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Exercise: Due MONDAY, 10/19/2020 before class

Controller Design: PI Controller using Angle Deficiency

Having developed a plant transfer function for the **MOTOR-SPRING-LOAD** configuration in Lab 5, let us assume that with the **RIGID SHAFT** installed instead of the flexible spring-shaft, the motor's open loop behavior is adequately represented by the **FIRST ORDER PART** of the **MOTOR-SPRING-LOAD** model you derived. If you make this assumption, you can design a PI controller on velocity for the lab rig in the **MOTOR-RIGID SHAFT- LOAD** configuration.

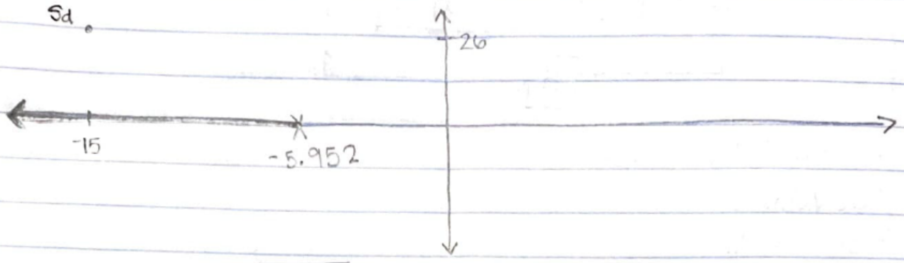
Using the angle deficiency method for controller design, design a controller such that the closed loop system's dominant eigenvalue pair is underdamped with $\zeta = 0.5$ and $\omega_n = 30 rad/s$. In this case, if you like, you could also check your solutions by designing the controller using the "direct method" because the closed loop system is only second order.

- Place your hand calculations in the Markdown cell below using typeset equations or a picture (small file size please! Screenshots work best).
- Place MATLAB code in the Octave cell below that shows a simulation of your system's closed-loop step response for a 25 rad/s step change in requested velocity. **Also plot a simulation of the predicted voltage u(s) sent from the Arduino to the PWMA**. Indicate whether you think this design is feasible with your lab hardware. If it is feasible, what is the maximum step size you'd recommend to ensure that a linear simulation would represent actual system behavior? If it is not feasible, why not?

STUDENT ANSWER: 10 POINTS POSSIBLE

Week 9 wednesday

$GH = \frac{66.64}{s+5.952}$ → no zeros
pole at $s=-5.952$



$S_d = -\zeta\omega_n + \omega_n\sqrt{1-\zeta^2}\,j = -(0.5)(30) + \sqrt{1-(0.5)^2}\,j = -15+26j$

w/ PI control...

$GH_{CL} = \frac{66.64(s+z)}{s(s+5.952)}$ where $z = \frac{K_i}{K_p}$

$\angle GH|_{s=s_d} = \alpha_z - \angle p_1 - \angle p_2 = \alpha_z - \tan^{-1}\left(\frac{26}{-15}\right) - \tan^{-1}\left(\frac{26}{-9.048}\right) = \pm 180$

$\rightarrow \alpha_z - 119.9816° - 109.1879 = \pm 180 \rightarrow \alpha_z = 49.1695°$
(zero to left of Sd)

$\tan(\alpha_z) = \frac{h}{d} \rightarrow d = \frac{h}{\tan(\alpha_z)} = \frac{26}{\tan(49.1695°)} = 22.45$

$z = 15 + 22.45 \Rightarrow \boxed{z = 37.5 \text{ rad/s}}$

$|GH|_{s=s_d} = \left|\frac{66.64(-15+26j+37.5)}{(-15+26j)(-15+26j+5.952)}\right|$

$= \left|\frac{1496.1+1731.4j}{-593.3-624.8j}\right| = \frac{\sqrt{(1496.1)^2+(1731.4)^2}}{\sqrt{(-593.6)^2+(-624.8)^2}}$

$= 2.77$

$K = K_{sum}K_p = 1/2.77 = 0.36$

assuming $K_{sum} = 1$ ... $\boxed{K_p = 0.36}$

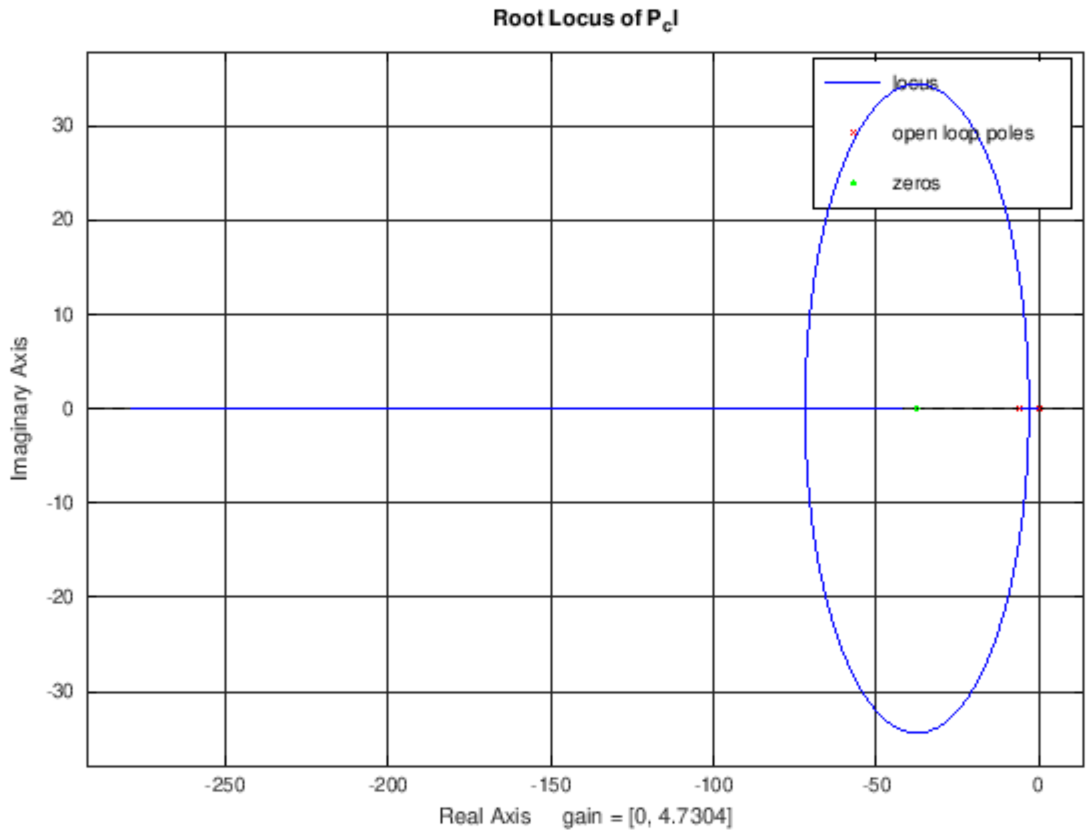
$\frac{K_i}{K_p} = z \rightarrow \boxed{K_i = 13.5}$

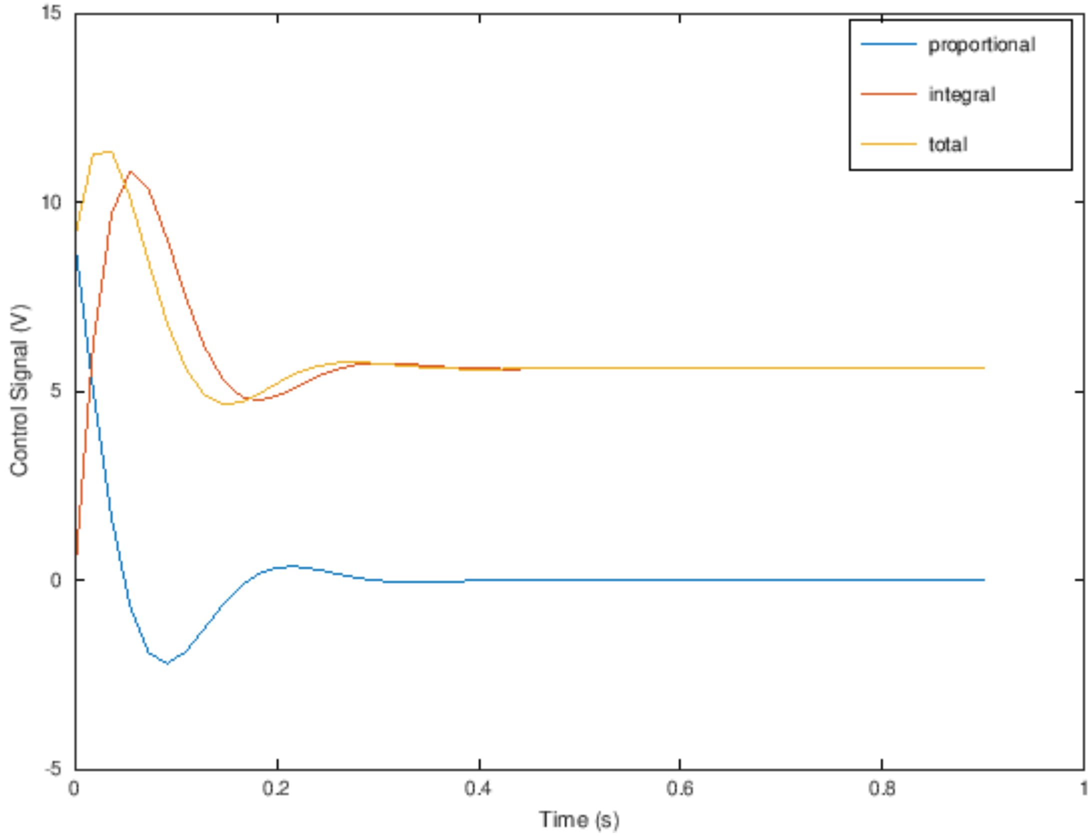
STUDENT ANSWER: 10 POINTS POSSIBLE

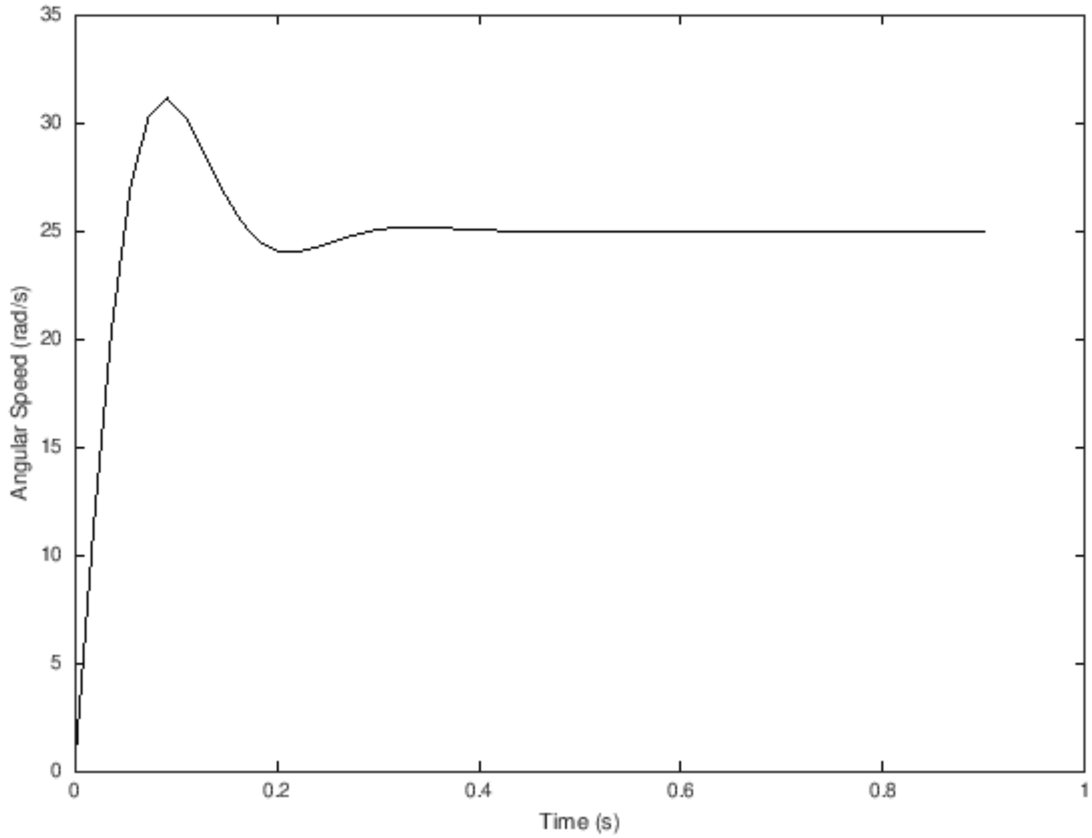
In […

```
%%octave
s = tf('s');
H = 1;
P = 66.64/(s+5.952);
Ksum = 1;
Ki = 13.508;
Kp = 0.360693;
z = 37.5;

C = Ksum*(Kp+Ki/s);

G_closedloop = C*P/(1+C*P*H);
Pclosedloop = (66.64*(s+z))/(s*(s+5.952));
figure()
rlocus(P_cl)

%check eigenvalues
[num,den] = tfdata(G_closedloop,'v');
roots(den);

%magnitude of our step input
r_mag = 25;
%simulate step response
[y,t] = step(r_mag*G_closedloop);

%calculate our error signal
e = r_mag-y;
%calculate error derivative
[u_i,t] = lsim(Ki/s,e,t);%this simulates the error signal going through
%calculate control voltage from each piece (proportional, derivative)
u_p = Kp*e;
u_total = Ksum*(u_p+u_i);

%plot our results
figure()
plot(t,y,'k')
xlabel('Time (s)')
ylabel('Angular Speed (rad/s)')
figure()
plot(t,u_p,t,u_i,t,u_total)
legend('proportional','integral','total')
xlabel('Time (s)')
ylabel('Control Signal (V)')
```

```
                                         Root Locus of
Pcl


|-------------------------------------------------------|
                   |       +        +        +     +-----------$$$$$$-----
+|
                30 |-+&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&|###?###loc$$    $$
+||
                   |       &        &        &     |   B   open loop
poles||
                20 |-+&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&+---F---zeros------$$--
+|
                   |       &        &        &        &     $  &       $&
|
                10 |-+&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&$&&&&&&&&&&$$
+-|
                   |       &        &        &        &    $$  &        $
|
                 0 |**#####################################$$$$$$
$F****BB**|
                   |       &        &        &        &    #   &        #
|
                   |       &        &        &        &    ##  &        #
|
               -10 |-
+&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&#&&&&&&&&&&##+-|
                   |       &        &        &        &     #  &       #&
|
               -20 |-
+&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&##&&&&&&&&##&+-|
                   |       &        &        &        &      # &      # &
|
               -30 |-
+&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&&##&&&&##&&&+-|
                   |       +        +        +        +        ######   +
|

|-------------------------------------------------------|
                         -250     -200     -150     -100      -50
0
                                 Real Axis     gain = [0,
4.7304]
```


Root Locus of $P_c I$
locus
open loop poles
zeros
Imaginary Axis
Real Axis gain = [0, 4.7304]
30
20
10
0
-10
-20
-30
-250
-200
-150
-100
-50
0


proportional
integral
total
Control Signal (V)
Time (s)
15
10
5
0
-5
0
0.2
0.4
0.6
0.8
1



Angular Speed (rad/s)
Time (s)
35
30
25
20
15
10
5
0
0
0.2
0.4
0.6
0.8
1

This design is most likely not feasible on our lab hardware. The maximum voltage that we can provide to the motor with our current setup is 5V. This controller would require initially sending approximately 10 V to the motor and then decreasing it to 5 V in order to maintain the desired angulat velocity. Since our lab setup can only produce a maximum of a 5V input, implementing this PI controller would result in nonlinearities and behavior that deviates from the model prediction.

```
In [ ]:
```

ONLY FOR THIS SIZE STEP. FEASIBLE STILL...

19/20