INTEGRATING OPEN SOFTWARE AND OPEN TEXTBOOKS IN UNDERGRADUATE MATHEMATICS COURSES

Proposal for a National Science Foundation
Course, Curriculum, and Laboratory Improvement Type 2 Award

Contents

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1. Project Description

Software for mathematical explorations, often known as computer algebra systems (CAS), have held great promise for education since their first appearance in the 1960s (Reduce, Macsyma [19, 23]) and accelerated with the introduction of Maple and Mathematica in the 1980s [17, 16]. Currently, CAS are widely found in the undergraduate classroom and a considerable amount of mathematics education research has focused on the use of CAS in the learning of undergraduate mathematics. Studies have documented the improvement in students' understanding [9, 10, 15, 20]. Computer algebra systems are now common in the undergraduate classroom, yet we rarely see a seamless integration with the curriculum. To accommodate competing commercial systems which divide the market, textbooks typically offer supplements for several different CAS or generic "technology" sidebars; otherwise the textbook can only be marketed to the subset of the audience that licenses a particular computer algebra system. Licensing restrictions for campus use, the expense of personal copies, and underpowered hardware often means that students can only work with commercial software in campus labs. Institutions with less funds may not even be able to afford the licensing costs to build such labs for their students. Open software is free and available to everyone, and the development is often driven by teacher-authors.

An example of an experiment to seamlessly integrate curriculum and software is the Calculus & Mathematica project, which generated much excitement in the early 1990s. However, today its use seems limited to the two institutions where it originated [5, 6]. Likewise, the creation of interactive Java applets to support teaching mathematics does not seem to have been widely adopted [2, 11, 13]. The undergraduate curriculum has not seen the broad transformative effect of these powerful tools for increasing the learning and understanding of mathematics. What is needed to increase the uptake of software for mathematical exploration and see the advantages that isolated, successful pilot programs predict across the spectrum of undergraduate education?

1.1. **An open approach.**

Our proposition is that open software, open textbooks, open standards, and open licenses can allow mathematical software to effect the long-envisioned transformation of undergraduate mathematics curriculum. We will test this hypothesis using the open source mathematics software Sage [1, 24], integrated into existing open textbooks and other curricular materials. By creating a model of how to easily integrate textbooks and other materials with electronic Sage worksheets, we will place the full computational power of Sage *directly into* a student's text, usable at all times and from anywhere, simply via a web browser. For the institution and instructor, the cost and time-consuming inconveniences of commercial software are removed by free software, installed on servers accessible with no license restrictions. Furthermore, the essential nature of open software means that curricular decisions and needs can drive the development of the software, with the classroom teacher actively advising, or actually doing, the software development. Leveraging these inherent advantages of an open approach to software and text, the promise of mathematical software for education can be fully realized by faculty and students.

More specifically, we will build and test a model for easily integrating mathematics software and open educational resources into the mathematics curriculum and classroom by:

- converting existing mature open textbooks to Sage worksheets containing new interactive demonstrations and live Sage code, creating new similar curricular materials, and building a system that makes it easy for other authors to do the same;
- partnering with diverse institutions to test these materials, and the tools for their creation, in a wide variety of courses, while providing support for teachers to test, refine, and disseminate;
- conducting formative and summative evaluations to determine the effectiveness of our model in making integrated open curricular materials and open software easy to create and adopt, and measuring the resulting impact on teaching practices and the learning of mathematics.

The model we build for creating or enhancing curricular materials will be simple and easy for teachers to use. Materials created or enhanced in our project will be distributed with open licenses and made freely available on Sage's website (90,000 visitors a month), distributed with every copy of Sage (over 6,000 downloads a month), or on the American Institute of Mathematics open textbook site, which will be initiated as part of this grant.

2. Mathematics Software Today

2.1. **Three crises in the academy.** The crisis in research publication is well-known [3]. Faculty have come a long way in their efforts to return scientific publication to a free exchange of ideas. Electronic journals and public repositories now publish articles with open licenses that explicitly allow for sharing new results among research communities and others, without copyright fees and with ubiquitous access via the Internet. Government initiatives, such as the Public Access Policy of the National Institutes of Health [18] and the Policy Forum on Public Access of the White House Office of Science and Technology [31], are working to accelerate this trend.

Following on the sea change in research publication, the next wave is open textbooks. Faculty are all too familiar with the problems that plague commercial textbook publication, such as high prices, edition churn, and orphaned works. Open licenses are now being used to assert control of these critical resources for education, with faculty in mathematics, computer science and business as the leaders. Government is poised to similarly accelerate this trend at all levels, with Senator Durbin proposing legislation directing the Department of Education to award grants for the creation of open textbooks by faculty [7], Washington State's initiative to provide open textbooks for the eighty highest-enrollment courses in their community college system [32], and California's initiative to create free digital textbooks for its high schools [4].

With the emergence of viable and comprehensive open source software for mathematics, there is now a spectacular opportunity for mathematics teachers to use, extend, and create this critical tool for mathematics education and shape it to reach its full potential in the service of educating students. This project will create an easy path for faculty to make the transition to open software, open textbooks, and open curricular materials in their courses. This path will be built by using open standards and freely-available tools for authoring materials, existing mature textbooks, and software built from open components and an industry-standard language. This path will help diverse schools and faculty, regardless of circumstances, to use mathematical software to transform the classroom into the interactive laboratory which takes student learning to a new level.

2.2. **Computer algebra systems.** We have had computer algebra systems for over forty years now, with mature commercial products available for the past twenty years. Though education research indicates the value of these tools for teaching and learning, faculty and institutions still struggle with cost, logistics and curriculum when it comes to decisions about adopting these commercial offerings. In particular, the closed development process for commercial software creates a high barrier for teachers and students to alter or extend the software to meet their curricular needs. For example, external extensions to proprietary systems, such as libraries or packages, can extend proprietary systems, but must be purchased, distributed, and installed by every end-user. However, an open development process, such as the open source model that Sage follows, allows students and faculty to shape the core technology in a timely fashion to support the transformation of teaching and learning of mathematics on a broad scale. Open source software, such as Sage, allows the teaching and learning of mathematics to drive the technology, rather than the technology driving the teaching and learning.

2.3. **Choosing a computer algebra system.** Institutional, departmental and course-by-course decisions about the use of a CAS require a commitment to a particular vendor's vision for the curriculum and in some cases may even require a choice of supported operating systems. With a significant investment in hardware, training of faculty, and bringing a group of students up-to-speed on a particular product, there is a significant penalty to switching to another product that might be superior for certain courses. With open software, hardware choices and expenses become irrelevant when it may only require a student to have access to a web browser and the Internet, and similarly the choice of operating system becomes irrelevant. As individuals and institutions extend the software, the entire mathematics community benefits from a comprehensive program that student and faculty can employ in courses ranging from pre-calculus and introductory statistics through to advanced courses like abstract algebra and number theory.

3. Why Sage Now?

Sage is a natural choice for software to realize the benefits of an open approach to the undergraduate mathematics curriculum. An open development process, the intuitive and familiar notebook interface, tight integration with LaTeX, a standard programming language and a modular design philosophy are simultaneously consequences of a steadfast commitment to open design principles and concrete examples of the benefits of this approach. With an open license, Sage removes substantial financial and logistical barriers to classroom use of mathematics software, such as purchase cost, restrictive licensing, expensive hardware, and limited availability on mobile devices.

3.1. **Building the car, not reinventing the wheel.** The genius of Sage is the way it unifies over one hundred mature, best-of-breed, open source packages, each excelling at specific types of computations (e.g., the Integer Matrix Library [12] for fast and exact matrix computations over the integers or rationals) or complete libraries for specific areas of mathematics (e.g., R for statistics, GAP for group theory, and SciPy for scientific computations [8, 22, 29]) into a single open source system with a consistent interface. This unification greatly simplifies the use of mathematical software for both students and teachers.

3.2. **A standard programming language.** The many components of Sage are held together with a significant library of new code, written in the industry-standard language Python. More and more packages are appearing for mathematics and science written in Python, often with open licenses. So Sage continues to benefit by easily integrating these new packages, bringing new functionality, or improving existing functionality, all with little disruption to the user.

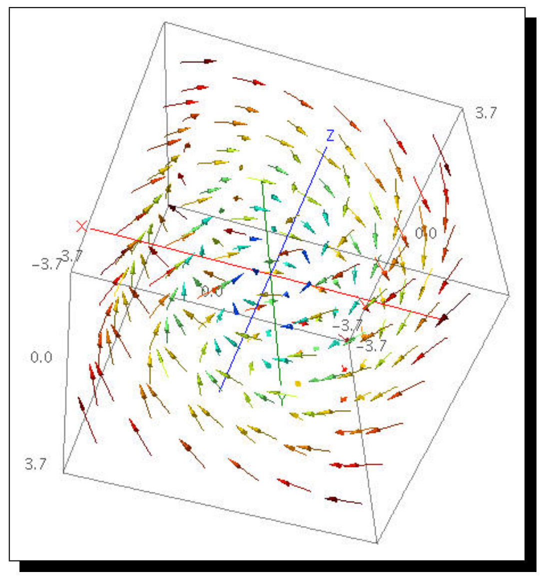
Most importantly, with the decision to use Python to integrate the many components of Sage, it was entirely natural to make Python also the user language within Sage. This is in contrast to other comprehensive programs for mathematics that have chosen to create and develop their own languages, which are of little use outside of the program itself. A user with knowledge of Python is ready to be productive in Sage immediately, while a student new to programming receives a basic education in Python as a by-product of learning Sage — a skill that is readily transferable to a wide variety of applications in mathematics, science and engineering.

3.3. **A comprehensive program.** Sage aims to be a comprehensive program for mathematics. Its design allows numerical, exact and symbolic approaches to mathematics to coexist equally. Mathematical objects, such as functions, differential equations, rings, fields, modules, and vector spaces are “objects” that look and behave like their abstract mathematical definitions intend. While the relative strength of Sage in particular topics may vary through the course of its development, it does not rest on a foundation such as pattern-matching, which favors symbolic computation, or floating-point numbers, which favors numerical work. With a modular design that allows for packages specializing in different approaches, Sage can seamlessly support many approaches to the application of computation to mathematical problems. Further, additions and extensions to Sage, driven by real curricular needs can be easily and quickly incorporated.

3.4. **Curriculum and open development.** Like mathematics, the Sage model for development is driven by collaboration, sharing and openness. Just as major research problems drive the direction of pure mathematics, the curriculum and needs of the users directly drives the development of open mathematics software. We give two examples of this important principle.

3.4.1. *3D vector fields.* Jason Grout was asked one day by a student if Sage had a 3D vector field plotting function. Grout quickly wrote a simple one. Open development communities encourage sharing even experimental code, so he posted his code online in Sage's publicly-accessible database of enhancements and bugs. Robert Bradshaw, a graduate student at University of Washington, who had written most of the 3D graphing code in Sage, suggested a small change that greatly increased the efficiency of the function. Several months later, another person authored a very similar function which had slightly improved options and published their code to the world on the public Sage notebook server. In Fall 2008, Grout needed the 3D vector field plotting function for his multivariate class. He made a few suggested changes to the function and posted it for his class to use. In a closed proprietary system, this is where the story likely would have ended.

In the next few months, many improvements were made to the internals of Sage 3D graphics (many by William Cauchois, a University of Washington freshman supported for a summer by an NSF VIGRE grant). Some of these improvements were directly the result of wanting to make the 3D vector field plotting function easier to write. Jason then taught the same class again. He incorporated the best ideas from the various sources and posted a documented, efficient version of the function, this time asking the community for a formal review (a prerequisite for his code to be added to the core Sage library). Marshall Hampton, an Assistant Professor at University of Minnesota, Duluth, who has also been active in the Sage project, reviewed the function immediately, with the comment "Very nice, positive review. This is great timing since I am about to teach vector fields in a week or two." The function was incorporated into the next release of Sage, two weeks later, ready for immediate use by the entire Sage user community.



The vector field $\vec{f}(x, y, z) = y\vec{\imath} + z\vec{\jmath} + x\vec{k}$

Now not only do the original students benefit from the invested effort, but the collaborative effort of at least five people from around the world has directly led to a function that will serve all students and teachers using Sage.

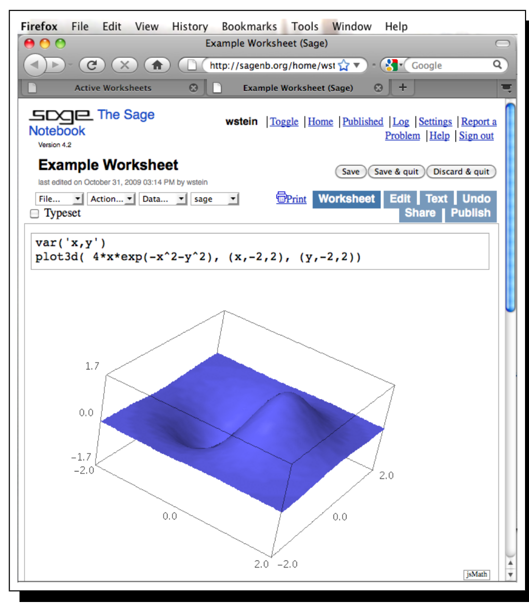
3.4.2. *Dicyclic groups.* When Rob Beezer teaches introductory group theory, as he did in Fall 2009 with Tom Judson's open source text, he includes classifying all the finite groups of order less than 16 as a way of exhibiting the power of the theorems developed through the term. Each of these groups is easily constructed and realized in Sage as a permutation group, except for one of the three non-abelian groups of order 12. This fall, he decided to add this group to the collection of constructions in Sage, but on a Friday evening he first polled the Sage development mailing list about the best way to realize this group.

John Palmieri, Associate Professor at the University of Washington, and an expert in algebraic topology, responded 90 minutes later with information about the dicyclic groups, a sequence of groups of order $4m$, including the quaternion group of order 8, the group of order 12 in question, and the generalized quaternions of order 2^n. David Joyner, Professor at the US Military Academy, Sage developer #2, and implementer of much of Sage's support for group theory, joined the conversation an hour later. Jaap Spies weighed in from Holland with a source for more information. With this help, in the next few days Beezer was able to easily create an efficient permutation representation of these groups, and included comprehensive information and references about the group's construction and properties to Sage's extensive documentation. Joyner reviewed the code and documentation, providing a positive review, several days later. Two weeks later this new code was incorporated in a Sage release available for public testing. Within a month of conception, Mike Hansen, a postdoctoral researcher at the University of Washington, working from Thailand, included the code in a general stable release of Sage.

This example illustrates many strengths of Sage development, such as individuals bringing their talents as educators, mathematicians and programmers together to make Sage an amazing product of open development. In this case, the net result is new capabilities for Sage, relevant to the undergraduate curriculum, with associated mathematical explanations in the Sage documentation. Beezer's course was improved by this exercise, and the benefits are now available to all through Sage. The dicyclic groups would also now appear to be a worthwhile addition for Judson's text, completing a virtuous cycle.

3.5. **Sage anytime, anywhere.** Sage's notebook interface is perhaps its most impressive feature, especially for use in education. Much of the power of the Sage notebook is a direct result of decisions to employ open software and open standards in its design. From within a standard web browser (preferably one supporting open standards, such as Firefox), a user can create a "worksheet" and interactively edit and execute new commands, with computations performed remotely on a server, and output returned to the worksheet. The interface makes it easy to view graphical output, annotate a worksheet, receive help on commands, browse the manuals, browse the source code, manage multiple worksheets, publish completed worksheets, or share and collaborate on worksheets.

The Sage notebook interface relies heavily on industry-standard Javascript, acting as an application similar to GMail or Google Wave. Much of the development work on the notebook has been student designed and implemented, including at the extreme very significant contributions this past fall by Tim Dumol, a high school student in the Philippines.



The Sage notebook interface

The Sage project currently hosts a public Sage server at `sagenb.org` that has over 20,000 accounts. This powerful hardware was purchased in January 2009 as part of a $110,000 NSF SCREMS grant (NSF Grant No. DMS-0821725) and an NSF FRG grant[i]. Anybody in the world with a network connection can productively harness the full power of Sage — even with minimal hardware or mobile devices, such as an under-powered desktop, a laptop, netbook, smartphone, or cell phone (see [27] for cell phone interface). Publicly-accessible notebook servers continue to appear throughout the world (most recently in Hungary [26]), and many more run behind campus firewalls for dedicated use. Public or private servers make hardware and operating decisions irrelevant. Underpowered hardware can control a web browser sufficiently to handle communications with a Sage server, and the use of portable devices means students can access Sage from anywhere they can get a network connection. Notice too, that it is straight-forward to install and run a Sage server on your own machine, so Sage can also be used productively on reasonably modern hardware without a network connection.

[i]Grant number for this one?

3.6. **Communicating mathematics.** TeX, with its add-on package LaTeX, is another open source success story and is the typesetting language of choice for mathematicians and other technical disciplines. As a result, there is an impressive array of extensions to LaTeX. Sage also includes extensive support for LaTeX, through every mathematical object being *required* to have a LaTeX representation. The Sage notebook then uses jsMath [14] to render mathematics beautifully in a Sage worksheet (within a standard web browser) via Javascript control of the necessary fonts and their accurate placement. Furthermore, the Sage notebook interface allows a user to insert new text, including LaTeX code that jsMath will similarly render accurately. This is another example of how open standards and open software serendipitously combine to make powerful tools, which is especially relevant for our plan to convert LaTeX documents into Sage worksheets.

3.7. **Freedom and liberty.** Open source software is more than about being free, as in no cost. It is about freedom — the liberty to use the software the way you want to use it, and in ways the developer might not have ever even imagined. If you make improvements, or worthwhile extensions, then it should be easy (as it is with Sage) to contribute those improvements back. At a minimum, license terms often require that if you make changes to the software and publicly distribute the resulting modified software in binary form, then you must also make the source code of your changes publicly available under a compatible license.

3.8. **Freedom and budgets.** Sage is also free. There is no cost to download and install Sage, install add-on packages, or upgrade to the latest version. There is no reason to have incompatible versions installed in different locations. So even institutions with highly constrained budgets can have their students work on public servers such as `sagenb.org` if they have even a minimal network connection. Imagine students at the poorest universities running the same version of Sage, on the same hardware cluster at the University of Washington, side-by-side with leading researchers in computational number theory. This $100,000 rack of servers was fully funded by the NSF and should be a viable resource for many years to come.

Certainly, there are costs to installing a campus Sage server. But the hardware does not need to be particularly expensive or extravagant. There are costs to learning a new program for faculty, and for students. However, this project will make it simple for an institution, department or course to enjoy the benefits of open mathematics software and open curricular materials. With an open approach, our successes will be easy to distribute and replicate.

4. Implementation

Our goal is to have a broad, positive influence on the teaching and learning of mathematics. By using open curricular materials, integrated with powerful and comprehensive open software, we will realize a transformative effect. Project Sage will revolve around three coordinated efforts: (1) creating Sage-enhanced curricular materials, (2) working with and providing classroom support for teachers using these materials in the classroom for the first time, and (3) providing the Sage infrastructure to support these materials. More specifically, our project can be divided into five main activities.

(1) Create a model for Sage-integrated textbooks by converting existing open textbooks to Sage worksheets, and incorporate interactive demonstrations powered by Sage, along with guidance on how to use Sage to further understand the topic of the textbook. The first courses we will address are linear algebra, abstract algebra, number theory, and combinatorics.
(2) Create a model for Sage-integrated modules by writing new curricular materials for standard undergraduate courses, especially courses not covered by the textbooks we are converting. These materials will include subject-specific guides to introduce Sage to both faculty and students in the context of a particular topic. Possible topics include calculus, differential equations, and complex variables.

(3) Improve the Sage library and its surrounding infrastructure (e.g., server design, notebook usability, and collaboration tools) where the improvements have a direct and obvious benefit for the use of Sage in educational settings.
(4) Identify and work closely with ten different institutions to test, refine, improve, and extend the work described above. We will solicit teachers from a broad range of institutions, especially from those with diverse student profiles.
(5) Disseminate our materials and the results of this project through Sage Days workshops, MAA PREP workshops, and MAA minicourses (national and regional) as opportunities to train faculty (outside of the ten test sites) in the use of these materials for their courses, and in the creation of new materials. Our materials will all carry open licenses and be available either in Sage itself or through the Sage website, which has a global audience.

4.1. **Sage-enhanced open textbooks.**

4.1.1. *Imagine this.* A student is learning about row-reducing matrices in a beginning linear algebra course. The electronic version of their textbook is an online Sage worksheet they can view from anywhere. Mathematical equations, with publication-quality formatting, describe the procedure. An interactive demonstration, embedded in the text where an example normally would be, allows the student to step through row-reducing a matrix, generated on-the-fly (see Appendix A). When prepared to guide the procedure themselves, the student may choose the row operation to apply at each step of the reduction. In either case, the correct notation for the operation is displayed and the operation itself is highlighted with color-coded entries of the displayed matrix. Another example shows the student how to use a built-in command to row-reduce a matrix. With a single click, the student creates an empty code cell under the example to experiment with the command.

In class, the instructor does one simple example on the board. Then the instructor opens a Sage worksheet, displaying the same demonstration that is in their electronic textbook. Students guide the choice of row operations at each step for several example matrices of increasing complexity. At the end of class, the instructor clicks a button to publish the work from that day's class session to the class Sage server. After class, students view the worksheet as they review their notes. They can also, with a click, copy the worksheet into their notes, as well as annotate or change it.

The book's exercises include an interactive problem generator that creates matrices of student-specified sizes and complexity for the student to row-reduce. There is a "Solution" button which generates a step-by-step solution. A group exercise asks students to collaboratively write and test a short function to implement row-reduction using row operations. The student clicks a button to open a new Sage worksheet and types in a few lines using Sage matrices and commands in Python. The student evaluates the code with a single click. Satisfied with their work, the student clicks another button, publishing the worksheet to the rest of the group for testing. Later that evening, the student checks the worksheet and sees that other students have tested the function, found a typo, and fixed it.

An advanced exercise guides a student through investigating numerical issues that arise in row reduction. By changing one parameter within a command, the matrices track numeric error bounds via interval arithmetic, and the student discovers that numerical errors can be a significant problem for some matrices.

The next week, the instructor goes to the online server and looks over all of the published worksheets for the group projects. The instructor annotates each worksheet online, using an embedded word processor that supports mathematical typesetting.

The instructor writes a quiz covering row-reduction techniques. Inside of the LaTeX code, the instructor embeds a Sage command to create a simple matrix in reduced-row echelon form, which will be the *answer* to a quiz problem. The instructor then calls Sage (automatically from within his LaTeX document using Dan Drake's SageTeX package [28]) to convert the answer to a matrix that forms the question and only requires a short sequence of simple row operations for a solution. The question matrix is automatically incorporated in the printed quiz, while the answer matrix and a detailed solution are automatically incorporated in the answer key.

4.1.2. *Infrastructure for textbooks.* Our project proposes creating and fully testing a model for integrating textbooks and mathematics software. We will convert existing mature open textbooks into folders of interactive Sage worksheets. The tex4ht translator (NSF Grant No. IIS-0312487) [30] can convert extremely complex LaTeX into jsMath, which is then simple to convert to the Sage worksheet format since it relies heavily on HTML and jsMath. It is possible to insert Sage code into a LaTeX source file, and have it migrate to the eventual Sage worksheet as an input cell. The end result is a folder of Sage worksheets, each a mixture of text, high-quality typeset mathematics, Sage input cells and Sage interactive demonstrations. Because LaTeX is the accepted standard for authoring mathematics, we expect this process to be applicable to a broad range of current and future open mathematics textbooks.

4.1.3. *Targeted textbooks.* Appendix B lists many open textbooks that could be converted by our process. However, conversion is more than a technical process. We plan to incorporate interactive demonstrations, live Sage code, and guidance on the Sage library itself. So there is a significant care and thought required to integrate these uses of mathematical software properly.

Listed below are our initial candidates for modeling how we can incorporate live code and interactive demonstrations to bring the full power of mathematics software to the student *directly within their book.* The open textbook landscape is rapidly growing, and beyond the following list we will make final decisions based on the most promising open textbooks that will help us achieve the project's goals. We expect to have the three texts below available for use in the Fall 2011 term at our test sites.

- Beezer's *A First Course in Linear Algebra.* This is a mature textbook designed from the start to be open source, and thus has a highly modular design. The first version of this book was released in December 2006, and has been used as the primary text in courses at 13 different universities. The author has taught from various versions of the book ten times. It will be an excellent choice as a full-scale test of the technical conversion process, the integration of Sage code and classroom use.
- Stein's *Elementary Number Theory.* This text already has an extensive collection of Sage examples, and the author is the founder of the Sage project who has taught from the book four times, making it another ideal candidate for early conversion. It is currently being published by Springer-Verlag, but Stein has the rights to make it freely available starting May 2010, so will be able create a Sage-enhanced version for free distribution. While it will not carry a true open license that allows modifications by others, it will be a good test of the technical aspects of the conversion process and will be made freely available to our test sites and other institutions.
- Judson's *Abstract Algebra.* This is another text authored by a project member. It was originally published by PWS-Kent in the early 1990s, but the author has received the copyright back and released it under an open source license. Beezer has taught from this text the past two years, creating supplementary material utilizing Sage, in addition to already contributing code to the Sage library to support the book. First released as open source in Spring 2009, eight universities adopted the book for the Fall 2009 term. As a year-long course, this is a larger project to tackle next, and will be a collaborative project between two team members.

4.2. **Sage-enhanced curricular materials.** This project is not about creating new textbooks, which is why we are starting with existing open books. However, targeted materials will make it easier for faculty to quickly use Sage in their courses.

4.2.1. *Subject-specific guides.* For selected courses where we do not provide a Sage-enhanced textbook, or no suitable open book currently exists, we will create subject-specific tutorials. These short guides will explain how to use Sage to investigate questions in a standard course, following a typical order for such a course. Existing examples of these materials include multivariate calculus worksheets by Jason Grout and Ben Woodruff [ii] and John Perry's course notes for a Mathematical Computing course [21]. While not a textbook, an excellent text for a curious undergraduate is the book by Stein and Mazur on the Riemann Hypothesis. This text is richly illustrated with Sage output and contains extensive Sage code, so would be another addition to the curriculum that we could provide in a Sage-enhanced form. **ii**

Stipends will be provided to faculty outside the project team, ideally from our test sites, to author such materials. These will be designed from the start for conversion to Sage worksheets, providing a test of our system for this conversion.

4.2.2. *Interactive demonstrations.* Sage has a powerful, but extremely simple-to-use framework for creating web-based interactive demonstrations. Input boxes, sliders, selectors, and other controls are simple to create. Output can use HTML, tables, and typeset mathematics, and 2D or 3D graphics. Computations have the full Sage library at their disposal. An example of what an interact would look like to a student is in Appendix A. Stein created the first implementation and continues to improve it with significant contributions from Grout and many other Sage developers. Current work by Mitesh Patel, supported by an NSF FRG grant, will allow embedding these interactive demonstrations into standard web pages, independent of a Sage worksheet.

For select courses we will create comprehensive collections of demonstrations for inclusion in Sage-enhanced textbooks and into Sage itself. We will also create a easily-searchable repository of high quality, reviewed Sage interacts, which will be included in every copy of Sage.

4.3. **Sage into the classroom.** The heart of this project is getting Sage-enhanced materials in the hands of teachers and students to assist them in the teaching and learning of mathematics. We will work with selected faculty at partner institutions providing support for using Sage in their classrooms, offering technical support for Sage, assisting with the Sage-enhanced materials that we have created, and instructing them how to write their own Sage-enhanced materials. Our teacher-authors and their students will receive access to a Sage on a server dedicated to Project Sage.

We will recruit two cohorts of teacher-authors for Project Sage with the first cohort beginning Summer 2011 and the second beginning the following summer. Teacher-authors will receive a stipend for each year for their participation in Project Sage and support to attend a workshop on using Sage in the classroom. In return, the teacher-authors will meet the following expectations.

- Teacher-authors will commit to using Sage in their classrooms during their time with Project Sage. More specifically, they will be expected to use Sage enhanced materials extensively in a least one single semester course during the academic year.
- Teacher-authors will write and test a Sage-enhanced model for the class that they are teaching.
- Teacher-authors will submit annual reports during the commitment to Project Sage.
- Teacher-authors will offer training to others at their institution on how to use Sage and how to use Sage-enhanced materials in the classroom. Such training could be offered as departmental workshops or seminars.
- Teacher-authors from the first cohort will be selected to mentor the new teacher-authors in the second cohort.

[ii] Jason: publish these as a set of worksheets

4.3.1. *Recruitment and selection plan.* We will recruit two cohorts for our test sites with the first cohort beginning Summer 2011 and the second beginning the following summer. In order to forge a strong relationship with the teacher-authors and provide the best possible support, we will select five teacher-authors each year, and each teacher-author will partner with one of the investigators.

We will recruit teacher-authors from diverse institutions, especially from those institutions with diverse student profiles. The existing Sage community is a natural place to look for participants, and faculty from several institutions have already expressed interest, and letters of interest/support can be found in the appendix.

The following criteria will be used in the selection of the teacher-authors for Project Sage:

- Narrative statement regarding rationale for applying and the course or courses they plan to use Project Sage materials and their familiarity with Sage
- Limited familiarity with Sage, since we are establishing a model to get Sage into *all* classrooms.
- A letter of support from the teacher-author's department chair.

Once the teacher-authors have been selected, departments will be required to present written commitments indicating support for teacher-authors as they carry out their responsibilities integrating Sage into the classroom. Five teacher-authors will be selected to begin work Summer 2011 for a two-year period, and five more to start Summer 2012 for a one-year period.

4.4. **Sage infrastructure.** While the heart of Sage is its library of commands, the notebook interface and server configurations are key elements of a successful experience for students and faculty using Sage in their courses. So a portion of our work will be to improve Sage itself, in those areas where the improvements *directly support* educational applications of Sage. Professors at dozens of universities around the world are getting excited about how they can leverage the Sage notebook in their teaching.

> "With some colleagues in our university we have built a project around Sage for undergraduate students... *And the University has decided to support this project.* Good news.
>
> Now we will be facing the problem to build a Sage configuration which will work for say 200 students at the same time (students will use the notebook), and prepare professors for Sage teaching. There are 'some' technical problems to solve..." (Thierry Dumont, University of Lyon, France, on the Sage mailing list.)

4.4.1. *The Sage library.* Experience has shown us that teaching with Sage invariably suggests new capabilities or exposes needed commands. We will continue to refine and enhance Sage, at an accelerated pace supported by this grant. All changes will undergo the existing code review process, and will then become permanent contributions to Sage, maintained and further extended by the larger Sage community. We will unify and extend the code relative to the specific targeted courses we will be testing. We have already identified specific changes for linear algebra, abstract algebra, calculus, differential equations, number theory, and combinatorics, as well as broader areas like 2D and 3D graphics. Targeted fixes and improvements can greatly improve the educational experience for students. In many cases, undergraduate students can, and already have, contributed code and other suggestions; we plan to employ several in these efforts.

4.4.2. *Notebook development.* The Sage notebook interface allows students to use the full power of Sage on a remote server with only a web browser and a network connection. Even when Sage runs on a single unnetworked machine, the notebook is a powerful tool for experimentation and collaboration. So continued improvement of the notebook will translate directly into an improved student experience.

Using hardware purchased using an NSF SCREMS grant (DMS-0821725), William Stein substantially expanded the free public Sage notebook server at `sagenb.org` in 2009. Stein worked fulltime during Fall 2009, supported by the University of Washington, on vastly improving the robustness and scalability of the notebook. This work came at the right time, because use of the Sage notebook server has been growing rapidly. In Fall 2009, we have typically seen two thousand new accounts created on the public server every month.

4.4.3. *Sage servers the easy way.* Sage servers are critical for user's independence from specific hardware and operating systems, given the prevalence of web browsers and network connections. So initiating and maintaining a server should be as simple as possible, making it easy for faculty and system administrators everywhere to get their students started with Sage.

We will create a Virtual Box Sage notebook server appliance with a graphical interface, two virtual machines that provides a rock solid secure Sage notebook server setup. Users will be able to very easily install this appliance on Windows, Linux, OS X, and Solaris (x86) servers. The management interface will provide clear and easy documentation about setting up this server, creating new notebook servers for specific classes or instructors, starting and stopping notebook servers, monitoring resource usage, adding users and authentication frameworks (such as LDAP), and upgrading Sage with minimal user downtime.

For users at our selected test sites we will create a new mailing list for server administration issues, and in addition to the help we provide ourselves, we will hire a knowledgeable employee for a certain number of hours per day to answer support questions.

Several specific goals for notebook and server development include:

- Improve the notebook so that it will robustly handle up to 1000 simultaneous users when running on a single high-end server, as demonstrated by a robust automated test suite
- Implement management tools so administrators can manage the notebook load and better balance resources
- Create easy-to-use research tools for educators, so they can understand how their students use Sage

In summary, we propose to provide high-quality enterprise level software infrastructure and support to educators. What we propose is far from free or easy — it is in fact expensive. But with support now, literally millions of students will benefit at no cost to them. This is an excellent investment in mathematics education for the National Science Foundation.

5. Preparation, Qualifications

The five faculty in the Project Sage team together have many years of experience teaching undergraduates at a wide range of institutions, four are active Sage developers (including its founder), three are authors of open textbooks, all have significant mathematics research experience, and one specializes in mathematics education research. Everyone is familiar with, and has extensive experience using, open source software and tools in their teaching and other professional activities. Besides the applicability of this experience to the project's central theme, they are also extremely adept at working collaboratively while still being separated geographically. Working together, they have the wide range of complementary experience and skills that will make this project a success.

5.1. **Personnel.**

Dr. William A. Stein, Associate Professor of Mathematics at the University of Washington, is a leader in both computational and theoretical number theory, and the author of two published number theory books. He started the Sage project in 2005, and has passionately led its development ever since, including co-organizing over 25 workshops during the last 4 years and directing dozens of Sage development projects by students at Harvard, UC San Diego, and U of Washington. Dr. Stein will implement improvements to Sage, direct work by students, contribute curricular materials he has authored, supervise test sites, and co-organize workshops.

Dr. Kiran S. Kedlaya, Associate Professor of Mathematics at Massachusetts Institute of Technology and University of California, San Diego, is a leading researcher in arithmetic algebraic geometry. He has received an NSF CAREER grant, a PECASE award, and a Sloan Fellowship, and was selected to speak at the 2010 International Congress of Mathematicians. Dr. Kedlaya brings extensive experience in undergraduate and graduate education at all levels, from calculus to mentoring Ph.D. students. He is also deeply involved with mathematics competitions and other programs for exceptional students, such as the USA Math Olympiad, the Putnam competition, and the Art of Problem Solving Foundation. In addition, Dr. Kedlaya brings extensive knowledge of the Sage system, having been an active user and developer since 2005. Dr. Kedlaya will review improvements for Sage related to the project, supervise test sites, and provide additional overall guidance.

Dr. Jason Grout, Assistant Professor of Mathematics at Drake University, is an undergraduate teacher, an active researcher in combinatorial matrix theory and graph theory, and a Sage developer. Dr. Grout has contributed extensively to the Sage code base over the last three years in the linear algebra, graph theory, plotting, and notebook interface components of Sage. For example, in the notebook, he helped design the framework for interactive demonstrations and integrated an open source Javascript word processor into the notebook. Dr. Grout has used Sage extensively in research and in teaching a number of undergraduate courses. He has also given numerous presentations and tutorials on Sage, and in Summer 2010 will be co-directing an MAA PREP workshop with Dr. Beezer and Dr. Karl-Dieter Crisman, to help participants develop new curricular materials in Sage. Dr. Grout will help direct and implement the infrastructure changes needed for Sage to display and use online textbooks and enhance Sage's usefulness to students and faculty in the classroom. He will also help direct the interactions with the mathematics community via workshops and presentations. Dr. Grout will also contribute a large and growing personal library of Sage curricular materials for undergraduate classes, and test and refine the process for converting textbooks to Sage-enhanced materials by converting a set of extensive curricular materials for multivariate calculus that he has been developing with Dr. Ben Woodruff.

Dr. Robert Beezer, Professor of Mathematics at the University of Puget Sound, is a undergraduate teacher with 31 years of experience, an active researcher in algebraic graph theory, one of the first open textbook authors and a Sage developer. He began writing his open source linear algebra textbook in 2004 and has assisted Dr. Judson with the recent release of his very successful open source abstract algebra text. He began using Sage in 2007 and began contributing code in early 2009. He will lead the technical process of converting textbooks from LaTeX to Sage worksheets, producing a simple system for other authors to use. The pedagogical implications of this new capability will be explored as he incorporates Sage into existing textbooks on topics he teaches frequently, such as linear algebra, abstract algebra, combinatorics, calculus and cryptography. He will continue to contribute code to Sage where the new functionality enables a more complete experience for undergraduate students, and will suggest, review and test the project's improvements to interactive demonstrations and the notebook interface.

Dr. Thomas Judson, Associate Professor of Mathematics at Stephen F. Austin Sate University, is an active researcher in both mathematics and mathematics education, with 32 years teaching experience. He is the author of an open source undergraduate abstract algebra textbook, and will work with Dr. Beezer to produce Sage-enhanced materials for abstract algebra. Dr. Judson has worked extensively with undergraduate mathematics teachers and has mentored graduate students in the teaching of mathematics. In addition to his teaching and research accomplishments, Dr. Judson brings added experience in working with diverse groups both in the US and abroad. As such, he will work with Dr. Hassi to guide research and evaluation efforts for Project Sage. With experience in working with undergraduate faculty and graduate student mentoring, Dr. Judson will also provide insight and assistance in the implementation of Sage-enhanced materials in the undergraduate classroom. Project Sage efforts connected to research in mathematics education will be under the direction of Dr. Judson.

Members of the evaluation team are profiled in Section 6.3.

5.2. **Preparation.** Sage has come a long way in a short time. There are still many rough edges, some of which we will polish as part of this project. But with over 200 developers world-wide and tens of thousands of users, there is great potential for the pace of its development to increase. Members of the project team have significant experience with open textbooks, with using Sage in the classroom, and combining the two. For example, as a pilot project, a short primer on using Sage to study group theory was designed by Beezer to accompany Judson's abstract algebra text. This experiment clearly shows the feasibility of this process and clearly identifies the technical changes needed to make the process simple and routine for an author.

So the time is right to translate the experience of the project team into forms that faculty at other institutions can use to similarly benefit from teaching mathematics to their students with open mathematics software and open textbooks. Further, members of the project team, as Sage developers, are in a position to quickly and accurately shape changes in Sage based on the experience of working with faculty at other institutions that are new to Sage. National organizations, such as the MAA, are recognizing this trend by their sponsorship of WebWork and their support of a PREP workshop for Sage.

5.3. **Prior and current support.**

- Stein has successfully administered many grants supporting Sage development from organizations such as UC San Diego, U of Washington, Google, Sun, Microsoft, and the US Department of Defense. National Science Foundation grants include awards from the SCREMS program for the computing cluster that hosts `sagenb.org`, the FRG program (through the American Institute for Mathematics) and the COMPMATH program to fund two postdoctoral positions.
- Judson has-been/is on several NSF education related grants.[iii] **iii**
- Beezer has been awarded a competitive year-long sabbatical leave from the University of Puget Sound for the 2010-11 academic year. This will allow him to begin on converting textbooks, increasing the possible textbooks available for test sites to use beginning in the Fall 2011 term.
- Some of Grout's recent work on Sage been partially supported by an NSF postdoctoral appointment in Summer 2008.
- Beezer and Grout will co-direct (with Karl-Dieter Crisman) an MAA PREP workshop on Sage in Summer 2009, as part of a program funded by the NSF (Grant No. DUE-0817071)

6. Evaluation

Formative and summative evaluation of the proposed study is an important element of the project and will be conducted at different phases of the project. The evaluation study will address questions about the effectiveness of the project in creating and implementing the designed model and materials for undergraduate mathematics teaching and learning. The first aspect of the evaluation will focus on use of the developed Sage tools, the second aspect will evaluate the effectiveness of the designed materials in curriculum development, and the final third aspect will gather formative and summative information about the impacts of the tool and materials on teachers pedagogy and their students learning of undergraduate mathematics. This knowledge will guide future development of open curricular materials enhanced with open software.

Evaluation questions to this project include:

[iii]Need Tom's details

(1) What aspects of the developed Sage tool are beneficial to teachers, what challenges they face, and what kinds of support they need in using these tools?
(2) How do teachers use and apply the developed Sage-enhanced curriculum materials, such as Sage integrated with open source textbooks, and what are the benefits of these to their teaching of mathematics?
(3) How do the tools and materials impact teachers content knowledge, pedagogical content knowledge, classroom instructional practice and their students learning of undergraduate mathematics?

Information will be gathered from both the processes and outcomes of the project at different stages. Reports of the teaching and learning outcomes will be targeted to several case studies of participating teachers and their students. The formative components of the evaluation will focus on monitoring and gathering feedback on the quality of project activities from project participants, enabling the project to make mid-course corrections and plans for future development. The summative components will focus on the impact of the project on teachers instruction and student learning at the test sites. The basis for selection of participating teachers and classrooms will be based on interest in the project and support from the teachers institution.

6.1. **Study Design.** The study design consists of pre/post surveys, follow-up surveys included in yearly self-reports, and interviews for the participating teachers (10-20 teachers). In addition, students will answer an online post-survey focusing on their experiences and gains in learning mathematics with the use of the Sage tool and materials by their teachers. The design is informed by previous evaluation studies of professional development, education, and workshops in STEM fields [33, 34, 38, 39, 40, 42] and also of student outcomes of active instructional methods in undergraduate mathematics[35, 36].

6.1.1. *Pre-survey.* While registering for the project, the participants will complete a short pre-survey to gather demographic and contact data, basic information about their classes and institution, current teaching practices, and pedagogical needs related to the project. This information will also help the project leaders to address the needs for the workshop and later support.

6.1.2. *Post-survey and feedback.* Participants will complete a survey that gathers their feedback for the project leaders so that they can make adjustments for their future workshop and support during the implementation. The survey will also ask about participants plans for using the Sage tools and material in the future, which helps to guide the follow-up components of the study. They will also be asked to sign a consent form to participate in a follow-up interview. Evaluators will observe the summer workshop.

6.1.3. *Follow-up survey and reports.* After using the tool and materials for one year, the participants will complete a brief follow-up survey and file a report on their implementation. They will report their use of the tool and materials in their own classrooms and their future plans. This information will also help to sample the teachers for an interview.

6.1.4. *Student learning assessment.* Student learning will be evaluated after the first year of implementation with a post-survey based on the well-developed SALG instrument [41], in which students report about their experiences of and gains from a mathematics course and implemented instructional practices. This will provide information of student outcomes and also formative feedback for the teachers using the Sage tool and material in their classroom.

6.1.5. *Follow-up interviews.* Based on teacher responses in the follow-up survey and report and also on students self-reports, about a half of the teachers will be interviewed to study factors that affect their success in implementing the Sage tool and material. The interviews will explore their use of the tool and material, impacts of these on their instructional practices, and also their perception of students responses.

6.2. **Management and dissemination.** Drs. Judson and Hassi will collaborate with the project director, members of the project, and partner institutions to conduct the evaluation study and consult with Dr. Sandra Laursen (Ethnography & Evaluation Research (E&ER), Center to Advance Research and Teaching in the Social Sciences (CARTSS), University of Colorado) who has extensive experience evaluating other large mathematics and science education projects (NSF Awards # 0920126, # 0723600, # 0450088). Dr. Hassis (E&ER) strong expertise is in research and evaluation of mathematics education and her latest large evaluation study focuses on inquiry-based learning and teaching of undergraduate mathematics in four large research universities [36, 37], followed by an evaluation study of related workshops for teachers (NSF CCLI grant).

Dr. Judson and the project PIs will conduct site visits to partner institutions to observe and document the use of Sage-enhanced materials as the project progresses. The surveys and interviews will be prepared and conducted by Drs. Hassi and Judson. They will analyze all data by using appropriate quantitative and qualitative methods and will prepare one annual report per year documenting the evaluation activities and results. This will provide formative evaluation feedback to the project PIs and participating teachers in presentations and correspondence timed to inform design decisions and mid-course corrections. A final report will gather results of the different phases of the evaluation study after the implementation of the tools and material by the second cohort of teachers in 2013. These findings will be shared also with the broader mathematics education community by presentations and articles about the impacts on the teaching and learning of undergraduate mathematics from using materials integrated with open software.

6.3. **Evaluation personnel.**

Dr. Marja-Liisa Hassi, Associate Reseacher in Ethnography & Evaluation Research (E&ER/CARTSS) at the University of Colorado at Boulder, has a Masters Degree in Mathematics and a Ph.D. in Education. She has a strong experience in mathematics education research related both to the theory and various research and evaluation methods. Dr. Hassi has a long experience in conducting seminars and teaching courses in education, mathematics education, and research methods for undergraduate and graduate students. She has collaborated internationally with education and mathematics education researchers, had presentations in international conferences, and had published articles in conference proceedings and journals of mathematics education. Dr. Hassi has also served as a referee and associate editor for several education and mathematics education journals and conferences.

Dr. Sandra Laursen, is. . .

7. Dissemination

This project is principally about dissemination: how can we enable the widespread implementation of approaches which have already been proven to work. In addition to the working with our test sites, we will broadly disseminate the results of Project Sage through a variety of forums, several of which are already in place. The Project Sage curricular materials will be accessible on the Sage website, `sagemath.org`, where users can read documentation, make contributions to Sage, download Sage for free, and keep abreast of new developments in Sage.

7.1. **Sage website.** The Sage website [24] is an established venue for sharing materials related to Sage. This central location receives 90,000 visitors a month, with 42% of the visitors from the Americas (North, Central, and South) and 45% of the visitors from Europe. What we learn and create will be made available here with open licenses. Further, some of our work will be incorporated into Sage itself, which already has an elaborate distribution system, designed to make it extremely easy to download and run, on a personal machine or a server. Complete, integrated documentation ships with every copy of Sage (such as the 4,641 page reference manual), and it is possible that much of our material, *including complete textbooks*, could ship as a core component of Sage itself. Standard interactive demonstrations may also find a home in Sage itself where they could be organized topically, be searchable, and benefit from Sage's automated test-suite that users routinely run on a wide variety of hardware.

7.2. **Sage Days workshops.** Sage Days, which are a series of conferences and workshops devoted to the development of Sage, are an ideal venue for dissemination. To date there have been nineteen Sage Days conferences, with nine more planned. These workshops now include sessions dedicated to using Sage in the classroom, with the first such session taking place in December 2009 at the Clay Mathematics Institute on the final day of a week-long workshop on Sage and number theory. It attracted roughly thirty college faculty (and a few high school faculty) from around the Northeast, all eager to learn more about the use of Sage in educational settings.

A Sage Days devoted to training others to use our materials, and receiving and incorporating feedback and new ideas, will be entirely isomorphic to the way Sage Days have been used to drive Sage development. We will have a Sage Days event in Summer 2012, involving the project team and all ten test sites, as an opportunity to discuss the project at the midpoint of our experiences in the classroom. Another Sage Days event at the American Institute of Mathematics in Summer 2013 will be an opportunity to more generally review the experiences of the project and plan for the future of integrating open software and open textbooks into the undergraduate mathematics curriculum.

7.3. **National workshops.** We will apply to offer special sessions and workshops on the use of Sage in the classroom at the Joint Mathematics Meetings and MathFest, where we will be able to share the results of Project Sage with the undergraduate teaching community. We will also publish articles about the impact of using our materials on the teaching and learning of undergraduate mathematics. Drs. Beezer and Grout are part of a team delivering a workshop on how to use Sage in the classroom under the MAA's Professional Enhancement Program (PREP). We will apply to lead future PREP workshops that will incorporate the results of Project Sage.

7.4. **AIM open textbook initiative.** The American Institute of Mathematics will execute a pilot project to test the feasibility of sponsoring a series of open mathematics textbooks. A hindrance to the adoption of open textbooks is the lack of an acknowledged authority to vouch for the content. Usually this is a role played by a publisher, so a recognized organization within the mathematics community with a trusted reputation, such as AIM, will be another component to disseminating our materials broadly.

8. Budget Overview

(Save this for later)

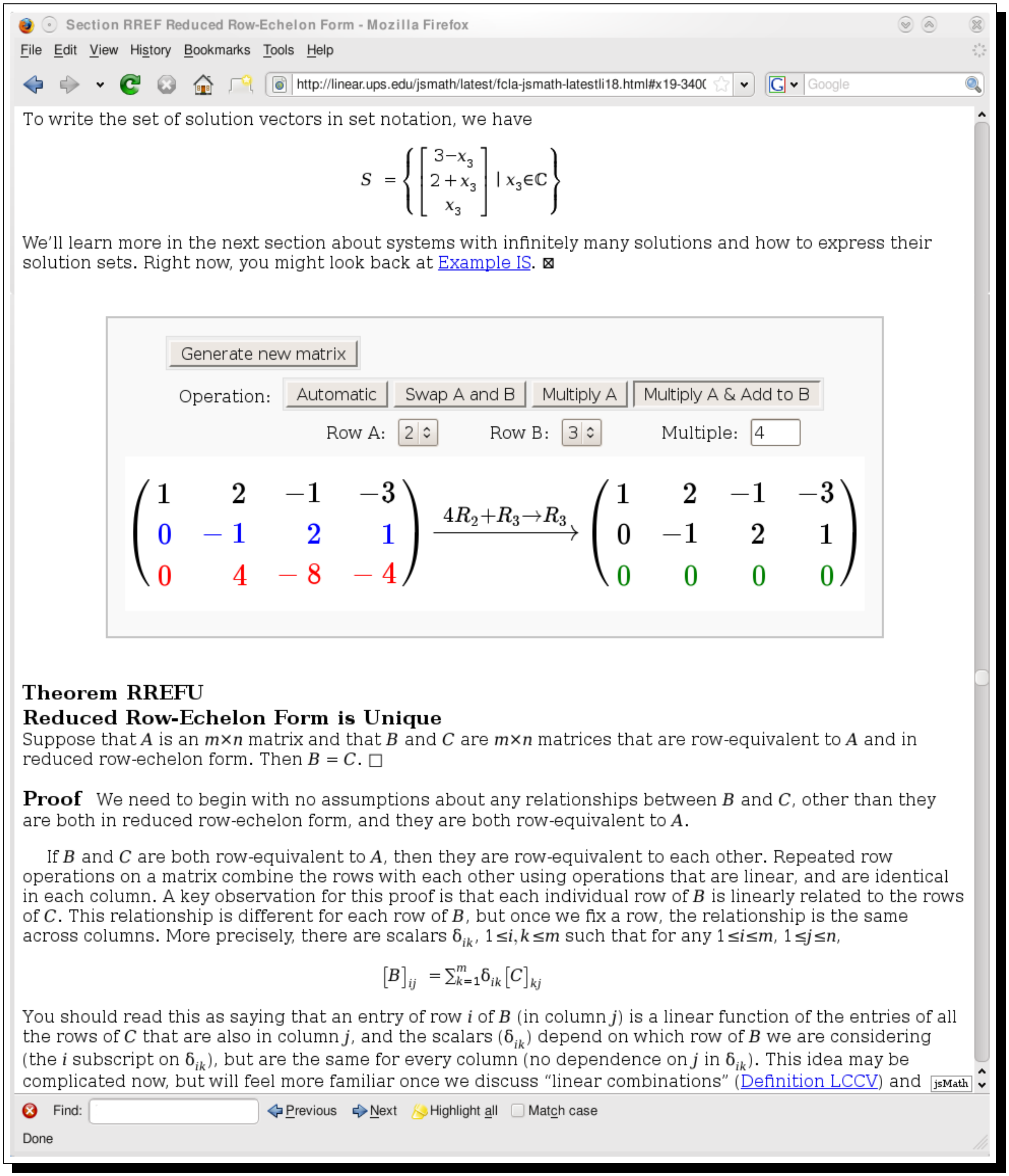
- Total Salaries
- Total Stipends to test sites (travel money here?)
- Stipends for curricular materials
- Total travel, equipment
- Total package for infrastructure

9. Future Directions

The limited scope of this proposal suggests several broader new directions. In particular, the project team was intrigued by the possibilities for using Sage to generate and check homework through integration with a system like WebWork, but decided such an initiative was too ambitious and disjoint to be included in this proposal.

- Create new open textbooks with Sage enhancements designed in from the start.
- Create Sage-enhanced open textbooks for every standard course in the undergraduate mathematics curriculum.
- Integrate Sage with an open source homework delivery system such as WebWork.

- Integrate collaboration features of the Sage notebook with a course management system such as Moodle.
- More????



To write the set of solution vectors in set notation, we have

$$S = \left\{ \begin{bmatrix} 3-x_3 \\ 2+x_3 \\ x_3 \end{bmatrix} \mid x_3 \in \mathbb{C} \right\}$$

We'll learn more in the next section about systems with infinitely many solutions and how to express their solution sets. Right now, you might look back at Example IS. ⊠

$$\begin{pmatrix} 1 & 2 & -1 & -3 \\ 0 & -1 & 2 & 1 \\ 0 & 4 & -8 & -4 \end{pmatrix} \xrightarrow{4R_2+R_3 \to R_3} \begin{pmatrix} 1 & 2 & -1 & -3 \\ 0 & -1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Theorem RREFU
Reduced Row-Echelon Form is Unique
Suppose that A is an $m \times n$ matrix and that B and C are $m \times n$ matrices that are row-equivalent to A and in reduced row-echelon form. Then $B = C$. □

Proof We need to begin with no assumptions about any relationships between B and C, other than they are both in reduced row-echelon form, and they are both row-equivalent to A.

If B and C are both row-equivalent to A, then they are row-equivalent to each other. Repeated row operations on a matrix combine the rows with each other using operations that are linear, and are identical in each column. A key observation for this proof is that each individual row of B is linearly related to the rows of C. This relationship is different for each row of B, but once we fix a row, the relationship is the same across columns. More precisely, there are scalars δ_{ik}, $1 \le i, k \le m$ such that for any $1 \le i \le m$, $1 \le j \le n$,

$$[B]_{ij} = \sum_{k=1}^{m} \delta_{ik} [C]_{kj}$$

You should read this as saying that an entry of row i of B (in column j) is a linear function of the entries of all the rows of C that are also in column j, and the scalars (δ_{ik}) depend on which row of B we are considering (the i subscript on δ_{ik}), but are the same for every column (no dependence on j in δ_{ik}). This idea may be complicated now, but will feel more familiar once we discuss "linear combinations" (Definition LCCV) and

Prototype of a Sage-enhanced mathematics open textbook

Appendix B. Open Source Mathematics Textbooks

Beyond our initial candidates there is a wide range of books available that could be converted to cover portions of the standard undergraduate curriculum. Except where noted, the textbooks below are licensed with a GNU Free Documentation Licenses (GFDL) or a Creative Commons License that allows anyone to distribute modified versions, typically only requiring attribution of the original author's work and the use of the same license. New open source textbooks continue to be released and evidence suggests many authors are considering the option seriously.

- *A First Course in Linear Algebra*, Robert Beezer `http://linear.pugetsound.edu/`
  Second-year university level text, concentrating on proof techniques. Used at thirteen schools since Fall 2007. Contains some Sage commands.
- *Abstract Algebra: Theory and Applications*, Tom Judson `http://abstract.ups.edu/`
  Standard upper-division treatment of groups, rings, domains, fields, Galois Theory. Used by seven schools in its first year available as open source.
- *Combinatorics Through Guided Discovery*, Kenneth Bogart
  `http://www.math.dartmouth.edu/news-resources/electronic/kpbogart/`
  Problem book in combinatorics with roughly 400 problems designed to teach the subject. Released as open source. (NSF Grant No. DUE-0087466)
- *Elementary Number Theory: Primes, Congruences, and Secrets*, William Stein
  `http://wstein.org/ent/`
  Classical elementary number theory and elliptic curves, with applications and extensive Sage examples. Published by Springer-Verlag, freely available May 2010. (NSF Grant No. 0653968)
- *Vector Calculus*, Michael Corral `http://www.mecmath.net/`
  Standard treatment of multivariate calculus. Topics include vector calculus, partial derivatives, multiple integrals, theorems of Green and Stokes.
- *Trigonometry*, Michael Corral `http://mecmath.net/trig/`
  An in-depth, comprehensive and unified treatment of the standard topics.
- *Cryptography*, David Kohel
  `http://echidna.maths.usyd.edu.au/~kohel/tch/Crypto/crypto.pdf`
  Classical ciphers and their cryptanalysis, modern stream ciphers and public-key cryptography. Significant appendices on Sage.
- *What is Riemann's Hypothesis?*, Barry Mazur, William Stein `http://wstein.org/rh/`
  Richly illustrated elementary treatment of a central problem of modern mathematics, designed for an undergraduate audience. Backed up with extensive Sage worksheets already. (NSF Grant No. 0653968)
- *Numerical Analysis*, Steven Pav `http://scicomp.ucsd.edu/~spav/pub/numas.pdf`
  Comprehensive book-quality lecture notes. Extensive use of Octave, an open source clone of Matlab.
- *Calculus: Modeling and Application*, David A. Smith, Lawrence C. Moore
  `http://www.math.duke.edu/education/calculustext/`
  Calculus textbook with significant electronic components. Mathematical Association of America project with "all rights reserved" (i.e. does not currently have an open license). (NSF Grant No. 0231083)

References

[1] Beezer, Robert A., *Sage (Version 3.4)*. SIAM Review, **51** (2009) 785–807.

[2] Banchoff, Thomas. *Interactive Geometry and Multivariable Calculus on the Internet*, in Ki-hyong Ko, Deane Arganbright (Eds.), Enhancing university mathematics: Proceedings of the first KAIST international symposium on teaching, Conference Board of the Mathematical Sciences, Issues in Mathematics Education, **14** (2007) 17–31.

[3] Branin, Joseph J., Mary Case, *Reforming Scholarly Publishing in the Sciences: A Librarian Perspective*. Notices of the American Mathematical Society, 45 (1998) 475–486. `http://www.ams.org/notices/199804/branin.pdf`

[4] Schwarzenegger, Governor Arnold. Press Release, Free Digital Textbook Initiative `http://gov.ca.gov/press-release/12225/`

[5] *Calculus & Mathematica*, University of Illinois at Urbana-Champaign, `http://www-cm.math.uiuc.edu`

[6] *Calculus & Mathematica*, The Ohio State University, `http://socrates.math.ohio-state.edu/`

[7] Durbin, Senator Dick, News Release, October 6, 2009. `http://durbin.senate.gov/showRelease.cfm?releaseId=318797`

[8] GAP—Groups, Algorithms, Programming—a System for Computational Discrete Algebra, `http://www.gap-system.org/`

[9] Heid, M. Kathleen, *The technological revolution and the reform of school mathematics*. American Journal of Education, **106** (1997) 5–61.

[10] Heid, M. Kathleen, Michael T. Edwards, *Computer algebra systems: revolution or retrofit for today's mathematics classrooms?* Theory into Practice, **40** (2001) 128–136.

[11] *Interactive Internet-Based Teaching and Learning in Mathematics*, NSF Grant No. 0428280.

[12] Integer Matrix Library, `http://www.cs.uwaterloo.ca/~astorjoh/iml.html`

[13] Java Components for Mathematics, NSF Grant No. DUE-9950473, `http://math.hws.edu/javamath/`

[14] jsMath: A Method of Including Mathematics in Web Pages, `http://www.math.union.edu/~dpvc/jsMath/`

[15] Leigh-Lancaster, David, et al., *The 2007 Common Technology Free Examination for Victorian Certicate of Education (VCE) Mathematical Methods and Mathematical Methods Computer Algebra System (CAS)*, in M. Goos, R. Brown, K. Makar (Eds.), Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia, 2008.

[16] Mathematica, Wolfram Research. `http://www.wolfram.com/products/mathematica/index.html`

[17] Maple, Maplesoft. `http://www.maplesoft.com/Products/Maple/`

[18] National Institutes of Health Public Access Policy, `http://publicaccess.nih.gov/`

[19] The Macsyma Saga. `http://maxima-project.org/wiki/index.php?title=The_Macsyma_Saga`

[20] Palmiter, Jeanette R., *Effects of computer algebra systems on concept and skill acquisition in calculus*. Journal for Research in Mathematics Education, **22** (1991) 151–156.

[21] Perry, John, Course notes for MAT 305, Mathematical Computing, University of Mississippi, `http://www.math.usm.edu/perry/mat305fa09/index.html`.

[22] The R Project for Statistical Computing, `http://www.r-project.org/`

[23] REDUCE, A portable general-purpose computer algebra system. `http://reduce-algebra.sourceforge.net/`

[24] Sage, `hhttp://www.sagemath.org`

[25] Sage Notebook Server, `http://sagenb.org`

[26] Sage Server (public), Hungary, `http://sage.math.u-szeged.hu/`

[27] Sage Server, cell-phone interface, Sungkyunkwan University, Korea, `http://math1.skku.ac.kr/wap_html`

[28] SageTEX, `http://tug.ctan.org/pkg/sagetex`

[29] SciPy, Scientific Tools for Python. `http://www.scipy.org/`

[30] TeX4ht: LaTeX and TeX for Hypertext, `http://www.cse.ohio-state.edu/~gurari/TeX4ht/`

[31] White House Office of Science and Technology Policy, Public Consultation on Public Access Policy, `http://www.whitehouse.gov/blog/2009/12/09/ostp-launch-public-forum-how-best-make-federally-funded-research-results-available-f`

[32] Washington State Student Completion Initiative, Washington State Community and Technical Colleges. `http://www.gatesfoundation.org/press-releases/Pages/grant-to-launch-washington-state-student-completion-initiative-091014.aspx` [iv] **iv**

[33] Burke, K. A., Greenbowe, T. J. Gelder, J., *The Multi-Initiative Dissemination Project Workshops: Who attends and how effective are they?* Journal of Chemical Education, **81** (6) 2004, 897–902.

[34] Gafney, L., Varma-Nelson, P. *Peer-led team learning: Evaluation, dissemination and institutionalization of a college-level initiative.* Innovations in Science Education and Technology, vol. 16. (2008) New York: Springer.

[iv] Evaluation references below need to be merged above.

[35] Hassi, M.-L. *Empowering undergraduate students through mathematical thinking and learning.* Safford-Ramus, K. (ed.), A Declaration of Numeracy: Empowering Adults through Mathematics Education, Proceedings of the 15th International Conference of Adults Learning Mathematics (ALM) (2009) 53–69. Lancaster, PA: DEStech Publications.

[36] Hassi M-L., Laursen, S. *Studying undergraduate mathematics: Exploring students' beliefs, experiences and gains.* In S.L. Swars, D.W. Stinson, S. Lemons-Smith (Eds.). Proceedings of the 31st Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (PME-NA), (2009) 113–121. Atlanta, GA: Georgia State University.

[37] Laursen, S. L., Hassi, M.-L. *Inquiring about inquiry: Progress on research and evaluation studies of Inquiry-Based Learning in undergraduate mathematics at four campuses.* Presented at the Joint Mathematics Meetings, January 5–8, 2009, Washington, DC.

[38] Lewis, S. E., Lewis, J. E.. *Effectiveness of a workshop to encourage action: Evaluation from a post-workshop survey.* Journal of Chemical Education, **83** (2) (2006) 299–304. `http://www.wcer.wisc.edu/publications/LEADcenter/nt-wkshop.pdf`

[39] Sell, G. R. *A review of research-based literature pertinent to an evaluation of workshop programs and related professional development activities for undergraduate faculty in the sciences, mathematics and engineering.* Report commissioned by SRI International as part of an evaluation for the Undergraduate Faculty Enhancement (UFE) program for the National Science Foundation. (1998) Available from the author.

[40] Smith, L. K., Laursen, S., Schott, C. *ReSciPE for scientific inquiry: Professional development for scientists to support their work with education.* Eos Trans. AGU **87**(52) (2006), Fall Meet. Suppl., Abstract U51D-05

[41] Student Assessment of their Learning Gains (SALG) (n.d.). `www.salgsite.org`

[42] Thiry, H., Laursen, S. L., Hunter, A.-B. *Professional development needs and outcomes for education-engaged scientists: A research-based framework and its application.* Journal of Geoscience Education, **56**(3) (2008) 235–246.