sd11 talk

Sage: Introduction and Status Report

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Sage Days 11

```
E = EllipticCurve('37a') ; E
```
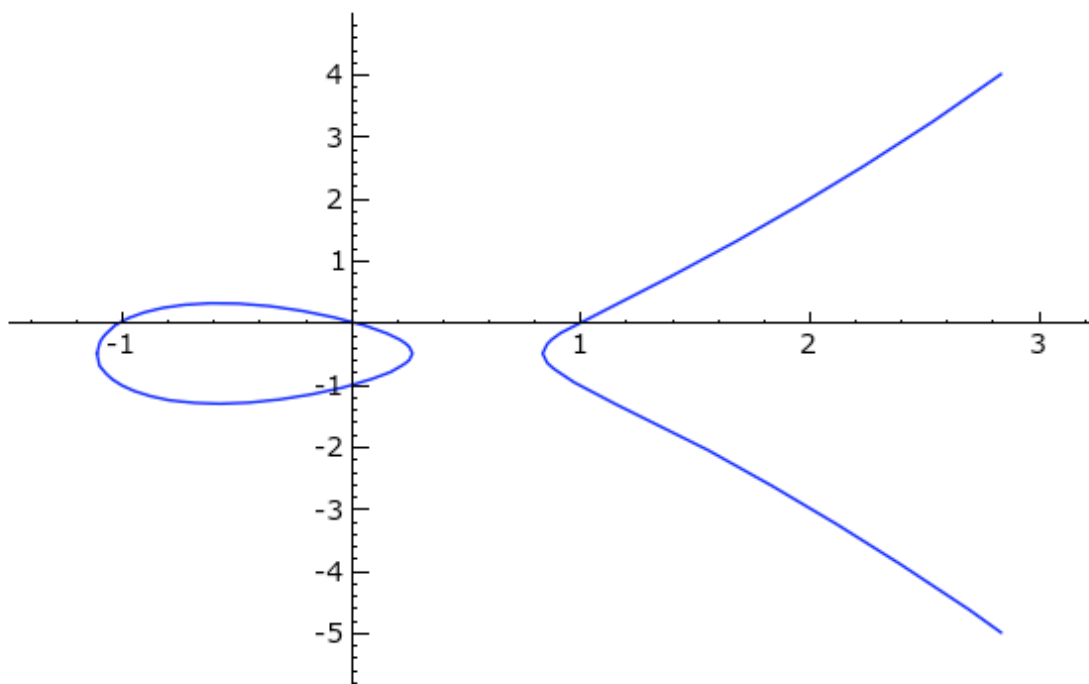   Elliptic Curve defined by y^2 + y = x^3 - x over Rational Field

```
E.c_invariants()
```
    (48, -216)

```
show(E)
```

$$y^2 + y = x^3 - x$$

```
show(plot(E))
```



```
delta_qexp(50)
```

```
        q − 24*q^2 + 252*q^3 − 1472*q^4 + 4830*q^5 − 6048*q^6 − 16744*q^7 +
        84480*q^8 − 113643*q^9 − 115920*q^10 + 534612*q^11 − 370944*q^12 −
        577738*q^13 + 401856*q^14 + 1217160*q^15 + 987136*q^16 −
        6905934*q^17 + 2727432*q^18 + 10661420*q^19 − 7109760*q^20 −
        4219488*q^21 − 12830688*q^22 + 18643272*q^23 + 21288960*q^24 −
        25499225*q^25 + 13865712*q^26 − 73279080*q^27 + 24647168*q^28 +
        128406630*q^29 − 29211840*q^30 − 52843168*q^31 − 196706304*q^32 +
        134722224*q^33 + 165742416*q^34 − 80873520*q^35 + 167282496*q^36 −
        182213314*q^37 − 255874080*q^38 − 145589976*q^39 + 408038400*q^40 +
        308120442*q^41 + 101267712*q^42 − 17125708*q^43 − 786948864*q^44 −
        548895690*q^45 − 447438528*q^46 + 2687348496*q^47 + 248758272*q^48 −
        1696965207*q^49 + O(q^50)
```

```
%time E4 = eisenstein_series_qexp(4,50000)
```

```
        CPU time: 0.43 s,  Wall time: 0.43 s
```

```
ls = E4.list()
ls[:30]
```

```
        [1/240, 1, 9, 28, 73, 126, 252, 344, 585, 757, 1134, 1332, 2044,
        2198, 3096, 3528, 4681, 4914, 6813, 6860, 9198, 9632, 11988, 12168,
        16380, 15751, 19782, 20440, 25112, 24390]
```

```
ls[-30:]
```

```
        [141515639939520, 129404204871032, 142408295876742, 125256166932960,
        145605951613080, 125819082506000, 142616700266040, 129628631599434,
        140439456663030, 124855477070688, 149328544489488, 124857593850784,
        140473649431368, 129497452357896, 142827602514912, 125943933626280,
        145903606879842, 125269163112608, 142475493385602, 129562979818720,
        141664967010000, 124932512149272, 148040896807440, 124947507349658,
        140984218923552, 130898657464848, 142549755366240, 125002968793794,
        145882205016912, 124992500150000]
```

```
eisenstein_series_qexp
```

```
        <function eisenstein_series_qexp at 0x64511f0>
```

```
# Yoda! -- over 50,000 triangles.
from scipy import io
x = io.loadmat(DATA + 'yodapose.mat')
from sage.plot.plot3d.index_face_set import IndexFaceSet
V = x['V']; F3=x['F3']-1; F4=x['F4']-1
Y = IndexFaceSet(F3,V,color=Color('#00aa00')) +
IndexFaceSet(F4,V,color=Color('#00aa00'))
Y = Y.rotateX(-1)
Y.show(aspect_ratio=[1,1,1], frame=False, figsize=4)
```

```
%cython
a = 2
print 1+a
```
    3
    __Users_cr...1_code_sage28_spyx.c   __Users_cr...ode_sage28_spyx.html

```
def mysum(N):
    s = 0
    for k in range(1,N):
        s += k
    return s
```

```
time mysum(10^6)
```
    499999500000
    CPU time: 1.76 s, Wall time: 1.78 s

```
%cython

def mysum(N):
    s = 0
    for k in range(1,N):
        s += k
    return s
```
    __Users_cr...1_code_sage32_spyx.c   __Users_cr...ode_sage32_spyx.html

```
time mysum(10^6)
```
    499999500000L
    CPU time: 0.18 s, Wall time: 0.18 s

```
%cython
def mysum(N):
    cdef int k
    cdef int my_N = N
```

```
    cdef long long s = 0
    for k from 1 <= k < my_N:
        s += k
    return s
```

__Users_cr...1_code_sage35_spyx.c   __Users_cr...ode_sage35_spyx.html

```
time mysum(10^6)
```

    499999500000L
    CPU time: 0.00 s, Wall time: 0.00 s

Interact

```
@interact
def gfan_browse(p1 = input_box('x^3+y^2',type = str, label='polynomial
1: '),
                p2 = input_box('y^3+z^2',type = str, label='polynomial
2: '),
                p3 = input_box('z^3+x^2',type = str, label='polynomial
3: ')):
    R.<x,y,z> = PolynomialRing(QQ,3)
    try:
        p1 = R(p1); p2 = R(p2); p3 = R(p3)
    except TypeError:
        print "Unable to parse input."
        return
    i1 = ideal(p1,p2,p3)
    alarm(2)
```

```
    try:
        gf1 = i1.groebner_fan()
        testr = gf1.render()
        html('Groebner fan of the ideal generated by: $%s, %s, %s$'%(
                latex(p1), latex(p2), latex(p3)))
        show(testr, axes = False, figsize=[8,8*(3^(.5))/2])
    except:
        print "Computation took more than 2 seconds... and I know you
are impatient, so stopping"
```
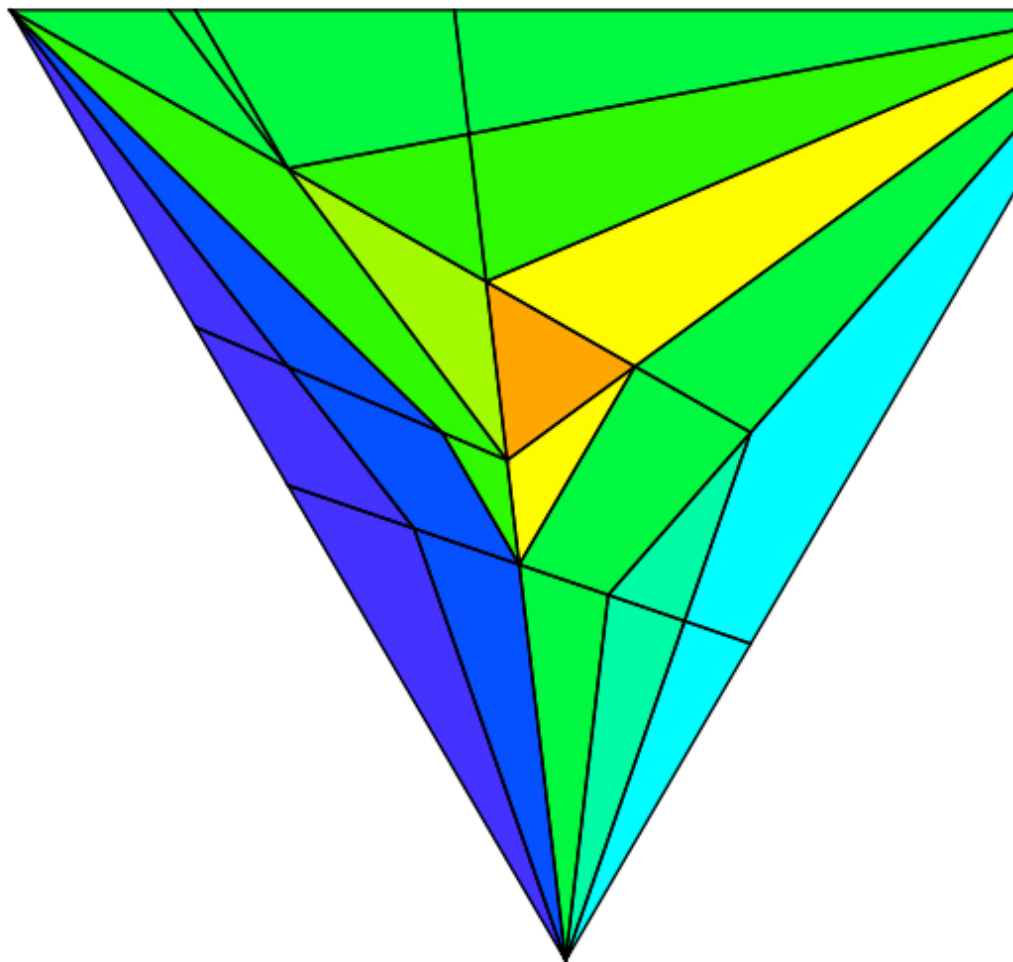
polynomial 1: `x^3+y^2`

polynomial 2: `y^3+z^2`

polynomial 3: `z^3+x^2`

Groebner fan of the ideal generated by: $x^3 + y^2, y^3 + z^2, x^8 + z^3$

```
var('x')
x0  = 0
f   = sin(x)*e^(-x)
p   = plot(f,-1,5, thickness=2)
dot = point((x0,f(x0)),pointsize=80,rgbcolor=(1,0,0))
@interact
def _(order=(1..12)):
 ft = f.taylor(x,x0,order)
 pt = plot(ft,-1, 5, color='green', thickness=2)
 html('$f(x)\;=\;%s$'%latex(f))
 html('$\hat{f}(x;%s)\;=\;%s+\mathcal{O}(x^{%s})$'%
```
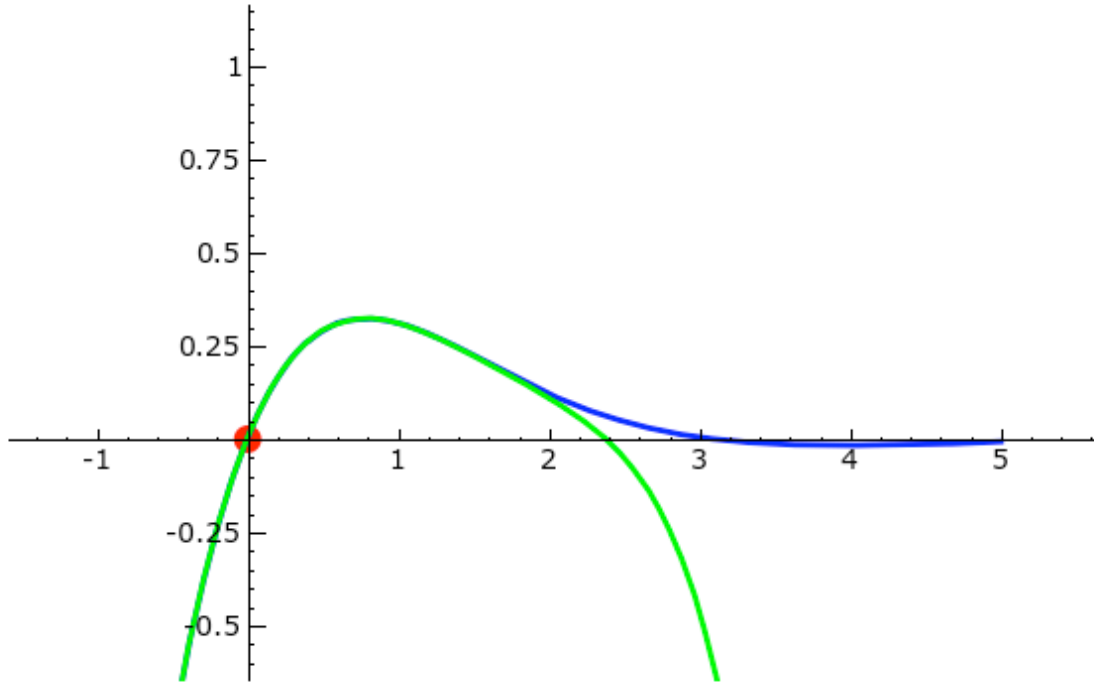
```
(x0,latex(ft),order+1))
 show(dot + p + pt, ymin = -.5, ymax = 1)
```

order

$$f(x) \;=\; e^{-x}\sin{(x)}$$
$$\hat{f}(x;0) \;=\; x - x^2 + \frac{x^3}{3} - \frac{x^5}{30} + \frac{x^6}{90} - \frac{x^7}{630} + \mathcal{O}(x^9)$$
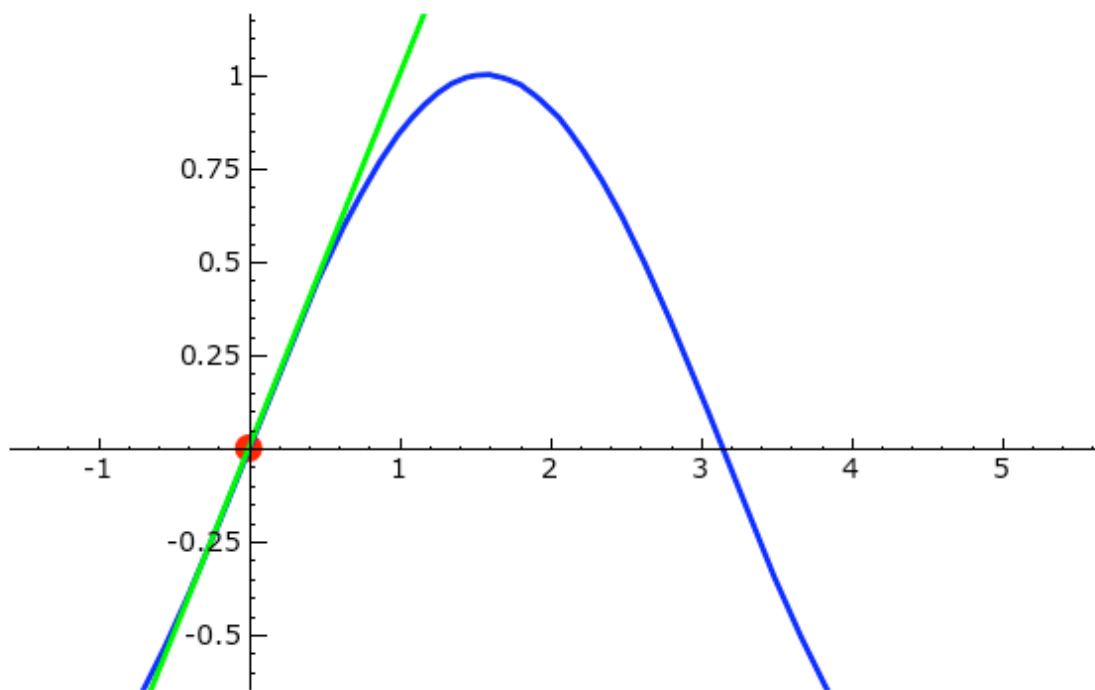


```
var('x')
x0  = 0
@interact
def _(f=input_box(sin(x)*e^(-x)),order=(1..12)):
 p   = plot(f,-1,5, thickness=2)
 dot = point((x0,f(x0)),pointsize=80,rgbcolor=(1,0,0))
 ft = f.taylor(x,x0,order)
 pt = plot(ft,-1, 5, color='green', thickness=2)
 html('$f(x)\;=\;%s$'%latex(f))
 html('$\hat{f}(x;%s)\;=\;%s+\mathcal{O}(x^{%s})$'%
(x0,latex(ft),order+1))
 show(dot + p + pt, ymin = -.5, ymax = 1)
```

f e^(-x)*sin(x)

order

$$f(x) \;=\; \sin{(x)}$$
$$\hat{f}(x;0) \;=\; x + \mathcal{O}(x^2)$$

Parallel Computing

```
N = 2^217-1
M = 2^218-1
```

```
ls = [N,M]
```

```
%time factor(N)
```
    127 * 5209 * 62497 * 2147483647 * 6268703933840364033151 *
    378428804431424484082633
    CPU time: 1.00 s, Wall time: 1.07 s

```
def f(n):
    return factor(n)
```

```
%time [ f(x) for x in ls ]
```
    [127 * 5209 * 62497 * 2147483647 * 6268703933840364033151 *
    378428804431424484082633, 3 * 104124649 * 745988807 *
    870035986098720987332873 * 2077756847362348863128179]
    CPU time: 2.29 s, Wall time: 2.43 s

```
@parallel(2)
def f_para(n):
    return factor(n)
```

```
%time list(f_para(ls))
```
    [(((2106245833371143733958360553673408646377901908010982225086219550\
    71,), {}), 127 * 5209 * 62497 * 2147483647 * 6268703933840364033151
    * 378428804431424484082633),
    (((42124916667422874679167211073468172927558038160219644501724391014\
    3,), {}), 3 * 104124649 * 745988807 * 870035986098720987332873 *
    2077756847362348863128179)]
    CPU time: 0.03 s, Wall time: 1.41 s