Module ⑥ Numerical Methods

Bisection method :-

Q) find the real root of equation x^3-2x-5 using bisection method in the interval $(2,3)$ 3 iteration

$f(x) = x^3 - 2x - 5$

$f(2) = 2^3 - 2(2) - 5 = -1 < 0$ (-ve)

$f(3) = 3^3 - 2(3) - 5 = 16 > 0$ (+ve)

$f(x) = 0$ has roots in the interval $(\overset{a}{2}, \overset{b}{3})$

Iteration	First	2nd	3rd	
a (-ve)	2	2	2	
b (+ve)	3	2.5	2.25	
formula	$-\frac{a+b}{2} = \frac{2+3}{2}$ $= 2.5 = x_1$	$x_2 = \frac{2+2.25}{2}$ $= 2.25$	$\boxed{x_3 = 2.125}$	
F(x)	$f(2.5)$ $= x^3 - 2x - 5$ $= (2.5)^3 - 2(2.5) - 5$ $= 5.625$ (+ve)	$f(2.25)$ $= 1.890625$		

Matrices

Types of matrices & operations

$$\begin{array}{ccc} C_1 & C_2 & \\ \left[\begin{matrix} 2 & 3 \\ 4 & -1 \end{matrix}\right] & & \begin{matrix} R_1 \\ R_2 \end{matrix} \end{array} \qquad \begin{bmatrix} 1 & 5 \\ 3 & 2 \end{bmatrix} \qquad \begin{bmatrix} 2 & 3 \\ 4 & 1 \\ 6 & 9 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 & 3 \\ 4 & -3 & 2 \\ 0 & 1 & 2 \end{bmatrix}$$

2×2 order of Matrix 2×2 3×2 3×3

$$\begin{bmatrix} 2 & 3 & -1 \\ -5 & 4 & -2 \\ 1 & 4 & 2 \end{bmatrix} \; 3 \times 3$$

Types of matrices:-

① Row & column matrix

$A = [1, 3, 5]$

$$A = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$$

① Square Matrix

nos of Rows = nos of columns

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2}$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 3 \\ -1 & 2 & 3 \end{bmatrix}$$

③ Diagonal Elements

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 4 \\ 2 & -1 & 3 \end{bmatrix}$$

④ Diagonal matrix

Non diagonal element must be 0

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -5 \end{bmatrix}$$

⑤ Trace of an matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 0 & 5 \\ 6 & 4 & 3 \end{bmatrix} \Rightarrow 4$$

Sum of diagonal elements

⑥ Determinant of an square matrix

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{bmatrix}$$

$$|A| = \begin{array}{ccc} + & - & + \end{array}$$
$$\begin{vmatrix} 2 & 3 & 1 \\ 1 & 2 & 3 \\ 3 & 1 & 2 \end{vmatrix}$$

$$2(4-3) - 3(2-9) + 1(1-6)$$

$$\Rightarrow 2 - 3(-7) + -5$$

$$\Rightarrow 2 + 21 - 5 = 18$$

⑦ Singular matrix

$$\boxed{|A| = 0}$$

⑧ Non Singular matrix

$$\boxed{|A| \neq 0}$$

⑨ Zero/ null matrix

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

⑩ unit matrix or identity matrix

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$$

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}_{3\times 3}$$

⑪ Scalar matrix

$$A = \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$$

diagonal element must be same

⑫ Transpose of matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 9 \end{bmatrix}$$

$$A^T = A' = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$

⑬ Triangular matrices

upper

$$A = \begin{bmatrix} 1 & 5 & 6 \\ 0 & 2 & 9 \\ 0 & 0 & 9 \end{bmatrix}$$

lower

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 6 & 6 & 9 \end{bmatrix}$$

⑭ Conjugate of a matrix

$$A = \begin{bmatrix} 2 & i \\ 2+i & 5-i \end{bmatrix}$$

$$z = 3+5i$$

$$\bar{z} = 3-5i$$

$$\bar{A} = \begin{bmatrix} 2 & -i \\ 2-i & 5+i \end{bmatrix}$$

15. Transposed Conjugate of matrix

$$A = \begin{bmatrix} 2 & 3+i \\ 3+6i & 9i \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 2 & 3-i \\ 3-6i & -9i \end{bmatrix} \Rightarrow \bar{A}^T = \begin{bmatrix} 2 & 3-6i \\ 3-i & -9i \end{bmatrix}$$

$$\boxed{(\bar{A})^T = A^\theta}$$

$$A = \begin{bmatrix} 5i & 6i \\ 3 & 2 \end{bmatrix} \quad \text{find } A^\theta = ?$$

$$\bar{A} = \begin{bmatrix} -5i & -6i \\ 3 & 2 \end{bmatrix} = A^\theta = (\bar{A})' = \begin{bmatrix} -5i & 3 \\ -6i & 2 \end{bmatrix}$$

16. Skew Symmetric & Symmetric matrix -

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 1 & 4 & 6 \\ 5 & 6 & 9 \end{bmatrix} \qquad A^T = \begin{bmatrix} 1 & 1 & 5 \\ 3 & 4 & 6 \\ 5 & 6 & 9 \end{bmatrix}$$

Symmetric matrix $\boxed{A^T = A}$

Skew Symm matrix

$$A = \begin{bmatrix} 0 & 5 & -4 \\ -5 & 0 & 3 \\ 4 & -3 & 0 \end{bmatrix}$$

$$\boxed{A = -A^T}$$

$$A^T = \begin{bmatrix} 0 & -5 & 4 \\ 5 & 0 & -3 \\ -4 & 3 & 0 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 0 & 5 & -4 \\ -5 & 0 & 3 \\ 4 & -3 & 0 \end{bmatrix} \qquad \boxed{A^T = -A}$$

(17) Hermitian & Skew hermition

$$\boxed{A^\theta = A} \qquad \boxed{A^\theta = -A}$$

Operations on Matrices –

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 3 & 6 \\ 2 & 8 \end{bmatrix}$$

$$A = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 5 \\ 0 & 0 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} 2 & 0 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} 1 & 5 \\ 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} R_1 \times C_1 & R_1 \times C_2 \\ R_2 \times C_1 & R_2 C_2 \end{bmatrix}$$

$$= \begin{bmatrix} 2+0 & 10+0 \\ 1+0 & 5+0 \end{bmatrix} - \begin{bmatrix} 2 & 10 \\ 1 & 5 \end{bmatrix}$$

Orthogonal Matrices

orthogonal matrix,

$$\boxed{A A^T = I}$$

$A = [\ \ ]$

$A^T = [\ \ ]$

Rows → Columns

$A A^T = [\ \ ][\ \ ] = I$

to find $A^{-1} = ?$

$$\boxed{A^T = A^{-1}}$$

If orthogonal matrix then is true

Q1) PT that the following matrix is orthogonal hence find A^{-1}

$$A = \frac{1}{3}\begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix}$$

$$\boxed{A^T = \frac{1}{3}\begin{bmatrix} -2 & 2 & 1 \\ 1 & 2 & -2 \\ 2 & 1 & 2 \end{bmatrix} = A^{-1}}$$

consider $A A^T = \frac{1}{9}\begin{bmatrix} -2 & 1 & 2 \\ 2 & 2 & 1 \\ 1 & -2 & 2 \end{bmatrix}\begin{bmatrix} -2 & 2 & 1 \\ 1 & 2 & -2 \\ 2 & 1 & 2 \end{bmatrix}$

$$= \frac{1}{9}\begin{bmatrix} 4+1+4 & -4+2+2 & -2-2+4 \\ -4+2+2 & 4+4+1 & 2-4+2 \\ -2-2+4 & 2-4+2 & 1+4+4 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

- Matrix A is orthogonal

Q2) if $A = \begin{bmatrix} 1/3 & 2/3 & a \\ 2/3 & 1/3 & b \\ 2/3 & -2/3 & c \end{bmatrix}$ is orthogonal find a, b, c

$$A = \begin{bmatrix} 1/3 & 2/3 & a \\ 2/3 & 1/3 & b \\ 2/3 & -2/3 & c \end{bmatrix}$$

A is orthogonal matrix $AA^T = I$

$$\begin{bmatrix} 1/3 & 2/3 & a \\ 2/3 & 1/3 & b \\ 2/3 & -2/3 & c \end{bmatrix}\begin{bmatrix} 1/3 & 2/3 & 2/3 \\ 2/3 & 1/3 & -2/3 \\ a & b & c \end{bmatrix}$$

$$= \begin{bmatrix} \overset{a_{11}}{1} & 0 & 0 \\ 0 & \overset{a_{22}}{1} & 0 \\ 0 & 0 & \underset{a_{33}}{1} \end{bmatrix}$$

$$\frac{1}{9} + \frac{4}{9} + a^2 = 1 \quad , \quad \frac{4}{9} + \frac{1}{9} + b^2 = 1 \quad , \quad \frac{4}{9} + \frac{4}{9} + c^2 = 1$$

$$\frac{5}{9} + a^2 = 1 \qquad b^2 = \sqrt{\frac{4}{9}} \qquad \frac{8}{9} + c^2 = 1$$

$$a^2 = 1 - \frac{5}{9} \qquad \boxed{b = \pm \frac{2}{3}} \qquad c^2 = \frac{1}{9}$$

$$\boxed{a = \pm \frac{2}{3}} \qquad \boxed{c = \pm \frac{1}{3}}$$

Q3) find a, b, c when

$$A = \begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix}$$ is orthogonal

Matrix A is orthogonal

$$A A^T = I$$

$$\begin{bmatrix} 0 & 2b & c \\ a & b & -c \\ a & -b & c \end{bmatrix} \begin{bmatrix} 0 & a & a \\ 2b & b & -b \\ c & -c & c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 4b^2 + c^2 & 2b^2 - c^2 & -2b^2 + c^2 \\ 2b^2 - c^2 & a^2 + b^2 + c^2 & a^2 - b^2 - c^2 \\ -2b^2 + c^2 & a^2 - b^2 - c^2 & a^2 + b^2 + c^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$2 \times \frac{1}{6} = c^2$$

$4b^2 + c^2 = 1$ — I $\qquad a^2 + b^2 + c^2 = 1$ — III

$2b^2 - c^2 = 0$ — II

$$\boxed{b = \pm \frac{1}{\sqrt{6}}}$$

$$2\left(\frac{1}{\sqrt{6}}\right) - c^2 = 0 \qquad \boxed{c = \pm \frac{1}{\sqrt{3}}}$$

Unitary Matrix

Q.) TPT the matrix $\frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary

$\boxed{AA^{\theta} = I}$ → Primary Condition for unitary matrix

$\boxed{A^{-1} = A^{\theta}}$

let $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$

$\bar{A} = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1-i \\ 1+i & -1 \end{bmatrix}$ $\quad (\bar{A})^T = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix} = A^{\theta}$

Consider $AA^{\theta} \Rightarrow \frac{1}{3}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$

$$\Rightarrow \frac{1}{3}\begin{bmatrix} 1+(1+i)(1-i) & (1+i) - (1+i) \\ (1-i)-(1-i) & (1-i)(1+i)+1 \end{bmatrix}$$

$$\Rightarrow \frac{1}{3}\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\boxed{AA^{\theta} = I}$

∴ A is an unitary matrix

Q2) Show that the matrix $A = \frac{1}{2}\begin{bmatrix} \sqrt{2} & -i\sqrt{2} & 0 \\ i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$ is unitary & find A^{-1}

$$\bar{A} = \begin{bmatrix} \sqrt{2} & +i\sqrt{2} & 0 \\ -i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$(\bar{A})^T = \frac{1}{2}\begin{bmatrix} \sqrt{2} & -i\sqrt{2} & 0 \\ i\sqrt{2} & \sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix} = A^\theta$$

Consider $A A^\theta$

$$= \frac{1}{4}\begin{bmatrix} \sqrt{2} & -i\sqrt{2} & 0 \\ i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix} \times \begin{bmatrix} \sqrt{2} & -i\sqrt{2} & 0 \\ i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \frac{1}{4}\begin{bmatrix} 2-2i^2 & -2i+2i & 0 \\ 2i-2i & -2i^2+2 & 0 \\ 0 & 0 & 4 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A A^\theta$$

$\boxed{A A^\theta = I}$ $\therefore$ A is an unitary matrix

$$A^{-1} = A^{\theta} = \frac{1}{2}\begin{bmatrix} \sqrt{2} & -i\sqrt{2} & 0 \\ i\sqrt{2} & -\sqrt{2} & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

lec (04) Rank of matrix

Normal form :-

Q1) Reduce the following matrix to normal form & find it's rank

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$R_2 \leftrightarrow R_1$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 2 & 3 & -1 & -1 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

$R_2 - 2R_1, \quad R_3 - 3R_1, \quad R_4 - 6R_1$

$$\begin{bmatrix} 1 & -1 & -2 & -4 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 12 \end{bmatrix}$$

$C_3 + C_1, \quad C_3 + 2C_1, \quad C_4 + 4C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 5 & 3 & 7 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix} \Rightarrow$$

$R_2 / 5$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 3/5 & 7/5 \\ 0 & 4 & 9 & 10 \\ 0 & 9 & 12 & 17 \end{bmatrix}$$

$R_3 - 4R_2, \quad R_4 - 9R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 3/5 & 7/5 \\ 0 & 0 & 33/5 & 22/5 \\ 0 & 0 & 33/5 & 22/5 \end{bmatrix} \Rightarrow$$

$C_3 - 3/5\, C_2 \qquad C_4 - 7/5\, C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 33/5 & 22/5 \\ 0 & 0 & 33/5 & 22/5 \end{bmatrix}$$

$R_3 / 33/5$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2/3 \\ 0 & 0 & 33/5 & 22/5 \end{bmatrix}$$

$R_4 - 33/5\, R_3$

$C_4 - 2/3\, C_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2/3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\boxed{\text{Rank of an matrix } 3}$

Lecture (05) Echlon form

only Row transformation no column transformation

$$\begin{bmatrix} a & b & c \\ 4 & b & 3 \\ 5 & a & 2 \end{bmatrix} \longrightarrow \text{Convert}$$

Rank = nos of non zero Part

$$\begin{bmatrix} 1 & 4 & 3 \\ 0 & 0 & 1 \\ 2 & 0 & 3 \end{bmatrix} \quad \text{rank} = 3$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{bmatrix} \quad \text{rank} = 2$$

$\Downarrow$

nos of non zero Part

Q.1) Reduce the following to echlon form & find it's rank

$$\begin{bmatrix} 1 & 2 & 3 & -1 \\ -2 & -1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_2 + 2R_1$, $R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 3 & 3 & -3 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_2 / 3$

$$\Rightarrow \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & -2 & -2 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

$R_3 + 2R_2$, $R_4 - R_2$

$$\begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank of an matrix $= 2$

Lec 06 Rank of a matrix &

Reduction of Matrix A to normal form

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 4 & 6 & 9 \\ 3 & 2 & 1 \end{bmatrix}$$

$\boxed{A = I_m A I_n}$

I_m - nos of rows
I_n = nos of column

$\boxed{A = I_3 A I_3}$

Q.1) If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$, find 2 matrices P & Q such that PAQ are in normal form

$A = I_3 A I_3$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 - 1R_1$, $R_3 - 3R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & -2 \\ 0 & -2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} \times A \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Normal form convert $C_1 \to 0$ then $C_2 \to 0$, $C_3 \to 0$

$C_2 - 1C_1$, $C_3 - 1C_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & -2 \\ 0 & -2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_2 / -2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & -1/2 & 0 \\ -3 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$R_3 + 2R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 1/2 & -1/2 & 0 \\ -2 & -1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -1 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$C_3 - 1C_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \underset{P}{\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & -1/2 & 0 \\ -2 & -1 & 1 \end{bmatrix}} A \underset{Q}{\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}}$$

Lec 06 Rank of a matrix & Reduction of Matrix A to normal form

Part 2

Q1) find non singular mtrices P & Q $A = \begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$

$A = I_3 A I_4$

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_2 - 2R_1 \;,\; R_3 - R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 0 & -1 & -1 & -3 \\ 0 & 1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_2 - 2C_1 \;,\; C_3 - 3C_1 \;,\; C_4 - 2C_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & -1 & -3 \\ 0 & 1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -2 & -3 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_2 \times (-1)$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 3 \\ 0 & 1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -2 & -3 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$R_3 - R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ -3 & 1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -2 & -3 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$C_3 - C_2 \;,\; C_4 - 3C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & -1 & 0 \\ -3 & 1 & 1 \end{bmatrix} A \begin{bmatrix} 1 & -2 & -1 & 4 \\ 0 & 1 & -1 & -3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Non Homogenous equation

Condition of consistency

$\boxed{Ax = B}$ (only Row transformation)

→ convert A to echlon form always

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 9 \\ 6 & 7 & 8 \end{bmatrix}$$

0 0 0

basically lower △gular matrix =c

Same sign (−)

opp sign (+)

Non homogenous :-

$x + y + z = 2$

$\_\_\_\_\_\_ = 3$

$\_\_\_\_\_\_ = -2$

$\_\_\_\_\_\_ = 6$

if thes is =0 then homogenous

$Ax = B$

$$\underbrace{\begin{bmatrix} 1 & 5 & 9 \\ 0 & 5 & 2 \\ 0 & 0 & 0 \end{bmatrix}}_{\text{rank} = 2} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}}_{\text{no of unknown} = 3} = \begin{bmatrix} 2 \\ 3 \\ 5 \end{bmatrix}$$

Rank $[A|B]$ ← augmented matrix

$$= \left[\begin{array}{ccc|c} 1 & 5 & 9 & 2 \\ 0 & 5 & 2 & 3 \\ 0 & 0 & 0 & 5 \end{array}\right]$$

Rank of AR = 3

Q) Test for Consistency & Solve

$x + y + z = 6$

$x - y + 2z = 5$

$3x + y + z = 8$

$2x - 2y + 3z = 7$

$Ax = B$

$$\begin{array}{c} \\ 0 \leftarrow \\ 0 \leftarrow \\ 0 \leftarrow \end{array} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ 3 & 1 & 1 \\ 2 & -2 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ 8 \\ 7 \end{bmatrix}$$

$R_2 - R_1$, $R_3 - 3R_1$, $R_4 - 2R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 1 \\ 0 & -2 & -2 \\ 0 & -4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \\ -10 \\ -5 \end{bmatrix}$$

$2R_3 - 2R_2$, $12R_4 - 2R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \\ -9 \\ -3 \end{bmatrix}$$

$3R_4 - R_3$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ -1 \\ 9 \\ 0 \end{bmatrix}$$

Rank of $A = r = 3$

Rank of $AB = 3$

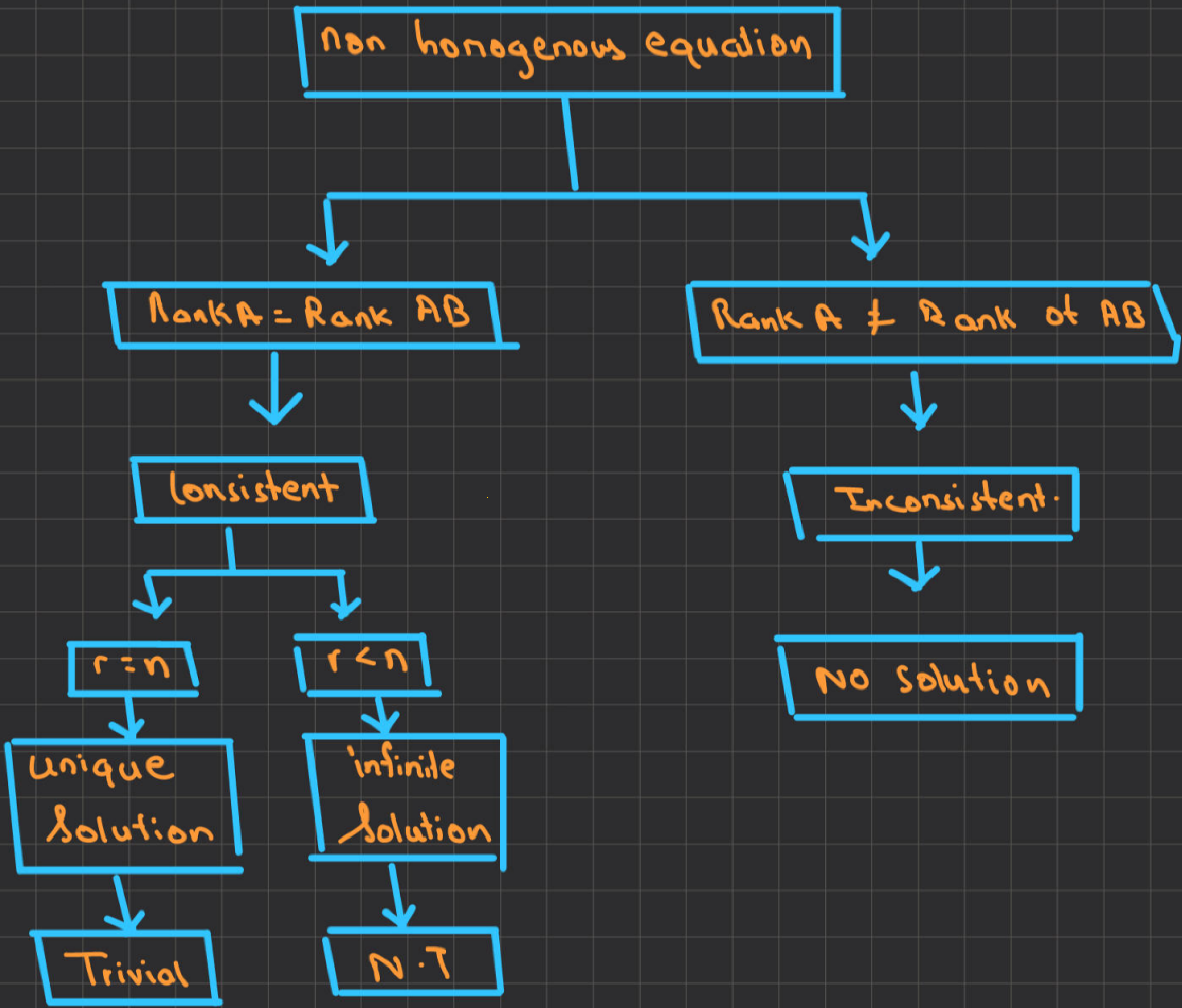
} Rank of A = Rank of AB
equations are consistent

$\boxed{r = n}$ (unique solution)

$x + y + z = 6$ | $-2y + z = -1$ | $-3z = -9$

$\boxed{x = 1}$ | $\boxed{y = 2}$ | $\boxed{z = 3}$

Condition of Consistency



Q) Investigate for what values of λ & is the equations

$x+y+z=6$, $x+2y+3z=10$, $x+2y+\lambda z=\mu$ have

i) No solution, ii) a unique solution iii) ∞ nos of solution

$Ax = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \\ 1 & 2 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 10 \\ \mu \end{bmatrix}$$

$R_2 - R_1$, $R_3 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & \lambda-1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ \mu-6 \end{bmatrix}$$

$R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & \lambda-3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ \mu-10 \end{bmatrix}$$

Augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & \mu-10 \end{array}\right]$$

$\boxed{\text{Rank A = Rank AB}}$

unique solution

$\lambda \neq 3$ $\quad$ $\mu =$ Any values

No solution

$\boxed{\text{Rank A} \neq \text{Rank AB}}$

$\lambda = 3$ $\quad$ $\mu \neq 10$

∞ Many Solution

$\boxed{\text{① Shoud be Consistent}}$

$\boxed{\text{② } r < n}$

$\lambda = 3$, $\mu = 10$

Created by Notein

Homogenous equation

$\underline{\qquad\qquad\qquad} = 0$

$\underline{\qquad\qquad\qquad} = 0$

$\underline{\qquad\qquad} = 0$

Q1) Solve $x_1 + x_2 - x_3 + x_4 = 0$, $x_1 - x_2 + 2x_3 - x_4 = 0$,

$3x_1 + x_2 + x_4 = 0$

$Ax = B$

$$\begin{bmatrix} 1 & -1 & -1 & 1 \\ 1 & -1 & 2 & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 - R_1 \quad R_3 - 3R_1$

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & -2 & 3 & -2 \\ 0 & -2 & 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 - R_2$

$$\begin{bmatrix} 1 & 1 & -1 & 1 \\ 0 & -2 & 3 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

rank of $A = 2$

nos of unknown $= 6$

$r < n$ Infinitely many Solution.

Parameter $= n - r = 4 - 2 = 2$

$x_1 - x_2 - x_3 + x_4 = 0$

let $x_4 = t_1$, $x_3 = t_2$

$-2x_2 + 3t_2 - 2t_1 = 0$

$-2x_2 = 2t_1 - 3t_2$

$-2x_2 + 3x_3 - 2x_4 = 0$

$x_2 = \frac{2t_1}{-2} - \frac{3t_2}{-2}$

$$\boxed{x_2 = -t_1 + \frac{3}{2}t_2}$$

$x_1 + x_2 - x_3 + x_4 = 0$

$x_1 + \left(-t_1 + \frac{3}{2}t_2\right) - t_2 - t_1 = 0$

$x_1 - t_1 + \frac{3}{2}t_2 - t_2 + t_1 : 0$

$x_1 + \frac{1}{2}t_2 = 0$

$$\boxed{x_1 = -\frac{1}{2}t_2}$$