boxed{\text{z - axis.}}$ $\Rightarrow$ $(0, 0, 1)$

formula $\nabla\phi = \dfrac{\partial\phi}{\partial x}\hat{i} + \dfrac{\partial\phi}{\partial y}\hat{j} + \dfrac{\partial\phi}{\partial z}\hat{k}$

$$\nabla\phi = (4z^2 - 6xyz^2)\hat{i} + (-3x^2z^2)\hat{j} + \hat{k}(8xz - 6x^2yz)$$

$$\nabla\phi_{(2,-1,2)} = (4(2)^2 - 6(2)(-1)(2)^2)\hat{i} + (-3(2)^2(2)^2)\hat{j} + \hat{k}(8(2)(2) - 6(2)^2(-1)(2))$$

$$\nabla\phi_{(2,-1,2)} = 64\hat{i} - 48\hat{j} + 80\hat{k}$$

Let $\vec{a} = 2\hat{i} - \hat{j} + 2\hat{k}$ $\quad\quad$ $\overline{AB} = \bar{b} - \bar{a}$

$\vec{b} = \hat{k}$ $\quad\quad$ $\Rightarrow 2\hat{i} - \hat{j} + \hat{k}$

Directional derivative $\Rightarrow \dfrac{\nabla\phi \; \overrightarrow{AB}}{|\overrightarrow{A \cdot B}|}$

$$\frac{(64\hat{i} - 48\hat{j} + 80\hat{k}) \times 2\hat{i} - \hat{j} + \hat{k}}{\sqrt{2^2 + (-1)^2 + (1)^2}}$$

$$\Rightarrow \frac{128 + 48 + 80}{\sqrt{4}} \Rightarrow \underline{\underline{128}} = DD$$

(d) $\phi = xy^2 + yz^3$ at the point P(2,-1,1) in the direction of the normal to the surface $x \log z - y^2 + 4 = 0$ at (-1, 2, 1). $\rightarrow$ $\underline{\underline{\nabla\phi}} \Rightarrow \underline{\underline{\nabla a}} \Rightarrow$

let $\nabla\phi = \hat{i}\dfrac{\partial\phi}{\partial x} + \hat{j}\dfrac{\partial\phi}{\partial y} + \hat{k}\dfrac{\partial\phi}{\partial z}$

$$\nabla\phi = \hat{i}(y^2 + 0) + \hat{j}(2xy + z^3) + \hat{k}(3yz^2)$$

$$\nabla\phi = \hat{i}y^2 + \hat{j}(2xy + z^3) + \hat{k}(3yz^2)$$

$$\nabla\phi_{(2,-1,1)} = (-1)^2\hat{i} + (2(2)(-1) + (1)^3)\hat{j} + [3(-1)(1)^2\hat{k}]$$

$$= \hat{i} + (-3)\hat{j} + -3\hat{k}$$

$$\nabla\phi_{(2,-1,1)} = \hat{i} - 3\hat{j} - 3\hat{k}$$

(e) $\phi = \dfrac{y}{x^2+y^2}$ at the point (0,1) making an angle 30° with the positive X-axis.

formula $\cos\theta = \dfrac{\bar{a}\cdot\bar{b}}{|\bar{a}||\bar{b}|}$

$$\nabla\phi = \hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y}$$

$$\Rightarrow y\left(\frac{1}{x^2+y^2}\right) \Rightarrow \frac{y(2x)}{x^2+y^2}\hat{i}$$

$$\frac{\partial\phi}{\partial y} \Rightarrow \frac{y}{x^2+y^2} \quad \left(\frac{u}{v}\right) \quad \frac{vu' + uv'}{v^2}$$

$$\Rightarrow \frac{(x^2+y^2)(1) + (2y)(y)}{(x^2+y^2)^2} = \frac{(x^2+y^2)+2y^2}{(x^2+y^2)^2} = \frac{3y^2+x^2}{(x^2+y^2)^2}$$

$$\Rightarrow \frac{y(2x)}{x^2+y^2}\hat{i} + \frac{3y^2+x^2}{(x^2+y^2)^2}\hat{j}$$

$$\Rightarrow \cancel{0}^{\,0}\,\hat{i} + \frac{3}{(1)}\hat{j} = 3\hat{j}$$

$$\frac{3\hat{j}(a\hat{i}+b\hat{j}+c\hat{k})}{9(\sqrt{a^2+b^2+c^2})} = \frac{\sqrt{3}}{2}$$

2. In what direction from (3,1,-2) is the directional derivative of $\phi = x^2y^2z^4$ maximum and what is its magnitude?

$\phi = x^2y^2z^4$

$\nabla\phi = \hat{i}\dfrac{\partial\phi}{\partial x} + \hat{j}\dfrac{\partial\phi}{\partial y} + \hat{k}\dfrac{\partial\phi}{\partial z}$

$\nabla\phi = \hat{i}(2xy^2z^4) + \hat{j}(x^2 2yz^4) + \hat{k}(x^2y^2 4z^3)$

$\nabla\phi_{(3,1,-2)} \Rightarrow \hat{i}(2(3)(1)^2(-2)^4) + \hat{j}(3^2 2(1)(-2)^4)$

$+ \hat{k}((3)^2(1)^2 4(-2)^3)$

$\nabla\phi_{(3,1,-2)} = +96\hat{i} + 288\hat{j} - 288\hat{k}$

Directional Derivative $= \dfrac{\nabla\phi \, \overline{AB}}{|\overline{AB}|}$

This stuff is the actual directional derivative

now upon simplifying

$\nabla\phi = (96, 288, -288) \Rightarrow (1, 3, -3)$

the magnitude $|\nabla\phi| = \sqrt{96^2 + 288^2 + (-288)^2}$

$\boxed{|\nabla\phi| = 96\sqrt{19}}$

the direction $\Rightarrow (1,3,-3)$ & magnitude $96\sqrt{19}$

3. What is the greatest rate of increase of $u = x^2 + yz^2$ at the point (1,-1,3)?

$$\nabla \phi = \hat{i}\frac{\partial \phi}{\partial x} + \hat{j}\frac{\partial \phi}{\partial y} + \hat{k}\frac{\partial \phi}{\partial y}$$

let $\phi = u$ $\qquad \nabla u = \hat{i}\frac{\partial u}{\partial x} + \hat{j}\frac{\partial u}{\partial y} + \hat{k}\frac{\partial u}{\partial z}$

$$\Rightarrow \nabla u = \hat{i}(2x) + \hat{j}(z^2) + \hat{k}(2zy)$$

$$\Rightarrow \nabla u_{(1,-1,3)} = 2(1)\hat{i} + (3)^2\hat{j} + 2(3)(-1)\hat{k}$$

$$\nabla u_{(1,-1,3)} = 2\hat{i} + 9\hat{j} - 6\hat{k}$$

The greatest rate of increase would be the $|\nabla u|$

$$|\nabla u| = \sqrt{2^2 + 9^2 - 6^2} = 11$$

$\therefore$ The greatest rate of increase at the point $(1,-1,3) = 11$

4. The temperature at a point (x, y, z) in space is given by $T(x, y, z) = x^2 + y^2 - z$. A mosquito located at (1,1,2) desires to fly in such a direction that it will get warm as soon as possible. In what direction should it fly?

let $T(x,y,z)$ of $\nabla T(x,y,z) = x^2 + y^2 - z$

$$\nabla T_{(x,y,z)} = \hat{i}\frac{\partial T}{\partial x} + \hat{j}\frac{\partial T}{\partial y} + \hat{k}\frac{\partial T}{\partial z}$$

$$= \hat{i}(2x) + \hat{j}(2y) + \hat{k}(-1)$$

$$= 2x\hat{i} + 2y\hat{j} - \hat{k}$$

$$\nabla T(x,y,z)_{(1,1,2)} = 2(1)\hat{i} + 2(1)\hat{j} - \hat{k}$$

$$= 2\hat{i} + 2\hat{j} - \hat{k}$$

It basically means that it should fly to the point $(2, 2, -1)$ or any other scalar multiple of it

5. Find a and b such that $\vec{F} = (axy + z^3)i + x^2j + bz^2xk$ is irrotational. **(Dec 2022)**

Problem of curl $\bar{f}$. $\bar{f} = f_1\hat{i} + f_2\hat{j} + f_3\hat{k}$

$f_1 = (axy + z^3)$, $f_2 = x^3$, $f_3 = bz^2x$

we know that condition if $\bar{f}$ is irrotational or conservative

$\nabla \bar{f} = 0$

$$\nabla \times \bar{f} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix}$$

$$\nabla \times \bar{f} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix}$$

$$\Rightarrow \hat{i}\left(\frac{\partial}{\partial y} f_3 - \frac{\partial}{\partial z} f_2\right) - \hat{j}\left(\frac{\partial}{\partial x} f_3 - \frac{\partial}{\partial z} f_1\right) + \hat{k}\left(\frac{\partial}{\partial x} f_3 - \frac{\partial}{\partial y} f_1\right)$$

$f_1 = (axy + z^3)$, $f_2 = x^3$, $f_3 = bz^2x$

$$\hat{i}(\ ax\ ) - \hat{j}(bz^2 - 3z^2) + \hat{k}(bz^2 - ax)$$

now equal have to each of the $f_1\ f_2\ \&\ f_3 = 0$

$ax = 0$

$bz^2 - 3z^2 = 0 \quad \Rightarrow \quad b - 3 = 0$

$\boxed{b = 3}$

$bz^2 - ax = 0$ $\quad a = 0$

$\rightarrow 3z^2 - ax$

$\therefore \boxed{z = 0}$

$\boxed{a = 0}$, $\boxed{b = 3}$, $\boxed{z = 0}$

6. Show that $\bar{F} = (x + 2y + 4z)\hat{i} + (2x - 3y - z)\hat{j} + (4x - y + 2z)\hat{k}$ is both solenoidal and irrotational. **(Dec 2024)**

In the case of Solenoidal $\bar{f} = f_1\hat{i} + f_2\hat{j} + f_3\hat{k}$

$f_1 = x + 2y + 4z$, $f_2 = 2x - 3y - z$, $f_3 = 4x - y + 2z$

$$\text{div}\,\bar{f} = \nabla \bar{f} = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$$

$\bar{f}$ is solenoidal if $\nabla \bar{f} = 0$

$\Rightarrow f_1 = x + 2y + 4z$, $f_2 = 2x - 3y - z$, $f_3 = 4x - y + 2z$

$\Rightarrow \boxed{\nabla \bar{f} = \hat{i} - 3\hat{j} + 2\hat{k}}$

for the case of solenoidal sum of all of the internal parts must add upto "0"

$1 - 3 + 2 = \underline{\underline{0}}$

hence it's definately solenoidal

irrotational part $\Rightarrow$

$$\nabla \times \bar{f} \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix}$$

$$\Rightarrow \hat{i}\left(\frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z}\right) - \hat{j}\left(\frac{\partial f_3}{\partial x} - \frac{\partial f_1}{\partial z}\right) + \hat{k}\left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right)$$

$$\Rightarrow \hat{i}\,\cancelto{0}{(-1 - (-1))} - \hat{j}\,\cancelto{0}{(4 - 4)} + \hat{j}\,\cancelto{0}{(2 - 2)}$$

since all of these are $= 0$ then these are definately irrotational

7. Find the value of constant $'a'$ such that $\bar{A} = (ax + 4y^2 z)\hat{i} + (x^3 \sin z - 3y)\hat{j} - (e^x + 4\cos x^2 y)\hat{k}$ is solenoidal.

$\bar{f} = f_1\hat{i} + f_2\hat{j} + f_3\hat{k}$

$f_1 = ax + 4y^2 z$, $f_2 = x^3 \sin z - 3y$, $f_3 = -e^x - 4\cos x^2 y$

$\text{div}\,\bar{f} = \nabla \cdot \bar{f} = \frac{\partial}{\partial x} f_1 + \frac{\partial}{\partial y} f_2 + \frac{\partial}{\partial z} f_3$

$\Rightarrow a + (-3) + 0 = 0$

$\boxed{a = 3}$ for solenoidal $\boxed{\nabla \cdot \bar{f} = 0}$

8. If $\bar{F} = (x + 2y + az)\hat{i} + (bx - 3y - z)\hat{j} + (4x + cy + 2z)\hat{k}$ is irrotational then find the value of a, b, c. **(May 2023)**

$\bar{F} = f_1\hat{i} + f_2\hat{j} + f_3\hat{k}$

$f_1 = (x + 2y + az)$, $f_2 = (bx - 3y - z)$, $f_3 = (4x + cy + 2z)$

$$\nabla \times \bar{f} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix}$$

$\Rightarrow \hat{i}\left(\frac{\partial f_3}{\partial y} - \frac{\partial}{\partial z} f_2\right) - \hat{j}\left(\frac{\partial f_3}{\partial x} - \frac{\partial}{\partial z} f_1\right) + \hat{k}\left(\frac{\partial f_2}{\partial x} - \frac{\partial}{\partial y} f_1\right)$

$\Rightarrow \hat{i}\,(c - (-1)) - \hat{j}\,(4 - a) + \hat{k}\,(b - 2)$

$\boxed{c = 1}$ $\boxed{a = 4}$, $\boxed{b = 2}$

10. Show that the vector field $\vec{V} = (\sin y + z)\hat{i} + (x\cos y - z)\hat{j} + (x - y)\hat{k}$ is irrotational. Hence find its scalar potential.

$$\bar{f} - f_1\hat{i} + f_2\hat{j} + f_3\hat{k}$$

$$f_1 = \sin y + z, \quad f_2 = x\cos y - z, \quad f_3 = (x - y)$$

$$\nabla \bar{f} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix}$$

$$\nabla \bar{f} = \hat{i}\left(\frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z}\right) - \hat{j}\left(\frac{\partial f_3}{\partial x} - \frac{\partial f_1}{\partial z}\right) + \hat{k}\left(\frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right)$$

$$= \hat{i}\cancel{\left(-1 - (-1)\right)}^{0} + \hat{j}\cancel{\left((1) - (1)\right)}^{0} + \hat{k}\cancel{\left(\cos y) - \cos y\right.}^{0}$$

hence proved this is irrotational

Scalar potential if $\bar{f}$ is irrotational

$$d\phi = f_1 dx + f_2 dy + f_3 dz$$

$$d\phi = (\sin y + z)\,dx + (x\cos y - z)\,dy + (x - y)\,dz$$

$$d\phi = \frac{\sin^2 y}{2} + \frac{z^2}{2} + x(-\sin y) - \frac{z^2}{2} + \frac{x^2 - y^2}{2}$$

$$\phi = \frac{\sin^2 y + \cancel{z^2}}{2} - \sin y + \frac{x^2 - y^2 - \cancel{z^2}}{2}$$

11. Show that the vector field $\vec{A}$, where $\vec{A} = (x^2 - y^2 + x)\hat{\imath} - (2xy + y)\hat{\jmath}$ is irrotational and find its scalar potential. Also find the work done by $\bar{F}$ in moving a particle from A(1,1,1) to B(1,2,3) along the straight line AB.

$\vec{A} = (x^2 - y^2 + x)\hat{i} - (2xy + y)\hat{j}$

. $\nabla \vec{f} = 0$ condition for irrotational

$$\nabla \vec{f} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f_1 & f_2 & f_3 \end{vmatrix}$$

$f_1 = (x^2 - y^2 + x)$

$f_2 = (-2xy - y)$

$$\hat{i}\left(\frac{\partial}{\partial y} f_3 - \frac{\partial}{\partial z} f_2\right) - \hat{j}\left(\frac{\partial}{\partial x} f_3 - \frac{\partial}{\partial z} f_1\right) + \hat{k}\left(\frac{\partial}{\partial x} f_2 - \frac{\partial}{\partial y} f_1\right)$$

$\Rightarrow$

1. $If\ u = x^y,\ Show\ that\ \dfrac{\partial^3 u}{\partial x^2 \partial y} = \dfrac{\partial^3 u}{\partial x \partial y \partial x}$

Starting off with LHS.

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)\right) = \underset{u}{x^y}\ \underset{v}{\log x} \qquad (u \cdot v = u \cdot v' - v u')$$

$$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = x^y \cdot \frac{1}{x} - \log x \cdot y \cdot x^{y-1}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)\right) = x^{y-1} - \underset{u}{\log x} \cdot \underset{v}{y x^{y-1}}$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)\right) = x^{y-1} \log x - \log x\,(y-1) \cdot y \cdot x^{y-2} - y x^{y-1} \frac{1}{x}$$

$$= x^{y-1} \log x - \log x\,(y-1)\, y\, \frac{x^{y-1}}{x} - y x^{y-1} \frac{1}{x}$$

$$= x^{y-1} \log x \left(1 - \frac{(y-1)}{x} y - \frac{y}{x}\right)$$

Moving onto RHS $\dfrac{\partial^3 u}{\partial x \cdot \partial y \cdot \partial x} = \dfrac{\partial}{\partial x}\left(\dfrac{\partial}{\partial y}\left(\dfrac{\partial u}{\partial x}\right)\right)$ $\boxed{u = x^y}$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right)\right) = \underset{u}{y}\,\underset{v}{x^{y-1}} \qquad (u \cdot v = u \cdot v' - v \cdot u')$$

$$\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) = y \cdot \underset{u}{\underline{x^{y-1}}}\ \underset{v}{\underline{\log x}} - x^{y-1} \cdot (1)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right)\right) = y\left(x^{y-1} \cdot \frac{1}{x} - y - 1\, x^{y-2} \log x\right) - y - 1 x^{y-2}$$

now since LHS = RHS hence Proved.

2. Verify that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$, for (i) $u = x^y + y^x$ (ii) $u = x^3 y + e^{xy^2}$

$u = x^3 y + e^{xy^2}$

(i) $u = x^y + y^x$ TPT LHS = RHS $\frac{\partial^2 u}{\partial x \cdot \partial y} = \frac{\partial^2 u}{\partial y \cdot \partial x}$.

Starting off with LHS $\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)$

$\frac{\partial u}{\partial y} = \underbrace{x^y}_{u}\underbrace{\log x}_{v} + \underbrace{x}_{u}\underbrace{y^{x-1}}_{v}$ $(u \cdot v = u \cdot v' - v u')$

$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = \left(x^y \frac{1}{x} - \log x \cdot y \cdot x^{y-1}\right) + \left(x\, y^{x-1} \log y - y^{x-1}(1)\right)$

Solving for the RHS Part. $\frac{\partial^2 u}{\partial y \cdot \partial x}$ $u = x^y + y^x$

$\Rightarrow \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) \Rightarrow \frac{\partial u}{\partial x} = \underbrace{y}_{u} \cdot \underbrace{x^{y-1}}_{v} + \underbrace{y^x}_{u} \cdot \underbrace{\log y}_{v}$ $(u \cdot v = u \cdot v' - v \cdot u')$

$\Rightarrow \frac{\partial^2 u}{\partial y \cdot \partial x} = y \cdot x^{y-1} \log x - x^{y-1}(1) + y^x \cdot \frac{1}{y} - \log y \, x y^{x-1}$

$\therefore$ LHS = RHS Hence Proved.

(ii) $u = x^3 y + e^{xy^2}$ $\frac{\partial^2 u}{\partial x \cdot \partial y} = \frac{\partial^2 u}{\partial y \cdot \partial x}$. $\Leftarrow$ TPT

$\therefore$ Starting from the LHS $\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right)$

$\frac{\partial u}{\partial y} = x^3 + \underbrace{e^{xy^2}}_{u} \cdot \underbrace{2yx}_{v}$ $\Rightarrow$

$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial y}\right) = 3x^2 + \left(e^{xy^2} \cdot 2y - 2xy\, e^{xy^2}\right)$

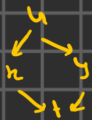
now the RHS Part. $\frac{\partial^2 u}{\partial y \cdot \partial x}$ $\Rightarrow \frac{\partial u}{\partial x} \Rightarrow 3x^2 y + \underbrace{e^{xy^2}}_{u} \cdot \underbrace{y^2}_{v}$

$(u \cdot v) = u \cdot v' - v u'$

$$\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x}\right) = 3x^2 + e^{xy^2} 2y - y^2 . e^{xy^2} . 2xy$$

$\therefore$ Since LHS = RHS hence proved.

3. $If\ u = tan^{-1}(\frac{x}{y}),\ where\ x = 2t,\ y = 1 - t^2,\ find\ \frac{du}{dt}$



$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} . \frac{\partial x}{\partial t} + \frac{\partial u}{\partial y} . \frac{\partial y}{\partial t}$$

$$\frac{\partial u}{\partial x} = \frac{1}{1+\left(\frac{x}{y}\right)^2} . \left(\frac{1}{y}\right) \ ; \ \frac{\partial x}{\partial t} \Rightarrow 2 \ ; \ \frac{\partial y}{\partial t} = -2t.$$

$$\frac{\partial u}{\partial y} = \frac{1}{1+\left(\frac{x}{y}\right)^2} - \left(\frac{x}{y^2}\right)$$

$$\therefore \frac{\partial u}{\partial t} = \frac{1}{1+\left(\frac{x}{y}\right)^2}\left(\frac{1}{y}\right)(2) + \frac{1}{1+\left(\frac{x}{y}\right)^2}\left(\frac{x}{y^2}\right) 2t$$

$$\boxed{\frac{\partial u}{\partial t} = 2\frac{1}{y}\left(\frac{1}{1+\left(\frac{x}{y}\right)^2}\right)\left(1 + \left(\frac{x}{y}\right)t\right)}$$

4. $If\ z = sin^{-1}(x - y),\ x = 3t,\ y = 4t^3,\ Prove\ \frac{dz}{dt} = \frac{3}{\sqrt{1-t^2}}$

TPT $\frac{\partial z}{\partial t} = \frac{3}{\sqrt{1-t^2}}$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} . \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} . \frac{\partial y}{\partial t}$$

$$\frac{\partial x}{\partial t} = 3 \qquad \frac{\partial z}{\partial x} = \frac{1}{\sqrt{1-(x-y)^2}} \qquad \frac{\partial y}{\partial t} = 12t^2 \qquad \frac{\partial z}{\partial y} = \frac{1}{\sqrt{1-(x-y)^2}} . (-1)$$

$$\frac{\partial z}{\partial t} = \frac{1}{\sqrt{1-(x-y)^2}}(3) - \frac{1}{\sqrt{1-(x-y)^2}} . (12t^2)$$

$$\frac{\partial z}{\partial t} = \frac{3}{\sqrt{1-(x-y)^2}}(1-4t^2)$$

5. $If\ u = log(x^3+y^3-x^2y-xy^2), \quad Prove\ that$
$$\frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = \frac{-4}{(x+y)^2}$$

$$\frac{\partial^2 u}{\partial x^2} \Rightarrow \frac{\partial u}{\partial x} = \frac{1}{x^3+y^3-x^2y-xy^2}(3x^2-2xy-y^2) \quad \left(\frac{u}{v}\right) = \frac{v\cdot u' - u\cdot v'}{v^2}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{(x^3+y^3-x^2y-xy^2)(6x-2y) - (3x^2-2xy-y^2)(3x^2-2xy-y^2)}{(x^3+y^3-x^2y-xy^2)^2}$$

$$2\frac{\partial^2 u}{\partial x\cdot\partial y} \Rightarrow \frac{\partial}{\partial x}\left(\frac{\partial}{\partial y}\right) \Rightarrow \frac{\partial u}{\partial y} = \frac{1}{(x^3+y^3-x^2y-xy^2)}\cdot(3y^2-x^2-2xy) \left(\left(\frac{u}{v}\right) = \frac{v\cdot u' - u\cdot v'}{v^2}\right)$$

$$2\frac{\partial^2 u}{\partial x\cdot\partial y} = 2\left(\frac{(x^3+y^3-x^2y-xy^2)(-2x-2y) - (3y^2-x^2-2xy)(3x^2-2xy-y^2)}{(x^3+y^3-x^2y-xy^2)}\right.$$

$$\frac{\partial^2 u}{\partial y^2} \Rightarrow \frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right) \Rightarrow \frac{\partial u}{\partial y} = \frac{1}{x^3+y^3-x^2y-y^2x}(3y^2-x^2-2yx) \quad \left(\frac{u}{v}\right) = \frac{v\cdot u' - u\cdot v'}{v^2}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{(x^3+y^3-x^2y-y^2x)(6y-2x) - (3y^2-x^2-2yx)(3y^2-x^2-2yx)}{(x^3+y^3-x^2y-y^2x)^2}$$

$$\text{now} = \frac{\partial^2 u}{\partial x^2} + 2\frac{\partial^2 u}{\partial x\cdot\partial y} + \frac{\partial^2 u}{\partial y^2} \Rightarrow$$

$$\frac{(x^3+y^3-x^2y-xy^2)(6x-2y) - (3x^2-2xy-y^2)(3x^2-2xy-y^2)}{(x^3+y^3-x^2y-xy^2)^2} +$$

$$2\left(\frac{(x^3+y^3-x^2y-xy^2)(-2x-2y) - (3y^2-x^2-2xy)(3x^2-2xy-y^2)}{(x^3+y^3-x^2y-xy^2)}\right.$$

$$\frac{(x^3+y^3-x^2y-y^2x)(6y-2x) - (3y^2-x^2-2yx)(3y^2-x^2-2yx)}{(x^3+y^3-x^2y-y^2x)^2}$$

upon adding all of these 3 together we get $\Rightarrow \frac{-4}{(x+y)^2}$

6. $If\ u = log(x^3 + y^3 + z^3 - 3xyz),\quad Prove\ that$

$$\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$$

The formula for $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$$\Rightarrow \left(\underset{a}{\frac{\partial}{\partial x}} + \underset{b}{\frac{\partial}{\partial y}} + \underset{c}{\frac{\partial}{\partial z}}\right)^2 = \left(\frac{\partial}{\partial x}\right)^2 + \left(\frac{\partial}{\partial y}\right)^2 + \left(\frac{\partial}{\partial z}\right)^2 + 2\left(\frac{\partial}{\partial x}\right)\left(\frac{\partial}{\partial y}\right) + 2\left(\frac{\partial}{\partial y}\right)\left(\frac{\partial}{\partial z}\right) + 2\left(\frac{\partial}{\partial z}\right)\left(\frac{\partial}{\partial z}\right)$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} + 2\frac{\partial^2 u}{\partial x \cdot \partial y} + 2\left(\frac{\partial^2 u}{\partial y \cdot \partial z}\right) + 2\left(\frac{\partial^2 u}{\partial x \cdot \partial z}\right)$$

$$u = \log(x^3 + y^3 + z^3 - 3xyz)$$

$$\frac{\partial^2 u}{\partial x^2} \Rightarrow \frac{\partial u}{\partial x} \Rightarrow \frac{1}{(x^3+y^3+z^3-3xyz)} \cdot (3x^2 - 3yz) \quad \left(\frac{u}{v}\right) = \frac{v \cdot u' - u \cdot v'}{v^2}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{(x^3+y^3+z^3-3xyz)(6x) - (3x^2-3yz)(3x^2-3yz)}{(x^3+y^3+z^3-3xyz)^2}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial y} = \frac{1}{x^3+y^3+z^3-3xyz}($$

7. *If* $f = \phi(u, v)$ *and* $u = x^2 - y^2$, $v = 2xy$,

Prove that $(\frac{\partial f}{\partial x})^2 + (\frac{\partial f}{\partial y})^2 = 4(u^2+v^2)^{\frac{1}{2}}[(\frac{\partial f}{\partial u})^2 + (\frac{\partial f}{\partial v})^2]$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial x} \qquad \frac{\partial u}{\partial x} = 2x \, ; \; \frac{\partial v}{\partial x} = 2y$$

$$\begin{tikzcd}[column sep=small] & & f \arrow[dl] \arrow[dr] & & \\ & u \arrow[dl] \arrow[d] & & v \arrow[d] \arrow[dr] & \\ x & y & & x & y \end{tikzcd}$$

$$\frac{\partial f}{\partial x} = \frac{\partial f}{\partial u}(2x) + \frac{\partial f}{\partial v}(2y)$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial f}{\partial v} \cdot \frac{\partial v}{\partial y} \qquad \frac{\partial u}{\partial y} = -2y \quad \frac{\partial v}{\partial y} = 2x$$

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial u}(-2y) + \frac{\partial f}{\partial v}(2x)$$

$$\frac{\partial f}{\partial y} = -2\frac{\partial f}{\partial u}(y) + 2\frac{\partial f}{\partial v}(x)$$

$$= \left(\frac{\partial f}{\partial u}(2x) + \frac{\partial f}{\partial v}(2y)\right)^2 \Rightarrow \frac{\partial^2 f}{\partial u^2}(4x^2) + 8\cancel{\frac{\partial f}{\partial u}\frac{\partial f}{\partial v}xy} + \frac{\partial^2 f}{\partial v^2}(4y^2)$$

$$\left(-2\frac{\partial f}{\partial u}(y) + 2\frac{\partial f}{\partial v}(x)\right)^2 \Rightarrow \frac{\partial^2 f}{\partial u^2}4y^2 - 8\cancel{\frac{\partial f}{\partial u}xy} + \frac{\partial^2 f}{\partial v^2}4x^2$$

$$+$$

$$\frac{\partial^2 f}{\partial u^2}(4x^2+4y^2) + \frac{\partial^2 f}{\partial v^2}(4y^2+4x^2)$$

$$\Rightarrow (4x^2+4y^2)\left(\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2}\right)$$

now $u^2+v^2 = (x+y)^2 + 2xy$

upon solving $= 4(x^2+y^2) = (u^2+v^2)^{1/2}$

Since LHS = RHS hence Proved.

now starting off with the RHS Part. $4(u^2+v^2)^{1/2}\left[\left(\frac{\partial f}{\partial u}\right)^2 + \left(\frac{\partial f}{\partial v}\right)^2\right]$

8. $If\ z = f(u,v), u = lx+my,\ v = ly-mx,$

$$Prove\ that\ \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (l^2+m^2)(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2})$$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x} . \qquad \frac{\partial u}{\partial x} = l ; \ \frac{\partial v}{\partial x} = -m$$

$$\begin{tikzcd}[column sep=small] & & z \arrow[dl] \arrow[dr] & & \\ & u \arrow[dl] \arrow[d] & & v \arrow[d] \arrow[dr] & \\ x & y & & x & y \end{tikzcd}$$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}(l) + \frac{\partial z}{\partial v}(-m) \Rightarrow \frac{\partial z}{\partial u}(l) - \frac{\partial z}{\partial v}(m)$$

Subject:- Applied Mathematics -I

Roll nos :- 47 Semester:- Ist Syllabus:- NEP-2020

Branch:- Mechanical

Q1) Show that the matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix}1 & 1+i\\ 1-i & -1\end{bmatrix}$ is unitary and hence find A^{-1}.

Condition of unitary matrix $= A \cdot A^{\theta} = I$ where $A^{\theta} = A^{-1}$

$$= \begin{bmatrix}-1 & 1+i\\ -1+i & +1\end{bmatrix} \cdot \begin{bmatrix}1 & 1+i \\ 1-i & -1\end{bmatrix} = \begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$= \begin{bmatrix} -1+(1-i)(1-i) & -1-i\ -1-i \\ (-1+i)(1-i) & (1+i)^2 - 1\end{bmatrix} = \begin{bmatrix}1&0\\0&1\end{bmatrix}$$

$$= \begin{bmatrix}\cancel{1}+\cancel{1}+2-1 & 0 \\ 0 & \cancel{1}+2\cancel{1}-1=1\end{bmatrix} = \begin{bmatrix}1&0\\0&1\end{bmatrix}$$ hence it's an unitary matrix

& $A^{-1} = \begin{bmatrix}-1 & 1+i\\ -1+i & +1\end{bmatrix}$

e) If $u = 2(ax+by)^2 - k(x^2+y^2)$ and $a^2+b^2 = k$, find $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$.

$u = 2(ax+by)^2 - (a^2+b^2)(x^2+y^2)$

$u = 2(a^2x^2 + 2axby + b^2y^2) - a^2x^2 + a^2y^2 + b^2x^2 + b^2y^2$

$\frac{\partial^2 u}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) \Rightarrow \frac{\partial u}{\partial x} = 4a^2x + 4aby - 2a^2x + 2b^2x$

$\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x}\right) = 4a^2 - 2a^2 + 2b^2$

$\frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial y} = 4axb + 4b^2y + 2a^2y + 2b^2y$

$$\frac{\partial^2 u}{\partial y^2} = 4b^2 + 2a^2 + 2b^2$$

$$\therefore \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 4a^2 - \cancel{2a^2} + 2b^2 + 4b^2 + \cancel{2a^2} + 2b^2$$

$$= \underline{\underline{4a^2 + 8b^2}}$$

(f) Using Newton-Raphson Method, find the real root of $x^3 - 2x - 5 = 0$ correct to three decimal places.

The interval lies in between $(2, 2.5)$

$f'(x_n) = 3x^2 - 2$

$f(2) = -1 < 0$; $f(2.5) = 5.625 > 0$

$f'(x_n) = 3(2) - 2$

$= 4$

$\therefore$ It'll be in the immediate neighbourhood (2)

the formula $x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$ $\boxed{x_0 = 2}$

$$x_1 = 2 + \frac{-1}{4} = \boxed{1.75} > 0$$

$$f(x_1) = \underline{\underline{-3.14}} < 0$$

$x_2 = x_1$

Numerical Methods

1. Solve the following equations by Bisection method:

(a) $3x + \sin x - e^x = 0$ ↳ what to take th

Step ① Calculate at which interval does it become that of opposite sign.

The root lies in between the interval $(0.5, 1)$

$f(0.5) = -0.139$; $f(1) = 0.7091$

Step ② to calculate the midpoints of the interval $c = \frac{a+b}{2} = \frac{0.5+1}{2} = 0.75$

now the newer interval would be $= (0.5, 0.75)$

$f(c) = f(0.75) = 0.146$

we're taking this upto 3 iterations

the newer $c = \frac{(0.5+0.75)}{2} = 0.625 > 0$

$f(0.625) = 0.01766$

$\therefore$ the newer interval becomes $(0.5, 0.625)$

the newest $c = \frac{(0.5 + 0.625)}{2} = \underline{\underline{0.5625}}$

$f(0.5625) = -0.057737 < 0$

$\therefore$ the newest interval becomes $(0.5625, 0.625)$

Q2) to solve by using bisection method.

(b) $x^3 - 4x - 9 = 0$

Step ① to find the interval.

$\therefore$ the root lies in between $(2.5, 3)$

$f(2.5) = -3.375$, $f(3) = 6$

Step ② to calculate the midpoints between them $\Rightarrow \left(\frac{2.5+3}{2}\right) = 2.75$

$f(2.75) = 0.796 > 0$

$\therefore$ the newer interval lies in between $(2.5, 2.75)$

the newer $c = \dfrac{(2.5 + 2.75)}{2} = \underline{\underline{2.625}}$

$f(2.625) = -1.4121 < 0$

$\therefore$ the latest interval would be $= (2.625, 2.75)$

$\therefore$ the newer c would be $= \dfrac{(2.625 + 2.75)}{2} = 2.6875$

$f(2.6875) = -0.33911$

$\therefore$ the newer interval would be $= (2.6875, 2.75)$

2. Solve the following equations by Regula-Falsi method:

(a) $x^3 = 3x - 4$

$f(x) = x^3 - 3x + 4$

the interval lies in between $(\overset{a}{-2.5}, \overset{b}{-2})$ $f(a) = -4.125$ $f(b) = 2$

$f(-2.5) = -4.125$; $f(-2) = 2$

$$x_1 = \frac{-2.5(2) - (-2)(-4.125)}{2 - (-4.125)} = \underline{\underline{-2.1632}}.$$

$f(x_1) = f(-2.1632) = 0.3670$

$\therefore$ the interval lies in between $(\overset{a}{-2.1632}, \overset{b}{-2.5})$ $f(a) = 0.3670$ $f(b) = -4.125$

now since the signs of $f(a)$ & $f(x_1)$ are opposite hence the root

$$x_2 = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)} = \frac{-2.1632(-4.125) - (-2.5)(0.3670)}{-4.125 - 0.3670}$$

$x_2 = -2.1907$

$f(x_2) = -2.1907^3 - 3(-2.1907) + 4 = 0.05856 > 0$

now this (x_2, b) will be applicable $(\overset{a}{-2.1907}, \overset{b}{-2.5})$

$x_3 = \dfrac{af(b) - bf(a)}{f(b) - f(a)} = \dfrac{-2.1907 \times (-4.125) - (-2.5)(0.05856)}{-4.125 - 0.05856}$

$x_3 = -2.195029$; $f(x_3) = -2.1950^3 - 3(2.1950) + 4$

$\boxed{f(x_3) = 0.004435}$

(b) $x^3 - x - 1 = 0$ by using regula falsi method

S① use the calculator to find $f(a)$ & $f(b)$ then $x_1 = \dfrac{af(b) - bf(a)}{f(b) - f(a)}$

The interval should lie between $(\overset{a}{1}, \overset{b}{1.5})$; $f(a) = f(1) = -1$;
$f(b) : f(1.5) = 0.875$

$x_1 = \dfrac{1(0.875) - (1.5)(-1)}{0.875 - (-1)} = 1.26666 > 0$ $f(x_1) : f(1.2666) = -0.2346$
$1.2666^3 - 1.2666 - 1 = -0.2346$

$\therefore$ the newer interval would lie in between $= (\underset{a}{1.2666}, \underset{b}{1.5})$

$f(a) = f(1.2666) = -0.2346$; $f(b) = f(1.5) = 0.875$

$x_2 = \dfrac{a.f(b) - b.f(a)}{f(b) - f(a)} = \dfrac{1.2666(0.875) - 1.5(-0.2346)}{0.875 - (-0.2346)} = 1.3159$

$f(x_2) = f(1.3159) = 1.3159^3 - 1.3159 - 1 = -0.03729 < 0$

$\therefore$ the newer interval would lie between $(\overset{a}{1.3159}, \overset{b}{1.5})$

$f(a) = f(1.3159) = -0.03729$; $f(b) = f(1.5) = 0.875$

$x_3 = \dfrac{a.f(b) - b.f(a)}{f(b) - f(a)} = \dfrac{1.3159(0.875) - 1.5(-0.03729)}{0.875 - (-0.03729)} = 1.3234$

$f(x_3) : f(1.3234) = 1.3234^3 - 1.3234 - 1 = \underline{\underline{-0.00561}}$

(c) $xe^x = \sin x$

$f(x) = xe^x - \sin x = 0$

The interval must lie in between $(\overset{a}{-0.5}, \overset{b}{0})$ $f(a) = f(-0.5) = -0.244$ $f(b) = f(0) = 0$

$$x_1 = \frac{a \cdot f(b) - b \cdot f(a)}{f(b) - f(a)} = \frac{-0.5(0) - (0)(0)}{0 - (-0.244)} = \underline{\underline{0}}$$

This is the fucking interval.

3. Solve the following equations by Newton Raphson Method:

(a) $e^{-x} = \sin x$

$$f(x) = e^{-x} - \sin(x)$$

The interval at which it has opposite signs are $(2.5, 3)$ $f(2.5) = 0.0384$ $f(3) = -2 \times 10^{-3}$

$x_0 = 2.5$

$\therefore$ the formula $= x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

$f'(x_n) = -e^{-x} - \cos x$

$f'(2.5) = -e^{-2.5} - \cos(2.5)$

$f'(2.5) = -1.0811$

$$= x_1 = 2.5 - \frac{0.0384}{-1.0811}$$

$= x_1 = 2.5355$ $\qquad |x_1 - x_0| = |2.5355 - 2.5| = 0.0355$

$$= x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$f(x_1)$ $e^{-2.5355} - \sin(+2.5355) = 0.0349$

$f'(x_1) = -e^{-2.5355} - \cos(+2.5355) = -1.078$

$$x_2 = 2.5355 - \frac{0.0349}{-1.07824}$$

$x_2 = 2.5678$ $\qquad |x_2 - x_1| = |2.5678 - 2.5355| = 0.0323$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$f(x_2) = e^{-2.5678} - \sin(+2.5678) = 0.03190$

$f'(x_2) = -e^{-2.5678} - \cos(2.5678) = -1.0757$

$$x_3 = 2.5678 - \frac{0.03190}{-1.0757} = 2.5974$$

$$|x_3 - x_2| = |2.5974 - 2.5678| = \underline{\underline{0.0296}}.$$

(c) $x^3 + 2x - 5 = 0$ by using the newton Raphson method.

$f(x) = x^3 + 2x - 5 = 0$

The interval lies in between $(1, 1.5)$ $f(1) = -2$, $f(1.5) = 1.375$

$\therefore x_0 = 1$ $f(x_0) = f(1) = -2$ $|x_1 - x_0| = |1.4 - 1| = \underline{\underline{0.4}}$

The formula is $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ $f(x_n) = -2$

$f'(x_n) = 3x^2 + 2 = 3(1) + 2 = \underline{\underline{5}}$

$x_1 = 1 - \dfrac{(-2)}{5} = 1.4$ $f(x_1) = f(1.4) = 1.4^3 + 2(1.4) - 5 = 0.544$

$f'(x_1) = f'(1.4) = 3(1.4)^2 + 2(1.4) = 8.68$

$x_2 = 1.4 - \dfrac{0.544}{8.68} = 1.33732$ $|x_2 - x_1| = |1.33732 - 1.4| = -0.06288$

$x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)}$ $f(x_2) = f(1.3372) = 1.3372^3 + 2(1.3372) - 5 = 0.0654$

$f'(x_2) = f'(1.3372) = 3(1.3372)^2 + 2 = 7.3643$

$x_3 = 1.4 - \dfrac{0.0654}{7.3643} = 1.39111$ $|x_3 - x_2| = |1.39111 - 1.33732| = 0.05378$

(b) $x^4 = 5x + 5$ Solve by using newton raphson method.

$f(x) = x^4 - 5x - 5$

The interval lies in between $(-1, -0.5)$ $f(-1) = 1$; $f(-0.5) = -2.437$.

the $x_0 = -1$ the formula is $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$ $f(x_0) = f(-1) = 1$

$f'(x_0) = 4x^3 - 5 = 4(-1)^3 - 5 = -9$

$x_1 = x_0 + \dfrac{f(x_0)}{f'(x_0)} = -1 + \dfrac{1}{-9} = -1.111$

$x_1 = -1.1111$

$f(x_1) = (-1.111)^4 - 5(-1.111) - 5 = -0.96854$

$f'(x_1) = 4(-1.111)^3 - 5 = -10.4853$

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)} = -1.111 - \dfrac{(-0.96854)}{-10.4853} = -1.2033$

$f(x_2) = (-1.2033)^4 - 5(-1.2033) - 5 = -1.08000$

$f'(x_2) = 4(-1.2033)^3 - 5 = -11.9691$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = -1.2033 - \frac{(-1.08000)}{-11.9691} = -1.2935$

40) To solve this using gauss Jacobi method (non updation method)

$\begin{array}{cccccccccccc} a_1 & b_1 & c_1 & d_1 & a_2 & b_2 & c_2 & d_2 & a_3 & b_3 & c_3 & d_3 \end{array}$

(a) $10x - 5y - 2z = 3;\ 4x - 10y + 3z = -3;\ x + 6y + 10z = -3$

$|10| > |-5| + |-2|$ $\quad |-10| > |4| + |3|$ $\quad |10| > |1| + |6|$

assume initial values to be x^0, y^0, z^0

$x^1 : \frac{1}{10}[3 - (-5) - (-2)] = 1$ $\quad z^1 = \frac{1}{10}[-3 - (1) - (6)] = -1$

$y^1 = \frac{1}{-10}[-3 - (4) - (3)] = 1$

A first formula then solve.

$x^2 = \frac{1}{10}[3 - (-5)(1) + (-2)(1)] = 0.6$

$y^2 = \frac{1}{-10}[-3 - 4(1) - 3(-1)] = 0.4.$

$z^2 = \frac{1}{10}[-3 - 1(1) - 6(1)] = 1$

Matrices Solving

1. If A is orthogonal, find a, b, c where $A = \frac{1}{9}\begin{bmatrix} -8 & 4 & a \\ 1 & 4 & b \\ 4 & 7 & c \end{bmatrix}$

Condition for orthogonal matrix = $A \cdot A^T = I$

$$\frac{1}{81}\begin{bmatrix} -8 & 4 & a \\ 1 & 4 & b \\ 4 & 7 & c \end{bmatrix}$$