Japan and the Allocation Puzzle in an Aging World [*]

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Abstract

Capital that flows from rich to poor countries driven by different capital labor ratios can be important in equalizing living standards across countries. Various factors, however, contribute to the 'Lucas Paradox' and the mixed empirical evidence. Differences in fundamentals, or simply TFPs, international financial market imperfections, differential demographic aging across regions of the world, or fiscal institutions affect the dynamics of capital flows. This paper builds a general equilibrium model of the world economy under imperfect capital mobility, populated by overlapping generations of individuals in three regions: the High-income (HI), Middle-income (MI) regions and Japan. By matching the observed dynamics of the Japanese net international investment position (NIIP), we show that TFP and aging are important factors to explain capital flows and that Japan will turn a net borrower in 2045 in a process where the MI region further develops the positive NIIP. A process of the MI region's catch-up in the generosity of the pension system and the TFP growth throughout the century could significantly improve the predicted Japanese NIIP.

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1 Introduction

A classic question in macro and development economics is 'why doesn't (more) capital flow from rich to poor countries?' Lucas (1990) argues that the (much) higher capital-labor ratios in developed economies would imply lower returns to capital in these countries and therefore capital ought to flow to developing economies. This flow of capital from rich to poor countries is important as it could possibly raise the living standards in poor countries. However, empirical evidence on the direction of flows is mixed at best.

There are two theoretical explanations for this 'Lucas Paradox'. First, there could be differences in 'fundamentals' such as human capital and institutions (or simply TFP), and second, there could be 'international financial market imperfections' that prevent capital from flowing to poor economies.

In addition, differential aging across economies with significant differences in the timing and severity of aging means that the relative capital-labor ratios are changing significantly over time. Finally, with higher rates of growth of TFPs in most of the developing economies, the TFP-advantage of richer economies is diminishing over time.

This paper develops a general equilibrium model of the world economy in which overlapping generations of individuals populate three regions that are connected through (imperfect) capital mobility but exhibit different demographic trends and TFP levels and growth rates. The 'high income' (HI) region is aging earlier and faster than the 'middle income' (MI) region, and has higher TFP but is facing slower growth of TFP and therefore (partial) convergence in living standards. Japan has been aging even earlier and faster than the HI region and for this reason we isolate Japan as a separate, third, region.

We contribute to the existing literature by 1) having Japan as a separate region and 2) developing a framework that quantifies the trade-off between different paths of aging and TFP growth rates across regions in determining the direction of capital flows. Isolating Japan as a separate region allows us to explicitly match data on the Net International Investment Position (NIIP) to Gross Domestic Product (GDP) for Japan. Our second modeling choice and the thought experiment we conduct allow us to determine the extent to which aging can help solve the 'allocation puzzle' making demographic projections in competition with (reasonably realistic) TFP growth paths in determining capital flows − an issue that the literature has not explicitly tackled tending to focus on aging in isolation.

Our setup consists of three regions populated with overlapping generations of individuals who face time-varying and random life spans with missing annuity markets. Labor is immobile but we assume that capital can move across regions in the open economy version of our model, subject to a certain imperfection. In addition, we assume that labor is exogenous in order to focus on the role of demographics and TFP in determining capital flows. Governments in the three regions have exogenous government purchases and transfer payments which increase over time as the populations get older. We also incorporate differences in the fiscal institutions across regions, captured by different degrees of generosity in the social security systems as well as taxes that support the programs. We assume that government debt to output ratios are held constant and lump sum taxes are

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used to achieve fiscal sustainability.

We calibrate the model under the assumption of an open economy using data from the World Bank, IMF, United Nations, and in the case of Japan, the National Institute of Population and Social Security Research (IPSS). We first compute equilibrium transitions from 1990 to a distant future in the three regions assuming that the economy is open and characterize the paths of equilibrium quantities such as output, capital and factor prices.[1] Second, under the same institutional settings we compute an equilibrium transition of the closed-economy version of the model, shutting down capital flows across regions.

Our quantitative analysis reveals three main findings on capital flows: 1) the MI region is always a net lender throughout our sample (1990-2015) and projection (2015-2100) periods, 2) the HI region is always a net borrower, and, 3) Japan starts as a net lender but becomes a net borrower in mid-century. These are novel results that suggest that efforts to improve TFP growth rates not only raise output per person but also attract foreign capital to further raise output in a region. This is especially true for Japan where policymakers have been trying to raise foreign direct investment in Japan, and, in the MI region to reduce the 'barriers' to development.

Finally, we calculate additional equilibrium transition paths under alternative assumptions on the projected demographics and on the future paths of regional total factor productivities. Our numerical results suggest that future TFP paths have a large impact on capital flows across borders, similar to that from the dynamics of demographics.

The paper is organized as follows. Section 2 demonstrates the differential timing and extent of demographic aging in our three regions and discusses roles of demographics and TFPs in determining capital flows across regions. Section 3 describes the related literature and summarizes the contributions of the paper. Section 4 describes the model and the economic environment. Section 5 summarizes our calibration with the details described in Appendix B. Section 7 discusses our main quantitative findings and alternative computations. Section 8 provides concluding remarks.

2   Differential Aging Across Regions, TFP Paths, and Capital Flows

2.1   Differential Aging Across Regions

The three regions in the model are High-income and Middle-income regions, and, Japan. For High-income region, we include North America (United States and Canada), Europe, Australia, and New Zealand. For Europe, we include 27 countries that are members of

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[1]The impact of demographic shocks, fiscal responses and TFPs is maximized in a closed economy, general equilibrium setting, with a direct link between the domestic capital labor ratio and factor rental rates. This mechanism is totally absent in a small, open economy, partial equilibrium model. Actual economies, however, are neither closed nor small, open, and depending on bilateral trade and capital flows, they may be closer to one extreme or the other.

European Union, plus the United Kingdom.

Middle-income region includes countries in Asia (China, Hong Kong, Taiwan, South-Korea, Singapore, Thailand, Indonesia, Malaysia, Philippines, Vietnam, India, Saudi Arabia, U.A.E., and Turkey), Mexico, Brazil, Russia, and South Africa.[2]

In order to see the differential demographic trends in the world, Figure 1 shows various aspects of the aging process and its implications in three regions: 1) HI region; 2) MI region; and 3) Japan.[3]
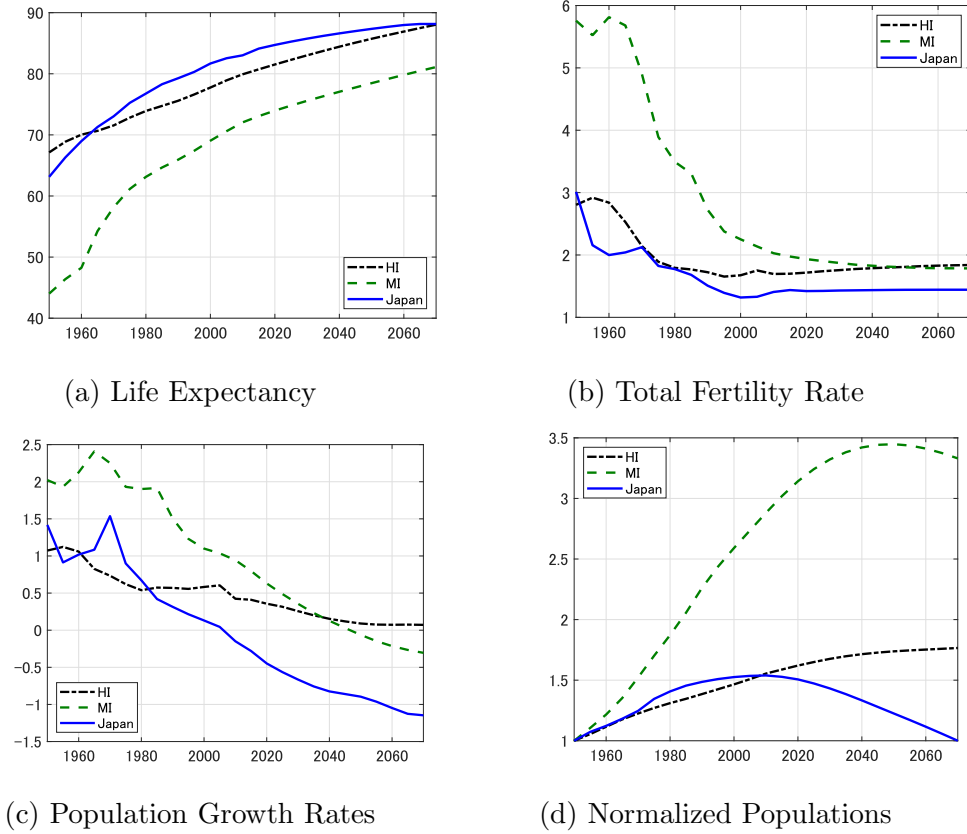

(a) Life Expectancy

(b) Total Fertility Rate

(c) Population Growth Rates

(d) Normalized Populations

Figure 1: Demographic Trends in the Three Regions

Figure 1a shows life expectancy at birth over time in the three regions. Longevity

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[2]We selected countries that have at least 100 billion yen (about $1 billion U.S.) of either foreign direct investment or portfolio investment according to the breakdown of Japanese foreign assets as of 2015 reported by the Bank of Japan. We exclude Cayman Islands from the list and also add Turkey to Middle-income region.

[3]The main source of demographic data is United Nation's *World Population Prospects: The 2017 Edition Revision* (2017), which provides harmonized data on population, fertility and life table projections for all countries from 1950 to 2100. The division of the first two regions is mainly based on the stages of overall economic growth and timing of demographic transition. The choice of countries included in these regions is based on the size of the economy, as well as the degree of investment and financial exposure to Japan. More details are provided in section 5.

is rising in all regions but significant differences are projected to persist well into the second half of the 21st century. Furthermore, Japan has and will continue to have higher longevity relative to the highly advanced economies during the next several decades. Figure 1b plots total fertility rates which have all come down from their earlier levels in 1960. In particular, fertility rates in HI and MI regions are projected to approximately converge (just below 2) whereas that in Japan is predicted to be significantly lower at around 1.5 for several more decades.

Given the significant fall in fertility rates, it is not surprising that population growth rates have fallen significantly since 1960 as Figure 1c shows. What stands out in this frame is that Japan's population growth has already turned negative in the late 2000s and the decline in the Japanese population is projected to accelerate over the next decades. According to Figure 1d, the Japanese population in 2070 will be about the same size as that in 1950. The populations in HI and MI regions will continue to rise, at least until the 2040s.

Combined, the fall in fertility and the rise in longevity imply significant increases in the old-age dependency ratio, defined as the ratio of individuals 65 or older to those between the ages of 20 and 64, shown in Figure 2. The dependency ratios show a gradual increase in all regions since 1950. In HI and MI regions, the main part of the increase is very recent and the projections put these ratios at about 55% and 45%, respectively, by 2070. In Japan, however, the sharp rise in the dependency ratio has started in the 1990s and it is projected to increase to 80% by 2050. In other words, the rise in the old-age dependency ratio in Japan has started much earlier and is predicted to reach unprecedented levels in a few decades.
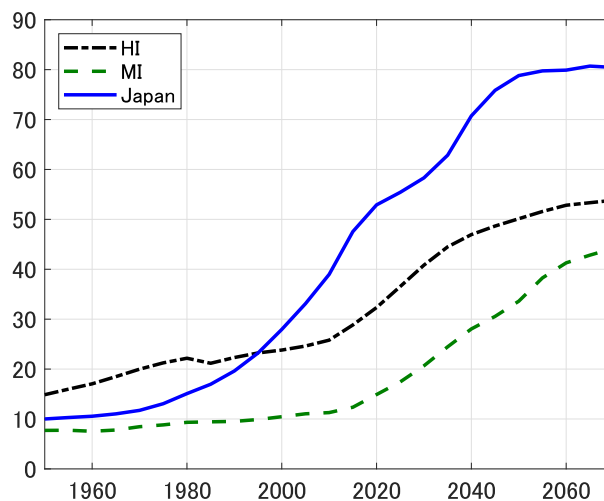

Figure 2: Dependency Ratios

2.2 Different TFP Paths Across Regions

The paths of factor prices are affected by the dynamics of aggregate capital and labor, as well as the projected trajectory of the TFPs. In our baseline calibration, we set the TFP growth rates in the 1990-2015 period as 0.99%, 1.98%, and 0.90% for the HI region, MI region, and Japan, respectively, to match the per capita growth rates of GDP in these regions for the same period. Beyond 2015, we assume that these rates converge to 1% eventually. This occurs in 2045 for the MI region, and 2100 for the HI region and Japan. After 2100, all regions' TFPs grow at 1% annually.
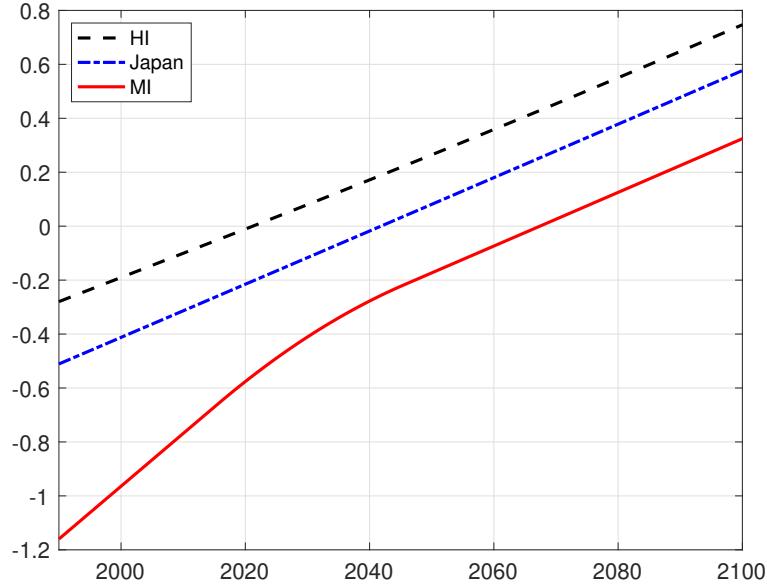

Figure 3: Total Factor Productivity in the Three Regions

Figure 3 depicts the time paths of (log)TFP in the three regions. Even though the TFP levels are increasing in all regions, as the growth rates converge to a common value in the long-run, the rate of increase stays higher in the MI region, allowing for some catching up in their living standards. In particular, the GDP per capita in the MI region rises from 28% in 2015 to 38% in 2045, relative to that in the HI region, in our baseline equilibrium transition path.

2.3 The Effects of Aging and TFP on Capital Flows

In order to provide intuition on how differential demographic trends and TFPs affect capital flows across regions, consider a constant returns to scale Cobb-Douglas production function $ZK^{\alpha}N^{1-\alpha}$ where K and N are aggregate capital and labor inputs, respectively,

α is capital's share of output, and Z is the TFP. Assume zero depreciation of capital. For two countries A and B, the rates of return to capital are given by

$$ret_A = (1 - \alpha)Z_A \left(K_A/N_A\right)^{\alpha-1},$$
$$ret_B = (1 - \alpha)Z_B \left(K_B/N_B\right)^{\alpha-1}.$$

Suppose that $Z_A = Z_B$. Now $ret_A < ret_B$ if $K_A/N_A > K_B/N_B$. That is, the closed economy rate of return in region B would be higher than that in region A if region A had a higher capital-labor ratio than that of region B. If region B is aging faster and accumulating capital faster or its aggregate labor input declining relative to that in region A, then the closed economy return to capital in region A may exceed that in region B in the future. In other words, both the current relative capital labor ratio and the future paths of the relative capital labor ratios are important in the closed economy return dominance between regions A and B. If capital is allowed to flow between regions, then capital would flow to the region where it earns a higher return until returns are equalized in the world financial market.

Similarly, both the relative level of and the future paths of relative TFPs are important in determining capital flows. Assuming that $K_A/N_A = K_B/N_B$, $ret_A > ret_B$ if $Z_A > Z_B$ in a closed economy setting. In this case, when capital is allowed to flow across borders, more of the world capital would be allocated to region A. In a dynamic setting, if Z_B grows faster than that in region A, as typically happens in growth models as part of catching up with the advanced economies, this flow may be reversed.

The allocation of capital in regions therefore depends on both the current and future values of TFPs and capital labor ratios. Richer countries with higher TFPs are also those where aging started earlier and progressed faster. In addition, the poorer countries are catching up with the richer countries (their TFP levels are partially converging) and they are also aging more rapidly lately.

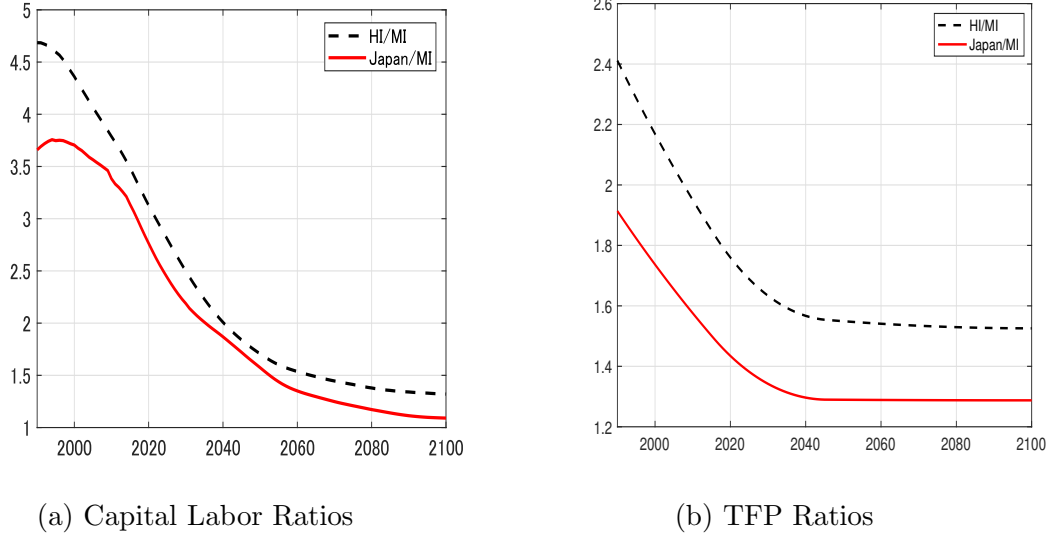|	
(a) Capital Labor Ratios	(b) TFP Ratios

Figure 4: Factors that Affect Closed Economy Rates of Return

Figure 4 shows the main mechanisms that drive the closed economy rates of return to capital and therefore the capital flows in an open economy equilibrium. Figure 4b plots the ratios of TFP paths in the HI and Japan regions relative to those in the MI region. First, the TFPs in both regions are much higher than those in the MI region, which means the closed economy returns to capital would be higher in HI and Japan regions. However, the TFP-advantage is getting smaller over time as the MI region experiences higher TFP growth rates in the future.

Figure 4a shows the ratios of capital labor ratios in the HI and Japan regions relative to those in the MI region. The capital labor ratios are higher in HI and Japan and this would tend to cause smaller rates of return to capital relative to the MI region. However, the MI region's capital labor ratio is rising relative to those in HI and Japan regions over time.

Given these mechanisms that put opposing forces on the closed economy rates of return to capital, we need a quantitative general equilibrium model in which these two mechanisms drive capital flows in an open economy setting.

We have reasonably accurate projections on population growth rates and conditional survival probabilities. However, TFP paths are very difficult to predict. In our quantitative work, we rely on the past two decades of average TFP growth rates and use these as our projections until they converge to their identical long run values when convergence of living standards stops. In order to study the sensitivity of capital flows to our TFP projection assumptions, we consider alternative paths where there is more or less convergence of living standards relative to our baseline assumptions.

3 Related Literature

Our paper contributes to the research that studies international capital flows using quantitative growth models. There is a large number of papers that model multiple regions of the world with overlapping generations and study flows of capital across regions, focusing on differences in the demographic structures and fiscal policies.[4]

Börsch-Supan et al. (2006) use a 7-region of the world economy with overlapping generations, consisting of the major European economies, the U.S. and others, and argue that capital flows out of fast aging countries like France, Germany and Italy until about 2050 and then the direction is reversed. Ferrero (2010) uses a Gertler (1999)-type model with age-independent survival (and retirement) probability and also finds a significant role for TFP differences in explaining the U.S. external imbalances, with the remaining portions explained by differences in life expectancies and fiscal policies.

Our paper is more closely related to Attanasio et al. (2006, 2007) that analyze the effects of demographic changes in developed and emerging economies. This research exploits the differential timing and nature of aging and different ways of dealing with pension programs in large, open economy models.

When an economy ages faster than others, then capital labor ratio will be larger than others since individuals would save more for a longer retirement period while the number of workers declines. As a result, the return to capital will decline by more. If capital is allowed to flow between economies, the fall in the return to capital in an aging economy will be accompanied by an outflow of capital seeking higher returns elsewhere. This is the main mechanism that determines capital flows in this quantitative literature. There is an effect that goes in the other direction. As the number of savers becomes smaller in an aging economy, the aggregate capital stock would fall as the fraction of retirees who do not save (much) increases. This effect on the interest rate appears quantitatively small relative to the first effect.[5]

Attanasio et al. (2006) develop a two-region model of the world economy with 'North' representing the more developed regions, according to the United Nations classification, and the 'South' standing for less developed regions. They calibrate their model so that the income per person in the North is 7 times that of the South initially, with the TFP of the South growing from 1.50% in the 1950-2000 period to 1.78% after 2000. There are also differences in the social security and fiscal policies. Their experiments are designed to quantify the roles of differences in survival probabilities, fertility rates and female labor supply participation rates in explaining capital flows. The general finding is that capital flows from the 'North' to the 'South' as aging occurs earlier and more severely in the

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[4]See, for example, Domeij and Floden (2006), Börsch-Supan et al. (2006), Krueger and Ludwig (2007), Chen et al. (2009), Backus et al. (2014), Lisack et al. (2017), Bárány et al. (2019), Papetti (2021), Auclert et al. (2021) and Sposi (2022). Liu (2021) provides a comprehensive literature review of modeling and empirical studies.

[5]As we show in section 7, interest rates in all closed and open economies of different regions decline under demographic aging.

North relative to that in the South.

Attanasio et al. (2007) use a similar framework to study whether the quantitative implications of various social security reforms for policy variables, factor prices, macroeconomic aggregates, and welfare of different cohorts in the North are sensitive to using a closed vs. open economy model. Their numerical findings suggest that either framework gives a similar answer to questions of fiscal sustainability but that factor prices and equilibrium quantities (and welfare) depend very much on the particular model used. Their model's prediction on capital flows is similar to that in their earlier paper. In the sensitivity analysis, they explore quantitative effects of alternative catching up scenarios of the South to the North in terms of the speed and level that it converges to, but the effect on capital flows is very small.

Our paper differs from this previous literature in several ways. First, we classify economies as HI vs MI in a different way than the North-South classification and as a result, our initial steady state and the first years, 1990-2015, of the transition to the eventual steady state produce the MI region starting from a positive Net International Investment Position (NIIP) and the HI region being a net borrower. Second, we emphasize not only the roles played by differences in the timing and extent of demographics and differences in fiscal policies (taxes, pensions, and indebtedness), but we also examine the role of TFP in attracting foreign capital. The latter is an essential channel in the literature that tries to explain the Lucas Paradox. Finally, our paper uses Japan as a separate 'region' and a laboratory to highlight the mechanisms at play because they are aging the fastest and earliest and we can more easily measure their NIIP with respect to the rest of the world.

Our paper is also related to the recent 'secular stagnation' papers such as Carvalho et al. (2016) and Eggertsson et al. (2017). Most of this research restricts attention to closed economy models and focus on understanding the mechanisms, including demographics, that contribute to the low real interest rates and economic growth in advanced economies.[6] Also, this paper extends the literature that studies fiscal sustainability issues and macroeconomic effects of aging population in Japan, including Hansen and İmrohoroğlu (2016), Braun and Joines (2015), Kitao (2015), and Kitao and Mikoshiba (2020). These models assume a closed economy and factor prices are determined solely by domestic factors. Our model allows for capital mobility into and out of Japan and quantifies how unsynchronized demographic aging in different parts of the world and different fiscal institutions could affect future paths of the Japanese economy.

Finally, this paper adds to the literature on the Lucas Paradox by highlighting the quantitative importance of the key mechanisms that determine capital flows. In a seminal paper, Lucas (1990) uses the standard Cobb-Douglas production function in the workhorse macro models to illustrate a paradox. These models predict that capital ought to flow to poor economies with higher rates of return such as developing economies with much lower capital labor ratios and total factor productivities. Capital flows, however, in the data

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[6]See also Gagnon et al. (2021), Ikeda and Saito (2014) and Sudo and Takizuka (2020) that study the effects of demographics on the real interest rates in the U.S. and Japan

seem to go in the other direction. Lucas (1990) suggests measuring and incorporating human capital in these calculations and also mentions the possibility of political risk as potential factors that may reconcile theory and data.[7]

4 The Model

4.1 Economic Environment

Preliminaries: Our model consists of Japan (J) and two regions: High-income (HI) and Middle-income (MI) regions. The two regions and Japan differ in their realized and projected demographic trends, total factor productivity levels and growth rates, and fiscal institutions. Our approach is to calculate a perfect foresight equilibrium transition path for the world economy from 2015 to a distant future steady state and characterize the transition path under the baseline and alternative economic settings. Let t denote time, j a household's age, and r the regions, with $r = HI, MI, J$.

Technology: In each region r, a constant returns to scale, aggregate production function $F(Z_t^r, K_t^r, N_t^r)$ produces output of a final good Y_t^r which can be used for consumption of households C_t^r and the government G_t^r or investment I_t^r. Among the arguments of the production function, Z_t^r denotes the total factor productivity level in region r at time t, N_t^r is the aggregate labor supply in efficiency units, and K_t^r is the aggregate stock of physical capital used in production. Physical capital depreciates at the same rate δ each period in all regions. The level of technology in region r grows exogenously at rate λ_t^r between t and $t+1$. The growth rates differ across regions during the transition, but in the long-run all regions grow at the same constant rate λ.

Demographics: Each region is populated by overlapping generations of ex-ante identical households who become active economically at age $j = 1$ and may live for a maximum of J periods. For households born in region r, $s_{j,t}^r$ denotes the probability of survival until age j at time t, conditional on being alive at time $t-1$. Hence, in region r, the

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[7]Caselli and Feyrer (2007) argue that a more comprehensive production function incorporating land and other resources may matter. In addition, they suggest that with the use of a multi-sector model in which the price of capital is higher in poor countries, and even with higher physical capital in rich countries, the return to capital may be close in these countries. Using a large international panel data set, Alfaro, Kalemli-Özcan, and Volosovych (2008) conduct an empirical analysis to study factors that explain capital flows across borders and find evidence that institutional quality explains the variation in net capital flows. Gourinchas and Jeanne (2013) studies the relationship of productivity growth and capital flows across developing countries and argues that the allocation puzzle is related to the pattern of growth, saving and international reserve accumulation. See also Kalemli-Özcan, Reshef, Sørensen, and Yosha (2010) that use data of capital flows across the U.S. states and Alfaro, Kalemli-Özcan, and Volosovych (2014) that use data of private and public capital flows for a large cross-section of developing economies and find that net international private capital flows are positively correlated with productivity growth.

unconditional probability of surviving j periods up to time t is given by

$$S_{j,t}^r = \prod_{k=1}^{j} s_{k,t+(k-j)}^r,$$

where $S_{1,t}^r = s_{1,t}^r \equiv 1$ for all t by definition. We denote by $\mu_{j,t}^r$ the size of the population of age j at time t in region r.

Household Preferences: Households of age j at time t in region r make consumption allocation decisions based on the instantaneous utility function

$$u(c_{j,t}) = \frac{\left(c_{j,t}^r\right)^{1-\theta}}{1-\theta}, \tag{1}$$

where $c_{j,t}^r$ denotes consumption of a household of age j at time t. The intertemporal preference ordering for households born at adult-age $j = 1$ at time t is given by

$$U^r = \sum_{j=1}^{J} \beta^{j-1} S_{j,t+j-1}^r \frac{\left(c_{j,t+j-1}^r\right)^{1-\theta}}{1-\theta}, \tag{2}$$

where β is the subjective discount factor. There is no explicit bequest motive driven by altruism. Accidental bequests left by the deceased are distributed as a lump-sum transfer, denoted as b_t^r.

Household Endowments: Households exogenously supply labor and derive no utility from leisure.[8] At age J_R^r, households are subject to compulsory retirement from market work. Households of age j at time t in region r are endowed with $\varepsilon_{j,t}^r$ efficiency units of labor for each unit of time worked in the market. Finally, we assume that the initial asset holdings of each household is zero, i.e. $a_{1,t}^r = 0$ for any t in all regions.

Household Budget Constraint: Let $a_{j,t}^r$ be the net asset holding of a household of age j at time t in region r.

$$\left(1 + \tau_{c,t}^r\right) c_{j,t}^r + a_{j+1,t+1}^r = y_{j,t}^r + \left[1 + \left(1 - \tau_{a,t}^r\right) ret_t^r\right] \left(a_{j,t}^r + beq_t^r\right) + p_{j,t}^r - \tau_{ls,t}^r. \tag{3}$$

We require households to die with non-negative wealth once they reach age J, but otherwise impose no borrowing constraint during their life. Net earnings $y_{j,t}^r$ accruing to households of age j in region r at time t are defined as

$$y_{j,t}^r = \begin{cases} \left(1 - \tau_{w,t}^r\right) w_t^r \varepsilon_{j,t}^r = \left(1 - \tau_{w,t}^r\right) \tilde{y}_{j,t}^r & \text{if } j < J_R^r, \\ 0 & \text{if } j \geq J_R^r, \end{cases} \tag{4}$$

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[8]We abstract from a labor-leisure choice in order to focus on the effects of saving and investment responses and capital flows across borders coming from exogenous demographic changes and saving/investment behavior.

If we model endogenous labor supply and distortionary taxation to finance rising expenditures from demographic aging, a rise in wages will be offset by an increase in taxes. Since what would affect individuals' labor supply is after-tax wages, households would not vary hours much under a reasonably calibrated labor supply elasticity (see, Attanasio et al, 2007).

where w_t^r is the market wage rate, $\varepsilon_{j,t}^r$ is the efficiency units of labor of a household of age j, and $\tilde{y}_{j,t}^r$ is the before-tax labor income. $p_{j,t}^r$ is pension income and takes a positive value for eligible individuals at and above social security's retirement age $j \geq j_{SS}^r$ and zero otherwise. ret_t^r denotes the real return to capital (net of depreciation) households in region r earn on their asset holdings.

Households pay proportional taxes at the rate of $\tau_{c,t}^r$ on consumption, $\tau_{a,t}^r$ on capital income, and $\tau_{w,t}^r$ on labor income and a lump-sum tax $\tau_{ls,t}^r$. Residents of region r pay capital income taxes in region r, independently of where capital was invested. Social security benefits are given by the formula

$$p_{j,t}^r = \kappa_t^r \frac{W_{j,t}^r}{J_{SS}^r - 1},$$

where κ_t^r is the replacement ratio of average past earnings. Cumulated past gross earnings $W_{j,t}^r$ are defined recursively as

$$W_{j,t}^r = \begin{cases} \tilde{y}_{1,t}^r & \text{if } j = 1 \\ \tilde{y}_{j,t}^r + W_{j-1,t-1}^r & \text{if } 1 < j < J_{SS}^r \\ W_{j-1,t-1}^r & \text{if } j \geq J_{SS}^r. \end{cases} \tag{5}$$

Government Budget Constraint: In each region r, public expenditures and social security program are administered by the government under a consolidated intertemporal budget constraint. The government can raise revenues by taxes on consumption, labor income, interest and capital income, and lump-sum taxes and can issue one-period risk-free debt, B_{t+1}^r. Government borrowing and tax revenues finance a stream of expenditures G_t^r and the PAYG social-security program described above. The consolidated government budget constraint reads as

$$G_t^r + (1 + ret_t^r) B_t^r + \sum_{j=J_{SS}^r}^{J} p_{j,t}^r \mu_{j,t}^r = \\ \tau_{w,t}^r w_t^r \sum_{j=1}^{J_R^r - 1} \mu_{j,t}^r \varepsilon_{j,t}^r + \sum_{j=1}^{J} \mu_{j,t}^r \tau_{a,t}^r ret_t^r \left(a_{j,t}^r + beq_t^r \right) + \sum_{j=1}^{J} \mu_{j,t}^r \tau_{c,t}^r c_{j,t}^r + \sum_{j=1}^{J} \mu_{j,t}^r \tau_{ls,t}^r + B_{t+1}^r. \tag{6}$$

Market Structure: There are three goods in the world economy: a final good which can be used either for consumption or investment, the services of labor and the services of capital. The price of the final good (homogeneous across the three regions) is used as the world numeraire. Labor is immobile, thus wages are determined independently in regional labor markets.

In the open economy, we assume that physical capital is mobile across the three regions, so there is one world market for capital where the world return to capital is determined. Let X_t^r denote the external wealth of region r at time t, that is, the stock of capital used in production in other regions and owned by households of region r. A negative value

indicates ownership of capital used for production in the region but owned by households of other regions. The sum of positive and negative external wealth across regions is zero in equilibrium, satisfying the condition $\sum_r X_t^r = 0$ at any time t. The markets where these goods and assets are traded are perfectly competitive. An intuitive no-arbitrage condition between assets and the absence of aggregate uncertainty imply that the return on the three regional bonds is equal to the return on physical capital, as we have already implicitly assumed when we wrote the budget constraints of the government and households.

In a closed-economy equilibrium, firms in each region maximize profits

$$
\begin{aligned}
w_t^r &= F_N(Z_t^r, K_t^r, N_t^r), \text{ for all } r \\
ret_t^r &= F_K(Z_t^r, K_t^r, N_t^r) - \delta.
\end{aligned}
$$

If capital were perfectly mobile across regions, we would have the world interest rate ret^* equal the interest rates of all regions, that is, $ret_t^* = ret_t^{HI} = ret_t^{MI} = ret_t^J$. In our baseline economy, however, we assume that there is an exogenous transaction cost that reduces the return that foreign (HI, J) savers earn on capital in MI. We include this transaction cost as a simple reduced form to capture region-specific expropriation risk typical of less developed financial markets. In addition, when MI households lend abroad, the differential captures the extra benefit for savers in MI to invest in safe assets abroad: $ret_t^* = ret_t^{HI} = ret_t^J$ and $ret_t^{MI} = ret_t^*/(1 - \phi)$, where $\phi \in (0, 1)$ captures the transaction cost. We will calibrate this parameter to match the average net external wealth of Japan in the 1996-2015 period.

4.2 Equilibrium

Before stating the definition of equilibrium, it is useful to point out that, without further restrictions, the equilibrium path of the fiscal variables $\left\{ G_t^r, \kappa_t^r, \tau_{w,t}^r, \tau_{a,t}^r, \tau_{c,t}^r, \tau_{ls,t}^r, B_t^r \right\}_{t=1}^\infty$ is indeterminate, as there is only one budget constraint we can operate on. In what follows, we define an equilibrium for the case where the paths of all fiscal variables are given, except for $\left\{ \tau_{ls,t}^r \right\}_{t=1}^\infty$. It is straightforward to extend this definition to the case where the path of a different set of government policies is given exogenously. Finally, for brevity we omit the definition of the closed-economy equilibrium and state directly the equilibrium conditions for the open economy.

For a given sequence of region-specific demographics, TFP levels $\{Z_t^r\}_{t=1}^\infty$, and fiscal variables $\left\{ G_t^r, \kappa_t^r, \tau_{a,t}^r, \tau_{c,t}^r, \tau_{w,t}^r, B_t^r \right\}_{t=1}^\infty$, a *Competitive Equilibrium of the Multi-Region Economy*, consists of sequences of: (i) households' choices $\left\{ \left\{ c_{j,t}^r, a_{j,t}^r \right\}_{j=1}^J \right\}_{t=1}^\infty$, (ii) lump-sum taxes $\left\{ \tau_{ls,t}^r \right\}_{t=1}^\infty$, (iii) wage rates $\{ w_t^r \}_{t=1}^\infty$, (iv) aggregate variables $\{ K_t^r, N_t^r, I_t^r, C_t^r \}_{t=1}^\infty$ in each region r, (v) world interest rates $\{ ret_t^* \}_{t=1}^\infty$, and (vi) external wealth positions $\{ X_t^r \}_{t=1}^\infty$ such that:

1. Households choose optimally consumption and wealth sequences $\left\{ \left\{ c_{j,t}^r, a_{j,t}^r \right\}_{j=1}^J \right\}_{t=1}^\infty$,

maximizing the objective function (2) subject to the budget constraint (3) and the income process (4).

2. Firms in each region maximize profits by setting the marginal product of each input equal to its price, i.e.

$$
\begin{aligned}
w_t^r &= F_N(Z_t^r, K_t^r, N_t^r), \text{ for all } r, & (7) \\
ret_t^* + \delta &= F_K(Z_t^r, K_t^r, N_t^r). & (8)
\end{aligned}
$$

3. The lump-sum transfer of accidental bequests equals the amount of assets left by the deceased, distributed equally to all households of the region.

$$
beq_t^r = \frac{\sum_{j=1}^{J-1} a_{j,t}^r (1 - s_{j+1,t}^r) \mu_{j,t-1}^r}{\sum_{j=1}^{J} \mu_{j,t}^r}
$$

4. The regional labor markets clear at wage w_t^r and aggregate labor supply in each region is given by

$$
N_t^r = \sum_{j=1}^{J_R^r - 1} \mu_{j,t}^r \varepsilon_{j,t}^r. \tag{9}
$$

5. The regional bond markets and the world capital market clear at the world interest rate ret_t^*, and the aggregate stocks of capital in the three regions satisfy

$$
K_t^r + X_t^r + B_t^r = \sum_{j=2}^{J} \mu_{j-1,t-1}^r a_{j,t}^r. \tag{10}
$$

The world interest rate and regional interest rates satisfy the relationships $ret_t^* = ret_t^{HI} = ret_t^J$ and $ret_t^{MI} = ret_t^*/(1-\phi)$, with the transaction cost $\phi \in (0,1)$.

6. The lump-sum taxes $\{\tau_{ls,t}^r\}_{t=1}^{\infty}$ satisfy the consolidated budget constraint (6) in each region.

7. The allocations are feasible in each region, i.e., they satisfy the regional aggregate resource constraints

$$
K_{t+1}^r - (1 - \delta) K_t^r + X_{t+1}^r - (1 + ret_t^*) X_t^r = F(Z_t^r, K_t^r, N_t^r) - C_t^r - G_t^r. \tag{11}
$$

Before concluding, it is useful to recall that aggregate gross investments in region r are given by

$$
I_t^r = K_{t+1}^r - (1 - \delta) K_t^r, \tag{12}
$$

whereas aggregate or national (private plus public) savings in the three regions are,

$$
S_t^r = F(Z_t^r, K_t^r, N_t^r) + ret_t^* X_t^r - C_t^r - G_t^r. \tag{13}
$$

As a result, the current account balances of the three regions equal their respective changes in net asset positions, and, are given by,

$$S_t^r - I_t^r = CA_t^r = X_{t+1}^r - X_t^r. \tag{14}$$

The sum of these current account balances is zero.

5 Calibration

We calibrate the initial steady-state using demographic and economic variables for the period 1990-2015 in the three regions. We assume that demographic parameters stabilize by 2100 and TFP growth rates in the three regions converge to the same values in the long run, and all regions eventually reach a balanced growth path some time after 2200. We then let our world economy transition between the two steady-states. The model's period is annual. Our calibration strategy is to match a set of moments in the data with the model's counterparts in the *open economy*. Appendix B provides more details of the calibration, including description of various databases used to compute statistics across regions. Calibrated parameters are summarized in Tables 1 to 4.

Technological Parameters: We assume a constant returns to scale production function

$$F(Z_t^r, K_t^r, N_t^r) = Z_t^r (K_t^r)^\alpha (N_t^r)^{1-\alpha},$$

with capital share $\alpha = 0.35$ in three regions, following Holmes, McGrattan, and Prescott (2015). Based on the World Bank's *World Development Indicators* (WDI), we obtain growth rates of output per capita in the three regions from 1990-2015, which stand at 1.1% in Japan, 1.4% in High-income region and 3.9% in Middle-income region. The growth rate of TFP λ_t^r in each region is set so that the region achieves the target average per-capita output growth rate during the same period. Values of the calibrated parameters are indicated in Table 4.

After 2015, we let λ_t^r of all regions converge smoothly to the common long-run growth rate of TFP, which we set to 1%.[9] We set the initial value of the Japanese TFP, Z_0^J, in order to normalize income per capita in Japan to 1 in 2015. Based upon the WDI, income per capita in High-income and Middle-income region in 2015 were 1.11 ($45,373) and 0.31 ($12,696) of that of Japan ($40,763), respectively, and we set Z_0^H and Z_0^M to match these ratios. The depreciation rate of capital is set to 6% per year and is common to all three regions.

---

[9]We assume that the growth rate of TFP in the Middle-income region converges over the next 30 years, by 2045, and those of Japan and High-income region by 2100, to the common value of 1%. The TFP levels in the long-run will differ across regions and the TFP levels of Japan and Middle-income region will be around 0.75 of that of the High-income region. We simulate the model under alternative assumptions about the future growth of TFP across regions in section 7.

Demographic Parameters: The population data is based on the estimates of the United Nations (2017) for the High-income and Middle-income regions and on the data and projections of the National Institute of Population and Social Security Research (IPSS) (2017) for Japan.[10] The population in 1990, the initial year of the transition, is 122 million in Japan, 778 million in High-income region, and 2,976 million in Middle-income regions. The survival probabilities are computed based on the population data and projections by age for each region, following the same procedure in Attanasio, Kitao, and Violante 2007. The population dynamics thereafter follow the projections of the United Nations and the IPSS.

We let households enter the economy at age $j = 1$, which corresponds to 20 years old, and live up to the maximum age of $J = 80$, up to 99 years old. We set the age to exit the labor force J_R^r at 46, which corresponds to 65 years old.[11]

Preferences and Endowments Parameters: Preferences are common across regions. The constant relative risk aversion parameter θ is the inverse of the intertemporal elasticity of substitution (IES) in consumption whose estimates vary considerably from about 0.2 to 2 (for a survey, see Attanasio, 1999). For our baseline calibration, we set the risk aversion parameter $\theta = 2$ and use $\theta = 1$ (log preferences) as part of a sensitivity analysis.

We set the subjective discount factor $\beta = 1.0296$ to match our target equilibrium world rate of return on capital of 4% in 2015.

The calibration of the age profile of efficiency units is done separately for each region. The age-efficiency profile for Japan is based on the earnings data from the Basic Survey of the Wage Structure (BSWS) in 2010. For High-income region, we use weekly wage data from the Consumer Expenditure Survey (CEX) for the period 1982-1999. For Middle-income region, we use estimates used in Attanasio, Kitao, and Violante (2007), an age-efficiency profile on Mexican data—precisely from the Encuesta Nacional de Ingreso y Gasto de los Hogares (ENIGH), which is the equivalent of the U.S. CEX, using 1989-2000 waves. For all data sources average earnings of male and female workers are used in the calibration.

Transaction Cost in the World Capital Market: The transaction cost ϕ is calibrated to match the external wealth to output ratio in Japan during the 1996-2015 period, which stands at 40%, based on the data from the Ministry of Finance. We assume

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[10]IPSS projections are updated periodically based on the Census and on detailed information about the current states of the demographics in Japan. Projections of the IPSS differ from those of the UN mainly because the latter assume that fertility rates of all countries converge to a common trend in the long-run. We use the IPSS projections since we consider that they are more consistent with the current situations in Japan. Ideally, we would use population projections of each country included in the HI and MI regions. We believe, however, statistical uncertainty at a country level is mitigated by combining a relatively large number of countries in the two regions, since the UN projections have been accurate at an aggregated level and most of the errors occur at an individual country level (see Attanasio et al. 2007).

[11]Note that the age to exit the labor force J_R^r, which is assumed to be exogenous in the model, can be different from the retirement age for the purpose of the social security system. The latter is a policy parameter as discussed below.

the same value of ϕ until 2020 and let it decline gradually to reach zero over a 80-year period and by the end of the century. In section 7, we simulate the transition under alternative assumptions about the transaction cost, including the case of zero cost and compare results to those of the benchmark transition.

Government Debt to Output Ratios: Government debt and expenditures as a fraction of output for High and Middle-income regions are computed from the IMF's World Economic Outlook database (WEO, 2017) as time-averages over the period 1990-2015.

The WEO data yield a ratio of government expenditures to output at 35%, 41% and 25% for Japan, High-income and Middle-income regions, respectively, in 1990-2015. These expenditures contain all expenditures including interest payments on the government debt and transfers such as social security benefits. We calibrate the ratio of the government expenditures in our model, G_t^r, to output to match these data and they turn out to be 25% in Japan, 34% in High-income and 20% in Middle-income region. The ratio of government debt B_t^r to output in 1990-2015 was 51% and 31% in High-income and Middle-income regions, respectively, based on the WEO database. The debt level of the Japanese government, based on data from the Ministry of Finance, greatly fluctuated during the period and rose from 14% of output in 1990-1995 to approximately 120% in 2010-2015 and we set the ratio to 100%.

Public Pension Replacement Rates: Pension replacement rates for the three regions are calculated using OECD Pensions at a Glance (2014). In particular, we compute 'coverage adjusted' net replacement rates (NRRs) by multiplying NRRs by active coverage (defined as total number of contributors divided by labor force), available from the World Bank Pensions database (2014). The GDP (current PPP) weighted, coverage adjusted NRRs are 47.8%, 26.8%, and 38.5% for High and Middle-income regions and Japan, respectively.

The statutory retirement age is 65, 66 and 56 in Japan, High-income and Middle-income regions, respectively, based on the population-weighted average of the World Bank's database and we set the retirement age J_{SS}^r in the model to these ages.

Tax Rates: Tax rates on consumption and assets are computed using the revenue data from the OECD Revenue Statistics, OECD National Accounts Statistics and UN National Account Statistics, following the method of Mendoza, Razin, and Tesar (1994). τ_a^r is set at 34.7%, 34.1% and 18.8% in Japan, High-income and Middle-income regions, respectively. Consumption tax rates are 10.9% and 12.7% for High-income and Middle income regions. The Japanese consumption tax rate had been zero until 1989, when it was set to 3% and increased to 5% in 1997, 8% in 2014 and 10% in 2019. We let $\tau_{c,t}^J$ increase accordingly and assume that it stays constant after 2019. Labor income tax rates τ_w^r are set to 32.8%, 17.0% and 29.8% for High-income, Middle-income and Japan, respectively. We adjust lump-sum tax $\tau_{ls,t}^r$ to satisfy the government budget constraint in each year and region. The paths of endogenously determined lump-sum taxes are reported in section 7.

Table 1: Demographic Parameters

Parameter and description		Value, source
$s_{j,t}^r$	Survival rates	United Nations (2017) and IPSS
n_t^r	Cohort growth rates	United Nations (2017) and IPSS
J_R^r	Age to retire from the labor force	46 (65 yrs)
J	Maximum age	80 (99 yrs)

Table 2: Calibrated Parameters: Preferences, Technology and Endowment

Parameter and description		Value	Target, source
β	Subj. discount factor	1.0296	Interest rate in 2015
θ	Risk aversion	2.0	Attanasio (1999)
ε_j^r	Wage profile	–	see text
α	Capital share	0.35	Holmes, McGrattan, and Prescott (2015)
δ	Depreciation rate	0.06	Holmes, McGrattan, and Prescott (2015)
ϕ	Transaction cost	0.41	Mean external wealth of Japan in 1996-2015

Note: values are on an annual basis.

Table 3: Calibrated Parameters: Government

Parameter and description		Target, source
D_t/Y_t	Debt to output ratio	IMF World Economic Outlook (2017)
G_t/Y_t	Gov. purch. to output ratio	IMF World Economic Outlook (2017)
κ_t^r	Pension replacement rate	OECD Pension at a Glance (2014)
J_{ss}^r	Pension retirement age	66 (HI), 56 (MI) and 65 (Japan)
		World Bank Pensions database
$\tau_{c,t}^r$	Consumption tax	OECD Revenue Stat., OECD/UN National Acct. Stat.,
		Mendoza, et al (1994)
$\tau_{a,t}^r$	Capital income tax	OECD Revenue Stat., OECD/UN National Acct. Stat.,
		Mendoza, et al (1994)
$\tau_{w,t}^r$	Labor income tax	OECD Revenue Stat., OECD/UN National Acct. Stat.,
		Mendoza, et al (1994)

Table 4: Growth Rate and Level of TFP

Region	GDP per capita growth, WDI (1990-2015), target	TFP growth rate λ_t^r(1990-2015) calibrated	GDP per capita level, WDI 2015, target	Initial TFP level Z_0^r calibrated
Japan	1.1%	0.99%	$40,763 (1.00)	0.60
High Income	1.4%	0.90%	$45,373 (1.11)	0.76
Middle Income	3.9%	1.98%	$12,696 (0.31)	0.31

6 Model Validation

In order to evaluate if our model and calibration produce a reasonable tool for the questions we are posing, we discuss some evidence in this section. Note that our open economy version is calibrated to produce a world return to capital of 4% in 2015 and the transaction cost parameter is calibrated so that our model generates an NIIP for Japan close to the average observed NIIP between 1996-2015. The other equilibrium quantities are not moments as targets for calibration.

Table 5: Capital Output Ratios

Region	PWT Data	Country Data	Model
High Income	3.64	3.23 (U.S.)	3.22
Middle Income	3.36	2.98 (China)	2.45
Japan	4.18	3.13 (Japan)	3.22

Table 5 displays the capital output ratios in the data and the baseline open economy simulation. The first column contains the data from the Penn World Table (PWT) between 1990 and 2017. The second column shows the capital output ratios using selected country-level data. In particular, we use the BEA data as in Koh, Santaeulàlia-Llopis, and Zheng (2020) for United States between 1990 and 2018, quarterly data between 1994 Q1 and 2019 Q2 from the Cabinet Office for Japan, and the PWT data for China between 1990 and 2017. Since none of these model quantities were targeted, we think the baseline model gets very close to measured capital output ratios, especially when we use country-level data as representative of the HI and MI regions.

However, there are significant drawbacks in comparing the data versus the model's current accounts and net foreign asset positions of the regions. First, the NIIP data present a country's position with respect to the rest of the world (RoW), thus including countries within the same region of our model or outside the model's world economy. Ideally, one would need data on countries' bilateral NIIPs to construct accurate regional net foreign assets positions. Second, coverage of China starts from 2004, restricting a

country with significant amount of foreign assets from our sample.[12]

The current account data from WDI has the same drawback, missing bilateral positions by each country necessary for us to construct a counterpart to our model's measure.

Given the difficulty of using the NIIP data with the lack of bilateral asset holdings aggregated to represent cleanly our three regions, we present the NIIP for the three regions using our (mis)measured NIIP as well as select economies to represent the regions. In particular, we will show the NIIP of the U.S. for the HI region and that of China for the MI region.

As shown in Figure 5, the model overstates the 'net borrower' position of the HI region but it is close to the U.S. NIIP. Similarly, the model overstates the 'net lender' position of the MI region and it is closer to the NIIP of China.

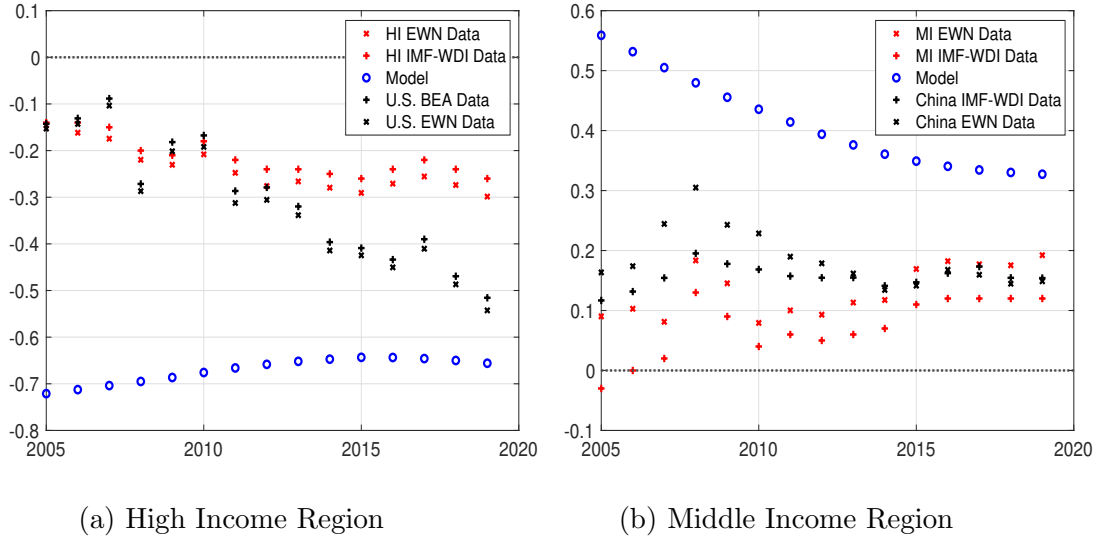(a) High Income Region   (b) Middle Income Region

Figure 5: Net International Investment Position to GDP Ratio

For Japan, we have less of an issue and therefore a comparison is far more meaningful. Figure 6 shows the data and model ratios of current account to output. The model seems to track the actual current account position of Japan reasonably well. Note that we targeted the average NIIP of Japan with our calibration of the single transaction cost parameter ϕ but not the current account directly.

---

[12]The External Wealth of Nations database has net NIIPs excluding gold since 1981. However, since China entered the WTO in 2001 and became a net lender in 1999, and a significant net lender much later on, after 2005, we show China's NIIP to GDP ratio in the data from 2005 onward in Figure 5b.

Figure 6: Current Account to GDP Ratio

Figure 7 displays the model's net foreign asset position of Japan and two data counterparts, the NIIP of Japan from the IMF-WDI data base, and that from the Bank of Japan (BoJ). With a single calibrated parameter, the model does a remarkable job of producing a net foreign asset position that is very close to the observed NIIP of Japan. This fit makes us optimistic that the model may produce projections that speak to the likely effects of demographics and TFP paths on capital flows, especially from a Japanese point of view.
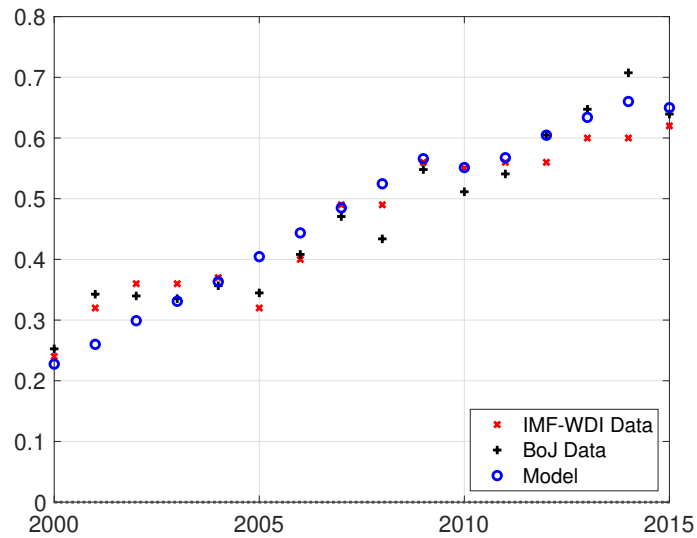
Figure 7: Net International Investment Position to GDP Ratio

6.1 Two Counterfactual Experiments

Another reason to have Japan as a separate region is that we can conduct counterfactual exercises and compare the results with the IMF-WDI and BoJ data depicted in Figure 7. In Figure 8, we add two counterfactual paths of NIIP to GDP ratios from the model to the baseline path and the two data series depicted in Figure 7.

The NIIP to GDP ratio labeled 'TFP' is generated under the counterfactual assumption that the rate of growth of TFP in Japan from 1990 to 2015 is 0.9% which is that of the HI region, instead of the 0.99% used in the baseline case. With a lower TFP growth rate, the closed economy return to capital would have been lower than that in the baseline case and therefore required more capital outflow in an open economy setting. Even though the growth rates 0.99% and 0.9% are only about 10% different in each year, the impact on capital flows seems relatively large. For example, in 2005, the NIIP position of Japan would have been 50% larger, 0.6 vs 0.4, under the counterfactual. This suggests a quantitatively important role for TFP.
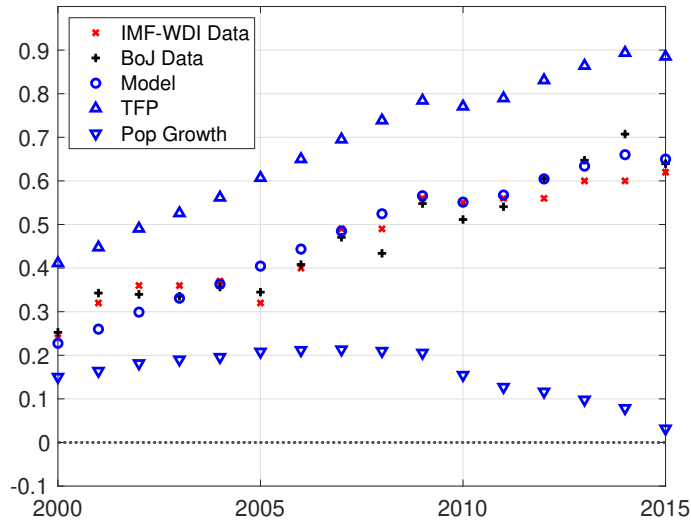

Figure 8: Counterfactuals: Net International Investment Position to GDP Ratio

In Figure 8, the NIIP to GDP ratio labeled 'Pop Growth' is the model's simulated path under the rather extreme assumption that Japan's population growth between 1990 and 2015 is the same as that of MI region. Clearly this counterfactual would have raised the labor supply significantly in Japan, lowering the capital-labor ratio and raising the closed economy return to capital which in turn would necessitate capital flowing into Japan, as shown in the figure. In this scenario, Japan would have had a much smaller NIIP to GDP ratio and would eventually become a zero net borrower by 2015.

These counterfactuals suggest that both TFP and demographics play a role in shaping the timing and the magnitude of international capital flows. In the next section we present our quantitative findings on how future capital flows depend on these mechanisms and how much fiscal pressure regional policymakers have to deal with, which is brought on by aging of populations.

7   Quantitative Findings

In this section, we present our numerical findings that characterize equilibrium transition paths from the initial steady state that represents the world economy in 1990-2015 to a final steady state in the distant future.[13]

These equilibrium paths are computed under the 'perfect foresight' assumption and are therefore deterministic. All households and firms have complete information on factor prices and fiscal policy parameters (tax rates, social security benefits, etc.) and make optimal decisions under new demographic variables, fiscal parameters and associated endogenous factor prices, given their asset holdings in the initial steady state. In other words, cohorts that are alive in 1990 re-optimize given their new demographic and policy environments. Along the transition, as in the initial and final steady states, it is assumed that a lump-sum tax adjusts in each time period to satisfy the government's budget, keeping debt to output ratios constant. These computations form our baseline results below.

In some of the figures below, we will show the equilibrium transition paths under two separate assumptions on the openness of the economies. First, we compute the transitions under the assumption that each region has been operating in a world economy in the initial steady state and will remain open forever. Alternatively, in the closed economy computations, we assume that there are three segmented regions in the steady states and along the transition path such that both labor and capital are immobile with segmented equilibria in three regions.

7.1   Baseline Transitions

Return to Capital and Capital Flows in the Open Economy: Figure 9 displays the returns to capital in the closed and open economy cases, and the NIIPs in the open economy case. The world real interest rate, shown in Figure 9a, starts at just above 6% in 1990 and falls to below 3% by 2030 and stays at around 2%-3% thereafter throughout the 'projection' period.[14] This finding is line with the previous research that demonstrated a fall in real interest rates over time primarily due to demographic aging.

---

[13]In the program, we compute transitions of about 400 periods (from 1990 to 2400), long enough to guarantee a smooth convergence of all variables to the final steady state.

[14]Note that we calibrate the subjective discount factor (common across the three regions) in order to obtain an equilibrium world return to capital equal to 4% in 2015 and use this value of β in all our closed and open economy computations.

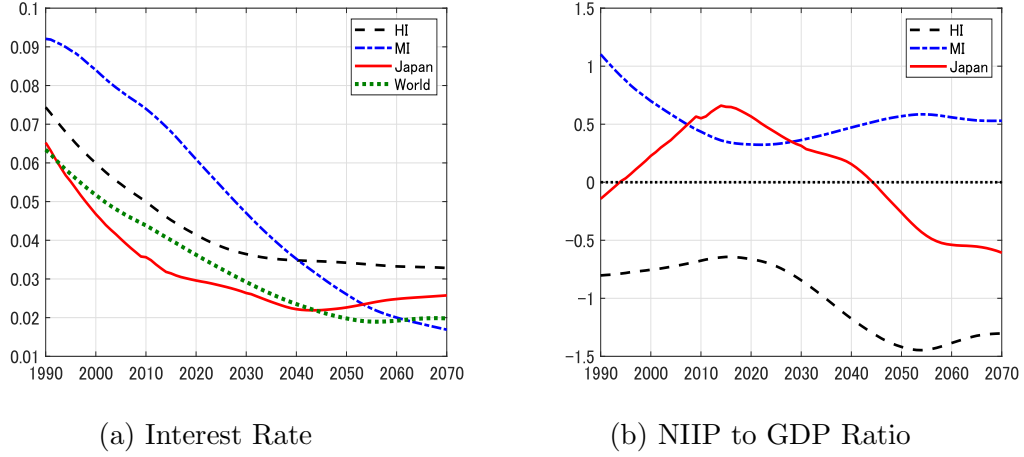(a) Interest Rate (b) NIIP to GDP Ratio

Figure 9: Interest Rate and NIIP to GDP Ratio

Figure 9b shows our main finding concerning the Net International Investment Positions (NIIP) of the three regions. In 2010, the MI region is a net lender to the HI region and Japan, which suggests that the higher TFP levels in the HI region and Japan necessitate a larger capital outflow from the MI region and a lower capital labor ratio. Thereafter, as the MI region starts to age faster, quickly catching up with the speed of aging in the HI region and Japan, the MI region's NIIP to GDP starts to rise despite the fact that the MI region is facing a higher TFP growth rate (until 2045), which has a tendency to reduce the NIIP to GDP ratio.

We would like to emphasize that our findings are different from the previous literature in that capital flows from the MI region to the HI region and Japan from the start and stay that way throughout the distant future.

Next we will examine the relative importance of the roles played by demographics and TFP in more detail.

7.2   The Role of Demographics in Capital Flows

In order to explore the role played by projected demographics in driving capital flows across regions, we present the numerical results from an alternative equilibrium transition path which assumes that individuals in the MI region experience much faster improvement in longevity than projected by the UN and eventually catch up with that in Japan in 2065. Note that this would place the individuals in the MI region on par in terms of conditional survival probabilities with those in Japan and essentially those in the HI region also.

Convergence of MI Survival Probabilities to HI and Japan Levels in 50 Years: To put this increase in longevity in perspective, recall Figure 1a. According to life expectancies at birth projected by the UN, our alternative calculations in this subsection

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assume an increase in longevity in the MI region from their mid 60s in 1990 to their late 80s by 2065 instead of the baseline projection of about 80, an increase of about 8 years.

Figure 10 shows the current account (as a ratio to GDP) in the HI and MI regions under the alternative assumption of enhanced longevity in the MI region and the baseline calculations. With faster aging in the MI region, the increase in the capital accumulation and capital outflow leads to a larger CA surplus in the MI region and a larger CA deficit in the HI region. The impact of higher survival probabilities on the current account can be as high as 2 percentage points for the HI.

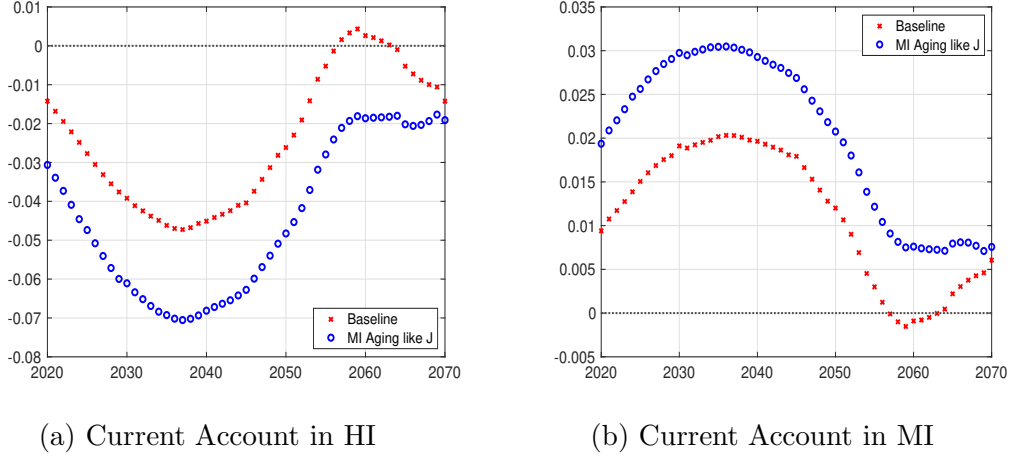(a) Current Account in HI

(b) Current Account in MI

Figure 10: Current Account to GDP Ratios in the HI and MI Regions

The stocks of external wealth show a similar result as Figure 11 indicates. As the MI region becomes a larger net lender, the HI region becomes a larger net borrower. Under the alternative assumption of MI quickly converging to the survival probabilities of the HI region and Japan, the increased NIIP of MI is 83% of GDP in 2050 compared to the 57% in the baseline case.[15]

It is useful to analyze Japan separately because our model validation suggested that our single parameter calibration of the transaction cost resulted in a good fit with the observed NIIP of Japan as we have cleaner data to compare with our model.

Figure 12a shows the paths of open economy interest rates in the baseline and under this alternative scenario together with the path of closed economy interest rates in Japan.

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[15]The increase in longevity that drives these results would require massive investments in health infrastructure, social insurance policy and large changes in human behaviors in order for the MI region to catch up with the HI region and Japan so quickly. Clearly this may not be very realistic but it still serves the purpose of showing the quantitative effect of significant demographic shifts on capital flows.
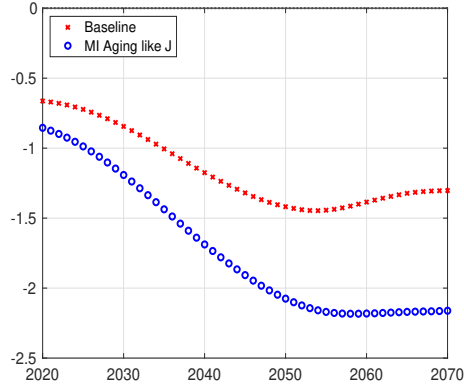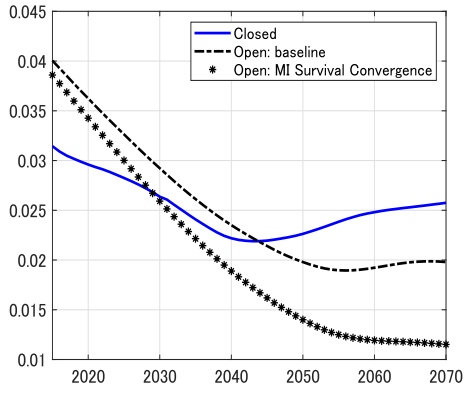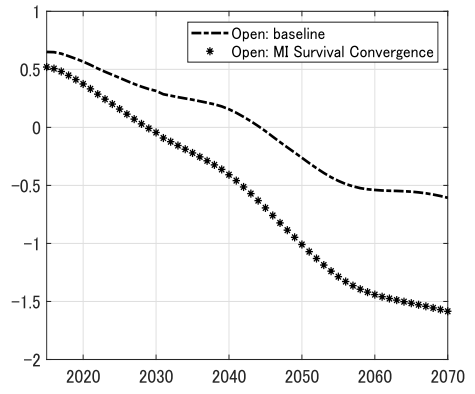
(a) NIIP in HI
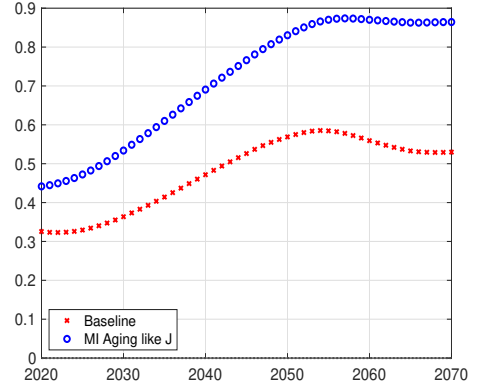
(b) NIIP in MI

Figure 11: NIIP to GDP Ratios in the HI and MI Regions

(a) Interest Rate in Japan

(b) NIIP to GDP Ratio in Japan

Figure 12: Convergence of Survival Rates of MI

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A rise in longevity leads to stronger saving motives for retirement and a larger aggregate capital will lower interest rate in the MI region. Given the large size of the MI region in the world economy, the open economy return to capital will be lower. Figure 12b shows that capital will flow to Japan much earlier and in larger quantities relative to the benchmark and Japan will be an even larger net borrower against the world. Both the timing and the magnitude are determined by the new development of demographic aging in other regions of the world. In particular, the speed with which the MI region catches up with the HI region and Japan in terms of longevity will expedite the flow of capital to both HI and Japan.

7.3 The Role of TFP in Capital Flows

The previous literature emphasized the role of differential aging across regions as a leading source of capital flows. But as Chen, İmrohoroğlu, and İmrohoroğlu (2006) and Ferrero (2010) argued in the context of external imbalances in the U.S. economy, TFP differences across economies are also important in determining the direction and magnitude of capital flows.

In a closed economy setting, the return to capital is determined by the capital labor ratio and the level of TFP. Over time, an aging economy typically faces an increase in the capital labor ratio which puts pressure on the return to fall making this economy less attractive to foreign investors if capital were allowed to cross borders. Therefore, once capital is mobile, it would tend to flow out to economies where returns would have been higher otherwise.

At the same time, differential growth rates of TFP also create differences in returns to capital independent from the aging process. If TFP grows faster in an economy, this puts pressure on the return to rise in a closed economy setting and when capital is allowed to move across borders this higher TFP would lead to a capital inflow.

Baseline vs Less/More Catching Up of TFP Levels: In the baseline transition path, we assume that the growth rate of TFP in the MI region converges to the long run value of 1% in 2045. As the MI region is assumed to grow faster than both HI region and Japan between 2015 and 2045, there is some catching up of the level of TFPs.

In two alternative computations, we assume that this convergence of growth rate to 1% occurs in 2035, which implies that the TFP of the MI region reaches a lower level compared to the baseline calculation, or, in 2055, which allows the MI to enjoy faster growth for another decade to get closer to the level of TFP, and therefore GDP per capita, in the HI region.

More catching up makes the current account deficit in the HI region smaller and less catching up makes the current account larger relative to the baseline case, as shown in Figure 13a. The intuition is that with a higher level of TFP in the MI region under the 'more catching up' assumption, the HI region and Japan are not as attractive as the baseline case regarding their closed economy interest rates (with a smaller gap in the

TFP levels) and therefore a smaller amount of capital is needed to flow out of the MI region and into the HI region and Japan to equalize the returns across regions in the world financial market. Conversely in the MI region, more catching up lowers the current account relative to the baseline case as Figure 13b shows.

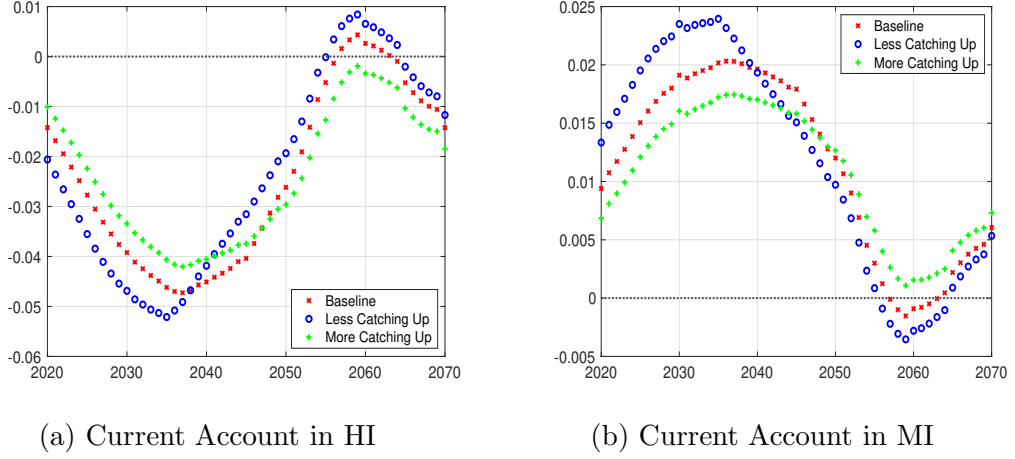

(a) Current Account in HI

(b) Current Account in MI

Figure 13: Current Account to GDP Ratios in the HI and MI Regions

Turning to the NIIPs, Figure 14 displays the accumulated effects of the annual capital flows depicted in Figure 13. More catching up makes the HI region less of a net borrower and the MI region less of a net lender.
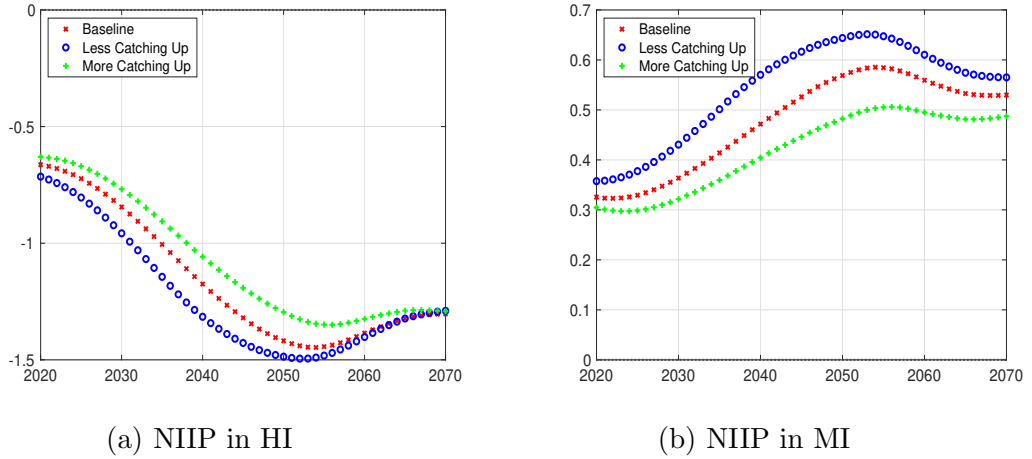

(a) NIIP in HI

(b) NIIP in MI

Figure 14: NIIP to GDP Ratios in the HI and MI Regions

Looking at Japan alone, the effects are similar to those in the HI region, as shown in Figure 15. With more catching up, Japan's current account position is higher by about half a percentage point and the year of turning into a net borrower is postponed to 2048. On the other hand, under less catching up, Japan becomes a net lender in 2040.

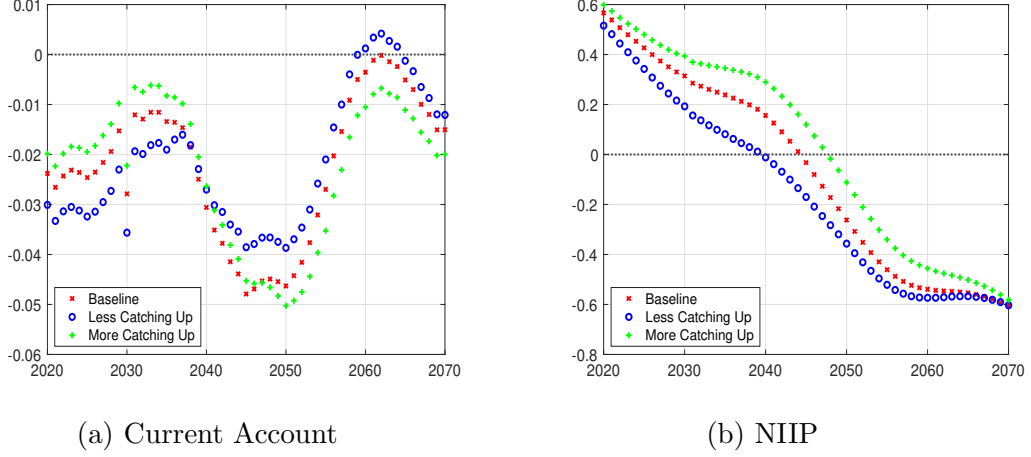(a) Current Account   (b) NIIP

Figure 15: Current Account and NIIP to GDP Ratios in Japan

Note that the quantitative difference in the current account positions (of all regions) between more or less catching up is not as large as that between the baseline case and the full convergence of survival probabilities case. The latter is admittedly an extreme assumption whereas the former is less so. The living standards as measured by GDP per capita relative to that in the HI region is 28% in the MI region in 2015 (our calibration). This rises to 38% in 2045 under the baseline assumption on the convergence of TFP growth rates. With more catching up, the convergence takes 10 years longer, allowing for another decade of faster growth in the MI region, raising their living standards to 42% compared to the less catching up, which raises GDP per capita in the MI region to only 34% of the HI region by 2045.

No TFP Level Convergence At All: We now compute the transition assuming that all three regions retain their relative TFP levels and therefore living standards as of 2015. Any changes in their NIIPs relative to those in the baseline case would come directly from differential aging which impacts the regional capital labor ratios differently and thereby leading to different capital movements across borders to equalize the returns in the world capital market.
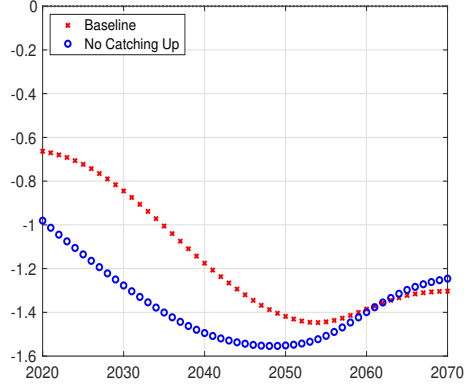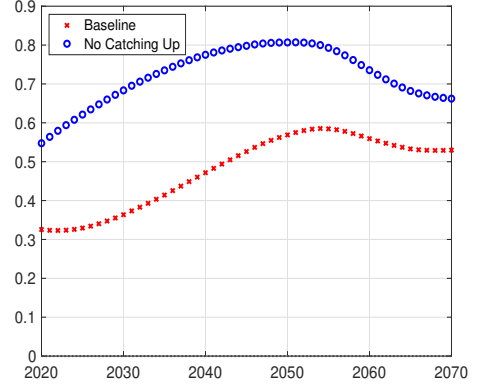
| (a) NIIP in HI | (b) NIIP in MI |

Figure 16: NIIP to GDP Ratios in the HI and MI Regions

According to Figure 16, the 'counterfactually' low TFP levels in the MI region would necessitate a massive capital outflow from MI to HI relative to the baseline transition. We mentioned before that comparing the NIIPs in the data to those in the model is problematic for the HI and MI regions since the data counterparts include capital flows into/from countries in the same region. For Japan, we are able to make a cleaner comparison and Figure 17 indicates that assumptions regarding alternative future paths of TFP make a significant difference in NIIP with respect to the rest of the world. In particular, Japan's NIIP turns negative much earlier relative to the baseline case as Japan's TFP remains much higher than that in the MI region due to 'no convergence' which necessitates larger capital inflows to Japan to equalize the returns across regions in the world financial market.
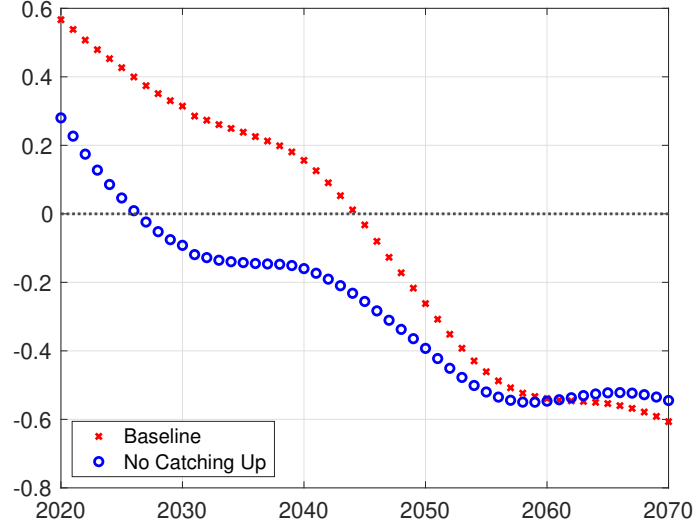
Figure 17: NIIP to GDP Ratio in Japan

The year of becoming a net borrower is 2027 with massive capital inflows from the MI region under this calculation compared to 2045 under the baseline assumptions. Again, this reflects the quantitatively powerful role played by TFP in shaping capital flows across borders.

7.4 Demographics vs TFP

It is not possible to conduct a truly counterfactual experiment to isolate the separate effects of demographics and TFP on future capital flows. The demographic projections come from the official sources and the TFP paths are determined by our assumptions on the extent and timing of catching up of living standards. Nevertheless, we present a few snapshots of some of the figures displayed in the previous subsections to get an idea about the relative strengths of differential aging versus TFP levels on future capital flows.

Table 6 shows the values of net foreign asset positions or NIIPs in 2030, 2040, and 2050 for the three regions: HI, MI and Japan. In addition to our baseline results, the table reports the 'MI High Longevity' case in which we assume that the conditional survival probabilities of individuals in the MI region increase very rapidly so that their longevity is as high as that in the HI region and Japan by 2045. As we mentioned before, this is against the projections by the United Nations and perhaps an extreme assumption. Table 6 also shows the NIIPs for the case of zero convergence of the MI living standards in which the typical MI individual remains at the 28% of the GDP per capita of the HI region individual forever. This is arguably also not realistic but these two rather extreme assumptions help us understand the relative strengths of demographics versus TFP in

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shaping international capital flows.[16]

Table 6: NIIP to GDP Ratios in the Three Regions

	2030	2040	2050
Baseline			
HI	−84%	−118%	−142%
MI	36%	47%	56%
Japan	31%	16%	−26%
MI High Longevity			
HI	−119%	−169%	−208%
MI	53%	69%	83%
Japan	−4%	−41%	−101%
MI No TFP Convergence			
HI	−128%	−150%	−155%
MI	68%	78%	81%
Japan	−9%	−16%	−39%

According to the quantitative findings in Table 6, alternative projections of aging and TFP in the MI region relative to the HI region and Japan produce somewhat similar changes in international capital flows. For the MI region, the range of NIIPs from 2030 to 2050 changes from 36%-56% in the baseline case to 53%-83% and 68%-81% for the 'aging fast' and 'stagnant per capita GDP' cases, respectively.

Although this is not quite an apples-to-apples comparison, our numerical results suggest that future TFP paths have as important quantitative effects on NIIPs as future demographic realizations.

7.5 Alternative Demographic and TFP Path Assumptions

In this subsection, we analyze the effects on capital flows from different aging and TFP paths in the future. Recall that the benchmark transition assumes the average rate of growth of TFP in Japan calibrated to the factual average rate of increase in output per person between 1990-2015 and the average growth rate of 20-year olds during the same 1990-2015 period is taken from the United Nations population data. In Table 7, we assume counterfactually that the TFP growth rate of Japan in the 1990-2015 period is only 0.9% (equal that of the HI region) and that the population growth rate of Japan during 1990-2015 is equal to that of the MI region.

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[16]Table 6 uses some of the numerical results in Figures 9, 11, 12, 16, and 17.

Table 7: NIIP to GDP Ratios in the Three Regions

	2010	2020	2030	2040	2050
Baseline					
HI	-68%	-66%	-84%	-118%	-142%
MI	44%	33%	36%	47%	57%
Japan	55%	57%	31%	16%	-26%
Growth of $Z_{1990-2015}^{J} = Z_{1990-2015}^{HI}$					
HI	-68%	-67%	-85%	-118%	-142%
MI	42%	31%	35%	46%	56%
Japan	77%	81%	55%	39%	-5%
$n_{1990-2015}^{J} = n_{1990-2015}^{MI}$					
HI	-66%	-63%	-81%	-113%	-139%
MI	46%	37%	42%	53%	62%
Japan	13%	-27%	-76%	-104%	-112%

The top panel in Table 7 shows the net asset positions under the baseline assumptions. The middle panel has Japan's output per person growing about 10% slower than that of the HI region. With slightly slower growth, the level of TFP in Japan would have been below that in the baseline case, leading to lower return to capital in the closed economy version, and therefore require higher capital outflows (to take advantage of relatively higher external returns) in an open economy equilibrium. Even though the differences in TFP growth rates seem quite small, there are still some sizable differences in the capital outflows. For example in 2040, the net asset position of Japan would be 39% compared to the baseline value of 16%.

The bottom three rows in Table 7 present the capital flows under the rather extreme counterfactual that the rate of growth of 20-year olds in Japan in 1990-2015 has been equal to that of the MI region. In this case, there would be a significant rise in the working age population and the labor input in Japan causing much higher closed economy returns to capital and requiring significant capital inflows in our open economy equilibrium transition path. According to this counterfactual, the high population growth in Japan in 1990-2015 would lead to counterfactually small/negative net asset position in 2010 and 2020 and a very large net borrower position soon after.

7.6 Sensitivity Analysis

7.6.1 Sensitivity to Transaction Costs

In the baseline simulation, the parameter ϕ that represents 'transaction costs of capital' is set to 0.41 and we assume that the cost will fall gradually to zero after 2020. The symmetric transaction cost is a reduced form device which approximates a cost that lenders in the international capital markets incur in investing in less developed financial markets

or an extra benefit that lenders in less developed markets derive from investing in 'safer' assets in high income economies. It is of course an open question how large such costs are or how they will evolve over time in the future. In this section we characterize equilibrium transitions under alternative scenarios about paths of transaction costs, assuming that they will fall but only by about one-half, to 0.20 in the long run, or they do not change and remain at 0.41.

In these new scenarios, households in the MI region will perceive investment in Japan (and HI) as providing the additional benefit longer and continue to lend additional amounts. Hence, as shown in Figure 18, the HI region will receive a greater amount of capital flow, thus experiencing larger current account deficits, and will have more negative net external wealth. The magnitude of the net external wealth differs but the direction of capital flows remains unchanged under alternative scenarios. Note that the effect on the magnitude of capital flows occurs in the distant future, starting just before 2050.
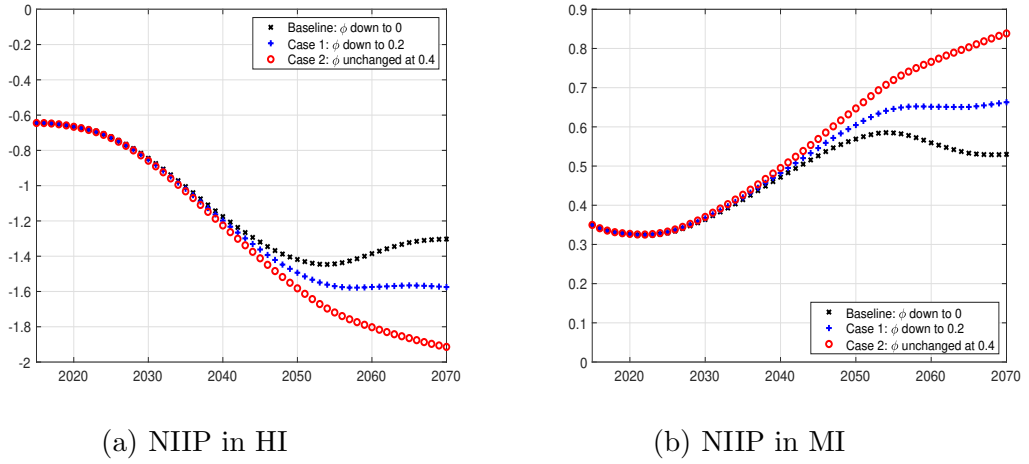

(a) NIIP in HI   (b) NIIP in MI

Figure 18: Alternative Capital Adjustment Costs and NIIP to GDP Ratios

The effect of alternative transaction costs on Japan's NIIP is similar to that on HI. There is hardly any effect over the next few decades and the year Japan becomes a net borrower, but after 2050 the non-declining appetite for Japan's domestic assets leads to a larger capital inflow. Figure 19 shows that the timing of the 'flip' from a net lender to a net borrower does not change but the magnitude of the negative NIIP does. If the friction in favor of the HI and Japan regions and against the MI region represented with the parameter ϕ does not disappear, as assumed in the baseline case, then more capital flows out of MI and into Japan (as well as HI).
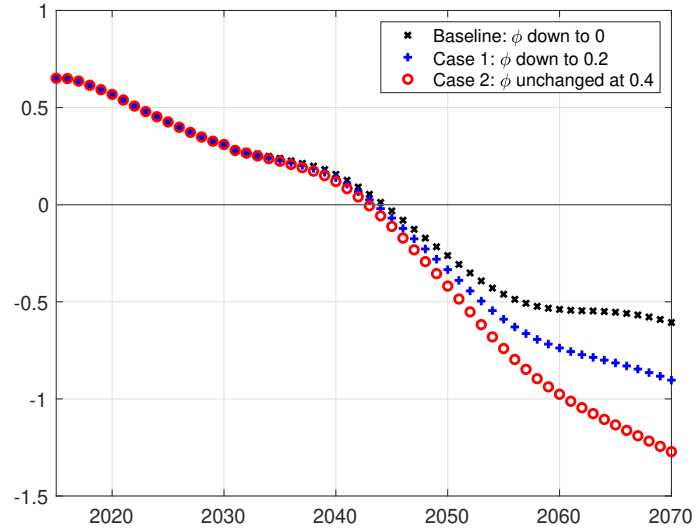
Figure 19: Alternative Adjustment Costs and NIIP to GDP Ratio in Japan

7.6.2 Sensitivity to CRRA

In this subsection, we recalibrate the model to achieve the same two targets 1) a 4% world real return to capital and 2) matching Japan's average NIIP to GDP ratio, by choosing the subjective discount factor and the transaction cost parameter, given a constant relative risk aversion coefficient of unity ($\theta = 1$). Under log preferences, we would expect to see the impact of aging on the real interest rate to be smaller as pointed out by Barany et al. (2019) and Auclert et al. (2021).[17]
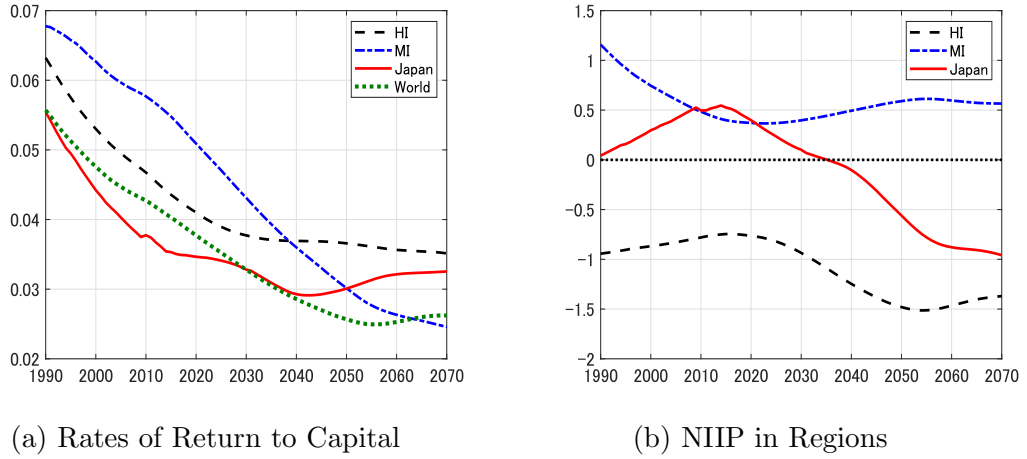

(a) Rates of Return to Capital

(b) NIIP in Regions

Figure 20: Log Preferences, Returns, and NIIP to GDP Ratios

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[17]We would like to thank an anonymous referee for pointing this out.

According to Figure 20, the closed economy return to capital in Japan falls from 5.5% to about 3.0% with $\theta = 1$, a decline much smaller than that under $\theta = 2$ where the return to capital falls from 6.5% to 2.2%. NIIPs move similarly as before. However, the timing of Japan's turning into a net borrower is now about a decade earlier. This is because MI's response in the interest rate is also milder, though not as much as that in Japan. Under $\theta = 1$, MI becomes a larger net lender and both HI and Japan become larger net borrowers and in Japan's case, the year of turning into a net borrower occurs about 10 years earlier (in 2035) than that under $\theta = 2$.

8 Conclusion

With improvements in longevity in all countries and declines in fertility in most regions, populations have been aging. The timing and the severity of these demographic trends are different across advanced and emerging economies. With existing differences in social security programs in place, the differential aging will impact saving and investment decisions and therefore capital flows in coming decades.

This paper develops a general equilibrium model populated with overlapping generations of individuals located in three regions that demonstrate large differences in their demographic trends and generosity of social security programs. Labor is immobile and there are segmented labor markets with regional wage rates. Capital, on the other hand, is allowed to move across borders with the return to capital determined in an imperfect world capital market subject to a symmetric transaction cost. Two of the three regions consist of a large number of High Income (HI) and Middle Income (MI) countries and the third region is Japan. This specification allows us to isolate a country that has experienced aging earliest among the advanced economies and where the aging is most severe. As a result, Japan serves as a laboratory case for our model's mechanisms in determining the future path of capital flows.

The demographic trends of countries in the HI and MI regions in the model are taken from United Nations' population projections whereas we rely on the Japanese government's projections for our third region. After calibrating the model to regional macroeconomic and fiscal indicators, we calculate equilibrium transition paths from the 1990s to a future balanced growth path. Our main quantitative finding is that the MI region is always a net lender to the rest of the world (RoW) while the HI region is a net borrower. The faster rise in the capital labor ratio in the HI region and Japan due to faster aging would lower the closed-economy returns to capital. On the other hand, higher TFP levels in the HI region and Japan would make these regions more attractive to investors. When capital is allowed to cross borders, albeit with a transaction cost, the MI region becomes a net lender. Japan starts as a net lender to the RoW but quickly starts to experience current account deficits, eventually becoming a net borrower by 2045. Even if the TFP level in Japan stays above that of the MI region, a rapid catch-up of demographic aging in the MI region leads to a reversal of capital flows. Our numerical results and additional cal-

culations suggest that differential demographic aging and TFP realizations across regions are important factors to determine future capital flows in the future.

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A   Computation of the Equilibrium

We describe the computation of the equilibrium in an open economy, where the labor income tax rate in each period is adjusted to achieve the government budget balance. All the other policy variables either remain fixed throughout the transition or move deterministically.

Step 1: Compute the initial and final steady-states of the model. Let the transition between the two steady states take T periods, long enough so the economy converges smoothly to the final steady state.

Step 2: Guess three T-dimensional vectors for the world interest rate, the lump-sum tax, and accidental bequests in each region. The first and last entry of these vectors are the initial and final stationary equilibrium values computed in Step 1.

Given the path for the interest rates, using the property of the constant returns to scale technology and the optimization conditions for the firm, sequences of wages in each region can be derived. the problem of the households can be solved in each region.

Step 3: Given prices and the sequence of tax rates and accidental bequests obtained in Step 2, solve households' problem. Recall the budget constraint of the household at time t:

$$\left(1 + \tau_{c,t}^r\right) c_{j,t}^r + a_{j+1,t+1}^j = y_{j,t}^r + \left[1 + \left(1 - \tau_{a,t}^r\right) ret_t^*\right]\left(a_{j,t}^r + b_t^r\right) + p_{j,t}^r - \tau_{ls,t}^r.$$

Denote net-of-tax gross interest rate as

$$R_t^r \equiv 1 + \left(1 - \tau_{a,t}^r\right) ret_t^*.$$

From the first order condition with respect to asset holdings next period, we obtain

$$\frac{c_{j+1,t+1}}{c_{j,t}} = \left[\beta s_{j+1,t+1}^r \frac{1 + \tau_{c,t}^r}{1 + \tau_{c,t+1}^r} R_{t+1}^r\right]^{\frac{1}{\theta}} \equiv g_{j+1,t+1}^c, \tag{15}$$

which is the optimal growth rate of consumption between age j and $j + 1$ and between time t and $t + 1$. Iterating backward over (15), we obtain that

$$c_{j+1,t+j} = c_{1,t} \prod_{k=1}^{j} g_{k+1,t+k}^c.$$

The discounted present value of the total (gross of taxes) lifetime consumption expenditures of the household of age 1 at time t is given as

$$\overline{c}_{1,t} = c_{1,t} \left[\left(1 + \tau_{c,t}^r\right) + \sum_{j=1}^{J-1}\left(1 + \tau_{c,t+j}^r\right) \prod_{k=1}^{j} \frac{s_{k+1,t+k}^r}{R_{t+k}^r} g_{k+1,t+k}^c\right]. \tag{16}$$

In general, the discounted present value of the total (gross of taxes) lifetime consumption expenditures of the household of age j^* at time t is

$$\overline{c}_{j^*,t} = c_{j^*,t} \left[\left(1 + \tau^r_{c,t}\right) + \sum_{i=j^*}^{J-1} \left(1 + \tau^r_{c,t+(i-j^*+1)}\right) \prod_{k=j^*}^{i} \frac{s^r_{k+1,t+(k-j^*+1)}}{R^r_{t+(k-j^*+1)}} g^c_{k+1,t+(k-j^*+1)} \right].$$
(17)

Define a variable $x^r_{j,t}$, as a sum of earnings, pensions and a bequest transfer net of taxes:

$$x^r_{j,t} = y^r_{j,t} + p^r_{j,t} + R^r_t b^r_t$$
(18)

The discounted present value of the total (net of taxes) lifetime earnings and bequest transfers of a household of age 1 at time t is:

$$\overline{x}_{1,t} = x^r_{1,t} + \sum_{j=1}^{J-1} \left(\prod_{k=1}^{j} \frac{s^r_{k+1,t+k}}{R^r_{t+k}} \right) x^r_{j+1,t+j},$$
(19)

where we are implicitly imposing the initial condition $a_1 = 0$. The discounted present value of the total (net of taxes) lifetime earnings of a household of age i^* at time t is:

$$\overline{x}_{j^*,t} = x^r_{j^*,t} + \sum_{j=j^*}^{J-1} \left(\prod_{k=j^*}^{j} \frac{s^r_{k+1,t+(k-j^*+1)}}{R^r_{t+(k-j^*+1)}} \right) x^r_{j+1,t+(j-j^*+1)} + R^r_t a_{j^*,t}.$$
(20)

Since individual optimization requires $\overline{c}_{j^*,t} = \overline{x}_{j^*,t}$ for each age j^* and time t, from (17) and (20), we obtain $c_{j^*,t}$ as

$$c_{j^*,t} = \frac{\overline{x}_{j^*,t}}{\left[\left(1 + \tau^r_{c,t}\right) + \sum_{i=j^*}^{J-1} \left(1 + \tau^r_{c,t+(i-j^*+1)}\right) \prod_{k=j^*}^{i} \frac{s^r_{k+1,t+(k-j^*+1)}}{R^r_{t+(k-j^*+1)}} g^c_{k+1,t+(k-j^*+1)} \right]}.$$

Note that $a_{j^*,t}$ in equation (20) is computed residually from $c_{j^*-1,t-1}$ and the budget constraint (3) :

$$a_{j^*,t} = x^r_{j^*-1,t-1} + R^r_{t-1} a_{j^*-1,t-1} - \left(1 + \tau^r_{c,t-1}\right) c_{j^*-1,t-1}.$$

Step 4: Aggregating asset holdings of all age groups and using equation (10) for each region, we obtain the implied sequence for external wealth of the region X^r_t. Together with the world capital market clearing condition (8), we arrive at a new guess for the sequence of the world interest rate. We use the government budget constraints (6) in each region with price sequences to update our guess for the tax rate. From the life-cycle asset decisions, we compute the accidental bequests left by the deceased in each region to update the guess for the bequests in the next iteration. If convergence is not reached, we restart from Step 3 with the new vector of guesses.

B   Summary of calibration targets

This section provides further details on the data used in the model calibration. Table B.1 reports a summary by region of the indicators used as targets in calibrating the model. Table B.2 presents a description of the macro data used and a short explanation of how calibration targets were computed; the sample period over which the statistics are calculated is also reported.

Table B.1: Summary of calibration targets

Parameter	Description	Target	Period	Value by region		
				Japan	HI[c]	MI[c]
Preferences and endowments						
β	Subj. discount factor	Interest rate, %	2015		4	
Production technology						
Z_0^r	TFP level (initial)	GDP per capita level, $ PPP	2015	40,763	45,373	12,696
λ_t^r	TFP growth rate	GDP per capita growth, %	1990-2015	1.1	1.4	3.9
Government						
D_t/Y_t	Debt to GDP ratio	General gov. net debt to GDP, %	1990-2015	100[a]	50.9	30.7
G_t/Y_t	Gov. purch. to GDP ratio	General gov. total expenditure to GDP, %	1990-2015	34.9	41.5	25.2
κ_t^r	Pension replacement rate	Net replacement rate, coverage adjusted	2014	38.5	47.8	26.8
τ_w^r	Labor income tax	Avg. eff. labor income tax, %	2000-2014	29.8	32.8	17.0
τ_a^r	Capital income tax	Avg. eff. capital income tax, %	2000-2014	34.7	34.1	18.8
τ_c^r	Consumption tax	Avg. eff. consumption tax, %	2000-2014	$3-8^b$	10.9	12.7

Notes: [a,b] See text for details on how targets were set; [c] High Income (HI): United States, Canada, Europe (UE 27, UK) Australia, New Zealand; Middle Income (MI): China, Hong-Kong, Taiwan, South Korea, Singapore, Thailand, Indonesia, Malaysia, Philippines, Vietnam, India, Saudi Arabia, United Arab Emirates, Turkey, Russia and South Africa.

Table B.2: Dataset description

Indicator	Period	Description	Sources
GDP per-capita (PPP, current international $)	2015	Gross domestic product divided by midyear population and converted to international dollars using purchasing power parity exchange rates. The target indicator by region is computed as a cross-country weighted mean, using GDP at current PPPs for weighting.	World Bank World Development Indicators (2017)
GDP per-capita growth (%)	1990-2015	The annual growth rate of GDP per-capita is calculated from GDP per-capita measured in constant local currency. The target indicator by region is a time average of the cross-country weighted mean. GDP at constant 2010 US$ is used for weighting.	World Bank World Development Indicators (2017)
General government total expenditure (% GDP)	1990-2015	Total spending of general government includes expenditures incurred by the central, state and local government, and social security funds. Total expenditure comprises current outlays, including interest payments on government debt and social transfers, and net investment in non-financial assets. The target total expenditure to GDP ratio by region is a time average of the cross-country weighted mean. GDP at constant 2010 US$ is used for within-region weighting.	IMF World Economic Outlook (2017)
General government net debt (% GDP)	1990-2015	Net debt of general government is given by gross debt minus financial assets corresponding to debt instruments (monetary gold and SDRs, currency and deposits, etc.). The target net debt to GDP ratio by region is a time average of the cross-country weighted mean. GDP at constant 2010 US$ is used for within-region weighting.	IMF World Economic Outlook (2017); Ministry of Finance for Japan
Labor income tax rate (%)	2000-2014	Average effective tax rate computed following the method by Mendoza, Razin and Tesar (1994). Tax revenue data and national account aggregates available from various sources. The target indicator by region is a time average of the cross-country weighted mean. GDP at constant 2010 US$ is used for within-region weighting.	OECD Revenue Statistics (2017), OECD National Accounts Statistics (2017), UN National Accounts Statistics (2017)
Capital income tax rate (%)	2000-2014	Average effective tax rate computed following the method by Mendoza, Razin and Tesar (1994). Tax revenue data and national account aggregates available from various sources. The target indicator by region is a time average of the cross-country weighted mean. GDP at constant 2010 US$ is used for within-region weighting.	OECD Revenue Statistics (2017), OECD National Accounts Statistics (2017), UN National Accounts Statistics (2017)

Indicator	Period	Description	Sources
Consumption tax rate (%)	2000-2014	Average effective tax rate computed following the method by Mendoza, Razin and Tesar (1994). Tax revenue data and national account aggregates available from various sources. The target indicator by region is a time average of the cross-country weighted mean. GDP at constant 2010 US$ is used for within-region weighting. For Japan, actual statutory tax rates are used, ranging from 3% in 1990 to 10% in 2019.	OECD Revenue Statistics (2017), OECD National Accounts Statistics (2017), UN National Accounts Statistics (2017); Ministry of Finance for Japan
Net replacement rate, coverage adjusted (%)	2014	The net replacement rate (NRR) is given by net pension entitlements divided by net pre-retirement lifetime earnings, thus accounting for individual income tax and social contributions (for a mean male earner). The adjustment for coverage is done by multiplying the NRR by active coverage (defined as the number of contributors to the social security system divided by labor force). GDP at current PPPs is used for within-region weighting.	OECD Pensions at a Glance (2014), World Bank Pensions Database (2014)
Pension retirement age	2013	It is the statutory retirement age (mean of females and males if they differ) at which people eligible to old-age pension start receiving benefits. Total population is used for within-region weighting.	World Bank Pensions Database (2014)
Total population by age groups	1990-2100	Historical data and projections (medium variant). Total population is used for within region weighting.	UN World Population Prospects (Rev. 2017); National Institute of Population and Social Security Research (IPSS) (2017) for Japan
Age-specific fertility rate	1990-2100	The age-specific fertility rate is the number of births to women in a particular age group divided by the number of women in that group. Historical data and projections (medium variant). Total population is used for within-region weighting.	UN World Population Prospects (Rev. 2017); National Institute of Population and Social Security Research (IPSS) (2017) for Japan
Current account (% GDP)	2000-2015	It shows the flows of goods, services, primary income, and secondary income between residents and non-residents.	World Bank World Development Indicators (2017); Ministry of Finance for Japan
Net External Wealth (% GDP)	1996-2019	It is the net international investment position (NIIP) of an economy, measuring at a point in time the difference between the value of financial assets of residents that are claims on non-residents (or are gold bullion held as reserve assets) and the liabilities of residents to non-residents.	IMF BoP and IIP Statistics (2020), World Bank World Development Indicators (2017); Bank of Japan
Capital-output ratio	1990-2017	Capital stock at current PPPs (in mil. 2011US$) divided by output-side real GDP at current PPPs; GDP at current PPPs is used for within region weighting.	Penn World Table (rel.9.1); World Bank World Development Indicators (2017)

C   Additional Computations

C.1   Fiscal Policy and Capital Flows

Fiscal Adjustments in the Three Regions: We now look at the magnitude of tax adjustments in these three regions to maintain fiscal sustainability in the future. In our baseline results, we assumed that the lump-sum tax is used to satisfy government budgets, holding the initial debt to output ratios and other tax rates constant. Figure 21 shows the time paths of lump-sum taxes that are required to satisfy the government budget constraint in these regions in the closed economy transitions.

Aging demographics will raise expenditures for old-age transfers and by more in economies that have more generous benefits. At the same time, a decline in the labor force would reduce tax revenues. As shown in the figure, Japan will experience a rapid increase in the fiscal burden during the next decades. The MI region has a lower replacement rate of social security and will experience a milder increase initially but tax will continue to rise as dependency ratios increase rapidly.[18]
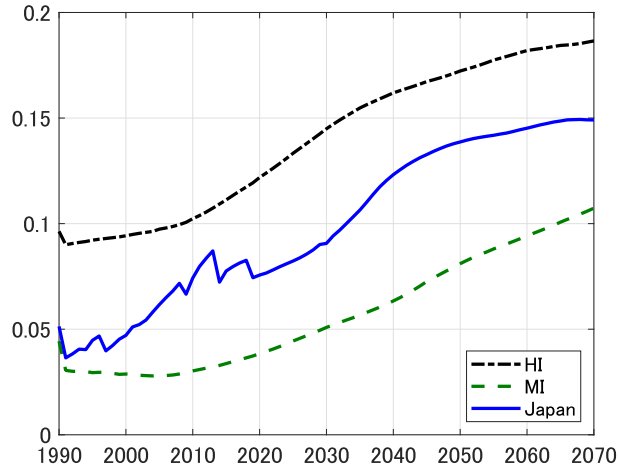

Figure 21: Total Lump-sum Taxes to GDP Ratio in Closed Economy
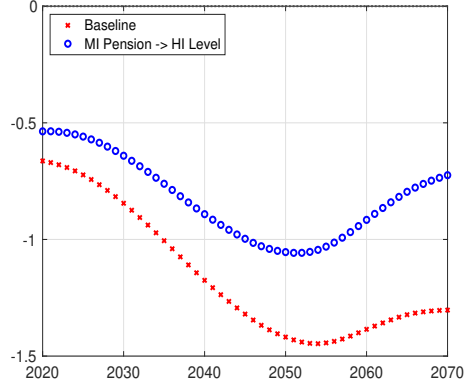
C.2   More Generous Pensions in the MI Region

The social security in the MI region is less comprehensive than in the HI region or Japan and the replacement rate of public pensions is lower. In this subsection we simulate a transition assuming that the MI region starts to provide more generous retirement benefits. More precisely, we let the replacement rate of the MI region, which stands at 26.8% in the baseline, increase to the level of HI, 47.8%, gradually over the next 20 years.

Under this alternative assumption, households in the MI region will have lower retirement saving motives and the resulting decline in their capital stock raises equilibrium open economy interest rates. Given higher interest rates within the MI region (in addition to the rest of the world), capital will flow less towards other regions.

As shown in Figure 22, NIIP in the HI region will be less negative and in the MI less positive. Net external wealth in Japan will turn negative by about ten years later than in the baseline model and the level of external wealth will be less negative thereafter as shown in Figure 23.

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[18]The figure for lump-sum taxes in the open economy is not included but the paths are very similar to those in the closed economy.

(a) NIIP in HI
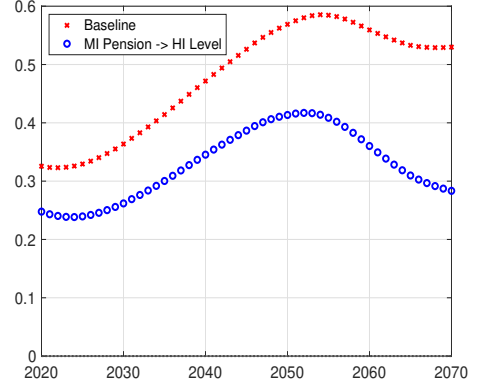
(b) NIIP in MI

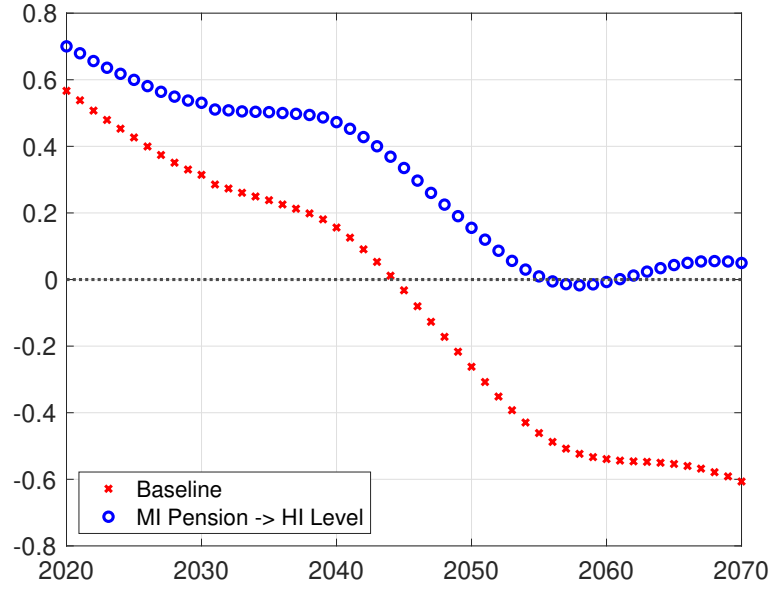Figure 22: NIIP to GDP Ratios in the HI and MI Regions

Figure 23: NIIP to GDP Ratio in Japan

Table C.3 reports a few snapshots of Figures 22 and 23. As pensions become more generous in the MI region, the negative impact on private saving leads to a reduction in capital accumulation. This reduction in the capital labor ratio in turn leads to a higher return to capital in the closed economy and requires a slowdown of capital flowing out of the MI region to equalize the regional returns in the world capital market. As a result, the net lender position of the MI region shrinks just as the net borrower position of the HI region. Japan enjoys a larger net lender position compared to the baseline case.

Table C.3: NIIPs to GDP Ratios in the Three Regions

	2030	2040	2050
Baseline			
HI	−84%	−118%	−142%
MI	36%	47%	56%
Japan	31%	16%	−26%
MI High Pensions			
HI	−64%	−89%	−105%
MI	26%	35%	41%
Japan	53%	47%	16%