

(k,m)-Segmentation Algorithm for Horizontal Lines

Lawrence P. Leipuner
Brookhaven Laboratories
Brookhaven National Lab
P.O. Box 5000

lleipuner@researchlabs.org

ABSTRACT

Division of an electrical signals into high and low, cuts a soccer video into filed of game and audiences shots... all of those are a thresholding problems. But when the noise get stronger the regular thresholding algorithm get weaker . In the km-segment mean problem we need to divide an input signal into a k-piecewise function when each piece is one of set M in size m.

we aproximate the solution for a constrained version of (k, m) -segmentation problem when the lines are horizontal . We can do that by trying every m size set in the input data as M and compute the k-segment of the signal while limit the segment to be from M set. then choose the one with minimum cost. What give us $cost < 4cost_{opt}$ by $O(n \text{ chose } m^*)$ time using corset tocompute k-segment (fl14).

we show how our algorithm can divides better than a regular threshold and keep divide when we increase the noises even after threshold totally failed.

1. INTRODUCTION

1.1 Main Contribution

The main contributions of the paper are: (i) A solution for a constrained version of $(k, 2)$ -segmentation problem(as given in 1.2) when the lines are horizontal .(ii)And to test this new solution on a Infra Red drone's conroler in order to do the first step of decoding this data.

1.2 Problem Statement

The (k, m) -segment mean problem optimally fits a given discrete time signal of n points by a set of k linear segments

over time, where $k \geq 1$ is a given integer. That is, we wish to partition the time into k consecutive time intervals such that the points in each interval are lying on a single line from one of m types of lines, where $m \geq 1$ is a given integer. (i)

We make the following assumptions with respect to the data:(i) We assume the data is represented by a feature space that suitably represents its underlying structure; (ii) The content of the data includes at most k segments ,from at most m (for this paper $m = 2$) types. that we wish to detect auto- matically; An example for this are scenes of the audience and the field in a soccer video, a "bits" of an Infra Red signal (as we show in Section 3), etc.

This motivates the following problem definition.

Definition. $Efi(k, m)$ -segment mean. A set P in \mathbb{R}^{d+1} is a signal if $P = \{(1, p_1), (2, p_2), \dots, (n, p_n)\}$ where $p_i \in \mathbb{R}^d$ is a point at time index i and $i = [n] = \{1, \dots, n\}$. For an integers $k \geq 1$, a (k, m) - segment is a k-piecewise linear function :

$$f : \mathbb{R} \rightarrow \mathbb{R}^d = \begin{cases} f'_1(x) & \text{if } x < d_1 \\ f'_2(x - d_1) & \text{if } d_1 \leq x < d_2 \\ \dots & \\ f'_k(x - d_{k-1}) & \text{if } d_{k-1} \leq x < d_k \end{cases}$$

where $\forall f_i : f'_i \in F$, and F is a group of lines from order m . this fuction f maps every time $i \in \mathbb{R}$ to a point $f(i)$ in \mathbb{R}^d .

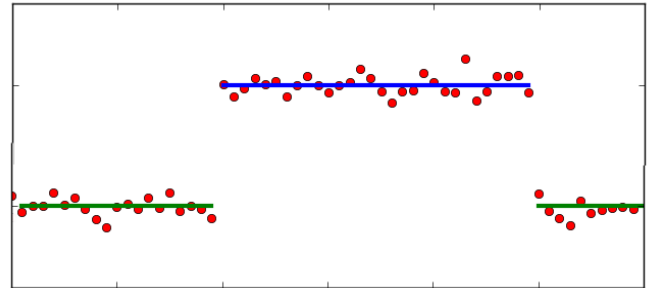


Figure 1: This figure is an exemple to the $(3, 2)$ -segment. the red dots are the input data and the lines are the segments when the blue is the low type and the grin is the high type.

The fitting error at time t is the squared distance between p_i and its corresponding projected point $f(i)$ on the (k, m) -segments. The fitting cost of f to P is the sum of these squared distances.

$$\text{cost}(P, f) = \sum_{i=1}^n \|p_i - f(i)\|_2^2.$$

where $\|\cdot\|$ denotes the *Euclidean* distance. The function f is a (k, M) -segment mean of P if it minimizes $\text{cost}(P, f)$ and conducts the conditions above.

In this paper we interest in a constrained version of $(k, 2)$ -segmentation ((k, m) -segmentation for $m = 2$) problem when the lines are horizontal.

2. THE ALGORITHM

If we look at the problem we can see that (i) if we had the correct set F so the problem was easy since all what we had to do is to compute the k -segment (see Definition 2) mean where each segment is in F . (ii) And if we had the dividers of the segments on the (k, m) -segment then each segment will be the mean height of the corresponding input points.

Definition 1: k -segment mean. The k -segment mean problem is a private case of the (k, m) -segment mean, when $k = m$.

Definition 2: α, β -approximation. For $\alpha, \beta > 0$, an (α, β) -approximation for the (k, m) -segment mean of P is a (k) -segment g such that $\text{cost}(P, g) \leq \alpha * \text{cost}(P, f)$.

THEOREM 1 (BICRITERIA APPROXIMATION [1]). *Let $P = \{(1, p_1), \dots, (k, p_k)\}$. Then an (α, β) -approximation for P can be computed in $O(dn)$ time where $\alpha = ..$ and $\beta = ..$*

For this solution to the km -segment problem we use i) Bicriteria. ii) BalancedPartition. As described in Feldman's 2014.

Algorithm 1 km - segmentation main

Input: A set $P = \{(1, p_1), \dots, (n, p_n)\}$ in \mathbb{R}^{d+1} and integer $k \geq 1$
Output: A 2-approximation to the (k, m) -segment mean of P
when $m = 2$.
Set $a \leftarrow \infty$
for $\{p_i, p_j \in P \mid 1 \leq i \leq j \leq n\}$ **do**
 Set $f \leftarrow \text{Bicriteria}(P, k, p_i, p_j)$
 $\text{coreset} = \text{BalancedPartition}(P, \varepsilon, \text{cost}(f), p_i, p_j)$
 $\text{temp} = \text{K-Segmentation}(\text{coreset}, k, p_i, p_j)$
 $\text{best_fit} = \text{Compute temp such that } \text{cost}(P, \text{temp}) \text{ is minimized.}$
end for
return best_fit

g_i is the line $y = p_i$

THEOREM 2 (SOLUTION COST). *This algorithm that we present will give a f' that $\text{cost}(P, f') \leq (2 + \varepsilon) * \text{cost}(P, f)$*

Algorithm 2 alternative for 1 - segment main

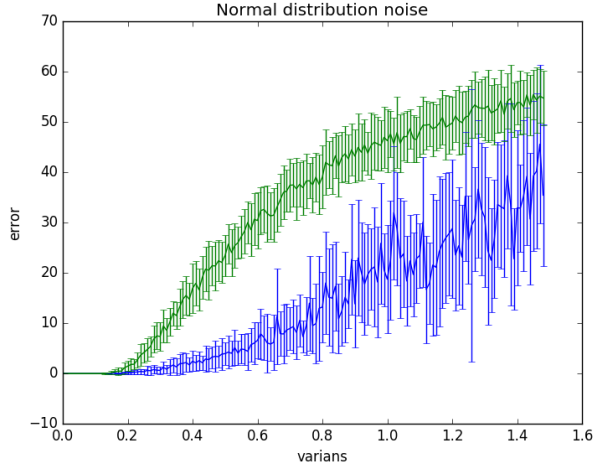
Input: A set $P \subseteq \mathbb{R}^{d+1}$ and two vectors $p_1, p_2 \in \mathbb{R}^d$
Output: An 2-approximation to the (k, m) -segment mean of P when $m = 2$.
if $\text{cost}(g_1, P) < \text{cost}(g_2, P)$ **then**
 return g_1
else
 return g_2
end if

PROOF. Without loss of generality if we look on all of the dots on the segments that from the higher type. Let p_{opt} be the mean of the corresponding of all those points, Let cost_{opt} be the average distance to p_{opt} and let p_s be the closest point to p_{opt} . Then the distance $\Delta(p_{opt}, p_s)$ is smaller or equal to cost_{opt} so from the triangle inequality for each point p in the data: $\Delta(p, p_s) < 2 * \text{cost}_{opt} (\Delta(p, p_{opt}) + \Delta(p_{opt}, p_s))$.

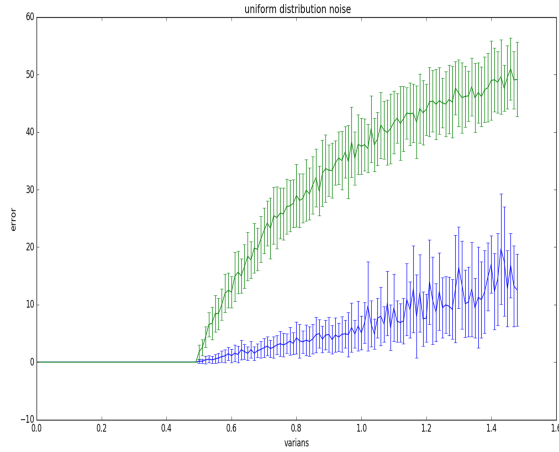
□

3. EXPERIMENTAL RESULTS

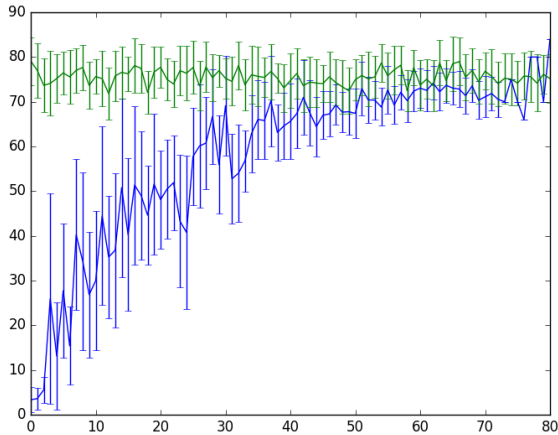
We now demonstrate the results of our algorithm on synthetic data in couple of tests. We compare our algorithms against regular thresholding algorithm. we also show our algorithm performs on a real life data taken from an ir remote controllers.



(a) Normal noise varians vs errorr



(b) Uniform noise varians vs errorr

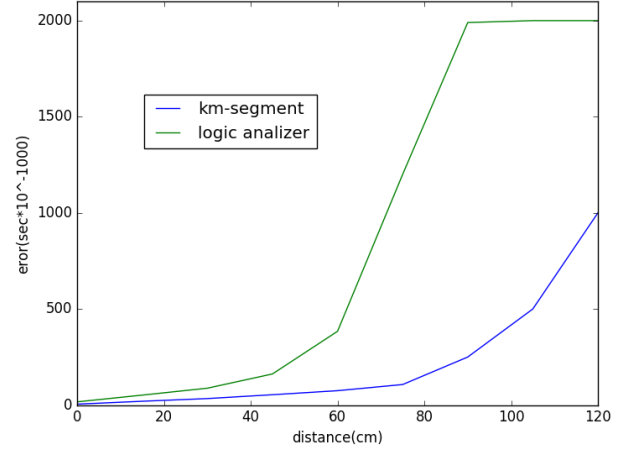


(c) Number of segment vs errorr

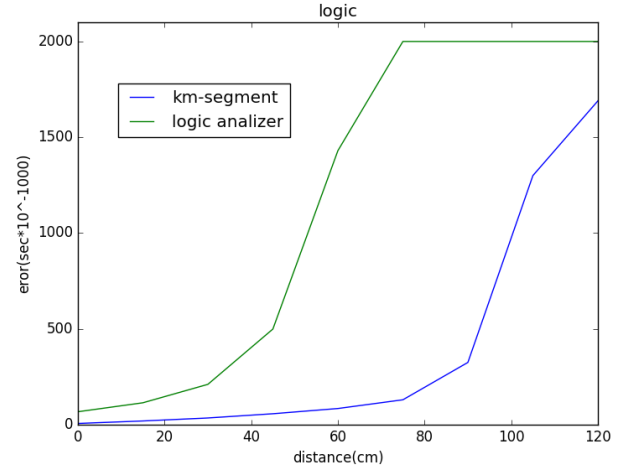
Figure 2: Figure 2a shows the division error as function of normal noise varians that added to the signal. For (k, m) -segmentation algorithm(blue) against threshold's(green). Figure 2b shows the same as 2a but for a uniform noise. Figure 2c shows the division error as function of the number of segments in the input signal (k, m) -segment(blue) against thresholding(green).

3.1 segmentation of data

We first examine the behavior of the algorithm on synthetic data which provides us with easy ground-truth, to evaluate the quality of the algorithm. We generate synthetic test data by drawing a discrete (k, m) -segment P with $k = 15$ and $m = 2$, and then we add Gaussian and normal noise(one in each experament).



(a) IR controler to IR resiver distance vs errorr



(b) IR controler to IR resiver distance vs errorr

Figure 3: Figures 3a and 3b shows the resultes of the expermaente that discribe in subsection 3.2 .the graphs shows taht the incrising of the division error(μsec) of the (k, m) -egmantation(blue) is slower the incrising of the logic analyzer(green) division error while we increase the distance between the IR controler and the IR resever(cm),for two diffrent key configurations .

3.2 real life data

We compare our algorithm agense Logic-analyzer divice. We conect an IR reciver circal that plot analog sinal to the Logic-analyzer and then ww comper the Logic

4. REFERENCES

- [1] D. Feldman, G. Rosman, M. Volkov, J.W. Fisher III, and D. Rus Proc. *Coresets for k -Segmentation of Streaming Data*. 27th Conference on Neural Information Processing Systems (NIPS) 2014