UTMOST: UNDERGRADUATE TEACHING IN MATHEMATICS WITH OPEN SOFTWARE AND TEXTBOOKS

Proposal for a National Science Foundation
Course, Curriculum, and Laboratory Improvement Phase 2 Award

1. Introduction

Software for mathematical explorations, including computer algebra systems (CAS), have held great promise for education since their first appearance in the 1960s (Reduce, Macsyma [27, 36]) and the introduction of Maple and Mathematica in the 1980s [26, 51]. Currently, CAS are widely found in the undergraduate classroom and a considerable amount of mathematics education research has focused on the use of CAS in the learning of undergraduate mathematics. Studies have documented the improvement in students' understanding [17, 18, 23, 32]. The use of technology allows students to visualize complicated surfaces in multivariable calculus, easily deal with larger matrices in linear algebra, and replaces tables of Laplace transforms in differential equations. There is strong agreement that most calculus students should learn the basic techniques of integration; however, CAS replaces the lengthy tables of integrals that were commonplace two decades ago. In addition, numerical software has allowed a qualitative approach to teaching differential equations. Most differential equations cannot be solved explicitly, but numerical and graphical software has enabled teachers to incorporate a qualitative approach in the courses that they teach [4]. The ability to make large computations, quickly and without errors, and often leading to a graphical output. can greatly aid a students' understanding of difficult ideas in mathematics and provides them with an incredible capacity for experimentation and conjecture.

Computer algebra systems are now common in the undergraduate classroom, yet we rarely see a seamless integration with the curriculum. To accommodate competing commercial systems which divide the market, textbooks typically offer supplements for several different CAS, or they are "technology-enhanced" with generic sidebars; otherwise the textbook can only be marketed to the subset of the audience that licenses a particular computer algebra system. Also, licensing restrictions for campus use, the expense of personal copies, and underpowered hardware often mean that students can only work with commercial software in campus labs. Some institutions are unable to afford the cost of building such labs for their students.

There have been efforts at seamless integration of technology and curriculum. An example of such an experiment is the Calculus & Mathematica project, which generated much excitement in the early 1990s and had demonstrable success in helping diverse students learn more effectively. However, today its use seems limited to the two institutions where it originated [30, 49]. Likewise, the use of interactive Java applets to support teaching mathematics (such as [2, 6, 19]) does not seem to have been widely adopted. The undergraduate curriculum has not seen the broad transformative effect of these powerful tools for increasing the learning and understanding of mathematics. What is needed to increase the uptake of software for mathematical exploration across the spectrum of undergraduate education so we will see the benefits that isolated, successful pilot programs have predicted?

1.1. **An open approach.**

Sage [3, 45] is free open-source software designed to be an alternative to Magma, Maple, Mathematica and Matlab. Coinciding with the development of Sage, there is a general movement to freely-available open textbooks that includes many quality texts in mathematics ([8, 31], Appendix B). Our proposition is that freely-available open software, open textbooks, open standards, and open licenses can allow teachers everywhere to transform the undergraduate mathematics curriculum by tightly and seamlessly integrating mathematics software with more traditional curricular materials. We will test this hypothesis by integrating Sage into existing open textbooks and other curricular materials. This will place the full computational power of Sage *directly into* a student's text, usable at all times and from anywhere, simply via a web browser. For the institution and the instructor, the cost and time-consuming inconveniences of commercial software are removed by free software, especially when installed on remote servers, freely accessible with no license restrictions. Furthermore, the essential nature of open software means that curricular decisions and needs can drive the development of the software, with the classroom teacher actively advising, or actually doing, the software development. Leveraging these inherent advantages of an open approach to software and curricular materials, the promise of mathematical software in education can be fully realized by faculty and students. Our work will create and disseminate a model for this integration, addressing both the pedagogical and technical aspects, so that other faculty authors may easily take the same approach on their own.

More specifically, UTMOST will build and test a model for easily integrating mathematics software and open educational materials into the mathematics curriculum and classroom as follows.

- We will create a system that makes it easy for authors to convert open textbooks and other curricular amterials to Sage worksheets interspersing runnable interactive demonstrations and live Sage code with publication-quality typeset mathematics.
- We will convert existing mature open textbooks to this format and create new curricular materials targeting this format, as demonstrations and tests of both the techical and peadagogical aspects of this new approach.
- We will partner with ten diverse institutions to test these materials in a wide variety of courses, while providing support for their use and assistance with the creation of new materials.
- We will evaluate the effectiveness of our model for making it easy to adopt open mathematics software and textbooks and making it easy to create integrated open curricular materials, and we will measure the resulting impact on teaching practices and the learning of mathematics with the expert assistance of professional evaluators from Ethnography & Evaluation Research at the University of Colorado at Boulder.

In addition to workshops and other presentations, materials created or enhanced by UTMOST will be widely distributed with open licenses and made freely available on Sage's website (90,000 visitors a month), distributed with every copy of Sage (over 6,000 downloads a month), and/or hosted on a newly-created open textbook website at the American Institute of Mathematics [1].

2. Trends in Scientific Communication

The cost of academic research journals, especially in science, accompanied by restrictions imposed by copyright law and new possibilities afforded by technology, are collectively referred to by librarians as the "serials crisis" [5]. Faculty have come a long way in their efforts to return scientific publication to a free exchange of ideas. Electronic journals and public repositories now publish articles with licenses that explicitly allow for sharing new results among research communities and others, without copyright fees and with ubiquitous access via the Internet. Government initiatives, such as the Public Access Policy of the National Institutes of Health [28] and the Policy Forum on Public Access of the White House Office of Science and Technology [29], are working to accelerate this trend.

Following on the sea change in research publication, the next wave is open textbooks. Faculty are all too familiar with the problems that plague commercial textbook publication, such as high prices, edition churn, and orphaned works. Open licenses are now being used to assert control of these critical resources for education, with faculty in mathematics, computer science and business as the leaders [12, 46]. Government is poised to accelerate this trend at all levels, with Senator Durbin proposing legislation directing the Department of Education to award grants for the creation of open textbooks by faculty [11], Washington State's initiative to provide open textbooks for the eighty highest-enrollment courses in their community college system [50], and California's initiative to create free digital textbooks for its high schools [41].

With the emergence of viable and comprehensive open source software for mathematics, there is now a spectacular opportunity for mathematics teachers to use, extend, and create this important software for mathematics education and shape it to reach its full potential in the service of educating students. UTMOST will create an easy path for all faculty to make the initial transition to open software, open textbooks, and open curricular materials in their courses. This path will make it easy for diverse schools and faculty to employ mathematical software to transform the classroom into an interactive laboratory which takes the study of mathematics to a new level.

3. Why Sage?

Sage is a natural choice for software to realize the benefits of an open approach to the undergraduate mathematics curriculum. Sage is a comprehensive program, with an open development process, an easy-to-use interface built on top of standard web browsers, tight integration with LaTeX, an industry-standard programming language, and a modular design philosophy. These are consequences of a steadfast commitment to open design principles and concrete examples of the benefits of this approach. With an open license and a platform-independent interface, Sage removes substantial financial and logistical barriers to classroom use of mathematics software. In this section we describe the many features of Sage that make it good choice for integrating mathematics software with open textbooks and other curricular materials.

3.1. A comprehensive program.

Sage is growing to be a comprehensive program for mathematics. Its modular design allows symbolic, exact, and numerical approaches to mathematics to coexist equally. Mathematical objects, such as functions, differential equations, rings, fields, modules, and vector spaces are "objects" that look and behave as their abstract mathematical definitions intend. Sage has been designed from the outset to incorporate many different computational strategies, and so does not rest on a foundation such as pattern-matching, which favors symbolic computation, or floating-point numbers, which favors numerical work.

Institutional, departmental, and course-by-course decisions about the use of a CAS require a commitment to a particular vendor's vision for the curriculum. With a modular design that allows for packages specializing in different approaches, Sage can seamlessly support many approaches to computationally exploring mathematical problems. Additions and extensions to Sage, driven by real curricular needs, can be easily and quickly incorporated. As individuals and institutions extend the software, the entire mathematics community benefits from a comprehensive program that student and faculty can employ in courses ranging from pre-calculus and introductory statistics through to advanced courses like abstract algebra and number theory.

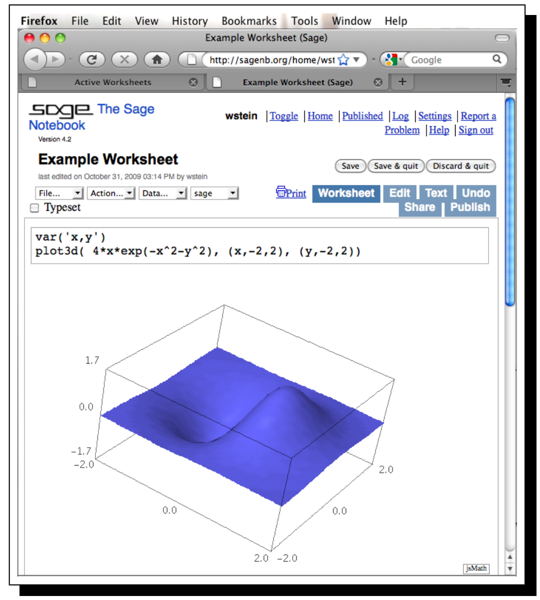
3.2. Curriculum and open development.

We have had computer algebra systems for over forty years now, with mature commercial products available for the past twenty years. The closed development process for commercial software creates a high barrier for teachers and students to alter or extend the software to meet their curricular needs. For example, while external extensions, such as libraries or packages, can extend proprietary systems, these must be purchased, distributed, and installed by every end-user. However, an open development process allows students and faculty to shape the core technology in a timely fashion to support the transformation of teaching and learning of mathematics on a broad scale. Software developed openly and collaboratively, such as Sage, allows the teaching and learning of mathematics to drive the technology, rather than the technology driving the teaching and learning. Just as major research problems drive the direction of pure mathematics, the curriculum and needs of the users directly drives the development of open mathematics software. Two concrete examples of this important principle are given in Section 4.

3.3. Sage anytime, anywhere.

Sage's notebook interface allows a student to communicate with a Sage server through any web browser — interactively running commands, viewing textual and graphic output, and annotating their results. So this removes many of the logistical barriers to using software in the study of mathematics. A student can employ the full power of Sage on a remote server with only a web browser and a minimal network connection, independent of the student's preference for an operating system. This is in contrast to an institution providing an expensive lab of machines with commercial software. In fact, a student does not need to purchase or configure any additional software at all to harness the full power of Sage.

The Sage notebook interface relies heavily on industry-standard Javascript and acts as an application similar to wiki software, Google Docs or GMail. Students have designed and implemented much of the current notebook. For example, Tim Dumol, a high school student in the Phillipines, has been a very active developer in Fall 2009.



The Sage notebook interface running in Mac OSX Safari

Then, what is required for an institution to establish a Sage server? Every copy of Sage comes ready to run as a server, and of course this software is free. Hardware can be chosen to meet the demands of the particular application. But what if an institution is unable to provide or maintain a server? Then there are publicly-available servers, the flagship server being `sagenb.org`, a \$100,000 rack of servers located at the University of Washington that was fully funded by a NSFSCREMS grant (NSF Grant No. DMS-0821725), and is currently home to over 20,000 user accounts. Imagine students at the poorest universities running the same version of Sage, on the same hardware cluster, side-by-side with leading researchers in computational number theory. Constructed in January 2009, this powerful server should be a viable resource for many years to come.

This is a key consequence of the design of Sage, and its free availablity, that cannot be underestimated for its use in education. Students can access Sage servers from any web browser to perform computations remotely using an intuitive and familiar interface, on devices including underpowered desktop machines or mobile devices like netbooks, smart phones, or simple cell phones. Students are not tied to a particular lab, or even to their campus. Right now, anyone, anywhere, can connect to `sagenb.org` and within minutes be productively using Sage to explore new mathematical ideas. At the extreme, see for example the cell phone interface [39] developed by students in Korea and backed by a publicly available server at their university. Publicly-accessible notebook servers continue to appear throughout the world (most recently in Hungary [38]), and many more run behind campus firewalls for dedicated use [40]. For occasions when there is no network access it is possible to simply install an additional copy of Sage directly onto a variety of different hardware (principally Linux, Mac, Windows).

3.4. **Building the car, not reinventing the wheel.** The genius of Sage is the way it unifies over one hundred mature, best-of-breed, open source packages. These packages range from focused libraries that excel at specific types of computations (e.g., the Integer Matrix Library for the fastest implemtations of solutions to linear systems over the rationals [20], or M4RI for the fastest implementations of exact computations with binary matrices [25]) to complete applications or general libraries for broad areas of mathematics (e.g., R for statistics, GAP for group theory, and SciPy for numeric scientific computations [14, 35, 42]). Sage ties these packages into a single open source system with a consistent interface, making it easy for a teacher or student to smoothly explore vast areas of mathematics.

As a student moves from course to course, the Sage notebook interface remains the same and the Sage command syntax remains the same, even if the particular mathematics relevant to a new course may be provided by an entirely different package. So once students and faculty become familiar and proficient with Sage it becomes trivial to use it again in another course. As the Sage user and developer communities grow, Sage will provide commands for even more areas of mathematics.

3.5. **Communicating mathematics.** TEX, with its add-on package LATEX, is another open source success story, and is the typesetting language of choice for mathematicians and other technical disciplines. Every mathematical object in Sage can be typeset in LATEX automatically. The Sage notebook uses jsMath [9] to typeset mathematics beautifully in a Sage worksheet (within a standard web browser). Furthermore, the Sage notebook interface allows a user to insert new text, including typeset mathematics. This is another example of how open standards and open software combine to make powerful tools, which is especially relevant for our plan to convert LATEX documents into Sage worksheets.

3.6. **A standard programming language.** The many components of Sage are held together with a significant library of new code, written in the industry-standard Python programming language. Through the power of Python, Sage brings these to the user by easily integrating these new packages, bringing new functionality, or improving existing functionality. More and more packages are appearing for mathematics and science written in Python, often with open licenses [34, 42].

Students may use Sage through point-and-click interactive demonstrations written by others, or they may execute a sequence of single-line commands in the notebook. However, for more involved computations, they can use Python, since it also serves as the user language in Sage. This is in contrast to other comprehensive programs for mathematics that have chosen to create and develop their own languages, which are of little use outside of the CAS itself. A user with knowledge of Python is ready to be incredibly productive in Sage immediately, while a student new to programming can receive a basic education in Python as a possible by-product of their experience with Sage — a skill that is readily transferable to a wide variety of applications in mathematics, science and engineering.

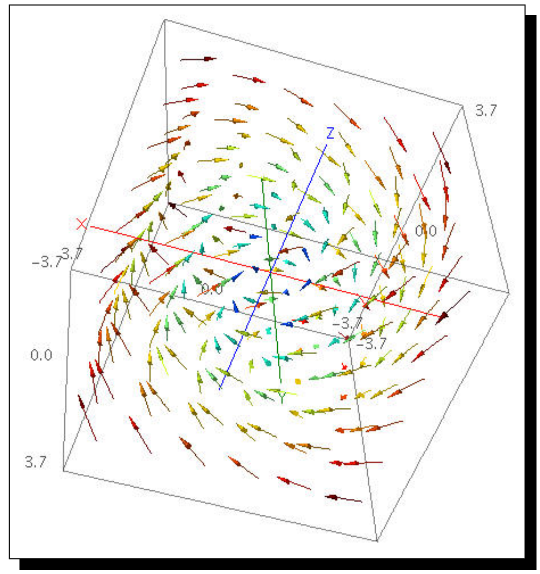
4. Two Examples of Open Development

In this section we provide two concrete examples of how an open development process applies classroom experiences directly to create better and more comprehensive tools for use in the teaching and learning of mathematics.

4.0.1. *3D vector fields.* Jason Grout, then a postdoctoral associate at Iowa State University, was asked one day by a student if Sage had a 3D vector field plotting function. Grout quickly wrote a simple one. Open development communities encourage sharing even experimental code, so he posted his code online in Sage's publicly-accessible database of enhancements and bugs. Robert Bradshaw, a graduate student at University of Washington who had written most of the 3D graphing code in Sage, suggested a small change that greatly increased the efficiency of the function. Several months later, another person authored a very similar function which had slightly improved options and published their code on the public Sage notebook server. In Fall 2008, Grout needed the 3D vector field plotting function for his multivariable class. He made a few suggested changes to the function and posted it for his class to use. If he had been using a commercial CAS, this is where the story likely would have ended.

In the next few months, improvements were made to the internals of Sage 3D graphics (many by William Cauchois, a University of Washington freshman supported for a summer by an NSF VIGRE grant). Some of these improvements were directly the result of wanting to make the 3D vector field plotting function easier to write. Jason then taught multivariable calculus again. He incorporated the best ideas from the various sources and posted a documented, efficient version of the function, this time asking the community for a formal review (a prerequisite for his code to be added to the core Sage library). Marshall Hampton, an Assistant Professor at University of Minnesota, Duluth, who has been active in the Sage project, reviewed the function immediately, with the comment "Very nice, positive review. This is great timing since I am about to teach vector fields in a week or two." The function was incorporated into the next release of Sage, two weeks later, ready for immediate use by the entire Sage user community.

Now not only do the original students benefit from the invested effort, but the collaborative effort of at least five teachers and students has directly led to a function that will serve all who use Sage.



$\vec{f}(x, y, z) = y\vec{\imath} + z\vec{\jmath} + x\vec{k}$ plotted with Sage's `plot_vector_field3d` command

4.0.2. *Dicyclic groups.* When Rob Beezer teaches introductory group theory, as he did in Fall 2009 with Tom Judson's open source text, he explores classifying all the finite groups of order less than 16 as a way of exhibiting the power of the theorems developed through the term. Each of these groups is easily constructed and realized in Sage as a permutation group, except for one of the three non-abelian groups of order 12. This fall, he decided to add this group to the collection of constructions in Sage, but first, on a Friday evening, he polled the Sage development mailing list about the best way to realize this group.

John Palmieri, an Associate Professor at the University of Washington who is an expert in algebraic topology, responded 90 minutes later with information about the dicyclic groups, a sequence of groups of order $4m$, including the group of order 12 in question, the quaternion group of order 8, and the generalized quaternions of order 2^n. David Joyner, a Professor at the US Military Academy who was the second Sage developer after Stein, and who implemented of much of Sage's support for group theory, joined the conversation an hour later. Jaap Spies weighed in from Holland with a source for more information. With this help, Beezer was able to easily create an efficient permutation representation of these groups in the next few days, and posted code and documentation, including comprehensive information and references about the group's construction and properties, to Sage's enhancement database. Joyner positively reviewed the code and documentation several days later. Two weeks later, this new code and documentation was incorporated in a preliminary Sage release. Within a month, Mike Hansen, a volunteer working from Thailand, oversaw the release of the next version of Sage, which included this code.

In this case, the net result is new capabilities for Sage, relevant to the undergraduate curriculum, with associated mathematical explanations in the Sage documentation. Beezer's course was improved by this exercise, and the benefits are now available to all through Sage. The dicyclic groups would also now appear to be a worthwhile addition for Judson's text, completing a virtuous cycle.

These two examples illustrate many strengths of the open Sage development process. Teachers, students, and individuals bring their talents as educators, mathematicians, and programmers together to make Sage an astonishing product of open development.

5. Implementation

Our goal is to have a broad, positive influence on the teaching and learning of mathematics. By using open curricular materials, integrated with powerful and comprehensive open software, we will realize a transformative effect. UTMOST will revolve around three coordinated efforts: (1) creating Sage-enhanced curricular materials, (2) working with, and providing classroom support for, teachers using these new materials in the classroom, and (3) providing the Sage infrastructure to support these materials. More specifically, UTMOST can be divided into five main activities.

(1) We will create a system to convert open mathematics textbooks to Sage worksheets that can be expanded with the addition of interactive demonstrations powered by Sage, and live Sage code. Our work to make these textbooks Sage-enhanced will provide a model for other authors to create Sage-enhanced textbooks. The first courses we will address are linear algebra, abstract algebra, number theory.
(2) We will futher describe our model for authoring Sage-enhanced curricular materials by creating modules for standard undergraduate courses, especially courses not covered by the textbooks we are converting. These materials will include subject-specific guides to introduce faculty and students to Sage in the context of a particular topic. These materials will be created by members of the UTMOST team and by faculty at our ten test sites. Possible topics include calculus, differential equations, and complex variables.
(3) We will improve the Sage library and its surrounding infrastructure (e.g., server design, notebook usability, and collaboration tools) where the improvements have a direct and obvious benefit for educational settings.
(4) We will identify and work closely with teachers at ten different institutions to test, refine, improve, and extend the work described above. We will select a broad range of institutions, including some with diverse student profiles. Sage Days workshops will provide opportunities for the UTMOST team and representatives from the test sites to all collaborate on the project's activities. These ten sites will provide extensive opportunities for our evaluation team to measure the success of our efforts and the impact on teaching and learning.
(5) Disseminate our materials and the results of UTMOST through Sage Days workshops, MAA PREP workshops, and MAA minicourses (national and regional). We will use these opportunities to train faculty outside of the ten test sites in the use of these materials for their courses, and in the process for creating their own new materials. Our materials will all carry open licenses and be available either in Sage itself, through the global Sage website, and/or a new open textbook website created at the American Institute of Mathematics as part of this grant.

5.1. **Sage-enhanced open textbooks.** Our principal activity is the design and creation of Sage-enhanced textbooks.

5.1.1. *Imagine this.* A student is learning about row-reducing matrices in a beginning linear algebra course. The electronic version of their textbook is an online Sage worksheet they can view from anywhere. Mathematical equations, with publication-quality typesetting, describe the procedure. An interactive demonstration, embedded in the worksheet where an example normally would be, allows the student to step through row-reducing a matrix that was generated on-the-fly (see Appendix A). When ready to guide the procedure themselves, the student may choose the row operation to apply at each step of the reduction. In either case, the correct notation for each selected operation is displayed and the operation itself is highlighted with color-coded entries in the displayed matrix. Another example shows the student how to use a built-in Sage command to row-reduce a matrix. With a single click, the student creates an empty code cell under the example to experiment with the command.

In class, the instructor does one simple example on the board. Then the instructor opens a Sage worksheet, displaying the same demonstration that is in every student's electronic textbook. Students guide the choice of row operations at each step for several example matrices of increasing complexity. At the end of class, the instructor clicks a button to publish the work from that day's class session to the class Sage server. After class, students view the worksheet as they review their notes. They can also, with a click, copy the worksheet into their notes, as well as annotate or change it.

The book's exercises include an interactive problem generator that creates matrices of student-specified sizes and complexity for the student to row-reduce. There is a "Solution" button which generates a step-by-step solution. An advanced exercise guides a student through investigating numerical issues that arise in row reduction. By changing one parameter within a command, the matrices track numeric error bounds via interval arithmetic, and the student discovers that numerical errors can be a significant problem for some matrices.

A group exercise asks students to collaboratively write and test a short function to implement row-reduction using row operations. The student clicks a button to open a new Sage worksheet and types in a few lines using Sage matrices and commands. The student evaluates the code with a single click. Satisfied with their work, the student clicks another button, publishing the worksheet to the rest of the group for testing. Later that evening, the student checks the worksheet and sees that other students have tested the function, found a typo, and fixed it.

The next week, the instructor goes to the Sage server and looks over all of the published worksheets for the group projects. The instructor annotates each worksheet online, using an embedded word processor that supports mathematical typesetting.

The instructor writes a quiz covering row-reduction techniques. Inside of the LaTeX code, the instructor embeds a Sage command to create a simple matrix in reduced-row echelon form, which will be the *answer* to a quiz problem. The instructor then uses Sage commands inside his LaTeX file to do several simple row operations using integers to transform the answer matrix to the matrix the students will row-reduce. The embedded Sage commands are executed automatically when the quiz is formatted using Dan Drake's SageTeX package [10], and the resulting question matrix is automatically incorporated in the printed quiz, while the answer matrix and a detailed solution are automatically incorporated in the answer key.

5.1.2. *Infrastructure for textbooks.* UTMOST proposes creating and fully testing a system for integrating mathematics software into open textbooks. Pilot projects have already demonstrated the feasibility of this process for short documents, while identifying the technical details that remain to have the system work on a large (book-length) document while remaining easy for authors to use. The primary tool is the existing tex4ht translator (NSF Grant No. IIS-0312487) [47] which can convert LaTeX into the jsMath format adopted by the Sage notebook for rendering publication-quality mathematics. The end result will be a folder of Sage worksheets, each a mixture of text, typeset mathematics, Sage input cells and Sage interactive demonstrations. Because LaTeX is the accepted standard for authoring mathematics, we expect this process to be applicable to a broad range of current and future open mathematics textbooks.

5.1.3. *Targeted textbooks.* Appendix B lists many open textbooks that could be converted by our process. However, conversion is more than a technical process. We plan to incorporate interactive demonstrations, live Sage code, and guidance on the Sage library itself. So there is a significant care and thought required to integrate these uses of mathematical software properly. Our work will provide a tested model other authors can consult when authoring new textbooks.

Listed below are our initial candidates for conversion to Sage-enhanced textbooks that bring the full power of mathematics software to the student *directly within their book.* The open textbook landscape is rapidly growing, and beyond the following list we will make final decisions based on the most promising open textbooks that will help us achieve the project's goals. We expect to have the three texts below available for use in the Fall 2011 term at our test sites.

- Beezer's *A First Course in Linear Algebra*. This is a mature textbook designed from the start to be open source, and thus has a highly modular design. The first version of this book was released in December 2006, and has been used as the primary text in courses at 13 different universities. The author has taught from various versions of the book ten times. It will be an excellent choice as a full-scale test of the technical conversion process, the integration of Sage code and classroom use.
- Stein's *Elementary Number Theory*. This text already has an extensive collection of Sage examples, and the author is the founder of the Sage project who has taught from the book four times, making it another ideal candidate for early conversion. It is currently being published by Springer-Verlag, but Stein has the rights to make it freely available starting May 2010, so will be able create a Sage-enhanced version for free distribution. While it will not carry a true open license that allows modifications by others, it will be a good test of the technical aspects of the conversion process and will be made freely available to our test sites and other institutions.
- Judson's *Abstract Algebra*. This is another text authored by a project member. It was originally published by PWS-Kent in the early 1990s, but the author has received the copyright back and released it under an open source license. Beezer has taught from this text the past two years, creating supplementary material utilizing Sage, in addition to already contributing code to the Sage library to support the book. First released as open source in Spring 2009, eight universities adopted the book for the Fall 2009 term. As a year-long course, this is a larger project to tackle next, and will be a collaborative project between two team members.

5.2. **Sage-enhanced curricular materials.** UTMOST is not about creating new textbooks, which is why we are starting with existing open books. However, targeted materials for other courses will make it easier for faculty to adopt and use Sage in their courses.

5.2.1. *Subject-specific guides.* For selected courses where we do not provide a Sage-enhanced textbook, or no suitable open book currently exists, we will create subject-specific tutorials. These short guides will explain how to use Sage to investigate questions in a standard course, following a typical order for such a course. Existing examples of these materials include multivariate calculus worksheets by Jason Grout and Ben Woodruff [i] and John Perry's course notes for a Mathematical Computing course [33]. While not a textbook, an excellent text for a curious undergraduate is the book by Stein and Mazur on the Riemann Hypothesis. This text is richly illustrated with Sage output and contains extensive Sage code, so would be another addition to the curriculum that we could provide in a Sage-enhanced form. **i**

Stipends will be provided to faculty outside the project team, ideally from our test sites, to author such materials. These will be designed from the start for conversion to Sage worksheets, providing a test of our system for this conversion.[ii] **ii**

5.2.2. *Interactive demonstrations.* Sage has a powerful, but extremely simple-to-use framework for creating web-based interactive demonstrations. Input boxes, sliders, selectors, and other controls are simple to create. Output can use HTML, tables, and typeset mathematics, and 2D or 3D graphics. Computations have the full Sage library at their disposal. An example of what an interact would look like to a student is in Appendix A. Stein created the first implementation and continues to improve it with significant contributions from Grout and many other Sage developers. Current

[i] Jason: publish these as a set of worksheets

[ii] Have we included this in the budget? Mention later in test-sites.

work by Mitesh Patel, supported by an NSF FRG grant, will allow embedding these interactive demonstrations into standard web pages, independent of a Sage worksheet.

For select courses we will create comprehensive collections of demonstrations for inclusion in our Sage-enhanced textbooks and into Sage itself. We will also create a easily-searchable repository of high quality, reviewed Sage interacts, which will be included in every copy of Sage.

5.3. **Sage into the classroom.** The heart of UTMOST is putting Sage-enhanced materials in the hands of teachers and students to assist them in the teaching and learning of mathematics. We will work with selected faculty at ten partner institutions, providing support for using Sage in their classrooms, offering technical support for Sage, assisting with the Sage-enhanced materials that we have created, and helping them write their own Sage-enhanced materials. Our teacher-authors and their students will receive access to Sage on a server dedicated to the UTMOST project, with dedicated technical support provided through this project.[iii] **iii**

We will recruit two groups of teacher-authors for UTMOST with the first group beginning in Summer 2011 for a two-year commitment and the second group beginning Summer 2012 for a one-year commitment, for a total of ten test sites. Teacher-authors will receive a stipend for their participation in UTMOST and support to attend a workshop on using Sage in the classroom. In order to forge a strong relationship with the teacher-authors and provide the best possible support, each teacher-author will be paired with a member of the UTMOST team who will serve as a contact and adviser. This adviser will also make visits to the test site. We will expect the following from these participants.

- Teacher-authors will commit to using Sage in their classrooms during their time with UTMOST. More specifically, they will be expected to use Sage-enhanced materials extensively in a least one single semester course during the academic year.
- Teacher-authors will write and test comprehensive Sage-enhanced curricular materials for the class that they are teaching.
- Teacher-authors will organize focus groups with their students as an opportunity of UTMOST to receive feedback on the design and use of Sage-enhanced curricular materials.
- Teacher-authors will offer training to others at their institution on how to use Sage and how to use Sage-enhanced materials in the classroom. Such training could be offered as departmental workshops or seminars.
- Teacher-authors from the first group will be selected to mentor the new teacher-authors in the second group.
- Teacher-authors will submit a report each year on their teaching experiences with Sage and new curricular materials.

5.3.1. *Recruitment plan.* Selection of the teacher-authors for UTMOST will be based on the following.

- A narrative statement regarding rationale for applying and the course or courses they plan to use UTMOST materials and their familiarity with Sage.
- Limited or no familiarity with Sage, since we aim to make it easy for all faculty to integrate mathematics software into their courses, not just those who already have this experience.
- A letter of support from the department chair, indicating the flexibility to schedule faculty for the appropriate courses, a commitment to using Sage and new curricular materials in their department, and awareness of evaluation activities.

5.4. **Sage infrastructure.** While the heart of Sage is its library of commands, the notebook interface and server configurations are key elements of a successful experience for students and faculty using Sage in their courses. So a portion of our work will be to improve Sage itself, in those areas where the improvements *directly support* educational applications of Sage. Discussions and

[iii] In the budget?

initial queries in the Sage-Edu discussion group indicate that faculty around the world are excited about setting up Sage servers for their students' use [40].

5.4.1. *The Sage library.* Experience has shown us that teaching with Sage invariably suggests new capabilities or exposes needed commands. Our work enhancing textbooks, and our teacher-authors at test sites, will certainly suggest extensions and corrections to the Sage library, so we will continue to refine and enhance Sage, but at an accelerated pace with support from this grant. All changes will undergo the existing code review process, and will then become permanent contributions to Sage, maintained and further extended by the larger Sage community. We have already identified specific changes for linear algebra, abstract algebra, calculus, differential equations, number theory, and combinatorics, as well as broader areas like 2D and 3D graphics. Targeted fixes and improvements will greatly improve the educational experience for students. In many cases, undergraduate students can, and already have, contributed code and other suggestions; we plan to employ several in these efforts.

5.4.2. *Notebook development.* The Sage notebook interface is a powerful tool for experimentation and collaboration. Stein worked fulltime during Fall 2009, supported by the University of Washington, on vastly improving the robustness and scalability of the notebook. This work came at the right time, because use of the Sage notebook server has been growing rapidly. In Fall 2009, we have typically seen two thousand new accounts created on `sagenb.org` every month. The notebook interface is the face of Sage for students and can be improved in many ways[iv] **iv**

5.4.3. *Sage servers the easy way.* Sage servers are critical for user's independence from specific hardware and operating systems, given the prevalence of web browsers and network connections. So initiating and maintaining a server should be as simple as possible, making it easy for faculty and system administrators everywhere to get their students started with Sage.

We will create a Virtual Box Sage notebook server appliance with a graphical interface, two virtual machines that provide a rock-solid secure Sage notebook server setup. Users will be able to very easily install this appliance on Windows, Linux, OS X, and Solaris (x86) servers. The management interface will provide clear and easy documentation about setting up this server, creating new notebook servers for specific classes or instructors, starting and stopping notebook servers, monitoring resource usage, adding users and authentication frameworks (such as LDAP), and upgrading Sage with minimal user downtime.

For users at our ten test sites, we will create a new mailing list for server administration issues, and in addition to the help we provide ourselves, we will hire a knowledgeable employee to answer support questions on a daily basis.

Several specific goals for notebook and server development include the following.

- We will improve the notebook so that it will robustly handle up to 1000 simultaneous users when running on a single high-end server, as demonstrated by a robust automated test suite.
- We will implement management tools so administrators can manage the notebook load and better balance resources.
- CWe will create easy-to-use research tools for educators, so they can understand how their students use Sage.

In summary, we propose to provide high-quality enterprise level software infrastructure and support to educators. What we propose is far from free or easy — it is in fact expensive. But with support now, literally millions of students will benefit at no cost to them. This is an excellent investment in mathematics education for the National Science Foundation.

[iv]William: collaboration and....

6. Qualifications and Preparation

The five faculty in the UTMOST team together have many years of experience teaching undergraduates at a wide range of institutions, four are active Sage developers (including its founder), three are authors of open textbooks, all have significant mathematics research experience, and one specializes in mathematics education research. As Sage developers, they are in a position to quickly and accurately shape changes in Sage based on the experience of working with faculty at other institutions that are new to Sage. Each is familiar with, and has extensive experience using, open source software and tools in their teaching and other professional activities. Besides the applicability of this experience to the project's central theme, they are also extremely adept at working collaboratively while still being separated geographically. Working together, they have the wide range of complementary experience and skills that will make UTMOST a success. So the time is right to translate the experience of the project team into forms that faculty at other institutions can use to similarly benefit from teaching mathematics to their students with open mathematics software and open textbooks.

6.1. Personnel.

Dr. William A. Stein, Associate Professor of Mathematics at the University of Washington, is a leader in both computational and theoretical number theory, and the author of two published number theory books. He started the Sage project in 2005, and has passionately led its development ever since, including co-organizing over 25 workshops during the last 4 years and directing dozens of Sage development projects by students at Harvard, UC San Diego, and U of Washington. Dr. Stein will implement improvements to Sage, direct work by students, contribute curricular materials he has authored, supervise test sites, and co-organize workshops.

Dr. Kiran S. Kedlaya, Associate Professor of Mathematics at Massachusetts Institute of Technology and University of California, San Diego, is a leading researcher in arithmetic algebraic geometry. He has received an NSF CAREER grant, a PECASE award, and a Sloan Fellowship, and was selected to speak at the 2010 International Congress of Mathematicians. Dr. Kedlaya brings extensive experience in undergraduate and graduate education at all levels, from calculus to mentoring Ph.D. students. He is also deeply involved with mathematics competitions and other programs for exceptional students, such as the USA Math Olympiad, the Putnam competition, and the Art of Problem Solving Foundation. In addition, Dr. Kedlaya brings extensive knowledge of the Sage system, having been an active user and developer since 2005. Dr. Kedlaya will review improvements for Sage related to the project, supervise test sites, and provide additional overall guidance.

Dr. Jason Grout, Assistant Professor of Mathematics at Drake University, is an undergraduate teacher, an active researcher in combinatorial matrix theory and graph theory, and a Sage developer. Dr. Grout has contributed extensively to the Sage code base over the last three years in the linear algebra, graph theory, plotting, and notebook interface components of Sage. Dr. Grout has used Sage in research and in teaching a number of undergraduate courses. He has also given numerous presentations and tutorials on Sage, and in Summer 2010 will be co-directing an MAA PREP workshop with Dr. Beezer and Dr. Karl-Dieter Crisman to help participants develop new curricular materials in Sage. Dr. Grout will implement improvements to Sage, direct work by students, supervise test sites, help organize workshops, and contribute curricular materials.

Dr. Robert Beezer, Professor of Mathematics at the University of Puget Sound, is a undergraduate teacher with 31 years of experience, an active researcher in algebraic graph theory, one of the first open textbook authors, and a Sage developer. He began writing his open source linear algebra textbook in 2004 and has assisted Dr. Judson with the recent release of his very successful open source abstract algebra text. He began using Sage in 2007 and began contributing code in early 2009.

He will lead the technical process of converting textbooks from LaTeX to Sage worksheets, producing a simple system for other authors to use. The pedagogical implications of this new capability will be explored as he incorporates Sage into existing textbooks on topics he teaches frequently, such as linear algebra, abstract algebra, combinatorics, calculus and cryptography. He will continue to contribute code to Sage where the new functionality enables a more complete experience for undergraduate students, and will suggest, review and test the project's improvements to interactive demonstrations and the notebook interface.

Dr. Thomas Judson, Associate Professor of Mathematics at Stephen F. Austin Sate University, is an active researcher in both mathematics and mathematics education, with 32 years of teaching experience. He is the author of an open source undergraduate abstract algebra textbook, and will work with Dr. Beezer to produce Sage-enhanced materials for abstract algebra. Dr. Judson has worked extensively with undergraduate mathematics teachers and has mentored graduate students in the teaching of mathematics. In addition to his teaching and research accomplishments, Dr. Judson brings added experience in working with diverse groups both in the US and abroad. He will work with Dr. Hassi to guide research and evaluation efforts for UTMOST. With experience in working with undergraduate faculty and graduate student mentoring, Dr. Judson will also provide insight and assistance in the implementation of Sage-enhanced materials in the undergraduate classroom. Project efforts connected to research in mathematics education will be under the direction of Dr. Judson.

Members of the evaluation team are profiled in Section 7.3.

6.2. Grant experience and support.

- Stein has successfully administered many grants supporting Sage development from organizations such as UC San Diego, U of Washington, Google, Sun, Microsoft, and the US Department of Defense. National Science Foundation grants include awards from the SCREMS program for the computing cluster that hosts `sagenb.org`, the FRG program (through the American Institute for Mathematics) and the COMPMATH program to fund two postdoctoral positions.
- Judson works with middle and high school mathematics teachers from high-needs school districts to help them become teacher-leaders in their schools and districts, and preparing them to deliver pedagogical content and mathematical content to their colleagues. He also collaborates with the PIs on these grants to help direct the mathematics education research component. (Texas Leadership Initiative: Mathematics Instruction Transformed (Texas LIMIT) NSF #0934878, Texas Middle and Secondary Mathematics Project - Supplemental Funds NSF #0227128)
- Beezer has been awarded a competitive year-long sabbatical leave from the University of Puget Sound for the 2010-11 academic year. This will allow him to begin converting textbooks, increasing the possible textbooks available for test sites to use beginning in the Fall 2011 term.
- Some of Grout's recent work on Sage been partially supported by an NSF postdoctoral appointment in Summer 2008.
- Beezer and Grout will co-direct (with Karl-Dieter Crisman) an MAA PREP workshop on Sage in Summer 2009, as part of a program funded by the NSF (Grant No. DUE-0817071)

7. Evaluation

Formative and summative evaluation will be conducted at different phases of the project to address questions about the effectiveness of the project in creating and implementing the UTMOST model and materials for undergraduate mathematics teaching and learning. Dr. Judson will coordinate internal evaluation data-gathering, and will serve as the liaison to the external evaluation team from Ethnography & Evaluation Research at the University of Colorado at Boulder. The evaluation questions include:

(1) What aspects of the Sage-based tools and materials are beneficial to instructors, what challenges do they face, and what kinds of support do they need in using these tools?
(2) How do instructors use and apply the Sage-integrated curriculum materials, and how do these benefit (or fail to benefit) their teaching of mathematics?
(3) How do the tools and materials impact instructors' content knowledge, pedagogical content knowledge, classroom instructional practice and their students' learning?

Information will be gathered on both the processes and outcomes of the project at different stages, and results will be organized as case studies of participating instructors and their students. Formative components will focus on monitoring the quality of project activities, enabling the project to make mid-course corrections and plan for future development. Summative components will focus on the impact of the project on instructors' instruction and student learning at the test sites. Instructors and classrooms will be sampled, taking into account instructor interest and local institutional cooperation. In addition to offering feedback on[v] v

7.1. **Study design.** The study design includes pre/post surveys, follow-up surveys included in yearly self-reports, and interviews of 10–20 participating instructors. In addition, students will answer an online post-survey focusing on their experiences and gains in learning mathematics, including their classroom use of the Sage tool and materials. The design is informed by previous evaluation studies on professional development, education, and workshops in STEM fields ([7, 13, 24, 43, 48] and on student outcomes of active instructional methods in undergraduate mathematics [15, 16].

7.1.1. *Pre-survey.* While registering for the summer workshops, participants will complete a short pre-survey to gather demographic and contact data, and information about their classes and institution, current teaching practices, and pedagogical needs. This information will also help the project leaders to plan the workshop and later support.

7.1.2. *Post-survey and feedback.* Participants will complete a survey on the summer workshops so that project leaders can make adjustments for their future workshop and implementation support. The survey will ask about participants plans for using the Sage tools and material, to help guide later components of the study. They will also be asked to sign a consent form to participate in a follow-up interview. Evaluators will observe the summer workshop and conduct a focus group with current and past participants.

7.1.3. *Follow-up survey and reports.* After using the tool and materials for one year, the participants will file a report on their implementation, including some follow-up survey questions. They will report their use of the tool and materials in their own classrooms and their future plans. Additional data on implementation will be gathered by the project team during site visits; they will conduct student focus groups using a protocol co-developed with the evaluators and share site visit notes that will be used as data sources.

7.1.4. *Student learning assessment.* Student learning will be evaluated after the first year of implementation with a post-survey based on the NSF-supported SALG instrument [44], in which students report about their experiences of and learning gains from their mathematics course. This will provide information on student outcomes as well as formative feedback for the instructors using the Sage tool and material in their classroom. A mathematics-specific version (SALG-M) has already been validated and used in a large evaluation study, and is sensitive to differences by student group and classroom practice [15, 22].

[v]never got the end of this sentence

7.1.5. *Follow-up interviews.* Based on instructor and student responses on other measures, about half of the instructors will be interviewed to study factors that affect their success in implementing. The interviews will explore classroom use of the tool and material, impacts on their instructional practices, and instructors' perception of students' response.

7.2. **Management and dissemination.** The evaluation will be led by Dr. Marja-Liisa Hassi from Ethnography & Evaluation Research (E&ER) at the University of Colorado at Boulder. E&ER has extensive experience evaluating other large mathematics and science education projects (NSF Awards # 0920126, # 0723600, # 0450088). Dr. Hassi is expert in research and evaluation of mathematics education and currently works with a large evaluation study focused on inquiry-based learning and teaching of undergraduate mathematics at four large research universities [16, 21]. She also works with an evaluation study of NSF DUE-funded workshops on inquiry-based learning for instructors.

Drs. Judson and Hassi will collaborate with the project director, members of the project, and partner institutions to conduct the evaluation study. Dr. Judson and the project PIs will conduct site visits to partner institutions to observe and document the use of Sage-enhanced materials. Dr. Judson will have primary responsibility for extracting information from the site visits and workshop data to feed back to developers to improve the technology and classroom resources. Surveys and interview protocols will be prepared and conducted by Drs. Hassi and Judson. They will analyze all data and will prepare an annual report to document the evaluation activities and results. This will provide formative evaluation feedback to the project PIs and participating instructors to inform design decisions and mid-course corrections. A final report will gather results after the implementation of the tools and material by the second group of instructors in 2013. These findings will be shared also with the broader mathematics education community through a presentation and a coauthored article about the impacts of using CAS-integrated materials on the teaching and learning of undergraduate mathematics.

7.3. **Evaluation personnel.**

Dr. Marja-Liisa Hassi, has a Master's Degree in Mathematics and a Ph.D. in Education. She has expertise in both theory and methodology of mathematics education research, as well as teaching experience in education, mathematics education, and research methods for undergraduate and graduate students. Her recent publications address theory and measurement of affect in undergraduate and adult mathematics learners and comparative student outcomes of inquiry-based and traditional undergraduate mathematics courses. She will be assisted by Dr. Sandra Laursen, co-director of E&ER and an experienced evaluator of STEM education initiatives in higher education.

8. Dissemination

UTMOST is principally about dissemination: how can we enable the widespread implementation of approaches which have already been proven to work. In addition to the working with our test sites, we will broadly disseminate the results of UTMOST through a variety of forums, several of which are already in place. The UTMOST curricular materials will be accessible on the Sage website, `sagemath.org`, where users can read documentation, make contributions to Sage, download Sage for free, and keep abreast of new developments in Sage.

8.1. **Sage website.** The Sage website [45] is an established venue for sharing materials related to Sage. This central location receives 90,000 visitors a month, with 42% of the visitors from the Americas (North, Central, and South) and 45% of the visitors from Europe. What we learn and create will be made available here with open licenses. Further, some of our work will be incorporated into Sage itself, which already has an elaborate distribution system, designed to make it extremely easy to download and run, on a personal machine or a server. Complete, integrated documentation ships with every copy of Sage (such as the 4,641 page reference manual), and it is possible that much of our material, *including complete textbooks*, could ship as a core component of Sage itself. Standard interactive demonstrations may also find a home in Sage itself where they could be organized topically, be searchable, and benefit from Sage's automated test-suite that users routinely run on a wide variety of hardware.

8.2. **Sage Days workshops.** Sage Days, which are a series of conferences and workshops devoted to the development of Sage, are an ideal venue for dissemination. To date there have been nineteen Sage Days conferences, with nine more planned. These workshops now include sessions dedicated to using Sage in the classroom, with the first such session taking place in December 2009 at the Clay Mathematics Institute on the final day of a week-long workshop on Sage and number theory. It attracted roughly thirty college faculty (and a few high school faculty) from around the Northeast, all eager to learn more about the use of Sage in educational settings.

A Sage Days devoted to training others to use our materials, and receiving and incorporating feedback and new ideas, will be entirely isomorphic to the way Sage Days have been used to drive Sage development. We will have a Sage Days event in Summer 2012, involving the UTMOST team and all ten test sites, as an opportunity to discuss the project at the midpoint of our experiences in the classroom. Another Sage Days event at the American Institute of Mathematics in Summer 2013 will be an opportunity to more generally review the experiences of the project and plan for the future of integrating open software and open textbooks into the undergraduate mathematics curriculum.

8.3. **National workshops.** We will apply to offer special sessions and workshops on the use of Sage in the classroom at the Joint Mathematics Meetings and MathFest, where we will be able to share the results of UTMOST with the undergraduate teaching community. We will also publish articles about the impact of using our materials on the teaching and learning of undergraduate mathematics. Drs. Beezer and Grout are part of a team delivering a workshop on how to use Sage in the classroom as part of the NSF-funded Mathematical Association of America Professional Enhancement Program (PREP). We will apply to lead future PREP workshops that will incorporate the results of UTMOST. Notably, Stein has recently been invited to propose an American Mathematical Society Short Course on Sage for the 2011 Joint Meetings.

8.4. **AIM open textbook initiative.** The American Institute of Mathematics is a respected institute that has supported both leading research mathematicians and mathematics education at all levels. As the sponsoring organization for this project AIM will conduct a pilot project to test the feasibility of sponsoring and supporting a series of open mathematics textbooks. A hindrance to the adoption of open textbooks is the lack of an acknowledged authority to vouch for the content. Usually this is a role played by a publisher, so a recognized organization within the mathematics community with a trusted reputation, such as AIM, will be another component to broadly disseminating our materials. This has the potential to positively impact open textbooks, in mathematics and more generally.

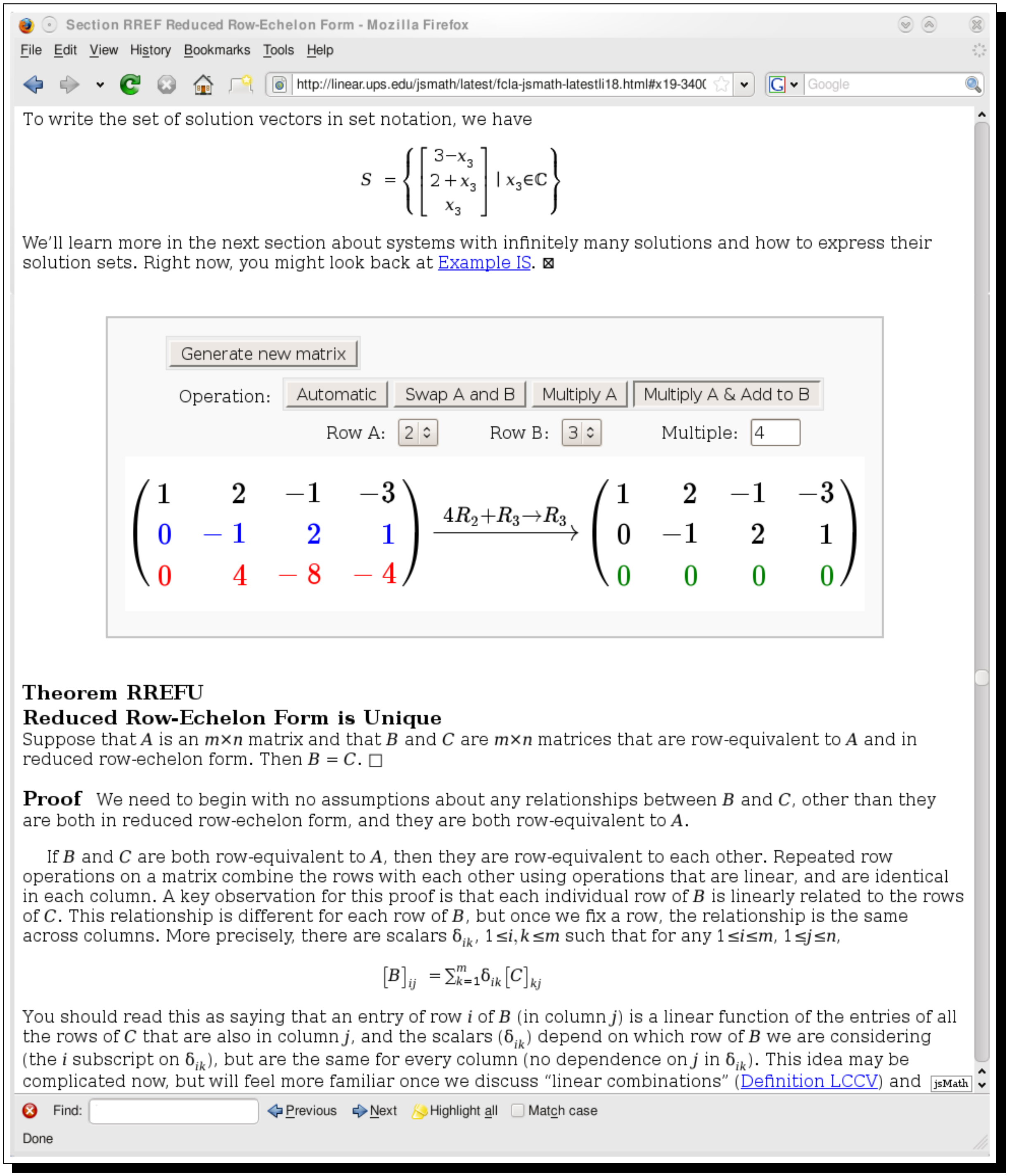
9. Future Directions

The limited scope of this proposal suggests several broader new directions. In particular, the project team was intrigued by the possibilities for using Sage to generate and check homework through integration with a system like WebWork, but decided such an initiative was too ambitious and disjoint to be included in this proposal. [vi] **vi**

- Create new open textbooks with Sage enhancements designed in from the start.
- Create Sage-enhanced open textbooks for every standard course in the undergraduate mathematics curriculum.
- Integrate Sage with an open source homework delivery system such as WebWork.

[vi] Delete this, or clean it up.

- Integrate collaboration features of the Sage notebook with a course management system such as Moodle.
- More????



Prototype of a Sage-enhanced mathematics open textbook

vii

vii

[vii] TODO for Jason: Make sure the color is dark enough to look good on black and white. Highlight the rows by surrounding the row with a box. Color the R_2 and R_3 above the arrow to emphasize which row is which.

Appendix B. Open Source Mathematics Textbooks

Beyond our initial candidates there is a wide range of books available that could be converted to cover portions of the standard undergraduate curriculum. Except where noted, the textbooks below are licensed with a GNU Free Documentation Licenses (GFDL) or a Creative Commons License that allows anyone to distribute modified versions, typically only requiring attribution of the original author's work and the use of the same license. New open source textbooks continue to be released and evidence suggests many authors are considering the option seriously.

- *A First Course in Linear Algebra*, Robert Beezer `http://linear.pugetsound.edu/`
  Second-year university level text, concentrating on proof techniques. Used at thirteen schools since Fall 2007. Contains some Sage commands.
- *Abstract Algebra: Theory and Applications*, Tom Judson `http://abstract.ups.edu/`
  Standard upper-division treatment of groups, rings, domains, fields, Galois Theory. Used by seven schools in its first year available as open source.
- *Combinatorics Through Guided Discovery*, Kenneth Bogart
  `http://www.math.dartmouth.edu/news-resources/electronic/kpbogart/`
  Problem book in combinatorics with roughly 400 problems designed to teach the subject. Released as open source. (NSF Grant No. DUE-0087466)
- *Elementary Number Theory: Primes, Congruences, and Secrets*, William Stein
  `http://wstein.org/ent/`
  Classical elementary number theory and elliptic curves, with applications and extensive Sage examples. Published by Springer-Verlag, freely available May 2010. (NSF Grant No. 0653968)
- *Vector Calculus*, Michael Corral `http://www.mecmath.net/`
  Standard treatment of multivariate calculus. Topics include vector calculus, partial derivatives, multiple integrals, theorems of Green and Stokes.
- *Trigonometry*, Michael Corral `http://mecmath.net/trig/`
  An in-depth, comprehensive and unified treatment of the standard topics.
- *Cryptography*, David Kohel
  `http://echidna.maths.usyd.edu.au/~kohel/tch/Crypto/crypto.pdf`
  Classical ciphers and their cryptanalysis, modern stream ciphers and public-key cryptography. Significant appendices on Sage.
- *What is Riemann's Hypothesis?*, Barry Mazur, William Stein `http://wstein.org/rh/`
  Richly illustrated elementary treatment of a central problem of modern mathematics, designed for an undergraduate audience. Backed up with extensive Sage worksheets already. (NSF Grant No. 0653968)
- *Numerical Analysis*, Steven Pav `http://scicomp.ucsd.edu/~spav/pub/numas.pdf`
  Comprehensive book-quality lecture notes. Extensive use of Octave, an open source clone of Matlab.
- *Calculus: Modeling and Application*, David A. Smith, Lawrence C. Moore
  `http://www.math.duke.edu/education/calculustext/`
  Calculus textbook with significant electronic components. Mathematical Association of America project with "all rights reserved" (i.e. does not currently have an open license). (NSF Grant No. 0231083)
- *Differential Calculus and Sage*, David Joyner, William Granville
  `http://sage.math.washington.edu/home/wdj/teaching/calc1-sage/`
  A classic text, now in the public domain, significantly expanded by a Sage developer with Sage code and graphics.

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