PARALLEL CIRCUITS, MESH ANALYSIS AND NODAL ANALYSIS

[PARALLEL CIRCUITS, KIRCHOFF'S CURRENT LAW (KCL), CURRENT DIVIDER RULE AND, LADDER NETWORKS]

APRIL 16, 2020

SAGE-MICHAEL BROWN

CST_122003.202020.Electric Circuits I Laboratory

Table of Contents

Table of Figures

Objectives

The objective of this lab is to investigate parallel circuit configurations to determine equivalent resistance of parallel resistors, application of Kirchhoff's Current Law (KCL) and the current divider rule (CDR). Similar to the voltage divider rule (VDR) in series circuits, there exists a current divider rule (CDR) for parallel circuits that can be used as a short-cut to obtain branch currents in the circuit without having to know the voltage value as necessary with Ohm's Law.

Parts List

DMM (Digital Multimeter), DC Power Supply, Breadboard, Multisim Software, 1kΩ, 2.2kΩ, 3.3kΩ, 4.7kΩ

General Understanding

a) What is Kirchhoff's Current Law (KCL)?

Kirchhoff's Current Law defines that the total current within a live circuit will stay constant as it travels in a closed loop, also while in a parallel network it discusses the spread of the current to each branch and the factor controlling the amount from the total is determined by the amount of resistance in the circuit. The following equation is the basis to show how it works:

$$I_1 = I_t - I_2$$

Assuming the Voltage and resistance value is known is known for each branch.

$$I_1 = (V_T/R_T) - (V_{I1}/R_{I1})$$

With this it can be shown that the increase in resistance will cause the decrease in current as the final quotient from both of the sub equations of the formula above.

b) What is the current divider rule (CDR)?

This rule states that the current is shared as a ratio of the total current using the same method as the voltage divider rule but instead of it being over different components of the circuit, it is over different branches of the circuit dividing in a portion over the branches with the high resistive branches getting the least amount of current and the less resistive getting the most current in the closed loop.

$$I_1 = \frac{R_1 * I_T}{{R_T}'}$$

Circuit 1: Parallel Circuits, KCL, Current Divider

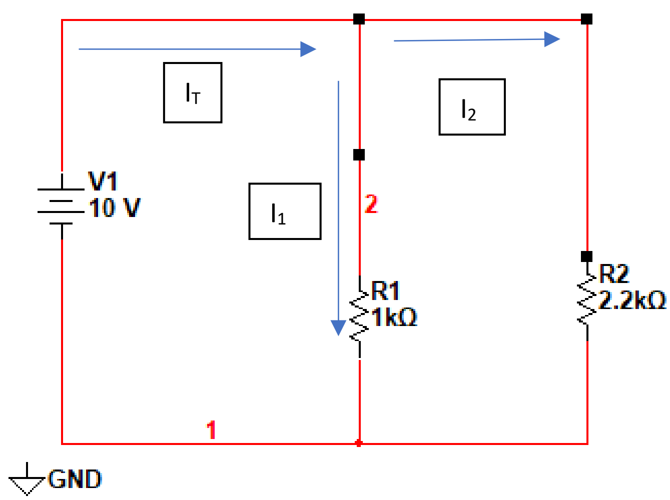

Figure 1: Circuit 1 Parallel Circuit Without Voltmeter and Ammeter

The circuit displayed for this part of the lab is a standard parallel circuit, with a parallel circuit the Voltage stays constant in each branch assuming that there is no component before the circuit has a junction with a single wire as a input and 2 or more wires as the output, causing a split in voltage but a constant supply of voltage. This also includes the assumption that no current objecting items such as diodes or insulators are within the circuit causing a opposition to the flow of the current.

With this is also the fact that the increase of resistance in each network causes a decrease in current across the network, so the higher a resistor voltage the lower the current input and output for that branch. This is the opposite of how voltage would function within a series circuit with the voltage constantly decreasing over each item in the circuit until the remaining voltage is 0 volts and current staying constant, but the higher the resistor value the greater the voltage requirement from the source (reference Figure 2: Proof of series circuit Current and voltage)

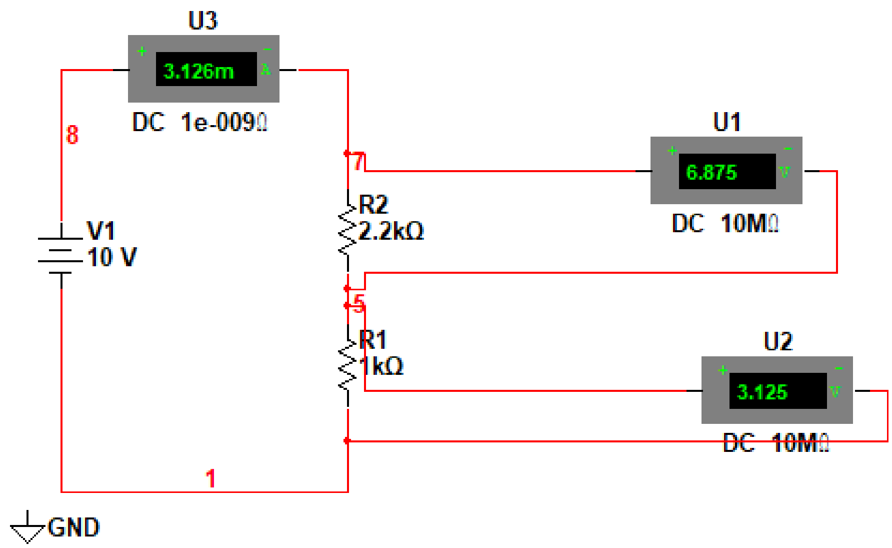

Figure 2: Proof of series circuit Current and voltage

Table 1: Circuit-Parallel Circuit

	Calculated	Multisim	Breadboard	%Error(BB vs Calculated
R_T	687.5Ω	687.5Ω	673.2Ω	2.08000%
I_T	0.014545 A	0.014548A	.01469A	0.99690%
I_1	0.01 A	.01A	0.0101A	0.99000%
I_2	.0045454A	.004546A	0.00459	0.98121%
I_1(CDR)	0.01 A	.01A	0.0101A	0.99000%
I_2(CDR)	.0045454A	.004546A	0.00459	0.98121%
E	10V	10V	9.937V	0.63000%
V_1	10V	10V	9.937V	0.63000%
V_2	10V	10V	9.937V	0.63000%
P_E	.14545W	.14545W	.1460186W	0.39090%
P_1	.1W	.1W	0.1000394W	0.03940%
P_2	0.045454W	0.045454W	.04561W	0.34320%

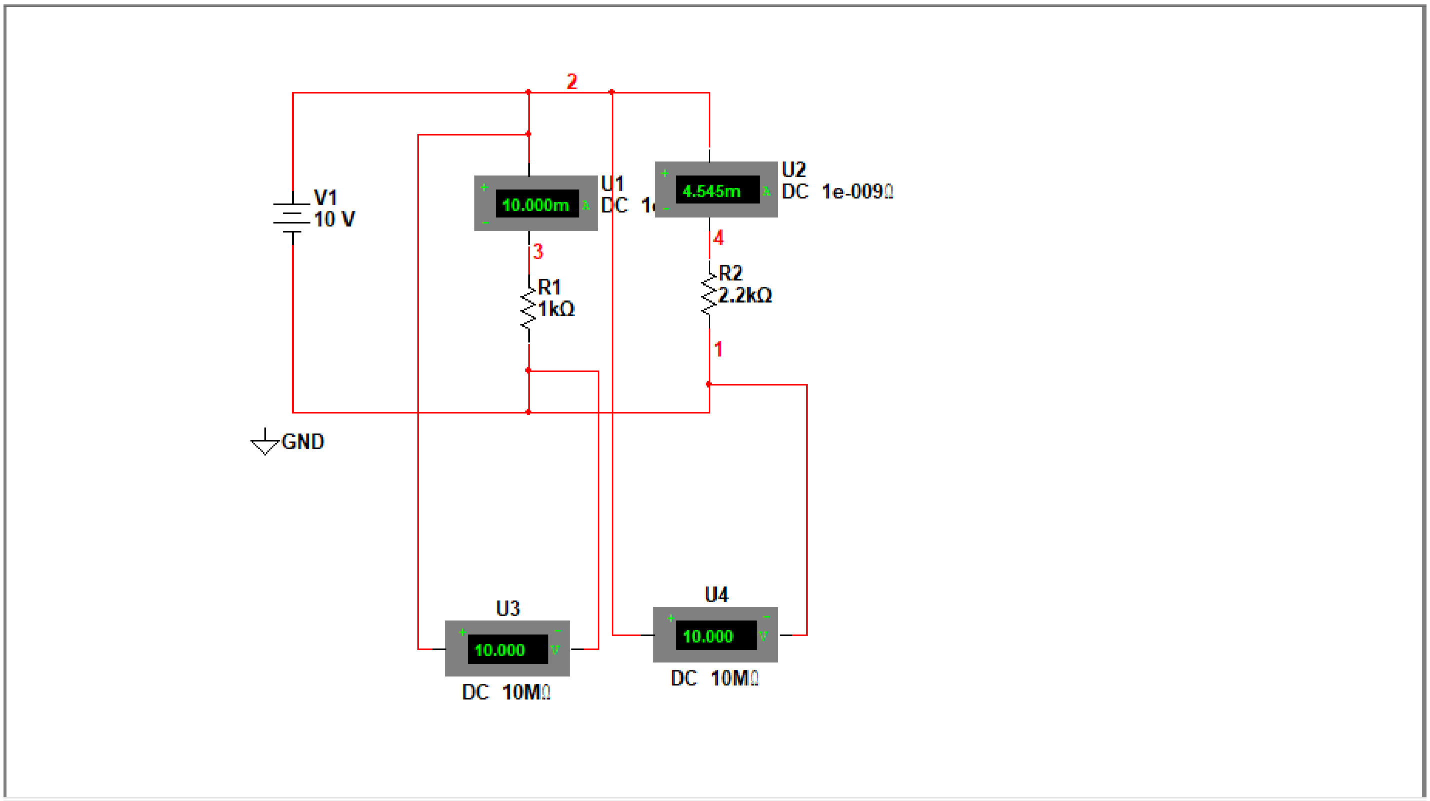

Figure 3: Circuit 1 Parallel Circuit With Ammeters and Voltmeters

Circuit 1 Calculations

First to find R_T in the parallel network is the first step to giving results for *Table 1*

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} =$$

$$R_T = \frac{1}{\frac{1}{1000} + \frac{1}{2200}} =$$

$$R_T = \frac{1}{.001 + .00045455} =$$

$$R_T = \frac{1}{.00145455} = 687.5\Omega$$

Then from this the total current in the circuit can be found using the voltage and resistor values in each branch to get the total current

$$V = IR$$

Since the total current is what is being looked for two equations can be used to find it and they will both eb equal to each other

$$I_T = I_1 + I_2 =$$

$$\frac{V_T}{R_T} = \frac{V_T}{R_1} + \frac{V_T}{R_2} =$$

$$\frac{10V}{687.5\Omega} = \frac{10V}{1000\Omega} + \frac{10V}{2200\Omega} =$$

$$.014545 = .01 + .00454545$$

$$I_T = .014545A$$

$$I_1 = .01A \ and \ \ I_2 = .004545A$$

Using the current divider rule

$$I_1 = \frac{I_T * R_T}{R_1} =$$

$$I_1 = \frac{.014545 * 687.5}{1000} =$$

$$I_1 = .0099996A \ or \ .001A$$

$$I_2 = \frac{I_T * R_T}{R_1} =$$

$$I_2 = \frac{I_T * R_T}{R_1} =$$

$$I_2 = \frac{.014545 * 687.5}{2200} =$$

$$I_2 = .004545A$$

Then finally the total power was calculated for the circuit and using a pair of equivalent equations the value for the power dissipated over both resistors can be calculated

$$P_E = P_2 + P_1 =$$

$$I_T * V_T = (V_T * I_{1)} + (V_T * I_2) =$$

$$.014545 * 10 = (.004545 * 10) + (.01 * 10) =$$

$$.14545 = .04545 + .1 =$$

$$P_T = .14545\text{W}$$

$$P_{R1} = .04545W \; and \; P_{R2} = .1W$$

Circuit 2: Parallel Circuits and KCL

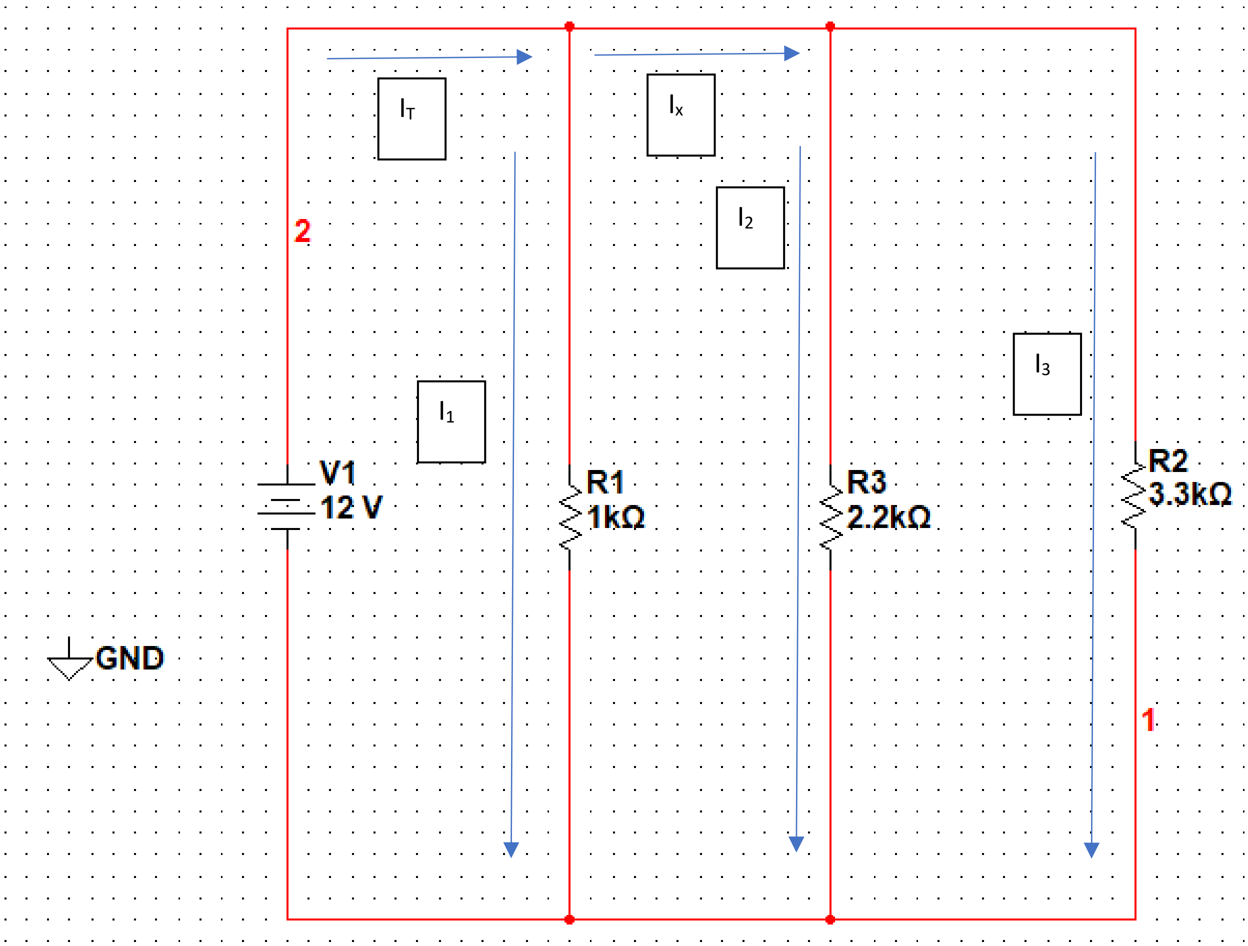

Figure 4: Circuit 2 Parallel Circuit Without Ammeters and Voltmeters

In comparison to the first circuit this one uses more than 3 branches, so equation such as the two resistor equation otherwise known as the product over sum equation (refer to appendix, Equation 4 below)could not be used in this case. So instead of the current splitting once as in the first circuit it spits at two points, with all 3 values not being equivalent to each other because of the differing resistor values on each branch, but the voltage value remaining constant over all branches.

At the point on the negative side of the cell the ingle wire leading to the battery will be the same current value as the total current the circuit started with. The lower the resistor value the greater the current value in the branch.

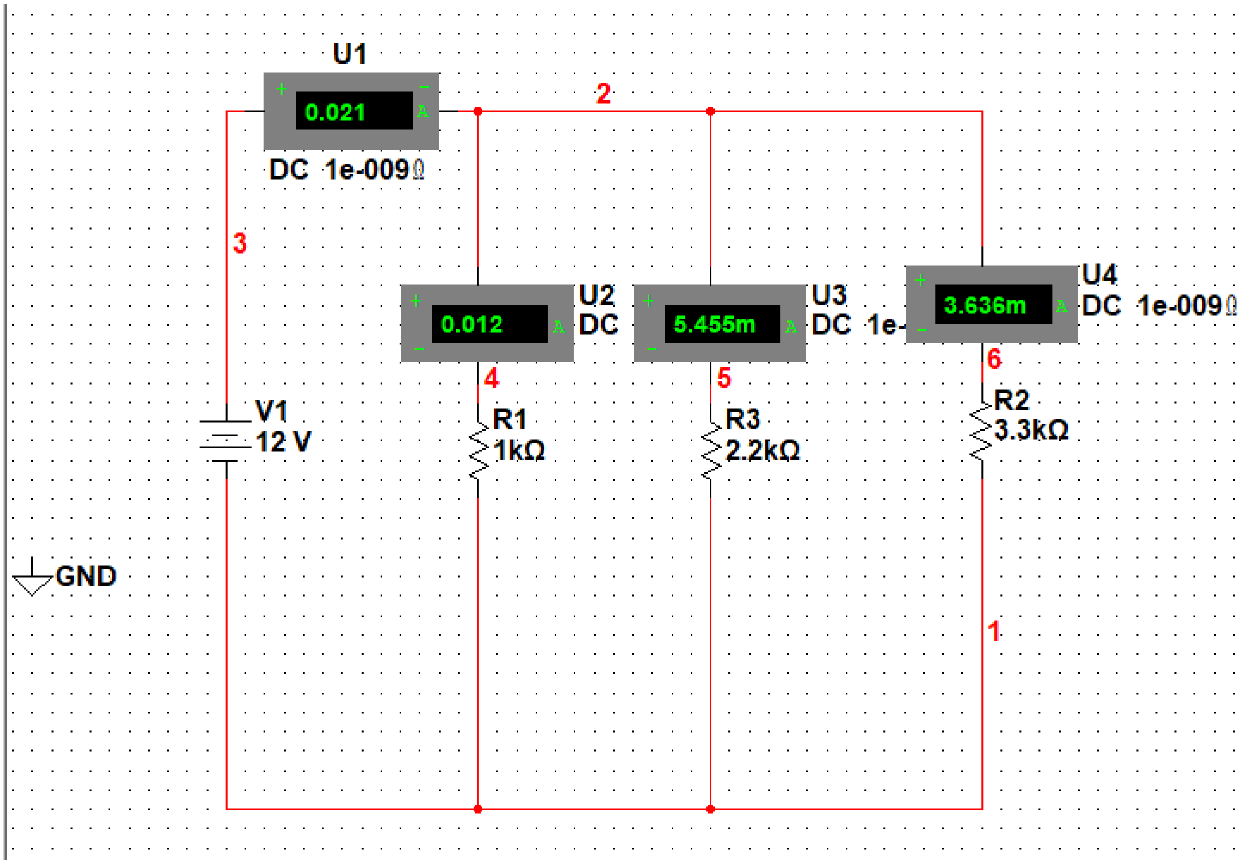

Figure 5: Circuit 1 Parallel Circuit With Ammeters and Voltmeters

Table 2:Circuit II – Parallel Circuit

A1	B	C	D
2		Calculated	Multisim
3	R_T	568.9655Ω	568.9655Ω
4	I_T	0.02109090909A	0.02109090909A
5	I_1	0.01200000000A	0.01200000000A
6	I_2	0.00545454545A	0.00545454545A
7	I_3	0.00363636364A	0.00363636364A
8	I_X	0.00909090909A	0.00909090909A
9	E	12.00V	12.00V
10	V_1	12.00V	12.00V
11	V_2	12.00V	12.00V
12	V_3	12.00V	12.00V
13	P_E	0.25309W	0.25309W
14	P_1	0.14400W	0.14400W
15	P_2	0.06545W	0.06545W
16	P_3	0.04364W	0.04364W

Circuit 2 Calculations

First as with the circuit before the total resistance need to be calculated

$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} =$$

$$R_T = \frac{1}{\frac{1}{1000} + \frac{1}{2200} + \frac{1}{3300}} =$$

$$R_T = \frac{1}{.001 + .00045455 + .00030303} =$$

$$R_T = \frac{1}{.00175758} =$$

$$R_T = 568.964\Omega$$

With this we can find the total current in the circuit and the total current in each branch of the circuit

$$I_T = I_1 + I_2 + I_3$$

$$\frac{V_T}{R_T} = \frac{V_T}{R_1} + \frac{V_T}{R_2} + \frac{V_T}{R_3} =$$

$$\frac{12}{568.964} = \frac{12}{1000} + \frac{12}{2200} + \frac{12}{3300} =$$

$$.02109097 = .012 + .00545455 + .00363636$$

$$I_T = .02109097A$$

$$I_1 = .012A, \quad I_2 = .00545455A \; and, \quad I_3 = .00363636A$$

$$I_x = I_T - I_1 \text{ or } I_x = I_2 + I_3$$

$$I_x = .0210907 - .012$$

$$I_x = .00090907A$$

The power within the circuit is the final calculation of the circuit

$$P_E = P_1 + P_2 + P_3 =$$

$$V_T * I_T = (V_T * I_1) + (V_T * I_2) + (V_T * I_3)$$

$$(12 * .02109097) = (12 * .012) + (12 * .00545455) + (12 * .00363636)$$

$$.25309164 = .144 + .0654546 + .04363632$$

$$P_E = .25309164W$$

$$P_1 = .144, P_2 = .0654546\ and, P_3 = .04363632$$

Circuit 3: Ladder Networks

(Series Parallel – Ladder Networks)

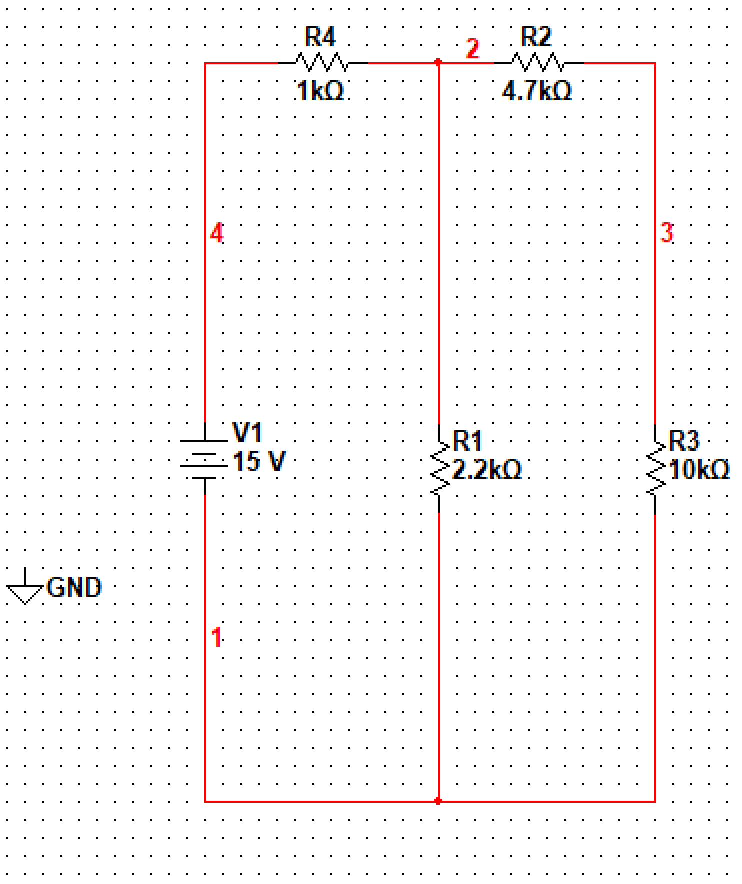

Figure 6: Circuit 3: Series Parallel Circuit without ammeter and voltmeter

The circuit above is a series parallel network with a resistor in series to a parallel network that has a series network within the parallel network. This allows for the control of voltage before it enters the components after the first resistor, that lowers the total voltage before it is able to enter the parallel network. The current for this network stays constant in the first resistor but after that resistor the current splits with a higher value to the lower resistor, being the 2200Ω in comparison to the 4700Ω resistor which is in series with the 10000Ω resistor

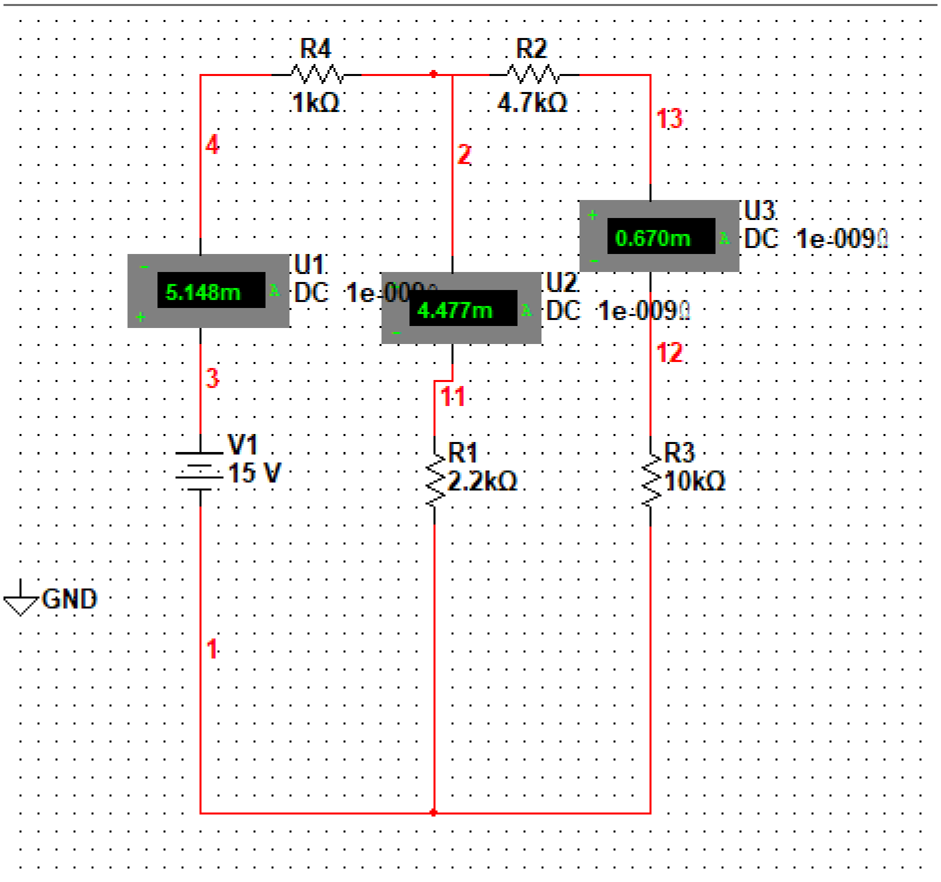

Figure 7:Circuit III: Series Parallel Circuit with voltmeter and ammeter

Table 3: Circuit III - Series Parallel

A1	B	C	
2		Calculated	MultiSim
3	R_T	2913.60947Ω	2913.60947Ω
4	I_T	0.00514825A	0.00514800A
5	I_1	0.00510000A	0.00514800A
6	I_2	0.00447700A	0.00447700A
7	I_3	0.00067000A	0.00067000A
8	I_4	0.00067000A	0.00067000A
9	E	15.00000V	15.00000V
10	V_1	5.14800V	5.14800V
11	V_2	9.85200V	9.85200V
12	V_3	3.15000V	3.15000V
13	V_4	6.70200V	6.70200V
14	P_E	0.07722380W	0.07722380W
15	P_1	0.02625480W	0.02625480W
16	P_2	0.04410740W	0.04410740W
17	P_3	0.00211050W	0.00211050W
18	P_4	0.00449034W	0.00449034W

Circuit 3 Calculations

As the beginning of most if not all the prior circuits, we first find the total resistance in the circuit

R1 is in series with parallel network $R_2|R_{3,4}$

$$R_T = R_1 + \left(\frac{1}{\left(\frac{1}{R_2} + \frac{1}{R_3 + R_4}\right)}\right) =$$

$$R_T = 1000 + \left(\frac{1}{\left(\frac{1}{2200} + \frac{1}{4700 + 10000}\right)}\right) =$$

$$R_T = 1000 + \frac{1}{.00052257} =$$

$$R_T = 1000 + 1913.60947 =$$

$$R_T = 2913.60947\Omega$$

Using this value of R_T, I_T can be found by either using Ohms Law

$$I_T = \frac{V_T}{R_T} =$$

$$I_T = \frac{15}{2913.60947} =$$

$$I_T = .00514825A$$

Using the voltage from the source we can find the amount of voltage used across R_1

$$V_1 = V_T * \frac{R_1}{R_T} =$$

$$V_1 = 15 * \frac{1000}{2913.60947} =$$

$$V_1 = 5.1482V$$

With this the value of R_2 and $R_{3,4}$ for the voltage across both networks since voltage at a junction is the same on all branches which the junctions is attached to

$$V_2 \; and \; V_{3,4} = V_{T} - V_1 =$$

$$V_2 = 15 - 5.1482 =$$

$$V_2 = 9.852V \;\; and \; V_{3,4} = 9.852\text{V}$$

Using this the voltage across the network the voltage for each resistor on the parallel network $R_{3,4}$

$$V_3 = V_{3,4} * \frac{R_3}{R_3 + R_4} =$$

$$V_3 = 9.852 * \frac{4700}{10000 + 4700} =$$

$$V_3 = 9.852 * \frac{4700}{14700} =$$

$$V_3 = 3.15V$$

The voltage for the other resistor can be found through subtracting the voltage of the other resistor that was in series with it

$$V_4 = V_{3,4} - V_3 =$$

$$V_4 = 9.852 - 3.15 =$$

$$V_4 = 6.702V$$

The current across all the resistors can be found with the values given

$$I_T = I_1$$

$$I_1 = .00514825A$$

$$I_2 = \frac{V_2}{R_2} =$$

$$I_2 = \frac{9.852}{2200} =$$

$$I_2 = .00447818A$$

$$I_3 = I_4$$

$$I_{3,4} = \frac{V_{3,4}}{R_{3,4}} =$$

$$I_{3,4} = \frac{9.852}{4700 + 10000} =$$

$$I_{3,4} = \frac{9.852}{14700} =$$

$$I_3 = .0006702A \; and \; I_4 = .00006702A$$

The total power in the circuit is as shown

$$V_T * I_T = (V_1 * I_T) + (V_2 * I_2) + (V_3 * I_3) + (V_4 * I_3)$$

15*.00514825=5.1482*.00514825+.00447818*9.852+3.15*.0006702+6.702*.0006702

$$.07722375 = .02650422 + .04411903 + .00211113 + .00449168$$

Mesh Analysis

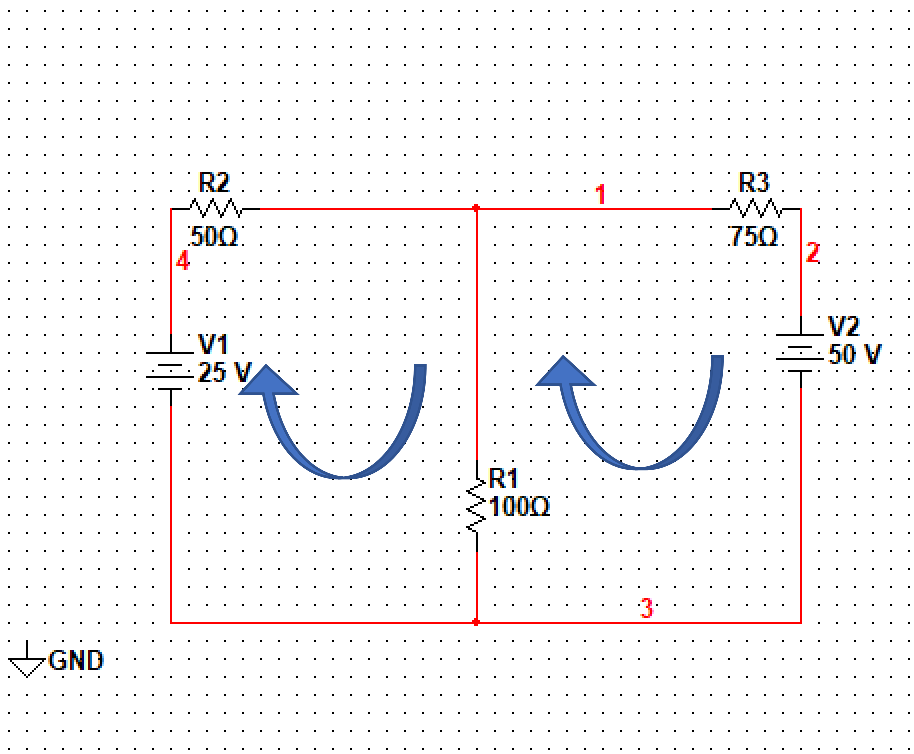

Figure 8: Circuit IV: Series Parallel Circuit with voltmeter and ammeter

For this part of the lab a circuit with more than one voltage sources connected in parallel is constructed with a single branch in-between both the resistors. The voltage along the single middle branch cannot be calculated using the normal method of ohms law and neither can it be calculated using the voltage sources by adding them because they are not in series. To find which way current goes and which direction current enters the branch from is found using the voltage sources connection to the single resistor by putting the values into a matrix to calculate the total current along each branch of the circuit

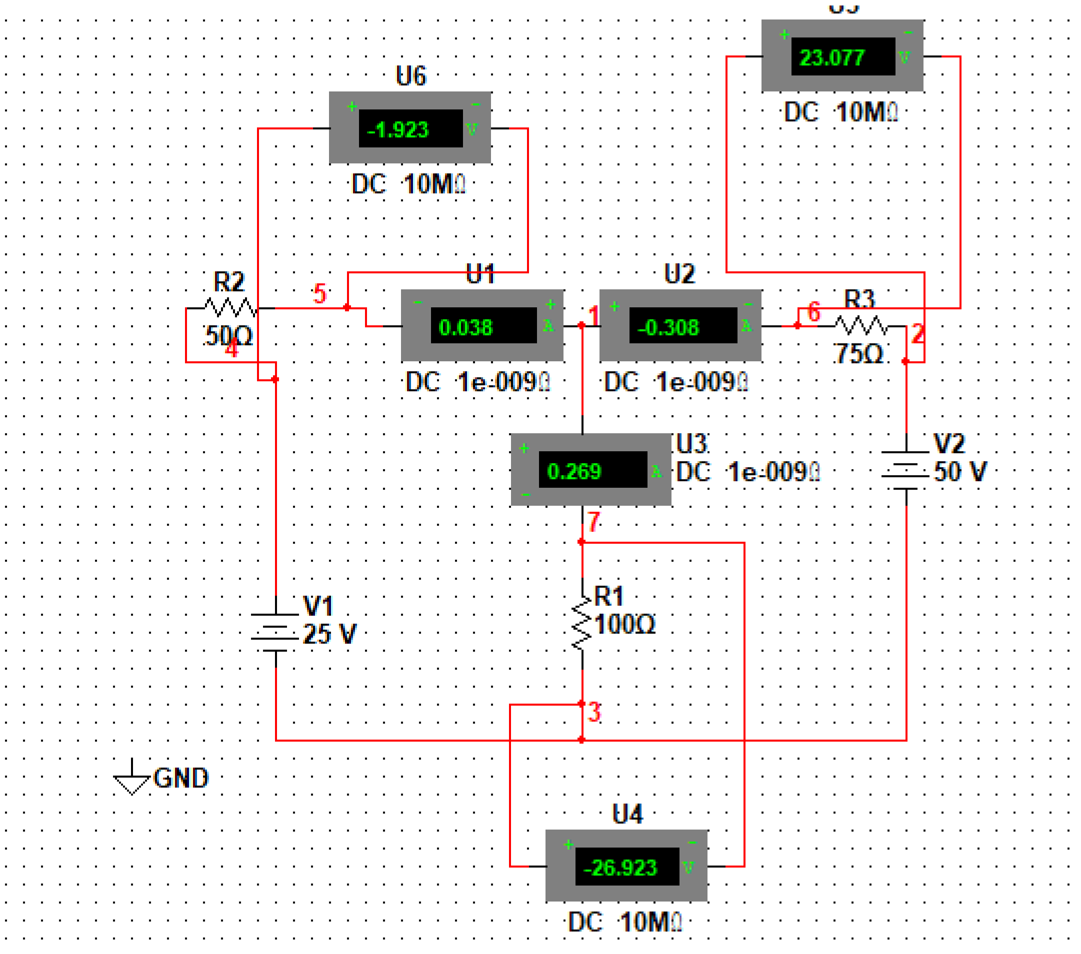

Figure 9: Circuit IV: Series Parallel Circuit without voltmeter and ammeter

Table 4: Circuit IV- Series Parallel

A1	B	C	D
2		Calculated	Multisim
3	Loop I1	25 -100 -50 175	25 -100 -50 175
4	Loo I2	150 -25 -100 50	25 -100 -50 175
5	IR1	.03846154A	0.038A
6	IR2	.30769231A	0.308A
7	IR3	0.26923077A	0.269A
8	V1	1.9V	1.9V
9	V2	23.077V	23.077V
10	V3	26.923V	26.923V

Circuit 4 Calculations

First the following simultaneous equations are made

LOOP I_1

$$I_1(R_1 + R_2) - I_2(R_2) = E_1$$

LOOP I_2

$$-I_1(R_2) + I_2(R_2 + R_3) = -E_2$$

Using these equations the values can be placed into the equations for the resistor and voltage source

LOOP I_1

$$I_1(50 + 100) - I_2(100) = 25$$

LOOP I_2

$$-I_1(100) + I_2(75 + 100) = -50$$

Matrix can be set up using the values put forward using Cramer's rule

$$\begin{bmatrix} 150 & -100 \\ -100 & 175 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{matrix} 25 \\ -50 \end{matrix}$$

$$\text{Det(A)} \begin{bmatrix} 150 & -100 \\ -100 & 175 \end{bmatrix} = 150 * 175 - -100 * -100 = 16250$$

$$Dx = \begin{bmatrix} 25 & -100 \\ -50 & 175 \end{bmatrix} = 25*175 - -100*-50 = -675$$

$$Dy \begin{bmatrix} 150 & 25 \\ -100 & -50 \end{bmatrix} = 150 * -50 - 25 * -100 = -5000$$

$$I_1 = \frac{Dx}{Det(A)}$$

$$I_1 = \frac{-675}{16250}$$

$$I_1 = -.03846154A$$

$$I_2 = \frac{Dy}{Det(A)}$$

$$I_2 = \frac{-5000}{16250}$$

$$I_2 = -.30769231A$$

With his I_3 can be found

$$I_3 = I_2 - I_1$$

$$I_3 = .30769231 - .03846154$$

$$I_3 = .26923077A$$

Now the voltage across each resistor can be calculated

$$V_1 = I_1 * R_1 =$$

$$V_1 = .038461544 * 50$$

$$V_1 = 1.923V$$

$$V_2 = I_2 * R_2$$

$$V_1 = .30769 * 75$$

$$V_1 = 23.077V$$

$$V_3 = I_3 * R_3$$

$$V_1 = .269 * 100$$

$$V_1 = 26.9V$$

Conclusion

To conclude this lab, it has been proven that with different ways in setting up parallel circuits come varying changes in both things that are normally constant and the varying may be constant at different points of the circuit, unlike the series circuit the parallel circuit variation is very abundant. The parallel circuit is also effective based of its ability to maintain voltage with the varying current in each branch. Because of this lights that function in class rooms do not completely go out because of each branch being separate from each other in comparison to a series circuit where it is in a singular loop and once one point in the circuit is broken the whole circuit will stop functioning.

Parallel circuits are also efficient when it comes to alternating current in the case of current flowing in multiple directions both negative and positive from each terminal. With this items such as medical equipment can continuously function while a error occurs. Parallel circuits also aid in the product lines for building items from cars to the smallest of toys, one single item within the product line going out will not cause the whole lien to be interfered with and cause It to come to a absolute halt.

Appendix

Images

Figure 10 Multimeter

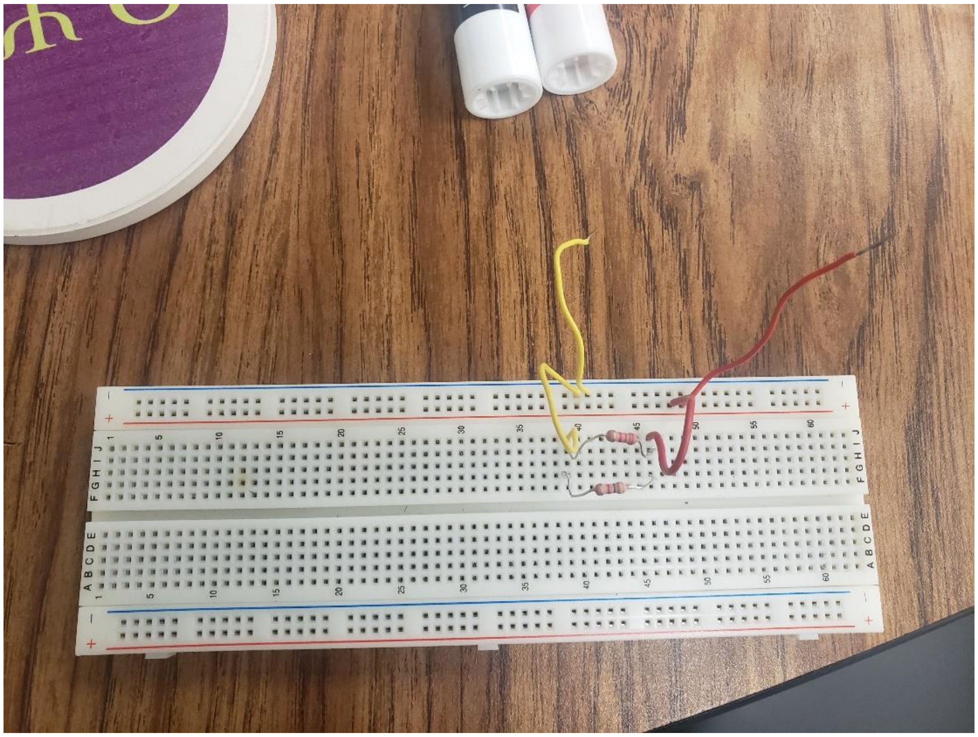

Figure 11 Breadboard

Equations

Equation 1

$$V = IR$$

Equation 2

$$I_T = I_1 + I_2$$

Equation 3

$$R_T = R_1 + R_2 + R_3$$

Equation 4

$$R_T = \frac{R_1 * R_2}{R_2 + R_1}$$

Equation 5

$$Det(A) = \begin{bmatrix} a & b \\ d & c \end{bmatrix} = ab - cd$$

Equation 6

$$V_1 = I_1 * \frac{R_1}{R_T}$$

Equation 7

$$P = VI$$