On the Trends of Technology, Family Formation, and Women's Time Allocation*

Sagiri Kitao[†] Kanato Nakakuni[‡]

December 29, 2023

Abstract

Over the past 50 years, Japan has witnessed a dramatic decline in fertility and marriage rates, along with a rise in educational attainment, particularly among women. Married women now dedicate significantly less time on market work and housework and more time on leisure and childcare. We develop a model that allows for various forms of technological change and relative prices surrounding families, and quantify their roles to account for the trends of family formation and time allocation. We find that neutral productivity growth leads to an increase in leisure time and a decrease in work hours. Technological changes that favor female labor supply and rises in the time and financial costs of childcare are the main factors contributing to the decline in fertility and marriage rates. Skill-biased technological change contributes to the rise in education levels, while advancements of home production technology explain the shift in married women's time allocation from housework to the market work.

Keywords: Fertility, Marriage, Home Production, Women's Time Allocation, Skill-biased Technological Change, Gender-biased Technological Change, Japan.

JEL Classification: D10, E10, J10, O11

^{*}We are grateful for useful comments from Toni Braun, Ricardo Marto, Kazushige Matsuda, Naoki Takayama, Satoshi Tanaka, Ken Yamada, and Minchul Yum, and participants of seminars at Hitotsubashi University, National Graduate Institute for Policy Studies (GRIPS), OzMac Workshop at Monash University, RIETI, Summer Workshop on Economic Theory (SWET), and Tohoku University. Kitao acknowledges financial support from the Grant-in-Aid for Scientific Research (B) 22H00831 and International Leading Research 22K21341.

[†]University of Tokyo, Research Institute of Economy, Trade and Industry (RIETI), and Centre for Applied Macroeconomic Analysis (CAMA). Email: sagiri.kitao@gmail.com

[‡]University of Tokyo, Email: k-nakakuni@g.ecc.u-tokyo.ac.jp

1 Introduction

Over the past several decades, advanced economies have undergone a secular transformation of the family structure and time allocation within households, as well as a substantial shift in the wage structure. Marriage rates have declined and couples are having fewer children, while dedicating a larger portion of their resources toward sending their children to college for skill acquisition, despite the escalating costs of education. Educational attainment has risen, particularly among women, leading to a narrowing of the gender gap. This implies that women face higher opportunity costs in allocating their time on childcare and domestic tasks. Additionally, advancements in home production technology have played a role in alleviating time constraints for household members.

All of these factors including demographics, family formation, technology both at home and in the market, and time allocation, are interconnected and should be simultaneously considered within a unified framework to comprehensively explain observed trends. The objective of this study is to make such an attempt.

We build a model in which individuals initially enter the economy as single and subsequently encounter a potential partner, making decisions regarding marriage. They allocate their disposable time among various activities, including market work, home production, leisure, and childcare. Married couples make choices concerning the number of children and the level of education for them, considering quantity-quality trade-offs (Becker and Barro 1988; Barro and Becker 1989).

Wages are determined in the competitive market, and the production function differentiates between high-skilled and low-skilled labor, as well as between men and women, as potentially distinct factors of production. In our model, the technology of market production can grow through three factors: the growth of factor-neutral technology or total factor productivity (TFP), skill-biased technological change (SBTC), and gender-biased technological change (GBTC). The SBTC can also be decomposed into two parts, a general one and one that is specific to female labor supply.

In the quantitative analysis, we calibrate the model using data from Japan, a country that has experienced a significant transformation over the last half century. Japan witnessed a dramatic decrease in total fertility rate from above 2.1 in 1970 to around 1.3 in 2020, as well as a simultaneous decline in marriage rates. While the gender gap in wage and education level narrowed, substantial disparities in wages and time allocation between men and women remain. To obtain relevant information on time allocation, we use various micro-level datasets such as the Survey on Time Use and Leisure Activities. Additionally, we utilize the Employment Status Survey to acquire wage rates by gender and skill to calibrate the underlying technological parameters. Furthermore, we use the price data of housework-assisting durable goods to quantify the technological advancements in home production.

We find that the increase in women's wages constitutes a major contributor to the trend of family formation. Technological changes that favor female labor supply, which include the SBTC specific to women and the GBTC, result in an increase in the opportunity cost of women, leading to a decline in fertility and marriage rates.

Female-specific SBTC and GBTC also lead to a rise in work hours of married women, along with a decline in housework and leisure time. Compared to the baseline economy, we find that, without the GBTC, married women would work 31% less on average, the marriage rate would be 9 percentage points higher, and the total fertility rate would be 1.56, as opposed to 1.33 in the baseline economy in 2020. While the general equilibrium effects partially offset the negative impact of GBTC due to the lower labor supply of women leading to higher wages, the qualitative effects remain unchanged.

Both general and women-specific SBTC contribute to a rise in married couples' investment in skill development for their children. Without the SBTC, the college enrollment rate would have been significantly lower than current rate and the impact can be attributed to the absence of a rise in returns on educational investment and the loss of income effect.

Neutral technological growth leads to an increase in the leisure time of married women and a decrease in work hours. Gender-neutral SBTC has similar effects caused by the income effect from higher household income.

Furthermore, the decline in the prices of housework-assisting goods, such as washing machines and vacuum cleaners, has contributed to a decrease in the time that is allocated to housework. Without this decline in prices, married women would have spent 15% more time on housework in 2020. Moreover, it has a positive impact on work hours of married women.

In our model, we incorporate fixed costs associated with basic childcare per child, which include both parental time and financial resources. There are also financial costs of education. We assume that these per-child basic costs and per-unit education costs are exogenous to parents, and let them decide on the number of children and allocation of financial resources toward their children's education, which subsequently enhances the children's quality through skill acquisition.

We find that an increase in financial and time costs of basic childcare leads to a decline in fertility rates, but encourages more investment in skills in the smaller number of children, thereby shifting the trade-off between quantity and quality of children toward the latter. An increase in education costs has the opposite effect, resulting in couples having a larger number of children and a decrease in the education level along the simulated transition path.

The study contributes to the growing literature on family and macroeconomics, which emphasizes the significance of intra-household decisions in explaining the dynamics of key variables that shape the macroeconomy.¹ Doepke et al. (2023) review recent research regarding the economics of fertility and highlight that the conventional theory, which suggests a negative relationship between income and fertility, no longer holds. They argue that various factors influencing the alignment of family objectives and women's careers play critical roles in understanding fertility trends.²

In a quantitative approach, Caucutt et al. (2002) construct a model with endogenous fertility, marriage, and women's labor supply, and demonstrate that the increase in returns to women's labor market experience is responsible for the observed delay in fertility in the U.S. Greenwood et al. (2005) incorporate home production and demonstrate that the burst of technological progress in the household sector contributed to the baby boom. Cordoba et al. (2019) study cross-country data on education investment and fertility, and investigate the determinants of the quantity-quality trade-off across different countries. Baudin et al. (2015) replicates the cross-sectional observations regarding the family formation and education in the U.S., and investigate the determinants of the decision to not have children.

Kim et al. (2023) build a model incorporating status externalities in education and endogenous fertility to establish a connection between the education fever and low fertility rate in Korea. Yamaguchi (2019) estimates a model using Japanese data to investigate the effects of parental leave policies on fertility and maternal employment. ³

Greenwood et al. (2016) develop a model that examines the dynamics of the marriage, divorce, and income inequality. They emphasize the roles played by home production technology and the wage structure. As they primarily focus on the marriage market and labor supply, their model abstracts from fertility decisions.⁴ The closest to our paper is the study by Greenwood et al. (2023), who propose a theory of endogenous marriage, fertility, and labor supply that incorporates technological development both in the market

¹A comprehensive survey of this literature is provided by Doepke and Tertilt (2016).

²For a theoretical treatment of fertility decisions in the context of the macroeconomy and economic growth, refer to Galor and Weil (1996, 2000), de la Croix and Doepke (2003) and Jones et al. (2010).

³There are also studies that utilize a life-cycle framework to model family decisions. Santos and Weiss (2016) develop an equilibrium search model of overlapping generations and demonstrate that the increase in income volatility explains a significant portion of the decline and delay of marriage in the U.S. Eckstein et al. (2019) find that advancements in contraception technology accounts for half of the fertility decline between the two cohorts born in 1935 and 1975, while improved labor market opportunities play a crucial role in the decline in marriage rates. Daruich and Kozlowski (2020) model household decisions regarding fertility and family transfers to analyze their interactions with integenerational mobility. Nakakuni (2023) constructs a life-cycle model calibrated to the Japanese economy to assess the impact of child benefit policies.

⁴There is a growing body of literature that focuses on marriage dynamics, including divorce. Besides Greenwood et al. (2023), see, for example, Goussé et al. (2017) that builds a search model of marriage and study how wages, education and family attitudes affect marital decisions and intra-household allocation. Voena (2015) estimate a model of couple's savings and labor supply during marriage to study the effects of divorce laws on their behavior and divorce decisions.

and at home. Their model is calibrated to capture the family trends in the U.S. since the late 19th century, providing a framework to analyze long-term trends of the structural change and household behavior in a tractable manner. However, the model does not distinguish between men and women, which is an essential distinction in our model for understanding the time trends of family in Japan and potentially in other countries as well.

Our study also focuses on the time allocation of married women and builds upon the literature that examines the determinants of female labor supply, including the wage structure and fiscal policies.⁵ Men and women exhibit various differences in our model and in the data. In Japan, married women bear a disproportionatelly large burden of childcare responsibilities. The significant cost of childcare greatly influences family decisions regarding fertility and labor supply. As mentioned earlier, educational attainment and wages have also followed distinct trajectories for men and women, although the gap has narrowed in recent decades. The advancements in home production technology have contributed to a decline in hours spent on household chores, particularly for women (Greenwood et al. 2005). Considering how these differences evolved over time is crucial for understanding the evolution of family behavior.

There is also a large body of literature that investigates the roles of technological change to account for the dynamics of the wage structure (Katz and Murphy 1992; Krusell et al. 2000). Heathcote et al. (2010) and Abbott et al. (2019) consider the technology that distinguishes among labor supply of different skills and genders, accounting for the rich heterogeneity in the dynamics of wage inequality. Kawaguchi and Mori (2016) contrast the evolution of skill premium in the U.S. and Japan and Taniguchi and Yamada (2023) investigate the gender and skill premium across OECD countries and estimate a model of factor-biased technological progress.⁶

The market wage structure not only influences the career plans of men and women, but also shapes their decisions regarding family formation. Wages of men and women represent opportunity costs of family members as they choose how to allocate disposable time to various activities, including work at home and in the market, childcare, and leisure. We also aim to contribute to the literature on the evolution of factor-biased technology, by studying the roles of various forms of technological growth on the decisions of families.

To the best of our knowledge, this study is the first attempt to construct a unified model that encompasses fertility, marriage, education, and women's time allocation, while considering the endogenous wage structure. We aim to demonstrate the interconnections among these factors and highlight the critical role each factor plays in jointly explaining

⁵See also, Attanasio et al. (2008), Albanesi and Olivetti (2009), Guner et al. (2012), Jones et al. (2015), Bick and Fuchs-Schundeln (2017), Borella et al. (2023), and Kitao and Mikoshiba (2023).

⁶Jang and Yum (2022) study nonlinear relationship between wages and work hours in some occupations and argue that the rise in experience premiums has a negative impact on the participation of women.

the observed trends in both family dynamics and the macroeconomy.

The rest of this paper is organized as follows. Section 2 presents the relevant data and discusses the trends in family and macroeconomic variables of interest. Section 3 describes the quantitative model, while Section 4 discusses the model's parametrization. The numerical results are presented in Section 5 and Section 6 presents a conclusion.

2 The Time Trends of Family Facts

In this section, we examine the trends of various facts surrounding families in Japan. Figure 1(a) shows the path of the total fertility rates in Japan, which represents the average number of children that each woman gives birth to over her life-cycle. In the early 1970s, the fertility rate exceeded the replacement rate that is needed to keep the population from decreasing, but started to decline quickly thereafter. By the mid-2000s, it had fallen below 1.3 and stayed at around 1.3 to 1.4 until 2020.

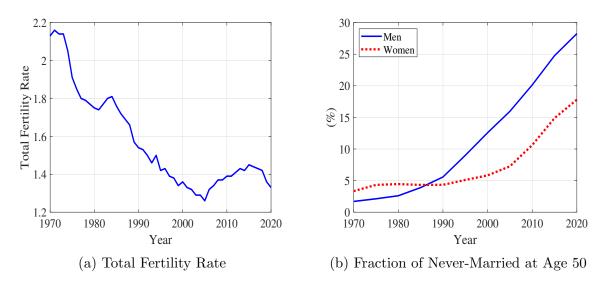


Figure 1: Fertility and Marriage

Source: Vital Statistics, Ministry of Health, Labour and Welfare, and Population Census, Ministry of Internal Affairs and Communications.

While fertility rates declined rapidly, marriage rates also declined.⁸ Figure 1(b) shows the fraction of men and women aged 50 who have never been married in their life. The

⁷More precisely, it is computed as the average number of children that a hypothetical cohort of women would have if they were subject to the fertility rates of a given year during their whole lives.

⁸Note that out of wedlock birth is uncommon in Japan and we do not include them in our analysis. According to the OECD Family Data in 2020, the share of births outside of marriage is 2.5% in Korea and 2.4% in Japan, the lowest among the OECD countries. The share is 41% for the U.S. and 42% for the EU countries on average. See also, Myong et al. (2021) for a structural model that explicitly incorporates stigma associated with out of wedlock births.

share was less than 5% until the late 1980s for men and until the early 1990s for women, but it rose quickly and reached 28% for men and 18% for women by 2020.

The cost of raising a child also increased during the last decades. As shown in Figure 2, the college enrollment rate increased from less than 30% in 1970 to almost 60% for men in 2020. The rise is more dramatic for women. Less than 10% of women went to college in 1970, but the share exceeded 50% in 2020. The cost of attending college also increased at the same time, as shown in the path of the average college tuition fee in Figure 2(a).

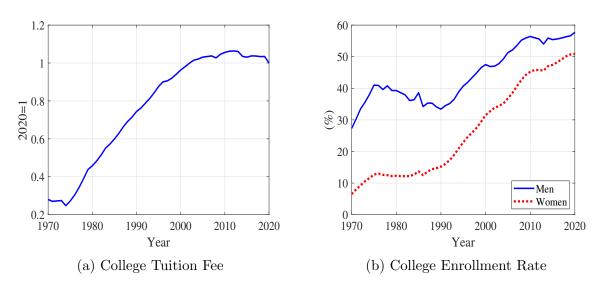


Figure 2: College Costs and Enrollment

Source: College tuition fee is based on the Consumer Price Index, Ministry of Internal Affairs and Communications. (a) depicts the price index of college tuition fees relative to the headline CPI. The data on 4-year college enrollment rates are from the School Basic Survey by the Ministry of Education, Culture, Sports, Science and Technology.

Not only the financial cost, but also the time that parents spend on raising children increased in the last 50 years. Table 1 shows the time allocation of married men and women of working age in the late 1970s and 2010s. Both men and women increased the share of time spent on childcare, and women especially allocate a much larger share of their time on childcare than did men, and the share increased from 7.1% to 13.9%. This occurred at the same time as the number of children decreased and childcare time per child rose even more rapidly. 10

⁹The data is based on the Survey on Time Use and Leisure Activities. The Survey is conducted every five years. Although the data for 2021 is available, we use the 2016 data as the most recent one to focus on the long-term trend and avoid being influenced by the short-term effects of the COVID shock.

¹⁰Similar trends have been observed to varying degrees in other countries. See Aguiar and Hurst (2007), Ramey and Ramey (2010), Doepke and Zilibotti (2017), and Kim et al. (2023). Guryan et al. (2008) show that the trend of rising childcare time is more pronounced among more educated and higher-income parents in the U.S. and that a similar pattern is observed within and across countries in a sample of 14

Table 1 shows that while married women spend more time on childcare, they allocate a significantly smaller fraction of their time to housework and market work. The time for leisure increased from 17.5% to 23.1%.

Table 1: Time Allocation of Married Men and Women (% of disposable time)

	M	en	Woı	Women			
	1976 2016		1976	2016			
Market work	77.45	71.52	37.22	30.61			
Housework	0.71	2.11	38.19	32.32			
Childcare	0.49	2.72	7.14	13.93			
(per child)	(0.22)	(1.68)	(3.24)	(8.61)			
Leisure	21.34	23.65	17.46	23.14			
Total	100.0%	100.0%	100.0%	100.0%			

Source: Survey on Time Use and Leisure Activities. Ministry of Internal Affairs and Communications. Average time use of married men and women aged 25-59.

The decline in the hours dedicated to housework occurred at the same time as the advancement in home production technology. Figure 3 shows the price index of major household appliances, such as refrigerators, washing machines, and vacuum cleaners, relative to the headline CPI. The relative price declined at an annual rate of 4 to 8%. The price indices of those goods are summarized as the index of housework-assisting durable goods. Figure 3(b) shows the path of their relative price, which declined at an annual rate of 5.75% between 1970 and 2020, while the relative prices of other items such as food and houses did not grow significantly.

countries.

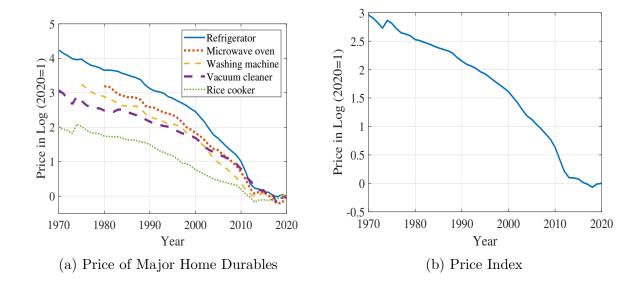


Figure 3: Price of Housework-assisting Durable Goods

Source: Consumer Price Index, Ministry of Internal Affairs and Communications. Each plotted line indicates the price index relative to the headline CPI. "Durable Goods" in the legend on (b) represents the price index of *housework-assisting durable goods*, including refrigerators, microwave ovens, washing machines/dryers, vacuum cleaners, rice cookers, and gas stoves, relative to the headline CPI.

Over time, as the family structure and time allocation changed, macroeconomic environment and wage structure shifted. Table 2 shows the real wage rates by gender and skill in 1970 and 2020, where high-skill represents those with college degrees or above.¹¹ The wage rate of low-skilled women in 1970 is set to 1 for normalization. Wages increased for all groups, with women's wage growth being higher than men's within each skill group. The average wage of women grew at an annual rate of 1.37%, while men's increased at 0.91%. Since there is also a rise in the share of high-skilled workers among both men and women, the average growth rate is higher than the growth rates within skill groups.

The wage growth of low-skilled women is higher than that of high-skilled women. As a result, the women's skill premium, defined as the ratio of high-skill wage to low-skill wage, declined from 1.65 to 1.55 in 2020, as shown in the bottom section of Table 2. Men's skill premium has not changed much between 1970 and 2020. Taniguchi and Yamada (2021) also demonstrate similar trends of men and women's skill premiums in Japan since 1980 using the EU KLEMS database.

The gender gap is defined as the ratio of women's wages to men's wages, and it has

¹¹We use the Employment Status Survey (ESS) data between 1982 and 2017, which contain information about work hours and annual earnings by gender and education level. The ESS data is based on statistical products provided by the Statistics Center, an independent administrative agency based on the Statistics Act

We use the real wage index of the Monthly Labour Survey since 1970 to extrapolate the wage in the 1970s.

narrowed over the last five decades, from 0.51 to 0.64 on average. Note that the change in gender gap is larger on average than the gender gap within the skill groups, since there was also a change in the composition of worker skills by gender.

Table 2: Wages by Gender and Skill, Gender Gap, and Skill Premium

		1970	2020	Ann. Growth
Women	Low Skill	1.00	1.66	1.02%
	High Skill	1.65	2.57	0.89%
	Weighted Avg.	1.03	2.03	1.37%
Men	Low Skill	1.83	2.72	0.79%
	High Skill	2.49	3.67	0.78%
	Weighted Avg.	2.01	3.17	0.91%
Gender Gap	Low Skill	0.55	0.61	_
	High Skill	0.66	0.70	_
	Weighted Avg.	0.51	0.64	_
Skill Premium	Men	1.36	1.35	_
	Women	1.65	1.55	_
	Weighted Avg.	1.46	1.42	_

Source: Employment Status Survey (ESS) and Monthly Labour Survey (MLS). Wage of low-skilled women in 1970 is set to 1.0 for normalization. *Gender gap* is defined as the ratio of women's wages to men's wages. *Skill premium* is defined as the ratio of high-skill wages to low-skill wages.

As shown in Table 3, the share of female workers in the labor force increased from 31% in 1970 to above 45% in 2020, and the rise is driven by an increase in the number of high-skilled women in the labor force. Low-skilled male workers represented 50% of the labor force in 1970, and the share declined to 29% over the last 50 years. Men, both low and high-skilled, constitute 54% of the labor force in $2020.^{12}$

We use these observations to calibrate the technological progress during the last half century in Section 4. It is important to consider the dramatic change in the composition of labor supply to account for the shift in the wage structure and productivity of different types of labor inputs.

¹²For structural models with endogenous labor participation decisions of women in Japan, see, for example, Kitao and Mikoshiba (2023) and Yamaguchi (2019).

Table 3: Distribution of Workers by Gender and Skill

	1970	2020
Women		
Low Skill	29.7%	28.5%
High Skill	1.4%	17.4%
Total	31.1%	45.9%
Men		
Low Skill	50.2%	29.0%
High Skill	18.7%	25.2%
Total	68.9%	54.1%
Total	100.0%	100.0%

Source: Population Census, Ministry of Internal Affairs and Communications.

3 Model

3.1 Overview

An individual of our model enters the economy as single and is matched with another single person upon entry. The pair chooses to get married if the value of marriage exceeds the value of staying single and remain single otherwise.

A married couple chooses consumption of market goods and non-market goods. The latter is produced at home, using durable goods and housework time as inputs. The couple also decides how many children to have. Parents derive utility from the quantity and quality of children, taking into account the time and money cost of raising children and educating them. The household allocates disposable time toward leisure, home production, market work, and childcare. Single individuals consume market and non-market goods and allocate their time to leisure, home production, and market work. Market goods are produced using skilled and unskilled labor and wages are determined in the competitive market.

The framework is static to keep the model tractable. Individuals make one-time decisions about marriage, and then about consumption, time allocation and fertility in the case of married couples. The model aims to examine how time-varying factors influence the evolving patterns of decision making by families and we abstract from roles of heterogeneity and uncertainty within cohort.

3.2 Preferences

Married households derive utility from a couple's consumption of market goods c, non-market goods n, total leisure time of husband and wife $l = l_m + l_f$, the number of children k, and the quality of children q. The utility function is denoted as $u^M(c/\eta, n/\eta, l, k, q)$, where η represents the equivalence scale of consumption goods for the couple, and is given as follows.

$$u^{M}(c/\eta, n/\eta, l, k, q) = \alpha \frac{(c/\eta)^{1-\rho} - 1}{1-\rho} + \beta \frac{(n/\eta)^{1-\nu} - 1}{1-\nu} + \mu \frac{l^{1-\lambda} - 1}{1-\lambda} + \phi \frac{k^{1-\kappa} - 1}{1-\kappa} + \xi \frac{q^{1-\psi} - 1}{1-\psi}, (1)$$

Parameters α , β , μ , ϕ , and ξ represent the weight attached to utilities from the consumption of market and non-market goods, leisure time, and the number and quality of children, respectively. Parameters ρ , ν , λ , κ , and ψ represent the curvature of the utility function and affect how households respond to a changing environment by reallocating resources to maximize the utility.

A single individual of gender $g \in \{m, f\}$ derives utility from the consumption of market goods c, non-market goods n, and leisure l. The utility function is denoted as $u_a^S(c, n, l)$ and given as:

$$u_g^S(c, n, l) = \alpha_g \frac{c^{1-\rho} - 1}{1 - \rho} + \beta_g \frac{n^{1-\nu} - 1}{1 - \nu} + \mu_g \frac{l^{1-\lambda} - 1}{1 - \lambda}$$
 (2)

Parameters α_g , β_g , and μ_g denote the preference weight on consumption of market goods, non-market goods and leisure time, respectively, which may depend on gender g.

3.3 Children

A married couple derives utility from both the number of children k, as well as the quality of children q. Raising children is costly for parents in two ways: time and money for basic childcare and money for education. Basic childcare is required for all children and education investment is based on the choice of the family. For basic childcare, a married couple must spend a financial cost b per child, and each parent of gender g must spend time ζ_g per child.

Parents choose how many financial resources to invest in child education, which raises the quality of children. We assume that parents choose a mix of skills for their children, with a fraction s of high skill and 1-s of low skill. It costs χs per child to equip them with skill s, which enables them to enjoy the skill premium, denoted as w_h/w_l , where w_h and w_l represent high and low-skill wages, respectively.¹³

The quality of children is denoted as q and it increases utility of parents. We assume that the quality depends on the skill level of the children s that parents choose to endow

 $^{^{13}}$ In what follows, we call the choice of children's skill level s also as schooling decision made by parents. As discussed in more detail in Section 4, s corresponds to college enrollment rates in calibration.

children with, and also on how much the skill is valued in the market. We define the quality of children as

$$q = \frac{w_h}{w_l} s,\tag{3}$$

where the first part $\frac{w_h}{w_l}$ represents the value of high-skill given in the market relative to the value of low-skill, and the second part s is the skill level. Skill premium is exogenous to parents, but it is determined endogenously in the labor market as a function of the supply of the skill and exogenous technological change, as discussed in Section 3.6.

We assume that parents do not differentiate educational investment by the gender of children. Consequently, the gender of children does not enter the problem of married individuals.

3.4 Home Production

Home goods n are produced according to the following function with two inputs, durable goods (d) and housework hours (h):

$$n = \left[\omega d^{\sigma} + (1 - \omega)h^{\sigma}\right]^{1/\sigma} \tag{4}$$

 σ is the parameter that determines the elasticity of substitution between durable goods and labor input. Durable goods are priced at π per unit. For married households, h is the sum of housework hours supplied by husband and wife, $h = h_m + h_f$.

3.5 Household Problems

Single Households: Single individuals allocate their disposable time, normalized to 1, to leisure l, home production h, and market work 1 - l - h. They allocate income to the consumption of market goods c, and durable goods d priced at π .

The value function of single individuals of gender g is given as follows.

$$S_g = \max_{c,d,l,h} \left\{ u_g^S(c,n,l) \right\} \tag{5}$$

s.t.

$$c + \pi d = w_a (1 - l - h) \tag{6}$$

where w_g denotes the wage rate of individuals of gender g. Note that we abstract from heterogeneity within cohort, including difference in education levels. The wage of each gender is computed as the weighted average of low and high-skill wages determined in the labor market, as discussed in more detail in Section 3.6.

Married Households: Married couples allocate earnings of husband and wife net of costs of childcare to consumption of market goods c and durable goods d. Married households also choose the number of children k and education investment for children s. The household decision for the investment in education determines the quality q of children.

We assume that the time allocation of married men is exogenous and they supply labor at home and in the market inelastically, and their time contribution to the home production and childcare is also exogenously given. Therefore, the time allocation decision of the couple is in regard to the wife's time for leisure l_f , home production h_f , and market work, which is given by $(1 - \zeta_f k - l_f - h_f)$, the total disposable time net of time spent on childcare, leisure, and home production.

The value function of married households is defined as

$$M = \max_{c,d,l_f,h_f,k,s} \left\{ u^M(c/\eta, n/\eta, l, k, q) \right\}$$
 (7)

s.t.

$$c + \pi d + \chi sk + bk = \sum_{g} w_g (1 - \zeta_g k - l_g - h_g)$$
 (8)

where the housework is given by $h = h_m + h_f$, leisure $l = l_m + l_f$ and the quality of children $q = (w_h/w_l)s$.

The value of a married individual of gender g is given by

$$\widehat{M}_{g} = \widehat{u}_{q}^{M}(c^{*}/\eta, n^{*}/\eta, l_{q}^{*}, k^{*}, q^{*}), \tag{9}$$

where a variable with an asterisk denotes the optimal choice from the above problem of married households. Utility function \widehat{u}_g^M of a married individual is defined similarly to (1), with leisure l_g of each individual, rather than that of a household. This value is relevant in the decision of marriage as discussed below.

Marriage Decision: Upon individuals' entry to the economy, each individual is matched with a potential partner. The pair makes a draw of a common joy shock r from the distribution F(r).

Given that we focus on the time allocation decision of married women, and for simplicity, we focus on the marriage decision of women and abstract from men's decision to marry. Women choose to marry if the sum of the value of marriage and the joy shock exceeds the value of staying single. Otherwise the pair will not marry and remain single. The decision rules are given as follows:

$$\begin{cases} \text{ marry if } \widehat{M}_f + r \ge S_f \\ \text{ single if } \widehat{M}_f + r < S_f \end{cases}$$

3.6 Market Production and Wages

The output is produced using labor supplied by men and women that consists of two different skill levels: high and low. We allow for imperfect substitutability among the four types of labor input and for the growth rates of the productivity that potentially differ across them.

From the wage rates for each factor determined in the labor market, we compute the average wage for male and female workers, w_g for $g \in \{m, f\}$, which individuals take as given in the household problems described in Section 3.5. We also compute the average wages for high and low-skilled labor, w_s for $s \in \{l, h\}$, which are used to compute the skill premium (3).

A representative firm produces output Y with unskilled labor L and skilled labor H according to the production function:

$$Y = F(L, H) = Z \left[L^{\varphi} + AH^{\varphi} \right]^{1/\varphi}, \tag{10}$$

where Z represents the neutral technology level and A governs the gender-neutral skill-biased technological change (SBTC). L and H are composites of male and female labor of each skill type, L_g and H_g for $g \in \{m, f\}$, and they are defined as follows:

$$L = \left[L_m^{\gamma} + B L_f^{\gamma} \right]^{1/\gamma} \tag{11}$$

$$H = \left[H_m^{\gamma} + A_f B H_f^{\gamma}\right]^{1/\gamma} \tag{12}$$

B and A_f govern the GBTC and SBTC specific to female workers, respectively.

The firm rents labor from individuals at the market wage rates. The labor market is competitive and the wages for unskilled and skilled labor of each gender are determined to equate supply and demand. A firm's problem is given as follows:

$$\max_{L_m, L_f, H_m, H_f} \left\{ F(L, H) - \sum_{g} (L_g w_{g,l} + H_g w_{g,h}) \right\}$$

In equilibrium, market wages are given as marginal product of each type of labor.

$$w_{m,l} = F_{L_m} = \widetilde{Z}L^{\varphi-\gamma}L_m^{\gamma-1}$$

$$w_{m,h} = F_{H_m} = \widetilde{Z}AH^{\varphi-\gamma}H_m^{\gamma-1}$$

$$w_{f,l} = F_{L_f} = \widetilde{Z}L^{\varphi-\gamma}BL_f^{\gamma-1}$$

$$w_{f,h} = F_{H_f} = \widetilde{Z}AH^{\varphi-\gamma}A_fBH_f^{\gamma-1}$$

$$(13)$$

where $\widetilde{Z} = Z \left[L^{\varphi} + AH^{\varphi} \right]^{\frac{1}{\varphi} - 1}$.

4 Calibration

We would like the model presented above to align with the facts presented in Section 2. We assign some parameter values directly from the data, and determine other parameter values to fit the transition path of the data. In particular, we focus on the trends between 1970 and 2020 and set some of the parameter values to match the transition patterns of data between the two periods. This strategy follows the method undertaken by Greenwood et al. (2023). Appendix A provides additional details of the data used in the calibration.

4.1 Preference Parameters

4.1.1 Equilibrium Conditions and Calibration Strategy

We use the optimality conditions of the household problems presented in Section 3.5 to pin down main preference parameters that enter the utility functions (1) and (2). We do so to match the target moments in two periods, 1970 and 2020, as explained in more detail below.

Substituting the home production equation (4) and the budget constraint (8) in the utility function of married households (1), the value function reads as

$$M = \max_{d,l_f,h_f,k,s} \left\{ \alpha \frac{\left\{ \left[\sum_g w_g (1 - \zeta_g k - l_g - h_g) - \pi d - \chi s k - b k \right] / \eta \right\}^{1-\rho} - 1}{1 - \rho} + \beta \frac{\left\{ \left[(\omega d^{\sigma} + (1 - \omega)h^{\sigma})^{1/\sigma} \right] / \eta \right\}^{(1-\nu)} - 1}{1 - \nu} + \mu \frac{l^{1-\lambda} - 1}{1 - \lambda} + \phi \frac{k^{1-\kappa} - 1}{1 - \kappa} + \xi \frac{q^{1-\psi} - 1}{1 - \psi} \right\},$$

The first order conditions of married households problem with respect to d, l_f , h_f , k, and s, respectively, are defined as follows:

$$d: \quad \alpha(1/\eta)^{1-\rho} c^{-\rho} \pi = \beta(1/\eta)^{1-\nu} n^{1-\sigma-\nu} \omega d^{\sigma-1}$$
 (14)

$$l_f: \quad \alpha(1/\eta)^{1-\rho} c^{-\rho} w_f = \mu(l_m + l_f)^{-\lambda}$$
 (15)

$$h_f: \quad \alpha(1/\eta)^{1-\rho} c^{-\rho} w_f = \beta(1/\eta)^{1-\nu} n^{1-\sigma-\nu} (1-\omega) (h_m + h_f)^{\sigma-1}$$
 (16)

$$k: \quad \alpha(1/\eta)^{1-\rho} c^{-\rho} (w_m \zeta_m + w_f \zeta_f + \chi s + b) = \phi k^{-\kappa}$$
 (17)

s:
$$\alpha (1/\eta)^{1-\rho} c^{-\rho} \chi k = \xi q^{-\psi} (w_h/w_l)$$
 (18)

The equation (14) represents the trade-off between the benefit obtained from additional home goods consumption by an extra unit of durable goods purchase, and the loss from market goods consumption. The conditions (15) and (16) equate the marginal benefit of an additional hour on leisure and housework, with the cost from reduced work hours and lower consumption.

The condition (17) equates the marginal benefit of having a child through direct utility from the larger number of children, with the marginal cost of raising a child, the time and

money to spare for basic childcare and education. In equation (18), the marginal cost of expenditures to educate children is equated with the higher utility from the better quality of children.

Turning to the problem of single individuals, substituting the equation for the home production (4) and the budget constraint (6) in the utility function (2), the value function reads as

$$S_g = \max_{d,l,h} \left\{ \alpha_g \frac{\left[w_g (1 - l - h) - \pi d \right]^{1 - \rho} - 1}{1 - \rho} + \beta_g \frac{\left[\omega d^{\sigma} + (1 - \omega) h^{\sigma} \right]^{\frac{1 - \nu}{\sigma}} - 1}{1 - \nu} + \mu_g \frac{l^{1 - \lambda} - 1}{1 - \lambda} \right\}$$

The first order conditions of single households problem with respect to d, l, and h, respectively, are given as follows.

$$d: \quad \alpha_a c^{-\rho} \pi = \beta_a n^{1-\sigma-\nu} \omega d^{\sigma-1} \tag{19}$$

$$l: \quad \alpha_g c^{-\rho} w_g = \mu_g l^{-\lambda} \tag{20}$$

$$h: \quad \alpha_g c^{-\rho} w_g = \beta_g n^{1-\sigma-\nu} (1-\omega) h^{\sigma-1}$$
 (21)

Similar to the problem of married households, the first order conditions represent the tradeoff between the utility from the consumption of market and home goods (19), time for leisure (20), and housework (21).

4.1.2 Married Households

For the utility function of married households (1), we set the weight parameter α to 1 for normalization and the risk aversion parameter to 2. The rest of the preference parameters, which include four weight parameters and four curvature parameters, are pinned down to match target data moments of (1) leisure (μ and λ), (2) fertility (ϕ and κ), (3) schooling (ξ and ψ), and (4) housework hours (β and ν). We exploit changes in these data between 1970 (t=0) and 2020 (t=1) to pin down curvature parameters and the levels in a specific year to set the weight parameters.

Leisure: From the first order conditions (15) and (8),

$$\mu(l_{m,t} + l_{f,t})^{-\lambda} = \alpha(1/\eta)^{1-\rho} c_t^{-\rho} w_{f,t}$$
(22)

where

$$c_t = \sum_{g} w_{g,t} (1 - \zeta_{g,t} k_t - l_{g,t} - h_{g,t}) - \pi_t d_t - \chi_t s_t k_t - b_t k_t$$

Taking the ratio of (22) at time t = 0 and 1,

$$\left(\frac{l_{m,1} + l_{f,1}}{l_{m,0} + l_{f,0}}\right)^{-\lambda} = \left(\frac{c_1}{c_0}\right)^{-\rho} \frac{w_{f,1}}{w_{f,0}} \tag{23}$$

The condition (23) describes the response of leisure to the changes in consumption and wages, representing income and substitution effects, respectively. The parameter λ plays a key role in representing elasticitieses of leisure. (23) pins down λ to match the target response in leisure, $l_{f,1}/l_{f,0}$, to changes in consumption and wages. Once λ is set, the preference weight on leisure μ is derived by from (22) evaluated at t = 0.

Fertility: From the first order conditions (17) with (8) and (15),

$$\phi k_t^{-\kappa} = \mu (l_{m,t} + l_{f,t})^{-\lambda} (w_{f,t} \zeta_{f,t} + \chi_t s_t + b_t) / w_{f,t}$$
(24)

In equilibrium, marginal utility from having another child on the left side of the equation is equated with marginal cost on the right. The latter is the sum of the time spent on childcare and financial costs of basic childcare and an optimally chosen education level, evaluated in terms of the lost utility from reduced leisure to cover such costs.

Using the condition for t = 0 and 1,

$$\left(\frac{k_1}{k_0}\right)^{-\kappa} = \left(\frac{l_{m,1} + l_{f,1}}{l_{m,0} + l_{f,0}}\right)^{-\lambda} \frac{(w_{f,1}\zeta_{f,1} + \chi_1 s_1 + b_1)/w_{f,1}}{(w_{f,0}\zeta_{f,0} + \chi_0 s_0 + b_0)/w_{f,0}}$$
(25)

From (25), we pin down κ to match the response in fertility to a change in leisure time and the total marginal cost of childcare in terms of wives' wages. With the value of κ , we compute ϕ from (24) for t = 0.

Education: Using the first order condition (18) with (3) and (15),

$$\xi q_t^{-\psi}(w_{h,t}/w_{l,t}) = \mu(l_{m,t} + l_{f,t})^{-\lambda} \chi_t k_t / w_{f,t}$$
(26)

Taking the ratio of the condition at t = 0 and 1,

$$\left(\frac{q_1}{q_0}\right)^{-\psi} \frac{w_{h,1}/w_{l,1}}{w_{h,0}/w_{l,0}} = \left(\frac{l_{m,1} + l_{f,1}}{l_{m,0} + l_{f,0}}\right)^{-\lambda} \frac{\chi_1 k_1/w_{f,1}}{\chi_0 k_0/w_{f,0}}$$
(27)

The equation (27) shows that the growth of the investment in children's quality is positively associated with the growth of the skill premium and the couple's leisure time, while negatively related to the cost of education in terms of married women's wages. We compute ψ from (27) and ξ from (26).

Housework Hours: Combining the first order conditions (16) and (15),

$$\beta(1/\eta)^{1-\nu} \left[\omega d_t^{\sigma} + (1-\omega)(h_{m,t} + h_{f,t})^{\sigma}\right]^{\frac{1-\sigma-\nu}{\sigma}} (1-\omega)(h_{m,t} + h_{f,t})^{\sigma-1} = \mu(l_{m,t} + l_{f,t})^{-\lambda}$$
(28)

A rise in d_t implies a decline in h_t , provided $(1 - \sigma - \nu) < 0$, and the effect is larger if either σ is larger $(h_t$ and d_t are more substitutable) or ν is larger. Taking the ratio of the conditions in two periods,

$$\left(\frac{\omega d_1^{\sigma} + (1 - \omega)(h_{m,1} + h_{f,1})^{\sigma}}{\omega d_0^{\sigma} + (1 - \omega)(h_{m,0} + h_{f,0})^{\sigma}}\right)^{\frac{1 - \sigma - \nu}{\sigma}} \left(\frac{h_{m,1} + h_{f,1}}{h_{m,0} + h_{f,0}}\right)^{\sigma - 1} = \left(\frac{l_{m,1} + l_{f,1}}{l_{m,0} + l_{f,0}}\right)^{-\lambda}$$
(29)

The parameter value of ν is set from (29) and β is set from (28).

4.1.3 Single Households

We set the weight parameter $\alpha_g = 1$ for normalization and set the weight parameters β_g and μ_g for each gender g to match the target time allocation of single individuals at time t = 0 (1970). We assume the same values for the curvature parameters ρ , ν and λ as married households.

4.1.4 Marriage Decisions

We assume that joy shock is drawn from the Gumbel distribution F(r) and calibrate the two parameters that define the distribution to match the fraction of married individuals and the change in the share of married households between the two time periods.

Each matched pair of man and woman draws a joy shock r from the Gumbel distribution. Define r^* as

$$r^* = S_f - \widehat{M}_f,$$

the joy added to the value of being married, which would equalize the value of staying single and the value of getting married, for a female individual.

Therefore, the pair will marry if $r \geq r^*$ and remain single otherwise. The fraction of individuals that are single is given as

$$1 - m = G(r^*) = \exp\left\{-\exp\left[-\frac{r^* - \mathbf{a}}{\mathbf{d}}\right]\right\}$$

Taking logs twice,

$$\ln[-\ln(1-m)] = -\frac{r^* - \mathbf{a}}{\mathbf{d}} \tag{30}$$

Taking the ratio of (30) for t = 0 (1970) and 1 (2020),

$$\frac{\ln[-\ln(1-m_1)]}{\ln[-\ln(1-m_0)]} = \frac{r_1^* - \mathbf{a}}{r_0^* - \mathbf{a}}$$
(31)

We choose **a** to match the change in the marriage rate m_t , using (31). Given the value of **a**, we set the scale parameter **d** to match the share of single individuals at t = 0 from (30).

4.1.5 Data Targets

As described above, we need data for target moments related to fertility, marriage, time allocation and schooling to pin down the values of preference parameters from the first order conditions.

Fertility and Marriage Rates: The average numbers of children per married couple, k_0 and k_1 , are computed as $k_t = TFR_t/m_t$, where TFR_t represents the total fertility rate at time t and m_t is the fraction of married individuals. The total fertility rates are 2.13

and 1.33 in 1970 and 2020, respectively, based on the Vital Statistics. The marriage probabilities are 0.967 and 0.822 in 1970 and 2020, respectively, based on the Population Census data. These imply $k_0 = 2.203$ and $k_1 = 1.618$.

Time Allocation: For data on the time allocation of single and married men and women, we use the Survey on Time Use and Leisure Activities and the data from individuals aged between 25 and 59.¹⁴ The time allocation of married and single men and women for market work hr, housework h, leisure l and basic childcare ζ is reported in Table 4, expressed in terms of the share of the total disposable time.

Schooling: We use the School Basic Survey to obtain the college enrollment rates of 0.169 and 0.543 in 1970 and 2020, respectively, computed as the mean of men and women's college enrollment rates each year. These values correspond to s_t in our model.

Table 4: Preference Parameters: Data Targets and Model

		Data		Mc	del	
Parameter	Description	1970	2020	1970	2020	
Marriage a	nd Fertility					
$\mid m \mid$	Fraction of married	0.967	0.822	0.967	0.822	
$\mid k \mid$	Number of children	2.203	1.618	2.203	1.618	
_	Total fertility rate	2.130	1.330	2.130	1.330	
Time Alloce	ation of Married Households					
hr	Market work hours (men)	0.775	0.715	0.775	0.715	
	Market work hours (women)	0.372	0.306	0.372	0.306	
h	Housework hours (men)	0.007	0.021	0.007	0.021	
	Housework hours (women)	0.382	0.323	0.382	0.323	
$\mid l \mid$	Leisure (men)	0.213	0.237	0.213	0.237	
	Leisure (women)	0.175	0.231	0.175	0.231	
Time Alloce	ation of Single Households					
hr	Market work hours (men)	0.715	0.663	0.715	0.604	
	Market work hours (women)	0.639	0.584	0.639	0.501	
h	Housework hours (men)	0.037	0.030	0.037	0.033	
	Housework hours (women)	0.148	0.119	0.148	0.138	
l	Leisure (men)	0.249	0.307	0.249	0.363	
	Leisure (women)	0.213	0.297	0.213	0.362	
Schooling	Schooling					
s	Fraction of college graduates	0.169	0.543	0.169	0.543	

¹⁴https://www.stat.go.jp/data/shakai/2016/index.html

4.2 Home Production and Durable Goods

For the home production function (4), we follow McGrattan et al. (1997) and Greenwood et al. (2005) and set σ to 0.282. ω is set for normalization so that the durable goods consumption in 1970 is 1.

The price index of housework-assisting durable goods declined at an annual rate of -5.75% between 1970 and 2020 and we use this value as the growth rate of durable goods price pi_t . We set the price level in 1970 so that the average share of household expenditures of durable goods matches the data.

4.3 Costs of Childcare

The time for basic childcare, ζ_g , is computed based on the Survey on Time Use and Leisure Activities. We divide the average time spent for childcare by men and women, respectively, by the number of children per married couple k_t in each year.

The financial cost of basic childcare b is computed based on the sum of the fees for school and extracurricular activities. The data is obtained from the Survey on Children's Learning Expenses conducted by the Ministry of Education, Culture, Sports, Science and Technology.

The cost of education χ represents the costs of sending a child to college. We use the data from the Student Life Survey conducted by the Japan Student Services Organization (JASSO), and compute it as the sum of the tuition fees and living costs of a student enrolled in a college. See Appendix A for more details about the data source and composition of the cost of education.

4.4 Production Technology

There are four different technological parameters that represent the productivity level in the production function. First, Z_t stands for the level of general productivity, or what is referred to in this study as TFP. Second, A_t and $A_{f,t}$ represent the productivity levels specific to skilled-labor, which govern the SBTC. A_t applies to both men and women's skilled-labor inputs and $A_{t,f}$ applies only to women's. Lastly, B_t represents the productivity level specific to female labor supply of both skill types, which governs the GBTC.

The data from the Employment Status Survey (ESS) is used to obtain the wage of workers by skill and gender, as well as the average work hours of each group.¹⁵ We then use the Population Census data for the total number of workers by skill and gender in 1970 and 2020. We assume that total work hours change at a constant rate between the

¹⁵The ESS has information about the gender-specific wage for the skill types that correspond to our definition of low and high skills since 1982 only. Therefore we use the real wage index of the Monthly Labour Survey in the 1970s to extrapolate the wage rate of each type.

two time periods and compute the path of labor supply, $L_{g,t}$ and $H_{g,t}$ for $g \in \{m, f\}$ for low and high-skilled workers. We then compute the paths of the productivity levels, Z_t , A_t , $A_{f,t}$, and B_t , using the labor supply of men and women and the two skill types based on the set of wage equations (10), (11) and (12). See Appendix B for more details of the computation.

For other parameters in the production function, we set φ to 0.7 following the estimates used in Abbott et al. (2019). The value implies an elasticity of substitution between low and high skill labor of around 3.3, which is in the range of estimates in the literature. We set γ , which is related to the elasticity between male and female labor, to 0.45, following Abbott et al. (2019), who estimate the production function that consists of three levels of education.

For the equivalence scale η , we assume the OECD equivalence scale and set it to 1.5 for married households.

Table 5: Calibration Parameters

Parameter	Description	Value
Preference		
ρ, α	Curvature and weight: consumption (married)	2.0, 1.0
ν, β	Curvature and weight: home goods (married)	$2.061,\ 0.047$
λ, μ	Curvature and weight: leisure (married)	0.290, 0.368
κ, ϕ	Curvature and weight: child (married)	1.452, 0.148
ψ, ξ	Curvature and weight: child quality (married)	0.251,0.030
β_g	Weight: home goods (single)	0.001 (men)
		0.049 (women)
μ_g	Weight: leisure (single)	0.652 (men)
		1.537 (women)
Childcare C	Costs	
$\zeta_{m,t}$	Basic childcare time (men)	0.002 (1970), 0.017 (2020)
$\zeta_{f,t}$	Basic childcare time (women)	$0.032\ (1970),\ 0.086\ (2020)$
b_t	Basic childcare fin. cost	$0.053\ (1970),\ 0.081\ (2020)$
χ_t	Education cost	$0.059\ (1970),\ 0.124\ (2020)$
Home Prod	uction	
σ	EOS b/w durables and housework	0.282
ω	Share of durables	0.0161
π	Durable goods price	-0.0575 (growth)
Market Pro	duction and Technology	
Z_t	Neutral technology	0.0022 (growth)
A_t	High-skill productivity (SBTC)	0.0041 (growth)
$A_{f,t}$	High-skill productivity (women) (SBTC)	0.0152 (growth)
B_t	Women's productivity (high) (GBTC)	0.0055 (growth)
φ	EOS b/w low and high-skill labor	0.70
γ	EOS b/w men and women	0.45
Other Para	meters	
η	Equivalence scale	1.5
d, a	Marriage joy shock distribution	0.895, -0.900

5 Numerical Results

This section presents the numerical results of our model. First, we present the outcome of the baseline model, and then show how various factors of the model contribute to the trends of key variables. We achieve this by simulating the transition while eliminating or changing the magnitude of each factor one by one, keeping all the other elements

unchanged from the baseline model. We will also consider the roles of the technological changes and costs related to childcare including the education spending.

5.1 Baseline Model

Figure 4 shows the model's predicted paths of married women's time allocation, which are compared with the data. The model generates a secular decline in married women's time spent on housework, a mild decline in their market work and a gradual increase in leisure time in line with the transition of the data. The time for childcare increases throughout the transition, while the number of children declines between 1970 and 2020.

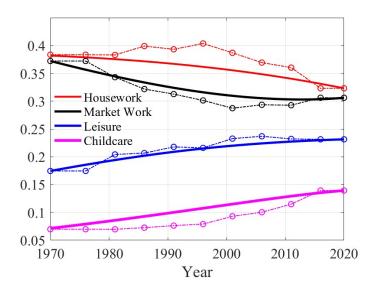


Figure 4: Time Allocation of Married Women: Baseline Model (lines) and Data (circles)

Figure 5 shows the trend of family formation and educational attainment in the model and data, represented by the shift in total fertility rates, college enrollment rates and marriage rates in each plot. Married couples choose to have fewer children over time, which is driven by the evolution of the financial cost of childcare and the opportunity cost of raising children, as represented by the change in women's wages driven by the skill and gender-biased technological change.

The marriage rates decline over time, as shown in Figure 5(c), as the relative attractiveness of being married wanes. The merit of marriage stems from the possibility of having children and enjoying their quantity and quality, and the ability of sharing resources to exploit economies of scale in consumption of home and market goods. The decline in the optimal number of children, higher wage rates due to technological growth, and cheaper input of home production to substitute housework all work in favor of deciding to remain single in our model.

The model also generates the rising investment in education, following the upward trajectory of the share of college graduates in the data, as shown in Figure 5(b). Given the rising fixed cost of children and higher income, parents choose to give children higher education over having more children, tilting the trade-off toward quality.

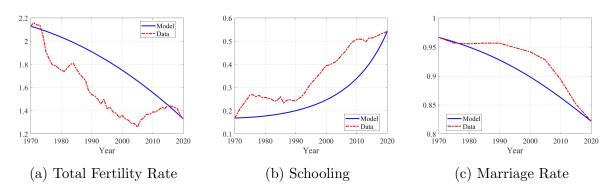


Figure 5: Fertility, Marriage and Schooling: Baseline Model and Data

As described in Section 4.4, we compute the technology level B_t , $A_{f,t}$, A_t , and Z_t from the gender and skill-specific wages and labor supply of each type of worker. Using data from 1970 and 2020 and assuming that wages and labor supply grow at a constant rate, the paths of the four technology levels are given as in Figure 6. The annualized growth rates between 1970 and 2020 are 0.22%, 0.41%, 1.52%, and 0.55% for Z_t , A_t , $A_{f,t}$, and B_t , respectively, as reported in Table 5. In the next section, we simulate various scenarios in which the technological growth is assumed to follow alternative paths.

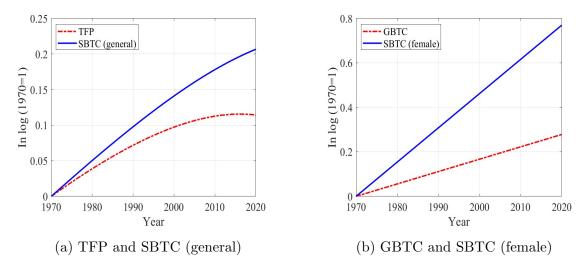


Figure 6: Technological Growth

5.2 Roles of Technology

In this section, we investigate how the technological progress during the last half century may have affected the trends of women's time allocation and family formation. We consider alternative paths of wage rates by assuming a different technological process and simulate the transition of the model which is otherwise identical to the baseline model.

In the first set of experiments, we adjust the wage rates solely by different technology levels and without considering the possible effects of a shift in the labor supply. We name them "partial equilibrium (PE)" simulations. More precisely, in the wage equations (13), Z_t , A_t , $A_{f,t}$ and B_t take alternative values, while all other variables remain unchanged from the baseline model.

In the second set, we also run the "general equilibrium (GE)" version of experiments, in which wages also adjust to endogenous changes in labor supply. Distribution of labor supply evolves according to individuals' endogenous responses in work hours and a shift in skill distribution driven by the education investment.

We run counterfactual experiments under five alternative scenarios about technology. First, we assume that the level of general technology, or the TFP, will remain at the same level as in 1970, that is, $Z_t = Z_{1970}$ for all t. Second, we mute the general SBTC and set $A_t = A_{1970}$, and third, we set $A_{f,t} = A_{f,1970}$ throughout the transition. Fourth, we assume that there is no GBTC and set $B_t = B_{1970}$. Finally, we let the home production technology remain constant and the price of housework-assisting durable goods stay at the same level as in 1970, $\pi_t = \pi_{1970}$ for all t.

Figure 7 shows the paths of married women's time allocation to market work, home production, and leisure, as well as the paths of marriage rates, total fertility rates, and schooling under the first two experiments about technological progress. The simulations assume adjustment of wages under partial equilibrium. When there is no TFP growth or general skill-biased technological change, household income will be lower as husbands' earnings decline. Although women's wages are also lower, income effects dominate and married women increase work hours and reduce leisure time.

Under both experiments, schooling is also lower than in the baseline, as they are able to afford less education costs given the lower household income. The decline is more pronounced without the general SBTC, since households not only suffer from low income but also no longer enjoy the higher return of skill premium from their investment in children's education. Households have lower incentives to spend financial resources on education. Fertility rates do not change much. Income effects lower fertility rates in the same direction as they reduce leisure and schooling, but the decline in the opportunity costs of childcare time has positive effects on fertility and offsets the negative effects, resulting in a muted response.

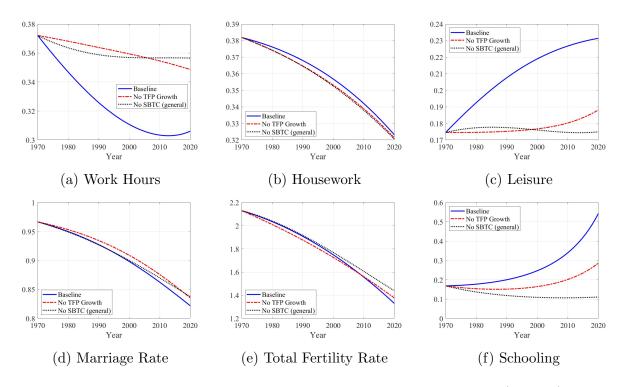


Figure 7: Roles of Technology: No TFP Growth and No SBTC (general)

Note: "No TFP Growth" and "No SBTC (general)" show the paths of variables when the TFP level and general skill-biased technology are fixed at 1970 levels, respectively. The top three panels show married women's time allocation to work hours, housework and leisure.

Figure 8 shows the results when there is no skill-biased technological change specific to female workers (female SBTC) and when there is no gender-biased technological change (GBTC). Under these two scenarios, women's productivity in the market declines while mens' productivity remains unaffected. Couples respond to this change by reducing work hours of women and allocating more time to housework and leisure. As summarized in Table 6, married women's market work in 2020 shifts from 30.6% of total time in the baseline to 19.1% without the female SBT and to 21.0% without the GBTC. Hours for leisure will increase from 23.1% in the baseline model to 31.7% and 30.5% without the female SBT and the GBTC, respectively. Time for housework also increases by approximately 2 percentage points in both scenarios.

The lack of female SBTC and GBTC lowers women's wage and more directly deteriorates the economic conditions of single women compared to those of married women. Therefore, marriage becomes more attractive to women, resulting in higher marriage rates as shown in Figure 8(d). Under these scenarios, women's wages are lower, and married couples choose to have a larger number of children and married women spend more time on childcare since the opportunity cost of their staying away from the labor market is low. The time married women spend on childcare increases from 13.9% of their time in the 2020 baseline economy to 15.1% and 14.7% under the two experiments, respectively.

Regarding the decision to educate, Figure 8(f) shows that schooling declines during the transition with no female SBTC and GBTC. The effect is more pronounced under the scenario of no female SBTC, in which the productivity decline is concentrated among high-skilled women. If women's wages are low, the time cost of basic childcare is lower in terms of their lost market opportunities, and the demand of families regarding children tilts toward quantity of children.

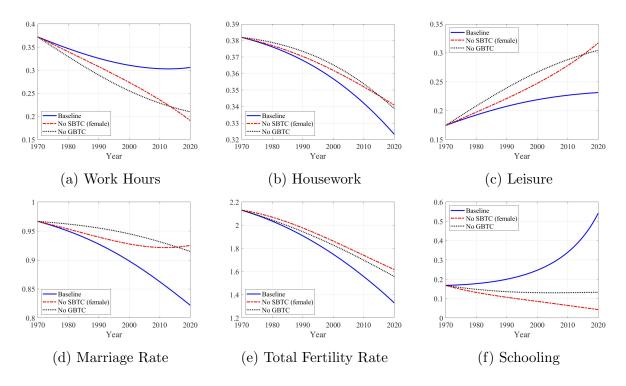


Figure 8: Roles of Technology. No SBTC (female) and No GBTC

Note: "No SBTC (female)" and "No GBTC" show the paths of variables when the skill-biased technology for female and gender-biased technology are fixed at 1970 levels, respectively. The top three panels show married women's time allocation to work hours, housework and leisure.

Table 6: Roles of Technology. Partial Equilibrium

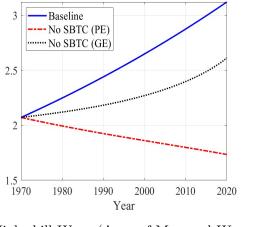
				SBT	SBT		Dur.
	1970	2020	TFP	general	female	GBT	Price
Family and Educ	cation						
Fertility (TFR)	2.130	1.330	1.377	1.439	1.618	1.558	1.336
Schooling	0.169	0.543	0.287	0.111	0.044	0.133	0.526
Marriage	0.967	0.822	0.836	0.838	0.925	0.915	0.828
Time Allocation	of Marr	ried Wor	\overline{nen}				
Work Hours	0.372	0.306	0.349	0.357	0.191	0.210	0.277
Leisure	0.175	0.231	0.188	0.175	0.317	0.305	0.214
Housework	0.382	0.323	0.321	0.321	0.341	0.339	0.370
Childcare	0.071	0.139	0.142	0.148	0.151	0.147	0.139

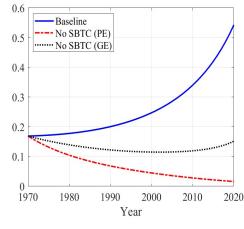
Note: One of the technological process is held fixed in each experiment in 2020.

Partial vs General Equilibrium Analysis: In the above experiments, we only considered the direct effects of technological change on wages. The aim of this section is to investigate full general equilibrium effects of the technological change, allowing changes in the market work hours and education investment to affect distribution of labor inputs and wages.

In general, if lower wages that result from a decline in productivity reduce work hours, incorporating general equilibrium effects mitigates the negative effects on wages. Behavioral responses under partial equilibrium are therefore weakened. We consider the case of no SBTC (both general and female) to highlight such effects. For the results of all the experiments, see Table 7.

Figure 9(a) compares the paths of high-skill wage under the baseline and scenarios without SBTC, under partial and general equilibrium. Removing the SBTC lowers high-skill wages and reduces education investment. In general equilibrium, a decline in the supply of high-skilled labor increases the equilibrium high-skill wages and this effect partially restores the incentive to invest in the education of children as shown in Figure 9(b), although not as significantly as in the baseline model.





- (a) High-skill Wage (Avg. of Men and Women)
- (b) Schooling

Figure 9: Roles of SBTC: Partial and General Equilibrium

Table 7: Roles of Technology. General Equilibrium

		SBT	SBT		Dur.	
	TFP	general	female	GBT	Price	
Family and Educ						
Fertility (TFR)	1.324	1.369	1.508	1.406	1.319	
Schooling	0.456	0.330	0.245	0.391	0.549	
Marriage	0.846	0.846	0.902	0.895	0.822	
Time Allocation	Time Allocation of Married Women					
Work Hours	0.342	0.341	0.212	0.261	0.288	
Leisure	0.200	0.197	0.307	0.270	0.205	
Housework	0.323	0.323	0.337	0.334	0.369	
Childcare	0.135	0.139	0.144	0.135	0.138	

Note: One of the technological processes is fixed in each experiment in 2020.

Roles of Home Production Technology: We now consider the roles of the advancement of home production technology. As shown in Figure 3, the price of housework-assisting durable goods rapidly decreased throughout the last half century and contributed to a decline in the cost of producing home goods. To quantify the effects of the price change of housework-assisting durable goods, we simulate the transition assuming that the price remains unchanged, that is, $\pi_t = \pi_{1970}$, throughout the transition.

As shown in the middle panel of Figure 10, the large decline in the housework hours of married women disappears. The rise of housework hours relative to the baseline model is compensated by a decrease in work hours and leisure. The additional hours for housework reduces the time married women can allocate to childcare, and the fertility rate slightly declines, as shown in the last column of Table 6.

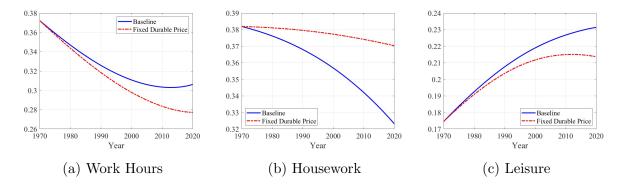


Figure 10: Roles of Home Production Technology

5.3 Childcare Costs

As discussed in Section 2, households have faced rising costs of childcare both in terms of financial expenses and parental time during the last several decades. In this section, we consider three alternative scenarios in which the rise of these costs is mitigated during the transition. Long-run effects of key variables are summarized in Table 8.¹⁶

In the first scenario, we assume that the increase in married women's childcare time per child is limited to half of the increase in the baseline model. More precisely, we let the time for childcare increase by 48% between 1970 and 2020 in the experiment instead of 96%. In the second scenario, we assume that the rise in the basic financial cost of childcare b_t is half of that of the baseline model.

In the third scenario, we let the cost of education χ_t needed to equip children with the high skill rise at the same speed as the average earnings of married couples in the baseline economy. In other words, there is no relative inflation of the education cost and it moves alongside the income level of parents. The education costs of this scenario would rise by 67% over the 50-year period, instead of the 125% of the baseline model.

As shown in Figure 11, when the education cost is lower, married couples would raise the investment in education. They allocate more resources on quality rather than quantity and the total fertility rate is lower than in the baseline transition under the scenario of low education cost.

However, the opposite responses occur when the cost of basic childcare is lower, both in terms of time and money. As shown in Figure 11, parents would have more children compared to the baseline transition and instead reduce their investment in education of each child.

In terms of married women's time allocation, when they face lower time costs of basic childcare, they are able to spend more time on market work, whereas the opposite is true when the financial cost of basic childcare is low. Higher fertility in the latter scenario is

¹⁶In the experiments on childcare costs, we abstract from implementing general equilibrium adjustment of wages.

accompanied by more time spent by married women on childcare and a decline in work hours. When they face lower costs of education per child, they allocate the saved time from fewer childcare hours toward market work.

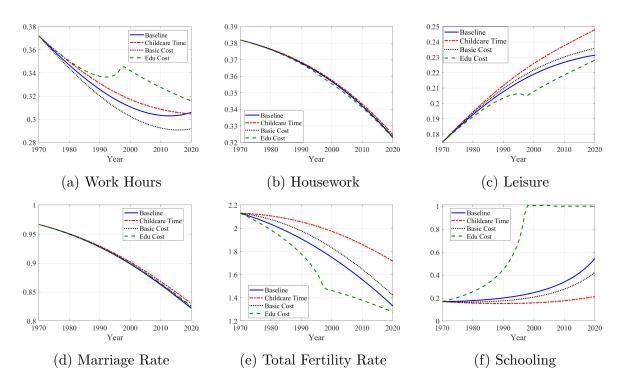


Figure 11: Roles of Childcare Cost

Table 8: Roles of Childcare Costs

			Childcare Cost			
			Basic	Basic	Edu	
	1970	2020	Time	Money	Cost	
Family and Educ	cation					
Fertility (TFR)	2.130	1.330	1.717	1.422	1.277	
Schooling	0.169	0.543	0.213	0.424	1.000	
Marriage	0.967	0.822	0.830	0.824	0.826	
Time Allocation	of Mar	ried Wo	men			
Work Hours	0.372	0.306	0.305	0.292	0.316	
Leisure	0.175	0.231	0.248	0.236	0.228	
Housework	0.382	0.323	0.325	0.324	0.323	
Childcare	0.071	0.139	0.123	0.149	0.133	

Note: Lower childcare costs are assumed in each experiment in 2020.

It is important to note that the quantitative results presented in this section should be interpreted with caution. We focus on the decisions made by households, assuming that the changes in wages, durable goods prices, and time and financial costs of childcare, and education costs are taken as given. However, the time and money parents spend on basic childcare for each child may well be endogenously chosen by them. Furthermore, the prices of childcare or education may well be determined in the market, reflecting the shifts in underlying fundamentals, similar to how wages are determined in the labor market.¹⁷

If the government were to implement policies aimed at reducing the time or financial costs of childcare, our model suggests that parents would reallocate the saved resources toward some other economic activities. However, in a model with endogenous childcare costs, parents may respond to the policy change by spending additional money and effort per child. Moreover, the increased demand for childcare resulting from the reform may influence the price of childcare services. The continuous increase in childcare time and financial costs over an extended period, at a rate surpassing income growth, suggests that such a response is possible.

The results of the experiments in this section can also be interpreted to imply that the fact that parents face the necessity to allocate more time and resources into raising a child compared to 50 years ago critically affects the optimal number of children that they choose to have. To fully account for the endogenous evolution of childcare costs and efforts, a richer model is needed and this is something we leave for future research.

6 Conclusion

Many developed countries have experienced the secular decline in fertility and marriage rates, as well as a shift in women's time allocation, over the past half century. Simultaneously, technological advancements drove the dynamics of the wage structure, driven by the general productivity growth and skill and gender-biased technological changes. These factors have influenced the trade-off involved in the time and resource allocation decisions of families.

We develop a tractable model in which men and women make decisions regarding marriage, fertility, and time allocation for various activities, including market work, home production, leisure, and childcare. Married couples determine the number of children they have and how much they invest in their skill development while considering the time and financial costs of childcare and education.

We calibrate the model using macro and micro data from Japan, a country that has witnessed a significant decline in fertility and marriage rates, and a reduction in the

¹⁷Studies have considered the factors contributing to the increasing costs of education observed over the past decades. Jones and Yang (2016) construct a general equilibrium model that incorporates skill and sector-biased technological changes to investigate the impact of technology on college costs and educational attainment in the U.S. Cai and Heathcote (2022) develop a model of the college market and demonstrate that the growing income inequality has been a significant driver of tuition hikes in the U.S. since 1990.

average family size over the past five decades. Our quantitative analysis reveals that technological progress that favors female labor supply contributes to declines in fertility and marriage rates. Neutral and general skill-biased technological growth leads to a decline in work hours and a rise in leisure time of married women. A rise in education levels is explained by the SBTC and a general increase in income level. Furthermore, we find that an increase in the financial and time costs of basic childcare results in lower fertility rates, while a rise in education costs has the opposite effect.

The analysis demonstrates that accounting for the trends of family formation and time allocation is not simple, and emphasizes the importance of considering the interaction of various micro and macro factors. Changes in technology and the wage structure play a crucial role in determining the dynamics of household income and the opportunity costs associated with childcare and home production. The significant, yet decreasing, gender wage gap influences decisions regarding women's time allocation among different activities. Furthermore, the advancement of home production technology has significantly reduced the burden of housework. Additionally, the increase in the time and financial costs of childcare directly affects the trade-off between quantity and quality faced by parents. This study presents a model that takes into account these forces and their interplay within a comprehensive framework.

There are still some factors considered as given in this study that may require a more careful explanation. Specifically, we did not explore the underlying reasons for the observed trends in factor-biased technology in the market and home production, as well as the shifts in various family constraints such as childcare and education costs. These are important topics that we leave for future research.

References

- Abbott, B., G. Gallipoli, C. Meghir, and G. Violante (2019). Education policy and intergenerational transfers in equilibrium. *Journal of Political Economy* 127(6), 2569–2624.
- Aguiar, M. and E. Hurst (2007). Measuring trends in leisure: The allocation of time over five decades. *Quarterly Journal of Economics* 122(3), 969–1006.
- Albanesi, S. and C. Olivetti (2009). Home production, market production and the gender wage gap: Incentives and expectations. *Review of Economic Dynamics* 12, 80–107.
- Attanasio, O. P., H. Low, and V. Sánchez-Marcos (2008). Explaining changes in female labour supply in a life-cycle model. *American Economic Review 98*(4), 1517–1552.
- Barro, R. and G. Becker (1989). Fertility choice in a model of economic growth. *Econometrica* 57(2), 481–501.

- Baudin, T., D. De La Croix, and P. E. Gobbi (2015). Fertility and childlessness in the United States. *American Economic Review* 105(6), 1852–1882.
- Becker, G. S. and R. Barro (1988). A reformulation of the theory of fertility. *Quarterly Journal of Economics* 103(1), 1–25.
- Bick, A. and N. Fuchs-Schundeln (2017). Taxation and labour supply of married couples across countries: A macroeconomic analysis. *Review of Economic Studies* 85(3), 1543–1576–292.
- Borella, M., M. De Nardi, and F. Yang (2023). Are marriage-related taxes and social security benefits holding back female labor supply? *Review of Economic Studies* 90(1), 102–131.
- Cai, Z. and J. Heathcote (2022). College tuition and income inequality. *American Economic Review* 112(1), 81–121.
- Caucutt, E. M., N. Guner, and J. Knowles (2002). Why do women wait? Matching, wage inequality, and the incentives for fertility delay. *Review of Economic Dynamics* 5(4), 815–855.
- Cordoba, J. C., X. Liu, and M. Ripoll (2019). Accounting for the international quantity-quality trade-off. Working Paper.
- Daruich, D. and J. Kozlowski (2020). Explaining intergenerational mobility: The role of fertility and family transfers. *Review of Economic Dynamics* 36, 220–245.
- de la Croix, D. and M. Doepke (2003). Inequality and growth: Why differential fertility matters. *American Economic Review* 93(4), 1091–1113.
- Doepke, M., A. Hannusch, F. Kindermann, and M. Tertilt (2023). The economics of fertility: a new era. In S. Lundberg and A. Voena (Eds.), *Handbook of the Economics of the Family*, Volume 1. Amsterdam: Elsevier. Chapter 4.
- Doepke, M. and M. Tertilt (2016). Families in macroeconomics. In J. B. Taylor and H. Uhlig (Eds.), *Handbook of Macroeconomics*, Volume 2B. Amsterdam: Elsevier. Chapter 23.
- Doepke, M. and F. Zilibotti (2017). Parenting with style: Altruism and paternalism in intergenerational preference transmission. *Econometrica* 85(5), 1331–1371.
- Eckstein, Z., M. Keane, and O. Lifshitz (2019). Career and family decisions: Cohorts born 1935–1975. *Econometrica* 87(1), 217–253.
- Galor, O. and D. N. Weil (1996). The gender gap, fertility, and growth. *American Economic Review* 86(3), 374–387.
- Galor, O. and D. N. Weil (2000). Population, technology, and growth: From malthusian stagnation to the demographic transition and beyond. *American Economic Review* 90(4), 806-828.

- Goussé, M., N. Jacquemet, and J.-M. Robin (2017). Marriage, labor supply, and home production. *Econometrica* 85(6), 1873–1919.
- Greenwood, J., N. Guner, G. Kocharkov, and C. Santos (2016). Technology and the changing family: A unified model of marriage, divorce, educational attainment, and married female labor-force participation. *American Economic Journal: Macroeconomics* 8(1), 1–41.
- Greenwood, J., N. Guner, and R. Marto (2023). The great transition: Kuznets facts for family-economists. In S. Lundberg and A. Voena (Eds.), *Handbook of the Economics of the Family*. Amsterdam: Elsevier.
- Greenwood, J., A. Seshadri, and G. Vandenbroucke (2005). The baby boom and baby bust. *American Economic Review* 95(1), 183–207.
- Greenwood, J., A. Seshadri, and M. Yorukoglu (2005). Engines of liberation. *Review of Economic Studies* 72(1), 109–133.
- Guner, N., R. Kaygusuz, and G. Ventura (2012). Taxation and household labour supply. Review of Economic Studies 79(3), 1113–1149.
- Guryan, J., E. Hurst, and M. Kearney (2008). Parental education and parental time with children. *Journal of Economic Perspectives* 22(3), 23–46.
- Heathcote, J., K. Storesletten, and G. L. Violante (2010). The macroeconomic implications of rising wage inequality in the United States. *Journal of Political Economy* 118(4), 681–722.
- Jang, Y. and M. Yum (2022). Nonlinear occupations and female labor supply over time. *Review of Economic Dynamics* 46, 51–73.
- Jones, J. B. and F. Yang (2016). Skill-biased technical change and the cost of higher education. *Journal of Labor Economics* 34(3), 621–662.
- Jones, L. E., R. E. Manuelli, and E. R. McGrattan (2015). Why are married women working so much? *Journal of Demographic Economics* 71(1), 75 114.
- Jones, L. E., A. Schoonbroodt, and M. Tertilt (2010). Fertility theories: Can they explain the negative fertility-income relationship? In J. Shoven (Ed.), *Demography and the Economy*, Chapter 2, pp. 43–100. Chicago: University of Chicago Press.
- Katz, L. F. and K. M. Murphy (1992). Changes in relative wages, 1963-1987: Supply and demand factors. *Quarterly Journal of Economics* 107(1), 35–78.
- Kawaguchi, D. and Y. Mori (2016). Why wage inequality have evolved so differently between Japan and the US? the role of supply of college educated workers. *Economics of Education Review* 52, 29–50.
- Kim, S., M. Tertilt, and M. Yum (2023). Status externalities and low birth rates in Korea. Working Paper.

- Kitao, S. and M. Mikoshiba (2023). Why women work the way they do in Japan: Roles of fiscal policies. Working Paper.
- Krusell, P., L. E. Ohanian, J.-V. Ríos-Rull, and G. L. Violante (2000). Capital-skill complementarity and inequality: A macroeconomic analysis. *Econometrica* 68(5), 1029–1053.
- McGrattan, E., R. Rogerson, and R. Wright (1997). Equilibrium model of the business cycle with household production and fiscal policy. *International Economic Review* 38(2), 267–290.
- Myong, S., J. Park, and J. Yi (2021). Social norms and fertility. *Journal of the European Economic Association* 19(5), 2429–2466.
- Nakakuni, K. (2023). Macroeconomic analysis of the child benefit: Fertility, demographic structure, and welfare. Working Paper.
- Ramey, G. and V. A. Ramey (2010). The rug rat race. *Brookings Papers on Economic Activity* 41, 129–176.
- Santos, C. and D. Weiss (2016). "Why not settle down already?" A quantitative analysis of the delay in marriage. *International Economic Review* 57(2), 425–452.
- Taniguchi, H. and K. Yamada (2021). The race between technological progress and female advancement: Changes in gender and skill premia in OECD countries. Working Paper.
- Taniguchi, H. and K. Yamada (2023). The race between technology and woman: Changes in gender and skill premia in OECD countries. Working Paper.
- Voena, A. (2015). Yours, mine, and ours: Do divorce laws affect the intertemporal behavior of married couples? *American Economic Review* 105(8), 2295–2332.
- Yamaguchi, S. (2019). Effects of parental leave policies on female career and fertility choices. Quantitative Economics 10(3), 1195-1232.

Appendix A Data Targets

Marriage Rates We use the Population Census data to construct the time series of marriage rates. The Census survey is conducted every five years, and it records the shares of individuals across four different marital statuses (never-married, currently married, divorced and currently not married, and widows/widowers) for each gender and age group. We compute the fraction of ever-married individuals at age 50 every five years between 1970 and 2020 and use the average of the ratios calculated separately for men and women aged 45-49 and 50-54 as the targets in the calibration of the distribution of marriage joy shocks, as discussed in Section 4.1.4.

Time Allocation: The Survey on Time Use and Leisure Activities provides detailed information on individuals' time allocation at five-year intervals since 1976. We use the data to draw the time allocation patterns for men and women with different marital statuses, following the steps outlined below.

First, for each year, we calculate the time spent on four different activities (housework, childcare, leisure, and market work) for the age groups between 25 and 59 and for the four groups of individuals by marital status and gender (married male, married female, unmarried male, and unmarried female). Note that we let "leisure" include the combined time spent on activities such as "rest and relaxation," "hobbies and amusement," "sports," "volunteering and social activities," and "social life."

Second, for each year and individual group, we calculate the average time spent on each activity across age groups and compute the average, as an approximation of the lifetime allocation over different activities.

Finally, we calculate the share of total disposable time spent on each activity for each year. This process yields a time-series of lifetime allocation every five years between 1976 and 2016. For the year 1970, we assume the time shares to be the same as in 1976. Similarly, for 2020, we use the 2016 data. We chose not to include the latest data for 2021 due to concerns that the data may be irregular, as it reflects a period shortly after the onset of the COVID-19 pandemic.

Costs of Childcare The financial costs of basic childcare b are computed using the Survey on Children's Learning Expenses, since 1994. It records the average education expenditures for children before high school graduation under several categories, such as school and extracurricular activities. The income share of average childcare expenditures remained almost constant between 1994 and 2018, the last year before the COVID-19 pandemic. More precisely, we compute the average education expenditures for a child until the age of 18 and express them as the ratio to the lifetime income for married households (total labor earnings from age 25 to 59), based on the Family Income and Expenditure Survey. The ratio amounts to about 2.9% in each year between 1994-2018.

Due to the data limitation for education expenditures in the years before 1994, we assume that the income share of child-related expenditures is constant across 1970-2020, including the years prior to 1994, since we do not have data for the period.

We use the Student Life Survey to set the education costs χ during the transition. According to the Survey's data in 2018, the average annual expenditure for college students amounted to 1.91 million yen, comprised of 0.93 million yen spent on college related-expenditures, such as tuition fees, and 0.98 million yen spent on living costs, such as housing and food expenses. While we obtained the time series of college tuition fees from the consumer price index, there are no other datasets that record other expenditures arising from college enrollment other than tuition fees, prior to 2004, when the survey started. Hence, we assume that the income share of the expenditures aside from tuition fees is constant throughout 1970-2020. Combining this and the price index for college tuition fees, we compute the total education costs χ for each year.

Appendix B Computation of Technological Growth

This appendix describes how we compute the transition paths of the technology levels, $\{Z_t, A_t, A_{f,t}, B_t\}$ between 1970 and 2020.

We compute the paths of gender-biased technology level B_t from the ratios of female and male low-skill wage equations:

$$\frac{w_{f,l,t}}{w_{m,l,t}} = B_t \left(\frac{L_{f,t}}{L_{m,t}}\right)^{\gamma - 1}$$

With B_t , the gender-specific skill-biased technology level $A_{f,t}$ is computed from:

$$\frac{w_{f,h,t}}{w_{m,h,t}} = A_{f,t}B_t \left(\frac{H_{f,t}}{H_{m,t}}\right)^{\gamma - 1}$$

We then compute the aggregate low and high-skill labor L_t and H_t for all t using (11) and (12), and obtain the general skill-biased technology level A_t from the ratios of low and high-skill wage equations:

$$\frac{w_{m,h,t}}{w_{m,l,t}} = A_t \left(\frac{H_t}{L_t}\right)^{\varphi - \gamma} \left(\frac{H_{m,t}}{L_{m,t}}\right)^{\gamma - 1}$$

Finally, we set Z_t to the path of $w_{m,l,t}$. Note that we set Z_t in the initial period of 1970 so that $w_{f,l,0} = 1$ for normalization.

$$w_{m,l,t} = Z_t \left[L_t^{\varphi} + A_t H_t^{\varphi} \right]^{\frac{1}{\varphi} - 1} L_t^{\varphi - \gamma} L_{m,t}^{\gamma - 1}$$