Problem Solving Framework

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Lecture 2

Polya's Framework*

(1) Understanding the problem

(2) Devising a Plan

(3) Carrying out the plan

(4) Looking back, i.e. verifying

**How To Solve It, by George Polya, 2nd ed., Princeton University Press, 1957, ISBN 0-69108097-*

Two types of Problem

- **Find type problem**
  - given some inputs and conditions, find the output
  - Example: "Given your birthday and the current date, calculate your age in days"

- **Proof type problem** 
  - Example: Fermat's Last Theorem states that no three positive integers a, b, and c can satisfy the equation $a^n + b^n = c^n$ for any integer value of n greater than two.

(1) Understanding Find type Problem

- Extract information from the problem
  - What is input?
  - What is unknown?
  - What is output?
  - What are the (hidden) assumptions?
  - What are the facts?
  - What are the conditions?
- But how to extract?

Extracting Information

- Read carefully (slowly and repeat if necessary)
- Consult definition for unfamiliar (or even familiar) terminologies
- Construct one or two simple example to illustrate what the problem says
- Restate the problem in your own words
- Think of the “Inventory of concepts”
- Think of a picture or diagram that might help understand the problem
- Write specification of the problem (and/or sub-problems

(2) Devising a Plan

- Try all possibilities (brute force)
- Eliminate possibilities
- Solve equations or formula
- Look for a pattern
- Solve a simpler problem
- Work backwards
- Draw a picture
- Divide and conquer

(3) Carrying Out the Plan

- Programming starts here
- Be patience
- Check each step

(4) Looking back

- Examine the solution
- Is it satisfied all the conditions?
- Can we derive the solution differently?

References

1. *How To Solve It, by George Polya, 2nd ed., Princeton University Press, 1957, ISBN 0-69108097*
2. http://math.berkeley.edu/~gmelvin/polya.pdf
3. http://www.cs.odu.edu/~toida/nerzic/content/problem_solving/problem_solving.html