```
Sage Quick Reference: Graph Theory
                                                                                                                 G.edge_cut(source, sink
                                                             G = Graph([(1,3,"Label"),(3,8,"Or"),(5,2)])
                                                             Incidence Matrix:
          Steven Rafael Turner(w/mod. by nu)
                                                             M = Matrix(2, [-1,0,0,0,1, 1,-1,0,0,0])
                    Sage Version 4.7
                                                             Graph6 Or Sparse6 string:
         http://wiki.sagemath.org/quickref
                                                             G=':IgMoqoCUOqeb\n :I'EDOAEQ?PccSsge\N \n '
  GNU Free Document License, extend for your own use
                                                             graphs_list.from_sparse6(G)
                                                                Above is a list of graphs using sparse6 strings.
                                                             NetworkX Graph:
グラフの生成 Constructing
                                                             g = networkx.Graph({0:[1,2,3], 2:[4]})
隣接写像を使う:
                                                             g_2 = networkx.MultiGraph({0:[1,2,3], 2:[4]}) Graph(g_2)
G=Graph([GF(13), lambda i, j: conditions on i,j])
                                                               Don't forget to import networkx
  入力に使うリストの最初の要素は頂点達、二番目の要素は隣
接関係を表す関数: [頂点のリスト,関数]
                                                        中心性の指標 Centrality Measures
隣接リストを使う:
                                                        G.centrality_betweenness(normalized=False)
G=Graph({0:[1,2,3], 2:[4]})
                                                        G.centrality_closeness(v=1)
G=Graph({0:{1:"x",2:"z",3:"a"}, 2:{5:"out"}})
                                                        G.centrality_degree()
  x, z, a などは辺のラベル. 重みとして使われる.
                                                              ..... ORGINAL TEXT
                                                             G.centrality_betweenness(normalized=False)
隣接行列を使う:
                                                             G.centrality_closeness(v=1)
A = \text{numpy.array}([[0,1,1],[1,0,1],[1,1,0]])
                                                             G.centrality_degree()
  NumPy の matrix や ndarray を使うには numpy をインポー
トしなければならない.
                                                        追加や削除 Graph Deletions and Additions
M = Matrix([(....), (....), . . . ])
                                                        G.add_cycle([vertices])
ラベルあり/なしの辺のリストを使う:
                                                        G.add_edge(edge)
G = Graph([(1,3,"Label"),(3,8,"Or"),(5,2)])
                                                        G.add_edges(iterable of edges)
                                                        G.add_path
接続行列を使う:
                                                        G.add_vertex(Name of isolated vertex)
M = Matrix(2, [-1,0,0,0,1, 1,-1,0,0,0])
                                                        G.add_vertices(iterable of vertices)
Graph6 や Sparse6 の文字列表現を使う:
                                                        G.delete_edge( v_1, v_2, 'label')
G=':IgMoqoCUOqeb\n :I'EDOAEQ?PccSsge\N \n '
                                                        G.delete_edges(iterable of edges)
graphs_list.from_sparse6(G)
                                                        G.delete_multiedge(v_1, v_2)
  上の例は sparse6 の書式.
                                                        G.delete_vertex(v_1)
NetworkX Graph:
                                                        G.delete_vertices(iterable of vertices)
g = networkx.Graph({0:[1,2,3], 2:[4]})
                                                        G.merge_vertices([vertices])
DiGraph(g)
g_2 = networkx.MultiGraph({0:[1,2,3], 2:[4]})
                                                             G.add_cycle([vertices])
                                                             G.add_edge(edge)
Graph(g_2)
                                                             G.add_edges(iterable of edges)
  networkx をインポートしないといけない.
                                                             G.add_path
                                                             G.add_vertex(Name of isolated vertex)
     Adjacency Mapping:
                                                             G.add_vertices(iterable of vertices)
    G=Graph([GF(13), lambda i, j: conditions on i, j])
                                                             G.delete_edge( v_1, v_2, 'label')
       Input is a list whose first item are vertices and the other is some
                                                             G.delete_edges(iterable of edges)
    adjacency function: [list of vertices, function]
                                                             G.delete_multiedge(v_1, v_2)
     Adjacency Lists:
                                                             G.delete_vertex(v_1)
    G=Graph({0:[1,2,3], 2:[4]})
                                                             G.delete_vertices(iterable of vertices)
    G=Graph({0:{1:"x",2:"z",3:"a"}, 2:{5:"out"}})
                                                             G.merge_vertices([vertices])
       x, z, a, and out are labels for edges and be used as weights.
     Adjacency Matrix:
                                                        連結性とカット Connectivity and Cuts
    A = numpy.array([[0,1,1],[1,0,1],[1,1,0]])
       Don't forget to import numpy for the NumPy matrix or ndar-
                                                        G.is_connected()
                                                        G.edge_connectivity()
    M = Matrix([(....), (....), . . . ])
                                                                                                                 グラフがある性質を満たすかのチェック Boolean Queries
```

Edge List with or without labels:

```
G.blocks_and_cut_vertices()
G.max_cut()
G.edge_disjoint_paths(v1, v2, method='LP')
  method にはLP (Linear Programming) もしくは FF (Ford-
Fulkerson) というオプションが指定できる.
vertex_disjoint_paths(v1, v2)
G.flow(1,2)
  たくさんのオプションがある(ドキュメントを参照されたい).
     G.is_connected()
     G.edge_connectivity()
     G.edge_cut(source, sink
     G.blocks_and_cut_vertices()
     G.max cut()
     G.edge_disjoint_paths(v1, v2, method='LP')
        This method can use LP (Linear Programming) or FF (Ford-
     Fulkerson)
     vertex_disjoint_paths(v1, v2)
     G.flow(1,2)
       There are many options to this function please check the doc-
     umentation.
变換 Conversions
G.to directed()
G.to_undirected()
G.sparse6_string()
G.graph6_string()
     G.to_directed()
     G.to_undirected()
     G.sparse6_string()
     G.graph6_string()
積 Products
G.strong_product(H)
G.tensor_product(H) G.categorical_product(H)
   テンソル積と同じ.
G.disjunctive_product(H)
G.lexicographic_product(H)
G.cartesian_product(H)
     G.strong_product(H)
     G.tensor_product(H)
     G.categorical_product(H)
       Same as the tensor product.
     G.disjunctive_product(H)
     G.lexicographic_product(H)
     G.cartesian_product(H)
```

```
DLX ではなく MILP も選べる.
G.is tree()
                                                                                                                           G.steiner_tree(g.vertices()[:10])
G.is_forest()
                                                           G.is_perfect(certificate=False)
                                                                                                                           G.spanning_trees_count()
G.is_gallai_tree()
                                                                                                                           G.edge_disjoint_spanning_trees(2, root vertex)
                                                                G.chromatic_polynomial()
G.is_interval()
                                                                                                                           G.min_spanning_tree(weight_function=somefunction,
                                                                G.chromatic_number(algorithm="DLX")
G.is_regular()
                                                                                                                           algorithm='Kruskal', starting_vertex=3)
                                                                   You can change DLX (dancing links) to CP (chromatic polyno-
                                                                                                                              Kruskal can be change to Prim_fringe, Prim_edge, or
G.is_chordal()
                                                                mial coefficients) or MILP (mixed integer linear program)
                                                                                                                           NetworkX.
G.is_eulerian()
                                                                G.coloring(algorithm="DLX")
                                                                   You can change DLX to MILP
G.is_hamiltonian()
                                                                G.is_perfect(certificate=False)
                                                                                                                      線形代数 Linear Algebra
G.is_interval()
                                                                                                                      行列:
G.is_independent_set([vertices])
                                                           平面性 Planarity
                                                                                                                      G.kirchhoff_matrix()
G.is_overfull()
                                                           G.is_planar()
                                                                                                                      G.laplacian_matrix()
G.is_regular(k)
  k-regular であるかを調べる. デフォルトでは k=None.
                                                           G.is_circular_planar()
                                                                                                                         キルヒホッフ行列に同じ.
                                                           G.is_drawn_free_of_edge_crossings()
                                                                                                                      G.weighted_adjacency_matrix()
                                                           G.layout_planar(test=True, set_embedding=True
    G.is tree()
                                                                                                                      G.adjacency_matrix()
    G.is_forest()
                                                           G.set_planar_positions()
                                                                                                                      G.incidence_matrix()
    G.is_gallai_tree()
    G.is_interval()
                                                                                                                       操作:
                                                                G.is_planar()
    G.is_regular()
                                                                G.is_circular_planar()
                                                                                                                      G.characteristic_polynomial()
    G.is_chordal()
                                                                G.is_drawn_free_of_edge_crossings()
    G.is_eulerian()
                                                                                                                      G.cycle_basis()
                                                                G.layout_planar(test=True, set_embedding=True
    G.is_hamiltonian()
                                                                G.set_planar_positions()
                                                                                                                      G.spectrum()
    G.is_interval()
    G.is_independent_set([vertices])
                                                                                                                      G.eigenspaces(laplacian=True)
    G.is_overfull()
                                                           検索と最短パス Search and Shortest Path
                                                                                                                      G.eigenvectors(laplacian=True)
    G.is_regular(k)
                                                           list(G.depth_first_search([vertices], distance=4)
       Can test for being k-regular, by default k=None.
                                                                                                                           Matrices:
                                                           list(G.breadth first search([vertices])
                                                                                                                           G.kirchhoff matrix()
                                                           dist,pred=graph.shortest_path_all_pairs(by_weight=
主な不変量 Common Invariants
                                                                                                                           G.laplacian_matrix()
                                                           True, algorithm="auto")
                                                                                                                              Same as the kirchoff matrix
G.diameter()
                                                                                                                           G.weighted_adjacency_matrix()
                                                              アルゴリズムを選べる: BFS, Floyd-Warshall-Python
G.average_distance()
                                                                                                                           G.adjacency_matrix()
                                                           G.shortest_path_length(v_1, v_2, by_weight=True
G.edge_disjoint_spanning_trees(k)
                                                                                                                           G.incidence_matrix()
G.girth()
                                                                                                                           Operations:
                                                           G.shortest_path_lengths(v_1)
                                                                                                                           G.characteristic_polynomial()
G.size()
                                                           G.shortest_path(v_1, v_2)
                                                                                                                           G.cycle_basis()
G.order()
                                                                                                                           G.spectrum()
                                                                list(G.depth_first_search([vertices], distance=4)
G.radius()
                                                                                                                           G.eigenspaces(laplacian=True)
                                                                list(G.breadth_first_search([vertices])
                                                                                                                           G.eigenvectors(laplacian=True)
                                                                dist,pred=graph.shortest_path_all_pairs(by_weight=True,
    G.diameter()
                                                                algorithm="auto")
    G.average_distance()
                                                                                                                      自己同型 Automorphism and Isomorphism Related
                                                                   Choice of algorithms: BFS or Floyd-Warshall-Python
    G.edge_disjoint_spanning_trees(k)
                                                                G.shortest_path_length(v_1, v_2, by_weight=True
                                                                                                                      G.automorphism_group()
    G.girth()
                                                                G.shortest_path_lengths(v_1)
    G.size()
                                                                                                                      G.is_isomorphic(H)
                                                                G.shortest_path(v_1, v_2)
    G.order()
                                                                                                                      G.is_vertex_transitive()
    G.radius()
                                                                                                                      G.canonical_label()
                                                           全域木 (スパニングツリー) Spanning Trees
                                                                                                                      G.minor(graph of minor to find)
彩色 Graph Coloring
                                                           G.steiner_tree(g.vertices()[:10])
G.chromatic_polynomial()
                                                           G.spanning_trees_count()
                                                                                                                           G.automorphism_group()
                                                                                                                           G.is_isomorphic(H)
G.chromatic_number(algorithm="DLX")
                                                           G.edge_disjoint_spanning_trees(2, root vertex)
                                                                                                                           G.is_vertex_transitive()
  DLX (dancing links) を CP (chromatic polynomial coeffi-
                                                           G.min_spanning_tree(weight_function=somefunction,
                                                                                                                           G.canonical label()
cients) や MILP (mixed integer linear program) に変更可能.
                                                           algorithm='Kruskal', starting_vertex=3)
                                                                                                                           G.minor(graph of minor to find)
G.coloring(algorithm="DLX")
                                                             Kruskal は Prim_fringe, Prim_edge, NetworkX に変更可
```

```
クラスタリング Generic Clustering
G.cluster_transitivity()
G.cluster_triangles()
G.clustering_average()
G..clustering_coeff(nbunch=[0,1,2],weights=True)
    G.cluster_transitivity()
    G.cluster_triangles()
    G.clustering_average()
    G..clustering_coeff(nbunch=[0,1,2],weights=True)
クリークの解析 Clique Analysis
G.is_clique([vertices])
G.cliques_vertex_clique_number(vertices=[(0, 1),
(1, 2)],algorithm="networkx")
  networkx は cliquer に変更可能.
G.cliques_number_of()
G.cliques_maximum()
G.cliques_maximal()
G.cliques_get_max_clique_graph()
G.cliques_get_clique_bipartite()
G.cliques_containing_vertex()
G.clique_number(algorithm="cliquer")
  cliquer は networkx に変更可能.
G.clique_maximum()
G.clique_complex()
    G.is_clique([vertices])
    G.cliques_vertex_clique_number(vertices=[(0, 1), (1, 2)
    ],algorithm="networkx")
       networks can be replaced with cliquer.
    G.cliques_number_of()
    G.cliques_maximum()
    G.cliques_maximal()
    G.cliques_get_max_clique_graph()
    G.cliques_get_clique_bipartite()
    G.cliques_containing_vertex()
    G.clique_number(algorithm="cliquer")
       cliquer can be replaced with networkx.
    G.clique_maximum()
    G.clique_complex()
連結成分について Component Algorithms
G.is_connected()
G.connected_component_containing_vertex(vertex)
G.connected_components_number()
G.connected_components_subgraphs()
G.strong_orientation()
```

G.strongly\_connected\_components()

```
G.strongly_connected_components_digraph()
G.strongly_connected_components_subgraphs()
G.strongly_connected_component_containing_vertex(
vertex )
G.is_strongly_connected()
     G.is connected()
     G.connected_component_containing_vertex(vertex)
     G.connected_components_number()
     G.connected_components_subgraphs()
     G.strong_orientation()
     G.strongly_connected_components()
     G.strongly_connected_components_digraph()
     G.strongly_connected_components_subgraphs()
     G.strongly_connected_component_containing_vertex(vertex)
     G.is_strongly_connected()
NP 問題 NP Problems
G.vertex_cover(algorithm='Cliquer')
   algorithm は MILP (mixed integer linear program) に変更
可能. ただし MILP を使うに GLPK か CBC をインポートしないと
いけない.
G.hamiltonian_cycle()
G.traveling_salesman_problem()
     G.vertex_cover(algorithm='Cliquer')
        The algorithm can be changed to MILP (mixed integer linear
     program). Note that MILP requires packages GLPK or CBC.
     G.hamiltonian_cycle()
     G.traveling_salesman_problem()
```