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Exam 2

In order to submit this exam in ME480 you are required to acknowledge your compliance with these academic honesty requirements by signing the statement below.

- Professor Brown and/or Professor Sabatino are the only people you are able to ask questions regarding the information on the exam.
- Consultation or collaboration with anyone else **are not permissible on take home exams.**
- You **ARE allowed to** consult your Jupyter notebooks, class notes, or other reference material.
- You may NOT consult any person other than your instructors about any aspect of this exam.

By typing your full name below, you acknowledge the following:

I affirm that I have had no conversation regarding this exam with any persons other than the instructors. Further, I certify that the attached work represents my own thinking. Any information, concepts, or words that originate from other sources are cited in accordance with Lafayette College guidelines as published in the Student Handbook. I am aware of the serious consequences that result from improper discussions with others or from the improper citation of work that is not my own.

Type your full name in the cell below to serve as an electronic signature

STUDENT ANSWER: 0 POINTS POSSIBLE

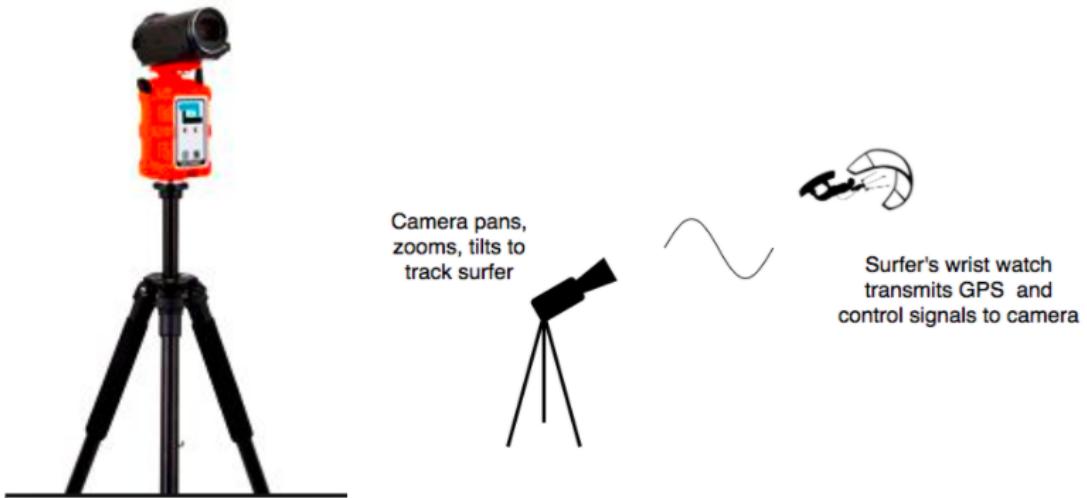
Sage Herz

Insert one image per cell.

You can add Markdown cells if you need to add images.

DO NOT DELETE OR SPLIT CELLS

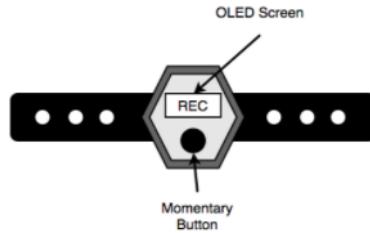
The following three exam problems all have to do with the design of a “robot cameraperson,” which automatically pans, tilts, and zooms to film users equipped with a custom GPS wristlet as they surf, ride horses, or careen around a racetrack. The wristlet controls the overall operation of the device (sets resolution, triggers recording, etc.) and the camera/stand system pans to align with the bearing (yaw) error between the unit and the GPS position reported by the user’s wristlet.



Your company is designing this product to catch up with competitors in this market sector, and is a little late to the game with products like the SoloShot® (shown above) and the Pixio® already in production. Therefore, you've been tasked with sketching out the product's overall operation and taking a first pass at the system's pan (yaw) control system while using as many off-the-shelf components as possible.

Problem 1

Design a state machine system that controls the overall operation of the product. For simplicity and ruggedness, the waterproof GPS wristlet only has one button and a simple OLED display screen.



Design Specifications:

1. In the default, "ready" mode the camera does not record and does not track the wristlet.
2. On a single, unique press and release of the wristlet button while the unit is in the ready mode will cause the system to wait for one second for an additional unique press.
 - A. If a second unique button press is detected, the unit will enter a time-lapse recording and track mode.
 - B. If one second passes without another unique button press the unit will enter video recording and tracking.
3. In either the video record mode or the time-lapse mode, any unique press of the button will exit to ready mode.
4. If video recording for 1 hour the unit will return to ready mode.

You will need to develop a state transition diagram, state transition table, and draft code for a state machine system that satisfies the requirements above.

Variables you may need for your program are provided below. **You may add and name any "intermediate" variables you need to describe states and/or transitions.**

Name	Description
SP0	True on first loop only
BTN	Input describing button state. LOW when not depressed, HIGH when depressed

You will submit a draft program that implements ONLY BLOCK 1, BLOCK 2, and Block 3 of the 4-block code structure for your design. Your code must implement any unique presses on the wristlet button and indicate when and how each timer/counter your system uses is active, but you do not have to code the internal functionality of any timers or counters (e.g. do not write code for how the timer counts or sets its "start time").

As in Exam 1, the operation of the timers and counters is described by the following variables.

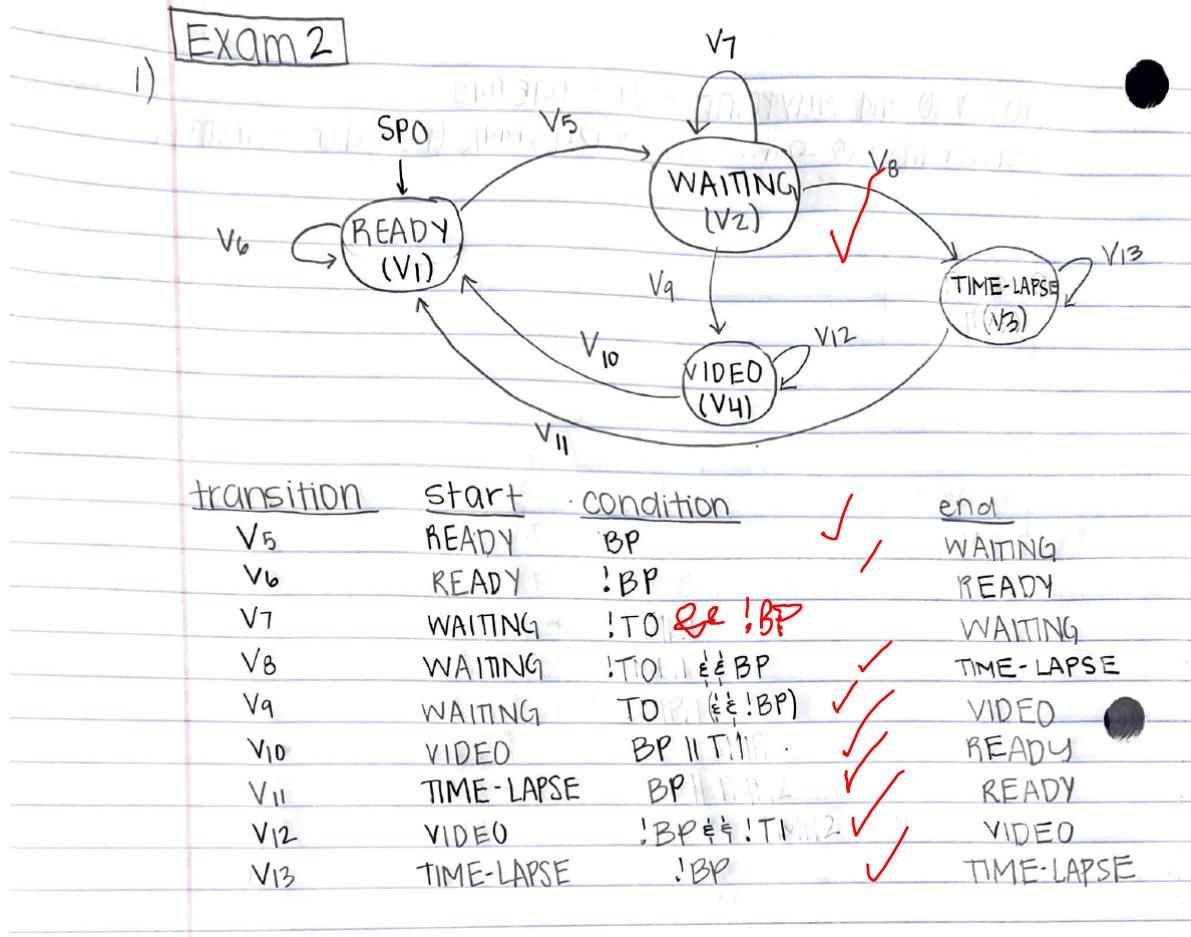
Name	Description
T0_EN,T0,TA0,T0_DUR	Timer 0 variables
T1_EN,T1,TA1,T1_DUR	Timer 1 variables
CT0_UP,CT0_DN,CT0,CT0_RST,CTA0,CT0_CNT	Counter 0 variables

Problem 1 Deliverables

Place your state transition diagram, table, and draft code below. You may add extra markdown cells **below** the one provided if you need to attach multiple images (one cell per image).

State Transition Diagram

STUDENT ANSWER: 10 POINTS POSSIBLE



State Transition Table and Variable Key

STUDENT ANSWER: 10 POINTS POSSIBLE

(State Transition Table is seen above)

Variable Key:

V1 represents the READY state

V2 represents the WAITING state

V3 represents the TIME-LAPSE recording and tracking state

V4 represents the VIDEO recording and tracking state

V5 represents the transition from the READY state to the WAITING state

V6 represents the latch on the READY state

V7 represents the latch on the WAITING state

V8 represents the transition from the WAITING state to the TIME-LAPSE recording state

V9 represents the transition from the WAITING state to the VIDEO recording state

V10 represents the transition from the VIDEO recording state to the READY state

V11 represents the transition from the TIME-LAPSE recording state to the READY state

V12 represents the latch on the VIDEO recording state

V13 represents the latch on the TIME-LAPSE recording state

BP represents a unique button press

TIMELAPSE_REC represents the camera recording in the time-lapse function

VIDEO_REC represents the camera recording in the video function

TRACKING represents the camera tracking the GPS in the wristlet

Draft code of Blocks 1, 2, and 3 ONLY You do not need to declare variables.

You are required to properly format your code by using tickmarks:

...

CODE HERE

...

STUDENT ANSWER: 10 POINTS POSSIBLE

```

//BLOCK 1
BP = BTN&&!BTN_OLD; ✓

//timer 1...
T0_DUR = 1000; //Timer 0 duration in milliseconds (1 second)
T0_EN = V2; ✓

//timer 2...
T1_DUR = 3600000; //Timer 1 duration in milliseconds (1 hour)
T1_EN = V4; ✓

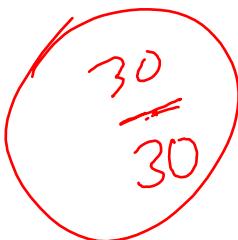
//BLOCK 2
V5 = V1&&BP;
V6 = V1&&!BP;
V7 = V2&&!T0;
V8 = V2&&!T0&&BP;
V9 = V2&&T0;
V10 = V4&&BP || V4&&T1; ✓
V11 = V3&&BP;
V12 = V4&&!BP&&!T1;
V13 = V3&&!BP;

//BLOCK 3
V1 = SP0 || V10 || V11 || V6; ✓
V2 = V5 || V7;
V3 = V8 || V13;
V4 = V9 || V12;

//BLOCK 4
TIMELAPSE_REC = V3;
VIDEO_REC = V4;
TRACKING = V3 || V4;

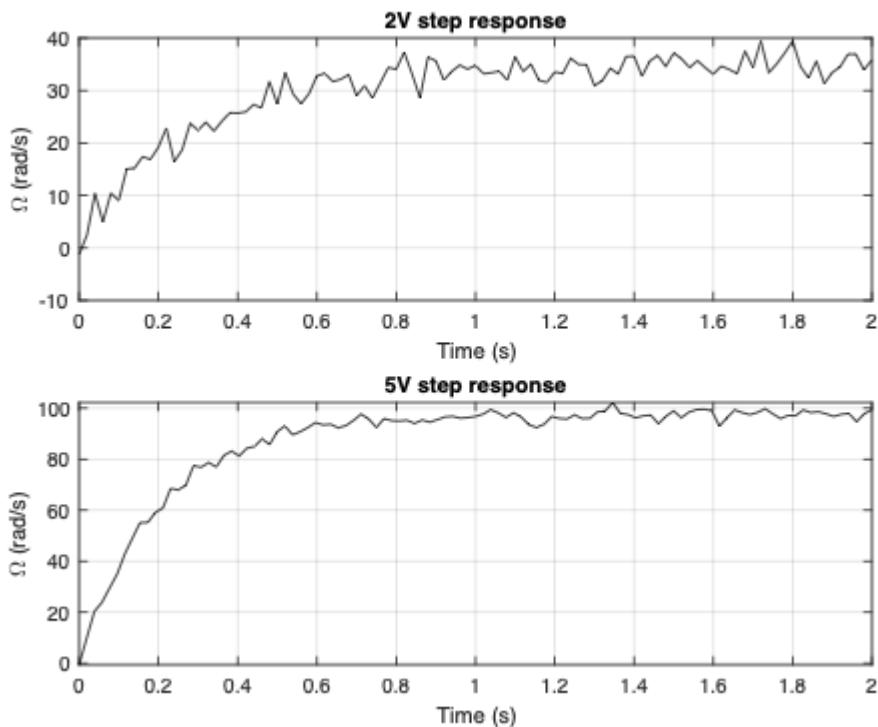
BTN_OLD = BTN;

```



Problem 2

Your company wishes to use a DC electric motor/amplifier for the pan tracking functionality of your robot camera. You decide on a motor, mount it on a prototype camera, and run a step response test on the motor-camera system. The results of this test are shown below. The motor's **angular velocity** (Ω) was measured.



Model Development: Motor-Camera Transfer Function

Clearly identify the input and output of this motor-camera system and produce a single transfer function describing the open loop behavior of the motor-camera system. Show all of your work below.

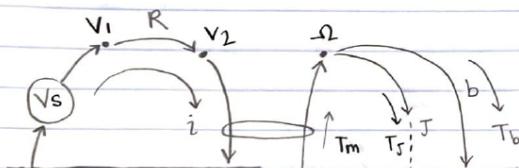
STUDENT ANSWER: 10 POINTS POSSIBLE

The input for the system is the supplied voltage to the motor (V_s) and the output is the angular speed of the motor (Ω).

The system appears to be first order and thus there should only be one independent energy storing element. The energy storing element is the rotational mass of the motor. This element was chosen as the only independent energy storing element in the system as the motor must have rotational inertia in order for us to see an output of the motor angular speed (Ω). There is also a dissipative element which causes the motor to approach a steady state when given an input voltage. This dissipative element is the damping between the motor and ground. This model must have a dissipative element or else the motor would be able to continuously spin when given a step input, making it a perpetual motion machine which is inherently impossible. There is also a transducer that converts between electrical and rotational mechanical energy. There is also a resistor in the electrical circuit between the source voltage and the voltage supplied to the transducer. A resistor is required because the voltage drop across the transducer is proportional to the output angular velocity of the motor. If there were to be no resistor and a theoretical infinite voltage source, there would be no voltage drop, resulting in an angular velocity of zero and infinite current. If there were an infinite current, the motor torque would subsequently also be infinite, and we know this is inherently impossible.

2) I would say this appears to be first order.

Linear Graph



element:

$$① V_{12} = iR$$

$$② K_t i = T_m$$

$$③ V_{2g} = K_t \Omega$$

$$④ T_b = b \Omega$$

$$⑤ T_J = J \ddot{\Omega}$$

node:

$$⑥ T_m = T_J + T_b$$

loop:

$$⑦ V_s = V_{12} + V_{2g}$$

NO PHYSICS - BASIC MODEL
NEED BUT SCOPING GOOD TO SEE

$$\textcircled{c} \quad T_m = T_J + T_b \rightarrow K_t i = J \dot{\Omega} + b \Omega \rightarrow \frac{V_{12}}{R} K_t = J \dot{\Omega} + b \Omega$$

(1)

$$\rightarrow \frac{K_t}{R} (V_s - V_{2g}) = J \dot{\Omega} + b \Omega \rightarrow \frac{K_t}{R} (V_s - K_t \Omega) = J \dot{\Omega} + b \Omega$$

(3)

$$\rightarrow J \dot{\Omega} + \left(b + \frac{K_t^2}{R}\right) \Omega = \frac{K_t}{R} V_s \rightarrow \dot{\Omega} + \left(\frac{K_t^2 + Rb}{RJ}\right) \Omega = \left(\frac{K_t}{RJ}\right) V_s$$

$$\mathcal{L}^{-1} \rightarrow s Y(s) + \left(\frac{K_t^2 + Rb}{RJ}\right) Y(s) = \left(\frac{K_t}{RJ}\right) U(s)$$

$$P(s) = \frac{Y(s)}{U(s)} = \frac{\frac{K_t}{RJ}}{s + \left(\frac{K_t^2 + Rb}{RJ}\right)} \cdot \frac{RJ}{RJ} \rightarrow P(s) = \boxed{\frac{K_t}{RJS + (K_t^2 + Rb)}}$$

*New A
Numerical T.F.*

Since we do not have datasheets, the model will subsequently be evaluated numerically using a general 1st order transfer function form.

$$P(s) = \frac{A}{s+a}$$

↳ find a from time constant (using 5V data)

$$P+a=0 \rightarrow P=-a \rightarrow T = -1/P = -1/(-a) = 1/a$$

$$T \approx 0.25s \therefore a = 4 \quad \text{du}$$

↳ find A from FVT...

$$\Omega_{ss} = \lim_{s \rightarrow 0} Y(s) = \lim_{s \rightarrow 0} s \cdot \frac{U(s)}{s+a} \cdot \frac{A}{s+a} \Rightarrow \Omega_{ss} = \frac{VsA}{a}$$

$$\Omega_{ss} \approx 35 \text{ for } Vs = 2V \quad \text{rad/s} \rightarrow A = 70$$

$$\therefore P(s) = \frac{70}{s+4}$$

Model Validation: Motor-Camera Transfer Function

Based on the tests above, evaluate your model and develop an expectation for how it will perform as part of a controller design. Specifically address:

- Does the **system** appear to be linear? If so, how do you know? If not, how do you know?
- How well do you expect the transfer function model you derived to match the motor's real behavior? Why?

STUDENT ANSWER: 10 POINTS POSSIBLE

This is ~~Answers~~
NOT TRUE -- evidence?
(-1) Dumb? B1M1?

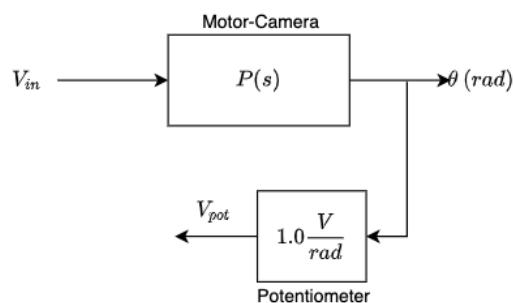
I'm concerned about the linearity of this model as it appears to have a significantly different time constant between the 2V input to the 5V input, where the time constant for the 5V input appears to be much smaller (faster) than the time constant for the 2V test.

Since there were no datasheets available, the numerical transfer function model was based off of rough estimates of the time constants as well as the steady state angular velocities for both tests.

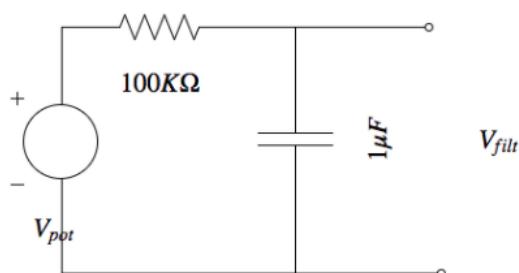
WHAT ABOUT SSGs? (-1)

Model Development: Filter and Measurement

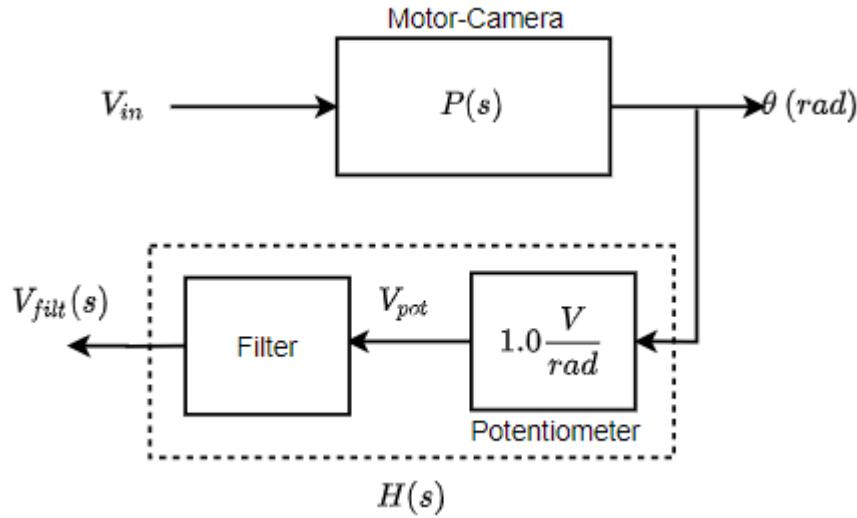
To track the wristlet, the closed loop controller will need to control **ANGULAR POSITION (\$\theta\$)** of the motor-camera system. In this case the angular position of the motor shaft will be read by a potentiometer. The potentiometer translates the shaft position in radians into a voltage. This can be represented by the following block diagram.



Your boss warns you that noise on V_{pot} might cause the camera pan system to shake as it tracks the GPS wristlet, so she suggests that you use your company's standard RC filter to smooth the potentiometer voltage before it is read by the controller. The RC filter you choose has the following schematic:



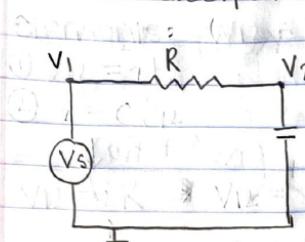
Derive a single transfer function H to represent the transformation from shaft angle θ to filtered voltage V_{filt} . Considering the potentiometer itself and the RC circuit above, the transfer function H could be represented in the following partial block diagram.



Finding $H(s)$ will require you to derive a physics-based model for the RC filter circuit you will use to smooth your potentiometer readings. Develop a physics-based model below, and present your final answer as a transfer function.

STUDENT ANSWER: 10 POINTS POSSIBLE

Filter → one energy storing element (capacitor) and one dissipative element (resistor)



$$\text{elements: } \begin{aligned} ① \quad & V_{12} = iR \\ ② \quad & i = C\dot{V}_{2g} \\ ③ \quad & V_s = V_{12} + V_{2g} \end{aligned}$$

$$① \quad V_{12} = iR \rightarrow V_{12} = RC\dot{V}_{2g} \rightarrow V_s - V_{2g} = RC\dot{V}_{2g}$$

$$V_s = V_{pot} \text{ and } V_{2g} = V_{filt} \rightarrow V_{pot} = RC\dot{V}_{filt} + V_{filt}$$

$$\rightarrow \dot{V}_{filt} + \left(\frac{1}{RC}\right)V_{filt} = \left(\frac{1}{RC}\right)V_{pot}$$

$$\mathcal{L}^{-1}[\cdot] \rightarrow sY(s) + \left(\frac{1}{RC}\right)Y(s) = \frac{1}{RC}U(s)$$

$$F(s) = \frac{V_{filt}(s)}{V_{pot}(s)} = \frac{\frac{1}{RC}}{s + \frac{1}{RC}} \cdot \frac{RC}{RC} \Rightarrow F(s) = \frac{1}{RCs + 1}$$

$$\begin{array}{c} V_{in} \rightarrow P(s) \xrightarrow{\theta} H \\ | \qquad | \\ V_{filt} \leftarrow \frac{1}{RCs+1} V_{pot} \xleftarrow[1.0 \frac{V}{rad}]{} \end{array}$$

$$H = \left(1.0 \frac{V}{rad}\right) \left(\frac{1}{RCs+1}\right)$$

$$H = \frac{1}{RCs+1}$$

$$H = \frac{1}{(100 \times 10^3 \Omega)(1 \times 10^{-6} F)s + 1}$$

$$\rightarrow H = \frac{1}{0.1s + 1} \rightarrow H = \frac{10}{s + 10}$$

28
30

Problem 3

Design a P, PD, or PI **ANGULAR POSITION** controller for the pan (yaw) tracking system on the robot camera to achieve smooth tracking of the GPS wristlet angle relative to the camera. Your controller should be able to achieve zero steady-state error for a unit step.

Use the DC motor transfer function P you derived using the block diagram from problem 2, and the sensor dynamics H you derived in problem 2. Assume that the math required to convert the wristlet position to a desired angle for your motor is already done, and that your controller has access to this desired angle θ_d for use in your controller software.

To keep the tracking smooth, like a skilled human camera operator, you wish to achieve a natural frequency of **no more than** 1.0Hz, with a damping ratio of 0.707.

Problem 3 Deliverables

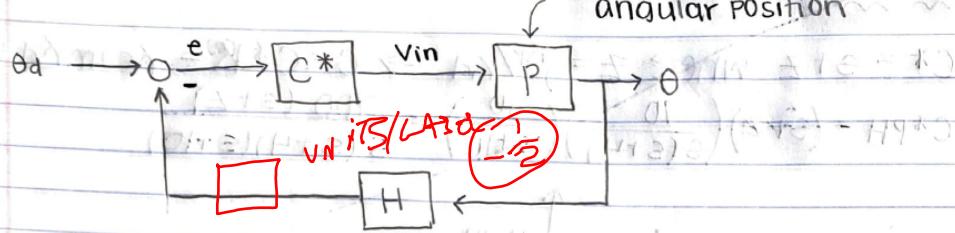
You may add more code or markdown cells below if needed.

Design a P,PD, or PI controller, and provide:

1. A block diagram of your control system, with all signals and blocks labeled.
2. A desired eigenvalue location for your control system.
3. A set of gains K_p , K_i , and K_d (as applicable) for your chosen controller. If you design your controller with a summing gain “ K_{sum} ,” assume that it is 1. **Use the Angle Deficiency Method** for full credit.
4. A **hand sketch** of the root locus for the system under control by your chosen controller. Show all of your hand calculations. **Do not** precisely compute any breakaway points if applicable. If applicable, you **do not** have to compute angles of departure or arrival. If the system ever goes unstable, you **do not need to** find the critical gain or crossover frequency. An Octave root locus plot can be provided, but it will not earn full credit.

STUDENT ANSWER: 25 POINTS POSSIBLE

3) Block Diagram:



note that this P is the plant TF for angular position

Plant TF for angular position:

$$P = \frac{\theta(s)}{V(s)} = \frac{\omega(s)}{V(s)} \cdot \frac{1}{s} = \frac{70}{s+4} \cdot \frac{1}{s} = \frac{70}{s(s+4)}$$

integrating ✓

the desired eigenvalue of the system can be expressed by

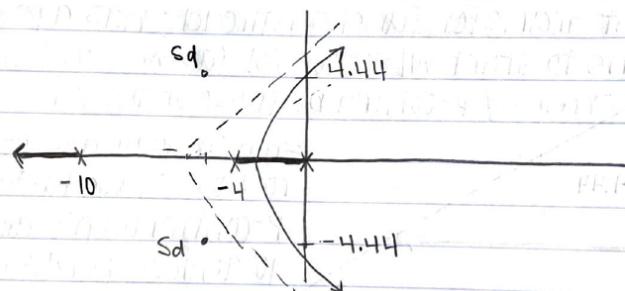
$$\sigma_d = -\zeta \omega_n \pm \omega_n \sqrt{1-\zeta^2} j \text{ where } \zeta = 0.707 \text{ and } \omega_n = 1.0 \text{ Hz}$$

$$\omega_n = 1.0 \frac{\text{rev}}{\text{s}} \times \frac{2\pi \text{ rad}}{1 \text{ rev}} \Rightarrow \omega_n = 2\pi \text{ rad/s}$$

$$\sigma_d = -(0.707)(2\pi) \pm (2\pi)(\sqrt{1-(0.707)^2})j \rightarrow \sigma_d = -4.44 \pm 4.44j$$

Start w/ P-control:

$$GH = \left(\frac{70}{s(s+4)} \right) \left(\frac{10}{s+10} \right) = \frac{700}{s(s+4)(s+10)}$$



$$a = n-m = 3-0 = 3$$

$$\theta_a = \frac{2k+1}{n-m} \pi \rightarrow \theta_a = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

$$\sigma_a = \frac{\sum \text{poles} - \sum \text{zeros}}{n-m} = \frac{0 + (-4) + (-10) - 0}{3-0} = -\frac{14}{3}$$

We need to bend RL to the left → PD control...

PD CONTROL: adding only a zero

$$C^* = s + z \text{ where } z = K_p/K_d \rightarrow C = C^*K = K_{sum}K_d(s+z)$$

$$C^*PH = (s+z) \left(\frac{70}{s(s+4)} \right) \left(\frac{10}{s+10} \right) = \frac{700(s+z)}{s(s+4)(s+10)}$$

using angle deficiency... $z = 4.4951$ (see code cell below)

```
In [1]: s = tf('s');

P = 70/(s*(s+4)); % plant transfer function for ang. position
H = 10/(s+10);
zeta = 0.707;
wn = 2*pi;
% desired eigenvalues...
sd1 = -zeta*wn + wn*sqrt(1-zeta^2)*j;
sd2 = -zeta*wn - wn*sqrt(1-zeta^2)*j;

% finding zero added from PD control using angle deficiency...
angle_p1 = atan2d(wn*sqrt(1-zeta^2), -zeta*wn); % angle from pole at origin
angle_p2 = atan2d(wn*sqrt(1-zeta^2), -zeta*wn+4); % angle from pole at target
angle_p3 = atan2d(wn*sqrt(1-zeta^2), -zeta*wn+10); % angle from pole at target
angle_z = -180+angle_p1+angle_p2+angle_p3; % angle from zero to target
d = (wn*sqrt(1-zeta^2))/(tand(angle_z));
z = zeta*wn + d % zero

P_PD = P*H*(s+z) % transfer function under PD control

% finding the gain (K) using magnitude criterion...
sd = -sd2;
Pmag_sd = abs((700*(sd+z))/(sd*(sd+4)*(sd+10)));
K = 1/Pmag_sd;
Ksum = 1.0;
Kp = K/Ksum;
Kd = Kp*z
z = 4.4951
```

SWITCHED!

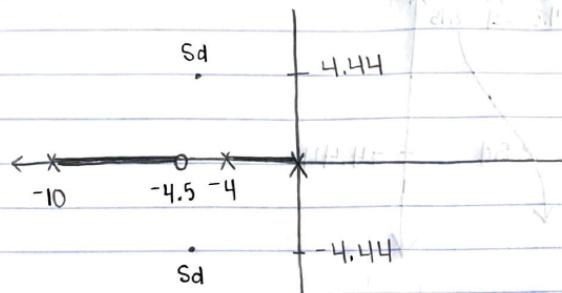
Transfer function 'P_PD' from input 'u1' to output ...

$$y1: \frac{700s + 3147}{s^3 + 14s^2 + 40s}$$

Continuous-time model.

$$\begin{aligned} K_p &= 0.12964 \\ K_d &= 0.58275 \end{aligned}$$

Sketching RL under PD control...



$a = n - m = 3 - 1 = 2$ asymptotes...

$$\theta_a = \frac{2k+1}{n-m}\pi \rightarrow \theta_a = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\sigma_a = \frac{\sum \text{poles} - \sum \text{zeros}}{n-m} = \frac{(0 + (-4) + (-10)) - (-4.4951)}{3-1} = -4.75$$

$$\frac{d}{ds} GH = 0 \rightarrow \frac{d}{ds} \left(\frac{700(s+4.4951)}{s(s+4)(s+10)} \right) = 0$$

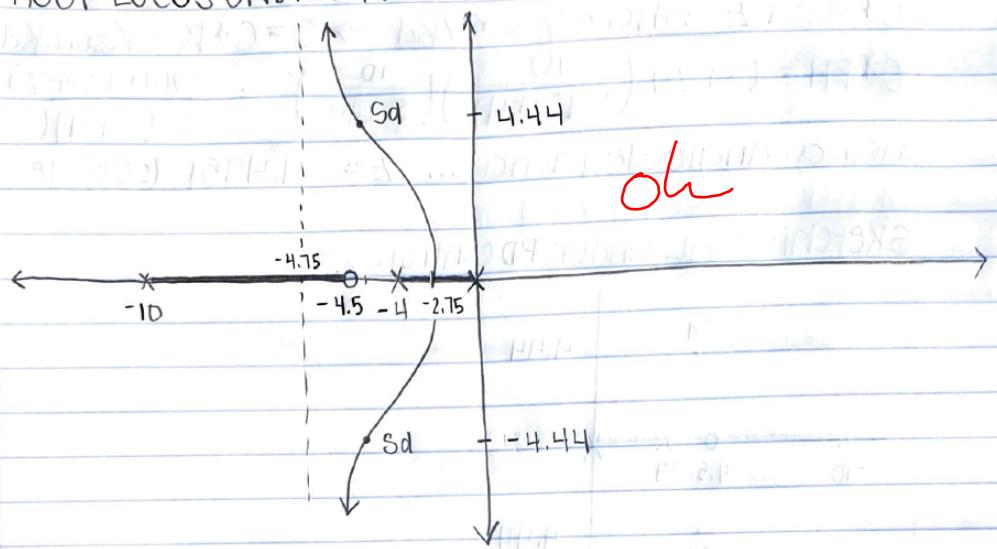
$$1400(s^3 + 13.7427s^2 + 62.9314s + 89.902) = 0$$

$$s^3 + 13.7427s^2 + 62.9314s + 89.902 = 0$$

$$s = -2.75$$

↖ breakaway / break-in point
oh

ROOT LOCUS UNDER PD CONTROL



Below, provide A short discussion of how changing the RC filter's time constant could influence your controller design, assuming that **your design goals do not change, but your gains can**. Specifically considerin the root locus, estimate *qualitatively* how your gain would have to change to achieve the same desired eigenvalue location. You do not have to do any numerical calculations, but use specific course concepts to make your case.

STUDENT ANSWER: 5 POINTS POSSIBLE

Currently, the RC filter's time constant is the smallest, thus it is faster and is not the dominant branch. If the time constant of the RC filter were to increase, thus moving the pole closer to the origin, it will effectively bend the root locus to the right. If the pole at -10 were to move more to the right (corresponding with the RC filter's time constant increasing and becoming slower), the angle that this pole to the desired eigenvalue would create would be larger than previously. This would add more "negative" when doing the angle deficiency method to find the angle that the zero makes with the desired eigenvalue as the angle deficiency formula is: $\angle GH = \angle(\text{zeros} \rightarrow s_d) - \angle(\text{poles} \rightarrow s_d)$. Therefore, the angle the zero would have to make with the desired eigenvalue would have to be larger. Making the angle that the zero makes with the desired eigenvalue larger would require moving the zero closer to the origin. Subsequently this would make the proportion gain larger, as the closed loop gain would increase because the magnitude of the transfer function evaluated at this desired eigenvalue would decrease ($\frac{1}{|GH|} = |GH|_{s=s_d}$). The derivative gain would decrease as the zero is getting smaller.

These patterns would flip if the time constant of the RC filter were decreasing or becoming faster.

In []:

more complex than this
since z moves too but on

