

# $(k, m)$ -Segmentation Algorithm for Horizontal Lines

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## ABSTRACT

Division of an electrical signals into high and low, cuts a soccer video into filed of game and audiences shots... all of those are a thresholding problems. But when the noise get stronger the regular thresholding algorithm get weaker . In the km-segment mean problem we need to divide an input signal into a k-piecewise function when each piece is one of set M in size m.

we aproximate the solution for a constrained version of  $(k, m)$ -segmentation problem when the lines are horizontal . We can do that by trying every m size set in the input data as M and compute the k-segment of the signal while limit the segment to be from M set. then choose the one with minimum cost. What give us  $cost < 4cost_{opt}$  by  $O(n \text{ chose } m^*)$  time using corset to compute k-segment (fl14 ).

we show how our algorithm can divides better than a regular threshold and keep divide when we increase the noises even after threshold totally failed.

## 1. INTRODUCTION

### 1.1 Main Contribution

The main contributions of the paper are: (i) A solution for a constrained version of  $(k, 2)$ -segmentation problem(as given in 1.2) when the lines are horizontal .(ii)And to test this new solution on a Infra Red drone's conroler in order to do the first step of decoding this data.

### 1.2 Problem Statement

The  $(k, m)$ -segment mean problem optimally fits a given discrete time signal of  $n$  points by a set of  $k$  linear segments over time, where  $k \geq 1$  is a given integer. That is, we wish to partition the signal into  $k$  consecutive time intervals such that the points in each time interval are lying on a single line from one of an  $m$  types of lines, where  $m \geq 1$  is a given integer to;

We make the following assumptions with respect to the data:(i) We assume the data is represented by a feature

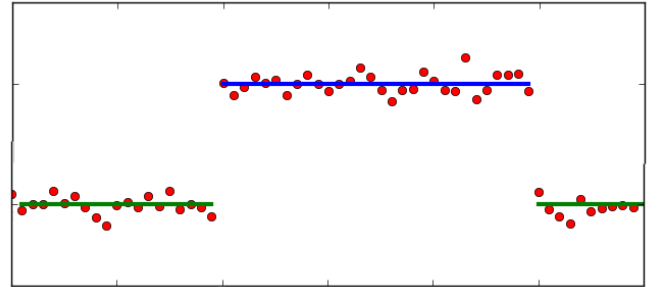
space that suitably represents its underlying structure; (ii) The content of the data includes at most  $k$  segments ,from at most  $m$  (for this paper  $m = 2$ ) types. that we wish to detect auto- matically; An example for this are scenes of the audience and the field in a soccer video, a "bits" of an Infra Red signal (as we show in Section 3), etc. This motivates the following problem definition.

*Definition 1.*  $(k, m)$ -segment mean.

A set  $P$  in  $\mathbb{R}^{d+1}$  is a signal if  $P = \{(1, p_1), (2, p_2), \dots, (n, p_n)\}$  where  $p_i \in \mathbb{R}^d$  is a point at time index  $i$  for evrey  $i = [n] = \{1, \dots, n\}$ . For an integers  $k \geq m \geq 1$ , a  $(k, m)$  - segment is a  $k$ -picewise linear function :

$$f : \mathbb{R} \rightarrow \mathbb{R}^d = \begin{cases} f'_1(x) & \text{if } x < d_1 \\ f'_2(x - d_1) & \text{if } d_1 \leq x < d_2 \\ \dots & \\ f'_k(x - d_{k-1}) & \text{if } d_{k-1} \leq x < d_k \end{cases}$$

where  $\forall f_i : f'_i \in F$ , and  $F$  is a group of lines from order  $m$ . this function  $f$  maps every time  $i \in \mathbb{R}$  to a point  $f(i)$  in  $\mathbb{R}^d$ .



**Figure 1:** This figure is an exemple to the  $(3, 2)$ -segment. the red dots are the input data and the lines are the segments when the blue is the low type and the grin is the high type.

The fitting error at time  $t$  is the squared distance between  $p_i$  and its corresponding projected point  $f(i)$  on the  $(k, m)$ -segments. The fitting cost of  $f$  to  $P$  is the sum of these squared distances.

$$cost(P, f) = \sum_{i=1}^n \| p_i - f(i) \|^2 .$$

where  $\|\cdot\|$  denotes the *Euclidean* distance. The function  $f$  is a  $(k, M)$ -segment mean of  $P$  if it minimizes  $cost(P, f)$  and conducts the conditions above.

In this paper we interest in a constrained version of  $(k, m)$ -segmentation problem when the lines are horizontal.

### 1.3 Related Work

## 2. THE ALGORITHM

If we look at the problem we can see that (i) if we had the corct set  $F$  so the problem was esy sins all what we had to is to compute the  $k$ -segment(see Definition 2) mean wehn eatch segment is in  $F$ .(ii) And if we had the dividers of the segments on the  $(k, m)$ -segmat then each segment will be the mean height of the corresponding input points.

*Definition Efi $k$ -segment mean.* The  $k$ -segment mean problem is a private case of the  $(k, m)$ -segment mean, when  $k = m$ .

*Definition EfiBicriteria or  $(\alpha, \beta)$ -approximation.* For  $\alpha, \beta > 0$ , an  $(\alpha, \beta)$ -approximation for the  $(k, m)$ -segmentmean of  $P$  is a  $(k)$ -segment  $g$  such that  $cost(P, g) \leq \alpha * cost(P, f)$ .

*Definition Efi $(\alpha, m)$ -centroid-set.* For  $\alpha > 0$ , integer  $m > 0$  and set  $P$ , an  $\alpha$ -centroid-set of  $P$  is a set of points that contain a subset  $C$  so that :  
 $cost(C, P) \leq (1 + \varepsilon) * cost(OPT\text{-centroid-set}, P)$

**THEOREM 1** (BICRITERIA APPROXIMATION [1]). *Let  $P = \{(1, p_1), \dots, (k, p_k)\}$ . Then an  $(\alpha, \beta)$ -approximation for  $P$  can be computed in  $O(dn)$  time where  $\alpha = ..$  and  $\beta = ..$*

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#### Algorithm 1 APPROX-HORIZONTAL( $P, k, m, \varepsilon$ )

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**Input:** A set  $P = \{(1, p_1), \dots, (n, p_n)\}$  in  $\mathbb{R}^{d+1}$ , pair of integers  $k \geq m \geq 1$  and a set  $F$  in size  $m$ .  
**Output:**  $(1 + \varepsilon)$ -approximation to the  $(k, m)$ -horizontal segment of  $P$ .  
Set  $G \leftarrow \varepsilon\text{-centroidset}(P)$ ; \*See Algorithm 2  
**for** evrey subset  $g$  of  $G$  **do**  
     $f \leftarrow K\text{-SEGMENTATION}(P, k, g)$   
     $best\_fit \leftarrow$  Compute  $f$  such that  $cost(P, f)$  is minimized.  
**end for**  
**return**  $best\_fit$

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#### Algorithm 2 2-APPROXIMATION TO $(k, m)$ - SEGMENTATION HORIZONTAL COST

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**Input:** A set  $P = \{(1, p_1), \dots, (n, p_n)\}$  in  $\mathbb{R}^{d+1}$  and paer of integers  $k \geq m \geq 1$   
**Output:** A 2-approximation of the cost of the  $(k, m)$ -horizontal segment of  $P$ .  
**for**  $\{F \subset P \mid |F| = m\}$  **do**  
     $best\_fit \leftarrow$  Compute a minimized  
     $cost(P, K\text{-SEGMENTATION}(coreset, k, F))$ .  
**end for**  
**return**  $best\_fit$

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#### Algorithm 3 $(\varepsilon, m)$ -CENTROID-SET

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**Input:** A set  $P$  of points, an integer  $m > 0$  and an error parameter  $\varepsilon \in (0, 1)$ .  
**Output:** a set  $G$  such that  $(1 + \varepsilon) * cost(Centroid\text{-Set-of-}P, P) \leq cost(G, P)$ .  
Set  $L \leftarrow \{l \mid l \in \mathbb{R}^d\}$

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#### Algorithm 4 ALTERNATIVE FOR 1-SEGMENT MAIN

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**Input:** A set  $P$  of points and a set  $G$  of centers.  
**Output:** the competitable center  $g$  in  $G$  to the set  $P$ .  
**if**  $cost(g_1, P) < cost(g_2, P)$  **then**  
    **return**  $g_1$   
**else**  
    **return**  $g_2$   
**end if**

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**THEOREM 2** (ALGORITEM 3 SOLUTION COST). *Algorithm 3 will give a  $(2 + \varepsilon)$ -approximation to  $cost_{opt}$*

**PROOF.** Without loss of generality if we lock on all of the dots on the segments that from the higher type . Let  $p_{opt}$  be the men of the corresponding of all those points, Let  $cost_{opt}$  be the average distance to  $p_{opt}$  and let  $p_s$  be the closest point to  $p_{opt}$ . Then the distanse  $\Delta(p_{opt}, p_s)$  is smoler or equal to  $cost_{opt}$  so from the triangle inequality for each point  $p$  in the data:  $\Delta(p, p_s) < 2 * cost_{opt} (\Delta(p, p_{opt}) + \Delta(p_{opt}, p_s))$ .

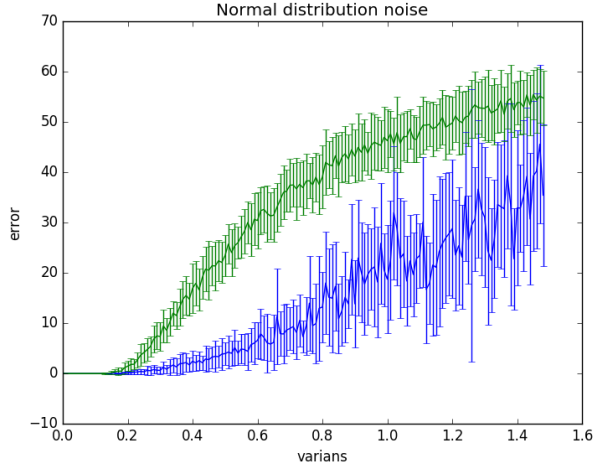
□

### 2.1 $(1 + \varepsilon)$ -Aproximation

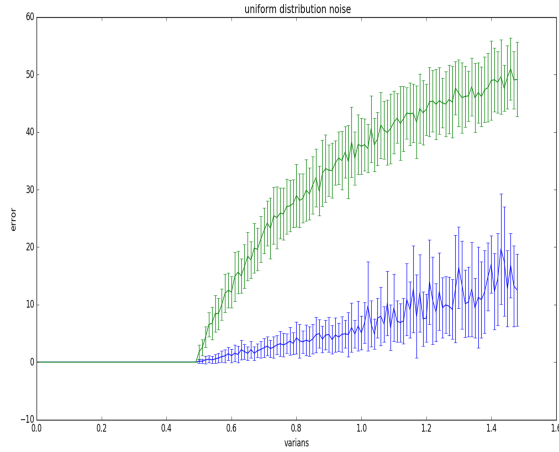
For the next part of the algorithm

## 3. EXPERIMENTAL RESULTS

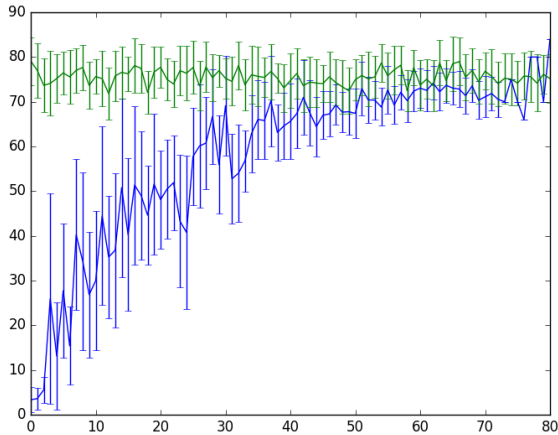
We now demonstrate the results of our algorithm on sin-tetic data in cuple of tests. We compare our algorithms against regular thresholding algorithm. we also show our algorithm preforms on a real life data token from an ir remote controlers.



(a) Normal noise varians vs errorr



(b) Uniform noise varians vs errorr



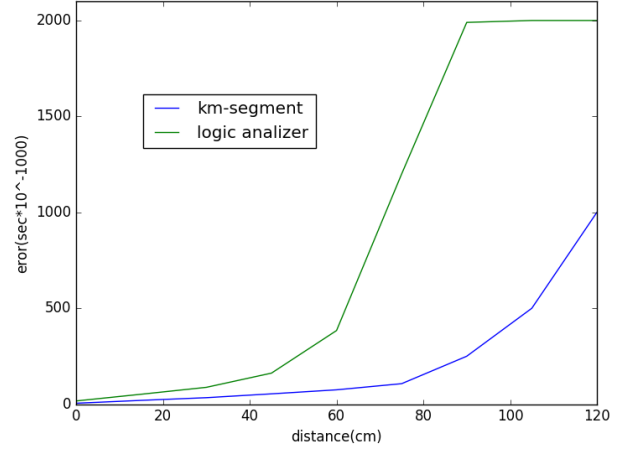
(c) Number of segment vs errorr

**Figure 2:** Figure 2a shows the division error as function of normal noise varians that added to the signal. For  $(k, m)$ -segmentation algorithm(blue) against threshold's(green). Figure 2b shows the same as 2a but for a uniform noise. Figure 2c shows the division error as function of the number of segments in the input signal  $(k, m)$ -segment(blue) against thresholding(green).

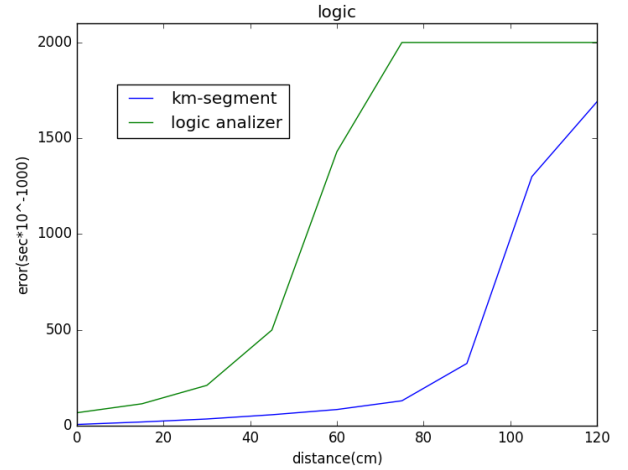
### 3.1 segmentation of data

We first examine the behavior of the algorithm on synthetic data which provides us with easy ground-truth, to evaluate the quality of the algorithm. We generate synthetic test data by drawing a discrete  $(k, m)$ -segment  $P$  with  $k = 15$  and  $m = 2$ , and then we add Gaussian and normal noise(one in each experament).

#### Resistance



(a) IR controler to IR resiver distance vs errorr



(b) IR controler to IR resiver distance vs errorr

**Figure 3:** Figures 3a and 3b shows the resultes of the expermaente that discribe in subsection 3.2 .the graphs shows taht the incrising of the division error( $\mu sec$ ) of the  $(k, m)$ -egmantation(blue) is slower the incrising of the logic analyzer(green) division error while we increase the distance between the IR controler and the IR resever(cm),for two diffrent key configurations .

### 3.2 real life data

We compare our algorithm agense Logic-analyzer device. We conect an IR reciver circal that plot anlog sinal to the Logic-analyzer and then ww comper the Logic

## 4. REFERENCES

- [1] D . Feldman, G. Rosman, M. Volkov, J.W. Fisher III, and D. Rus Proc. *Coresets for  $k$ -Segmentation of Streaming Data*. 27th Conference on Neural Information Processing Systems (NIPS) 2014