On the Trends of Technology, Family Formation, and Women's Time Allocations*

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Abstract

We develop a model that explains the secular trend of fertility, marriage, education and women's time allocations in Japan over the last fifty years. We use the model to quantify the effects of technological development and the costs associated with raising children on the time trends surrounding families. We let factor-biased technological progress determine the paths of gender and skill-specific wages, while allowing individuals to make decisions regarding family formation and allocation of time to market work, home production, leisure, and childcare. We find that skill-biased technological change contributes to a rise in fertility rates and investment in education. The growth of gender-biased technology has the opposite effect on the number of children, but increases educational attainment. Additionally, it leads to a decline in the marriage rate and an increase in married women's work hours. Neutral technological growth increases fertility, reduces education investment, and allows women to shift time from market work to childcare. A rise in the financial and time costs of basic childcare reduces fertility rates but increases skill investment, while a rise in education costs has the opposite effect.

Keywords: Fertility, Marriage, Home Production, Women's Time Allocations, Skill-biased Technological Change, Gender-biased Technological change, Japan.
JEL Classification: D10, E10, J10, O11

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1 Introduction

The past several decades have witnessed a secular transformation of the family structure and time allocations within households, as well as a substantial shift in the wage structure across advanced economies. Marriage rates have declined and couples are having fewer children, while dedicating a larger portion of resources to sending their children to college for skill acquisition, despite the escalating costs of education. Educational attainment has risen, particularly among women, leading to a narrowing of the gender gap. This implies that women face higher opportunity costs in terms of allocating their time on childcare and domestic tasks. Additionally, advancements of home production technology have played a role in alleviating time constraints for household members.

All of these factors - demographics, family formation, technology both at home and in the market, and time allocations - are interconnected and should be simultaneously considered within a unified framework to comprehensively explain the trend observed in any of these factors. The objective of this paper is to make such an attempt.

We build a model in which individuals initially enter the economy as single and subsequently encounter a potential partner, making decisions regarding marriage. They allocate their disposable time among various activities, including market work, home production, leisure, and childcare. Married couples make choices concerning the number of children and their level of education, considering the Barro-Becker trade-off (Becker and Barro 1988; Barro and Becker 1989).

Wages are determined in the competitive market, and the production function differentiates between high-skilled and low-skilled labor as well, as between men and women as potentially distinct factors of production. In our model, the technology of market production can grow through three factors: the growth of factor-neutral technology (or total factor productivity, TFP), skill-biased technological change (SBTC) and gender-biased technological change (GBTC).

In the quantitative analysis, we calibrate the model using data from Japan, a country that has experienced a significant decline in fertility and marriage rates over the past several decades. While Japan has observed a reduction in the gender gap, substantial disparities in wages and time allocation between men and women still persist, despite nearly equal college enrollment rates. To obtain relevant information on time allocations, we use various micro-level datasets such as the Survey on Time Use and Leisure Activities. Additionally, we utilize the Basic Survey on Wage Structure to acquire wage rates by gender and skill to calibrate the underlying technological parameters. Furthermore, we use the price data of house-assisting durable goods to quantify the technological advancements in home production.

We find that the SBTC contributes to an increase in fertility rates and investment in children's skills. For instance, without the SBTC, the college enrollment rate in 2020 would have been 75% lower than actual rate. This substantial impact can be attributed to the absence of education's increasing returns and the loss of income effect.

Conversely, the GBTC has an opposite effect on the number of children, while increasing educational attainment. The GBTC also leads to a decline in the marriage rate and an increase in work hours for married women, resulting in reduced time for leisure and housework. Compared to the baseline economy, we find that, without the GBTC, married women would work 12% less, the marriage rate would be 10 percentage points higher, and the total fertility rate would be 1.59, as opposed to 1.33 in the baseline economy. While the general equilibrium effects partially offset the negative impact of GBTC due to the lower labor supply of women leading to higher wages, the qualitative effects remain unchanged.

Neutral technological growth has several effects: it increases fertility, reduces investment in education, and enables women to allocate more time to childcare instead of market work. Furthermore, the decline in the prices of house-assisting goods, such as washing machines and vacuum cleaners, has contributed to a decrease in the time households dedicate to housework. Without this decline in prices, married women would have spent 18% more hours on housework in 2020. Moreover, this decline in prices has a positive impact on work hours of married women and fertility rates.

In our model, we incorporate fixed costs associated with basic childcare per child, which include both parental time and financial resources. These costs are considered exogenous to parents. Additionally, parents make choices regarding the allocation of financial resources towards their children's education, which subsequently enhances the children's quality. The costs per unit of education are also assumed exogenous to parents.

We find that an increase in financial and time costs of basic childcare leads to a decline in fertility rates but encourages greater investment in skills, thereby shifting the trade-off between quantity and quality of children towards the latter. Conversely, an increase in education costs has the opposite effect, resulting in a higher number of children and a decrease in the the education level along the simulated transition path.

The paper contributes to the growing literature of family and macroeconomics, which emphasizes the significance of intra-household decisions in explaining the dynamics of key variables that shape the macroeconomy. A comprehensive survey of this literature is provided by Doepke and Tertilt (2016). Additionally, Doepke et al. (2023) review recent research regarding the economics of fertility and highlights that the conventional theory, which suggests a negative relationship between income and fertility, no longer holds. They argue that various factors influencing the alignment of family objectives and with women's career play a critical role in understanding the fertility trends.[1]

In a quantitative approach, Caucutt et al. (2002) construct a model with endogenous

[1]For a theoretical treatment of fertility decisions in the context of the macroeconomy and economic growth, refer to Galor and Weil (1996, 2000), de la Croix and Doepke (2003) and Jones et al. 2010.

fertility, marriage, and women's labor supply, and demonstrate that the increase in returns to women's labor market experience is responsible for the observed delay in fertility in the U.S. Greenwood et al. (2005) incorporate home production and demonstrate that the burst of the technological progress in the household sector contributed to the baby boom. Cordoba et al. (2019) study cross-country data on education investment and fertility, and investigate the determinants of the quantity-quality trade-off across different countries. Baudin et al. (2015) replicates the cross-sectional observations regarding the family formation and education in the U.S., and investigate the determinants of childlessness.

Kim et al. (2023) constructs a model incorporating status externalities and endogenous fertility to establish a connection between the education fever and low fertility rate in Korea. Yamaguchi (2019) estimates a model using the Japanese data to investigate the effects of parental leave policies on fertility and maternal employment. These papers primarily focus on decisions within families and interactions among individuals, thereby not considering the roles of general equilibrium and the market wage structure in shaping the dynamics of family decisions.[2]

Greenwood et al. (2016) develop a model that examines the dynamics of the marriage, divorce, and income inequality. They emphasize the roles played by home production technology and the wage structure. As they primarily focus on the marriage market and labor supply, their model abstracts from fertility decisions.[3] The closest study to our paper is Greenwood et al. (2023). They propose a theory of endogenous marriage, fertility, and labor supply that incorporates production both in the market and at home. Their model is calibrated to capture the trends of these and other variables in the U.S. since the late 19th century, providing a framework to analyze interactions and long-term trends of these factors in a tractable manner. However, they abstract from gender differences, which are essential in our model for understanding the time trends of family in Japan.

Our paper also focuses on the time allocation of married women and builds upon existing literature that examines the determinants of female labor supply, including the

[2]There are also papers that utilize a life-cycle framework to model family decisions. Santos and Weiss (2016) develop an equilibrium search model of overlapping generations and demonstrate that the increase in income volatility explains a significant portion of the decline and delay in marriage in the U.S. Eckstein et al. (2019) find that advancements in contraception technology accounts for half of the fertility decline between the two cohorts born in 1935 and 1975, while improved labor market opportunities play a crucial role in the decline in marriage. Daruich and Kozlowski (2020) model household decisions regarding fertility and family transfers to analyze their interactions with integenerational mobility. Nakakuni (2023) constructs a life-cycle model calibrated to the Japanese economy to assess the impact of child benefit policies.

[3]There is a growing literature that focuses on the marriage dynamics including divorce. Besides Greenwood et al. (2023), see, for example, Goussé et al. (2017) that builds a search model of marriage and study how wages, education and family attitudes affect marital decisions and intra-household allocations. Voena (2015) estimate a model of couple's saving and labor supply during the marriage to study the effects of the divorce laws on their behavior and divorce decisions.

wage structure and fiscal policies.[4] Men and women exhibit various differences in our model and in the data. In Japan, married women bear a disproportionately large burden of childcare responsibilities. The significant cost of childcare greatly influences family decisions regarding fertility and labor supply. As mentioned earlier, educational attainment and wages have also followed distinct trajectories for men and women, although the gaps have narrowed in recent decades. The advancement of home production technology has facilitated a reduction in hours spent on household chores, particularly for women (Greenwood et al. 2005). Considering how these differences evolved over time is crucial for understanding the evolution of family behavior.

The market wage structure not only influences the career plans of men and women, but also shapes their decisions regarding the marriage. Once married, it further impacts choices related to fertility and childcare. The wages earned by men and women reflect the opportunity costs associated with allocating the available time within the family. In this context, we also aim to contribute to the literature on the evolution of factor-biased technology, changes in work hours of families, and the distribution of skills. These factors play a key role in understanding the dynamics of the wage structure (Krusell et al. 2000; Heathcote et al. 2010).[5]

To the best of our knowledge, our paper represents the first attempt to construct a unified model that encompasses fertility, marriage, education, and women's time allocations, while considering the endogenous wage structure. We demonstrate the interconnections among these factors and highlight the critical role each plays in jointly explaining the observed trends in both family dynamics and the macroeconomy.

The rest of the paper is organized as follows. Section 2 presents the relevant data and discusses the trends in family and macroeconomic variables of interest. Section 3 describes the quantitative model, while Section 4 discusses the model's parametrization. The numerical results are presented in Section 5, and Section 6 concludes.

2 The Time Trends of Family Facts

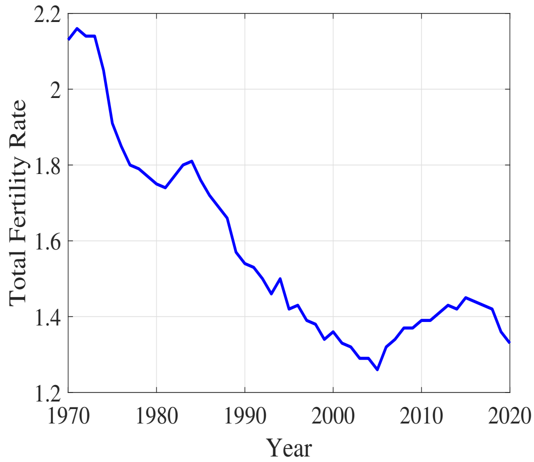
In this section, we examine the trends of various facts surrounding families in Japan. Figure 1(a) shows the path of the total fertility rates in Japan, which represents the average number of children that each woman gives birth to over her life-cycle.[6] In the early 1970s, the fertility rate exceeded the replacement rate that is needed to keep the population from decreasing, but started to decline quickly thereafter. By the mid-2000s,

[4] See, for example, Attanasio et al. (2008), Albanesi and Olivetti (2009), Guner et al. (2012), Jones et al. (2015), Bick and Fuchs-Schundeln (2017), Borella et al. (2023), and Kitao and Mikoshiba (2023).

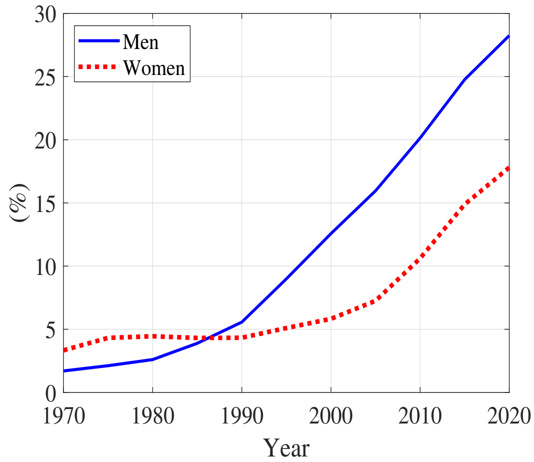
[5] See also Abbott et al. (2019), Kawaguchi and Mori (2016), and Taniguchi and Yamada (2023) for recent papers that estimate a model of factor-biased technological progress.

[6] More precisely, it is computed as the average number of children that a hypothetical cohort of women would have if she was subject to the fertility rates of a given year during her whole life.

it fell below 1.3 and stayed at around 1.3 to 1.4 until 2020.



(a) Total Fertility Rate



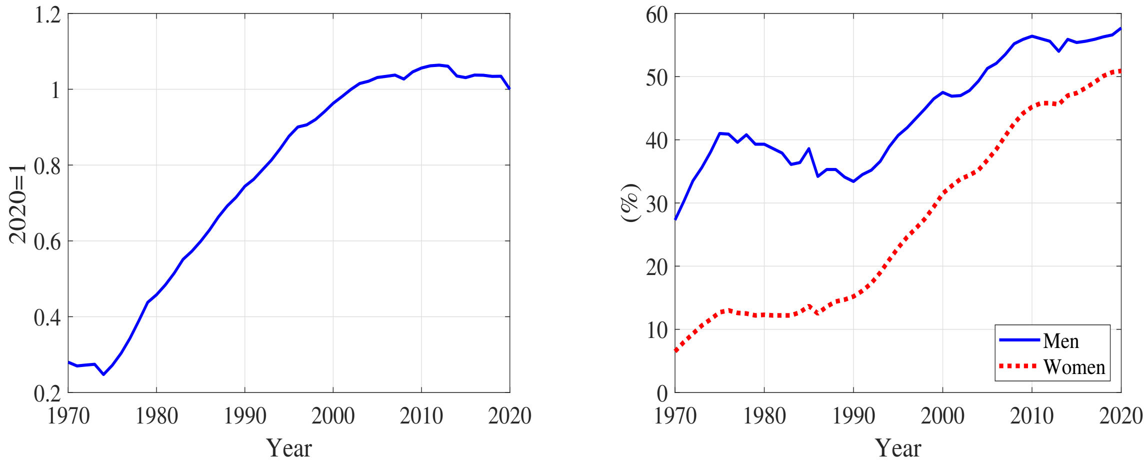
(b) Fraction of Never-Married at Age 50

Figure 1: Fertility and Marriage

Source: Vital Statistics, Ministry of Health, Labour and Welfare

The fall in fertility rates occurred at the same time as the marriage rates declined. Figure 1(b) shows the fraction of men and women aged 50 who are never married in their life. The share was less than 5% until the late 1980s for men and until the early 1990s for women, but it rose quickly and reached 28% for men and 18% for women by 2020.

The cost of raising a child also increased during the last decades. As shown in Figure 2, the college enrollment rate increased from less than 30% in 1970 to almost 60% for men in 2020. The rise is more dramatic for women. Less than 10% of women went to college in 1970, but the share exceeded 50% in 2020. The cost of attending college also increased at the same time, as shown in the path of the average college tuition in Figure 2(a).



(a) College Tuition

(b) College Enrollment Rate

Figure 2: College Costs and Enrollment

Source: College tuition is based on the Consumer Price Index, Ministry of Internal Affairs and Communications. (a) depicts the price index of college tuition fees relative to the headline CPI. 4-year college enrollment rates are from the Basic School Survey by the Ministry of Education, Culture, Sports, Science and Technology.

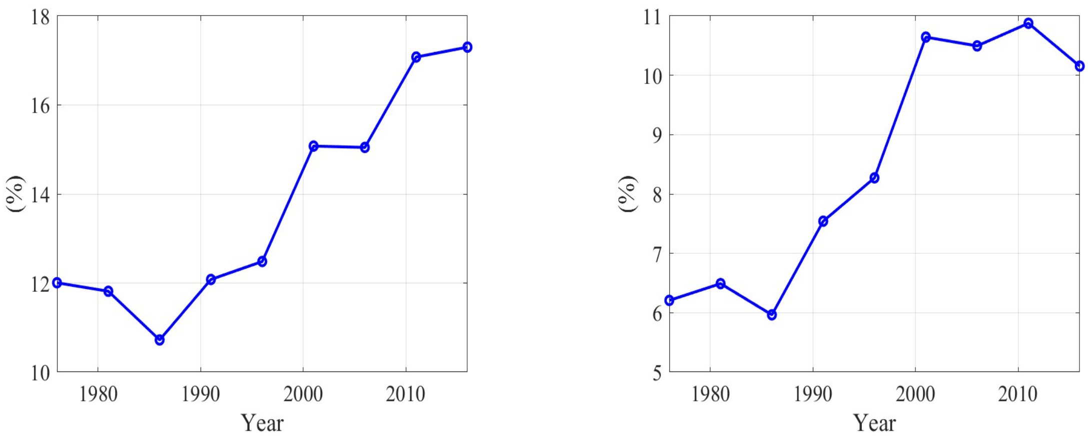
Not only the financial cost, but also the time that parents spend for raising children increased during the last fifty years. Table 1 shows the time allocation of married men and women of working age in the late 1970s and 2010s. Both men and women increased the share of time spent for childcare, and women especially allocate a much larger share of their time than men and the share increased from 12.0% to 17.3%. This occurred at the same time as the number of children decreased and childcare time per child rose even more rapidly, as shown in Figure 3.

Table 1 shows that while women spend more time on childcare, they allocate a significantly smaller fraction of their time on housework. The time for market work changed only slightly and the leisure time increased from 18.9% to 21.5%.

Table 1: Time Allocation of Married Men and Women (% of disposable time)

	Men		Women	
	1976	2016	1976	2016
Market work	76.90	71.17	27.71	26.60
Housework	0.59	2.26	41.35	34.57
Childcare	0.83	3.74	12.01	17.30
(per child)	(0.38)	(2.21)	(5.45)	(10.88)
Leisure	21.68	22.84	18.93	21.53
Total	100.0%	100.0%	100.0%	100.0%

Source: Survey on Time Use and Leisure Activities. Statistics Bureau, Ministry of Internal Affairs and Communications. Average time use of married men and women aged 25-59.



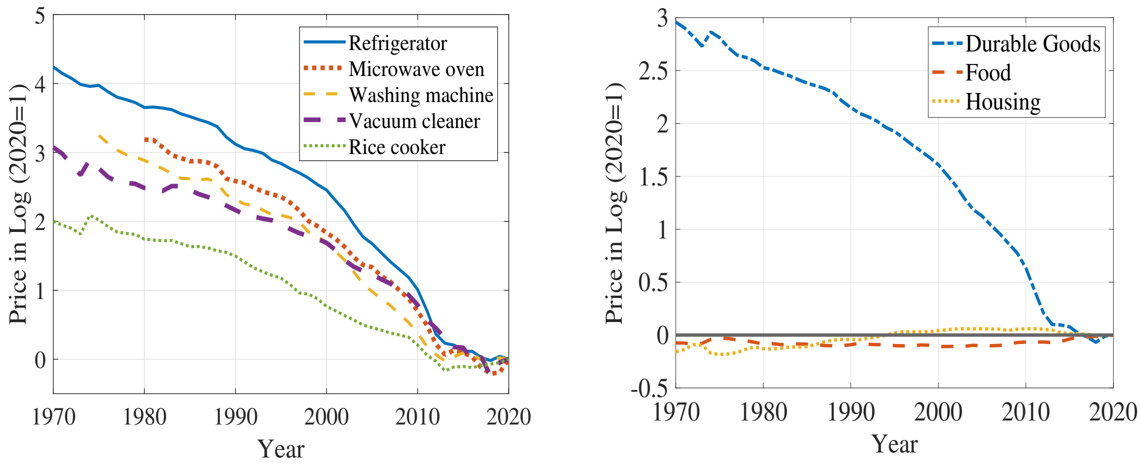
(a) Total Childcare Time

(b) Per Child

Figure 3: Time for Childcare: Married Women

Source: Survey on Time Use and Leisure Activities. Statistics Bureau, Ministry of Internal Affairs and Communications.

The decline in the hours of housework occurred at the same time as the advancement of home production technology. Figure 4 shows the price index of major household appliances, such as refrigerators, washing machines, and vacuum cleaners, relative to the headline CPI. The relative price declined at an annual rate of 4 to 8%. The price indices of those goods are summarized as the index of *durable goods assisting housework*. Figure 4(b) shows the path of its relative price, which declined at an annual rate of 5.75% between 1970 and 2020, while the relative prices of other items such as food and houses did not grow much.



(a) Price of Major Home Durables

(b) Price Index

Figure 4: Price of Housework-assisting Durable Goods

Source: Consumer Price Index (Statistics Bureau, Ministry of Internal Affairs and Communications). Each plotted line indicates the price index relative to the headline CPI. "Durable Goods" in the legend on (b) represents the price index of *durable goods assisting housework*, including refrigerator, microwave oven, washing machine/dryer, vacuum cleaner, rice cooker, and gas cooking table, relative to the headline CPI.

Over time, as the family structure and time allocations underwent changes, shifts in the macroeconomic environment and wage structure took place. Table 2 shows the real wage rates by gender and skill in 1970 and 2020, where high-skill represents those with college degree or above. The wage rate of low-skilled women in 1970 is normalized to 1. Wages increased for all groups, and the growth rate is higher for high-skill wage of both men and women. Women's wage growth is also higher than men's within each skill group.

The skill premium defined as the ratio of high-skill wage to low-skill wage was 1.44 for women in 1970 and rose to 1.53 in 2020, as shown in the bottom section of Table 2. The gender gap is defined as the ratio of women's wage to men's wage and it has narrowed during the five decades, from 0.55 to 0.74 on average. Note that the change in the gender gap is larger on average rather than that within the skill group, since there was also a change in the composition of worker skills by gender.

Table 2: Wages by Gender and Skill, Gender Gap and Skill Premium

		1970	2020	Ann. Growth
Women	Low Skill	1.00	1.74	1.11%
	High Skill	1.44	2.65	1.23%
Men	Low Skill	1.65	2.38	0.73%
	High Skill	2.35	3.40	0.75%
Gender Gap	Low Skill	0.60	0.73	–
	High Skill	0.61	0.78	–
	Weighted Avg.	0.55	0.74	–
Skill Premium	Men	1.42	1.43	–
	Women	1.44	1.53	–
	Weighted Avg.	1.43	1.47	–

Source: Basic Survey on Wage Structure (BSWS). Wage of low-skilled women in 1970 normalized to 1.0. *Gender gap* is defined as the ratio of women's wage to men's wage. *Skill premium* is defined as the ratio of high-skill wage to low-skill wage.

As shown in Table 3, the share of female workers in the labor force increased from less than 20% in 1970 to almost 30% in 2020, and the rise is driven by the rise in the number of high-skilled women in the labor force. Low-skilled men had a 55% share in the labor force in 1970 and the share declined by 20 percentage points over the last 50 years, and both low and high-skilled men constitute 35% of the labor force.

We use these observations to calibrate the technological progress during the last half century in Section 4. It is important to consider the dramatic change in the composition of labor supply to account for the shift in the wage and productivity of different types of labor inputs.

Table 3: Distribution of Workers by Gender and Skill

	1970	2020
Men		
Low Skill	55.4%	35.3%
High Skill	25.3%	35.4%
Total	80.6%	70.8%
Women		
Low Skill	17.8%	16.7%
High Skill	1.6%	12.5%
Total	19.4%	29.2%
Total	100.0%	100.0%

Source: Basic Survey on Wage Structure (BSWS).

3 Model

3.1 Overview

An individual of our model enters the economy as single and is matched with another single person upon entry. The pair chooses to get married if the value of marriage exceeds the value of staying single and remain single otherwise.

A married couple chooses consumption of market goods and non-market goods. The latter is produced at home, using durable goods and housework time as inputs. The couple also decides whether to have a child and if so how many. Parents derive utility from the quantity and quality of children, taking into account the time and money cost of raising children and educating them. The household allocates the disposable time to leisure, home production, market work, and childcare. Single individuals consume market and non-market goods and allocate their time to leisure, home production and market work. Marget goods are produced using skilled and unskilled labor and wages are determined in the competitive market.

The framework is static to keep the model tractable. Individuals make one-time decisions about marriage, and then about consumption, time allocation and fertility for the case of married couples. The model aims to examine how time-varying factors influence the evolving patterns of decision making by families and we abstract from roles of heterogeneity and uncertainty within cohort.

3.2 Preferences

Married households derive utility from a couple's consumption of market goods c, non-market goods n, total leisure of husband and wife $l = l_m + l_f$, the number of children k, and the quality of children q. The utility function is denoted as $u^M(c/\eta, n/\eta, l, k, q)$, where η represents the equivalence scale of consumption goods for the couple, and given as follows.

$$u^M(c/\eta, n/\eta, l, k, q) = \alpha\frac{(c/\eta)^{1-\rho}-1}{1-\rho}+\beta\frac{(n/\eta)^{1-\nu}-1}{1-\nu}+\mu\frac{l^{1-\lambda}-1}{1-\lambda}+\phi\frac{k^{1-\kappa}-1}{1-\kappa}+\xi\frac{q^{1-\psi}-1}{1-\psi}, \quad (1)$$

Parameters α, β, μ, ϕ and ξ represent the weight attached to utilities from consumption of market and non-market goods, leisure, and the number and quality of children, respectively. Parameters ρ, ν, λ, κ and ψ represent the curvature of the utility function and affect how households respond to changing environment by reallocating resources to maximize the utility.

A single individual of gender $g \in \{m, f\}$ derives utility from consumption of market goods c, non-market goods n, and leisure l. The utility function is denoted as $u^S_g(c, n, l)$ and given as:

$$u^S_g(c, n, l) = \alpha_g\frac{c^{1-\rho}-1}{1-\rho}+\beta_g\frac{n^{1-\nu}-1}{1-\nu}+\mu_g\frac{l^{1-\lambda}-1}{1-\lambda} \quad (2)$$

Parameters α_g, β_g and μ_g denote the preference weight to consumption of market goods, non-market goods and leisure, respectively, which may depend on gender g.

3.3 Children

A married couple derives utility from both the number of children k, as well as the quality of children q. Raising children is costly for parents in two ways: first, time and money for basic childcare and second, money for education. Basic childcare is required for all children and education investment is based on the choice of the family. For the basic childcare, a married couple must spend a financial cost b per child, and each parent of gender g must spend time ζ_g per child.

Parents choose how much financial resources to invest in child education, which raises the quality of children. We assume that parents choose the mix of skills for their children, with a fraction s of high skill and $1-s$ of low skill. It costs χs per child to equip them with skill s, which enables them to enjoy the skill premium, denoted as w_h/w_l, where w_h and w_l represent high and low-skill wages, respectively.

q denotes the quality of children that increases utility of parents. We assume that the utility increases in the skill level of their children and also depends on how the skill is valued in the market. We define the quality of children as

$$q = \frac{w_h}{w_l}s, \quad (3)$$

where the first term represents the skill premium and the second term is the skill level. Skill premium is exogenous to parents, but it is determined endogenously in the labor market as a function of the supply of the skill and exogenous technological change, as discussed in Section 3.5.

We assume that parents do not differentiate educational investment based on the gender of children and children's gender does not enter the problem of married individuals.

3.4 Home Production

Home goods n are produced according to the following home production function with two inputs, durable goods (d) and housework hours (h):

$$n = [\omega d^{\sigma} + (1-\omega)h^{\sigma}]^{1/\sigma} \tag{4}$$

σ is the parameter that determines the elasticity of substitution between durable goods and labor input. Durable goods are priced at π per unit. For married households, h is the sum of housework hours supplied by husband and wife, $h = h_m + h_f$.

3.5 Market Production and Wages

A representative firm produces output Y with unskilled labor L and skilled labor H according to the production function:

$$Y = F(L, H) = Z\left[(1-\theta_h)L^{\varphi} + \theta_h A H^{\varphi}\right]^{1/\varphi}, \tag{5}$$

where Z represents the neutral technology level and A governs the gender-neutral SBTC. L and H are composites of male and female labor of each skill type, L_g and H_g for $g \in \{m, f\}$, and they are defined as follows.

$$L = \left[(1-\theta_{l,f})L_m^{\gamma} + \theta_{l,f} B L_f^{\gamma}\right]^{1/\gamma} \tag{6}$$

$$H = \left[(1-\theta_{h,f})H_m^{\gamma} + \theta_{h,f} A_f B H_f^{\gamma}\right]^{1/\gamma} \tag{7}$$

B and A_f govern the GBTC and SBTC specific to female workers, respectively.

The firm rents labor from individual workers at the market wage rates. The labor market is competitive and the wages for unskilled and skilled labor of each gender are determined to equate supply and demand. A firm's problem is given as follows.

$$\max_{L_m, L_f, H_m, H_f} \left\{ F(L, H) - \sum_g (L_g w_{g,l} + H_g w_{g,h}) \right\}$$

In equilibrium, market wages are given as marginal product of each type of labor.

$$\begin{aligned} w_{g,l} &= F_{L_g} \\ w_{g,h} &= F_{H_g}, \end{aligned}$$

for $g \in \{m, f\}$.

3.6 Household Problems

Single Households: Single individuals allocate the disposable time normalized to 1 to leisure l, home production h, and market work $1 - l - h$. They allocate income to the consumption of market goods c, and durable goods d priced at π.

The value function of single individuals of gender g is given as follows.

$$S_g = \max_{c,d,l,h} \left\{ u_g^S(c, n, l) \right\} \tag{8}$$

s.t.

$$c + \pi d = w_g(1 - l - h) \tag{9}$$

where w_g denotes the wage rate of individuals of gender g.[7]

Married Households: Married couples allocate earnings of husband and wife net of costs of childcare to consumption of market goods c and durable goods d. Married households also choose the number of children k and education investment for children s. The household decision for the investment determines the quality q of children.

We assume that the time allocation of married men is exogenous and they supply labor at home and in the market inelastically and their time contribution to the home production and childcare is also exogenously given. Therefore, the time allocation decision of the couple is about the wife's time for leisure l_f, home production h_f, and market work, which is given by $(1 - \zeta_f k - l_f - h_f)$, the total disposable time net of time spent for childcare, leisure, and home production.

The value function of married households is defined as

$$M = \max_{c,d,l_f,h_f,k,s} \left\{ u^M(c/\eta, n/\eta, l, k, q) \right\} \tag{10}$$

s.t.

$$c + \pi d + \chi s k + b k = \sum_g w_g(1 - \zeta_g k - l_g - h_g) \tag{11}$$

where the housework is given as $h = h_m + h_f$, leisure $l = l_m + l_f$ and the quality of children $q = (w_h/w_l)s$.

The value of a married *individual* of gender g is given as

$$\widehat{M}_g = \widehat{u}_g^M(c^*/\eta, n^*/\eta, l_g^*, k^*, q^*), \tag{12}$$

where a variable with an asterisk denotes the optimal choice from the above problem of married households. Utility function $\widehat{u}_g^M$ of a married individual is defined similarly to

[7]Note that we abstract from heterogeneity within cohort including education levels. The wage of each gender is computed as the weighted average of low and high-skill wages determined in the labor market.

(1), with leisure l_g of each individual, rather than that of a household. This value is relevant in the decision of marriage as discussed below.

Marriage Decision: Upon individuals' entry to the economy, each individual is matched with a potential partner. The pair makes a draw of a common joy shock r from the distribution $F(r)$.

Given that we focus on the time allocation decision of married women, and also for simplicity, we focus on the marriage decision of women and abstract from that of men. Women choose to marry if the value of marriage for them plus the joy shock exceeds the value of staying single. Otherwise the pair will not marry and remain single. The decision rules are given as follows.

$$\begin{cases} \text{marry if} & \widehat{M}_f + r \geq S_f \\ \text{single if} & \widehat{M}_f + r < S_f \end{cases}$$

4 Calibration

We would like the model presented above to align with the facts presented in Section 2. We assign some parameter values directly from the data, and determine other parameter values to fit the transition path of the data. In particular, we focus on the trends between 1970 and 2020 and set some of the parameter values to match the transition patterns of data between the two periods. This strategy follows the method undertaken by Greenwood et al. (2023). Appendix A. provides additional details of the data used in the calibration.

4.1 Preference Parameters

4.1.1 Strategy

We use the optimality conditions of the household problems presented in Section 3.6 to pin down main preference parameters that enter the utility functions (1) and (2). We do so to match the target moments in two periods, 1970 and 2020, as explained in more details below.

Substituting the home production equation (4) and the budget constraint (11) in the utility function of married households (1), the value function reads as

$$\begin{aligned} M &= \max_{d,l_f,h_f,k,s} \left\{ \alpha \frac{\left\{ \left[\sum_g w_g(1-\zeta_g k - l_g - h_g) - \pi d - \chi s k - bk\right]/\eta \right\}^{1-\rho} - 1}{1-\rho} \right. \\ &+ \left. \beta \frac{\left\{ \left[(\omega d^\sigma + (1-\omega) h^\sigma)^{1/\sigma}\right]/\eta \right\}^{(1-\nu)} - 1}{1-\nu} + \mu \frac{l^{1-\lambda}-1}{1-\lambda} + \phi \frac{k^{1-\kappa}-1}{1-\kappa} + \xi \frac{q^{1-\psi}-1}{1-\psi} \right\}, \end{aligned}$$

The first order conditions of married households problem with respect to d, l_f, h_f, k, and s, respectively, are given as follows.

$$\begin{aligned}
d: &\quad \alpha(1/\eta)^{1-\rho}c^{-\rho}\pi = \beta(1/\eta)^{1-\nu}n^{1-\sigma-\nu}\omega d^{\sigma-1} &(13)\\
l_f: &\quad \alpha(1/\eta)^{1-\rho}c^{-\rho}w_f = \mu(l_m + l_f)^{-\lambda} &(14)\\
h_f: &\quad \alpha(1/\eta)^{1-\rho}c^{-\rho}w_f = \beta(1/\eta)^{1-\nu}n^{1-\sigma-\nu}(1-\omega)(h_m + h_f)^{\sigma-1} &(15)\\
k: &\quad \alpha(1/\eta)^{1-\rho}c^{-\rho}(w_m\zeta_m + w_f\zeta_f + \chi s + b) = \phi k^{-\kappa} &(16)\\
s: &\quad \alpha(1/\eta)^{1-\rho}c^{-\rho}\chi k = \xi q^{-\psi}(w_h/w_l) &(17)
\end{aligned}$$

The equation (13) represents the trade-off between the gain from additional home goods consumption by an extra unit of durable goods purchase, and the loss from market goods consumption. The conditions (14) and (15) equate the marginal benefit of an additional hour on leisure and housework, with the cost from reduced work hours and lower consumption.

The condition (16) equates the marginal benefit of having a child through direct utility from the larger number of the children, with the marginal cost of raising a child, the time and money to spare for the basic childcare and education. In equation (17), the marginal cost of expenditures to educate the children is equated with the higher utility from the better quality of children.

Turning to the problem of single individuals, substituting the equation for the home production (4) and the budget constraint (9) in the utility function (2), the value function reads as

$$S_g = \max_{d,l,h}\left\{\alpha_g\frac{[w_g(1-l-h)-\pi d]^{1-\rho}-1}{1-\rho} + \beta_g\frac{[\omega d^\sigma + (1-\omega)h^\sigma]^{\frac{1-\nu}{\sigma}}-1}{1-\nu} + \mu_g\frac{l^{1-\lambda}-1}{1-\lambda}\right\}$$

The first order conditions of single households problem with respect to d, l, and h, respectively, are given as follows.

$$\begin{aligned}
d: &\quad \alpha_g c^{-\rho}\pi = \beta_g n^{1-\sigma-\nu}\omega d^{\sigma-1} &(18)\\
l: &\quad \alpha_g c^{-\rho}w_g = \mu_g l^{-\lambda} &(19)\\
h: &\quad \alpha_g c^{-\rho}w_g = \beta_g n^{1-\sigma-\nu}(1-\omega)h^{\sigma-1} &(20)
\end{aligned}$$

Similarly to the problem of married households, the first order conditions represent the tradeoff between the utility from the consumption of market goods and the home goods (18), time for leisure (19), and housework (20).

4.1.2 Married Households

For the utility function of married households (1), we set the weight parameter α to 1 for normalization and the risk aversion parameter to 3. The rest of the preference parameters, which include 4 weight parameters and 4 curvature parameters, are pinned down to match target data moments of (1) leisure (μ and λ), (2) fertility (ϕ and κ), (3) schooling (ξ and ψ) and (4) housework hours (β and ν). We exploit changes in these data between 1970 ($t = 0$) and 2020 ($t = 1$) to pin down curvature parameters and the levels in one of the years to set the weight parameters.

Leisure: From the first order conditions (14) and (11),

$$\mu(l_{m,t} + l_{f,t})^{-\lambda} = \alpha(1/\eta)^{1-\rho} c_t^{-\rho} w_{f,t} \tag{21}$$

where

$$c_t = \sum_g w_{g,t}(1 - \zeta_{g,t}k_t - l_{g,t} - h_{g,t}) - \pi_t d_t - \chi_t s_t k_t - b_t k_t$$

Taking the ratio of (21) at time $t = 0$ and 1,

$$\left(\frac{l_{m,1} + l_{f,1}}{l_{m,0} + l_{f,0}}\right)^{-\lambda} = \left(\frac{c_1}{c_0}\right)^{-\rho} \frac{w_{f,1}}{w_{f,0}} \tag{22}$$

The condition (22) describes the response of leisure to the changes in consumption and wages, representing income and substitution effects, respectively. The parameter λ plays a key role in representing elasticitieses of leisure. (22) pins down λ to match the target response in leisure, $l_{f,1}/l_{f,0}$, to changes in consumption and wages. Once λ is set, the preference weight on leisure μ is derived by from (21) evaluated at $t = 0$.

Fertility: From the first order conditions (16) with (11) and (14),

$$\phi k_t^{-\kappa} = \mu(l_{m,t} + l_{f,t})^{-\lambda}(w_{f,t}\zeta_{f,t} + \chi_t s_t + b_t)/w_{f,t} \tag{23}$$

Using the condition for $t = 0$ and 1,

$$\left(\frac{k_1}{k_0}\right)^{-\kappa} = \left(\frac{l_{m,1} + l_{f,1}}{l_{m,0} + l_{f,0}}\right)^{-\lambda} \frac{(w_{f,1}\zeta_{f,1} + \chi_1 s_1 + b_1)/w_{f,1}}{(w_{f,0}\zeta_{f,0} + \chi_0 s_0 + b_0)/w_{f,0}} \tag{24}$$

From (24), we pin down κ to match the response in fertility to a change in leisure and the total marginal cost of childcare in terms of a wife's wage. With the value of κ, we compute ϕ from (23) for $t = 0$.

Education: Using the first order condition (17) with (3) and (14),

$$\xi q_t^{-\psi}(w_{h,t}/w_{l,t}) = \mu(l_{m,t} + l_{f,t})^{-\lambda}\chi_t k_t/w_{f,t} \tag{25}$$

Taking the ratio of the condition at $t = 0$ and 1,

$$\left(\frac{q_1}{q_0}\right)^{-\psi} \frac{w_{h,1}/w_{l,1}}{w_{h,0}/w_{l,0}} = \left(\frac{l_{m,1} + l_{f,1}}{l_{m,0} + l_{f,0}}\right)^{-\lambda} \frac{\chi_1 k_1/w_{f,1}}{\chi_0 k_0/w_{f,0}} \tag{26}$$

The equation (26) shows that the growth of the investment in children's quality is positively associated with the growth of the skill premium and the couple's leisure time while negatively related to the cost of education in terms of married women's wage. We pin down ψ from (26) and ξ from (25).

Housework Hours: Combining the first order conditions (15) and (14),

$$\beta(1/\eta)^{1-\nu}\left[\omega d_t^\sigma + (1-\omega)(h_{m,t}+h_{f,t})^\sigma\right]^{\frac{1-\sigma-\nu}{\sigma}}(1-\omega)(h_{m,t}+h_{f,t})^{\sigma-1} = \mu(l_{m,t}+l_{f,t})^{-\lambda} \quad (27)$$

A rise in d_t implies a decline in h_t, provided $(1-\sigma-\nu)<0$, and the effect is larger if σ is larger (h_t and d_t are more substitutable) or ν is larger. Taking the ratio of the conditions in two periods,

$$\left(\frac{\omega d_1^\sigma + (1-\omega)(h_{m,1}+h_{f,1})^\sigma}{\omega d_0^\sigma + (1-\omega)(h_{m,0}+h_{f,0})^\sigma}\right)^{\frac{1-\sigma-\nu}{\sigma}}\left(\frac{h_{m,1}+h_{f,1}}{h_{m,0}+h_{f,0}}\right)^{\sigma-1} = \left(\frac{l_{m,1}+l_{f,1}}{l_{m,0}+l_{f,0}}\right)^{-\lambda} \quad (28)$$

The parameter value of ν is set from (28) and β from (27).

4.1.3 Single Households

We set the weight parameter $\alpha_g = 1$ for normalization and set weight parameters β_g and μ_g for each gender g to match the target time allocations of single individuals at time $t=0$ (1970). We assume the same values for the curvature parameters ρ, ν and λ as married households.

4.1.4 Marriage Decisions

We assume that the joy shock is drawn from the Gumbel distribution $F(r)$ and calibrate the two parameters that define the distribution to match the fraction of married individuals and the change in the share of married households between the two time periods.

Each matched pair of a man and a woman draws a joy shock r from the Gumbel distribution. Define r^* as

$$r^* = S_f - \widehat{M}_f,$$

the joy added to the value of being married, which would equalize the values of staying single and getting married, for a female individual.

Therefore, the pair will marry if $r \geq r^*$ and remain single otherwise. The fraction of individuals being single is given as

$$1-m = G(r^*) = \exp\left\{-\exp\left[-\frac{r^*-\mathbf{a}}{\mathbf{d}}\right]\right\}$$

Taking logs twice,

$$\ln[-\ln(1-m)] = -\frac{r^*-\mathbf{a}}{\mathbf{d}} \quad (29)$$

Taking the ratio of (29) for $t = 0$ (1970) and 1 (2020),

$$\frac{\ln[-\ln(1-m_1)]}{\ln[-\ln(1-m_0)]} = \frac{r_1^* - \mathbf{a}}{r_0^* - \mathbf{a}} \tag{30}$$

We choose $\mathbf{a}$ to match the change in the marriage rate m_t, using (30). Given the value of $\mathbf{a}$, we set the scale parameter $\mathbf{d}$ to match the share of single individuals at $t = 0$ from (29).

4.1.5 Data Targets

As described above, we need data for target moments related to fertility, marriage, time allocation and schooling, to pin down the values of preference parameters from the first order conditions.

Fertility and Marriage Rates: The average numbers of children per married couple, k_0 and k_1, are computed as $k_t = TFR_t/m_t$, where TFR_t is the total fertility rate at time t and m_t is the fraction of married individuals. The total fertility rates are 2.13 and 1.33 in 1970 and 2020, respectively, based on the Vital Statistics. The marriage probabilities are 0.967 and 0.822 in 1970 and 2020, respectively, based on the Census data. These imply $k_0 = 2.203$ and $k_1 = 1.618$.

Time Allocations: For the data regarding time allocations of single and married men and women, we use the Survey on Time Use and Leisure Activities and the data for individuals aged between 25 and 59.[8] The time allocations of married and single men and women for market work hr, housework h, leisure l and basic childcare ζ are reported in Table 4, expressed in terms of the share of the total disposable time.

Schooling: We use the Basic School Survey to obtain the college enrollment rates of 0.169 and 0.543 in 1970 and 2020, respectively, computed as the mean of men's and women's college enrollment rates each year. These values correspond to s_t of our model.

[8] https://www.stat.go.jp/data/shakai/2016/index.html

Table 4: Preference Parameters: Data Targets and Model

		Data		Model	
Parameter	Description	1970	2020	1970	2020
Marriage and Fertility					
m	Fraction of married	0.967	0.822	0.967	0.822
k	Number of children	2.203	1.618	2.203	1.618
–	Total fertility rate	2.130	1.330	2.130	1.330
Time Allocation of Married Households					
hr	Market work hours (men)	0.769	0.712	0.769	0.712
	Market work hours (women)	0.277	0.266	0.277	0.266
h	Housework hours (men)	0.006	0.023	0.006	0.023
	Housework hours (women)	0.414	0.346	0.414	0.346
l	Leisure (men)	0.217	0.228	0.217	0.228
	Leisure (women)	0.189	0.215	0.189	0.215
Time Allocation of Single Households					
hr	Market work hours (men)	0.703	0.663	0.703	0.596
	Market work hours (women)	0.637	0.584	0.637	0.480
h	Housework hours (men)	0.034	0.030	0.034	0.033
	Housework hours (women)	0.140	0.119	0.140	0.147
l	Leisure (men)	0.263	0.307	0.263	0.371
	Leisure (women)	0.222	0.297	0.222	0.372
Schooling					
s	Fraction of college graduates	0.169	0.543	0.169	0.543

4.2 Home Production and Durable Goods

For the home production function (4), we follow McGrattan et al. (1997) and Greenwood et al. (2005) and set σ to 0.282. ω is set for normalization so that the durable goods consumption in 1970 is 1.

The path of durable goods price π is based on the Consumer Price Index data between 1970 and 2020, as shown in Figure 4. The price index of housework-assisting durable goods declined at an annual rate of 5.75% and we use this value as the growth rate of durable goods price. We set the price in 1970 so that the average share of household expenditures of durable goods matches the data.

4.3 Costs of Childcare

The time for basic childcare, ζ_g, is computed based on the Survey on Time Use and Leisure Activities. We divide the average time spent for childcare by men and women, respectively, by the number of children per married couple k_t in each year.

The financial cost of basic childcare b is computed based on the sum of the fees for school and extracurricular activities. The data is obtained from the Survey on Children's Learning Expenses conducted by the Ministry of Education, Culture, Sports, Science and Technology.

The education cost χ represents the costs of sending a child to college. We use the data from the Student Life Survey conducted by the Japan Student Services Organization (JASSO), and compute it as the sum of the tuition fees and living costs of a student enrolled in a college. See Appendix A. for more details about the data source and composition of the education cost.

4.4 Production Technology

There are three different technological parameters that represent the productivity level in the production function. First, Z_t stands for the level of general productivity, or what we call here TFP, and second, A_t and $A_{f,t}$ represent the productivity levels specific to skilled-labor, which govern the SBTC. A_t applies to both men's and women's skilled-labor inputs and $A_{t,f}$ applies only to women's. Lastly, B_t represents the productivity level specific to female labor supply of both skill types, which governs the GBTC.

We use data from the Basic Survey on Wage Structure (BSWS) to obtain the wage and distribution of workers by skill and gender, as well as the average work hours of each group, in 1970 and 2020, respectively. We assume that the distribution and average work hours change at a constant rate between the two time periods and compute the path of labor supply, L_g and H_g for $g \in \{m, f\}$ for low and high-skilled workers.[9]

We compute the paths of the productivity levels using the labor supply of men and women and the two skill types based on (5), (6) and (7). See Appendix B. for more details of the computation.

For other parameters in the production function, we set φ to 0.7 following the estimates used in Abbott et al. (2019). The value implies the elasticity of substitution between low and high skill labor of around 3.3, which is in the range of estimates in the literature. We set γ, that is related to the elasticity between male and female labor, to 0.55 and the share parameters θ_h, $\theta_{l,f}$ and $\theta_{h,f}$ are set to 0.55, 0.40 and 0.38, respectively, following

[9]Due to the data limitation, we have not been able to obtain wage rates by gender and skill in years between 1970 and 2020, and therefore assume constant growth rates. The BSWS started to ask the education level of "short-hour" workers (*tanjikan rodosha*) only after 2020 and we can use the information to compute the skill and gender-specific wages in 2020. Also, the BSWS had reported wage data for all workers by gender and skill, including short-hour workers, up to 1970 and we use this data for the wages in the model's initial year of 1970.

We also do not have data for average work hours of each type of workers in years between 1970 and 2020 and therefore assume that all variables including wage, hours and distribution change at a constant rate, in computing the path of the technology levels.

Abbott et al. (2019), who estimate the production function that consists of three levels of education. We adjust the estimates to our model of two education levels.

For the equivalence scale η, we assume the OECD equivalence scale and set to 1.5 for married households.

Table 5: Calibration Parameters

Parameter	Description	Value
Preference		
ρ, α	Curvature and weight: consumption (married)	3.0, 1.0
ν, β	Curvature and weight: home goods (married)	7.669, 0.0001
λ, μ	Curvature and weight: leisure (married)	3.946, 0.0168
κ, ϕ	Curvature and weight: child (married)	0.624, 0.1088
ψ, ξ	Curvature and weight: child quality (married)	0.5038, 0.0362
β_g	Weight: home goods (single)	>0.000 (men)
		>0.000 (women)
μ_g	Weight: leisure (single)	0.0042 (men)
		0.0097 (women)
Childcare Costs		
$\zeta_{m,t}$	Basic childcare time (men)	0.004 (1970), 0.023 (2020)
$\zeta_{f,t}$	Basic childcare time (women)	0.055 (1970), 0.107 (2020)
b_t	Basic childcare fin. cost	0.046 (1970), 0.084 (2020)
χ_t	Education cost	0.084 (1970), 0.190 (2020)
Home Production		
σ	EOS b/w durables and housework	0.282
ω	Share of durables	0.0135
π	Durable goods price, time-varying	−0.0575 (growth)
Market Production and Technology		
Z_t	Neutral technology	0.0023 (growth)
A_t	High-skill productivity (SBTC)	0.0042 (growth)
$A_{f,t}$	High-skill productivity (women) (SBTC)	0.0137 (growth)
B_t	Women's productivity (high) (GBTC)	0.0057 (growth)
φ	EOS b/w low and high-skill labor	0.70
γ	EOS b/w men and women	0.55
θ_h	Share of high-skill labor	0.55
$\theta_{l,f}$, $\theta_{h,f}$	Share of women (low, high)	0.40, 0.38
Other Parameters		
η	Equivalence scale	1.5
d, a	Marriage joy shock distribution	0.803, 0.659

5 Numerical Results

This section presents the numerical results of our model. We first present the outcome of the baseline model and then show how various factors of the model contribute to the trends of key variables. We do this by simulating the transition while eliminating or changing the magnitude of each factor one by one, keeping all the other elements unchanged from the baseline model. We will consider the roles of the technological changes and costs related to childcare including the education spending.

5.1 Baseline Model

Figure 5 shows the model's predicted paths of married women's time allocation, which are compared with the data. The model generates a secular decline in the housework, a mild decline in market work and a gradual increase in leisure in line with the transition of the data. The time for child care increases throughout the transition, while the number of children declines, as shown in Figure 6(a).

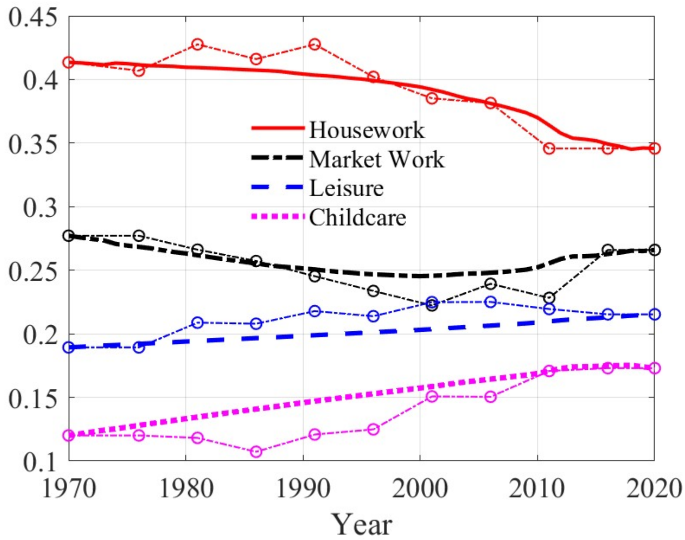
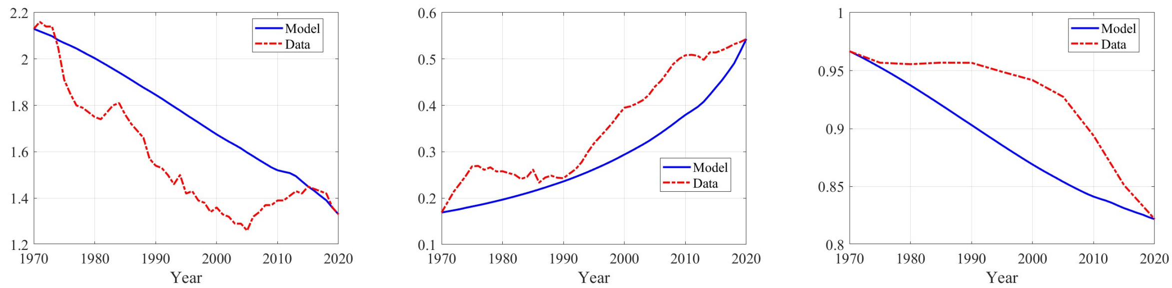


Figure 5: Time Allocations of Married Women: Baseline Model (lines) and Data (circles)

Figure 6 shows the trend of family formation and educational attainment in the model and data, represented by the shift in total fertility rates, college enrollment rates and marriage rates in each plot. Married couples choose to have fewer children over time, as the cost of childcare increases during the transition. The opportunity cost of raising children also increases as women's wages rise with the skill and gender-biased technological change.

The marriage rates decline over time as shown in Figure 6(c), as the relative attractiveness of being married wanes over time. The merit of the marriage stems from the possibility of having children and the ability of sharing resources to enjoy economies of scale. The decline in the optimal number of children, higher wage rates due to the technological growth, and cheaper input of home production to substitute housework all work in favor of remaining single.

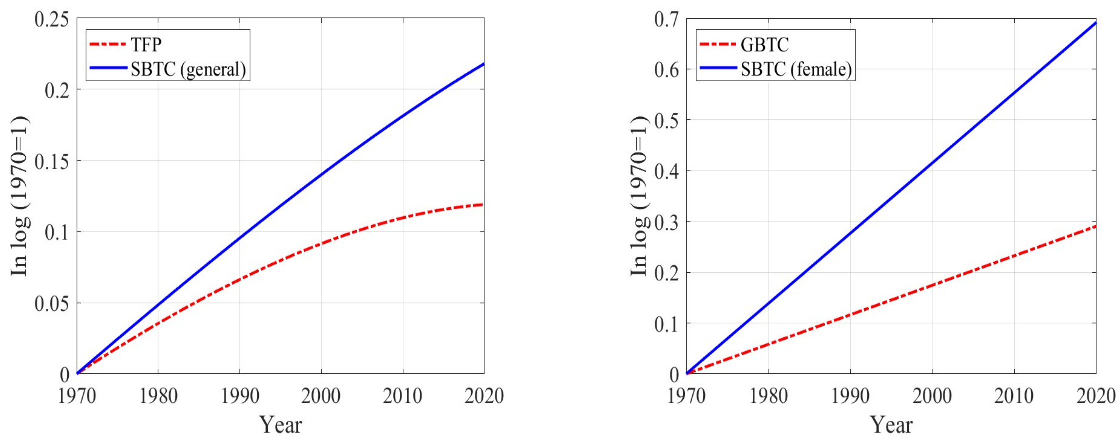
The model also generates the rising investment in education, following the upward trajectory of the share of college graduates in the data, as shown in Figure 6(b). Given the rising fixed cost of children and higher income, parents choose more education over more children, faced with the quantity and quality trade-off, tilting towards more quality.



(a) Total Fertility Rate (b) Schooling (c) Marriage Rate

Figure 6: Fertility, Marriage and Schooling: Baseline Model and Data

As described in Section 4.4, we compute the technology level B_t, $A_{f,t}$, A_t and Z_t from the gender and skill-specific wages and labor supply of each type of workers. The paths of the four technology levels are presented in Figure 7. The annualized growth rates between 1970 and 2020 are 0.23%, 0.42%, 1.37% and 0.57% for Z_t, A_t, $A_{f,t}$, and B_t, respectively, as also reported in Table 5. In the next section, we simulate various scenarios in which the technological growth is assumed to follow alternative paths.



(a) TFP and SBTC (general) (b) GBTC and SBTC (female)

Figure 7: Technological Growth

5.2 Roles of Technology

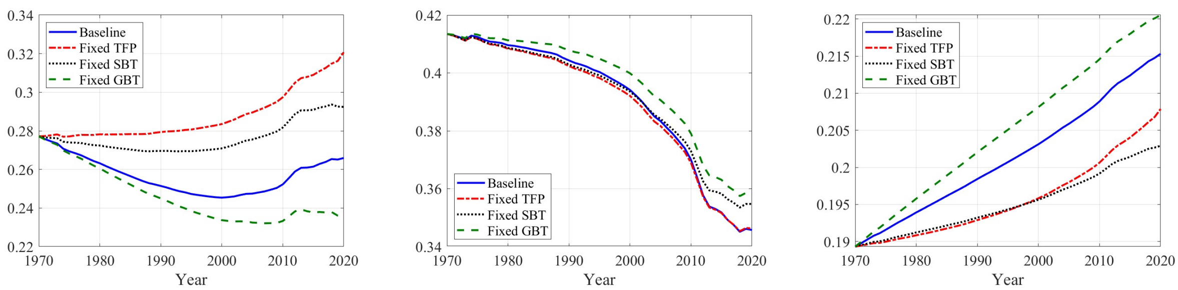
In this section, we consider how the technological progress during the last half century affects the trends of household decisions. We adjust the wage rates by shifting the values of technological levels and simulate the transition of the model which otherwise is identical to the baseline model.

In addition to the partial equilibrium (PE) simulations, where we adjust the wage rates solely by different technology levels and keep them fixed, we also run the general equilibrium (GE) version of experiments, in which wages also adjust because of an endogenous change of the labor supply driven by the change in the work hours and also because of the shift in the skill distribution due to a change in the education investment.

We run four sets of counterfactual experiments. First, we assume that the level of general technology, or the TFP, will remain at the same level as in 1970, that is, $Z_t = Z_{1970}$ for all t. Second, we mute the SBTC and set $A_t = A_{1970}$ and $A_{f,t} = A_{f,1970}$ throughout the transition. Third, we assume that there is no GBTC and set $B_t = B_{1970}$. Finally, we let the home production technology remain constant and the price of house-assisting durable goods stay at the level in 1970, $\pi_t = \pi_{1970}$ for all t.

Figure 8 shows the paths of married women's time allocations to market work, home production and leisure, under the three alternative scenarios about the technological progress. When there is no TFP or skill-biased technological growth, household income will be lower as husbands' earnings decline. Although women's wages are lower and the return from additional work declines, income effects dominate and married women increase work hours and reduce leisure.

When the gender-biased technological growth is absent, wages of women decline while mens' wages remain almost the same. Couples respond to this by reducing work hours of women and allocating more time to housework and leisure. As also summarized in Table 6, married women's market work in 2020 is lower at 23.5% of total time without the GBTC, which is about 12% lower than in the baseline model. Hours for leisure and housework will increase by 2.8% and 3.5%, respectively.



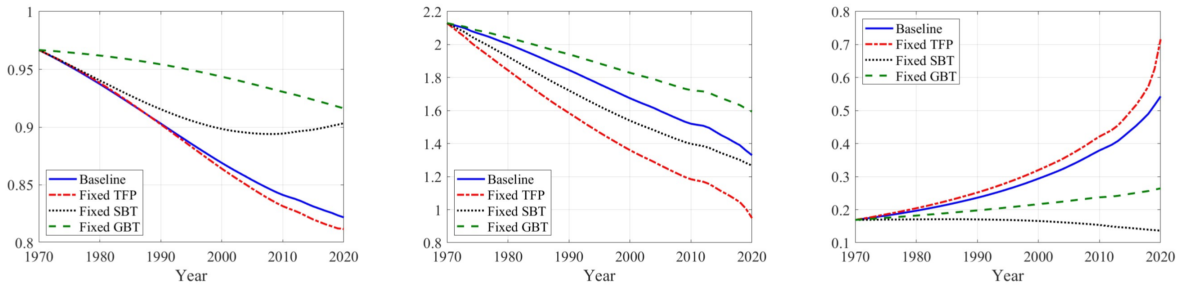
(a) Work Hours (b) Housework (c) Leisure

Figure 8: Roles of Technology (1)

Since the lack of SBTC and GBTC would lower the women's wages relative to men's and their economic conditions if they were to live by themselves, marriage becomes more attractive to women (recall that SBTC was stronger for women), resulting in the higher marriage rates as shown in Figure 9(a). Married households, however, become poorer under these experiments, and in the case of no SBTC, they can afford fewer children and total fertility rates decline. In the scenario of fixed GBT, although they are poorer, men's wage is not affected and women's wage is lower, they choose to have a larger number of children and spend more time on childcare. The time married women spend on childcare increases by 7.5%, from 17.3% of their time in the baseline to 18.6% under the experiment.

Turning to the education decision, Figure 9(c) shows that the schooling declines during the transition with no SBTC and GBTC. The decline is much more pronounced without the SBTC, since households no longer enjoy the higher return in the form of the skill premium from the education investment and the incentives to spend resources on education decline.

Without the GBT, women reallocate time from market work to housework, leisure, and childcare. Given the lower time cost of basic childcare, the demand of families regarding children tilts toward quantity from quality, and as a result, the schooling declines under the fixed GBT, as shown in Figures 9(b) and 9(c).



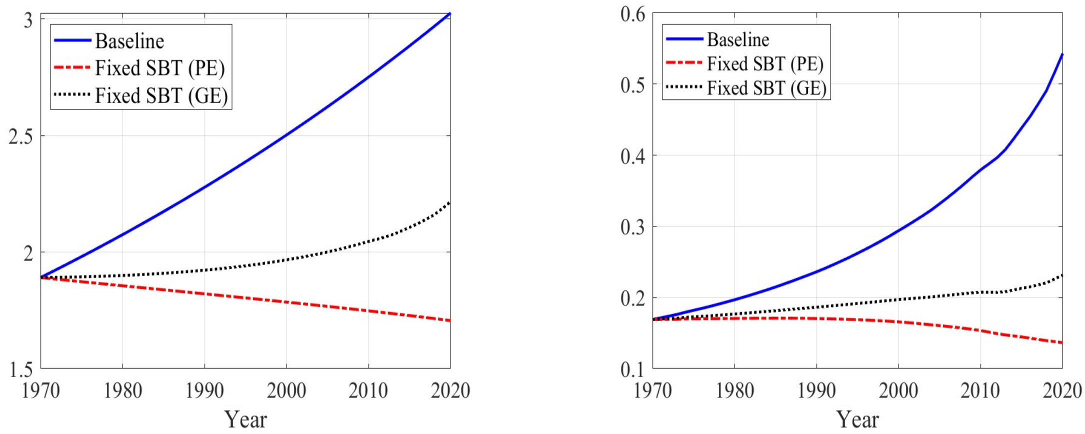
(a) Marriage Rate (b) Total Fertility Rate (c) Schooling

Figure 9: Roles of Technology (2)

Partial vs General Equilibrium Analysis: In the above experiments, we considered the effects of only the direct effects of the technological change on wages. We now consider full general equilibrium effects of the technology, allowing changes in the market work hours and education investment to affect labor inputs and wages.

In general, if lower wages due to a decline in the technology level reduce work hours or education investment, incorporating general equilibrium effects will mitigate the negative effects on wages. We consider the case of fixed SBTC to highlight such effects. For the results of other experiments, see the last columns of Table 6.

Figure 10(a) compares the baseline path of high-skill wage with other paths in two cases of no SBTC, with and without GE effects. As seen above, removing the SBTC increases work hours and reduce education investment. Since the latter effect quantitatively dominates the former with the significant disincentives to acquire skills, the high-skill labor supply declines. In general equilibrium, this effect increases the equilibrium wage of high-skill labor and mitigates the negative direct effect. The change will partially restore the incentive to invest in children's education as shown in Figure 10(b), though the level is still significantly lower than in the baseline model.



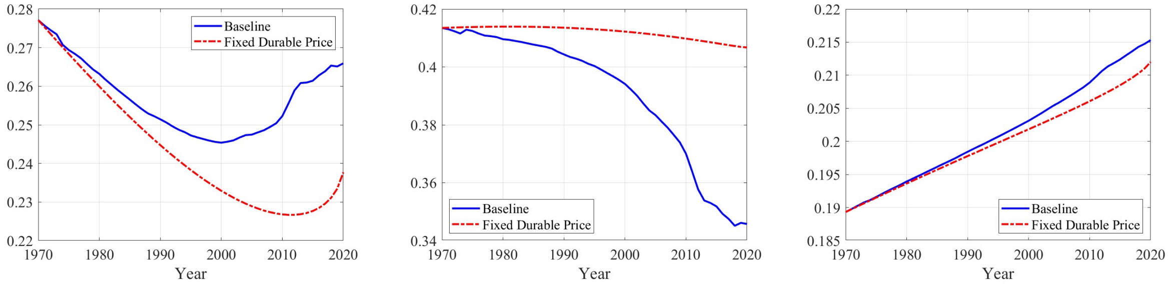
(a) High-skill Wage (Men & Women Avg.)

(b) Schooling

Figure 10: Roles of SBTC: Partial and General Equilibrium

Roles of Home Production Technology: We now consider the roles of the advancement of home production technology. As shown in Figure 4, the price of house-assisting durable goods rapidly decreased throughout the half century and contributed to a decline in the cost of home-produced goods. To quantify the effects of the price change of house-assisting durable goods, we simulate the transition assuming that the price remains unchanged, that is, $\pi_t = \pi_{1970}$ throughout the transition.

As shown in Figure 11(b), we no longer see a large decline in housework hours and this is compensated by a decrease in work hours and leisure relative to the baseline model. The additional hours for housework reduces time married women can allocate to childcare as well, and the fertility rate declines, as shown in Table 6. Parents instead spend more resources on education and the share of the high-skilled is higher than in the baseline model.



(a) Work Hours (b) Housework (c) Leisure

Figure 11: Roles of Home Production Technology

Table 6: Roles of Technology. Removing Factors One by One

			Partial Eq.: 2020				*General Eq.*: 2020			
	1970	2020	TFP	SBT	GBT	Dur. Pr.	TFP	SBT	GBT	Dur. Pr.
Family and Education										
Fertility (TFR)	2.130	1.330	0.949	1.267	1.594	1.093	1.081	1.371	1.402	1.156
Schooling	0.169	0.543	0.716	0.136	0.264	0.741	0.541	0.232	0.422	0.599
Marriage	0.967	0.822	0.812	0.903	0.916	0.814	0.825	0.927	0.905	0.796
Time Allocation of Married Women										
Work Hours	0.277	0.266	0.321	0.292	0.235	0.238	0.304	0.271	0.257	0.231
Leisure	0.189	0.215	0.209	0.203	0.221	0.212	0.209	0.213	0.221	0.209
Housework	0.414	0.346	0.346	0.354	0.358	0.407	0.348	0.358	0.357	0.405
Childcare	0.120	0.173	0.125	0.150	0.186	0.144	0.140	0.158	0.166	0.155

5.3 Childcare Costs

As we discussed in Section 2, households have faced rising costs of childcare both in terms of financial expenses and parental time during the last several decades. In this section, we consider and simulate three alternative scenarios in which the rise of these costs is mitigated during the transition. Long-run effects of key variables are summarized in Table 7.[10]

In the first scenario, we assume that the increase in the childcare time per child is limited to 50% of that in the baseline model. More precisely, we let the time for childcare increase by 48.1% between 1970 and 2020 in the experiment, instead of 96.2% in the baseline model.

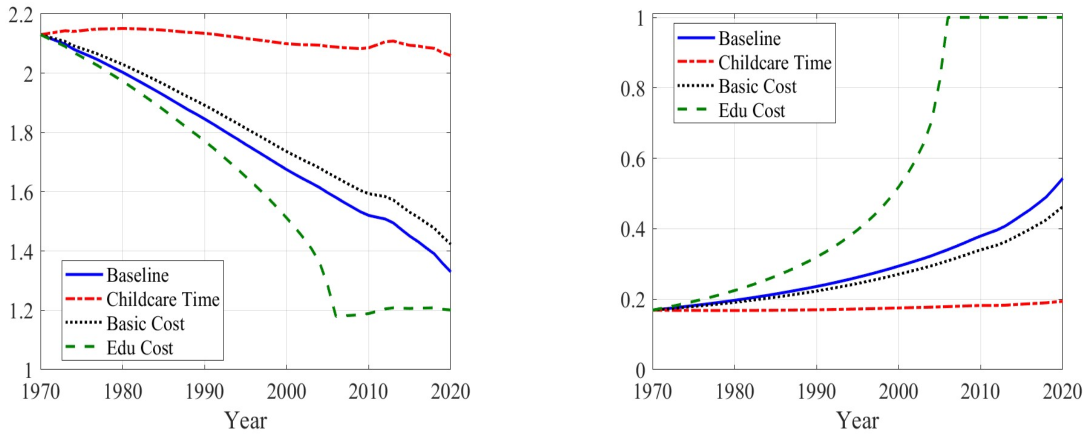
In the second and third scenarios, the basic financial cost of childcare b_t and education cost χ_t are assumed to increase at the same speed as the average earnings of married couples in the baseline economy. In other words, there is no "childcare cost inflation" and

[10]In the experiments of childcare costs, we abstract from general equilibrium through changes in factor prices and focus on the direct effects.

child-related costs move in parallel with the income levels of parents. We let the two types of financial costs rise by 66.7% in real terms over the 50-year period, instead of 81.0% and 125.1%, for the basic childcare and education costs, respectively, in the baseline model.

As shown in Figure 12(b), when the education cost is lower, married couples would raise the education investment. They allocate more resources on quality rather than quantity and total fertility rate is lower than in the baseline transition as shown in Figure 12(a).

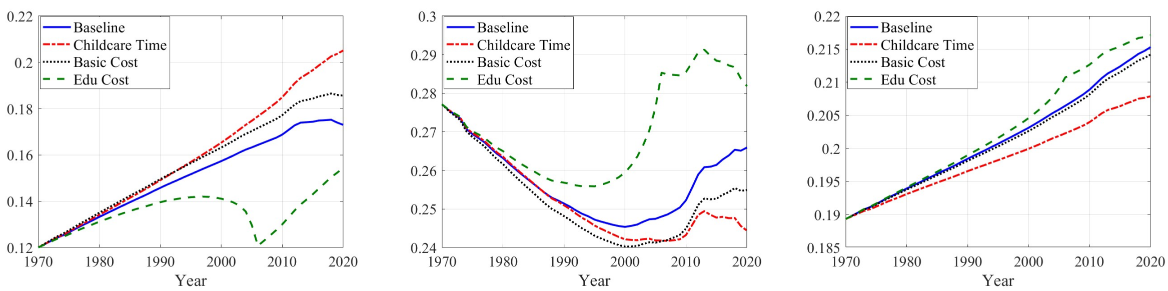
The responses, however, are opposite, when the cost of basic childcare is lower, both in terms of time and money. As shown in Figure 12, parents would increase the number of children compared to the baseline transition and instead reduce the education investment in each child.



(a) Total Fertility Rate (b) Schooling

Figure 12: Roles of Childcare Cost

In terms of married women's time allocations, they spend more time on childcare and reduce work hours and leisure when the basic childcare cost is lower, as shown in Figure 13. When they face lower education cost per child, they allocate the saved from fewer childcare hours towards market work and leisure.



(a) Childcare Time (b) Work Hours (c) Leisure

Figure 13: Roles of Childcare Cost

Table 7: Roles of Childcare Costs: Adjusting Factors One by One

			Childcare Cost: 2020		
			Basic	Basic	Edu
	1970	2020	Time	Money	Cost
Family and Education					
Fertility (TFR)	2.203	1.618	2.059	1.424	1.201
Schooling	0.169	0.543	0.194	0.463	1.000
Marriage	0.967	0.822	0.810	0.820	0.831
Time Allocation of Married Women					
Work Hours	0.277	0.266	0.244	0.255	0.282
Leisure	0.189	0.215	0.208	0.214	0.217
Housework	0.414	0.346	0.343	0.345	0.347
Childcare	0.120	0.173	0.205	0.186	0.155

We note that the quantitative results presented in this section should be interpreted with some caution. We focus on the decisions made by households regarding time allocations and family formation, assuming that households take as given the changes in wages, durable goods prices, and costs of childcare and education. However, time and money on basic childcare that parents spend on children may well be endogenously chosen by them. Furthermore, the prices of the childcare or education may also be determined in the market, reflecting the shifts in underlying fundamentals, similar to how wages are determined in the labor market.[11]

If the government were to implement a policy aimed at reducing the time or financial costs of childcare, our model suggests that parents would reallocate the saved resources towards something else. However, in a model with endogenous childcare, parents may respond to the policy by spending additional money and effort per child at the same time. Moreover, the increased demand for childcare resulting from the policy may influence the price of childcare services. The continuous increase in childcare time and financial costs over an extended period, at a rate surpassing income growth, indicates that such response is possible.

What the experiments conducted in this section highlight is that the fact that parents face the necessity to allocate more time and resources for raising a child compared to fifty years ago critically affects the optimal number of children that they choose to have. To

[11] There are some recent papers that consider the factors contributing to the increasing costs of education observed over the past decades. Jones and Yang (2016) construct a general equilibrium model that incorporates skill and sector-biased technological changes to investigate the impact of technology on college costs and educational attainment in the U.S. Cai and Heathcote (2022) develop a model of the college market and demonstrate that the growing income inequality has been a significant driver of tuition hikes in the U.S. since 1990.

fully account for the evolution of these childcare costs and efforts, a more comprehensive model is required and this is something we leave for future research.

6 Conclusion

Many developed countries have experienced the secular decline in fertility and marriage rates, as well as a shift in women's time allocations, over the past half century. Simultaneously, technological advancements drove the dynamics of the wage structure, driven by the general productivity growth and skill and gender-biased technological changes. These factors have influenced the trade-off involved in the time and resource allocation decisions of families.

We develop a tractable model in which households make decisions regarding marriage, fertility, and time allocations for various activities, including market work, home production, leisure, and childcare. Married couples determine the number of children and investment in their skills while considering time and financial costs of childcare and education.

We calibrate the model using macro and micro data from Japan, a country that has witnessed a significant decline in fertility and marriage rates, and a reduction in family size over the past five decades. Our quantitative analysis reveals that neutral and the skill-biased technological growth contributes to an increase in fertility rates and investment in children's skills. Conversely, gender-biased technological change leads to a decrease in the number of children, but an increase in educational attainment. Furthermore, we find that an increase in the financial and time costs of basic childcare results in lower fertility rates, while a rise in education costs has the opposite effect.

The analysis demonstrates that accounting for the trends of family formation and time allocations is not simple and emphasizes the importance of considering the interaction of various micro and macro factors. Changes in technology and the wage structure play a crucial role in determining the dynamics of the household income and the opportunity costs associated with childcare and home production. The significant, yet decreasing, gender wage gap influences the decisions regarding women's time allocations among different activities. Furthermore, the advancement of home production technology has had a significant impact on reducing the burden of housework. Additionally, the increase in time and financial costs of childcare directly affects the trade-off between quantity and quality faced by parents. This paper presents a model that takes into account these forces and their interplay within a comprehensive framework.

There are factors that we consider as given in this paper but may require a more careful explanation. Specifically, we did not explore the underlying reasons for the observed trends in factor-biased technology in the market and home production, as well as the shifts in various family constraints such as childcare and education costs. These are important topics that we leave for future research.

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Appendix A. Data Targets

Marriage Rates We use the Census data to construct the time series of marriage rates. The Census survey is conducted every five years, and it records the shares of individuals across four different marital statuses (ever-married, being married, divorced and currently not married, and widows/widowers) for each gender and age group. We compute the fraction of ever-married individuals at age 50 every five years between 1970 and 2020 and use the average of the ratios for men and women as the targets in the calibration of the distribution of marriage joy shocks, as discussed in Section 4.1.4.[12]

Figure 14 shows the fraction of never-married individuals at age 30, 40 and 50 between 1970 and 2020.

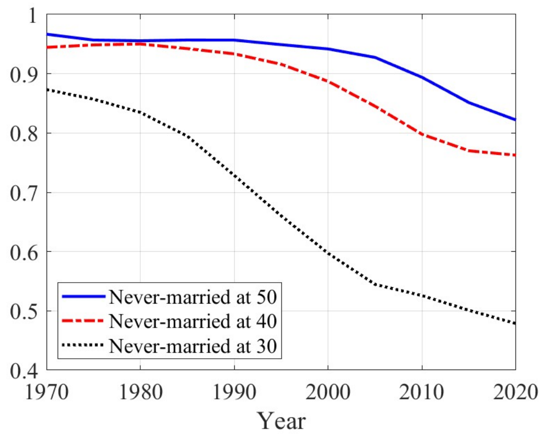

Figure 14: Fractions of Never-Married Individuals at age 30, 40 and 50. Source: The Census Data.

Time Allocations: The Survey on Time Use and Leisure Activities provides detailed information on individuals' time allocation at five-year intervals since 1976. We use the data to draw the time allocation patterns for men and women with different marital statuses, following the steps outlined below.

First, for each year, we calculate the time spent on four different activities (housework, childcare, leisure, and market work) for age groups of 25-29, 30-39, 40-49, and 50-59, separately for the four groups of individuals by marital status and gender (married male, married female, unmarried male, and unmarried female). Note that we let "Leisure" include the combined time spent on activities such as "Rest and relaxation," "Hobbies and amusements," "Sports," "Volunteer and social activities," and "Social life."

Second, for each year and individual group, we take a simple average of time spent on each activity over the four age groups between 25 and 59, as an approximation for the

[12]More specifically, we calculate the ever-married rate for men and women aged 45-49 and 50-54, and compute the simple average of the two values.

lifetime allocation over different activities.

Finally, we calculate the share of total disposable time spent on each activity for each year, which gives us time-series of lifetime allocations in every five years from 1976 to 2016. We chose not to include the latest data for 2021 due to concerns that the data may be irregular, as it reflects a period shortly after the onset of the COVID-19 pandemic. For 1970, we assume the time shares are the same as in 1976. Similarly, for 2020, we use the 2016 data.

Costs of Childcare The financial cost of basic childcare b is computed using the Survey on Children's Learning Expenses and the data is available since 1994. It records the average education expenditures for children before high school graduation under several categories, such as school and extracurricular activities. It turns out that the income share of average childcare expenditures remained almost constant between 1994 and 2018, the last year before the COVID-19 pandemic. More precisely, we compute the average education expenditures for a child until 18 years old and express them as the ratio to the lifetime income for married households (total labor earnings from age 25 to 59), based on the Family Income and Expenditure Survey. The ratio amounts to about 2.9% in each year between 1994-2018. Due to the data limitation for education expenditures in years before 1994, we assume that the income share of the child-related spending is constant across 1970-2020, including years prior to 1994, for which we do not have data.

We use the Student Life Survey to set the education costs χ during the transition. According to the Survey's data in 2018, the average annual expenditure for college students amounted to 1.91 million yen, comprised of 0.93 million yen of college related-expenditures, such as tuition and fees, and 0.98 million yen of living costs, such as housing and food expenses. While we have the time series of college tuition fees from the consumer price index, there are no other datasets that record other expenditures arising from college enrollment other than tuition and fees, prior to 2004, when the survey started. Hence, we assume that the income share of the expenditures aside from tuition and fees is constant throughout 1970-2020. Combining this and the price index for college tuition fees, we compute the total education costs χ for each year.

Appendix B. Computation of Technological Growth

This appendix describes how we compute the transition paths of the technology levels, $\{Z_t, A_t, A_{f,t}, B_t\}$ between 1970 and 2020.

We compute the paths of gender-biased technology level B_t from the ratios of female and male low-skill wage equations:

$$\frac{w_{l,f,t}}{w_{l,m,t}} = B_t \frac{\theta_{l,f}}{1-\theta_{l,f}} \left(\frac{L_{f,t}}{L_{m,t}}\right)^{\gamma-1}$$

With B_t, the gender-specific skill-biased technology level $A_{f,t}$ is computed from:

$$\frac{w_{h,f,t}}{w_{h,m,t}} = A_{f,t} B_t \frac{\theta_{h,f}}{1-\theta_{h,f}} \left(\frac{H_{f,t}}{H_{m,t}}\right)^{\gamma-1}$$

We then compute the aggregate low and high-skill labor L_t and H_t for all t using (6) and (7), and obtain the general skill-biased technology level A_t for from the ratios of low and high-skill wage equations:

$$\frac{w_{h,m,t}}{w_{l,m,t}} = A_t \frac{\theta_h}{1-\theta_h} \left(\frac{H_t}{L_t}\right)^{\varphi-\gamma} \frac{1-\theta_{h,f}}{1-\theta_{l,f}} \left(\frac{H_{m,t}}{L_{m,t}}\right)^{\gamma-1}$$

Finally, we set Z_t to the path of $w_{l,m,t}$. Note that we set Z_t in the initial period of 1970 so that $w_{l,f,0} = 1$ for normalization.

$$w_{l,m,t} = Z_t \left[(1-\theta_h)L_t^{\varphi} + \theta_h A_t H_t^{\varphi}\right]^{\frac{1}{\varphi}-1} (1-\theta_h) L_t^{\varphi-\gamma} (1-\theta_{l,f}) L_{m,t}^{\gamma-1}$$