Module 1: DC Circuits & Network Theorems

1.1: Kirchhoff's Laws, Series and Parallel DC Circuits, Voltage/Current Division Rule, Ideal and Practical sources, Source Transformation.
1.2: Mesh analysis, Nodal analysis.
1.3: Network Theorems: Superposition, Thevenin, Norton, Maximum Power Transfer
Self – Learning Topics: Concept of Star-Delta Transformation – Study the conversion between star and delta networks and their practical use in simplification of electrical networks.

Network terminologies

Circuit. A circuit is a closed conducting path through which an electric current either flows or is intended flow

Parameters. The various elements of an electric circuit are called its parameters like resistance, inductance, and capacitance. These parameters may be lumped or distributed.

Liner Circuit. A linear circuit is one whose parameters are constant i.e. they do not change with voltage or current.

Non-linear Circuit. It is that circuit whose parameters change with voltage or current

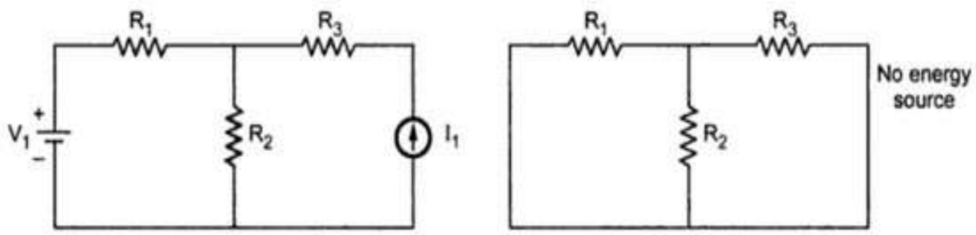
Bilateral Circuit. A bilateral circuit is one whose properties or characteristics are the same in either direction. The usual transmission line is bilateral, because it can be made to per- form its function equally well in either direction.

Unilateral Circuit. It is that circuit whose properties or characteristics change with the direction of its operation. A diode rectifier is a unilateral circuit, because it cannot perform rectification in both directions.

Electric Network. A combination of various electric elements, connected in any manner whatsoever, is called an electric network.

Passive Network is one which contains no source of e.m.f. in it.

Active Network is one which contains one or more than one source of e.m.f.



(a) Active network (b) Passive network

Node is a junction in a circuit where two or more circuit elements are connected.

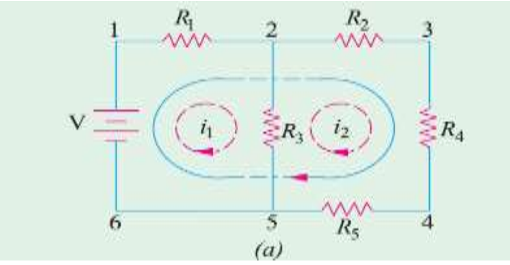
Junction is a node where three or more than three branches meet.

Branch is that part of a network which lies between two junctions.

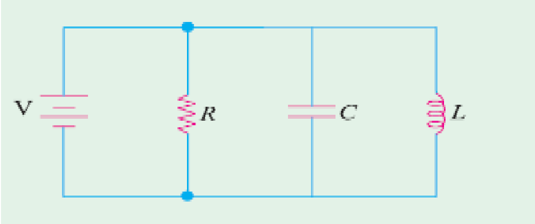
Loop. It is a close path in a circuit in which no element or node is encountered more than once.

Mesh. It is a loop that contains no other loop within it.

For example, the circuit of Fig (a) has three branches, two junctions, six nodes, three loops and two meshes whereas the circuit of Fig. (b) has four branches, two nodes, two junctions, six loops and three meshes.



(a)



(b)

Active and Passive Elements. Elements which supply the energy to the circuit are known as active elements. A network which contains active elements is known as active networks. Ex: batteries, generators, transistors etc. Elements which absorb the energy are known as passive elements. A network containing only passive elements is known as passive networks. Ex: resistors, inductors, and capacitors.

Unilateral/Bilateral Elements. An element whose operational behavior is dependent on the direction of flow of current through is known as unilateral elements. Elements like semiconductor diode, which allow the current to pass through them only in one direction.

An element whose behavior is same irrespective of the direction of flow of current through it is known as bilateral element. Passive elements that allow the current to pass through them in both directions are known as bilateral elements.

Lumped and Distributed Networks. Networks consisting of elements which can be physically separated are known as lumped networks. Most of the networks we deal with, are lumped in nature and consists of R, L, C and sources. Networks, like transmission lines, having inseparable elements are known as distributed networks.

Linear and Non-Linear Elements. A linear element is one which has linear output/input relation and always follows superposition and homogeneity principles. Ohm’s can be applied to such networks.

The element that which does not follow these is known as a nonlinear element. Ohm's law cannot be applied to such networks.

There are two general approaches to network analysis:

(i) Direct Method

Here, the network is left in its original form while determining its different voltages and currents. Such methods are usually restricted to simple circuits and include Kirchhoff's laws, Loop analysis, Nodal analysis, superposition theorem, Compensation theorem and Reciprocity theorem etc.

(ii) Network Reduction Method

Here, the original network is converted into a much simpler equivalent circuit for rapid calculation of different quantities. This method can be applied to simple as well as complicated networks. Examples of this method are: Delta/Star and Star/Delta conversions. Thevenin's theorem and Norton's Theorem etc.

The fundamental laws that govern electric circuits are the Ohm's Law and the Kirchoff's Laws.

Ohm's Law

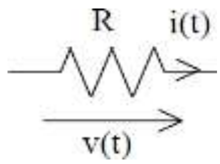
The potential difference across any two points on a conductor is directly proportional to the current flowing through it, provided the physical conditions, viz., material, length, cross-sectional area, and temperature of the conductor remains constant.

OR

Ohm's Law states that the voltage v(t) across a resistor R is directly proportional to the current i(t) flowing through it.

$$v(t) \propto i(t)$$

or $$v(t) = R \cdot i(t)$$



This general statement of Ohm's Law can be extended to cover inductances and capacitors as well under alternating current conditions and transient conditions. This is then known as the **Generalized Ohm's Law.**

Determination of Voltage Sign

In applying Kirchhoff's laws to specific problems, particular attention should be paid to the algebraic signs of voltage drops and e.m.f.s, otherwise results will come out to be wrong. Following sign conventions is suggested:

(a) Sign of Battery E.M.F.

A rise in voltage should be given a + ve sign and a fall in voltage a − ve sign. Keeping this in mind, as we go from the − ve terminal of a battery to its +ve terminal as in figure a below, there is a rise in potential, hence this voltage should be given a + ve sign. If, on the other hand, we go from +ve terminal to − ve terminal, then there is a fall in potential, hence this voltage should be preceded by a − ve sign.
It is important to note that the sign of the battery e.m.f. is independent of the direction of the current through that branch.

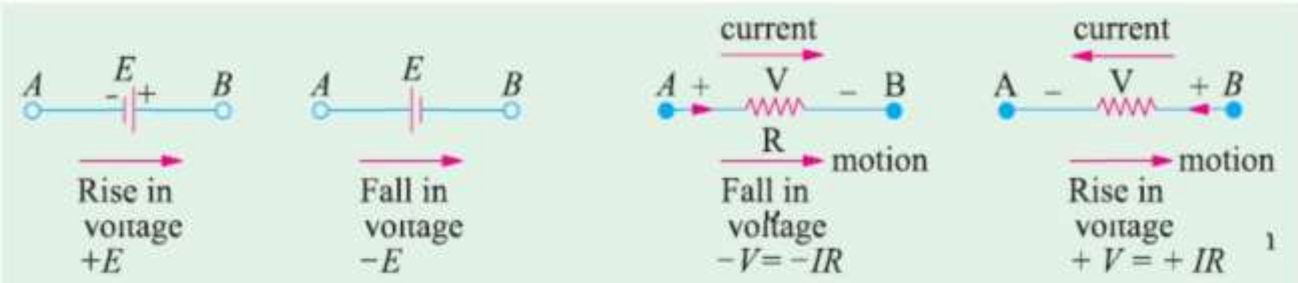


Figure a   Figure b

(b) *Sign of IR Drop*

Now, take the case of a resistor (Fig. b). If we go through a resistor in the same direction as the current, then there is a fall in potential because current flows from a higher to a lower potential. Hence, this voltage fall should be taken − ve. However, if we go in a direction opposite to that of the current, then there is a rise in voltage. Hence, this voltage rise should be given a positive sign.
The sign of voltage drop across a resistor depends on the direction of current through that resistor but is independent of the polarity of any other source of e.m.f. in the circuit under consideration.

Limitation of Ohm's Law

1] It is not applicable to the nonlinear devices like diodes, Zener dioses, voltage regulators etc.
2] It does not hold good for non-metallic conductors like silicon carbide. The law for such conductor is given by

V= k I m where k and m are constants.

Concept of emf and potential difference

Electric Potential:

When a body is charged, work is done in charging it. This work done is stored in the body in the form of potential energy. The charged body has the capacity to do work by moving other charges either by attraction or repulsion.

The capacity of a charged body to do work is called electric potential.

The greater the capacity of the charged body to do work, the greater is its electric potential. The work done to charge a body to 1C will be a measure of its electric potential.

$$V = \frac{Work\ Done}{Charge} = \frac{W}{Q} \quad (\underline{\textbf{Unit:}}\ Joules/Coulomb\ or\ Volt)$$

Potential Difference

The difference in the potentials of any two charged bodies is called potential difference.

Example:

Consider two bodies, 'A' and 'B' having potentials of 5V and 3V respectively as shown in the figure. Each Coulomb of charge on body A has an energy of 5J, while each Coulomb of charge on body B has a energy of 3J. Hence, body A is at a higher potential than body B.

If the two bodies are joined through a conductor, then electrons will flow from body B to body A. When the two bodies attain the same potential, the flow of current stops.
Conclusion: Current will flow in a circuit if potential difference exists. No potential difference, no current flow. Potential difference is sometimes called voltage. Unit of potential difference is Volt(V).

Resistance

The opposition offered by a substance to the flow of electric current is called resistance. As current is the flow of free electrons, resistance is the opposition offered by the substance to the flow of free electrons. This opposition occurs because atoms and molecules of the same substance obstruct the flow of these electrons.

Resistance is the electric friction offered by the substance and causes production of heat with the flow of electric current. The moving electrons collide with atoms or molecules of the substance, each collision resulting in the liberation of minute quantities of heat.

The unit of resistance is Ohm(Ω).

Factors on which resistance depends on:

$$R \; \alpha \; \frac{l}{a}$$

Resistance(R) of a conductor is:

- Directly proportional to its length(l).
- Inversely proportional to its cross-sectional area(a).
- Depends on the nature of its material.
- Nature of material.

$$R = \rho \frac{l}{a}$$

Where, is a constant and is known as the resistivity or specific resistance of the material.
Specific Resistance/Resistivity

$$R = \rho \frac{l}{a}$$

If, $l = 1m$ and $a = 1m^2$, then $R = \rho$.

Hence, specific resistance $\boldsymbol{\rho}$ of a material is the resistance offered by 1m length of wire of material having a cross-sectional area of $1m^2$.

Conductance(G): The conductance is defined as the reciprocal of resistance.

$$R = \rho \frac{l}{a}$$

$$G = \frac{1}{R} = \frac{1}{\rho} \frac{a}{l}$$

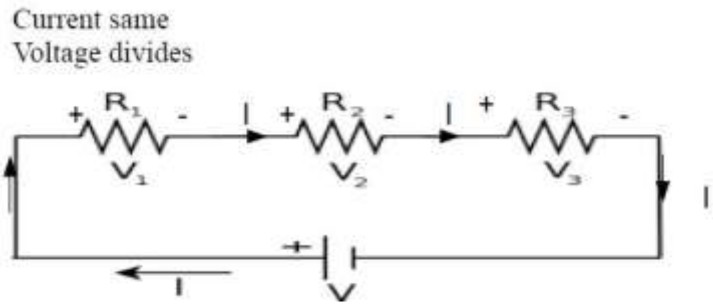
OR

$$G = \sigma \frac{a}{l}$$

Where, σ (sigma) = Conductivity/Specific conductance of the material. The unit of conductance is mho. SI unit of conductivity is mho/m or Siemens.

Resistances in Series

A series circuit is the one in which several resistances are connected one after the other. Such connections are also called as end to end connections or cascade connections. There is only one path for the flow of current. Consider the resistances as shown in the figure below.



The resistance, R_1, R_2 and R_3 are said to be in series. The combination is connected across a source of voltage(V) volts. Naturally, the current flowing through all of them is indicated as ’I’ Amperes. Eg: The chain of small lights used for the decoration purposes.

Voltage Distribution

Let, V_1, V_2 and V_3 be the voltages across the terminals of resistances R_1, R_2 and R_3 respectively.
Then, $V = V_1 + V_2 + V_3$
According to Ohm’s Law,

$V_1 = IR_1$, $V_2 = IR_2$ and $V_3 = IR_3$

Current through all of them is same,
$V = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3)$
Applying Ohm’s law to overall circuit, $V = IR_{eq}$
Where, R_{eq} is the equivalent resistance of the circuit.

Hence, total resistance or equivalent resistance of the series circuit is the arithmetic sum of the resistances connected in series. For ’n’ resistances connected in series,

$$R = R_1 + R_2 + ... + R_n$$

Characteristics of series circuits

1. The same current flows through each resistance.
2. The supply voltage, ’V’ is the sum of the individual voltage drops across the resistances.

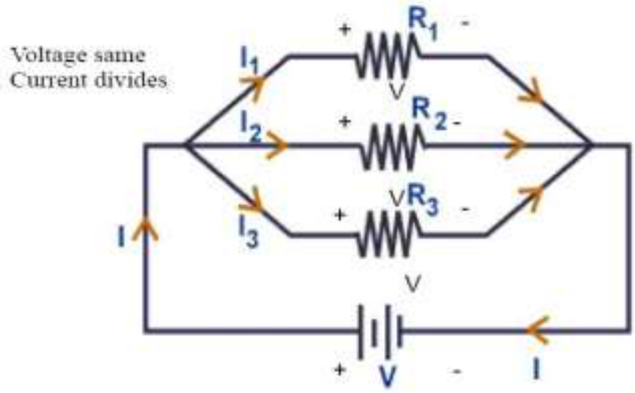
$$V = V_1 + V_2 + ... + V_n$$

3. The equivalent resistance is equal to the sum of the individual resistances.
4. The equivalent resistance is the largest of all the individual resistances.

$$R > R_1,\ R > R_2,\ ...\ R > R_n$$

Resistances in parallel

The parallel circuit is the one in which several resistances are connected across one another in such a way that one terminal of each is connected to form a junction point while the remaining ends are also joined to form another junction point. Consider a parallel circuit as shown in the figure.



In the parallel connection, the three resistances, R_1, R_2 and R_3 are connected in parallel and the combination is connected across a source of voltage, 'V'.

In a parallel circuit, current passing through each resistance is different. Let the total current drawn is say, 'I' as shown. There are 3 paths for the current, one through R_1, second through R_2 and the third through R_3. Depending upon the values of R_1, R_2 and R_3, the appropriate fraction of total current passes through them. These individual currents are shown as I_1, I_2 and I_3. The voltage across the two ends of each resistance R_1, R_2 and R_3 is the same and equals the supply voltage, 'V'.

Current Distribution, $V = I_1R_1$, $V = I_2R_2$ and $V = I_3R_3$

$I_1 = V/R_1$, $I_2 = V/R_2$ and $I_3 = V/R_3$

$I = I_1 + I_2 + I_3$

$I = V/R_1 + V/R_2 + V/R_3$

$I = V [1/R_1 + 1/R_2 + 1/R_3]$

For overall circuit, if Ohm's law is applied,

$V = IR_{eq}$

$I = V/R_{eq}$

Where, R_{eq} = Total or equivalent resistance of the circuit.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Where, 'R_{eq}' is the equivalent resistance of the parallel combination.

In general, if 'n' resistances are connected in parallel,

$$\boxed{\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\ldots\ldots+\frac{1}{R_n}}$$

Now, if n = 2, two resistances are in parallel, then,

$$\frac{1}{R}=\frac{1}{R_1}+\frac{1}{R_2}$$

$$\boxed{R=\frac{R_1 R_2}{R_1+R_2}}$$

Characteristics of parallel circuits

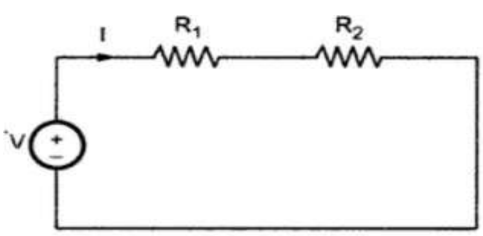
• The same potential difference gets across all the resistances in parallel.
• The total current gets divided into the number of paths equal to the no. of resistances in parallel. The total current is always the sum of all the individual currents.

$I = I_1 + I_2 + \ldots + I_n$

• The reciprocal of the equivalent resistance of a parallel circuit is equal to the sum of the reciprocal of the individual resistances.
• The equivalent resistance is the smallest of all the resistances. $R < R_1$, $R < R_2$, ..., $R < R_n$.
• The equivalent conductance is the arithmetic addition of the individual conductance.

Voltage division is series circuit of resistance.

Consider a series circuit consisting of two resistors R_1 and R_2 connected to a voltage source of V volts.



The potential difference across R_1 is $V_{R1} = IR_1$ and the voltage, and across R_2 is $V_{R2} = IR_2$ i.e., total voltage across the series combination is,

$$V = I R_1 + I R_2$$

$$\therefore \quad I = \frac{V}{R_1 + R_2}$$

Total voltage applied is equal to the sum of the voltage drops V_{R1} and V_{R2} across R_1 and R_2 respectively.

$$\therefore \quad V_{R1} = I \,.\, R_1$$

$$\therefore \quad \boxed{V_{R1} = \frac{V}{R_1 + R_2} \cdot R_1 = \left[\frac{R_1}{R_1 + R_2}\right] V}$$

Similarly, $V_{R2} = IR_2$

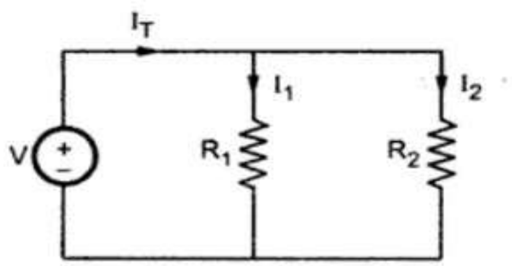
$$\boxed{V_{R2} = \frac{V}{R_1 + R_2} \cdot R_2 = \left[\frac{R_2}{R_1 + R_2}\right] V}$$

So, this circuit is a Voltage Divider Circuit

In general, voltage drop across any resistor or combination of resistors, in a series circuit is equal to the ratio of that resistance value to the total resistance, multiplied by the source voltage.

Current division is Parallel circuit of resistance.

Consider a parallel circuit consisting of two resistors R_1 and R_2 connected across a voltage source of V volts.



Current through R_1 is I_1, current through R_2 is I_2 and the total current is I_T

$$\therefore \quad I_T = I_1 + I_2$$

But $I_1 = \dfrac{V}{R_1}$, $\quad I_2 = \dfrac{V}{R_2}$

i.e. $V = I_1 R_1 = I_2 R_2$

$$\therefore \quad I_1 = I_2 \left(\frac{R_2}{R_1}\right)$$

Substituting value of I_1 in I_T,

$$\therefore \quad I_T = I_2 \left(\frac{R_2}{R_1}\right) + I_2 = I_2 \left[\frac{R_2}{R_1} + 1\right] = I_2 \left[\frac{R_1 + R_2}{R_1}\right]$$

$$\therefore \quad \boxed{I_2 = \left[\frac{R_1}{R_1 + R_2}\right] I_T}$$

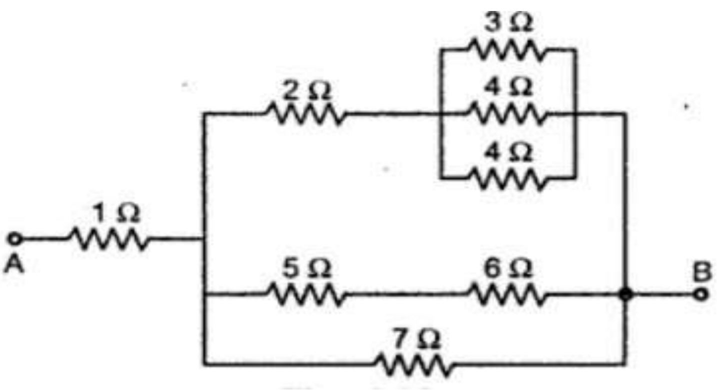
Now $\quad I_1 = I_T - I_2 = I_T - \left[\dfrac{R_1}{R_1 + R_2}\right] I_T$

$$\therefore \quad I_1 = \left[\frac{R_1 + R_2 - R_1}{R_1 + R_2}\right] I_T$$

$$\therefore \quad \boxed{I_1 = \left[\frac{R_2}{R_1 + R_2}\right] I_T}$$

In general, the current in any branch is equal to the ratio of opposite branch resistance to the total resistance value, multiplied by the total current in the circuit.

Example: Find the equivalent resistance between terminal A and B.



Solution: Identify the series and parallel combination of resistances.

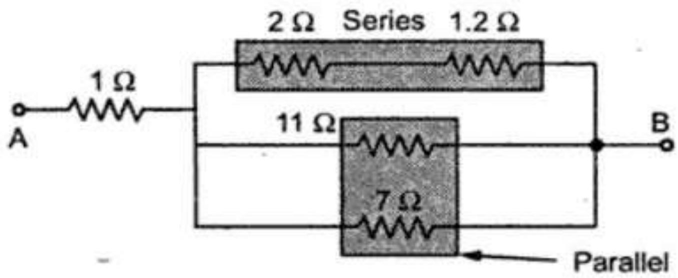
The resistance 5Ω and 6Ω are in series so they are going to carry the same current.
Total resistance 5+ 6= 11 Ω.

The resistance 3Ω, 4Ω and 4Ω are in parallel as the current divides but the voltage remains the same

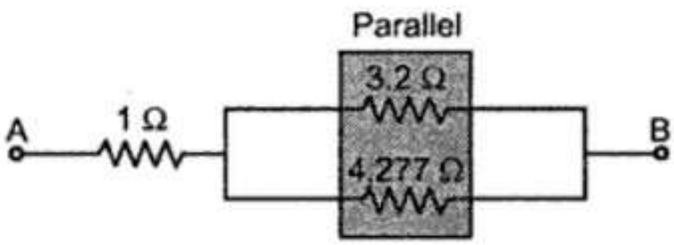
$$\frac{1}{R} = \frac{1}{3}+\frac{1}{4}+\frac{1}{4} = \frac{10}{12}$$

$$R = \frac{12}{10} = 1.2\ \Omega$$

Replacing these combinations and redrawing the circuit we get



Now 1.2 and 2 Ω are in series and 11 and 7 Ω are in parallel



Therefore, we get

$$\frac{3.2 \times 4.277}{3.2 + 4.277} = 1.8304\ \Omega$$

$$R_{AB} = 1 + 1.8304 = 2.8304\ \Omega$$

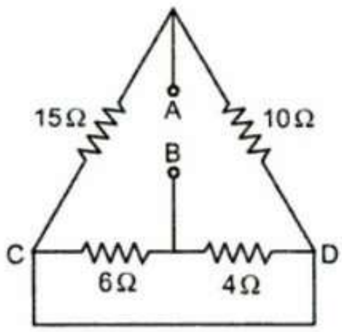
(c) Since $p = I^2R$,

$$R_1 = \frac{p_1}{I_1^2} = \frac{20}{2.222^2} = 4.05\ \Omega$$

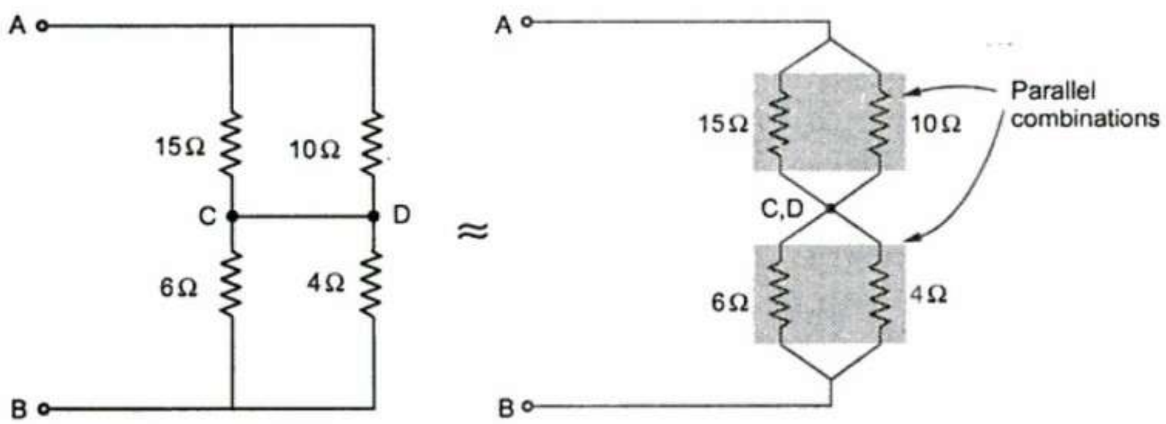
$$R_2 = \frac{p_2}{I_2^2} = \frac{15}{2.777^2} = 1.945\ \Omega$$

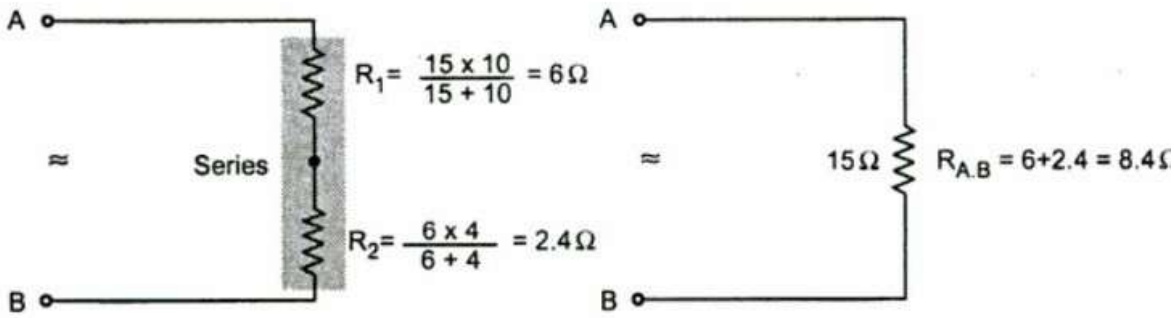
$$R_3 = \frac{p_3}{I_3^2} = \frac{10}{2.777^2} = 1.297\ \Omega$$

Example: Find the equivalent resistance between points A –B.

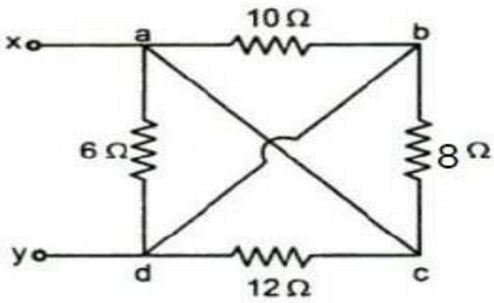


Solution: Redrawing the circuit.



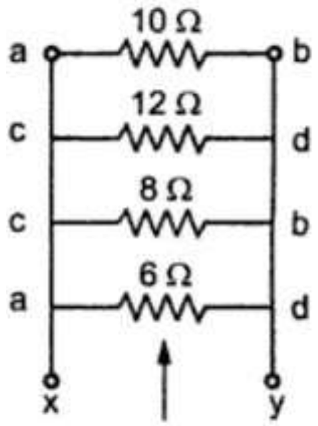


Example: Find the equivalent resistance across terminals X and Y.



$$\therefore \qquad R_{AB} = 8.4\ \Omega$$

Solution: Points x and a are at same potential.
Points y and d are at same potential
Points a and c are at same potential
Points d and b are at same potential
All resistances are in parallel.

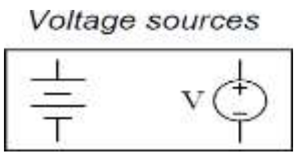


$$\therefore \qquad R_{xy} = \frac{1}{\frac{1}{6}+\frac{1}{8}+\frac{1}{12}+\frac{1}{10}} = 2.1053\ \Omega$$

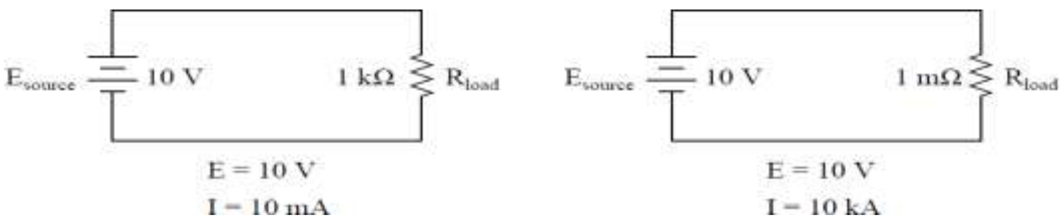
Voltage and Current sources

Ideal voltage source

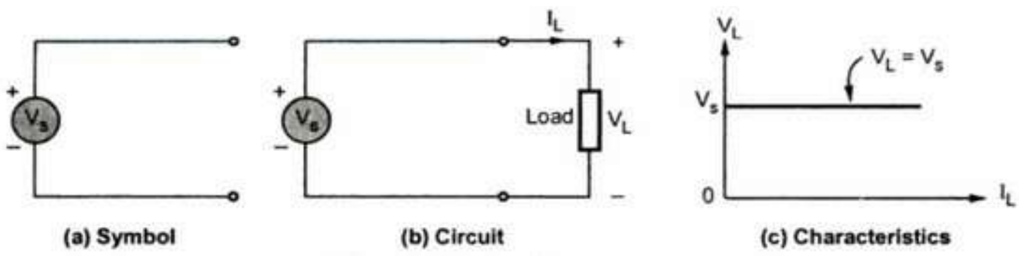
An ideal voltage source (also called constant voltage source) is one which maintains a constant terminal voltage no matter how much current is drawn from it.



The figure shows an ideal voltage source of 10V. Regardless of the value of load resistance R_L, the terminal voltage will remain 10V. An ideal voltage source has *zero or negligible internal resistance* so that there is negligible voltage drop in the internal resistance due to change in current. Consequently, terminal voltage remains constant.



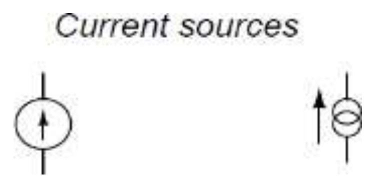
In real life, there is no such thing as a perfect voltage source, but sources having extremely low internal resistance come close.



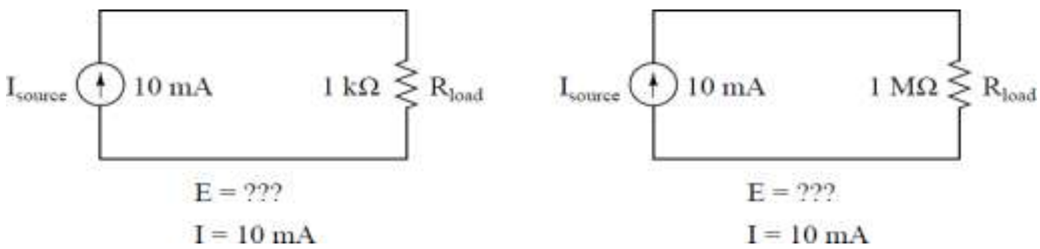
Example: Internal resistance of a lead-acid cell is very small (0.01) so it can be regarded as a constant voltage source for all practical purposes.

Ideal current source

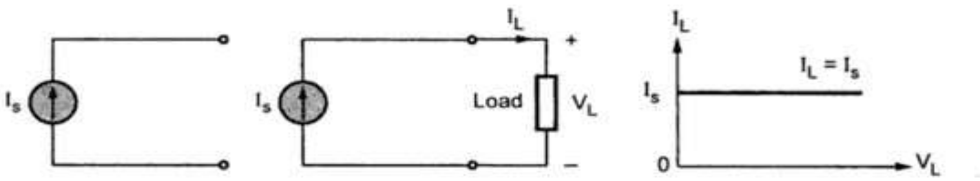
An ideal current source (also called constant current source) is one which will supply the same current to any resistance connected across its terminals.



The figure shows an ideal current source of 10mA. Regardless of the value of load resistance R_L, the source will supply a current of 10mA. The schematic symbol for a current source is a circle with an arrow in it.



The arrow shows the direction of the conventional current produced by the source.

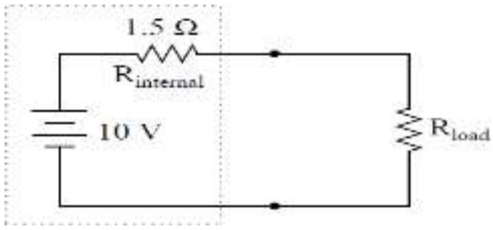


(a) Symbol (b) Circuit (c) Characteristics

Real (Practical) voltage and current sources

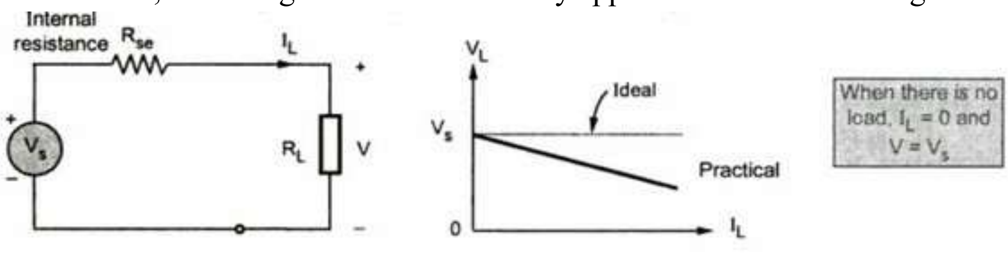
Real / practical voltage source

A real(non-ideal) voltage source has internal resistance that causes its terminal voltage to decrease when current is drawn from it. A real voltage source can be represented as an ideal voltage source in series with a resistor equal to its internal resistance (R_{int}).



This representation can be used to calculate the true terminal voltage of a voltage source when current is drawn from it. Note that internal resistance is an inherent property of a source; it is not a discrete component that can be measured with an Ohmmeter.

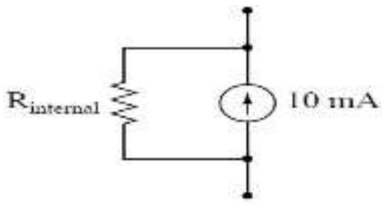
As R_{int} becomes small, the voltage source more closely approaches the ideal voltage source.



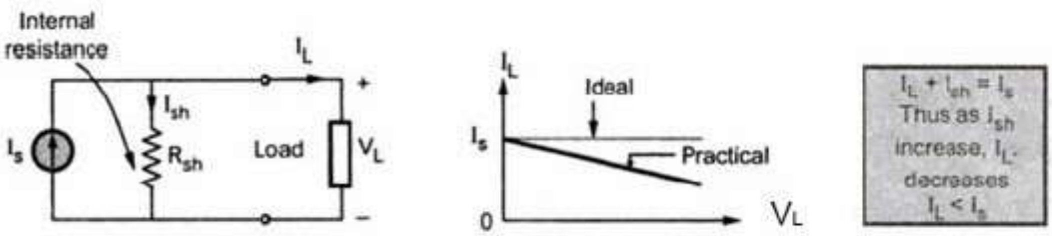
(a) Circuit (b) Characteristics

Real/ practical current source

A real current source can be represented as an ideal current source in parallel with an internal resistance (R_{int}) as shown in the figure.



When load resistance R_L is connected across the terminals, the current (I) produced by the source divides between R_{int} and R_L. Consequently, the load current is less than it would be if the source were ideal.



(a) Circuit (b) Characteristic

Example: A 12V voltage source has an internal resistance of 2Ω. Find the following parameters,
1. Terminal voltage when 22 load is connected across its terminals.
2. Load current.

Solution: Load current

$$I_L = \frac{12}{2+22} = 0.5A$$

Terminal Voltage, V_T = Load voltage
Hence, VT = $I_L * R_L$ = 0.5 * 22 = 11V

Example: When a 1k Ω load is connected across a 20mA current source, it is found that only 18mA flows in the load. What is the internal resistance of the source?

Solution:

Load Voltage = $I_L R_L$ = 18mA ***X*** 1k Ω = 18V

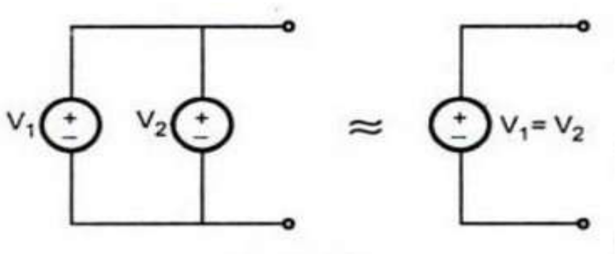
Current in the internal resistance (R_{int}),
I_{int} = 20 − 18 = 2mA
Since, R_{int} is in parallel with R_L, the voltage across R_{int} is the same as V_L = 18V.
Therefore,

$$R_{int} = \frac{V_L}{I_{int}} = \frac{18V}{2mA} = 9k\Omega$$

Voltage sources in series

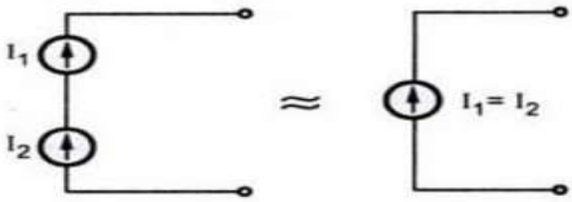
The voltage source connected in series must have the same current rating and their voltage rating may be same or different.

Voltage sources in parallel



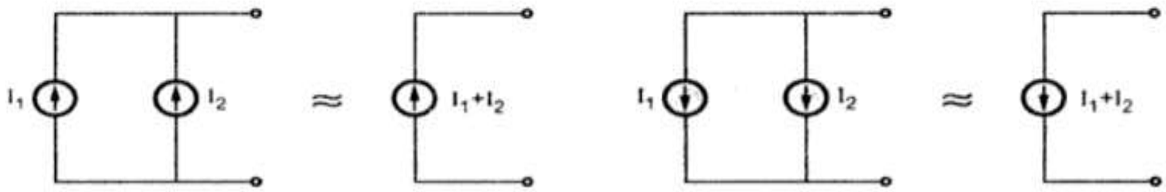
The voltage source connected in parallel must have the same voltage rating and their current rating may be same or different.

Current sources in Series



The current source connected in series must have the same current rating and their voltage rating may be same or different.

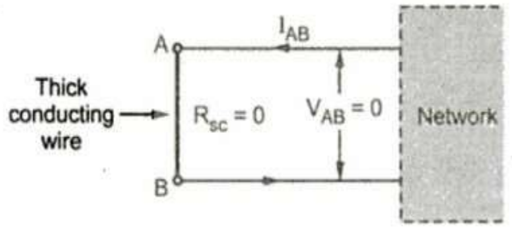
Current sources in Parallel



The current source connected in parallel must have the same voltage rating and their current rating may be same or different.

Concept of Short Circuit and Open Circuit

When any two points in a network are joined directly to each other with a thick metallic conducting wire, the two points are said to be **short circuited**. The resistance of such short circuit is zero.

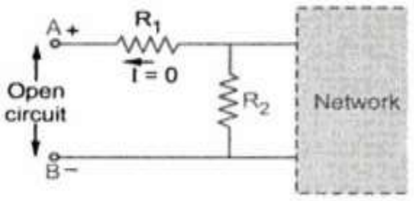


The resistance of branch AB is $R_{sc} = 0$

According to Ohm's law,

$$V_{AB} = R_{sc} \times I_{AB} = 0 \times I_{AB} = 0V$$

When there is no connection between two points in a network, having some voltage across the two points then the two points are said to be **open circuited**. There is no direct connection in OC and the resistance is ∞.



There exists a voltage called open circuit voltage V_{AB} and $R_{oc} = \infty$

According to Ohm's law,

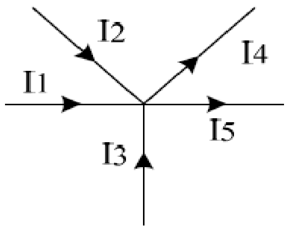
$$I_{oc} = \frac{V_{AB}}{R_{oc}} = \frac{V_{AB}}{\infty} = 0 \text{ A}$$

Kirchoff's Laws

Kirchoff's Current Law (KCL) states at any junction in an electric circuit the total current flowing towards that junction is equal to the total current flowing away from the junction,

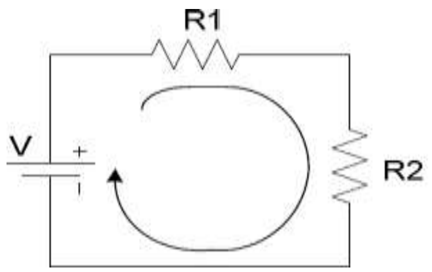
i.e. $\Sigma I = 0$

Thus, referring to figure:



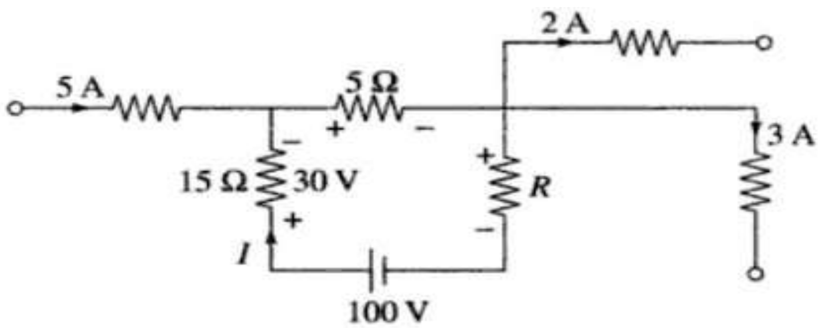
$$\sum \text{current towards} = \sum \text{current flowing away}$$
$$I_1 + I_2 + I_3 = I_4 + I_5$$
$$I_1 + I_2 + (-I_3) + (-I_4) + (-I_5) = 0$$
$$\sum I = 0$$

Kirchoff's Voltage Law (KVL) states in any closed loop in a network, the algebraic sum of the voltage drops (i.e. products of current and resistance) taken around the loop is equal to the resultant e.m.f. acting in that loop.



$$E = IR_1 + IR_2$$
$$E = I(R_1 + R_2)$$
$$E + (-IR_1) + (-IR_2) = 0$$

Example: The voltage drop across the 15Ω resistance is 30V, having polarity indicated. Find tthe value of R?



Solution: Current through the 15Ω resistance is given by

$$I = \frac{30}{15} = 2\text{ A}$$

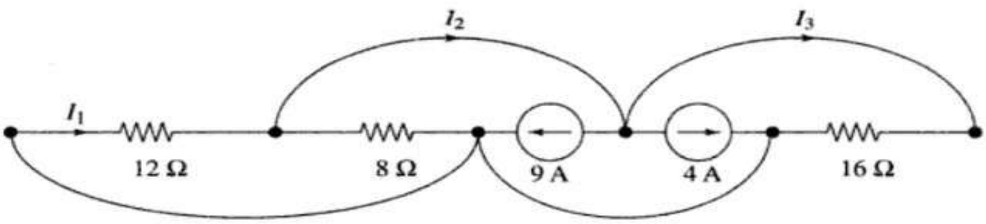
Current through the 5-Ω resistor is given by

$$= 5 + 2 = 7\text{ A}$$

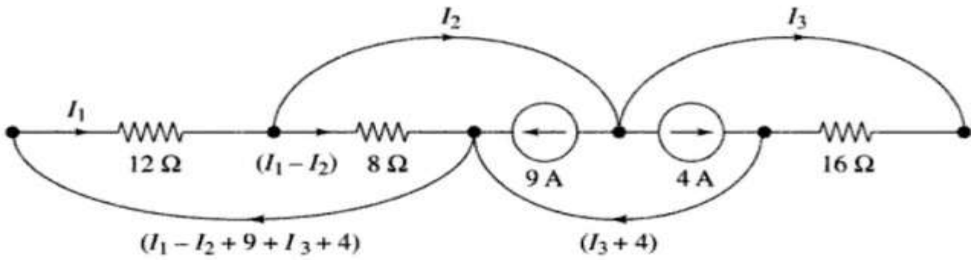
Applying KVL to the closed path,

$$-5(7) - R(I) + 100 - 30 = 0$$
$$-35 - 2R + 100 - 30 = 0$$
$$R = 17.5\ \Omega$$

Example: Determine the currents I_1, I_2 and I_3?



Solution: Assigning currents to all the branches.



$$I_1 = I_1 - I_2 + 9 + I_3 + 4$$
$$I_2 - I_3 = 13 \qquad \text{...... 1}$$

Also, $-12I_1 - 8(I_1 - I_2) = 0$

$$-20I_1 + 8I_2 = 0$$

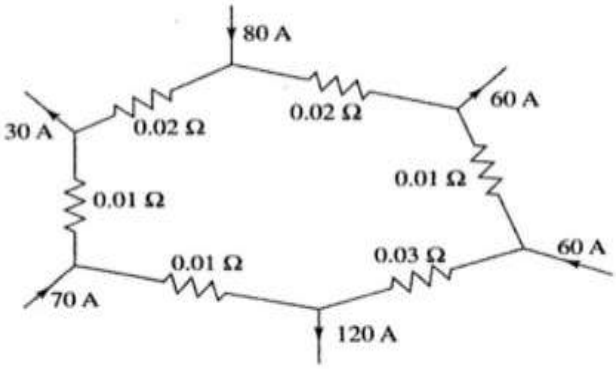
Also, $-12I_1 - 16I_3 = 0$

....2 &3

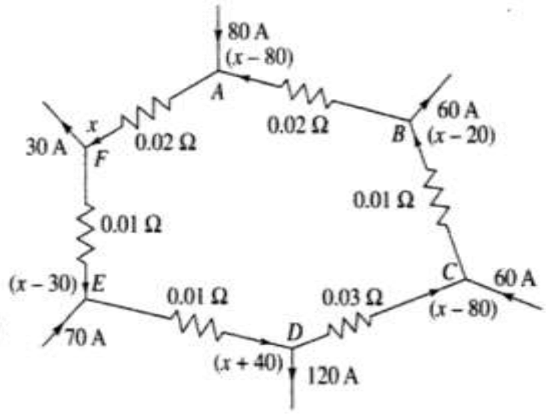
Solving Eqs (i), (ii) and (iii),

$$I_1 = 4\text{ A}$$
$$I_2 = 10\text{ A}$$
$$I_3 = -3\text{ A}$$

Example: **Find the current in all the branches of the network?**



Solution: Let



$$I_{AF} = x$$

then

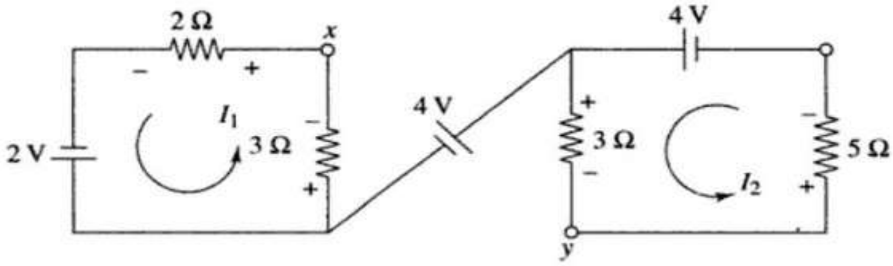
$$I_{FE} = x - 30$$
$$I_{ED} = x + 40$$
$$I_{DC} = x - 80$$
$$I_{CB} = x - 20$$
$$I_{BA} = x - 80$$

Applying KVL to the closed path *AFEDCBA*,

$$-0.02\,x - 0.01\,(x - 30) - 0.01\,(x + 40) - 0.03\,(x - 80)$$
$$-0.01\,(x - 20) - 0.02\,(x - 80) = 0$$

$$x = 41$$
$$I_{AF} = 41\text{ A}$$
$$I_{BA} = 41 - 80 = -39\text{ A}$$
$$I_{AB} = 39\text{ A}$$
$$I_{CB} = 41 - 20 = 21\text{ A}$$
$$I_{DC} = 41 - 80 = -39\text{ A}$$
$$I_{CD} = 39\text{ A}$$
$$I_{ED} = 41 + 40 = 81\text{ A}$$
$$I_{FE} = 41 - 30 = 11\text{ A}$$

Example: What is the potential difference between points x and y in the network?



Solution:

$$I_1 = \frac{2}{5} = 0.4 \text{ A}$$

$$I_2 = \frac{4}{8} = 0.5 \text{ A}$$

Potential difference between points x and y = $V_{xy} = V_x - V_y$

Writing KVL equation for the path x to y,

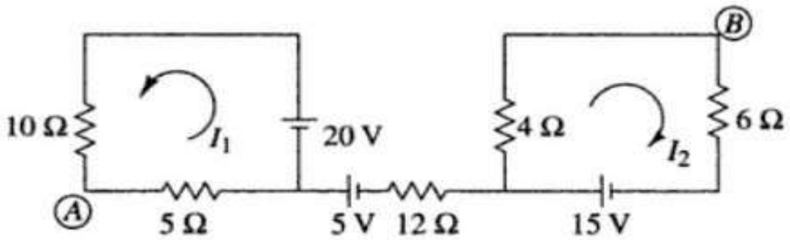
$$V_x + 3I_1 + 4 - 3I_2 - V_y = 0$$

$$V_x + 3(0.4) + 4 - 3(0.5) - V_y = 0$$

$$V_x - V_y = -3.7$$

$$V_{xy} = -3.7 \text{ V}$$

Example: In the network find the voltage between point A and B?

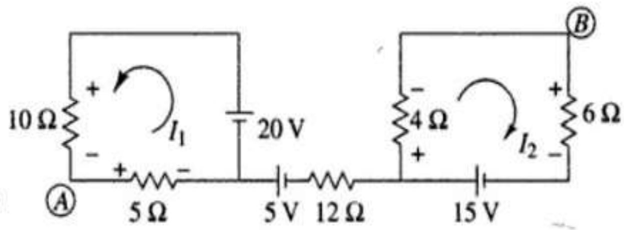


Solution:

$$I_1 = \frac{20}{15} = 1.33 \text{ A}$$

$$I_2 = \frac{15}{10} = 1.5 \text{ A}$$

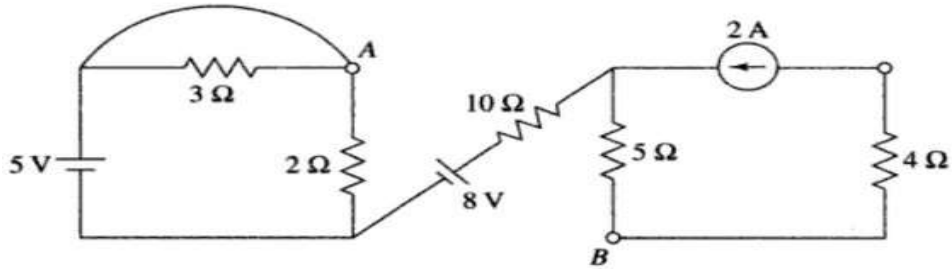
Voltage between points A and B = $V_{AB} = V_A - V_B$



Voltage between points A and $B = V_{AB} = V_A - V_B$
Writing KVL equation for the path A to B,

$$V_A - 5I_1 - 5 - 15 + 6I_2 - V_B = 0$$
$$V_A - 5(1.33) - 5 - 15 + 6(1.5) - V_B = 0$$
$$V_A - V_B = 17.65$$
$$V_{AB} = 17.65 \text{ V}$$

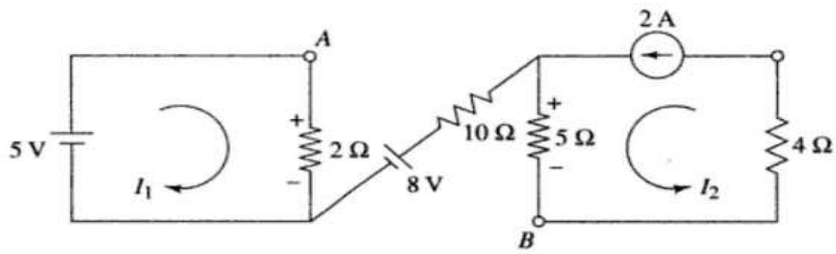
Example: Determine the potential difference V_{AB} for the given network.



Solution:

The resistance of 3 Ω is connected across a short circuit. Hence, it gets shorted.

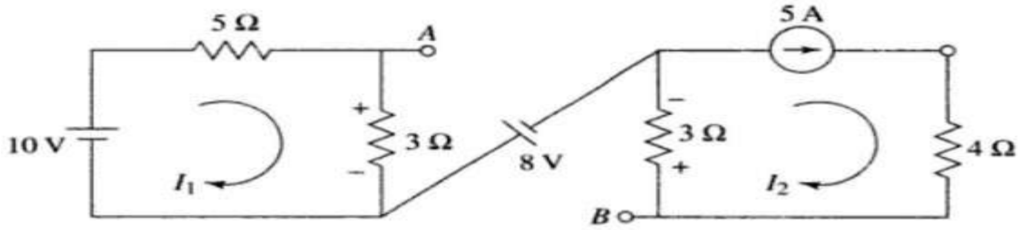
$$I_1 = \frac{5}{2} = 2.5 \text{ A}$$
$$I_2 = 2 \text{ A}$$



Potential difference $V_{AB} = V_A - V_B$
Writing KVL equation for the path A to B,

$$V_A - 2I_1 + 8 - 5I_2 - V_B = 0$$
$$V_A - 2(2.5) + 8 - 5(2) - V_B = 0$$
$$V_A - V_B = 7$$
$$V_{AB} = 7 \text{ V}$$

Example: Find the voltage of point A w.r.t B?



Solution:

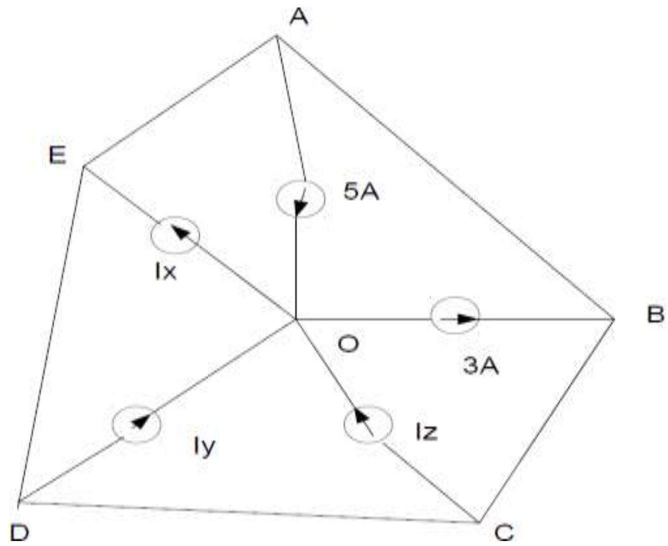
$$I_1 = \frac{10}{8} = 1.25 \text{ A}$$
$$I_2 = 5 \text{ A}$$

Applying KVL to the path from A to B,

$$V_A - 3I_1 - 8 + 3I_2 - V_B = 0$$
$$V_A - 3(1.25) - 8 + 3(5) - V_B = 0$$
$$V_A - V_B = -3.25$$
$$V_{AB} = -3.25 \text{ V}$$

Example:

Find (i) Ix, if Iy = 2A and Iz = 0 A
(ii) Iy, if Ix = 2A and Iz = 2 Iy A
(iii) Iz, if Ix = Iz = Iy



Solution:

(i) Ix, if Iy = 2A and Iz = 0 A
By applying KCL at node O, we get
5 + Iz + Iy = Ix + 3
5 +0 +2 = Ix +3

Ix = 4 A

(ii) Iy, if Ix = 2A and Iz = 2 Iy A

By applying KCL at node O, we get

5 + Iz + Iy = Ix + 3

5 + 2 Iy + Iy = 2 + 3

***Iy* = 0 A**

(iii) Iz, if Ix = Iz = Iy

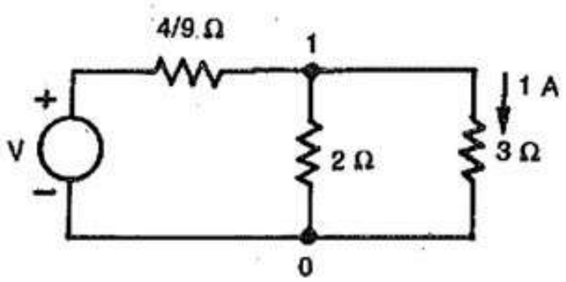
by applying KCL at node O, we get

5 + Iz + Iy = Ix + 3

5 + Iz +Iz = Iz + 3

***Iz* = - 2 A**

Example: For the resistive circuit find the value of V.



Solution:

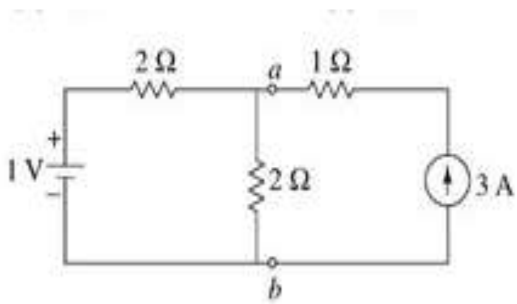
$V_{10} = 3 \times 1 = 3$ V, $\quad I(2\ \Omega) = 3/2$ A

$I(4/9\ \Omega) = 1 + 3/2 = 5/2$ A

$V = V_{10} + I(4/9\ \Omega) \times (4/9)$

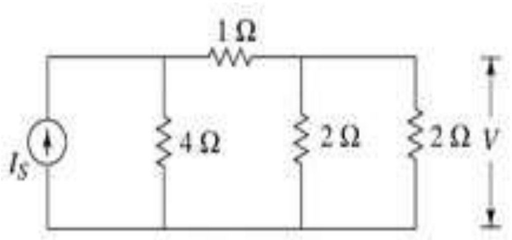
$= 3 + (4/9) \times (5/2) = 4.11$ V

Example: The voltage across terminal and b is _______________V



Solution: [Ans: 3.5V]

Example: If $V = 4V$, the value of I_s given ________



[Ans: 6 A]

Solution:

Mesh Analysis

Analysis using KVL to solve for the currents around each closed loop of the network and hence determine the currents through and voltages across each element of the network.

In this method, Kirchoff's voltage laws are applied to each mesh in terms of mesh currents instead of branch currents. Each mesh is assigned a separate mesh current. This mesh current is assumed to flow in the clockwise direction around the perimeter of the mesh without splitting at the junction into branch currents. Kirchoff's voltage law is applied to write equation in terms of unknown mesh currents. As the mesh currents are known, the branch currents can be easily determined.

Maxwell mesh current method consists of the following steps:

• Each mesh is assigned a separate mesh current. All mesh currents are assumed to flow in the clockwise direction (for convenience).

• If two mesh currents are flowing through a circuit element, the actual current in the circuit is the algebraic sum of the two.

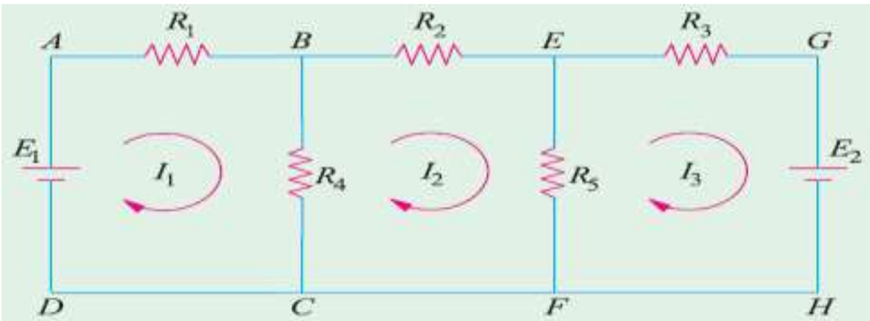


Figure shows two batteries E1and E2 connected in a network consisting of five resistors. Let the loop currents for the three meshes be I1, I2 and I3. It is obvious that current through R4 (when considered as a part of the first loop) is (I1−I2) and that through RS is (I2−I3). However, when R4 is considered part of the second loop, current through it is (I2−I1).

Similarly, when R5 is considered part of the third loop, current through it is(I3−I2). Applying Kirchhoff's voltage law to the three loops, we get,

$$E_1 - I_1R_1 - R_4\,(I_1 - I_2) = 0 \quad \text{or} \quad I_1\,(R_1 + R_4) - I_2\,R_4 - E_1 = 0 \qquad \text{...loop 1}$$

Similarly, $-I_2R_2 - R_5\,(I_2 - I_3) - R_4\,(I_2 - I_1) = 0$

or $I_2\,R_4 - I_2\,(R_2 + R_4 + R_5) + I_3R_5 = 0$ ...**loop 2**

Also $-I_3R_3 - E_2 - R_5\,(I_3 - I_2) = 0$ or $I_2R_5 - I_3\,(R_3 + R_5) - E_2 = 0$ ...**loop 3**

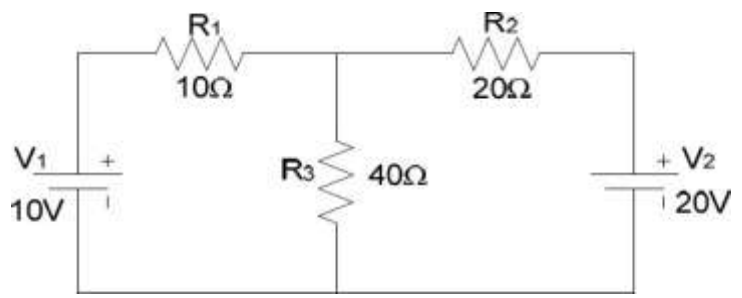
The above three equations can be solved not only to find loop currents but branch currents as well.

• If the value of any mesh current comes out to be negative in the solution, it means that true direction of that mesh current is anticlockwise, i.e., opposite to the assumed clockwise direction.

NOTE: Branch currents are the real currents because they flow in the branches and can be measured. However, mesh currents are fictitious quantities and cannot be measured directly. Hence, mesh current is concept rather than a reality.

Practice problems in Mesh Analysis

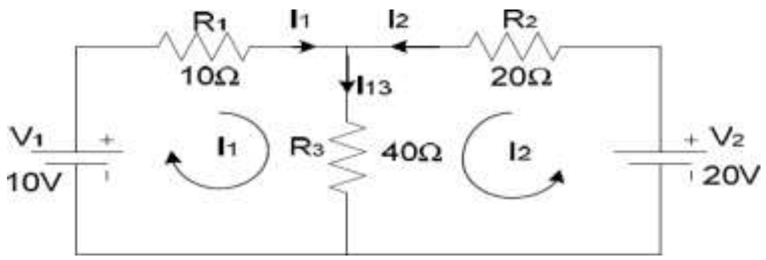
Example: Find the current flow through each resistor using **mesh analysis** for the circuit below.



Solution:

Step 1: Assign a distinct current to each closed loop of the network.

Take current in clock wise direction preferably



Step 2: Apply KVL around each closed loop of the network.

Loop 1:

$$I_1R_1 + I_1R_3 + I_2R_3 = V_1$$
$$10I_1 + 40I_1 + 40I_2 = 10$$
$$50I_1 + 40I_2 = 10 \quad \text{------------ equation 1}$$

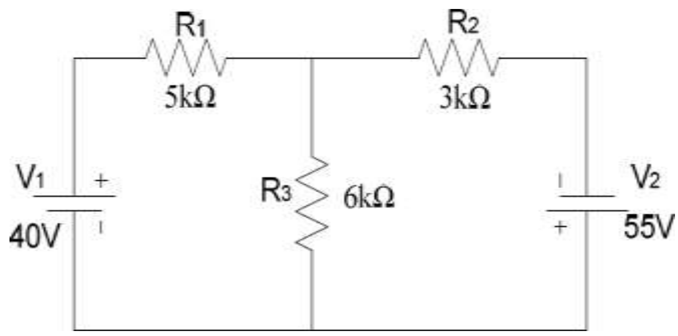
Loop 2:

$$I_2R_2 + I_2R_3 + I_1R_3 = V_2$$
$$20I_2 + 40I_2 + 40I_1 = 20$$
$$40I_1 + 60I_2 = 20 \quad \text{--------------- equation 2}$$

$$I_3 = I_1 + I_2 = -0.143 + 0.429 = 0.286A$$

Solving equation 1 and 2 we get I_1= -0.143 A and I_2 = 0.429 A and

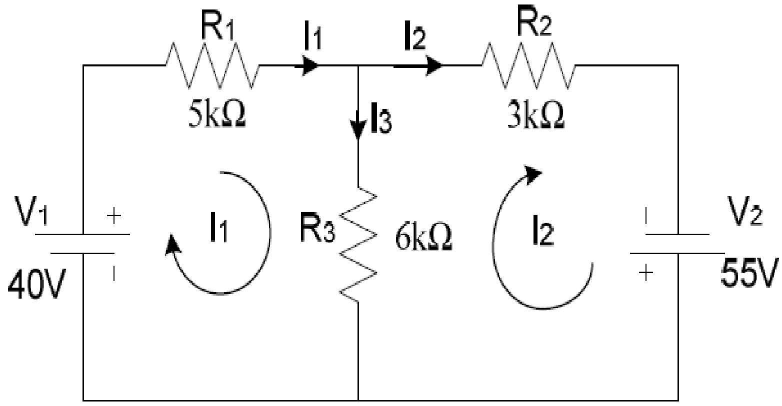
Example: Find the current flow through each resistor using mesh analysis for the circuit.



Take current in clock wise direction preferably

Solution:

Step 1: Assign a distinct current to each closed loop of the network.



Step 2: Apply KVL around each closed loop of the network.

Loop 1:

$I_1R_1 + I_1R_3 - I_2R_3 = V_1$

$5kI_1 + 6kI_1 - 6kI_2 = 40$

$11kI_1 - 6kI_2 = 40$ ------------ equation 1

Loop 2:

$I_2R_2 + I_2R_3 - I_1R_3 = V_2$

$3kI_2 + 6kI_2 - 6kI_1 = 55$

$-6kI_1 + 9kI_2 = 55$ --------------- equation 2

Step 3: Solve the resulting simultaneous linear equations 1 and 2 for the loop currents.

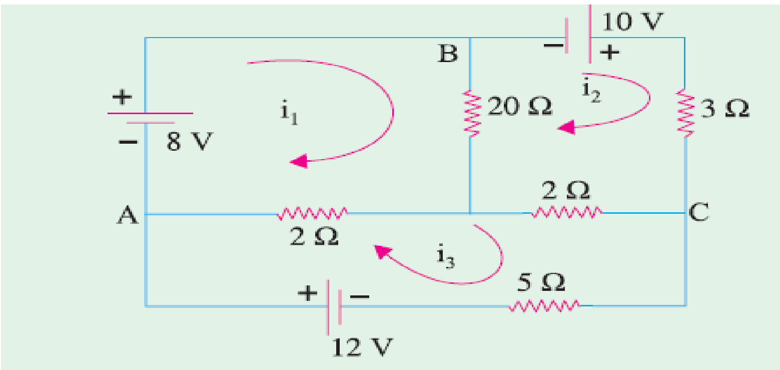
$$11kI_1 - 6kI_2 = 40$$
$$-6kI_1 + 9kI_2 = 55$$

$I_1 = 10.95$ mA

$I_2 = 13.41$ mA

$$I_3 = I_1 - I_2 = 10.95\text{mA} - 13.41\text{mA} = -2.46\text{mA}$$

Example: Determine the current in the 5Ω resistor using Mesh Analysis.



Solution: Consider the three Mesh current i_1, i_2 and i_3 for the three mesh

Writing KVL for Mesh 1

$+8 + 20(i_1-i_2) + 2(i_1-i3) = 0$ .... [1]

Writing KVL for Mesh 2

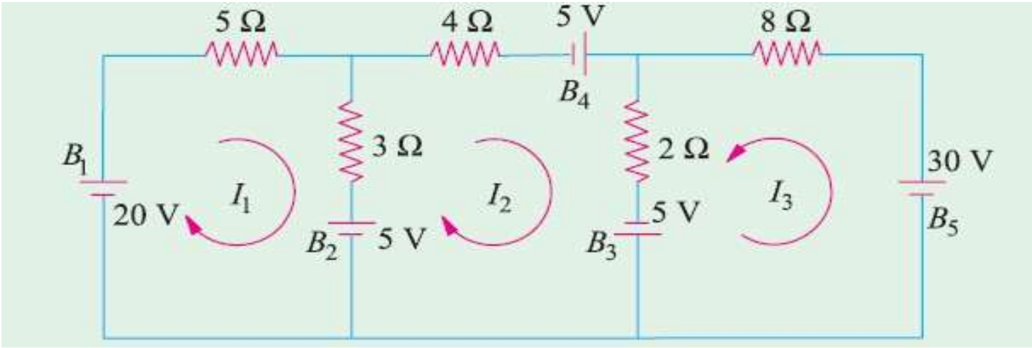
$+10 + 3\, i_2 + 2(i_2-i_3) + 20\,(i_2-i_1) = 0$ .... [2]

Writing KVL for Mesh 3

$+12 + 2(i_3-i_1) + 2(i_3-i_2) + 5\, i_3 = 0$ .... [3]

Solving for i_3 which is the current in 5Ω resistor we get 3.633 A

Example: Determine the current supplied by each battery in the circuit shown in Figure.



Solution:

For loop 1 we get

$20 - 5I_1 - 3(I_1 - I_2) - 5 = 0$ or $8I_1 - 3I_2 = 15$ ...(*i*)

For loop 2 we have

$-4I_2 + 5 - 2(I_2 - I_3) + 5 + 5 - 3(I_2 - I_1) = 0$ or $3I_1 - 9I_2 + 2I_3 = -15$ ...(*ii*)

Similarly, for loop 3, we get

$-8I_3 - 30 - 5 - 2(I_3 - I_2) = 0$ or $2I_2 - 10I_3 = 35$ ...(*iii*)

Eliminating I_1 from (*i*) and (*ii*), we get $63I_2 - 16I_3 = 165$ ...(*iv*)

Similarly, for I_2 from (*iii*) and (*iv*), we have $I_2 = 542/299$ A

From (*iv*), $I_3 = -1875/598$ A

Substituting the value of I_2 in (*i*), we get $I_1 = 765/299$ A

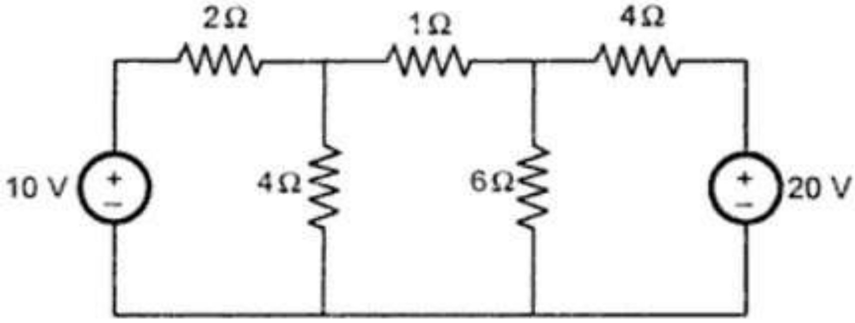
Discharge current of B_1 = **765/299A**

Charging current of $B_2 = I_1 - I_2 =$ **220/299 A**

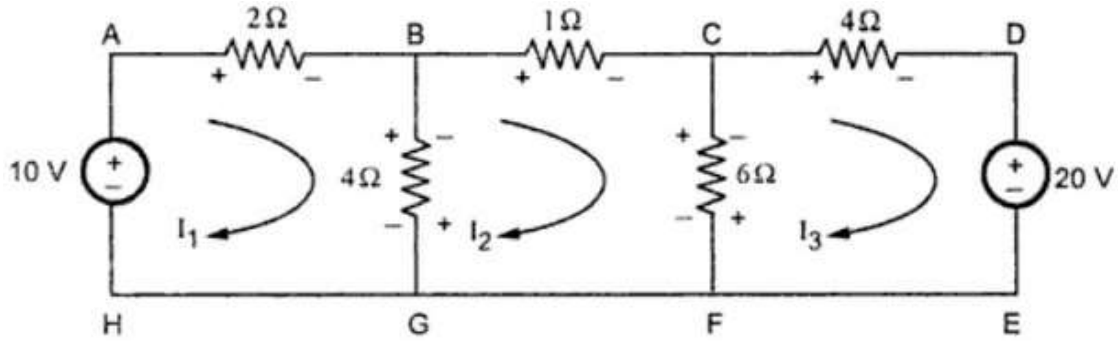
Discharge current of $B_3 = I_2 + I_3 =$ **2965/598 A**

Discharge current of $B_4 = I_2 =$ **545/299 A**; Discharge current of B_5 = **1875/598 A**

Example: Calculate the current through 6Ω resistance using mesh analysis.



Solution: Assuming mesh current I_1, I_2 and I_3 in the three mesh



Consider loop A - B - G - H - A, loop equation is,

$$-2I_1 - 4(I_1 - I_2) + 10 = 0$$

i.e. $$6I_1 - 4I_2 = 10 \quad \text{... (1)}$$

Consider loop B - C - F - G - B, loop equation is,

$$-1I_2 - 6(I_2 - I_3) - 4(I_2 - I_1) = 0$$

i.e $$4I_1 - 11I_2 + 6I_3 = 0 \quad \text{... (2)}$$

Consider loop C - D - E - F - C, loop equation is,

$$-4I_3 - 20 - 6(I_3 - I_2) = 0$$

i.e. $$6I_2 - 10I_3 = 20 \quad \text{... (3)}$$

Solving for I_2 and I_3 to get current through 6Ω we get

$I_2 = (-1.1267\ A)$

$I_3 = (-2.676\ A)$

Current through 6Ω resistance is **$I_2 - I_3$**

$= (-1.1267\ A) - (-2.676\ A)$

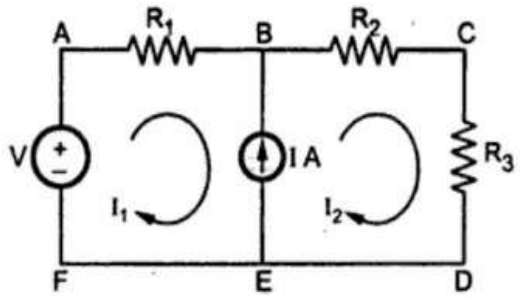
$= 1.5493\ A$ from C to F

Concept of Super Mesh. [only for understanding, not to be asked in exam]

Meshes that share a current source with other meshes, none of them which contains a current source in outer loop form a super mesh. A path around a super mesh does not pass through a current source. A path around each mesh contained within a super mesh passes through a current source. The total number of equations required for a super mesh is equal to the number of meshes contained in the super mesh. A super mesh requires one mesh current equations, that is, a KVL equation. The remaining mesh current equations are KCL equations.

OR

If there exists a current source in any of the branches of the network then a loop cannot be defined through the current source as drop across the current source is unknown from KVL point of view.

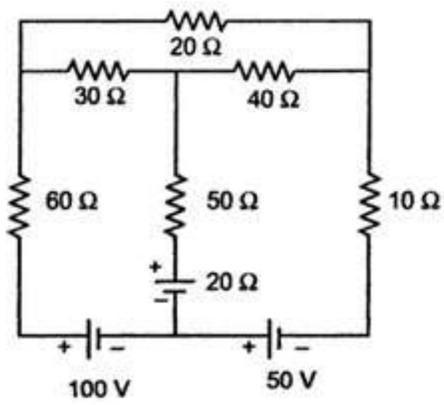


Branch containing the current source must be analyzed separately.

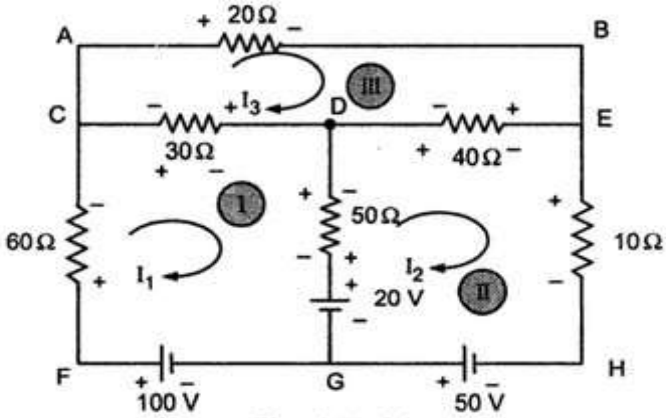
$$I = I_2 - I_1$$

We can write KVL for the super mesh ABCDEF which is the mesh existing around a current source which is common to the two loops.

Example: Calculate the current in the 50 Ω resistor in the network shown in the figure using mesh analysis



Solution: The various mesh current is shown in the diagram



Applying KVL to the three loops,

$$-30\ I_1 + 30\ I_3 - 50\ I_1 + 50\ I_2 - 20 + 100 - 60\ I_1 = 0 \qquad \text{... Loop I}$$

$$140\ I_1 - 50\ I_2 - 30\ I_3 = 80 \qquad \text{... (1)}$$

$$-40\ I_2 + 40\ I_3 - 10\ I_2 + 50 + 20 - 50\ I_2 + 50\ I_1 = 0 \qquad \text{... Loop II}$$

$$-50\ I_1 + 100\ I_2 - 40\ I_3 = 70 \qquad \text{... (2)}$$

$$-20\ I_3 - 40\ I_3 + 40\ I_2 - 30\ I_3 + 30\ I_1 = 0 \qquad \text{... Loop III}$$

$$30\ I_1 + 40\ I_2 - 90\ I_3 = 0 \qquad \text{... (3)}$$

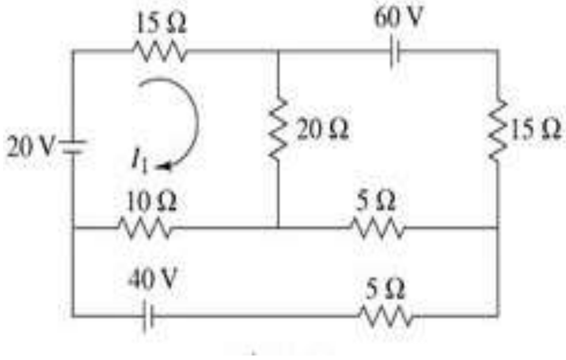
Solving equations (1), (2) and (3),

$$I_1 = 1.6489\ \text{A}\ ,\ I_2 = 2.1214\ \text{A}\ ,\ I_3 = 1.4925\ \text{A}$$

Thus the current through 50 Ω is,

$$I_{50} = I_1 - I_2 = 1.6489 - 2.121 = \mathbf{-\ 0.4721\ A} \quad \text{i.e. } \mathbf{0.4721\ A} \uparrow$$

Example: Find Current I_1 using mesh analysis.



[0.41A]

Nodal Analysis

Analysis using KCL to solve for voltages at each common node of the network and hence determines the currents through and voltages across each element of the network.

The node-equation method is based directly on Kirchhoff's current law unlike loop-current method which is based on Kirchhoff's voltage law.

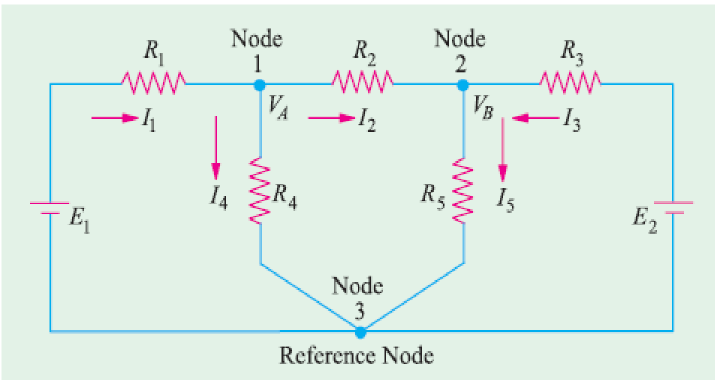
However, like loop current method, nodal method also has the advantage that a minimum number of equations need be written to determine the unknown quantities.

Moreover, it is particularly suited for networks having many parallel circuits with common ground connected such as electronic circuits.

For the application of this method, every junction in the network where three or more branches meet is regarded a node.
One of these is regarded as the reference node or datum node or zero-potential node.

Hence the number of simultaneous equations to be solved becomes (n − 1) where n is the number of independent nodes.

These node equations often become simplified if all voltage sources are converted into current sources.



Take all currents leaving the nodes

Consider the above circuit which has three nodes. One of these i.e. node 3 has been taken in as the reference node.

VA represents the potential of node 1 with reference to the datum node 3.
Similarly, VB is the potential difference between node 2 and node 3.
Let the current directions which have been chosen arbitrary be as shown in the Diagram.

For node 1, the following current equation can be written with the help of KCL

$$I_1 = I_4 + I_2$$

Now $\quad I_1R_1 = E_1 - V_A \quad \therefore\ I_1 = (E_1 - V_A)/R_1 \quad$ ...(*i*)

Obviously, $\quad I_4 = V_A/R_4 \quad$ Also, $I_2R_2 = V_A - V_B \quad (\because\ V_A > V_B)$

$\therefore \quad I_2 = (V_A - V_B)/R_2$

Substituting these values in Eq. (*i*) above, we get,

$$\frac{E_1 - V_A}{R_1} = \frac{V_A}{R_4} + \frac{V_A - V_B}{R_2}$$

Simplifying the above, we have

$$V_A\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right) - \frac{V_B}{R_2} - \frac{E_1}{R_1} = 0 \quad \text{...(ii)}$$

The current equation for node 2 is $I_5 = I_2 + I_3$

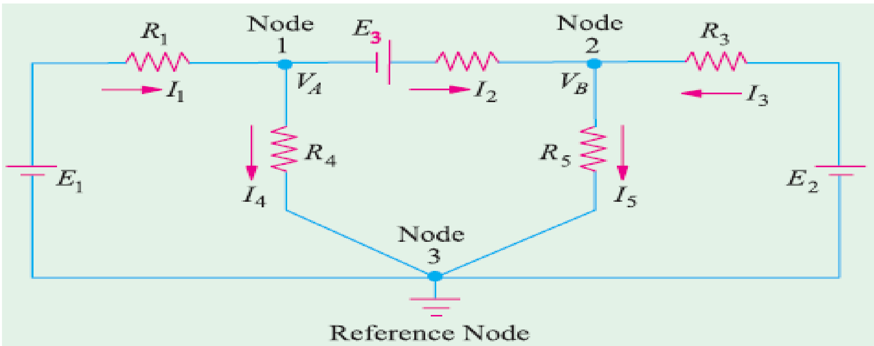
or $$\frac{V_B}{R_5} = \frac{V_A - V_B}{R_2} + \frac{E_2 - V_B}{R_3} \quad \text{...(iii)}$$

or $$V_B\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) - \frac{V_A}{R_2} - \frac{E_2}{R_3} = 0 \quad \text{...(iv)}$$

After finding different node voltages, various currents can be calculated by using Ohm’s law.

***** Now, consider the case when a third battery of e.m.f. E_3 is connected between nodes 1 and 2 as shown in Figure. It must be noted that as we travel from node 1 to node 2, we go from the − ve terminal of E_3 to its +ve terminal. Hence, according to the sign convention, E_3 must be taken as positive.

However, if we travel from node 2 to node 1, we go from the +ve to the − ve terminal of E_3. Hence, when viewed from node 2, E_3 is taken negative.



Take all currents leaving the nodes

For node 1

$$I_1 - I_4 - I_2 = 0 \text{ or } I_1 = I_4 + I_1 \text{ —as per KCL}$$

Now,
$$I_1 = \frac{E_1 - V_A}{R_1};\ I_2 = \frac{V_A + E_3 - V_B}{R_2};\ I_4 = \frac{V_A}{R_4}$$

$\therefore$
$$\frac{E_1 - V_A}{R_1} = \frac{V_A}{R_4} + \frac{V_A + E_3 - V_B}{R_2}$$

or
$$V_A\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) - \frac{E_1}{R_1} - \frac{V_B}{R_2} + \frac{E_2}{R_2} = 0 \qquad ...(i)$$

For node 2

$$I_2 + I_3 - I_5 = 0 \quad \text{or} \quad I_2 + I_3 = I_5 \quad \text{—as per } KCL$$

Now, as before,
$$I_2 = \frac{V_A + E_3 - V_B}{R_2},\ I_3 = \frac{E_2 - V_B}{R_3},\ I_5 = \frac{V_B}{R_5}$$

$\therefore$
$$\frac{V_A + E_3 - V_B}{R_2} + \frac{E_2 - V_B}{R_3} = \frac{V_B}{R_5}$$

On simplifying, we get
$$V_B\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_5}\right) - \frac{E_2}{R_3} - \frac{V_A}{R_2} - \frac{E_3}{R_2} = 0 \qquad ...(ii)$$

Points to remember for Nodal Analysis.

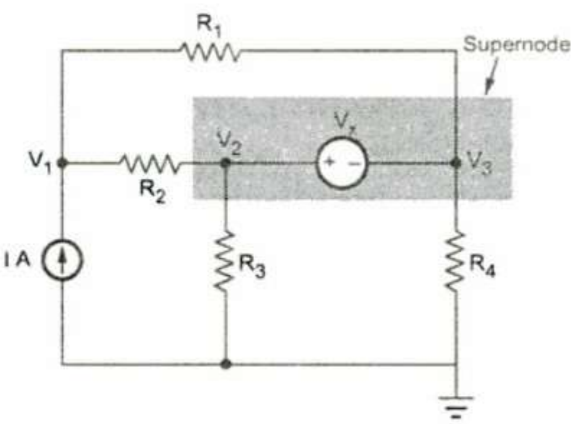
1] While assuming branch currents, make sure that each unknown branch current is considered at least once.

2] Convert the voltage source present into their equivalent current sources for node analysis, wherever possible.

3] Follow the same sign conventions, currents entering at node are to be considered positive, while currents leaving the node are to be considered as negative.

4] As far as possible, select the directions of various branch currents leaving the respective nodes. (***)

Super node [Only concepts, not to come in exam]

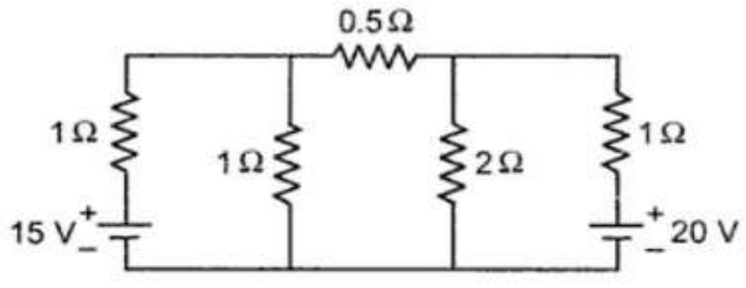


Here node V_2 and V_3 are directly connected to V_x without any circuit element.
The region surrounding the voltage source which connects two nodes directly is called as ***super node.***

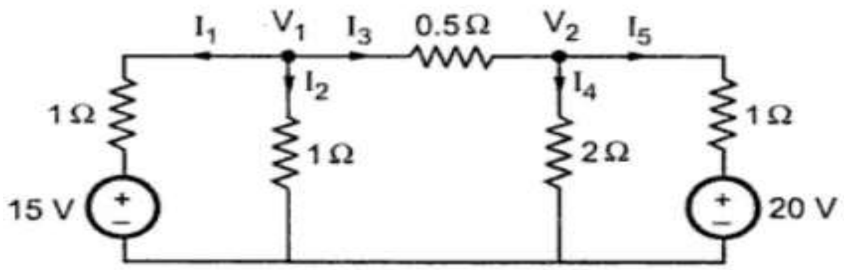
$$\mathbf{V_2 = V_3 + V_x}$$

Practice Problems on Nodal Analysis

Example: Find the current through each resistor of the circuit using Nodal Analysis.



Solution: The various node voltages and current are shown as



At node 1, $$-I_1 - I_2 - I_3 = 0$$

$$\therefore \quad -\left[\frac{V_1 - 15}{1}\right] - \left[\frac{V_1}{1}\right] - \left[\frac{V_1 - V_2}{0.5}\right] = 0$$

$$\therefore \quad -V_1 + 15 - V_1 - 2V_1 + 2V_2 = 0$$

$$\therefore \quad 4V_1 - 2V_2 = 15 \quad \ldots (1)$$

At node 2, $$I_3 - I_4 - I_5 = 0$$

$$\therefore \quad \frac{V_1 - V_2}{0.5} - \frac{V_2}{2} - \frac{V_2 - 20}{1} = 0$$

$$\therefore \quad 2V_1 - 2V_2 - 0.5\ V_2 - V_2 + 20 = 0$$

$$\therefore \quad 2V_1 - 3.5\ V_2 = -20 \quad \ldots (2)$$

Multiplying equation (2) by 2 and subtracting from equation (1) we get,

$$5V_2 = 55$$

$$\therefore \quad V_2 = 11\ \text{V}$$

and $$V_1 = 9.25\ \text{V}$$

Hence the various currents are,

$$I_1 = \frac{V_1 - 5}{1} = 9.25 - 15 = -5.75A \text{ i.e. } \mathbf{5.75\ A} \uparrow$$

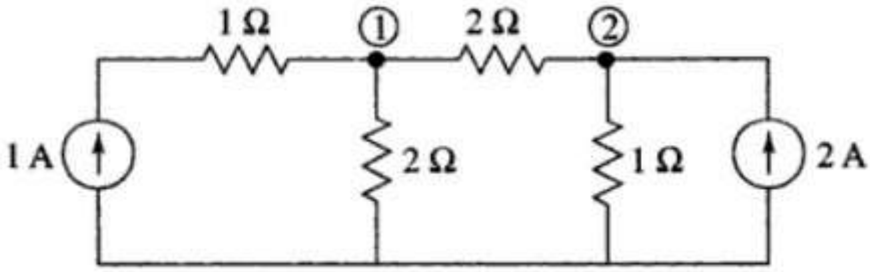
$$I_2 = \frac{V_1}{1} = \mathbf{9.25\ A}$$

$$I_3 = \frac{V_1 - V_2}{0.5} = -3.5A \text{ i.e. } \mathbf{3.5\ A} \leftarrow$$

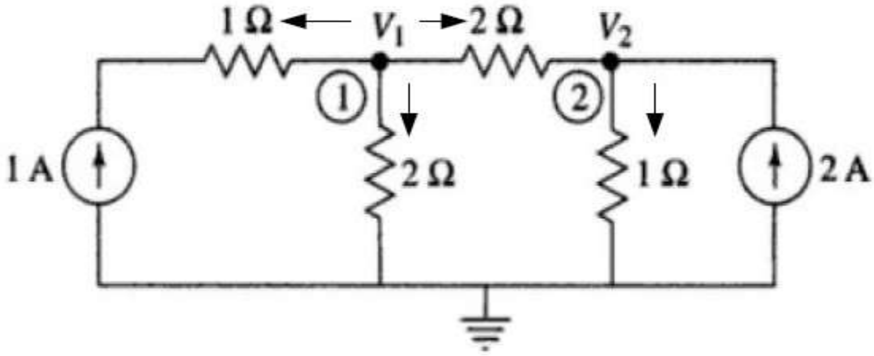
$$I_4 = \frac{V_2}{2} = \mathbf{5.5\ A}$$

$$I_5 = \frac{V_2 - 20}{1} = \frac{11 - 20}{1} = \mathbf{-9\ A \text{ i.e. } 9\ A} \uparrow$$

Example: Find voltages at node 1 and 2



Solution: Assuming node voltages V_1 and V_2 at nodes 1 and 2 respectively and assuming the currents to be leaving away from the nodes.



Applying KCL at Node 1,

$$1 = \frac{V_1}{2} + \frac{V_1 - V_2}{2}$$

$$2V_1 - V_2 = 2 \qquad ...(i)$$

Applying KCL at Node 2,

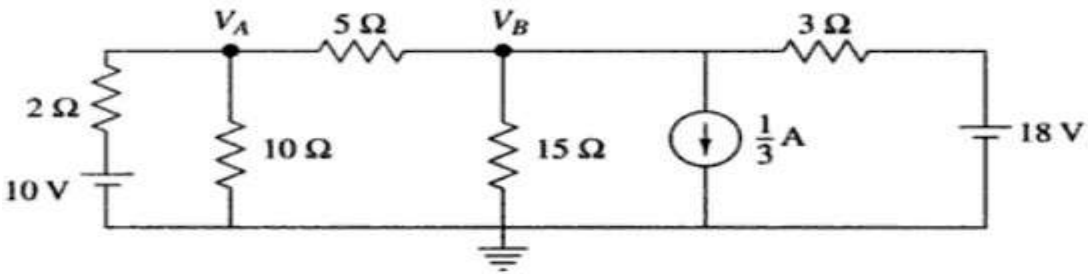
$$2 = \frac{V_2}{1} + \frac{V_2 - V_1}{2}$$

$$3V_2 - V_1 = 4 \qquad ...(ii)$$

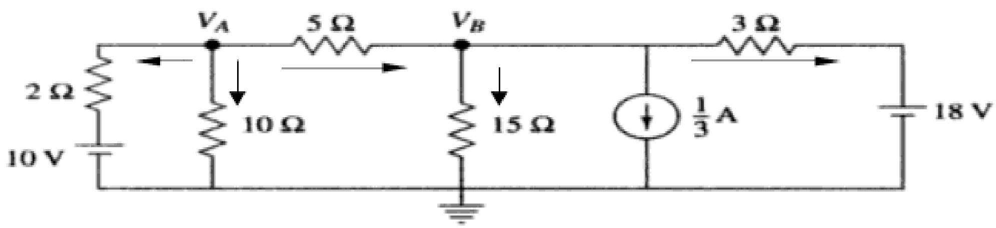
Solving Eqs (i) and (ii),

$$V_1 = 2\text{ V}$$
$$V_2 = 2\text{ V}$$

Example: Find V_A and V_B from the circuit.



Solution: Assume the currents are leaving away from the node.



Applying KCL at Node A,

$$\frac{V_A - 10}{2} + \frac{V_A}{10} + \frac{V_A - V_B}{5} = 0$$

$$\frac{5V_A - 50 + V_A + 2V_A - 2V_B}{10} = 0$$

$$8V_A - 2V_B = 50$$

Applying KCL at Node B,

$$\frac{V_B - V_A}{5} + \frac{V_B}{15} + \frac{1}{3} + \frac{V_B - 18}{3} = 0$$

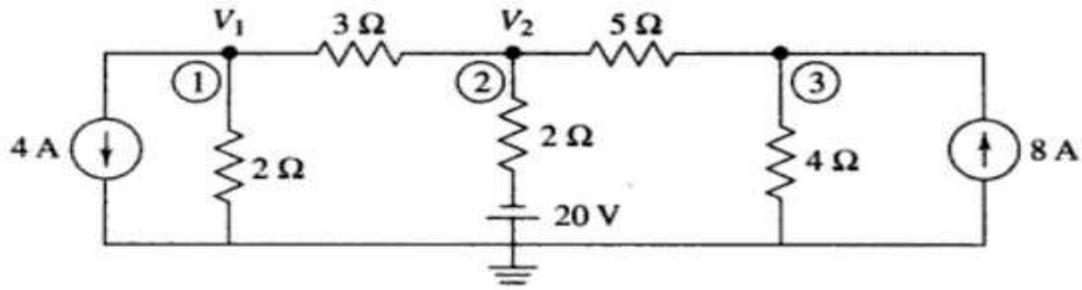
$$\frac{3V_B - 3V_A + V_B + 5 + 5V_B - 90}{15} = 0$$

$$-3V_A + 9V_B = 85$$

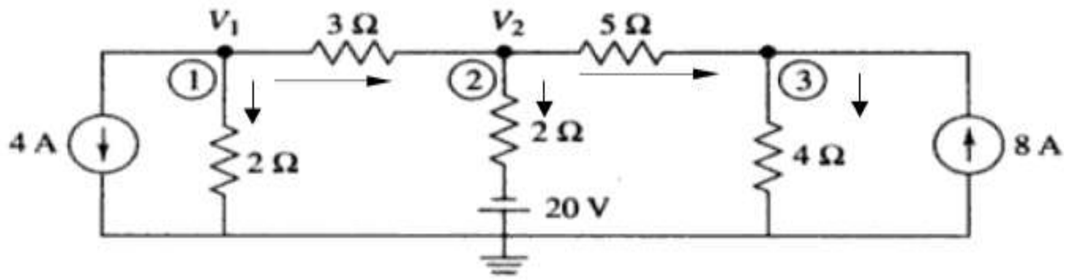
Solving Eqs (i) and (ii),

$$V_A = 9.39\text{ V}$$
$$V_B = 12.58\text{ V}$$

Example: Calculate the current through the 5Ω resistor using Nodal Analysis



Solution: Assuming the current directions to be moving away from the node.



Applying KCL at Node 1,

$$4+\frac{V_1}{2}+\frac{V_1-V_2}{3}=0$$

$$\frac{24+3V_1+2V_1-2V_2}{6}=0$$

$$5V_1-2V_2=-24$$

$$\frac{V_2-V_1}{3}+\frac{V_2-(-20)}{2}+\frac{V_2-V_3}{5}=0$$

…. **(i)**

$$\frac{10V_2-10V_1+15V_2+300+6V_2-6V_3}{30}=0$$

$$10V_1-31V_2+6V_3=300$$

Applying KCL at Node 3,

$$\frac{V_3-V_2}{5}+\frac{V_3}{4}=8$$

$$4V_3-4V_2+5V_3=160$$

$$-4V_2+9V_3=160$$

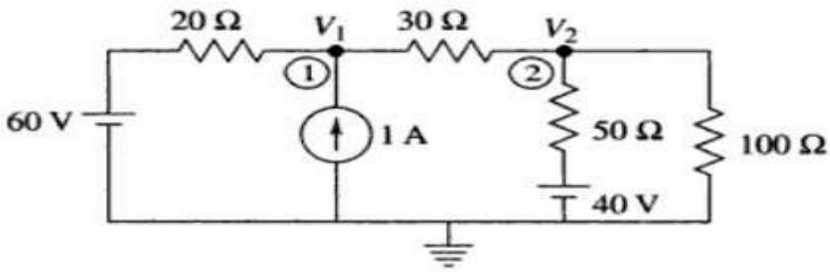
Solving Eqs (i), (ii) and (iii), …. **(ii)**

$$V_1=-8.77\text{ V}$$
$$V_2=-9.92\text{ V}$$
$$V_3=13.37\text{ V}$$

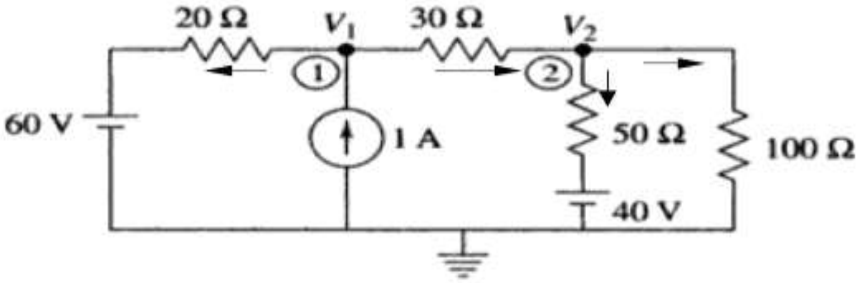
Current through the 5-Ω resistor= $\dfrac{V_3 - V_2}{5}$

$$= \frac{13.37 - (-9.92)}{5} = 4.66 \text{ A}$$

Example: Find the current in the 100Ω resistor.



Solution: Assuming the current directions to be leaving the node



Applying KCL at Node 1,

$$\frac{V_1 - 60}{20} + \frac{V_1 - V_2}{30} = 1$$

$$\frac{30V_1 - 1800 + 20V_1 - 20V_2}{600} = 1$$

$$50V_1 - 20V_2 = 2400$$

Applying KCL at Node 2,

....(i)

$$\frac{V_2 - V_1}{30} + \frac{V_2 - 40}{50} + \frac{V_2}{100} = 0$$

$$\frac{10V_2 - 10V_1 + 6V_2 - 240 + 3V_2}{300} = 0$$

$$-10V_1 + 19V_2 = 240$$

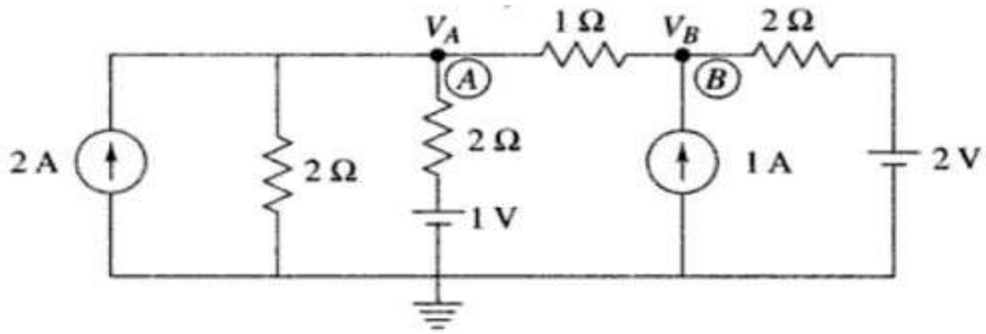
Solving Eqs (i) and (ii),

$$V_1 = 67.2 \text{ V}$$
$$V_2 = 48 \text{ V}$$

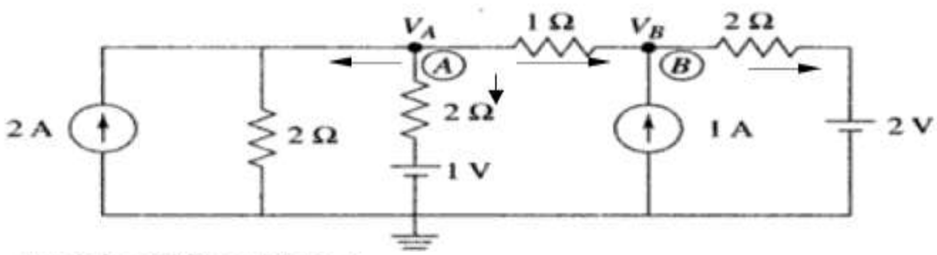
....(ii)

Current through the 100-Ω resistor $= \frac{V_2}{100} = \frac{48}{100} = 0.48$ A

Example: Find V_A and V_B using Nodal Analysis.



Solution: Assuming the current directions to be leaving away from the nodes



Applying KCL at Node *A*,

$$2 = \frac{V_A}{2} + \frac{V_A - 1}{2} + \frac{V_A - V_B}{1}$$

$$2 = \frac{V_A + V_A - 1 + 2V_A - 2V_B}{2}$$

$$4V_A - 2V_B = 5 \qquad \text{.... (i)}$$

Applying KCL at Node *B*,

$$\frac{V_B - V_A}{1} + \frac{V_B - 2}{2} = 1$$

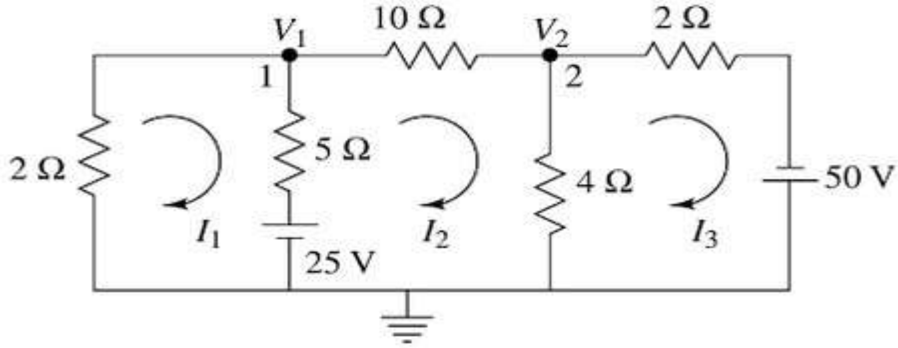
$$\frac{2V_B - 2V_A + V_B - 2}{2} = 1$$

$$-2V_A + 3V_B = 4 \qquad \text{.... (ii)}$$

Solving Eqs (i) and (ii),

$$V_A = 2.88 \text{ V}$$
$$V_B = 3.25 \text{ V}$$

Example: Find currents I_1, I_2 and I_3 using nodal analysis



Solution Assume that the currents are moving away from the nodes.

Applying KCL at Node 1,

$$\frac{V_1}{2}+\frac{V_1-25}{5}+\frac{V_1-V_2}{10}=0$$

$$\frac{5V_1+2V_1-50+V_1-V_2}{10}=0$$

$$8V_1-V_2=50 \qquad \text{...(i)}$$

Applying KCL at Node 2,

$$\frac{V_2-V_1}{10}+\frac{V_2}{4}+\frac{V_2-(-50)}{2}=0$$

$$\frac{2V_2-2V_1+5V_2+10V_2+500}{20}=0$$

$$-2V_1+17V_2=-500 \qquad \text{...(ii)}$$

Solving Eqs (i) and (ii),

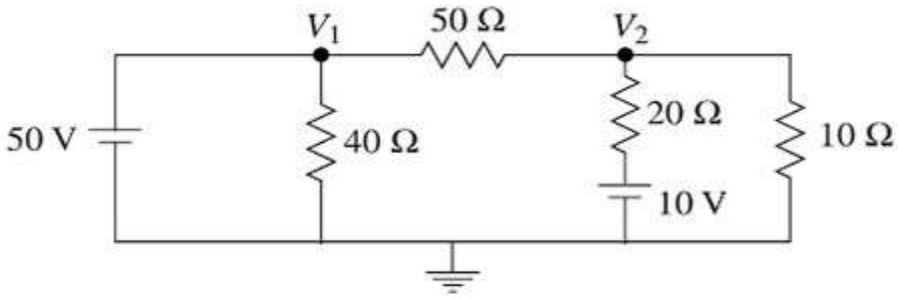
$$V_1 = 2.61 \text{ V}$$
$$V_2 = -29.1 \text{ V}$$

$$I_1=-\frac{V_1}{2}=\frac{-2.61}{2}=-1.31 \text{ A}$$

$$I_2=\frac{V_1-V_2}{10}=\frac{2.61-(-29.1)}{10}=3.17 \text{ A}$$

$$I_3=\frac{V_2+50}{2}=\frac{-29.1+50}{2}=10.45 \text{ A}$$

Example: Find the current in the 10 Ω resistor using nodal analysis



Solution Node 1 is directly connected to a voltage source of 50 V. Hence, we cannot write KCL equation at Node 1.

At Node 1,

$$V_1 = 50 \qquad \text{...(i)}$$

Assume that the current are moving away from the node.

Applying KCL at Node 2,

$$\frac{V_2 - V_1}{50} + \frac{V_2 - 10}{20} + \frac{V_2}{10} = 0$$

$$\frac{2V_2 - 2V_1 + 5V_2 - 50 + 10V_2}{100} = 0$$

$$-2V_1 + 17V_2 = 50 \qquad \text{...(ii)}$$

Solving Eqs (i) and (ii),

$$V_1 = 50 \text{ V}$$
$$V_2 = 8.82 \text{ V}$$

$$\text{Current in the 10-}\Omega\text{ resistor} = \frac{V_2}{10}$$

Solving Eqs (i) and (ii),

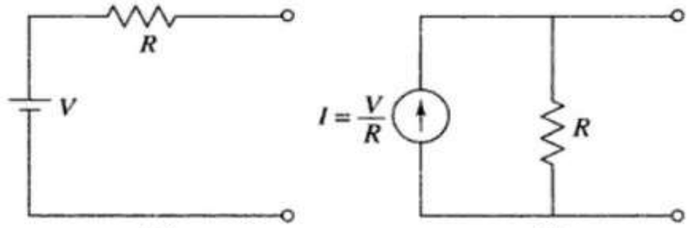
$$V_1 = 50 \text{ V}$$
$$V_2 = 8.82 \text{ V}$$

$$\text{Current in the 10-}\Omega\text{ resistor} = \frac{V_2}{10}$$

$$= \frac{8.82}{10} = 0.88 \text{ A}$$

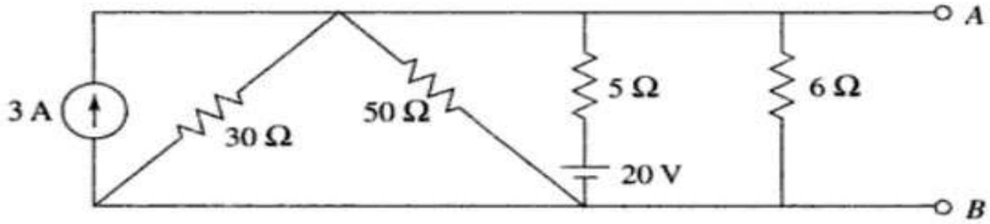
Source transformation

A voltage source with a series resistance can be converted to an equivalent current source with a resistance in parallel.
Conversely, a current source with a parallel resistance can be converted to a equivalent voltage source with a resistance in series.

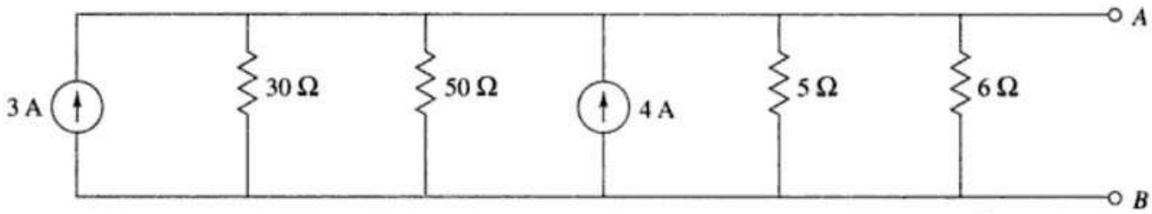


(a) (b)

Example: Replace the circuit between terminal A and B with a voltage source in series with a resistor?

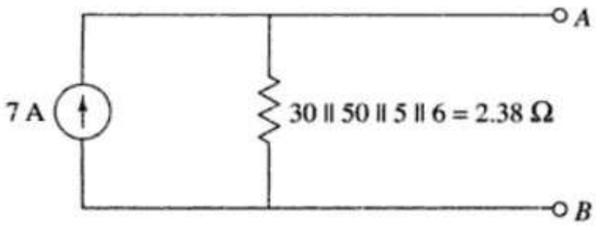


Solution: Converting the series combination of voltage source of 20V and a resistance of 5 Ω into

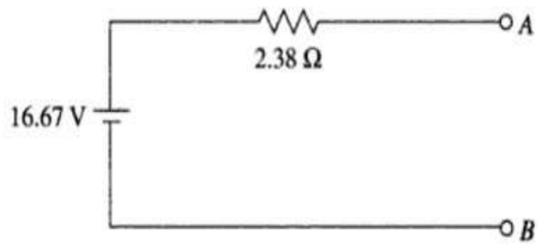


current source in parallel with a resistor gives.

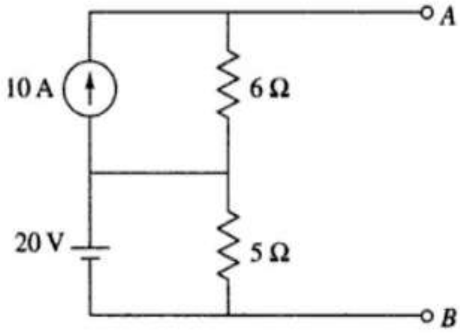
Adding the two current sources and simplifying the circuit,



By source conversion,



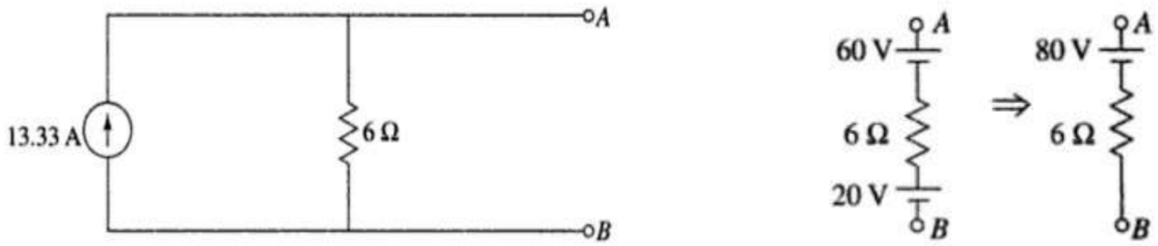
Example: Replace the given network with a single current source and a resistor.



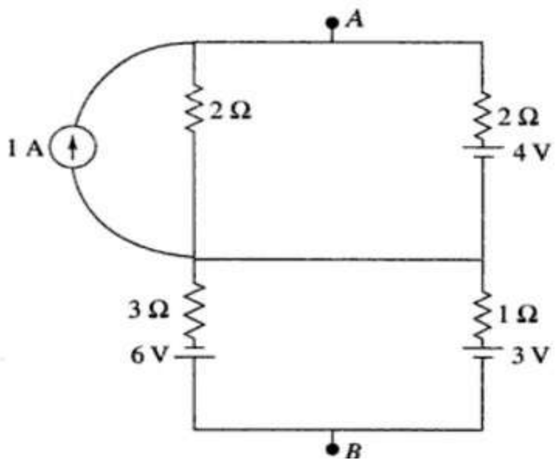
Solution: Since the resistance of 5 Ω is connected in parallel with the voltage source of 20V , it becomes redundant.

Converting the parallel combination of current source and resistance into equivalent voltage souce in series with a resistor we get:

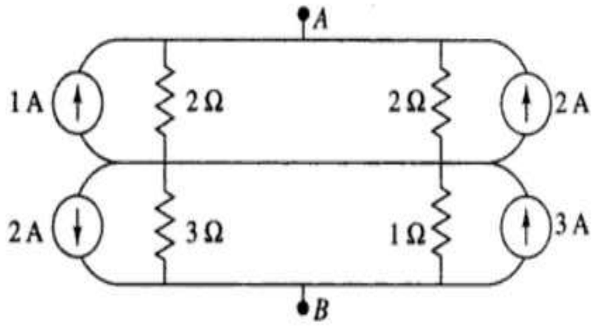
By source conversion,

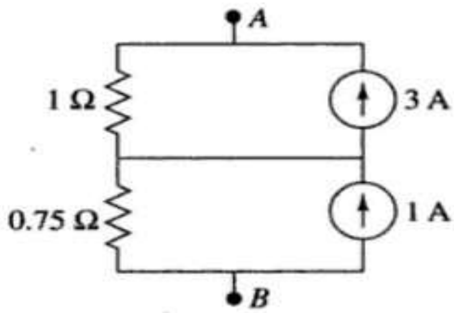


Example: Reduce the network into a single source and a single resistor between terminal A and B.

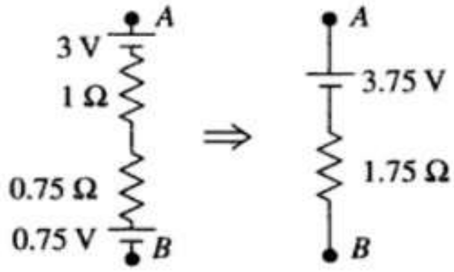


Solution: Converting all voltage source into equivalent current source.

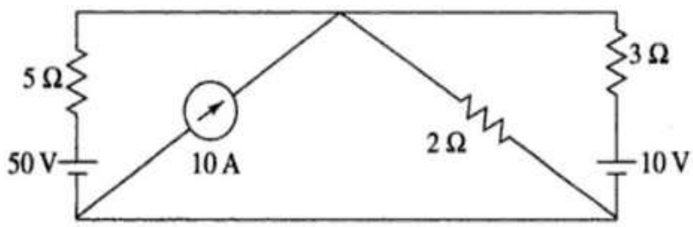




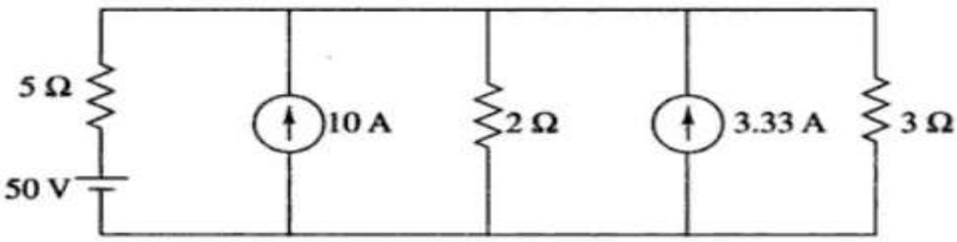
Converting the current source into equivalent voltage source.



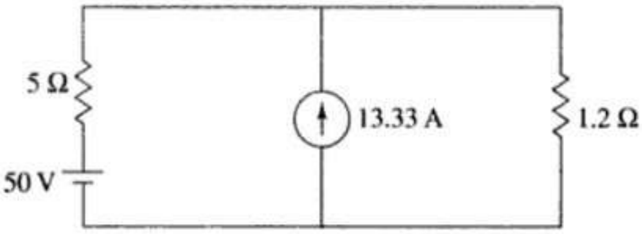
Example: Find the power delivered by the 50 V source in the circuit using source conversion?

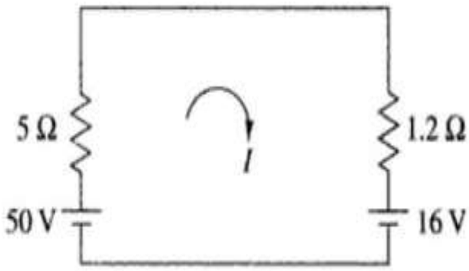


Solution: Converting the series combination of 10V and 3Ω resistor into current source in parallel with a resistor we get:



By source conversion





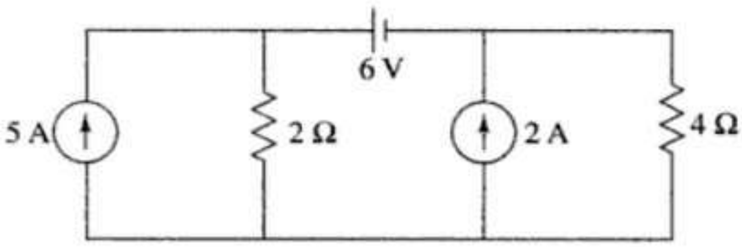
Applying KVL to the circuit,

$$50 - 5I - 1.2I - 16 = 0$$

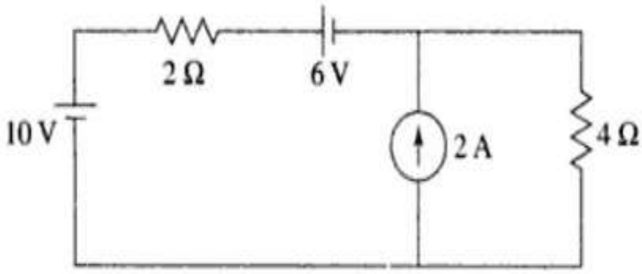
$$I = 5.48 \text{ A}$$

$$\text{Power delivered by the 50-V source} = 50 \times 5.48 = 274 \text{ W}$$

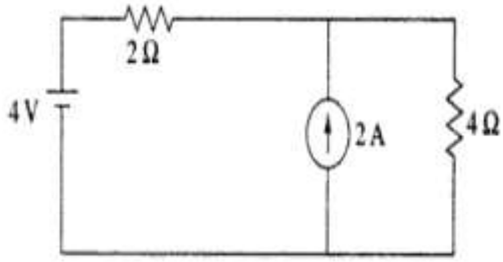
Example: Find the current in the 4 Ω resistor using source conversion.



Solution: Converting the parallel combination of 5A and 2 Ω into its equivalent voltage source and series resistor we get...



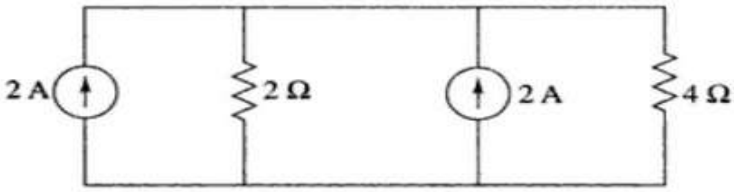
(1)

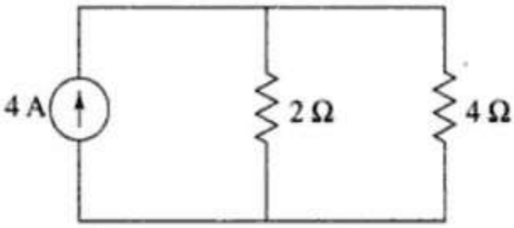


(2)

Again, by source conversion

(3)

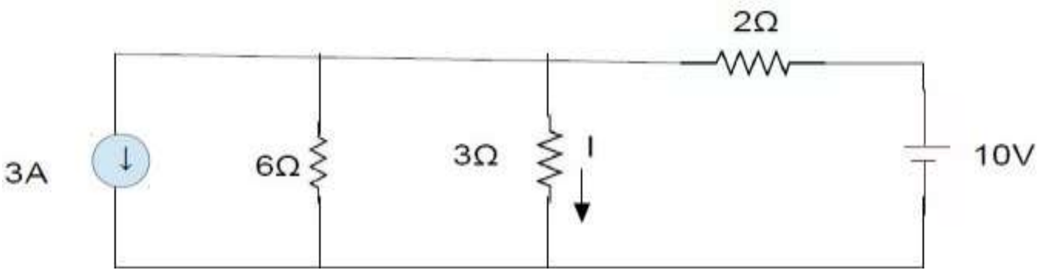




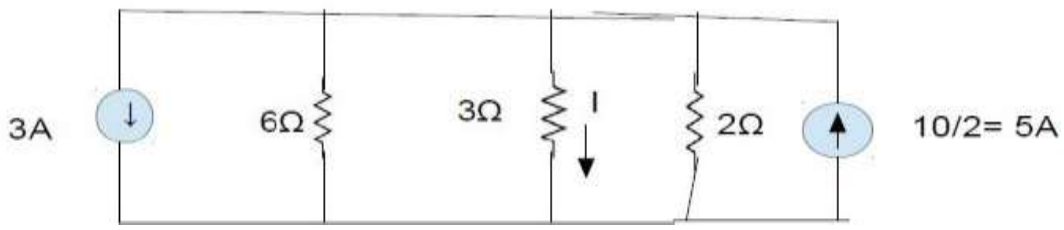
By current-division formula,

$$I_{4\Omega} = 4 \times \frac{2}{2+4} = 1.33 \text{ A}$$

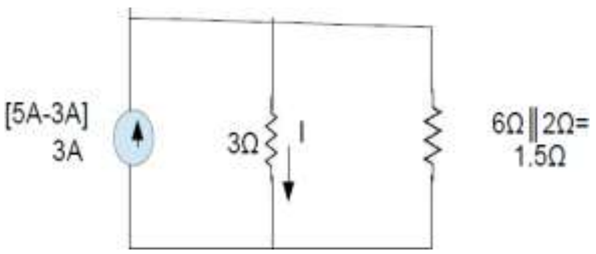
Question: Using source transformation find I.



Solution: Converting 10V and 2Ω series combination to current source with parallel resistance, we get



Combining 3A and 5 A current source we get,

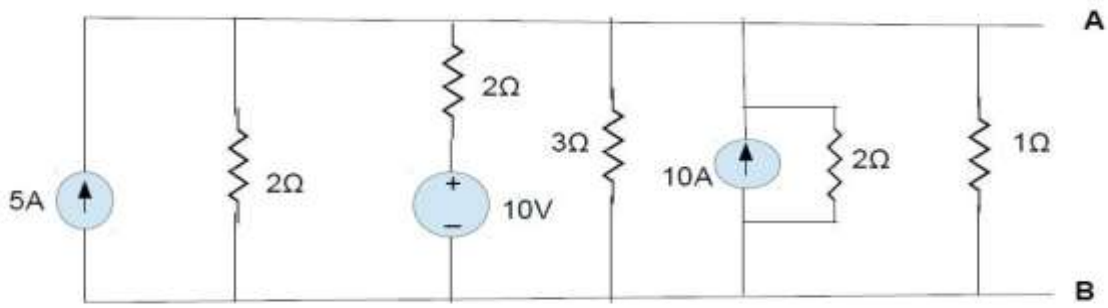


Using current division

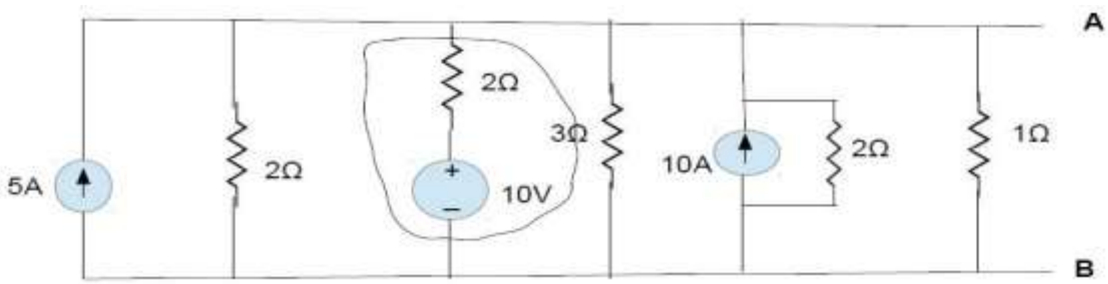
$I_{3\Omega} = \frac{2 \text{ x}1.5}{1.5+3}$

$= 0.67\text{A} \ (\downarrow)$

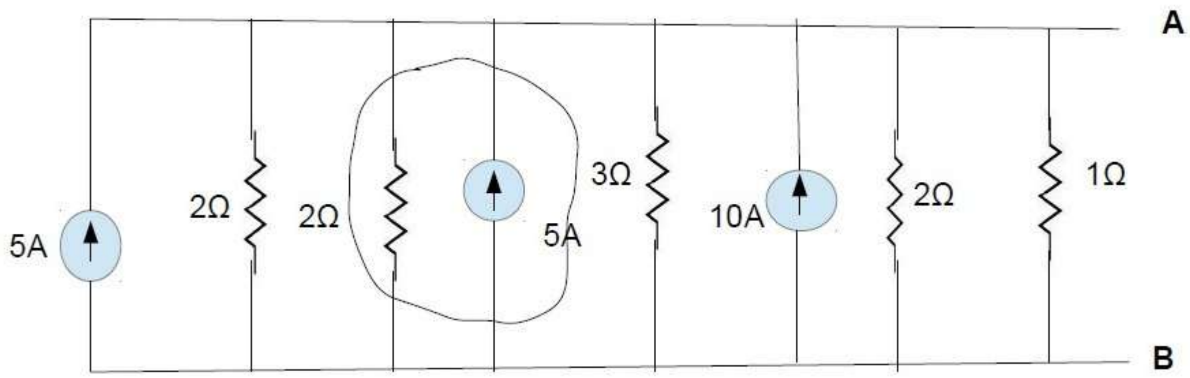
Question: Using source conversion, convert the circuit into a single voltage source.



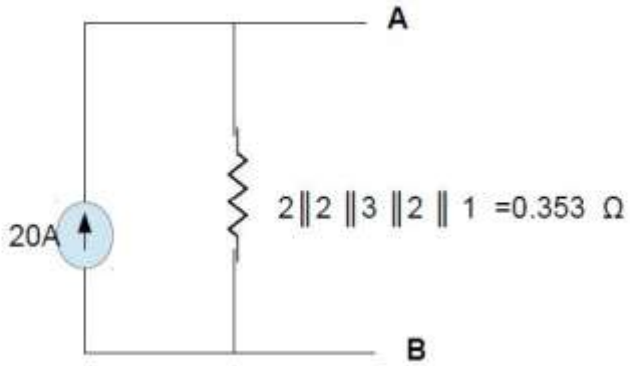
Solution: Converting the source of 10V and 2Ω to current source.



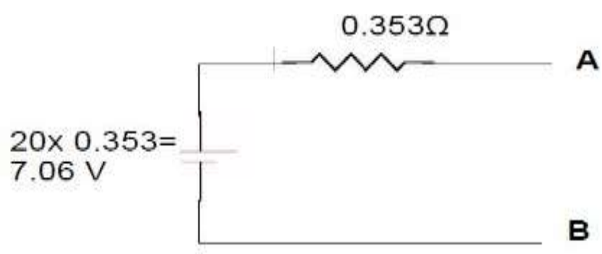
We get



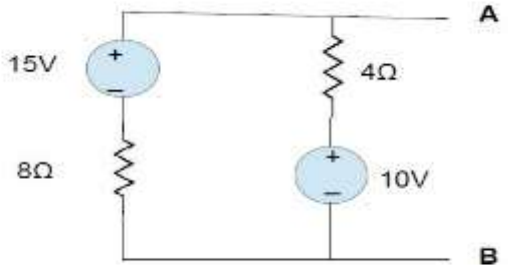
Now we have three current sources in same direction parallel. So, adding them up using kcl and all calculating the resistances in parallel we get



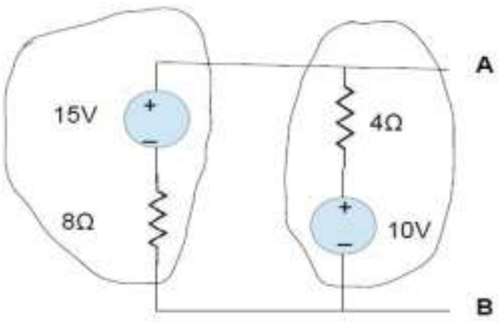
converting to voltage source we get

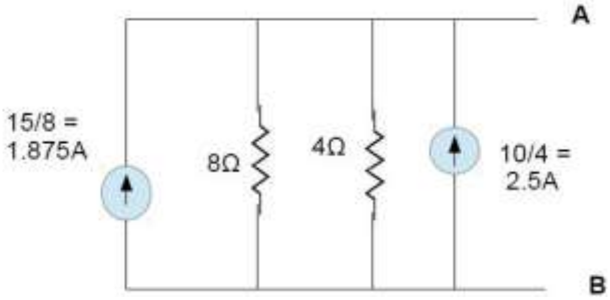


Question: Using source transformation convert the circuit to a single voltage source in series with a resistor.

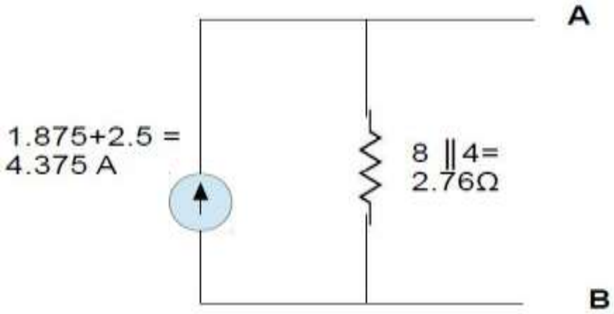


Solution: Transforming both the voltage sources to current sources we get

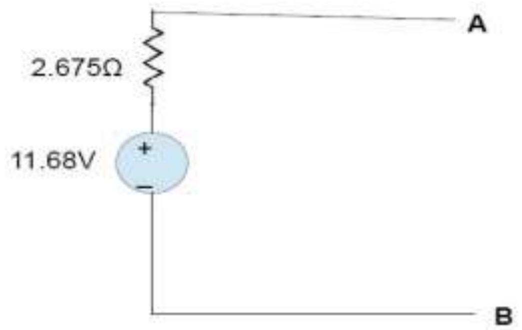




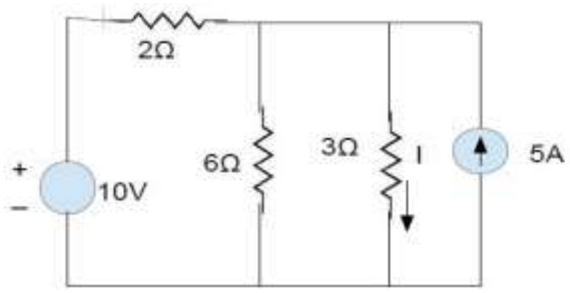
Combining current source and parallel resistances we get



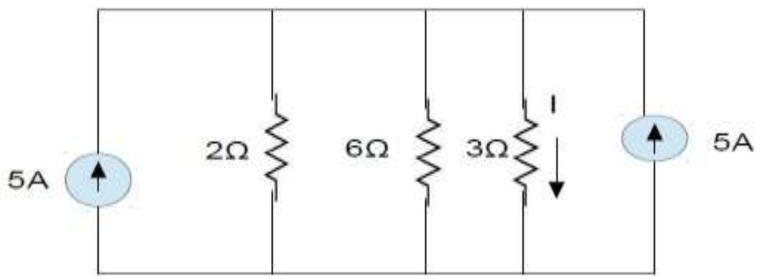
Converting current source back to voltage as is asked in question we get



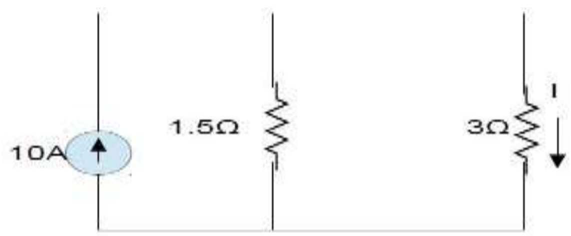
Question: Using source transformation find I?



Solution: Converting the 10 V voltage source and resistance of 2Ω to equivalent current source.



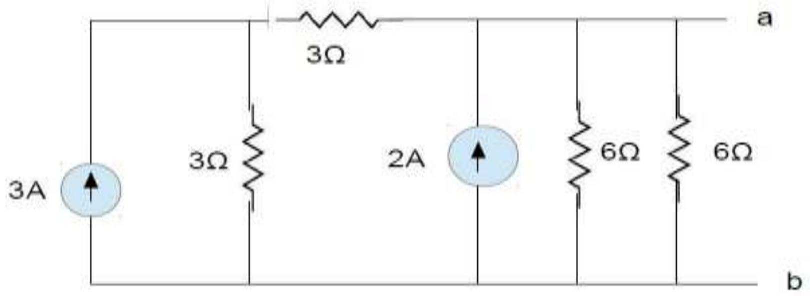
Combining the two current source and resistance in parallel we get



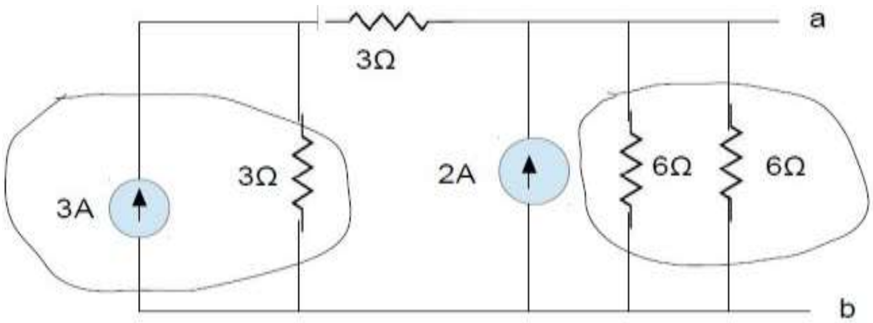
Using current division, we get current $I = \frac{10 \times 1.5}{1.5 + 3}$

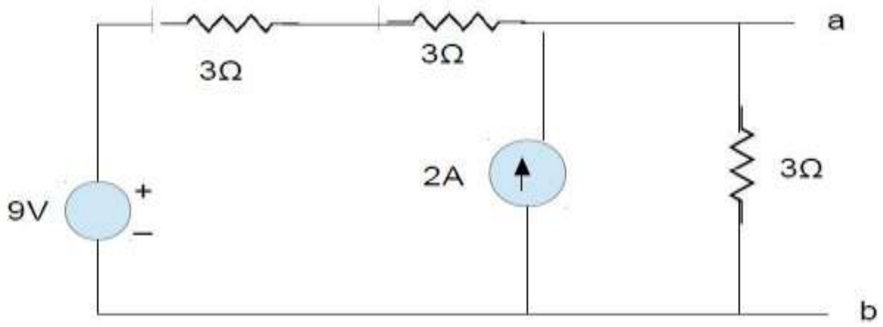
$= 3.334$ A (↓)

Question: Using source conversion, reduce the circuit into single current source in parallel with a single resistance.

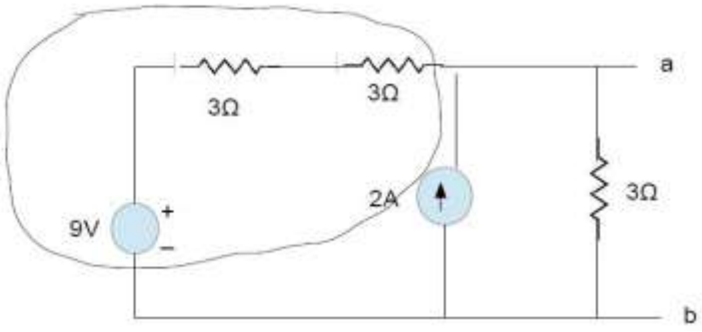


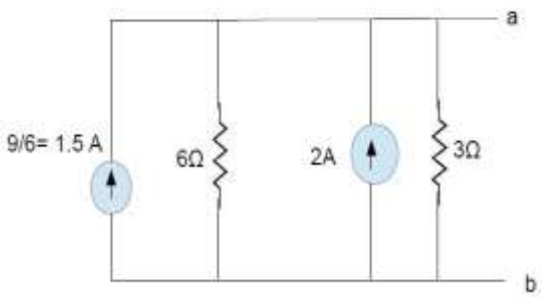
Solution: Converting 3A current source and 3Ω resistance into voltage source we get

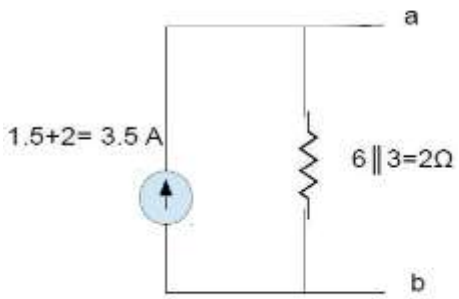




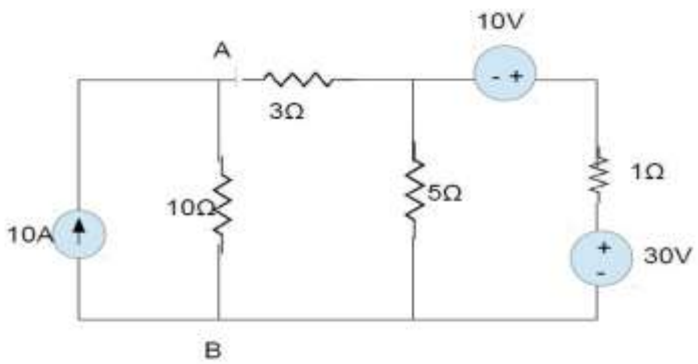
Again, converting voltage source to current source, we get



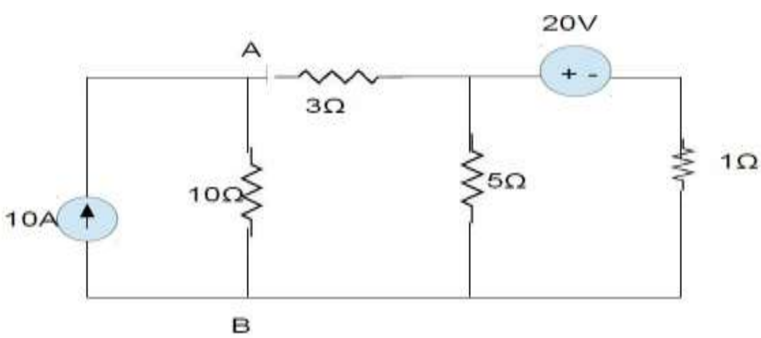




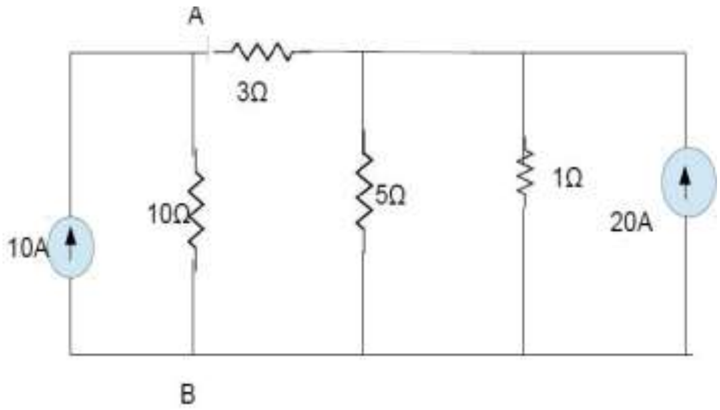
Question: Using source transformation find the current flowing through the 10Ω resistor.



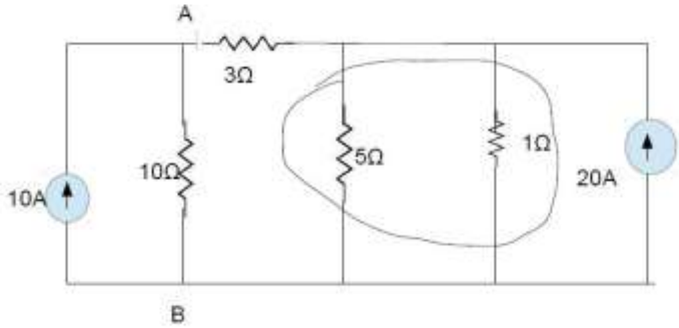
Solution: Calculating the effect of two series batteries, Combining 10V and 30V we get

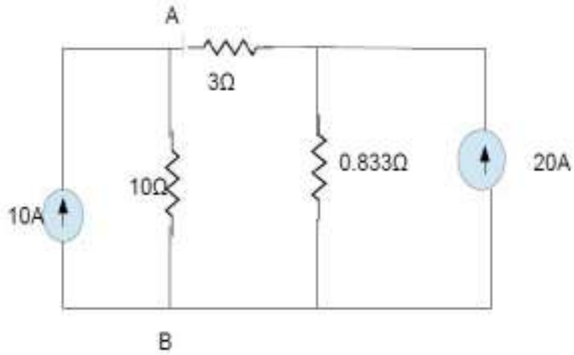


Source conversion of voltage

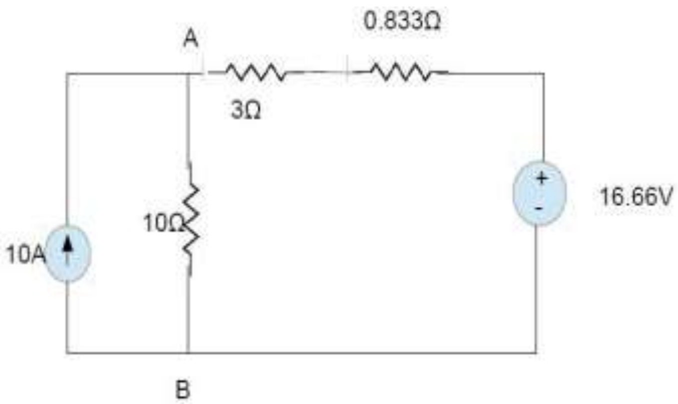


Combining parallel resistors

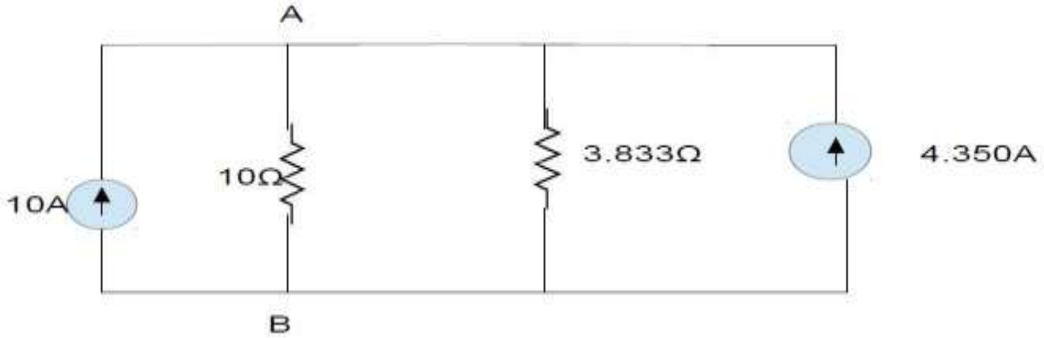




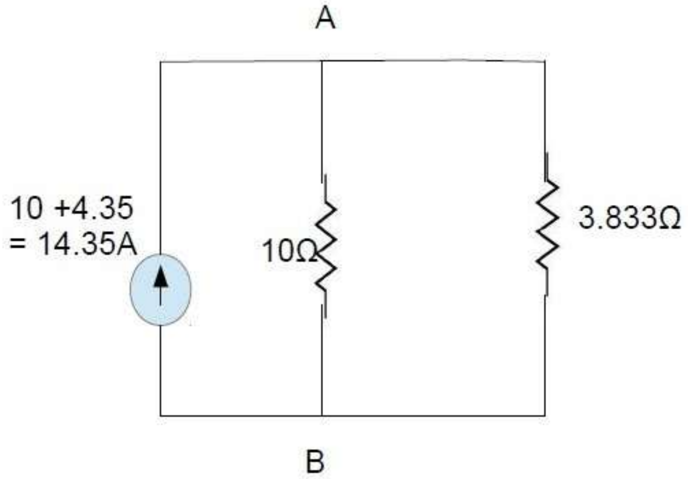
Source conversion of current



Combining the series resistance and again converting voltage to current source we get



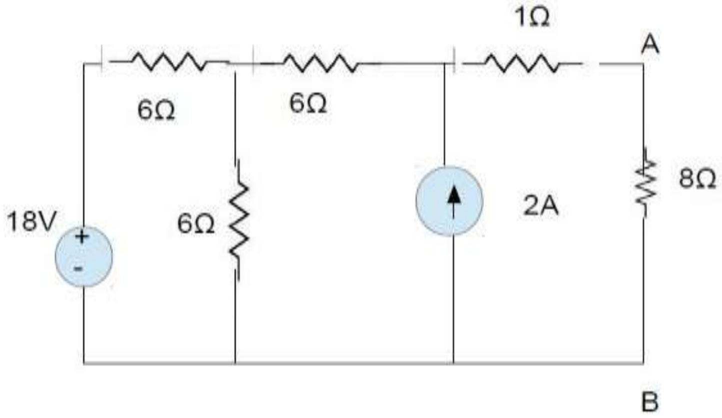
Combining the current sources, we get



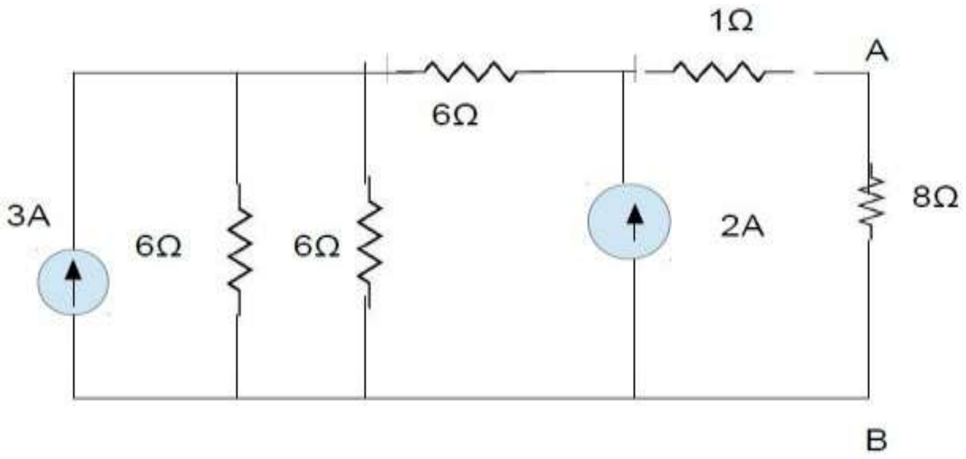
Using current division rule we get we get current through 10Ω resistor as

$$I_{10\Omega} = \frac{14.33 \times 3.83}{10 + 3.83} = 3.97A\ (\downarrow)$$

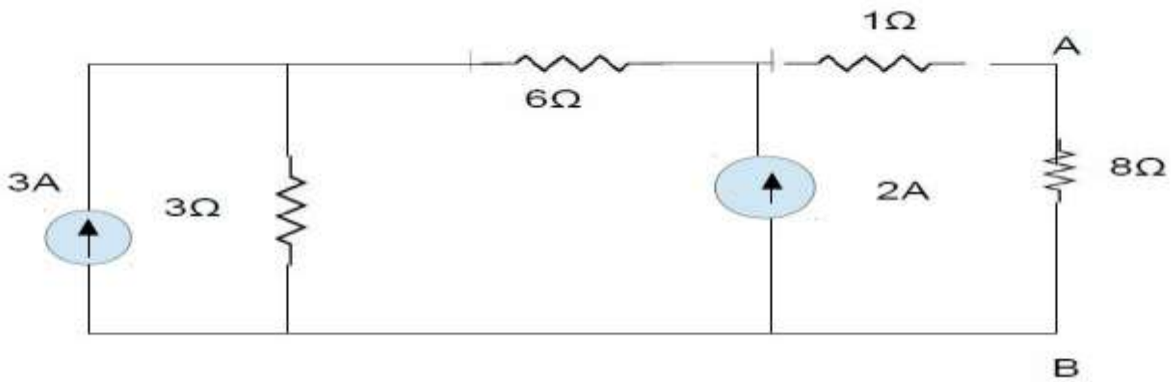
Question: Using source transformation find the current flowing through the 8 Ω resistor.



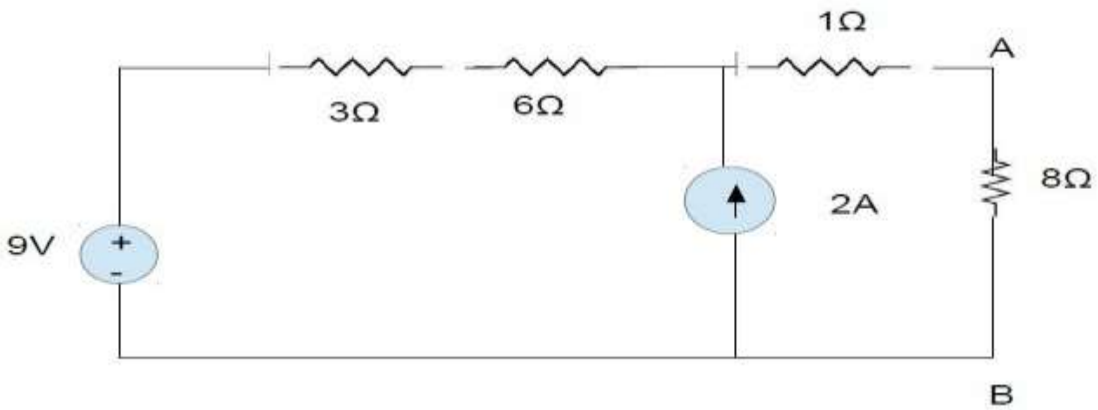
Solution: Converting 18V voltage source and 6 Ω resistance to equivalent current source we get



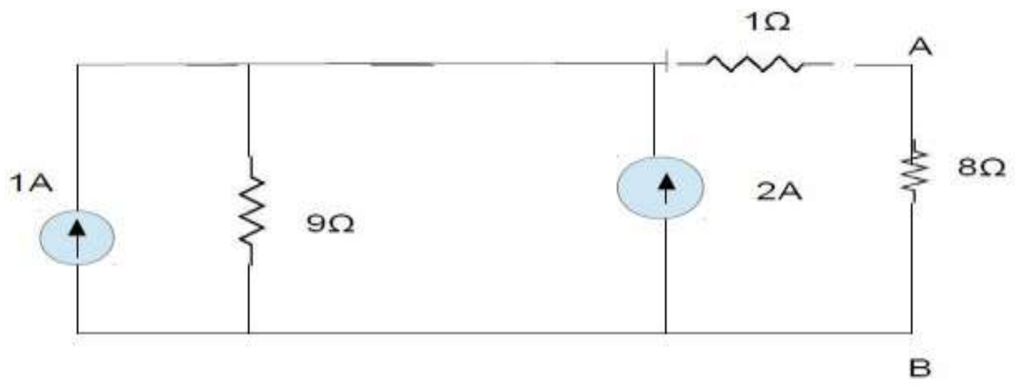
Merging the 6 Ω parallel resistors



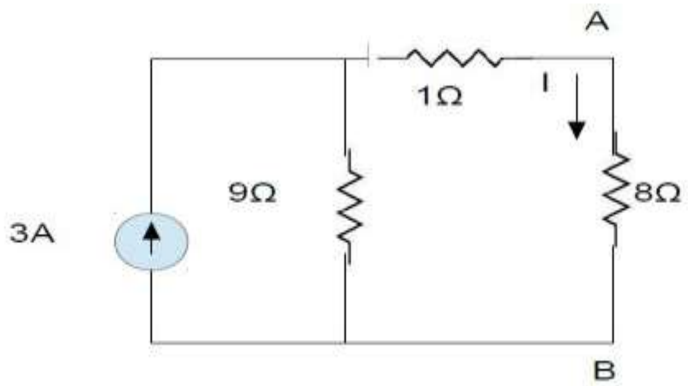
Source transformation of 3A current source to voltage source we get



Reconverting 9V to current source again we get

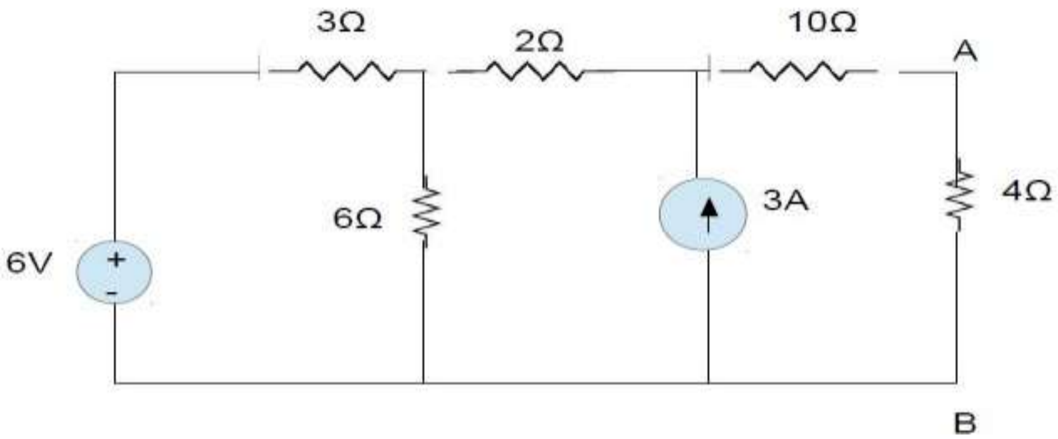


Merging the current source, we get

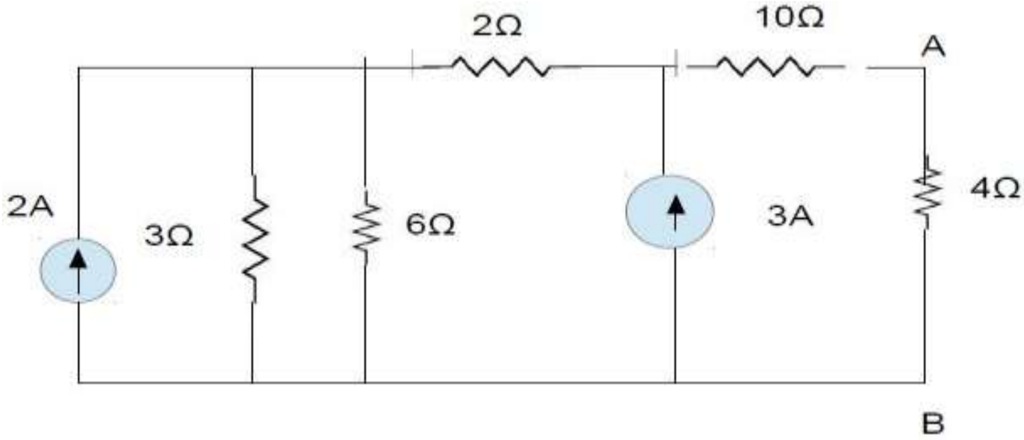


$$I = \frac{3 \times 9}{9+9} = 1.5\ A\ (\rightarrow)$$

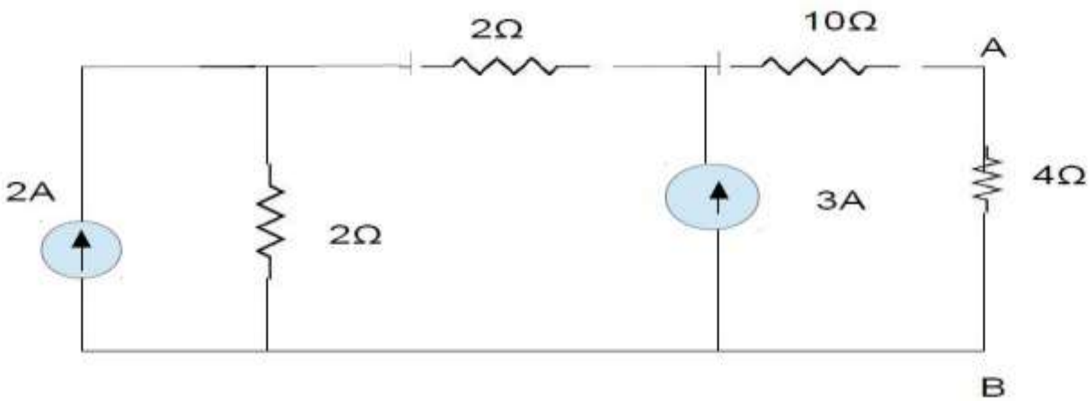
Question: Using source transformation calculate voltage across 4Ω resistor.



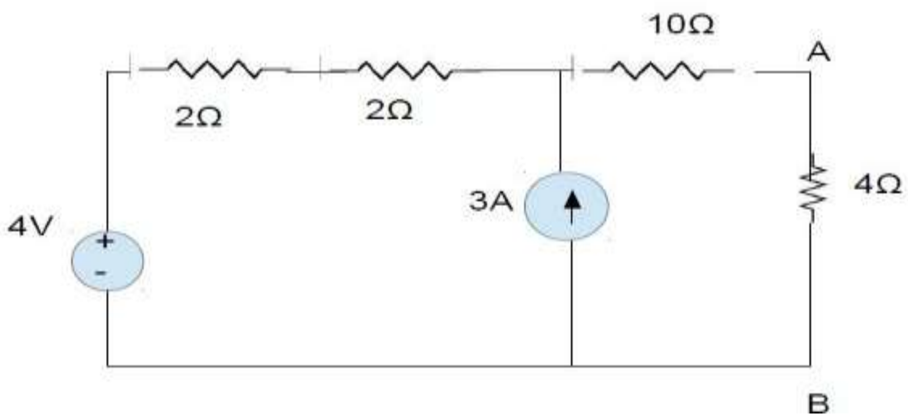
Solution: Converting 6V voltage source to current source we get



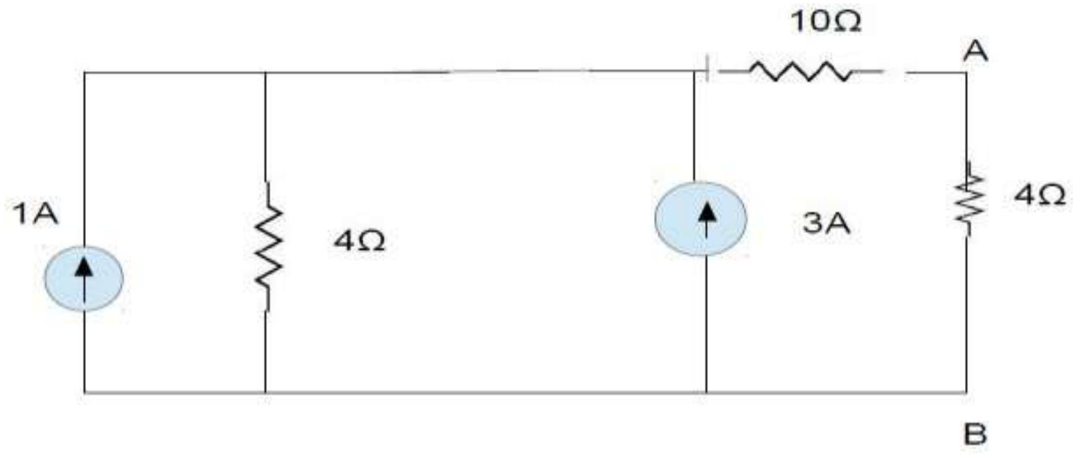
Merging the 3Ω and 6Ω parallel resistor



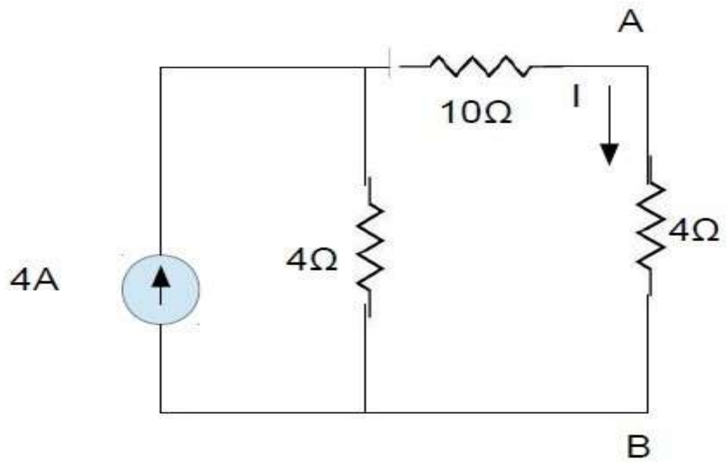
Converting the 2A current source we get



Source transformation of 4V gives



Combining the current sources



$$I = \frac{4 \times 4}{4+14}$$

$= 0.889$ A (→)

$V = I \times 4$

$= 4 \times 0.889$

$= 3.56$ V

Superposition Theorem

If a circuit has two or more independent sources, one way to determine the value of a specific variable (voltage or current) is to use nodal or mesh analysis. Another way is to determine the contribution of each independent source to the variable and then add them up. The latter approach is known as the superposition. The idea of superposition rests on the linearity property

Statement of Superposition Theorem:

"The superposition principle states that the voltage across (or current through) an element in a linear bilateral circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone, while considering one independent source at a time, and all other independent sources are replaced by their internal resistances. If internal resistance is not known we replace every voltage source by 0 V (or a short circuit), and every current source by 0 A (or an open circuit)."

Steps to Apply Superposition Principle:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using the techniques learnt.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources

Note: No Source Transformation is allowed in Case of superposition Theorem

Disadvantage of Superposition Theorem

Analyzing a circuit using superposition has one major disadvantage: It may very likely involve more work. If the circuit has three independent sources, we may have to analyze three simpler circuits each providing the contribution due to the respective individual source.

However, superposition does help reduce a complex circuit to simpler circuits through replacement of voltage sources by short circuits and of current sources by open circuits.

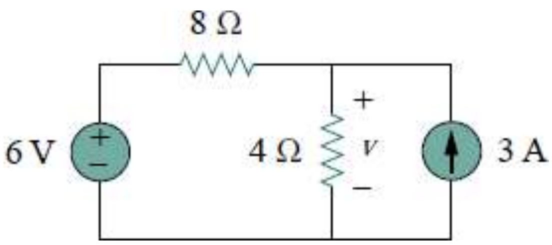
Keeping in mind that superposition is based on linearity. For this reason, it is not applicable to the effect on power due to each source, because the power absorbed by a resistor depends on the square of the voltage or current. If the power value is needed, the current through (or voltage across) the element must be calculated first using superposition.

Application of Superposition Theorem.

Superposition is not limited to circuit analysis but is applicable in many fields where cause and effect bear a linear relationship to one another.

Practice problem on Superposition Theorem

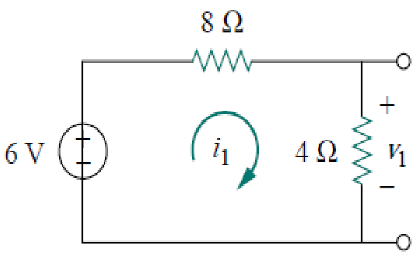
Example: Find the voltage across 4Ω resistor using superposition Theorem.



Solution: Since there are two sources let $v = V_1 + V_2$

Where V_1 is the contribution due to 6V voltage source
and V_2 is the contribution due to 3 A current source.

Step 1: To obtain V_1 we set the current source to zero (open circuit)



Step 2: Solving for V_1

$$12i_1 - 6 = 0 \qquad \Longrightarrow \qquad i_1 = 0.5 \text{ A}$$

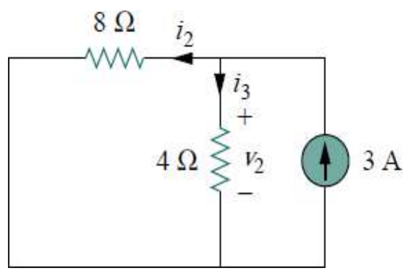
Thus,

$$v_1 = 4i_1 = 2 \text{ V}$$

We may also use voltage division to get v_1 by writing

$$v_1 = \frac{4}{4+8}(6) = 2 \text{ V}$$

Step 3: To obtain V_2 we short the 6V voltage source.



Step 4: Solving for V_2

$$i_3 = \frac{8}{4+8}(3) = 2 \text{ A}$$

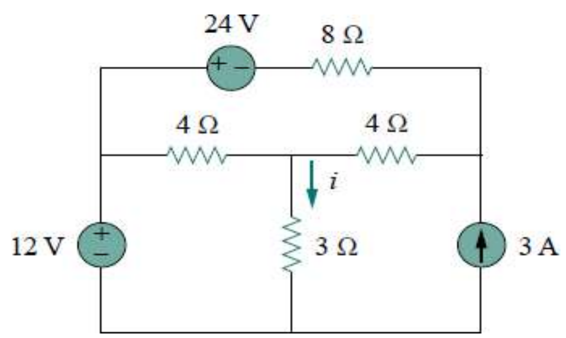
Hence,

$$v_2 = 4i_3 = 8 \text{ V}$$

Step 5: Finding *v*

$$v = v_1 + v_2 = 2 + 8 = 10 \text{ V}$$

Example: For the circuit in Figure use the superposition theorem to find ***i***.

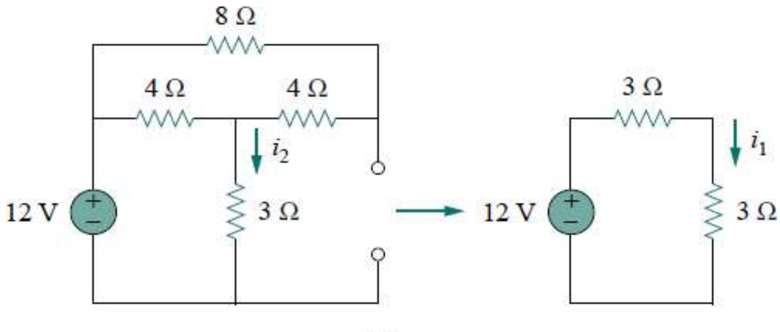


Solution: In this case, we have three sources. Let

$$i = i_1 + i_2 + i_3$$

where i_1, i_2 and i_3, and are due to the 12-V, 24-V, and 3-A sources respectively.

Step 1: To get i_1, consider the circuit in Figure (a) with 12V source.

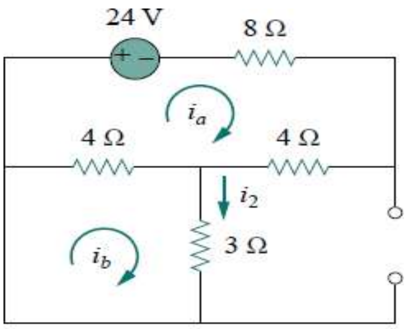


(a)

Combining 4Ω (on the right-hand side) in series with 8Ω gives 12Ω. The 12Ω is in parallel with 4Ω gives 3Ω.

$12 \times 4/16 = 3\ \Omega.$

Step 2: **T**o get ***i₂***, consider the circuit in Figure (b) with 24V source.



(b)

Thus,

$$i_1 = \frac{12}{6} = 2\text{ A}$$

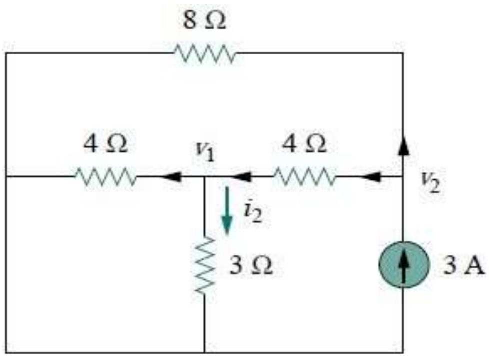
Applying Mesh analysis

$$16i_a - 4i_b + 24 = 0 \quad \Longrightarrow \quad 4i_a - i_b = -6$$

$$7i_b - 4i_a = 0 \quad \Longrightarrow \quad i_a = \frac{7}{4}i_b$$

$$i_2 = i_b = -1$$

Step 3: **T**o get ***i₃,*** consider the circuit in Figure (c) with 3A source



(c)

Applying Nodal Analysis

$$3 = \frac{v_2}{8} + \frac{v_2 - v_1}{4} \quad \Longrightarrow \quad 24 = 3v_2 - 2v_1$$

Therefore $v_1 = 3$

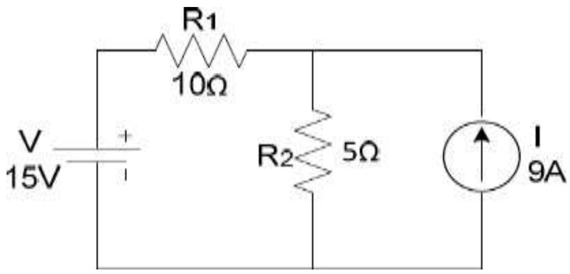
$$\frac{v_2 - v_1}{4} = \frac{v_1}{4} + \frac{v_1}{3} \quad \Longrightarrow \quad v_2 = \frac{10}{3}v_1$$

$$i_3 = \frac{v_1}{3} = 1\text{ A}$$

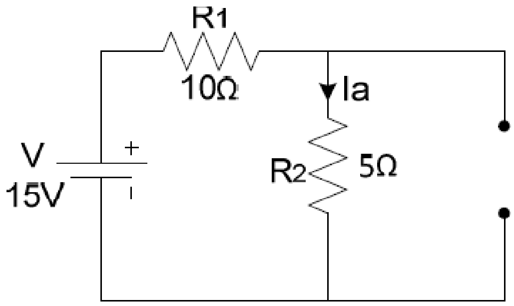
Step 4: Combining all the three results

$$i = i_1 + i_2 + i_3 = 2 - 1 + 1 = 2\text{ A}$$

Example: Determine the current through resistor R2=5Ω for the network in figure 36 using superposition theorem.

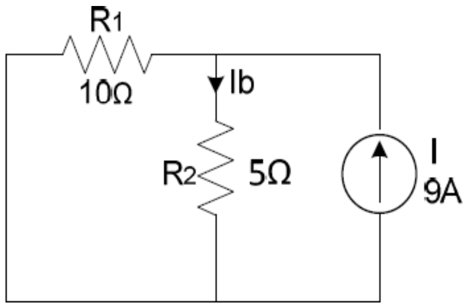


Solution: **Step 1**: V active, I inactive. So current source is open circuit



$$I_a = \frac{V}{R_1 + R_2} = \frac{15}{10 + 5} = 1A$$

Step 2: V inactive, I active. So, voltage source is short circuit.



Using CDR

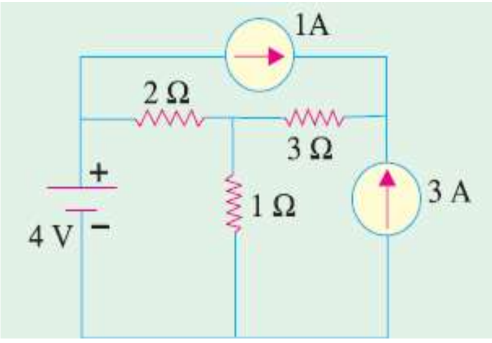
$$I_b = \frac{R_1}{R_1 + R_2} \times I = \frac{10}{10 + 5} \times 9 = 6A$$

Step 3: Total current through $R_2 = 5\Omega$.

Ia $\downarrow$ 1A   Ib $\downarrow$ 6A

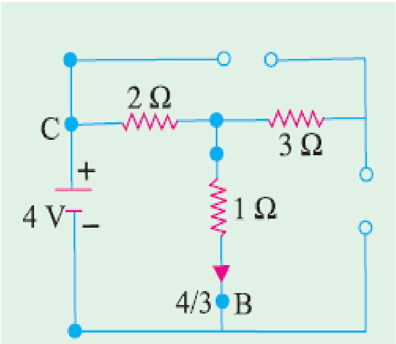
$$I_{R2} = I_a + I_b = 1 + 6 = 7A$$

Example: In the circuit, find current through 1Ω resistor using SUPERPOSITION theorem.



Solution: One source act at a time. Current through ***A-B (1 ohm)*** is to be calculated due to each source and finally all these contributions added.

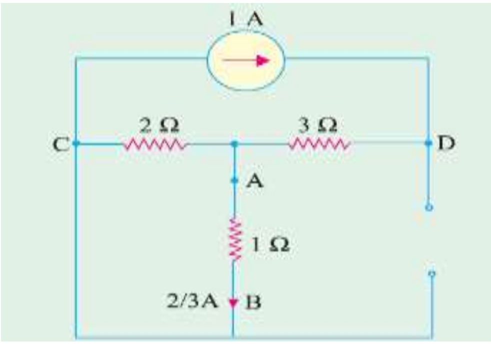
Step 1: Considering the 4V source



Due to 4 V using KVL ,1-ohm resistor carries a current of 4/3 amp

i_1= 4/3 A (from A to B)

Step 2: Considering 1A source

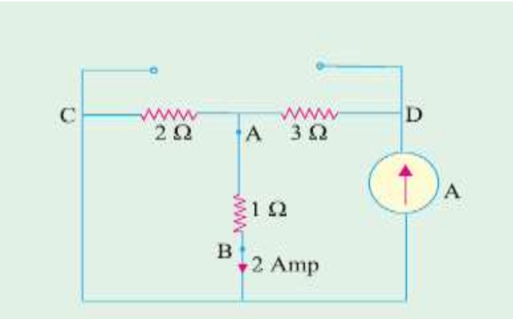


Due to 1-A source using CDR

i_2 = 1 X 2/ (1+2)

= **2/3 A** (from Ato B)

Step 3: Considering 3 A source



Due to 3-A source using CDR

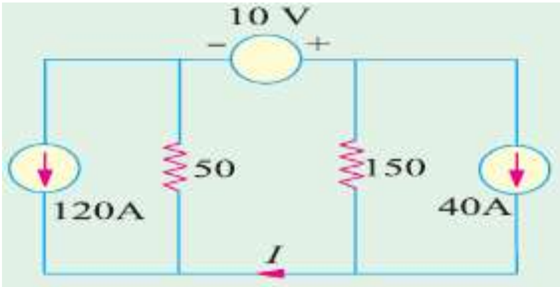
i_3 = 3 X 2/ (1+2)

=2 A (from Ato B)

Total current = $i_1 + i_2 + i_3$

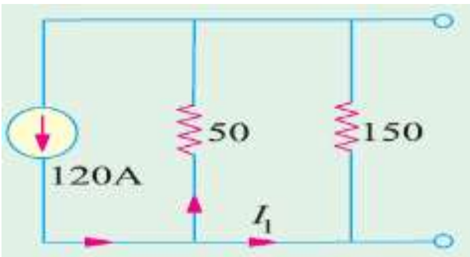
= 4 A (from Ato B)

Example: Use Superposition theorem to find current I in the circuit shown. All resistances are in ohms.



(a)

Solution: Step 1: Considering 120 A source

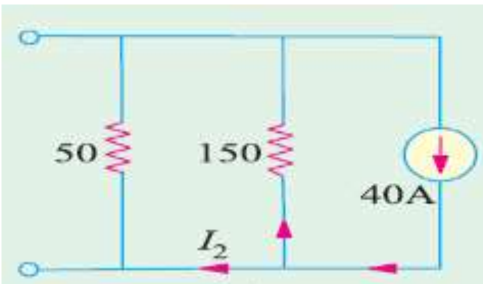


(b)

In Fig (b), the voltage source has been replaced by a short and the 40 A current sources by an open.

Using the current-divider rule,
we get I_1= 120 × 50/200 = 30 A.

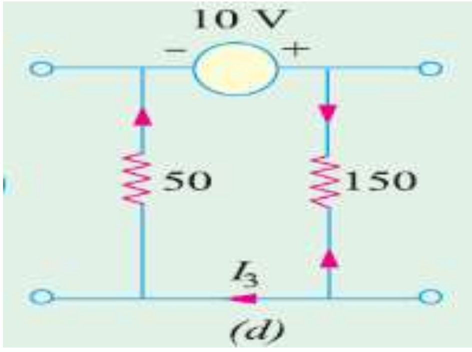
Step 2: Considering 40A source



(c)

In Fig (c), only 40 A current source has been considered.
Again, using current-divider rule
I_2= 40 × 150/200 = 30 A.

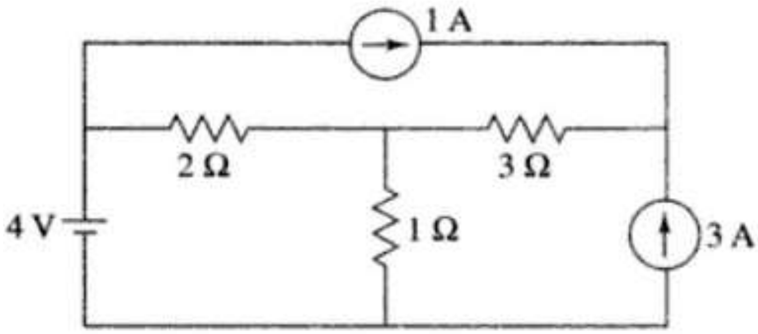
Step 3: Considering 10V voltage source



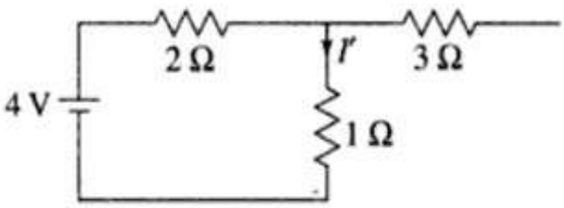
In Fig (d), only voltage source has been considered.
Using Ohm's law,
I_3 = 10/200 = 0.05 A.

Step 4: ***Since I_1 and I_2 cancel out, $I = I_3 = 0.005$ A.***

Example: Find the current in the 1Ω resistor using superposition theorem.

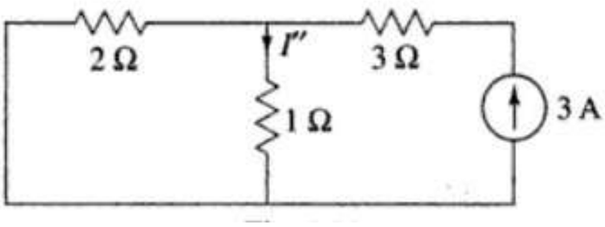


Solution: Step1: When 4V source is acting alone



$$I' = \frac{4}{2+1} = 1.33 \text{ A } (\downarrow)$$

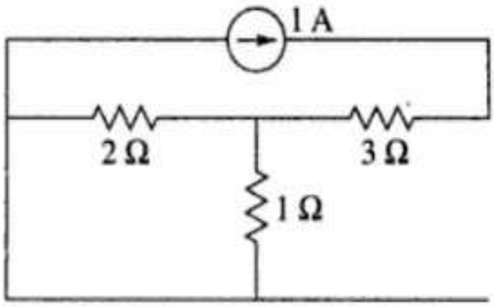
Step 2: When 3 A source is acting alone

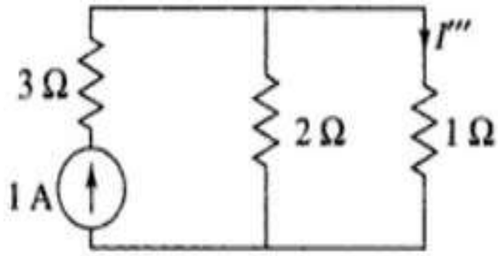


By current-division formula,

$$I'' = 3 \times \frac{2}{1+2} = 2\text{ A }(\downarrow)$$

Step 3: When 1 A source is acting alone the circuit redrawn as below





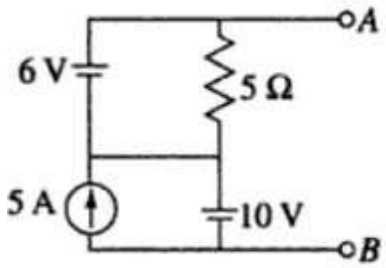
By current-division formula,

$$I''' = 1 \times \frac{2}{2+1} = 0.66\text{ A }(\downarrow)$$

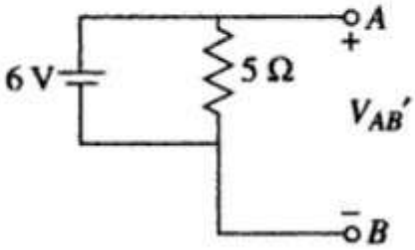
Using superposition theorem

$$I = I' + I'' + I'''$$
$$= 1.33 + 2 + 0.66 = 4\text{ A }(\downarrow)$$

Example: Find the voltage V_{AB}.

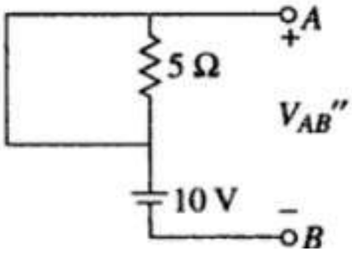


Solution: Step 1: When the 6V source is acting alone.



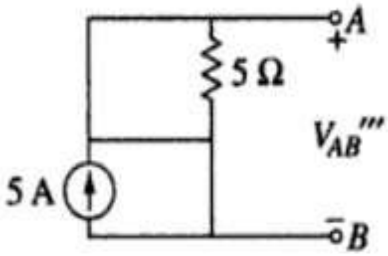
$V_{AB}' = 6$ V

Step 2: When the 10 V source is acting alone



5Ω is shorted out. **$V_{AB}'' = 10$ V**

Step 3: When the 5A source is acting alone



$$V_{AB}''' = 0 \text{ V}$$

Step 4: By superposition theorem

$$V_{AB} = V_{AB}' + V_{AB}'' + V_{AB}'''$$
$$= 6 + 10 + 0 = 16 \text{ V}$$

Thevenin's Theorem

It often occurs in practice that a particular element in a circuit is variable (usually called the load) while other elements are fixed.

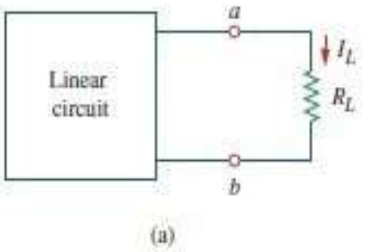
As a typical example, a household outlet terminal may be connected to different appliances constituting a variable load. Each time the variable element is changed, the entire circuit must be analyzed all over again. To avoid this problem, Thevenin's theorem provides a technique by which the fixed part of the circuit is replaced by an equivalent circuit.

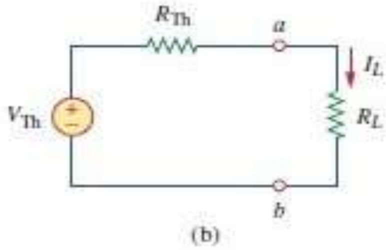
(a) (b)

Statement of the Thevenin's Theorem

It states that any two terminals (a-b) of a linear bilateral network can be replaced by an equivalent voltage source (V_{Th}) and an equivalent series resistance (R_{Th}). The voltage source is called as the open circuited voltage source i.e. the voltage across the two terminals with load, if any removed. The series resistance is the resistance of the network measured between two terminals with load removed and constant voltage source and constant current source being replaced by its internal resistance. If the internal resistances are not known, the voltage source is replaced by short circuit (zero resistance) and current source by open circuit (infinite resistance).

Consider a linear circuit terminated by a load, as shown in Fig.(a) below.





The current $\mathbf{I_L}$ through the load and the voltage $\mathbf{V_L}$ across the load are easily determined once the Thevenin equivalent of the circuit at the load's terminals is obtained, as shown in Fig.(b).
From Fig. (b), we obtain

$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

$$V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} V_{Th}$$

Thevenin's theorem is very important in circuit analysis. It helps simplify a circuit. A large circuit may be replaced by a single independent voltage source and a single resistor. This replacement technique is a powerful tool in circuit design.

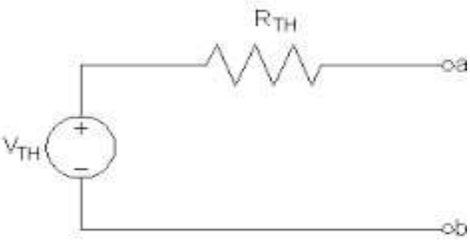
Steps to apply Thevenin's Theorem

Step1: Remove the branch (a-b) element through which current is required to be calculated.

Step 2: Calculate the voltage across these open circuited terminals, by using any of the network simplification techniques. This voltage is called Thevenin's equivalent voltage V_{Th} (Voc).

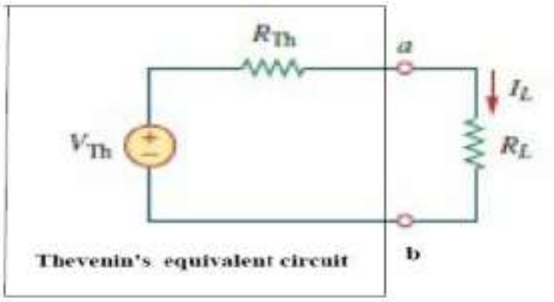
Step 3: Calculate the equivalent resistance R_{Th}, as viewed through the two terminals of the branch from which current is to be calculated by removing that branch resistance and replacing all the independent sources by their internal impedances.

Step 4: Draw the Thevenin's equivalent showing the voltage source V_{Th}, with the resistance R_{Th} in series with it, across the terminals of the branch through which the current is to be calculated.



Thevenin's equivalent circuit

Reconnect the load resistance R_L



The required current through the branch is

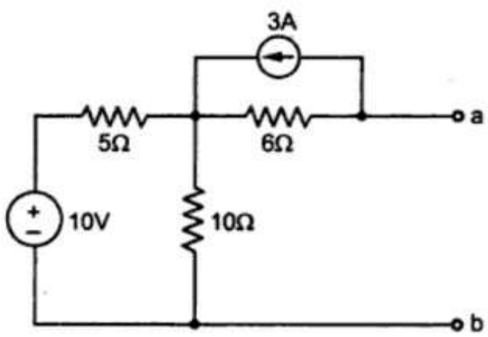
$$I_L = \frac{V_{Th}}{R_{Th} + R_L}$$

Limitation of Thevenin's theorem

1. Not applicable to circuits consisting of nonlinear elements.
2. Not applicable to unilateral circuits.
3. There should not be magnetic coupling between the load and the circuit, to be replaced by Thevenin's theorem
4. In the load side there should not be controlled sources, controlled from some other part of the circuit.

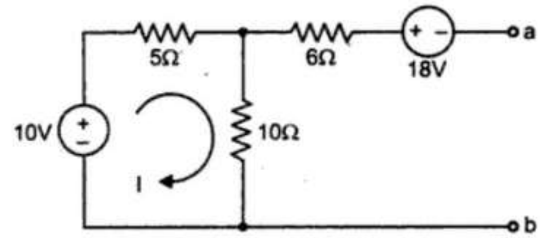
Practice Problems using Thevenin's theorem

Example: Determine the Thevenin's equivalent of the network.



Solution:

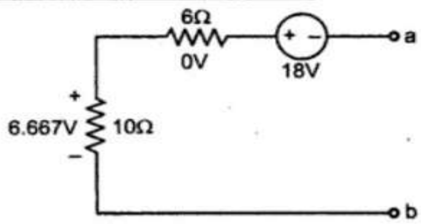
Step 1: Terminal a-b are open



Step 2 : Calculate V_{TH}.

Converting current source to voltage source.

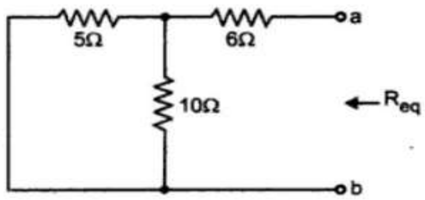
$$I = \frac{10}{5+10} = 0.6667 \text{ A}$$



No current flows through 6 Ω. The path a-b with different voltage drops is shown in the Fig. 2.59 (b).

$$V_{ab} = V_{TH} = -18 + 6.667$$
$$= -11.333 \text{ V}$$

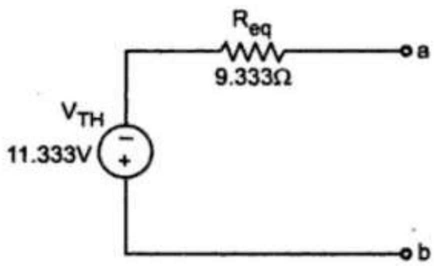
So V_{TH} = 11.333 V with a negative



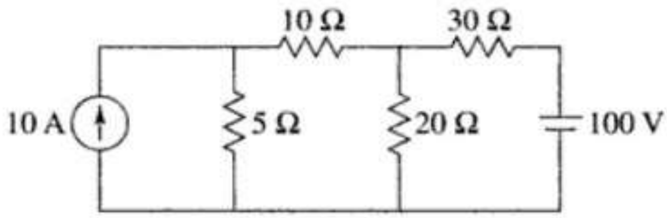
Step 3 : Find R_{eq}, replacing voltage sources by short.

$$\therefore R_{eq} = 6 + (5||10))$$
$$= 6 + 3.33$$
$$= 9.3333 \, \Omega$$

Step 4 : So Thevenin's equivalent is,

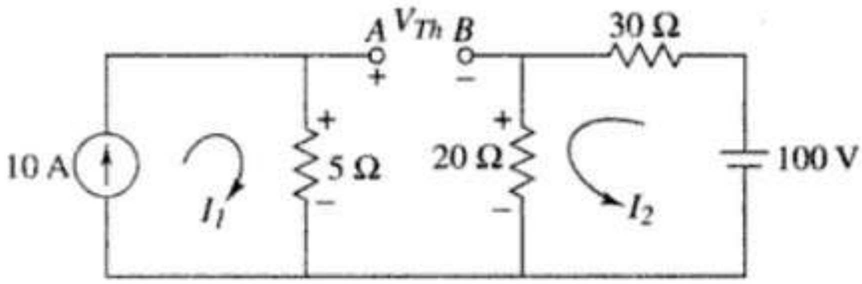


Example: Find the current through the 10 Ω resistor using Thevenin's theorem.



Solution: **Step 1**: Calculation of V_{Th}

Removing the 10 Ω resistor from the circuit.



For mesh 1

$$I_1 = 10 \text{ A}$$

Applying KVL to Mesh 2

$$100 - 30I_2 - 20I_2 = 0$$

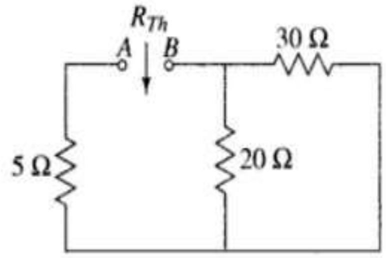
$$I_2 = 2 \text{ A}$$

Writing V_{Th} equation we get $\mathbf{5I_1 - V_{Th} - 20I_2 = 0}$

Therefore $\mathbf{V_{Th} = 20\ V}$

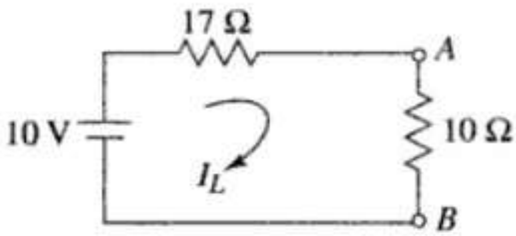
Step 2: Calculation of $\mathbf{R_{Th}}$

Replacing the current source of **10A** with an open circuit and the voltage source of **100V** with a short circuit,



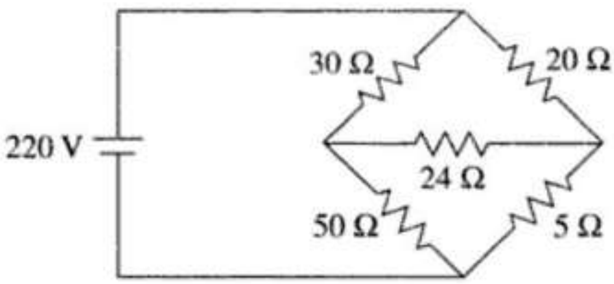
$$R_{Th} = 5 + (20 \| 30) = 17\ \Omega$$

Step 3: Calculation of I_L



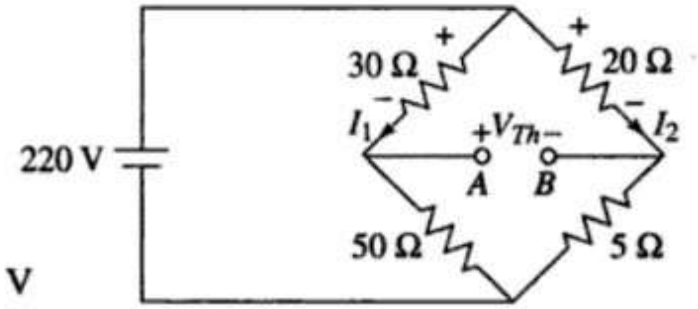
$$I_L = \frac{10}{17 + 10} = 0.37\ \text{A}$$

Example: Find the current through the 24- Ω resistor using Thevenin's theorem.



Solution: **Step 1:** Calculation of $\mathbf{V_{Th}}$

Removing the 24- Ω resistor from the network.



$$I_1 = \frac{220}{30+50} = 2.75 \text{ A}$$

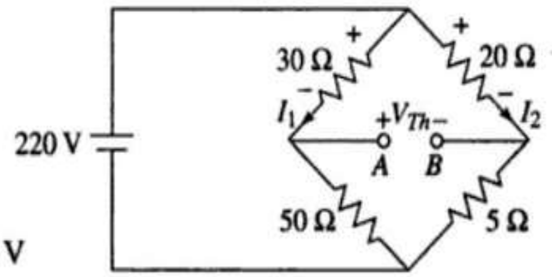
$$I_2 = \frac{220}{20+5} = 8.8 \text{ A}$$

Writing V_{Th} equation,

$$V_{Th} + 30I_1 - 20I_2 = 0$$

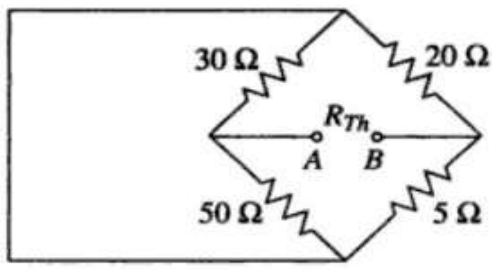
$$V_{Th} = 20I_2 - 30I_1$$

$$= 20\,(8.8) - 30(2.75) = 93.5 \text{ V}$$

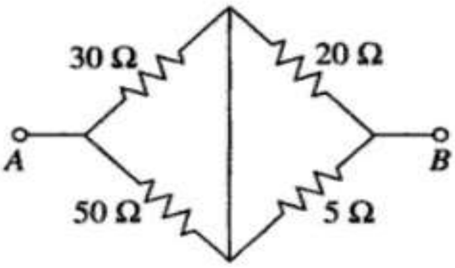


Step 2: Calculation of $\mathbf{R_{Th}}$

Replacing the 220-V source with short circuit,

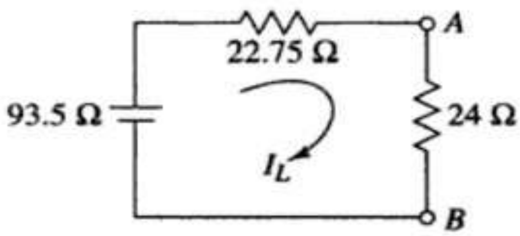


The circuit can be redrawn as shown:



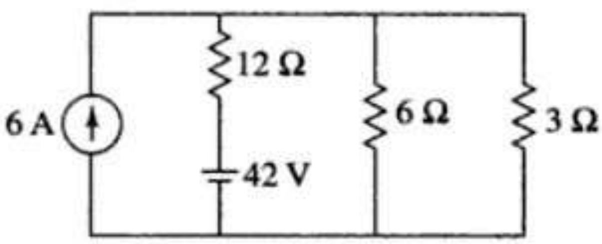
$$R_{Th} = (30 \,||\, 50) + (20 \,||\, 5) = 22.75\ \Omega$$

Step 3: Calculation of $\mathbf{I_L}$



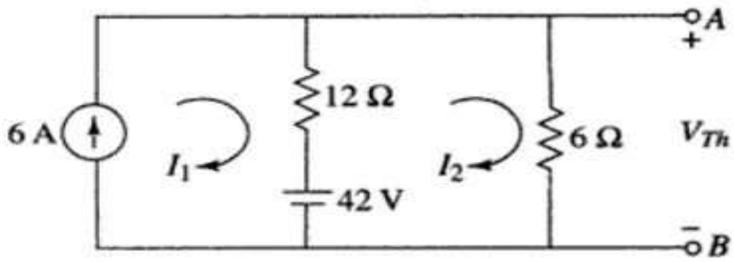
$$I_L = \frac{93.5}{22.75 + 24} = 2\text{ A}$$

Example: Find the current in the 3-Ω resistor using Thevenin's theorem.



Solution: **Step1:** Calculation of $\mathbf{V_{Th}}$

Removing the 3- Ω resistor from the network.



Writing KVL equation for Mesh 1

$$I_1 = 6 \qquad ...(1)$$

Applying KVL to Mesh 2,

$$42 - 12(I_2 - I_1) - 6\,I_2 = 0$$

$$-12\,I_1 + 18\,I_2 = 42 \qquad ...(2)$$

Substituting value of I_1 in Eq. (2),

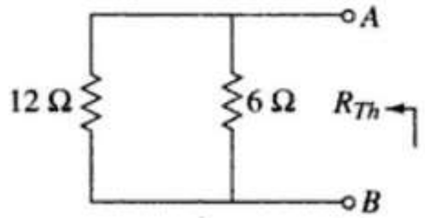
$$I_2 = 6.33\text{ A}$$

Writing V_{Th} equation,

$$V_{Th} = 6\,I_2 = 38\text{ V}$$

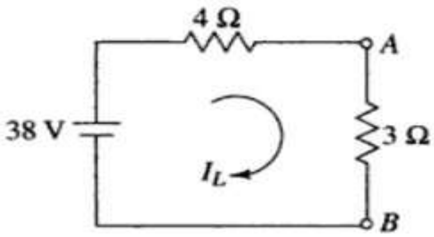
Step 2: Calculation of $\mathbf{R_{Th}}$

Replacing voltage source by short circuit and current source by open circuit,



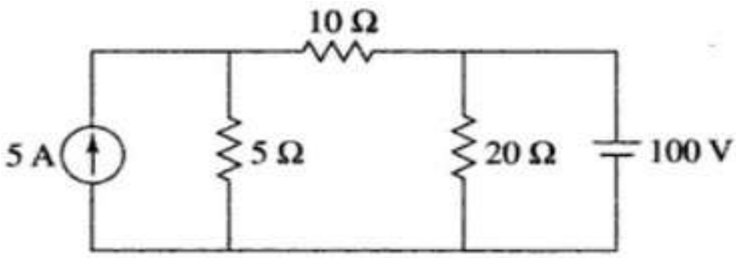
$$R_{Th} = 6 \parallel 12 = 4\ \Omega$$

Step 3: Calculation of $\mathbf{I_L}$



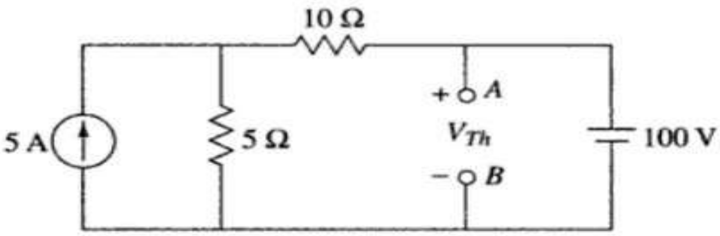
$$I_L = \frac{38}{4+3} = 5.43\text{ A}$$

Example: Find the current in the 20-Ω resistor using Thevenin's theorem.



Solution: **Step1:** Calculation of **V_{Th}**

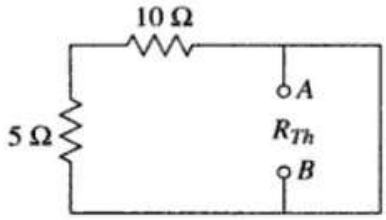
Removing 20-Ω resistor by the network.



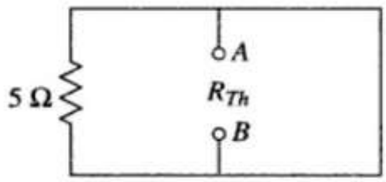
$V_{Th} = 100V$

Step 2: Calculation of **R_{Th}**

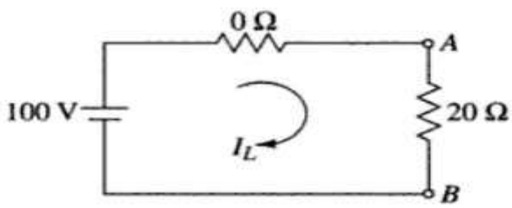
Replacing voltage source by short circuit and current source by open circuit,



$R_{Th} = 0\Omega$

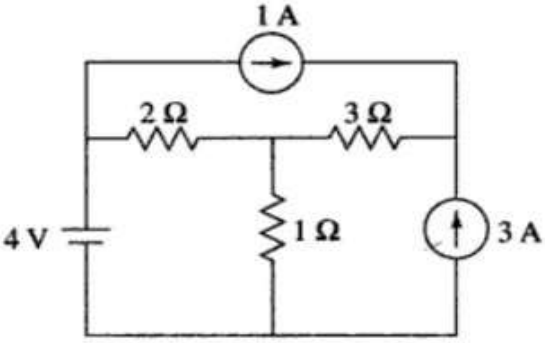


Step 3: Calculation of I_L



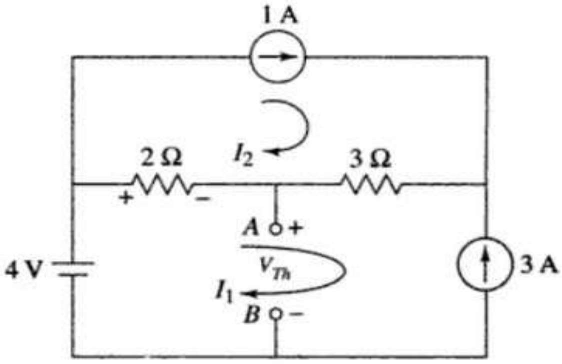
$$I_L = \frac{100}{20} = 5\text{ A}$$

Example: Find the current in the 1-Ω resistor using Thevenin's theorem.



Solution: **Step1:** Calculation of **V_{Th}**

Removing 1- Ω resistor by the network.



Writing the current equation for Meshes 1 and 2,

$$I_1 = -3 \text{ A}$$

$$I_2 = 1 \text{ A}$$

Writing V_{Th} equation,

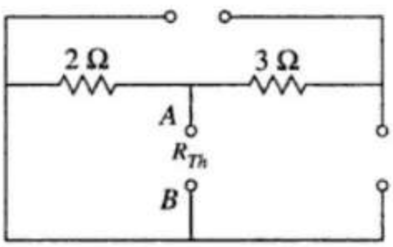
$$4 - 2(I_1 - I_2) - V_{Th} = 0$$

$$V_{Th} = 4 - 2(-3 - 1)$$

$$= 4 - 2(-4) = 12 \text{ V}$$

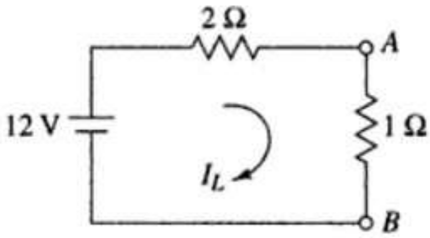
Step 2: Calculation of **R_{Th}**

Replacing voltage source by short circuit and current source by open circiut.



$R_{Th} = 2\Omega$

Step 3: Calculation of **I_L**

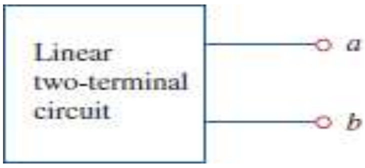


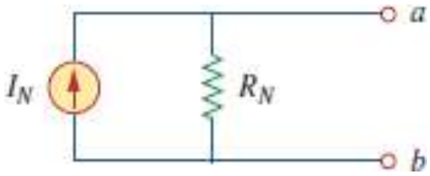
$$I_L = \frac{12}{2+1} = 4\ \text{A}$$

Norton's Theorem

Statement of the Norton's Theorem

It states that any two terminals (a-b) of a linear bilateral network can be replaced by an equivalent current source (I_N) and an equivalent parallel resistance (R_N). The current source is called as the short-circuited current source i.e. the current across two shorted terminal The parallel resistance is the resistance of the network measured between two terminals with load removed and constant voltage source and constant current source being replaced by its internal resistance. If the internal resistances are not known, the voltage source is replaced by short circuit (zero resistance) and current source by open circuit (infinite resistance).



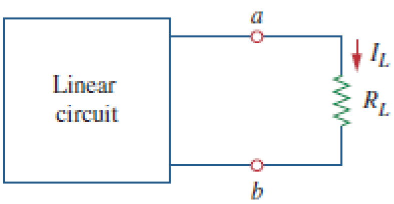


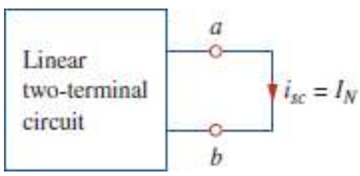
Using concept of source transformation, the Thevenin and Norton resistances are equal.

$$\boxed{R_N = R_{Th}}$$

Steps to apply Norton's Theorem

Step1: Short the branch (a-b) element through which current is required to be calculated.
Step 2: Calculate the current across these shorted terminals, by using any of the network simplification techniques. This voltage is called short circuit current **(I_{sc} /I_N).**
Step 3: Calculate the equivalent resistance **R_N /R_{Th}**, as viewed through the two terminals of the branch from which current is to be calculated by removing that branch resistance and replacing all the independent sources by their internal impedances.
Step 4: Draw the Norton's equivalent showing the current source **I_N,** with the resistance **R_N** in series with it, across the terminals of the branch through which the current is to be calculated.





$$I_L = \frac{I_{SC} R_N}{R_N + R_L}$$

$$\boxed{I_N = \frac{V_{Th}}{R_{Th}}}$$

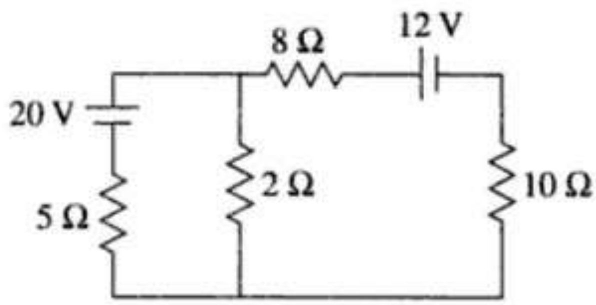
$$V_{Th} = v_{oc}$$

$$I_N = i_{sc}$$

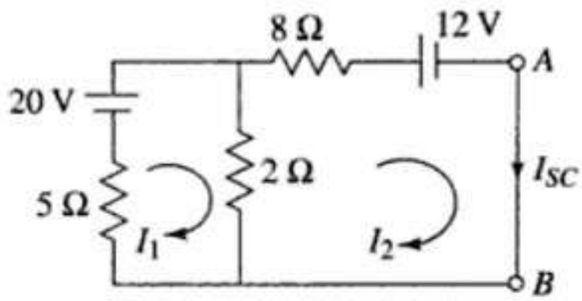
$$R_{Th} = \frac{v_{oc}}{i_{sc}} = R_N$$

Practice problem on Norton's Theorem

Example: Determine the current through 10 - Ω resistor using Norton's Theorem.



Solution: Replacing 10 - Ω resistor with a short circuit



Step I: *Calculation of I_{SC}*

Applying KVL to Mesh 1,

$$-5I_1 + 20 - 2(I_1 - I_2) = 0$$

$$7I_1 - 2I_2 = 20 \qquad ...(1)$$

Applying KVL to Mesh 2,

$$-2(I_2 - I_1) - 8I_2 - 12 = 0$$

$$-2I_1 + 10I_2 = -12 \qquad ...(2)$$

Solving Eqs (1) and (2),

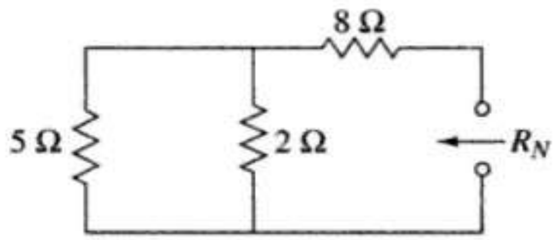
$$I_2 = -0.67 \text{ A}$$

$$I_{SC} = I_2 = -0.67 \text{ A}$$

Step II: *Calculation of R_N*

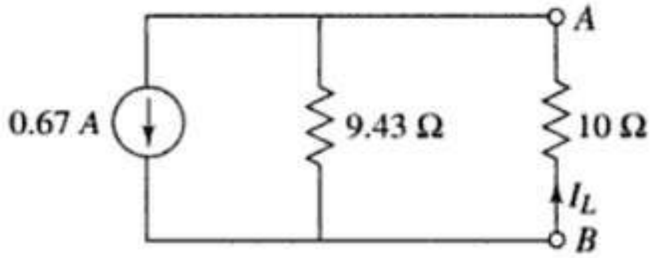
Replacing voltage sources with short circuits,

$$R_N = (5 \,||\, 2) + 8 = 9.43\ \Omega$$

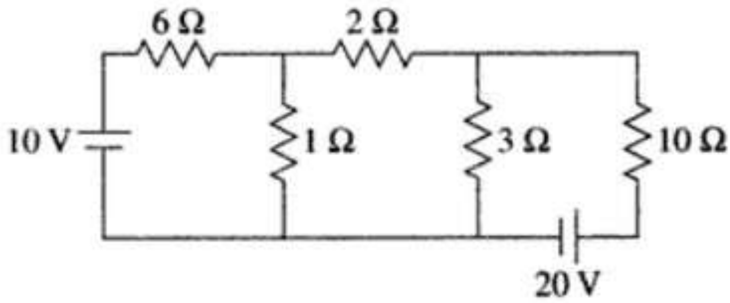


Step III: *Calculation of I_L*

$$I_L = 0.67 \times \frac{9.43}{9.43+10}$$
$$= 0.33\ \text{A}\ (\uparrow)$$

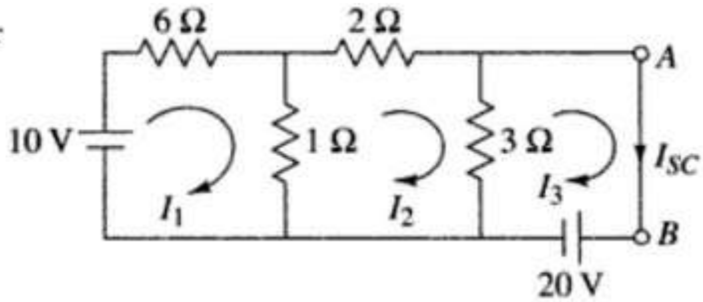


Example: Determine the current through 10 - Ω resistor using Norton's Theorem.



Solution:

Step I: *Calculation of I_{SC}*



Applying KVL to Mesh 1,

$$10 - 6I_1 - 1\,(I_1 - I_2) = 0$$

$$7I_1 - I_2 = 10 \qquad ...(1)$$

Applying KVL to Mesh 2,

$$-1\,(I_2 - I_1) - 2I_2 - 3(I_2 - I_3) = 0$$

$$-I_1 + 6I_2 - 3I_3 = 0 \qquad ...(2)$$

Applying KVL to Mesh 3,

$$-3\,(I_3 - I_2) - 20 = 0$$

$$3I_2 - 3I_3 = 20 \qquad ...(3)$$

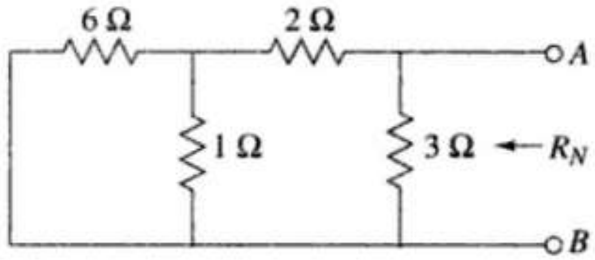
Solving Eqs (1), (2) and (3),

$$I_3 = -13.17 \text{ A}$$

$$I_{SC} = I_3 = -13.17 \text{ A}$$

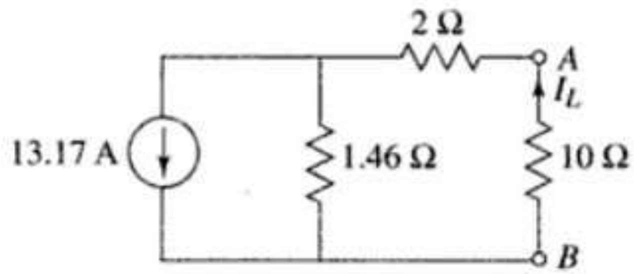
Step II: *Calculation of R_N*

Replacing voltage sources with short circuits,



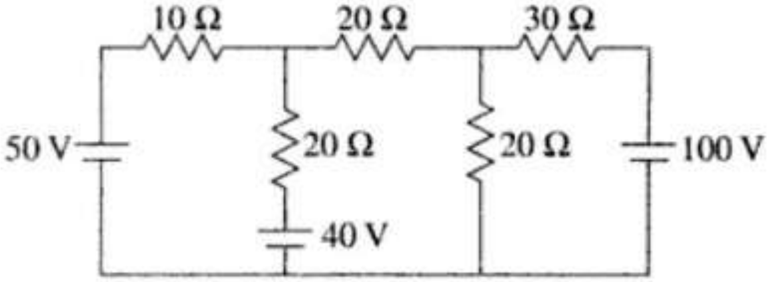
$$R_N = [(6 \parallel 1) + 2] \parallel 3 = 1.46\ \Omega$$

Step *III: Calculation of I_L*



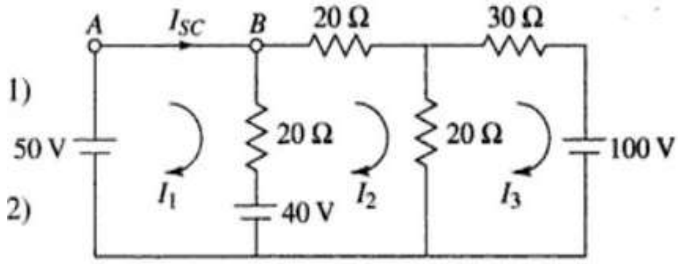
$$I_L = 13.17 \times \frac{1.46}{1.46 + 10} = 1.68 \text{ A } (\uparrow)$$

Example: Determine the current through 10 - Ω resistor using Norton's Theorem.



Solution:

Step *I Calculation of I_{sc}*



. .....(2)

Applying KVL to Mesh 1,

$$50 - 20\,(I_1 - I_2) - 40 = 0$$

$$20I_1 - 20I_2 = 10$$

Applying KVL to Mesh 2,

$$40 - 20\,(I_2 - I_1) - 20I_2 - 20\,(I_2 - I_3) = 0$$

$$-20I_1 + 60I_2 - 20I_3 = 40$$

Applying KVL to Mesh 3,

$$-20\,(I_3 - I_2) - 30I_3 - 100 = 0$$

$$-20I_2 + 50I_3 = -100 \quad ...(3)$$

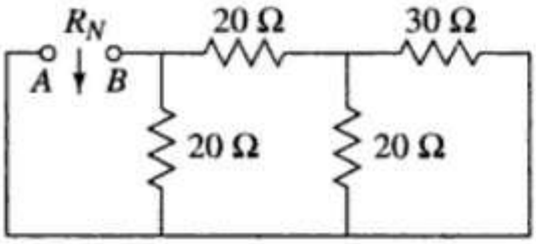
Solving Eqs (1), (2) and (3),

$$I_1 = 0.81\ \text{A}$$

$$I_{SC} = I_1 = 0.81\ \text{A}$$

Step II: *Calculation of R_N*

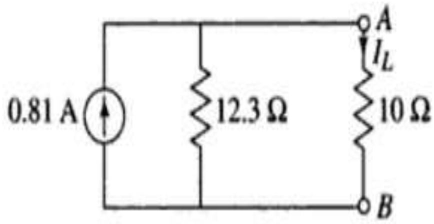
Replacing all voltage sources by short circuits,



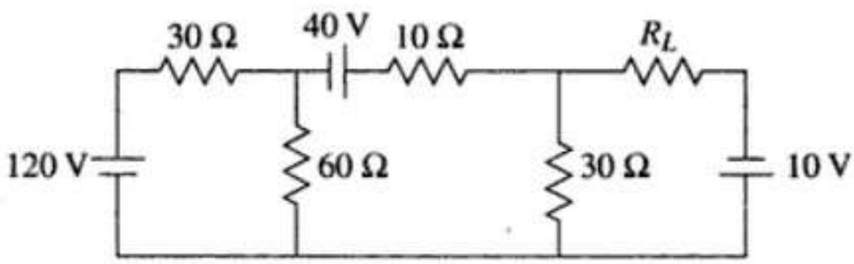
$$R_N = [(20 \parallel 30) + 20] \parallel 20 = 12.3\ \Omega$$

Step III: *Calculation of I_L*

$$I_L = 0.81 \times \frac{12.3}{12.3 + 10} = 0.45\ \text{A}$$



Example: Obtain Norton's equivalent circuit as seen by resistor $\mathbf{R_L}$



Solution:

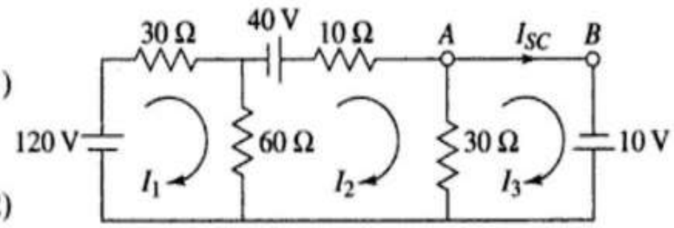
Step I: *Calculation of I_{SC}*

Applying KVL to Mesh 1,

$$120 - 30I_1 - 60(I_1 - I_2) = 0$$
$$90I_1 - 60I_2 = 120 \quad ...(1)$$

Applying KVL to Mesh 2,

$$-60(I_2 - I_1) + 40 - 10I_2 - 30(I_2 - I_3) = 0$$
$$-60I_1 + 100I_2 - 30I_3 = 40 \quad ...(2)$$



Applying KVL to Mesh 3,

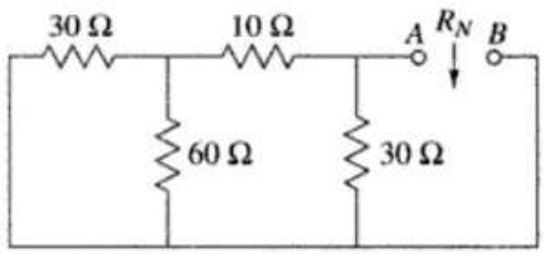
$$-30(I_3 - I_2) + 10 = 0$$
$$30I_2 - 30I_3 = -10 \quad ...(3)$$

Solving Eqs (1), (2) and (3),

$$I_3 = 4.67 \text{ A}$$
$$I_{SC} = I_3 = 4.67 \text{ A}$$

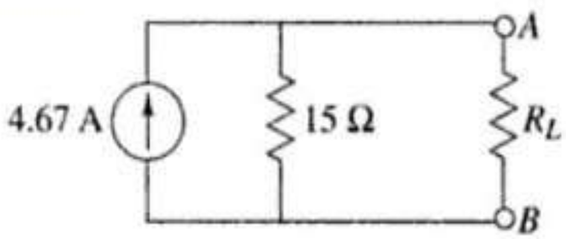
Step II: *Calculation of R_N*

Replacing voltage sources by short circuits,



$$R_N = [(30 \parallel 60) + 10] \parallel 30 = 15\ \Omega$$

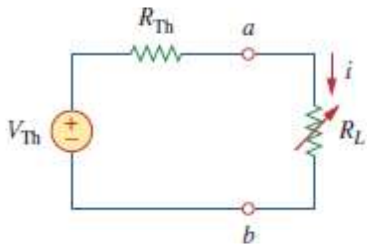
Step III: Norton's equivalent network is a shown below.



Maximum Power Transfer Theorem

In many practical situations, a circuit is designed to provide power to a load. There are applications in areas such as communications where it is desirable to maximize the power delivered to a load.

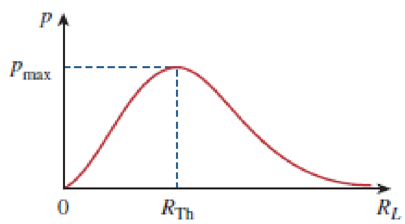
The Thevenin equivalent is useful in finding the maximum power a linear circuit can deliver to a load. We assume that we can adjust the load resistance **R_L**. If the entire circuit is replaced by its Thevenin equivalent except for the load, as shown in Figure, the power delivered to the load is



$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L$$

For a given circuit, **V_{Th}** and **R_{Th}** are fixed. By varying the load resistance **R_L**, the power delivered to the load varies as sketched in Figure below.

We notice from Figure that the power is small for small or large values of but maximum for some value of **R_L** between **0** and ∞. We now want to show that this maximum power occurs when **R_L** is equal to **R_{Th}**. This is known as the ***maximum power theorem.***



Power delivered to the load as a function of R_L.

Statement of Maximum power transfer theorem.

Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$).

Proof of Maximum power transfer theorem.

We have

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L \qquad ....(1)$$

We differentiate p in Equation 1 with respect to R_L and set the result equal to zero.

$$\frac{dp}{dR_L} = V_{Th}^2\left[\frac{(R_{Th}+R_L)^2 - 2R_L(R_{Th}+R_L)}{(R_{Th}+R_L)^4}\right]$$
$$= V_{Th}^2\left[\frac{(R_{Th}+R_L-2R_L)}{(R_{Th}+R_L)^3}\right] = 0$$

This implies that

$$0 = (R_{Th}+R_L-2R_L) = (R_{Th}-R_L) \qquad \text{....(2)}$$

which yields

$$\boxed{R_L = R_{Th}} \qquad \text{......(3)}$$

This shows that the maximum power transfer takes place when the load resistance R_L equals the Thevenin resistance R_{Th}. We can readily confirm that Eq. 3 gives the maximum power by showing that $d^2 p / dR_L < 0$.

The maximum power transferred is obtained by substituting Eq. 3 into Eq 1.

$$\boxed{p_{max} = \frac{V_{Th}^2}{4R_{Th}}}$$

Steps to be followed in maximum power transfer theorem.

1. Remove the variable load resistor R_L.
2. Find the open circuit voltage V_{Th} across points a and B.
3. Find the resistance R_{Th} as seen from points A and B with voltage source and current source replaced by internal resistance.
4. Find the resistance R_L for maximum power transfer

$$\boxed{R_L = R_{Th}}$$

5. Find the maximum power

$$I_L = \frac{V_{Th}}{R_{Th} + R_L} = \frac{V_{Th}}{2R_{Th}}$$

$$P_{\max} = I_L^2 R_L$$

$$= \frac{V_{Th}^2}{4R_{Th}^2} \times R_{Th} = \frac{V_{Th}^2}{4R_{Th}}$$

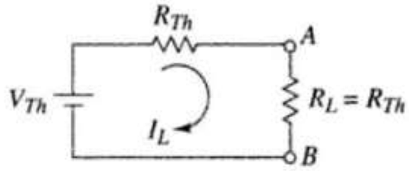
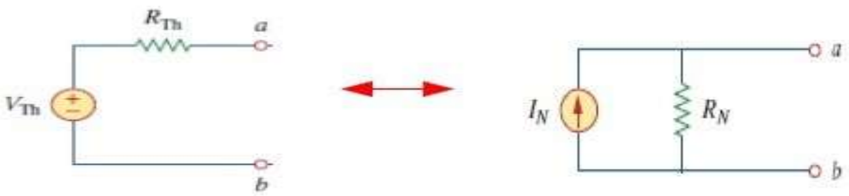


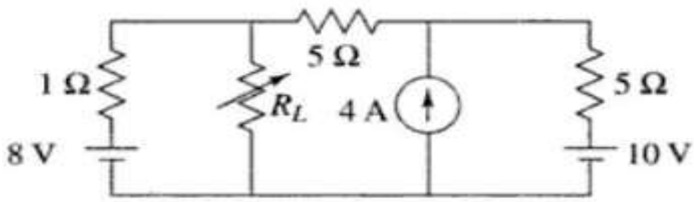
Fig. 3.177

Norton's theorem is also called the dual of Thevenin's theorem because the Thevenin's voltage source and equivalent resistance can be converted to Nortons current source in parallel with a resistance.



Practice problem using maximum power transfer theorem

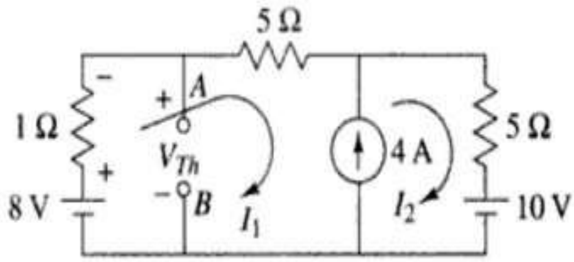
Example 11.1: For the circuit shown find the value of resistance $\mathbf{R_L}$ for maximum power and calculate the maximum power.



Solution:

Step I: Calculation of V_{Th}

Removing the variable resistor $\mathbf{R_L}$ from the network.



$$I_2 - I_1 = 4 \qquad \text{...(1)}$$

Applying KVL to the outer path,

$$8 - I_1 - 5I_1 - 5I_2 - 10 = 0$$

$$-6I_1 - 5I_2 = 2 \qquad \text{...(2)}$$

Solving Eqs (1) and (2),

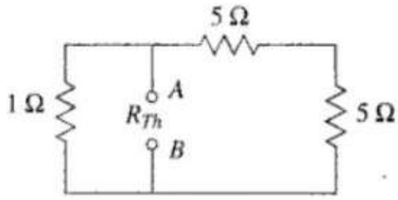
$$I_1 = -2 \text{ A}$$

$$I_2 = 2 \text{ A}$$

Writing V_{Th} equation,

$$8 - 1(I_1) - V_{Th} = 0$$

$$V_{Th} = 8 - I_1 = 8 - (-2) = 10 \text{ V}$$



Step II: *Calculation of R_{Th}*

Replacing the voltage sources by short circuits and current source by an open circuit,

$$R_{Th} = 10\ \Omega \parallel 1\ \Omega = 0.91\ \Omega$$

Step III: *Value of R_L*

For maximum power transfer,

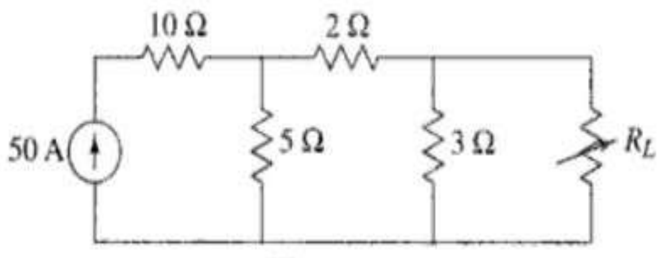
$$R_L = R_{Th} = 0.91\ \Omega$$

Step IV: *Calculation of P_{max}*

$$P_{\max} = \frac{V_{Th}^2}{4R_{Th}}$$

$$= \frac{(10)^2}{4 \times 0.91} = 27.47 \text{ W}$$

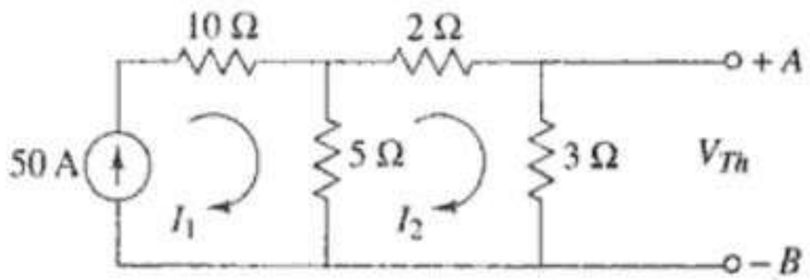
Example: For the circuit shown, find the value of the resistance R_L for maximum power and calculate the maximum power.



Solution:

Step I: Calculation of V_{Th}

Removing the variable resistor R_L from the circuit.



For Mesh 1,

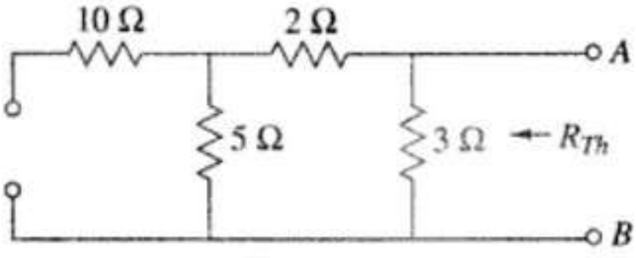
$$I_1 = 50 \text{ A}$$

Applying KVL to Mesh 2,

$$-5(I_2 - I_1) - 2I_2 - 3I_2 = 0$$
$$5I_1 - 10I_2 = 0$$
$$I_1 = 2I_2$$
$$I_2 = 25 \text{ A}$$
$$V_{Th} = 3I_2 = 3(25) = 75 \text{ V}$$

Step II: Calculation of R_{Th}

Replacing the current source of 50A with an open circuit.



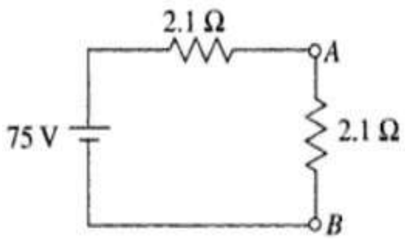
$$R_{Th} = 7 \parallel 3 = 2.1\ \Omega$$

Step III: Value of **R_L**

For maximum power transfer,

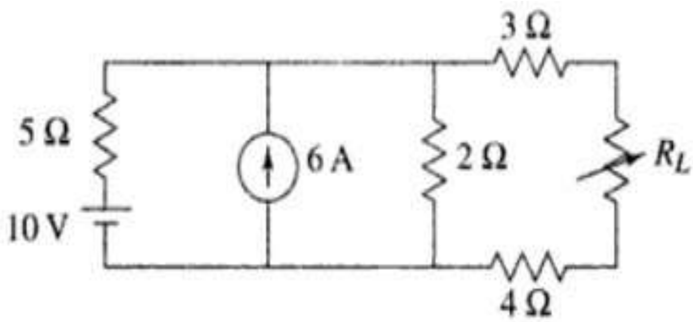
$$R_L = R_{Th} = 2.1\ \Omega$$

Step IV: Calculation of P_{max}



$$P_{\max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(75)^2}{4\times 2.1} = 669.64\ \text{W}$$

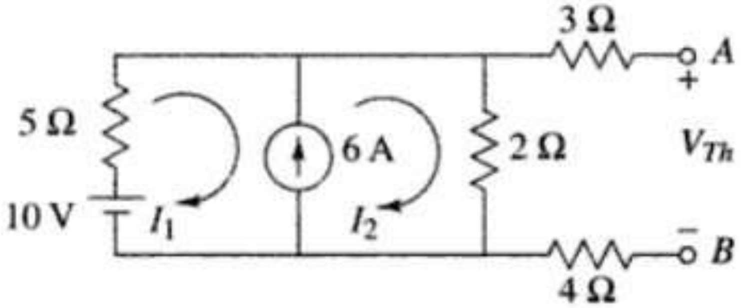
Example: For the circuit shown, find the value of the resistance **R_L** for maximum power and calculate the maximum power.



Solution:

Step I: Calculation of **V_{Th}**

Removing the variable resistor $\mathbf{R_L}$ from the circuit.



Writing the current equation for the supermesh,

$$I_2 - I_1 = 6 \qquad \text{...(1)}$$

Applying KVL to the supermesh,

$$10 - 5I_1 - 2I_2 = 0$$

$$5I_1 + 2I_2 = 10 \qquad \text{...(2)}$$

Solving equations (1) and (2),

$$I_1 = -0.29 \text{ A}$$

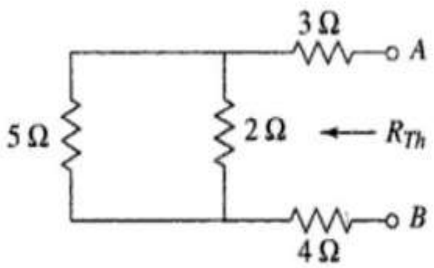
$$I_2 = 5.71 \text{ A}$$

Writing V_{Th} equation,

$$V_{Th} = 2I_2 = 11.42 \text{ V}$$

Step II: Calculation of $\mathbf{R_{Th}}$

Replacing the current source with an open circuit and voltage source with a short circuit.



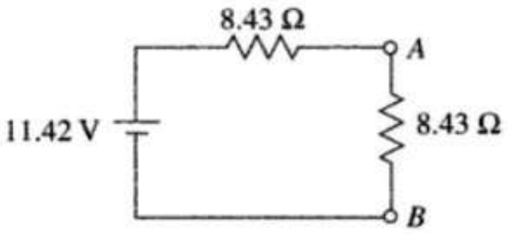
$$R_{Th} = (5 \parallel 2) + 3 + 4 = 8.43\ \Omega$$

Step III: Calculation of $\mathbf{R_L}$

For maximum power transfer

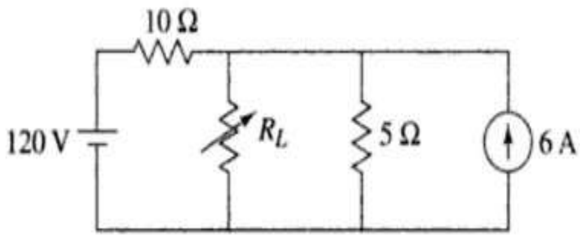
$$R_L = R_{Th} = 8.43\ \Omega$$

Step IV: Calculation of ***P*** ***max***



$$P_{\max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(11.42)^2}{4\times 8.43} = 3.87 \text{ W}$$

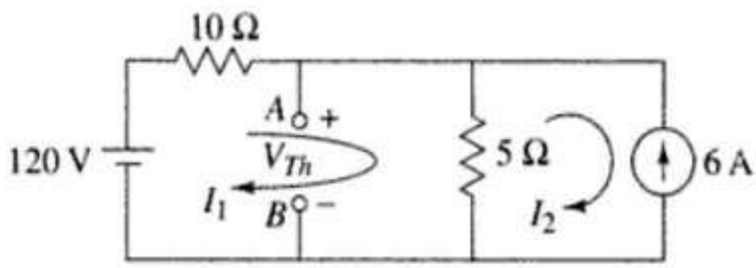
Example: For the circuit shown, find the value of the resistance **R_L** for maximum power and calculate the maximum power.



Solution:

Step I: Calculation of $\mathbf{V_{Th}}$

Removing the variable resistor $\mathbf{R_L}$ from the circuit.



Applying KVL to Mesh 1,

$$120 - 10I_1 - 5(I_1 - I_2) = 0$$

$$15I_1 - 5I_2 = 120 \qquad ...(1)$$

Writing current equation for Mesh 2,

$$I_2 = -6 \text{ A} \qquad ...(2)$$

Solving Eqs (1) and (2),

$$I_1 = 6 \text{ A}$$

Writing V_{Th} equation,

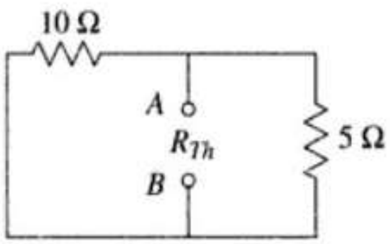
$$120 - 10I_1 - V_{Th} = 0$$

$$V_{Th} = 120 - 10\,(6)$$

$$= 60 \text{ V}$$

Step II: Calculation of $\mathbf{R_{Th}}$

Replacing the current source with an open circuit and voltage source with a short circuit.



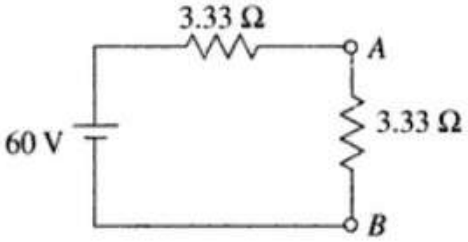
$$R_{Th} = 10 \,\|\, 5 = 3.33\ \Omega$$

Step III: Calculation of $\mathbf{R_L}$

Step IV: Calculation of $\boldsymbol{P_{max}}$

For maximum power transfer

$$R_L = R_{Th} = 3.33\ \Omega$$

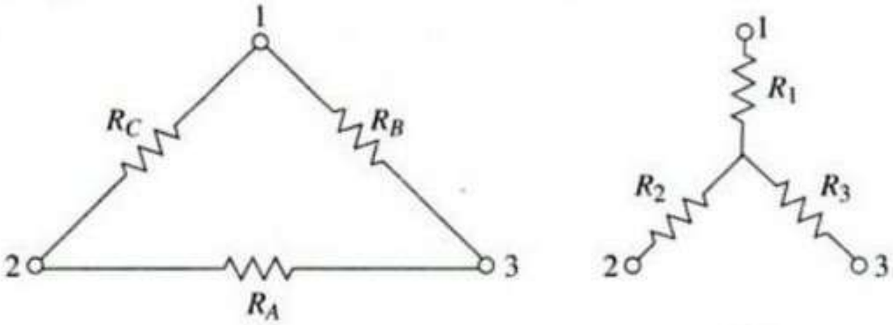


$$P_{max} = \frac{V_{Th}^2}{4R_{Th}} = \frac{(60)^2}{4 \times 3.33} = 270.27 \text{ W}$$

Star - Delta transformation (Self Study)

When a circuit cannot be simplified by normal series - parallel reduction technique, star-delta transformation is used.

Figure (a) shows three resistances R_A, R_B and R_C connected in delta.
Figure (b) shows three resistances R_1, R_2 and R_3 connected in star.



(a) (b)

These two networks will be electrically equivalent if the resistance as measured between any pair of terminals is same in both the arrangements.

Delta to Star transformation

Referring to the delta network shown in Figure (a)

The resistance between terminals 1 and 2 = $R_C \, || \, (R_A + R_B)$

$$= \frac{R_C(R_A + R_B)}{R_A + R_B + R_C} \qquad \text{...... (1)}$$

Referring to the star network shown in Figure (b)

The resistance between terminals 1 and 2 = $R_1 + R_2$ ....(2)

Since the two networks are electrically equivalent,

$$R_1 + R_2 = \frac{R_C(R_A + R_B)}{R_A + R_B + R_C} \qquad ....(3)$$

Similarly,

$$R_2 + R_3 = \frac{R_A(R_B + R_C)}{R_A + R_B + R_C} \qquad ....(4)$$

and

$$R_3 + R_1 = \frac{R_B(R_A + R_C)}{R_A + R_B + R_C} \qquad ....(5)$$

Subtracting equation (4) from (3)

$$R_1 - R_3 = \frac{R_B R_C - R_A R_B}{R_A + R_B + R_C} \qquad ......(6)$$

Adding equation (6) and (5)

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} \qquad ....(7)$$

Similarly,

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} \qquad ....(8)$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} \qquad ....(9)$$

Thus, star resistance connected to terminal is equal to the product of the two delta resistances connected to the same terminal divided by the sum of the delta resistances.

Star to Delta transformation

Multiplying the above equation

$$R_1 R_2 = \frac{R_A R_B R_C^2}{(R_A + R_B + R_C)^2} \quad \text{.......(10)}$$

$$R_2 R_3 = \frac{R_A^2 R_B R_C}{(R_A + R_B + R_C)^2} \quad \text{.......(11)}$$

$$R_3 R_1 = \frac{R_A R_B^2 R_C}{(R_A + R_B + R_C)^2}$$

$$\text{.....(12)}$$

Adding equation 10, 11 and 12 we get

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_A R_B R_C (R_A + R_B + R_C)}{(R_A + R_B + R_C)^2}$$

$$= \frac{R_A R_B R_C}{R_A + R_B + R_C} = R_A R_1$$

$$= R_B R_2$$

$$= R_C R_3$$

Hence $$R_A = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1}$$

$$= R_2 + R_3 + \frac{R_2R_3}{R_1}$$

$$R_B = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2}$$

$$= R_1 + R_3 + \frac{R_3R_1}{R_2}$$

$$R_C = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3}$$

$$= R_1 + R_2 + \frac{R_1R_2}{R_3}$$

Thus, the delta resistance between the two terminals is the sum of two-star resistances connected to the same terminals plus the product of the two resistances divided by the remaining third star resistances.

Note: When three equal resistances are connected in delta, the equivalent star resistance is given by

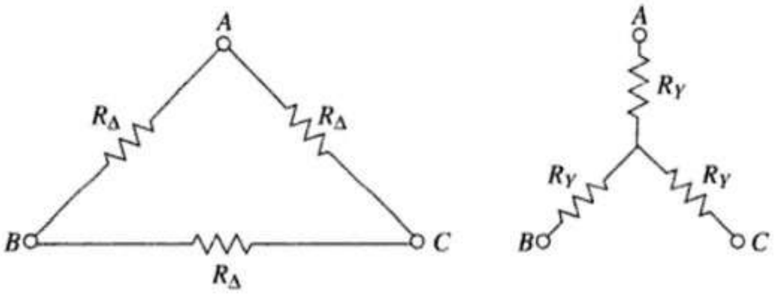


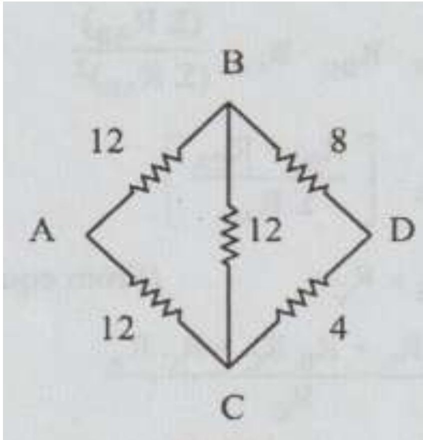
Fig. 2.2

$$R_Y = \frac{R_\Delta R_\Delta}{R_\Delta + R_\Delta + R_\Delta} = \frac{R_\Delta}{3}$$

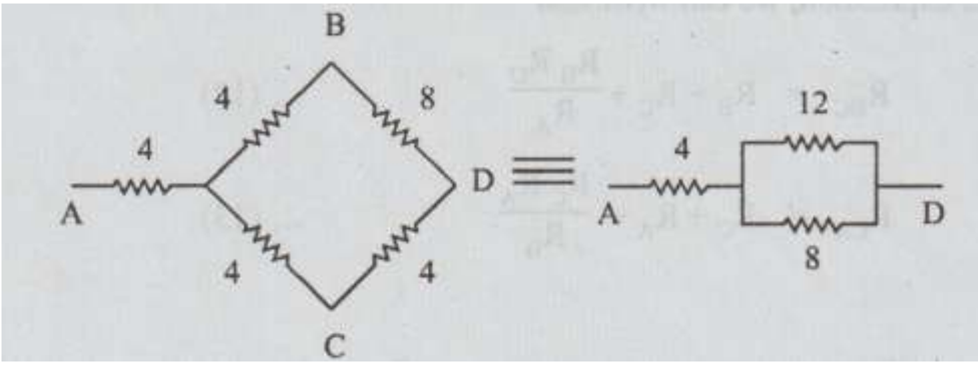
or $$R_\Delta = 3R_Y$$

Practice problems on Star - Delta analysis

Example: For the passive circuit consisting of resistances (in ohms) calculate the resistance between terminal A and D.



Solution: Between the terminals A and D in the circuit given, the combination is neither series nor parallel. Hence, simplification is not possible as it is. So, we can convert the delta between A, B and C into equivalent star. As a result, the circuit becomes as drawn below:



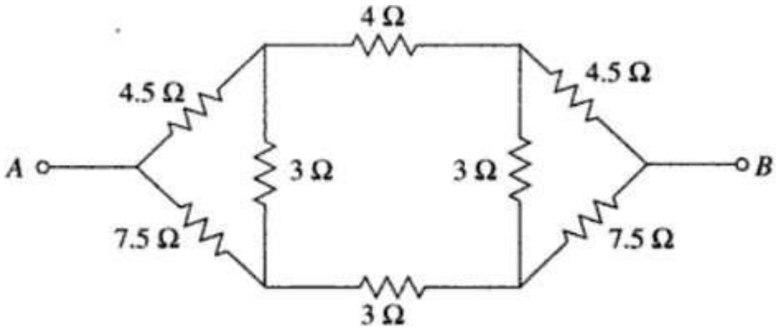
Now, it is series parallel combination of resistances. Hence the total resistance between A and D is equal to

$$R_{AD} = 4 + \frac{12 \times 8}{12 + 8}$$

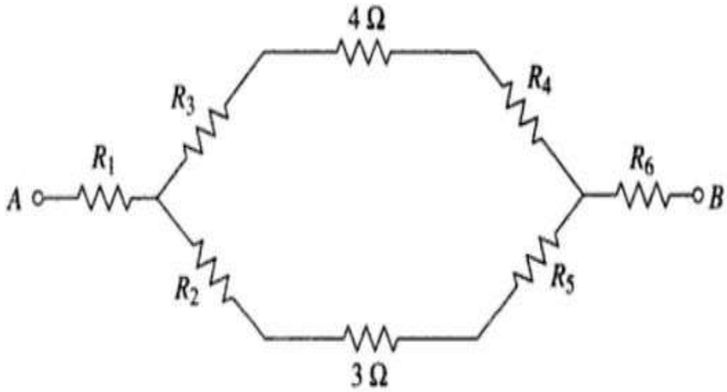
$$= 4 + 4.8$$

$$= 8.8\,\Omega$$

Example: Find the equivalent resistance between terminal A and B



Solution: Converting the two-delta network formed by 4.5 Ω ,7.5 Ω and 3 Ω resistance on either side into equivalent star....

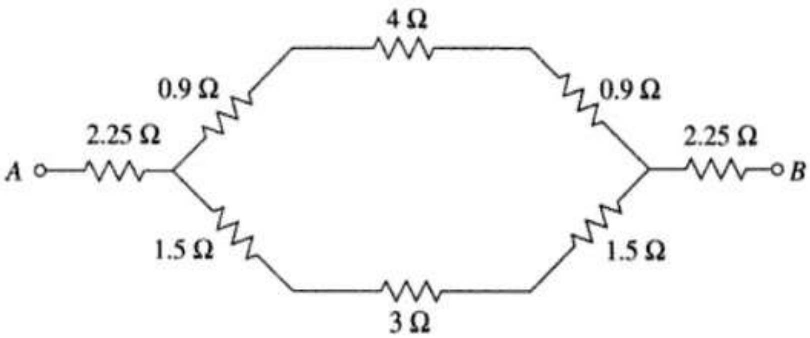


$$R_1 = R_6 = \frac{4.5 \times 7.5}{4.5 + 7.5 + 3} = 2.25\ \Omega$$

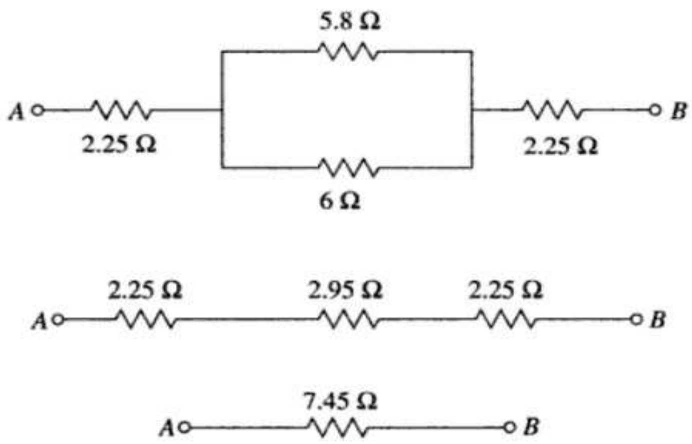
$$R_2 = R_5 = \frac{7.5 \times 3}{4.5 + 7.5 + 3} = 1.5\ \Omega$$

$$R_3 = R_4 = \frac{4.5 \times 3}{4.5 + 7.5 + 3} = 0.9\ \Omega$$

The simplified network is shown as

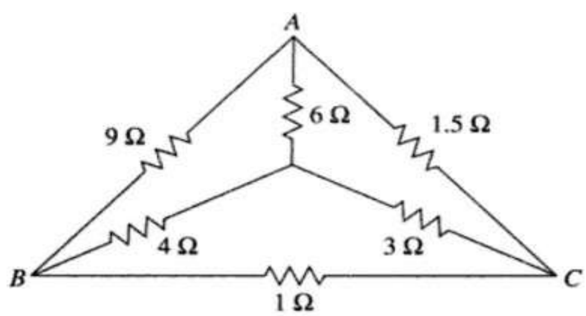


The network can be simplified as follows.

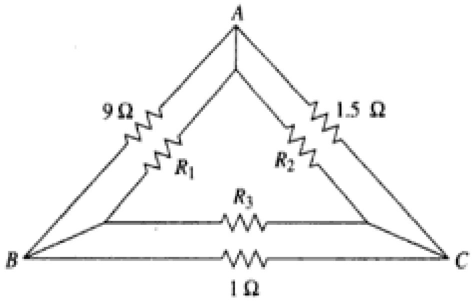


Therefore $R_{AB} = 7.5\ \Omega$

Example: Find the equivalent resistance between terminal A and B.



Solution: Converting the star network formed by 6Ω, 4 Ω and 3 Ω resistor into equivalent delta network.



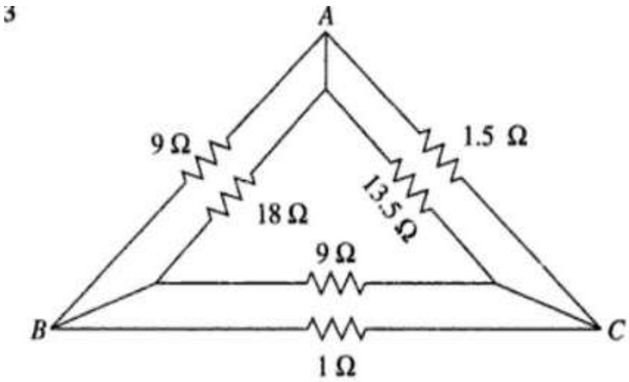
$$R_1 = 6 + 4 + \frac{6\times 4}{3} = 18\ \Omega$$

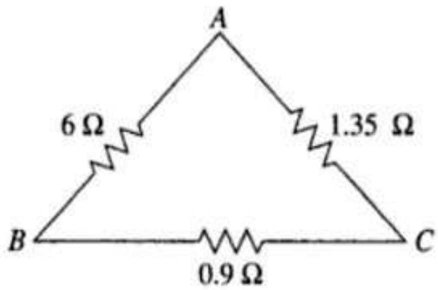
$$R_2 = 6 + 3 + \frac{6\times 3}{4} = 13.5\ \Omega$$

$$R_3 = 4 + 3 + \frac{4\times 3}{6} = 9\ \Omega$$

(1)

The network can be simplified as

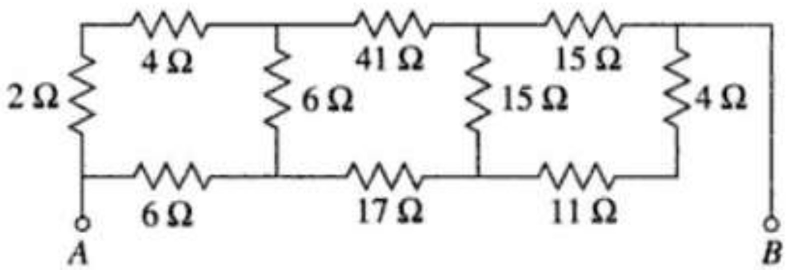




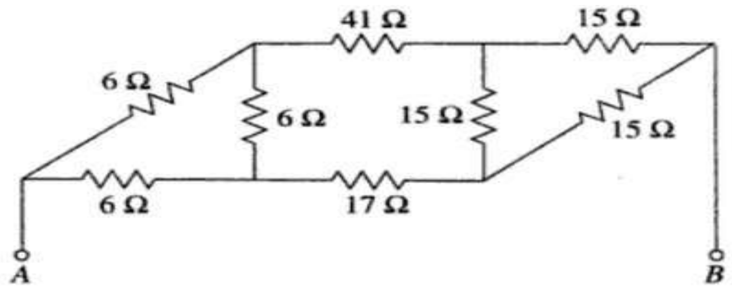
(2) (3)

$$R_{AB} = 6 \,||\, (1.35 + 0.9)$$
$$= 6 \,||\, 2.25$$
$$= 1.64\ \Omega$$

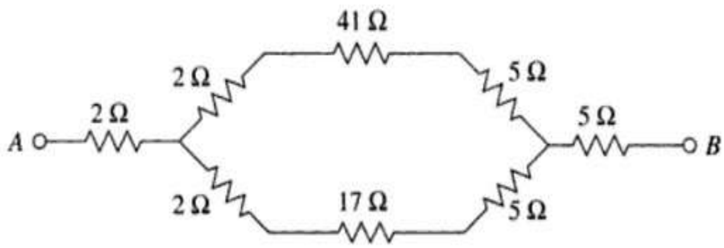
Example: Find the equivalent resistance between terminal A and B.



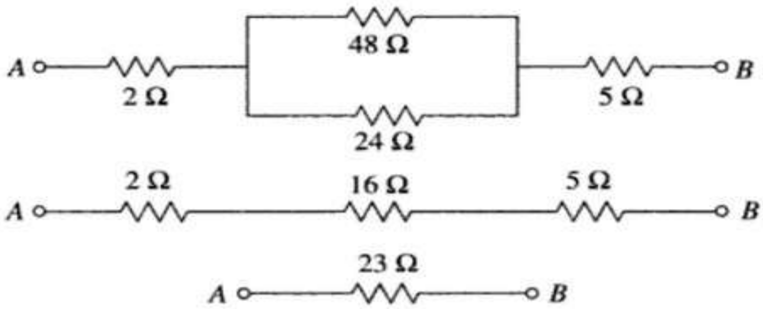
Solution: The resistance of 2Ω and 4Ω and 4Ω and 11Ω are in series.



Converting the two outer deltas into equivalent star networks gives...



The network can be further simplified as...



Therefore $R_{AB} = 23\Omega$

Star–Delta transformation is mainly used to simplify complex resistive networks where direct series or parallel reduction is not possible. It is particularly useful in solving bridge circuits such as the unbalanced Wheatstone bridge, in reducing complex resistive lattices, and in simplifying networks for Thevenin's or Norton's equivalent analysis. The key idea is to convert a star (Y) into a delta (Δ) or vice versa so that the resulting circuit can be reduced easily using ordinary series and parallel rules.