

# Aging, Factor Prices, and Capital Flows <sup>\*</sup>

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## Abstract

Advanced economies are aging earlier and faster than the developing and emerging economies, with Japan leading the way. In a world with integrated capital markets, these differences in demographic trends, with different social security environments and productivity growth rates, will have differential implications on national or regional capital accumulation, investment, factor prices and capital flows across borders. This paper develops a general equilibrium model of the world economy under imperfect capital mobility, populated by overlapping generations of individuals in three regions: the High-income (HI), Middle-income (MI) regions and Japan. We compute equilibrium transitions from the 1990s toward a future balanced growth path and numerically characterize the first few decades. Our findings highlight the quantitative importance of the differential aging mechanism and TFP in studying capital flows across regions. In particular, we find that the higher TFP levels in the HI region and Japan lead the MI region to be a net lender throughout the 1990-2070 period with the HI region remaining a net borrower. Japan starts out as a net lender but turns into a net borrower in 2045. We conduct alternative computations that suggest the future TFP paths are about as important as the differential aging in determining future capital flows.

**Keywords:** Capital Flows, Demographic Trends, Factor Prices, Japan.

**JEL Classification:** F21, F41, J11, H55.

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# 1 Introduction

One of the major challenges facing all economies is the aging of the populations. In virtually all countries, fertility rates and death rates have fallen and this demographic trend has been accompanied by a secular decline in real interest rates and slow economic growth. However, the timing and the severity of these demographic trends are quite different across countries and regions of the world. Advanced economies such as United States, Japan, and those in the Eurozone have started aging much earlier and more significantly than emerging and developing countries.

These large demographic shocks and the accompanying policy adjustments have already started to affect capital accumulation, labor supply and economic growth in all economies and will continue to do so in the next few decades. To the extent that capital markets are integrated across regions, different demographic trends and different fiscal responses to them will have differential implications on capital accumulation, labor supplies, and capital movements across regions. When viewed through the lens of a general equilibrium model of the world, these capital movements will have effects on the time paths of factor prices. The impact of demographic shocks and fiscal responses is maximized in a closed economy, general equilibrium setting, with a direct link between the domestic capital labor ratio and factor rental rates. This mechanism is totally absent in a small, open economy, partial equilibrium model. Actual economies, however, are neither closed nor small, open, and depending on bilateral trade and capital flows, they may be closer to one extreme or the other.

This paper develops a general equilibrium model of the world in which overlapping generations of individuals populate three regions that are connected through capital mobility. Our idea is to view our world economy as consisting of three regions that have very different demographic trends. The ‘high income’ (HI) region is aging earlier and faster than the ‘middle income’ (MI) region, and Japan has been aging even earlier and faster than the HI region. By isolating Japan as a separate region, our framework aims to highlight the quantitative importance of the differential aging mechanism in shaping capital flows and fiscal sustainability.

Each region in our model consists of overlapping generations of individuals who face complete markets except for missing annuity markets. Labor is immobile but we assume that capital can move across regions in the open economy version of our model. In addition, we assume that labor is exogenous in order to focus on the role of demographics in determining capital flows. Governments in the three regions have exogenous government purchases and transfer payments which increase over time as the populations get older. We also incorporate differences in the fiscal institutions across regions, captured by different degrees of generosity in the social security systems as well as taxes that support the programs. We assume that government debt to output ratios are held constant and lump sum taxes are used to achieve fiscal sustainability.

We calibrate the model under the assumption of an open economy using data from the World Bank, IMF, United Nations, and in the case of Japan, the Institute for Population

Studies. We first compute equilibrium transitions in the three regions assuming that the economy is open and characterize the paths of equilibrium quantities such as output, capital, wages and factor prices. Second, under the same institutional settings we compute an equilibrium transition of the closed-economy version of the model, shutting down capital flows across regions. We find that the return to capital in the world capital market falls from 4% in 2015 to about 2% by 2070. The capital-labor ratio rises significantly in the HI region and Japan until the 2040s but starts to stabilize thereafter. The increases, however, continue in the MI region well into the second half of the 21st century. This difference, and the higher TFP in Japan relative to the MI region, drive the reversal of the net foreign asset position of Japan. In particular, Japan starts running current account deficits, as its net foreign asset position worsens, and eventually becomes a net foreign borrower. The MI region is a net lender and the HI region a net borrower throughout our sample and projection periods 1990-2070.

We calculate additional equilibrium transition paths under alternative assumptions on the projected demographics and on the future paths of regional total factor productivities (TFPs). Our numerical results suggest that future TFP paths have a large impact on capital flows across borders, similar to that from demographics.

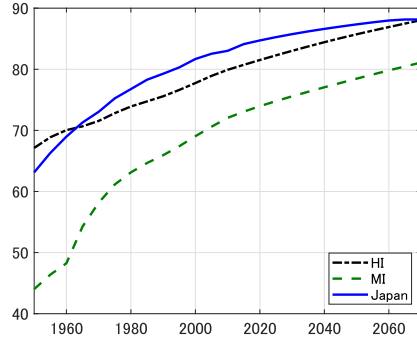
The paper is organized as follows. Section 2 demonstrates the differential timing and extent of demographic aging in our three regions. Section 3 describes the related literature and summarizes the contributions of the paper. Section 4 describes the model and the economic environment. Section 5 summarizes our calibration with the details described in Appendix B. Section 7 discusses our main quantitative findings and alternative computations. Section 8 provides concluding remarks.

## 2 Differential Aging Across Regions

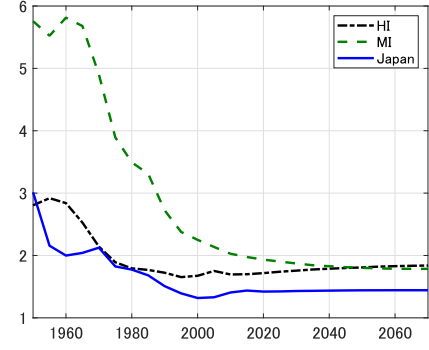
In order to see the differential demographic trends in the world, Figure 1 shows various aspects of the aging process and its implications in three regions: 1) HI region; 2) MI region; and 3) Japan.<sup>1</sup>

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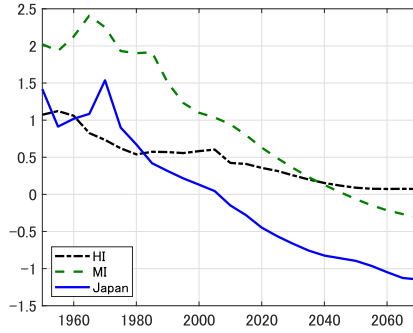
<sup>1</sup>The main source of demographic data is United Nation's *World Population Prospects: The 2017 Edition Revision* (2017), which provides harmonized data on population, fertility and life table projections for all countries from 1950 to 2100. The division of the first two regions is mainly based on the stages of overall economic growth and timing of demographic transition. The choice of countries included in these regions is based on the size of the economy, as well as the degree of investment and financial exposure to Japan. More details are provided in section 5.



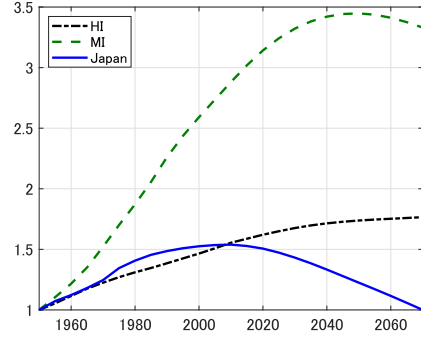
(a) Life Expectancy



(b) Total Fertility Rate



(c) Population Growth Rates



(d) Normalized Populations

Figure 1: Demographic Trends in the Three Regions

Figure 1a shows life expectancy at birth over time in the three regions. Longevity is rising in all regions but significant differences are projected to persist well into the second half of the 21st century. Furthermore, Japan has and will continue to have higher longevity relative to the highly advanced economies during the next several decades. Figure 1b plots total fertility rates which have all come down from their earlier levels in 1960. In particular, fertility rates in HI and MI regions are projected to approximately converge (just below 2) whereas that in Japan is predicted to be significantly lower at around 1.5 for several more decades.

Given the significant fall in fertility rates, it is not surprising that population growth rates have fallen significantly since 1960 as Figure 1c shows. What stands out in this frame is that Japan's population growth has already turned negative in the late 2000s and the decline in the Japanese population is projected to accelerate over the next decades. According to Figure 1d, the Japanese population in 2070 will be about the same size as that in 1950. The populations in HI and MI regions will continue to rise, at least until the 2040s.

Combined, the fall in fertility and the rise in longevity imply significant increases in the old-age dependency ratio, defined as the ratio of individuals 65 or older to those between the ages of 20 and 64, shown in Figure 2. The dependency ratios show a gradual increase

in all regions since 1950. In HI and MI regions, the main part of the increase is very recent and the projections put these ratios at about 55% and 45%, respectively, by 2070. In Japan, however, the sharp rise in the dependency ratio has started in the 1990s and it is projected to increase to 80% by 2050. In other words, the rise in the old-age dependency ratio in Japan has started much earlier and is predicted to reach unprecedented levels in a few decades.

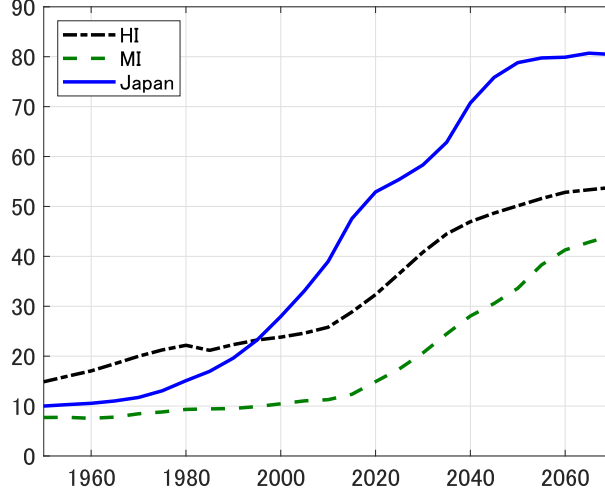


Figure 2: Dependency Ratios

### 3 Related Literature

Our paper contributes to the research that studies international capital flows using quantitative growth models. Börsch-Supan, Ludwig, and Winter (2006) use a 7-region of the world economy with overlapping generations, consisting of the major European economies, the U.S. and others, and argue that capital flows out of fast aging countries like France, Germany and Italy until about 2050 and then the direction is reversed.

Chen, İmrohoroğlu, and İmrohoroğlu (2009) develop a two-country open economy, representative agent model and study the effects of productivity, tax, and population growth rate differences on capital flows between the U.S. and the rest of the world (RoW). They argue that the factor that seems to have driven the capital to flow into the U.S. in the 1960-2004 period is faster productivity growth in the U.S. relative to the RoW, with differences in time-varying tax rates and depreciation rates playing a smaller role. Ferrero (2010) uses a Blanchard-type model with age-independent survival (and retirement) probability and also finds a significant role for TFP differences in explaining the U.S. external imbalances, with the remaining portions explained by differences in life expectancies and fiscal policies.<sup>2</sup>

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<sup>2</sup>Backus, Cooley, and Henriksen (2014) develop a 4-country overlapping generations model with Germany, Japan, the U.S. and China. In their model, the world interest rate is determined in equilibrium

Lisack, Sajedi, and Thwaites (2017) build an overlapping generations model with the OECD countries taken together forming a closed economy. The aging in their model implies a small reduction in the real interest rate over time. Their small open economy extension shows large capital inflows to the U.S. (much larger than those observed in the recent decades) and their simulated net foreign asset to output ratios explain about 28% of those in the data.

Krueger and Ludwig (2007) build a 3-country model of the world economy, with the U.S., EU and the rest of the OECD. In their calibrated overlapping generations model with incomplete markets, the initial TFP levels are different (reflecting the differences in incomes per person) but the growth rates are identical at 1.8%. They project that U.S. will run a small current account surplus between 2000 and 2015 which will then turn into a current account deficit until about 2040 and again into a surplus after that, essentially the mirror image of the CA movements of the rest of the OECD. These papers mostly look at OECD or advanced economies. Our paper looks at both developed or high income and developing or middle income economies.

Our paper is related to the recent ‘secular stagnation’ papers such as Ikeda and Saito (2014), Gagnon, Johnson, and Lopez-Salido (2016), Carvalho, Ferrero, and Nechio (2016) and Eggertsson, Mehrotra, and Robbins (2017). Most of this research restricts attention to closed economy models and focus on understanding the mechanisms, including demographics, that contribute to the low real interest rates and economic growth in advanced economies.<sup>3</sup>

Also, this paper extends the literature that studies fiscal sustainability issues and macroeconomic effects of aging population in Japan. Hansen and İmrohoroglu (2016) use a neoclassical growth model to quantify costs of aging demographics, showing a very large fiscal adjustment needed to stabilize the government debt. Braun and Joines (2015) and Kitao (2015) build a life-cycle model with details of the social insurance system in Japan and study effects of alternative policies including reforms of social security and health insurance programs. Kitao and Mikoshiba (2020) study effects of changes in the labor market on fiscal projections, focusing on participation of females and the elderly in Japan. All of these models assume a closed economy and factor prices are determined solely by domestic factors. Our model allows for capital mobility into and out of Japan and quantifies how unsynchronized demographic aging in different parts of the world and different fiscal institutions could affect future paths of the Japanese economy.

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with Germany, Japan and the U.S. participating in the single world financial market. Their quantitative predictions appear consistent with the experiences of the U.S. and Germany but not with those of Japan and China.

<sup>3</sup>For example, Eggertsson, Mehrotra, and Robbins (2017) build a three-period New Keynesian model to highlight the various mechanisms of their model (shocks to collateral constraints, demographics, zero lower bound, debt to output ratio, relative price of investment goods, labor share, nominal rigidities, productivity growth, hysteresis) to create secular stagnation and negative natural interest rates for the United States between 1970 and 2015, with projections to 2030. For Japan, their numerical example using a 3-period OLG model under a closed economy setup delivers persistent low growth, low inflation and low real interest rates in the 1970-2013 period.

Our paper is more closely related to Domeij and Floden (2006), and Attanasio, Kitao, and Violante (2006, 2007) that analyze the effects of demographic changes in developed and emerging economies. This research exploits the differential timing and nature of aging and different ways of dealing with pension programs in large, open economy models. Domeij and Floden (2006) develop a 19-country model of the world economy, 18 of the OECD economies and another economy representing the rest of the world (RoW), populated with overlapping generations. Their model's simulated international capital flows correlate with those in the data and explain a small but significant part of the flows. The results are driven by differences in the populations' age structure. The current account deficit of the rest of the world gets smaller and eventually reaches zero by 2050.

The mechanism in these papers is twofold. When an economy ages faster than others, then the rise in capital labor ratio will be larger than others and as a result the fall in the return to capital will be bigger. If capital is allowed to flow between economies, the fall in the return to capital in an aging economy will be accompanied by an outflow of capital seeking higher returns elsewhere. This is the simple mechanism that determines capital flows in this quantitative literature. A second effect goes in the other direction. As the number of savers gets smaller in an aging economy, the aggregate capital stock would fall as the fraction of retirees who do not save (much) increases. This composition effect appears quantitatively small relative to the first effect.

Attanasio, Kitao, and Violante (2006) develop a two-region model of the world economy with 'North' representing the more developed regions, according to the United Nations classification, and the 'South' standing for less developed regions. They calibrate their model so that the income per person in the North is 7 times that of the South initially, with the TFP of the South growing from 1.50% in the 1950-2000 period to 1.78% after 2000. There are also differences in the social security and fiscal policies. Their experiments are designed to quantify the roles of differences in survival probabilities, fertility rates and female labor supply participation rates in explaining capital flows. The general finding is that capital flows from the 'North' to the 'South' as aging occurs earlier and more severely in the North relative to that in the South. In particular, the North starts with a negative net foreign asset position in 1950, which initially becomes more negative until 2000 but from then on it rises over time and by 2040 it reaches a value of zero.

In a related paper, Attanasio, Kitao, and Violante (2007) use a similar framework to study whether the quantitative implications of various social security reforms for policy variables, factor prices, macroeconomic aggregates, and welfare of different cohorts in the North are sensitive to using a closed vs. open economy model. Their numerical findings suggest that either framework gives a similar answer to questions of fiscal sustainability but that factor prices and equilibrium quantities (and welfare) depend very much on the particular model used. Their model's prediction on capital flows is similar to that in their earlier paper. In the sensitivity analysis, they explore quantitative effects of alternative catching up scenarios of the South to the North in terms of the speed and level that it converges to, but the effect on capital flows is very small.

Our paper is different from this previous literature in several ways. First, we classify

economies as HI vs MI in a different way than the North-South classification and as a result our initial steady state and the first years, 1990-2015, of the transition to the eventual steady state produce the MI region starting from a positive Net International Investment Position (NIIP) and the HI region being a net borrower. Second, we emphasize not only the roles played by differences in the timing and extent demographics and differences in fiscal policies (taxes, pensions, and indebtedness), but we also examine the role of TFP in attracting foreign capital. The latter is an essential channel in the literature that tries to explain the Lucas Paradox. Finally, our paper uses Japan as a separate ‘region’ and a laboratory to highlight the mechanisms at play because they are aging the fastest and earliest and we can more easily measure their NIIP with respect to the rest of the world.

Finally, this paper adds to the literature on the Lucas Paradox by highlighting the quantitative importance of the key mechanisms that determine capital flows. In a seminal paper, Lucas (1990) uses the standard Cobb-Douglas production function in the workhorse macro models to illustrate a paradox. These models predict that capital ought to flow to poor economies with higher rates of return such as developing economies with much lower capital labor ratios and total factor productivities. Capital flows, however, in the data seem to go in the other direction. Lucas (1990) suggests measuring and incorporating human capital in these calculations and also mentions the possibility of political risk as potential factors that may reconcile theory and data.<sup>4</sup>

Using a large international panel data set, Alfaro, Kalemli-Özcan, and Volosovych (2008) conduct an empirical analysis to study what factors seem to explain capital flows across borders. They consider measures of fundamentals that raise productivity and an index of institutional quality in their panel regressions of fifty-eight countries during 1970-2000. They find evidence that institutional quality explains the variation in net capital flows.

In order to check whether a typical neoclassical model may be consistent with data from the fifty U.S. states, Kalemli-Özcan, Reshef, Sørensen, and Yosha (2010) develop a neoclassical model and use data from the states in the U.S. Their model predicts that capital flows to fast-growing states from slow-growing states, and therefore, high-growth states pay capital income to other states, and eventually become net debtors with persistent productivity shocks. Using data, they find evidence that suggests that the net capital flows and patterns of ownership seem consistent with their frictionless neoclassical model. They argue that the limited size and the ‘wrong’ direction of international capital flows may be due to frictions associated with national borders.

Alfaro, Kalemli-Özcan, and Volosovych (2014) construct measures of net private and public capital flows for a large cross-section of developing economies and using these constructs they find that net international private capital flows are positively correlated with productivity growth, consistent with neoclassical macro models.

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<sup>4</sup>Caselli and Feyrer (2007) argue that a more comprehensive production function incorporating land and other resources may matter. In addition, they suggest that with the use of a multi-sector model in which the price of capital is higher in poor countries, and even with higher physical capital in rich countries, the return to capital may be close in these countries.



Our model’s mechanism for capital flows consists of fundamentals such as regional capital labor ratios and total factor productivities; the former is affected by demographics. In addition, we introduce a ‘transaction cost’ in investing abroad in our calibration which is intended to represent frictions across regional borders and this allows us to match the net foreign asset position of Japan over 1990-2015. Our simulation results indicate that capital flows beyond 2015 are primarily driven by fundamentals. In section 7, we show that our quantitative model suggests that the timing and direction of capital flows are entirely driven by fundamentals such as TFP and demographics but the magnitude does depend on the size of the frictions across regional borders.<sup>5</sup>

## 4 The Model

### 4.1 Economic Environment

**Preliminaries:** Our model consists of Japan ( $J$ ) and two regions: High-income ( $HI$ ) and Middle-income ( $MI$ ) regions. The two regions and Japan differ in their realized and projected demographic trends, total factor productivity levels and growth rates, and fiscal institutions. Our approach is to calculate a perfect foresight equilibrium transition path for the world economy from 2015 to a distant future steady state and characterize the transition path under the baseline and alternative economic settings. Let  $t$  denote time,  $j$  a household’s age, and  $r$  the regions, with  $r = HI, MI, J$ .

**Technology:** In each region  $r$ , a constant returns to scale, aggregate production function  $F(Z_t^r, K_t^r, N_t^r)$  produces output of a final good  $Y_t^r$  which can be used for consumption of households  $C_t^r$  and the government  $G_t^r$  or investment  $I_t^r$ . Among the arguments of the production function,  $Z_t^r$  denotes the total factor productivity level in region  $r$  at time  $t$ ,  $N_t^r$  is the aggregate labor supply in efficiency units, and  $K_t^r$  is the aggregate stock of physical capital used in production. Physical capital depreciates at the same rate  $\delta$  each period in all regions. The level of technology in region  $r$  grows exogenously at rate  $\lambda_t^r$  between  $t$  and  $t + 1$ . The growth rates differ across regions during the transition, but in the long-run all regions grow at the same constant rate  $\lambda$ .

**Demographics:** Each region is populated by overlapping generations of ex-ante identical households who become active economically at age  $j = 1$  and may live for a maximum of  $J$  periods. For households born in region  $r$ ,  $s_{j,t}^r$  denotes the probability of survival until age  $j$  at time  $t$ , conditional on being alive at time  $t - 1$ . Hence, in region  $r$ , the

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<sup>5</sup>Our paper follows the tradition of Auerbach and Kotlikoff (1987) by using a quantitative general equilibrium model populated by overlapping generations, and incorporates life cycle institutions and demographic projections in detail to tease out the implications of aging on capital flows across borders. Earlier papers that studied the effects of aging on the economy focused on the effects arising from the social security budget in closed economy models. See, for example, Huang, İmrohoroglu, and Sargent (1997), De Nardi, İmrohoroglu, and Sargent (1999).

unconditional probability of surviving  $j$  periods up to time  $t$  is given by

$$S_{j,t}^r = \prod_{k=1}^j s_{k,t+(k-j)}^r,$$

where  $S_{1,t}^r = s_{1,t}^r \equiv 1$  for all  $t$  by definition. We denote by  $\mu_{j,t}^r$  the size of the population of age  $j$  at time  $t$  in region  $r$ .

**Household Preferences:** Households of age  $j$  at time  $t$  in region  $r$  make consumption allocation decisions based on the instantaneous utility function

$$u(c_{j,t}) = \frac{(c_{j,t}^r)^{1-\theta}}{1-\theta}, \quad (1)$$

where  $c_{j,t}^r$  denotes consumption of a household of age  $j$  at time  $t$ . The intertemporal preference ordering for households born at adult-age  $j = 1$  at time  $t$  is given by

$$U^r = \sum_{j=1}^J \beta^{j-1} S_{j,t+j-1}^r \frac{(c_{j,t+j-1}^r)^{1-\theta}}{1-\theta}, \quad (2)$$

where  $\beta$  is the subjective discount factor. There is no explicit bequest motive driven by altruism. Accidental bequests left by the deceased are distributed as a lump-sum transfer, denoted as  $b_t^r$ .

**Household Endowments:** Households exogenously supply labor and derive no utility from leisure.<sup>6</sup> At age  $J_R^r$ , households are subject to compulsory retirement from market work. Households of age  $j$  at time  $t$  in region  $r$  are endowed with  $\varepsilon_{j,t}^r$  efficiency units of labor for each unit of time worked in the market. Finally, we assume that the initial asset holdings of each household is zero, i.e.  $a_{1,t}^r = 0$  for any  $t$  in all regions.

**Household Budget Constraint:** Let  $a_{j,t}^r$  be the net asset holding of a household of age  $j$  at time  $t$  in region  $r$ .

$$(1 + \tau_{c,t}^r) c_{j,t}^r + a_{j+1,t+1}^r = y_{j,t}^r + [1 + (1 - \tau_{a,t}^r) r_t^r] (a_{j,t}^r + b_t^r) + p_{j,t}^r - \tau_{ls,t}^r. \quad (3)$$

We require households to die with non-negative wealth once they reach age  $J$ , but otherwise impose no borrowing constraint during their life. Net earnings  $y_{j,t}^r$  accruing to households of age  $j$  in region  $r$  at time  $t$  are defined as

$$y_{j,t}^r = \begin{cases} (1 - \tau_{w,t}^r) w_t^r \varepsilon_{j,t}^r = (1 - \tau_{w,t}^r) \tilde{y}_{j,t}^r & \text{if } j < J_R^r, \\ 0 & \text{if } j \geq J_R^r, \end{cases} \quad (4)$$

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<sup>6</sup>We abstract from a labor-leisure choice in order to focus on the effects of saving and investment responses and capital flows across borders coming from exogenous demographic changes and saving/investment behavior.

where  $w_t^r$  is the market wage rate,  $\varepsilon_{j,t}^r$  is the efficiency units of labor of a household of age  $j$ , and  $\tilde{y}_{j,t}^r$  is the before-tax labor income.  $p_{j,t}^r$  is pension income and takes a positive value for eligible individuals at and above social security's retirement age  $j \geq j_{SS}^r$  and zero otherwise.  $r_t^r$  denotes the interest rate households in region  $r$  earn on their saving.

Households pay proportional taxes at the rate of  $\tau_{c,t}^r$  on consumption,  $\tau_{a,t}^r$  on capital income, and  $\tau_{w,t}^r$  on labor income and a lump-sum tax  $\tau_{ls,t}^r$ . Residents of region  $r$  pay capital income taxes in region  $r$ , independently of where capital was invested. Social security benefits are given by the formula

$$p_{j,t}^r = \kappa_t^r \frac{W_{j,t}^r}{J_{SS}^r - 1},$$

where  $\kappa_t^r$  is the replacement ratio of average past earnings. Cumulated past gross earnings  $W_{j,t}^r$  are defined recursively as

$$W_{j,t}^r = \begin{cases} \tilde{y}_{1,t}^r & \text{if } j = 1 \\ \tilde{y}_{j,t}^r + W_{j-1,t-1}^r & \text{if } 1 < j < J_{SS}^r \\ W_{j-1,t-1}^r & \text{if } j \geq J_{SS}^r. \end{cases} \quad (5)$$

**Government Budget Constraint:** In each region  $r$ , public expenditures and social security program are administered by the government under a consolidated intertemporal budget constraint. The government can raise revenues by taxes on consumption, labor income, interest and capital income, and lump-sum taxes and can issue one-period risk-free debt,  $B_{t+1}^r$ . Government borrowing and tax revenues finance a stream of expenditures  $G_t^r$  and the PAYG social-security program described above. The consolidated government budget constraint reads as

$$G_t^r + (1 + r_t^r) B_t^r + \sum_{j=J_{SS}^r}^J p_{j,t}^r \mu_{j,t}^r = \tau_{w,t}^r w_t^r \sum_{j=1}^{J_R-1} \mu_{j,t}^r \varepsilon_{j,t}^r + \sum_{j=1}^J \mu_{j,t}^r \tau_{a,t}^r r_t^r (a_{j,t}^r + b_t^r) + \sum_{j=1}^J \mu_{j,t}^r \tau_{c,t}^r c_{j,t}^r + \sum_{j=1}^J \mu_{j,t}^r \tau_{ls,t}^r + B_{t+1}^r. \quad (6)$$

**Market Structure:** There are three goods in the world economy: a final good which can be used either for consumption or investment, the services of labor and the services of capital. The price of the final good (homogeneous across the three regions) is used as the world numeraire. Labor is immobile, thus wages are determined independently in regional labor markets.

In the open economy, we assume that physical capital is mobile across the three regions, so there is one world market for capital where the world return to capital is determined. Let  $X_t^r$  denote the external wealth of region  $r$  at time  $t$ , that is, the stock of capital used in production in other regions and owned by households of region  $r$ . A negative value indicates ownership of capital used for production in the region but owned by households

of other regions. The sum of positive and negative external wealth across regions is zero in equilibrium, satisfying the condition  $\sum_r X_t^r = 0$  at any time  $t$ . The markets where these goods and assets are traded are perfectly competitive. An intuitive no-arbitrage condition between assets and the absence of aggregate uncertainty imply that the return on the three regional bonds is equal to the return on physical capital, as we have already implicitly assumed when we wrote the budget constraints of the government and households.

In equilibrium, firms in each region maximize profits

$$\begin{aligned} w_t^r &= F_N(Z_t^r, K_t^r, N_t^r), \text{ for all } r \\ r_t^r &= F_K(Z_t^r, K_t^r, N_t^r) - \delta. \end{aligned}$$

If capital were perfectly mobile across regions, we would have the world interest rate  $r^*$  equal the interest rates of all regions, that is,  $r_t^* = r_t^{HI} = r_t^{MI} = r_t^J$ . In our baseline economy, however, we assume that there is an exogenous transaction cost that reduces the return that foreign ( $HI, J$ ) savers earn on capital in  $MI$ . We include this transaction cost as a simple reduced form to capture region-specific expropriation risk typical of less developed financial markets. In addition, when  $MI$  households lend abroad, the differential captures the extra benefit for savers in  $MI$  to invest in safe assets abroad:  $r_t^* = r_t^{HI} = r_t^J$  and  $r_t^{MI} = r_t^*/(1 - \phi)$ , where  $\phi \in (0, 1)$  captures the transaction cost. We will calibrate this parameter to match the average net external wealth of Japan in the 1996-2015 period.

## 4.2 Equilibrium

Before stating the definition of equilibrium, it is useful to point out that, without further restrictions, the equilibrium path of the fiscal variables  $\{G_t^r, \kappa_t^r, \tau_{w,t}^r, \tau_{a,t}^r, \tau_{c,t}^r, \tau_{ls,t}^r, B_t^r\}_{t=1}^\infty$  is indeterminate, as there is only one budget constraint we can operate on. In what follows, we define an equilibrium for the case where the paths of all fiscal variables are given, except for  $\{\tau_{ls,t}^r\}_{t=1}^\infty$ . It is straightforward to extend this definition to the case where the path of a different set of government policies is given exogenously. Finally, for brevity we omit the definition of the closed-economy equilibrium and state directly the equilibrium conditions for the open economy.

A *Competitive Equilibrium of the Multi-Region Economy*, for a given sequence of region-specific demographics, TFP levels  $\{Z_t^r\}_{t=1}^\infty$ , and fiscal variables  $\{G_t^r, \kappa_t^r, \tau_{a,t}^r, \tau_{c,t}^r, \tau_{w,t}^r, B_t^r\}_{t=1}^\infty$ , is a sequence of: (i) households' choices  $\left\{ \{c_{j,t}^r, a_{j,t}^r\}_{j=1}^J \right\}_{t=1}^\infty$ , (ii) lump-sum taxes  $\{\tau_{ls,t}^r\}_{t=1}^\infty$ , (iii) wage rates  $\{w_t^r\}_{t=1}^\infty$ , (iv) aggregate variables  $\{K_t^r, N_t^r, I_t^r, C_t^r\}_{t=1}^\infty$  in each region  $r$ , (v) world interest rates  $\{r_t^*\}_{t=1}^\infty$ , and (vi) external wealth positions  $\{X_t^r\}_{t=1}^\infty$  such that:

1. Households choose optimally consumption and wealth sequences  $\left\{ \{c_{j,t}^r, a_{j,t}^r\}_{j=1}^J \right\}_{t=1}^\infty$ , maximizing the objective function (2) subject to the budget constraint (3) and the income process (4).

2. Firms in each region maximize profits by setting the marginal product of each input equal to its price, i.e.

$$w_t^r = F_N(Z_t^r, K_t^r, N_t^r), \text{ for all } r, \quad (7)$$

$$r_t^r + \delta = F_K(Z_t^r, K_t^r, N_t^r). \quad (8)$$

3. The lump-sum transfer of accidental bequests equals the amount of assets left by the deceased, distributed equally to all households of the region.

$$b_t^r = \frac{\sum_{j=1}^{J-1} a_{j,t}^r (1 - s_{j+1,t}^r) \mu_{j,t-1}^r}{\sum_{j=1}^J \mu_{j,t}^r}$$

4. The regional labor markets clear at wage  $w_t^r$  and aggregate labor supply in each region is given by

$$N_t^r = \sum_{j=1}^{J_R^r-1} \mu_{j,t}^r \varepsilon_{j,t}^r. \quad (9)$$

5. The regional bond markets and the world capital market clear at the world interest rate  $r_t^*$ , and the aggregate stocks of capital in the three regions satisfy

$$K_t^r + X_t^r + B_t^r = \sum_{j=2}^J \mu_{j-1,t-1}^r a_{j,t}^r, \quad (10)$$

where

$$K_t^r = \sum_{j=1}^J \mu_{j,t}^r (a_{j,t}^r + b_t^r). \quad (11)$$

The world interest rate and regional interest rates satisfy the relationships  $r_t^* = r_t^{HI} = r_t^J$  and  $r_t^{MI} = r_t^*/(1 - \phi)$ , with the transaction cost  $\phi \in (0, 1)$ .

6. The lump-sum taxes  $\{\tau_{ls,t}^r\}_{t=1}^\infty$  satisfy the consolidated budget constraint (6) in each region.
7. The allocations are feasible in each region, i.e., they satisfy the regional aggregate resource constraints

$$K_{t+1}^r - (1 - \delta) K_t^r + X_{t+1}^r - (1 + r_t^r) X_t^r = F(Z_t^r, K_t^r, N_t^r) - C_t^r - G_t^r. \quad (12)$$

Before concluding, it is useful to recall that aggregate gross investments in region  $r$  are given by

$$I_t^r = K_{t+1}^r - (1 - \delta) K_t^r, \quad (13)$$

whereas aggregate or national (private plus public) savings in the three regions are,

$$S_t^r = F(Z_t^r, K_t^r, N_t^r) + r_t^r X_t^r - C_t^r - G_t^r. \quad (14)$$

As a result, the current account balances of the three regions equal their respective changes in net asset positions, and, are given by,

$$S_t^r - I_t^r = CA_t^r = X_{t+1}^r - X_t^r. \quad (15)$$

The sum of these current account balances is zero.

## 5 Calibration

We calibrate the initial steady-state using demographic and economic variables for the period 1990-2015 in the three regions. We assume that demographic parameters stabilize by 2100 and TFP growth rates in the three regions converge to the same values in the long run, and all regions eventually reach a balanced growth path some time after 2200. We then let our world economy transition between the two steady-states. The model's period is annual. Our calibration strategy is to match a set of moments in the data with the model's counterparts in the *open economy*. Appendix B provides more details of the calibration, including description of various databases used to compute statistics across regions. Calibrated parameters are summarized in Tables 1 to 4.

**The Three Regions:** As discussed above, there are three regions in the model, Japan, High-income and Middle-income regions. For High-income region, we include North America (United States and Canada), Europe, Australia, and New Zealand. For Europe, we include 27 countries that are members of European Union, plus the United Kingdom.

Middle-income region includes countries in Asia (China, Hong Kong, Taiwan, South-Korea, Singapore, Thailand, Indonesia, Malaysia, Philippines, Vietnam, India, Saudi Arabia, U.A.E., and Turkey), Mexico, Brazil, Russia, and South Africa.<sup>7</sup>

**Technological Parameters:** We assume a constant returns to scale production function

$$F(Z_t^r, K_t^r, N_t^r) = Z_t^r (K_t^r)^\alpha (N_t^r)^{1-\alpha},$$

with capital share  $\alpha = 0.35$  in three regions, following Holmes, McGrattan, and Prescott (2015). Based on the World Bank's *World Development Indicators* (WDI), we obtain growth rates of output per capita in the three regions from 1990-2015, which stand at 1.1% in Japan, 1.4% in High-income region and 3.9% in Middle-income region. The growth

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<sup>7</sup>We selected countries that have at least 100 billion yen (about \$1 billion U.S.) of either foreign direct investment or portfolio investment according to the breakdown of Japanese foreign assets as of 2015 reported by the Bank of Japan. We exclude Cayman Islands from the list and also add Turkey to Middle-income region.

rate of TFP  $\lambda_t^r$  in each region is set so that the region achieves the target average per-capita output growth rate during the same period. Values of the calibrated parameters are indicated in Table 4.

After 2015, we let  $\lambda_t^r$  of all regions converge smoothly to the common long-run growth rate of TFP, which we set to 1%.<sup>8</sup> We set the initial value of the Japanese TFP,  $Z_0^J$ , in order to normalize income per capita in Japan to 1 in 2015. Based upon the WDI, income per capita in High-income and Middle-income region in 2015 were 1.11 (\$45,373) and 0.31 (\$12,696) of that of Japan (\$40,763), respectively, and we set  $Z_0^H$  and  $Z_0^M$  to match these ratios. The depreciation rate of capital is set to 6% per year and is common to all three regions.

**Demographic Parameters:** The population data is based on the estimates of the United Nations (2017) for the High-income and Middle-income regions and on the data and projections of the National Institute of Population and Social Security Research (IPSS) (2017) for Japan. The population in 1990, the initial year of the transition, is 122 million in Japan, 778 million in High-income region, and 2,976 million in Middle-income regions. The survival probabilities are computed based on the population data and projections by age for each region. The population dynamics thereafter follow the projections of the United Nations and the IPSS.

We let households enter the economy at age  $j = 1$ , which corresponds to 20 years old, and live up to the maximum age of  $J = 80$ , up to 99 years old. We set the age to exit the labor force  $J_R^r$  at 46, which corresponds to 65 years old.<sup>9</sup>

**Preferences and Endowments Parameters:** Preferences are common across regions. We set the risk aversion parameter  $\theta = 2$  based on the estimates in the literature (for a survey, see Attanasio, 1999). We set the subjective discount factor  $\beta = 1.0296$  to match the target world rate of return on capital of 4% in 2015.

The calibration of the age profile of efficiency units is done separately for each region. The age-efficiency profile for Japan is based on the earnings data from the Basic Survey of the Wage Structure (BSWS) in 2010. For High-income region, we use weekly wage data from the Consumer Expenditure Survey (CEX) for the period 1982-1999. For Middle-income region, we use estimates used in Attanasio, Kitao, and Violante (2007), an age-efficiency profile on Mexican data—precisely from the Encuesta Nacional de Ingreso y Gasto de los Hogares (ENIGH), which is the equivalent of the U.S. CEX, using 1989-2000 waves. For all data sources average earnings of male and female workers are used in the calibration.

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<sup>8</sup>We assume that the growth rate of Middle-income region converges over the next 30 years, by 2045, and those of Japan and High-income region by 2100. The TFP level in the long-run will differ across regions and the levels of Japan and Middle-income region will be around 0.75 of that of the High-income region. We simulate the model under alternative assumptions about the future growth of TFP across regions in section 7.

<sup>9</sup>Note that the age to exit the labor force  $J_R^r$ , which is assumed to be exogenous in the model, can be different from the retirement age for the purpose of the social security system. The latter is a policy parameter as discussed below.

**Transaction Cost in the World Capital Market:** The transaction cost  $\phi$  is calibrated to match the external wealth to output ratio in Japan during the 1996-2015 period, which stands at 40%, based on the data from the Ministry of Finance. We assume the same value of  $\phi$  until 2020 and let it decline gradually to reach zero over a 80-year period and by the end of the century. In section 7, we simulate the transition under alternative assumptions about the transaction cost, including the case of zero cost and compare results to those of the benchmark transition.

**Government Debt to Output Ratios:** Government debt and expenditures as a fraction of output for High and Middle-income regions are computed from the IMF’s World Economic Outlook database (WEO, 2017) as time-averages over the period 1990-2015.

The WEO data yield a ratio of government expenditures to output at 35%, 41% and 25% for Japan, High-income and Middle-income regions, respectively, in 1990-2015. These expenditures contain all expenditures including interest payments on the government debt and transfers such as social security benefits. We calibrate the ratio of the government expenditures in our model,  $G_t^r$ , to output to match these data and they turn out to be 25% in Japan, 34% in High-income and 20% in Middle-income region. The ratio of government debt  $B_t^r$  to output in 1990-2015 was 51% and 31% in High-income and Middle-income regions, respectively, based on the WEO database. The debt level of the Japanese government, based on data from the Ministry of Finance, greatly fluctuated during the period and rose from 14% of output in 1990-1995 to approximately 120% in 2010-2015 and we set the ratio to 100%.

**Public Pension Replacement Rates:** Pension replacement rates for the three regions are calculated using OECD Pensions at a Glance (2014). In particular, we compute ‘coverage adjusted’ net replacement rates (NRRs) by multiplying NRRs by active coverage (defined as total number of contributors divided by labor force), available from the World Bank Pensions database (2014). The GDP (current PPP) weighted, coverage adjusted NRRs are 47.8%, 26.8%, and 38.5% for High and Middle-income regions and Japan, respectively.

The statutory retirement age is 65, 66 and 56 in Japan, High-income and Middle-income regions, respectively, based on the population-weighted average of the World Bank’s database and we set the retirement age  $J_{SS}^r$  in the model to these ages.

**Tax Rates:** Tax rates on consumption and assets are computed using the revenue data from the OECD Revenue Statistics, OECD National Accounts Statistics and UN National Account Statistics, following the method of Mendoza, Razin, and Tesar (1994).  $\tau_a^r$  is set at 34.7%, 34.1% and 18.8% in Japan, High-income and Middle-income regions, respectively. Consumption tax rates are 10.9% and 12.7% for High-income and Middle income regions. The Japanese consumption tax rate had been zero until 1989, when it was set to 3% and increased to 5% in 1997, 8% in 2014 and 10% in 2019. We let  $\tau_{c,t}^J$  increase accordingly and assume that it stays constant after 2019. Labor income tax rates  $\tau_w^r$  are set to 32.8%, 17.0% and 29.8% for High-income, Middle-income and Japan, respectively. We adjust lump-sum tax  $\tau_{ls,t}^r$  to satisfy the government budget constraint in each year and



region. The paths of endogenously determined lump-sum taxes are reported in section 7.

Table 1: Demographic Parameters

Parameter and description		Value, source
$s_{j,t}^r$	Survival rates	United Nations (2017) and IPSS
$n_t^r$	Cohort growth rates	United Nations (2017) and IPSS
$J_R^r$	Age to retire from the labor force	46 (65 yrs)
$J$	Maximum age	80 (99 yrs)

Table 2: Calibrated Parameters: Preferences, Technology and Endowment

Parameter and description	Value	Target, source
$\beta$ Subj. discount factor	1.0296	Interest rate in 2015
$\theta$ Risk aversion	2.0	Attanasio (1999)
$\varepsilon_j^r$ Wage profile	—	see text
$\alpha$ Capital share	0.35	Holmes, McGrattan, and Prescott (2015)
$\delta$ Depreciation rate	0.06	Holmes, McGrattan, and Prescott (2015)
$\phi$ Transaction cost	0.41	External wealth of Japan in 1996-2015

Note: values are on an annual basis.

Table 3: Calibrated Parameters: Government

Parameter and description		Target, source
$D_t/Y_t$	Debt to output ratio	IMF World Economic Outlook (2017)
$G_t/Y_t$	Gov. purch. to output ratio	IMF World Economic Outlook (2017)
$\kappa_t^r$	Pension replacement rate	OECD Pension at a Glance (2014)
$J_{ss}^r$	Pension retirement age	66 (HI), 56 (MI) and 65 (Japan) World Bank Pensions database
$\tau_{c,t}^r$	Consumption tax	OECD Revenue Stat., OECD/UN National Acct. Stat., Mendoza, et al (1994)
$\tau_{a,t}^r$	Capital income tax	OECD Revenue Stat., OECD/UN National Acct. Stat., Mendoza, et al (1994)
$\tau_{w,t}^r$	Labor income tax	OECD Revenue Stat., OECD/UN National Acct. Stat., Mendoza, et al (1994)

Table 4: Growth Rate and Level of TFP

Region	GDP per capita growth, WDI (1990-2015), target	TFP growth rate $\lambda_t^r$ (1990-2015) calibrated	GDP per capita level, WDI 2015, target	Initial TFP level $Z_0^r$ calibrated
Japan	1.1%	0.90%	\$40,763 (1.00)	0.60
High Income	1.4%	0.99%	\$45,373 (1.11)	0.76
Middle Income	3.9%	1.98%	\$12,696 (0.31)	0.31

## 6 Model Validation

In order to evaluate if our model and calibration produce a reasonable tool for the questions we are posing, we discuss some evidence in this section. Note that our open economy version is calibrated to produce a world return to capital of 4% in 2015 and the transaction cost parameter is calibrated so that our model generates an NIIP for Japan close to the average observed NIIP between 1996-2015. The other equilibrium quantities are not moments as targets for calibration.

Table 5: Capital Output Ratios

Region	PWT Data	Country Data	Model
High Income	3.64	3.23 (U.S.)	3.22
Middle Income	3.36	2.98 (China)	2.45
Japan	4.18	3.13 (Japan)	3.22

Table 5 displays the capital output ratios in the data and the baseline open economy simulation. The first column contains the data from the Penn World Table (PWT) between 1990 and 2017. The second column shows the capital output ratios using selected country-level data. In particular, we use the BEA data as in Koh, Santaaulàlia-Llopi, and Zheng (2020) for United States between 1990 and 2018, quarterly data between 1994 Q1 and 2019 Q2 from the Cabinet Office for Japan, and the PWT data for China between 1990 and 2017. Since none of these model quantities were targeted, we think the baseline model gets very close to measured capital output ratios, especially when we use country-level data as representative of the HI and MI regions.

However, there are significant drawbacks in comparing the data versus the model's current accounts and net foreign asset positions of the regions. First, the NIIP data present a country's position with respect to the rest of the world (RoW), thus including countries within the same region of our model or outside the model's world economy. Ideally, one would need data on countries' bilateral NIIPs to construct accurate regional net foreign assets positions. Second, coverage of China starts from 2004, restricting a country with significant amount of foreign assets from our sample.

The current account data from WDI has the same drawback, missing bilateral positions by each country necessary for us to construct a counterpart to our model's measure.

Given the difficulty of using the NIIP data with the lack of bilateral asset holdings aggregated to represent cleanly our three regions, we present the NIIP for the three regions using our (mis)measured NIIP as well as select economies to represent the regions. In particular, we will show the NIIP of the U.S. for the HI region and that of China for the MI region.

As shown in Figure 3, the model overstates the ‘net borrower’ position of the HI region but it is close to the U.S. NIIP. Similarly, the model overstates the ‘net lender’ position of the MI region and it is closer to the NIIP of China.

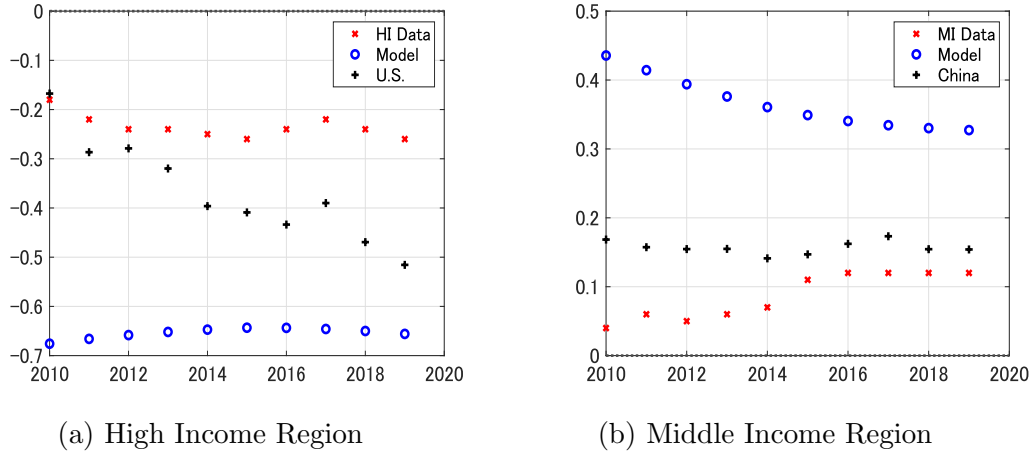


Figure 3: Net International Investment Position (% of GDP)

For Japan, we have less of an issue and therefore a comparison is far more meaningful. Figure 4 shows the data and model ratios of current account to output. The model seems to track the actual current account position of Japan reasonably well. Note that we targeted the average NIIP of Japan with our calibration of the single transaction cost parameter  $\phi$  but not the current account directly.

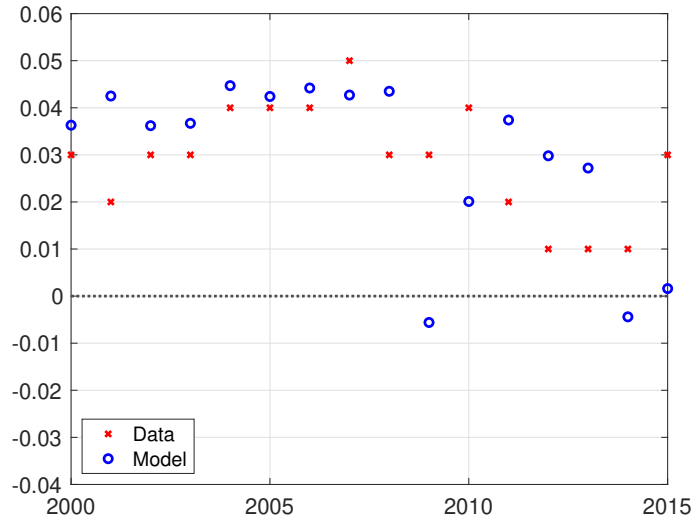


Figure 4: Current Account (% of GDP)

Figure 5 displays the model's net foreign asset position of Japan and two data counterparts, the NIIP of Japan from the IMF-WDI data base, and that from the Bank of Japan. With a single calibrated parameter, the model does a remarkable job of producing a net foreign asset position that is very close to the observed NIIP of Japan. This fit makes us optimistic that the model may produce projections that speak to the likely effects of demographics and TFP paths on capital flows, especially from a Japanese point of view.



Figure 5: Net International Investment Position (% of GDP)

## 7 Quantitative Findings

In this section, we present our numerical findings that characterize equilibrium transition paths from the initial steady state that represents the world economy in 1990-2015 to a final steady state in the distant future.<sup>10</sup>

These equilibrium paths are computed under the ‘perfect foresight’ assumption and are therefore deterministic. All households and firms have complete information on factor prices and fiscal policy parameters (tax rates, social security benefits, etc.) and make optimal decisions under new demographic variables, fiscal parameters and associated endogenous factor prices, given their asset holdings in the initial steady state. In other words, cohorts that are alive in 1990 re-optimize given their new demographic and policy environments. Along the transition, as in the initial and final steady states, it is assumed that a lump-sum tax adjusts in each time period to satisfy the government’s budget, keeping debt to output ratios constant. These computations form our baseline results below.

In some of the figures below, we will show the equilibrium transition paths under two separate assumptions on the openness of the economies. First, we compute the transitions under the assumption that each region has been operating in a world economy in the initial steady state and will remain open forever. Alternatively, in the closed economy computations, we assume that there are three segmented regions in the steady states and along the transition path such that both labor and capital are immobile with segmented equilibria in three regions.

### 7.1 Baseline Transitions

**Capital Labor Ratios in the Three Regions:** Figure 6 shows the paths of capital-labor ratios in the closed economy equilibria of the three regions. Capital labor ratios monotonically rise in the MI region because of strong growth of the capital stock that offsets a rise in labor supply. In the HI region, the ratios rise until around 2030 and stabilize thereafter. Japan shows a similar trend, but the ratio will decline after the 2040s as the capital stock falls faster than the labor supply.

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<sup>10</sup>In the program, we compute transitions of about 400 periods (from 1990 to 2400), long enough to guarantee a smooth convergence of all variables to the final steady state.

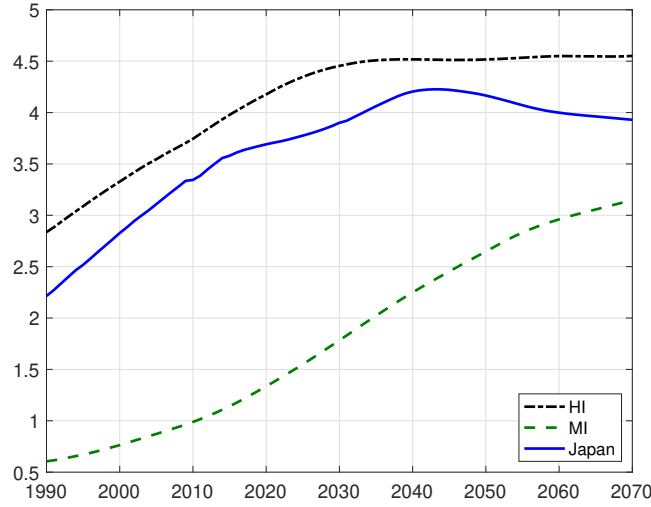


Figure 6: Capital Labor Ratios in Closed Economy

Although the capital labor ratios generally rise in all three regions until about 2040s, the gap between them shrinks. There is larger capital deepening in the MI region compared to the HI region and Japan, suggesting that the return to capital would be higher but would fall faster and longer in the MI region in the closed economy version of the model.

**TFPs and Interest Rates in the Closed Economy:** The paths of factor prices are affected by the dynamics of aggregate capital and labor, as well as the projected trajectory of the TFPs. We calibrated TFP growth rates in the 1990-2015 period as 0.99%, 1.98%, and 0.90% for the HI and MI regions and Japan, respectively, to match the per capita growth rates of GDP in these regions for the same period. Beyond 2015, we assume that these rates converge to 1% eventually. This occurs in 2045 for the MI region, and 2100 for the HI region and Japan.

Figure 7 depicts the time paths of TFP in the three regions. Even though the TFP levels are increasing in all regions, as the growth rates converge to a common value in the long-run, the rate of increase stays higher in the MI region, allowing for some catching up in their living standards. In particular, the GDP per capita in the MI region rises from 28% in 2015 to 38% in 2045, relative to that in the HI region, in our baseline equilibrium transition path.

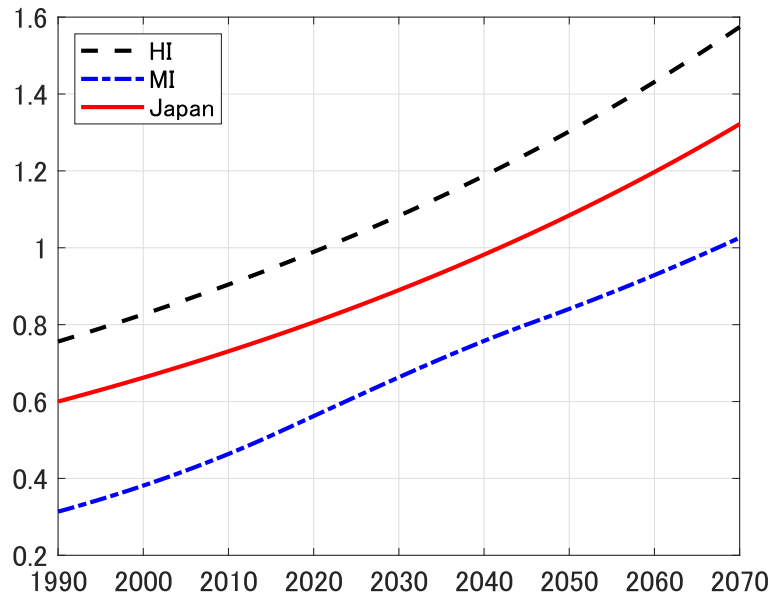


Figure 7: Total Factor Productivity in the Three Regions

The relative TFPs move more slowly after 2040. The slow convergence of TFP growth and the differential levels of TFPs that remain counter the incentive to move foreign capital out of the MI region that rising capital labor ratios would imply.

Figure 8 shows the returns to capital and labor under the closed economy of the three regions. In the HI region, the interest rate falls until 2030, stays fairly flat thereafter and remains almost unchanged until 2070. In the MI region, the interest rate monotonically falls from 6.7% in 2015 to 1.7% in 2070. In Japan, the closed economy interest rate is rather flat, similar to that in the HI region, but at a much lower level. These differences in the interest rate in the closed economy, as driven by the dynamics of capital labor ratios and TFP, are what will drive the flow of capital across regions in the open economy.

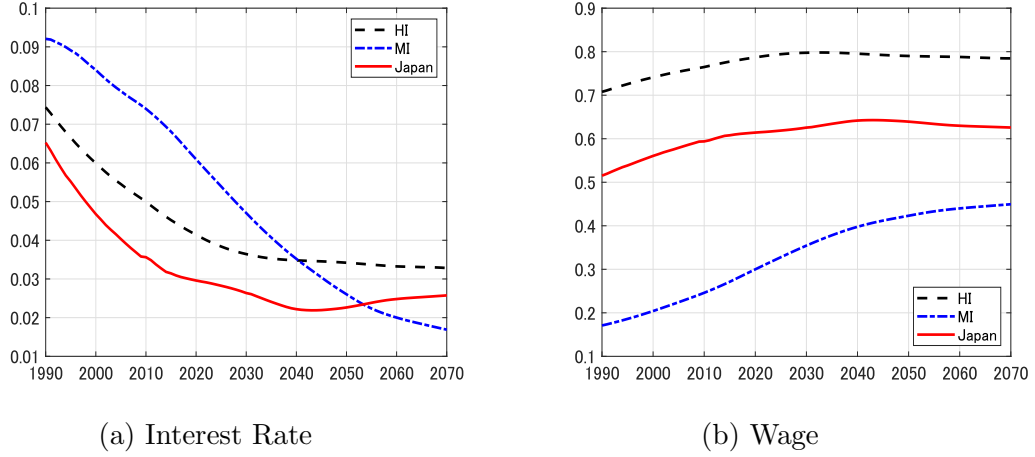


Figure 8: Factor Prices in Closed Economy

The only region that enjoys continuously rising wages is the MI region. Wages are flat for the most part of the period in the HI region. In Japan, wages rise initially but they decline slightly after 2040 following the trajectory of capital labor ratio.

**Return to Capital and Capital Flows in the Open Economy:** Figure 9 displays the returns to capital in the closed and open economy cases, and the NIIPs in the open economy case. The world real interest rate, shown in Figure 9a, starts at just above 6% in 1990 and falls to below 3% by 2030 and stays at around 2%-3% thereafter throughout the ‘projection’ period. This finding is line with the previous research that demonstrated a fall in real interest rates over time primarily due to demographic aging.

Figure 9b shows the Net International Investment Positions (NIIP) of the three regions. In 2010, the MI region is a net lender to the HI region and Japan, which suggests that the higher TFP levels in the HI region and Japan necessitate a larger capital outflow from the MI region and a lower capital labor ratio. As the aging continues in the HI region and Japan, and, as the TFP levels partially converge from their 2015 levels, the net lender role of the MI region slightly increases over time, the HI region becomes a larger net borrower, and Japan turns from a net lender to a net borrower in 2045.

We would like to emphasize that our findings are different from the previous literature in that capital flows from the MI region to the HI region and Japan from the start and stay that way throughout the distant future.

Next we will examine the relative importance of the roles played by demographics and TFP in more detail.



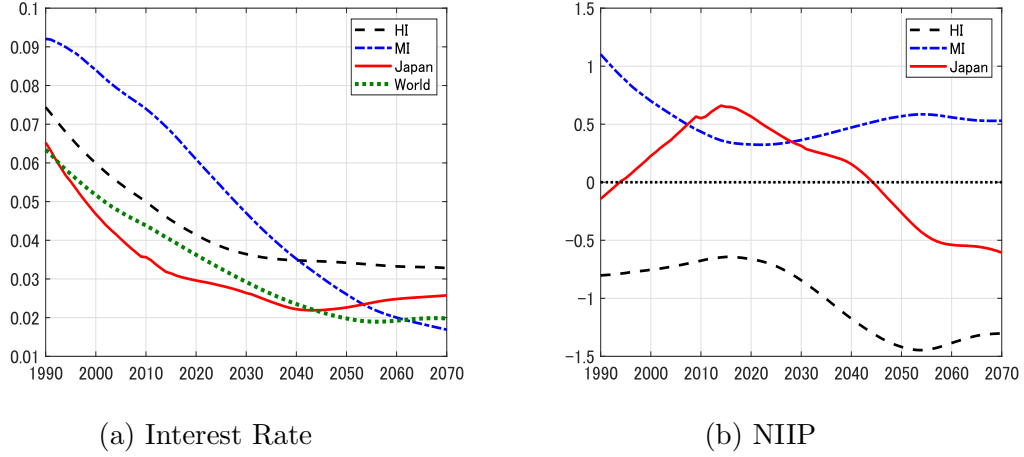


Figure 9: Interest Rate and NIIP (% of GDP)

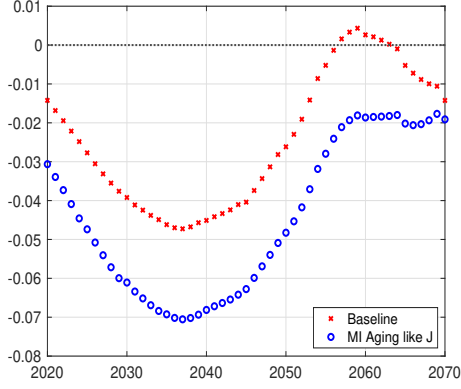
## 7.2 The Role of Demographics in Capital Flows

In order to explore the role played by projected demographics in driving capital flows across regions, we present the numerical results from an alternative equilibrium transition path which assumes that individuals in the MI region experience much faster improvement in longevity than projected by the UN and eventually catch up with that in Japan in 2065. Note that this would place the individuals in the MI region on par in terms of conditional survival probabilities with those in Japan and essentially those in the HI region also.

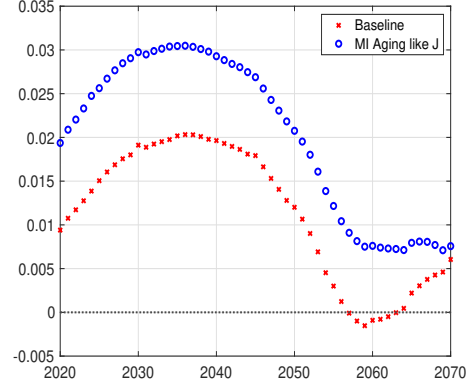
### Convergence of MI Survival Probabilities to HI and Japan Levels in 50 Years:

To put this increase in longevity in perspective, recall Figure 1a. According to life expectancies at birth projected by the UN, our alternative calculations in this subsection assume an increase in longevity in the MI region from their mid 60s in 1990 to their late 80s by 2065 instead of the baseline projection of about 80, an increase of about 8 years.

Figure 10 shows the current account (as % of GDP) in the HI and MI regions under the alternative assumption of enhanced longevity in the MI region and the baseline calculations. With faster aging in the MI region, the increase in the capital accumulation and capital outflow leads to a larger CA surplus in the MI region and a larger CA deficit in the HI region. The impact of higher survival probabilities on the current account can be as high as 2 percentage points for the HI.



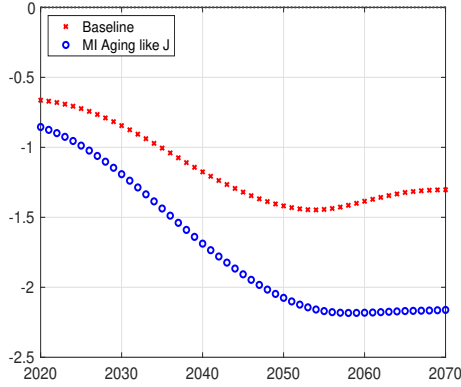
(a) Current Account in HI



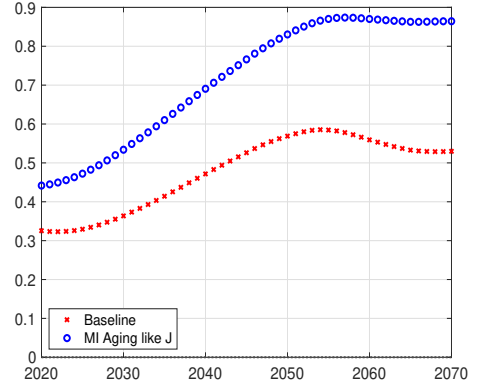
(b) Current Account in MI

Figure 10: Current Account (% of GDP) in the HI and MI Regions

The stocks of external wealth show a similar result as Figure 11 indicates. As the MI region becomes a larger net lender, the HI region becomes a larger net borrower. Under the alternative assumption of MI quickly converging to the survival probabilities of the HI region and Japan, the increased NIIP of MI is 83% of GDP in 2050 compared to the 57% in the baseline case.<sup>11</sup>



(a) NIIP in HI



(b) NIIP in MI

Figure 11: NIIP (% of GDP) in the HI and MI Regions

<sup>11</sup>The increase in longevity that drives these results would require massive investments in health infrastructure, social insurance policy and large changes in human behaviors in order for the MI region to catch up with the HI region and Japan so quickly. Clearly this may not be very realistic but it still serves the purpose of showing the quantitative effect of significant demographic shifts on capital flows.

It is useful to analyze Japan separately because our model validation suggested that our single parameter calibration of the transaction cost resulted in a good fit with the observed NIIP of Japan as we have cleaner data to compare with our model.

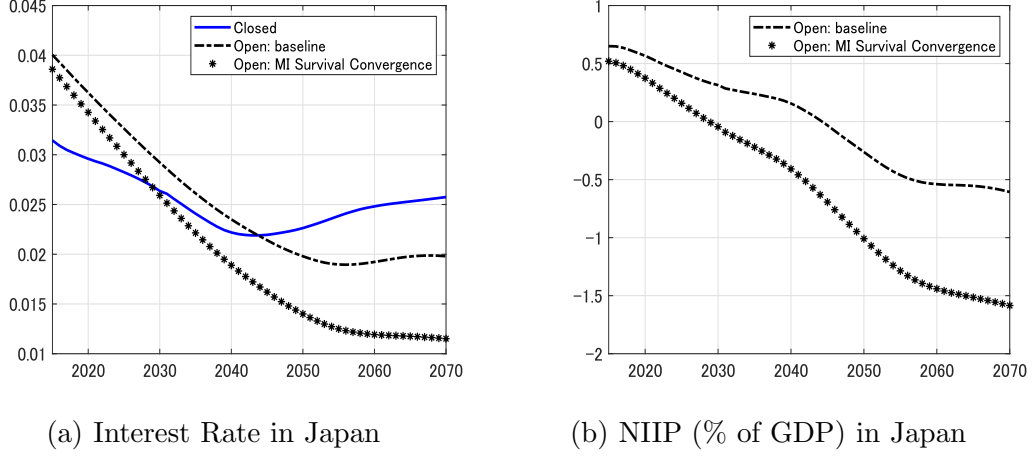


Figure 12: Convergence of Survival Rates of MI

Figure 12a shows the paths of open economy interest rates in the baseline and under this alternative scenario together with the path of closed economy interest rates in Japan. A rise in longevity leads to stronger saving motives for retirement and a larger aggregate capital will lower interest rate in the MI region. Given the large size of the MI region in the world economy, the open economy return to capital will be lower. Figure 12b shows that capital will flow to Japan much earlier and in larger quantities relative to the benchmark and Japan will be an even larger net borrower against the world. Both the timing and the magnitude are determined by the new development of demographic aging in other regions of the world. In particular, the speed with which the MI region catches up with the HI region and Japan in terms of longevity will expedite the flow of capital to both HI and Japan.

### 7.3 The Role of TFP in Capital Flows

The previous literature emphasized the role of differential aging across regions as a leading source of capital flows. But as Chen, İmrohoroglu, and İmrohoroglu (2006) and Ferrero (2010) argued in the context of external imbalances in the U.S. economy, TFP differences across economies are also important in determining the direction and magnitude of capital flows.

In a closed economy setting, the return to capital is determined by the capital labor ratio and the level of TFP. Over time, an aging economy typically faces an increase in

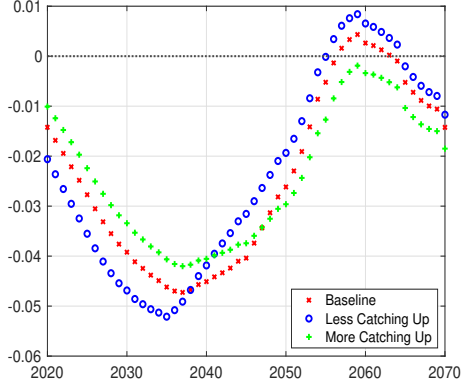
the capital labor ratio which puts pressure on the return to fall making this economy less attractive to foreign investors if capital were allowed to cross borders. Therefore, once capital is mobile, it would tend to flow out to economies where returns would have been higher otherwise.

At the same time, differential growth rates of TFP also create differences in returns to capital independent from the aging process. If TFP grows faster in an economy, this puts pressure on the return to rise in a closed economy setting and when capital is allowed to move across borders this higher TFP would lead to a capital inflow.

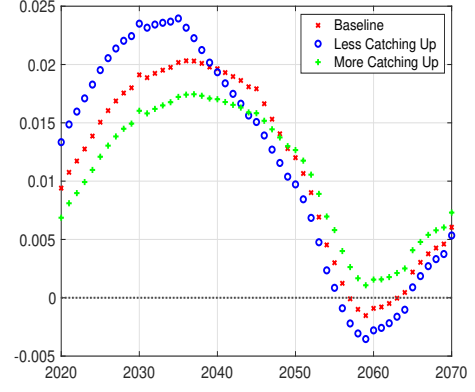
In the previous subsection, we present numerical results that suggest that there will be large international capital flows if the MI region attains the much higher longevity levels of the HI region and Japan. In this subsection, we display our quantitative findings from alternative calculations that allow living standards of the MI region to get closer to those in the HI region by assuming different years for the convergence of the regional TFP growth rates.

**Baseline vs Less/More Catching Up of TFP Levels:** In the baseline transition path, we assume that the growth rate of TFP in the MI region converges to the long run value of 1% in 2045. As the MI region is assumed to grow faster than both HI region and Japan between 2015 and 2045, there is some catching up of the level of TFPs. In two alternative computations, we assume that this convergence of growth rate to 1% occurs in 2035, which implies that the TFP of the MI region reaches a lower level compared to the baseline calculation, or, in 2055, which allows the MI to enjoy faster growth for another decade to get closer to the level of TFP, and therefore GDP per capita, in the HI region.

More catching up makes the current account deficit in the HI region smaller and less catching up makes the current account larger relative to the baseline case, as shown in Figure 13a. The intuition is that with a higher level of TFP in the MI region under the ‘more catching up’ assumption, the HI region and Japan are not as attractive as the baseline case regarding their closed economy interest rates (with a smaller gap in the TFP levels) and therefore a smaller amount of capital is needed to flow out of the MI region and into the HI region and Japan to equalize the returns across regions in the world financial market. Conversely in the MI region, more catching up lowers the current account relative to the baseline case as Figure 13b shows.



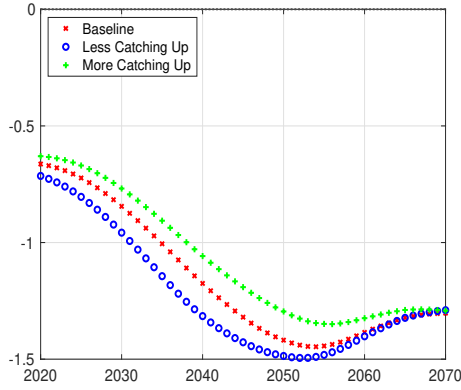
(a) Current Account in HI



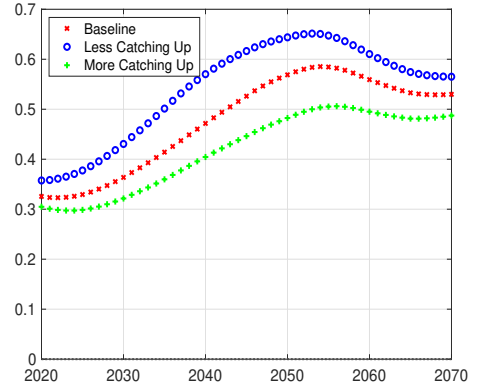
(b) Current Account in MI

Figure 13: Current Account in the HI and MI Regions (% of GDP)

Turning to the NIIPs, Figure 14 displays the accumulated effects of the annual capital flows depicted in Figure 13. More catching up makes the HI region less of a net borrower and the MI region less of a net lender.



(a) NIIP in HI



(b) NIIP in MI

Figure 14: NIIP in the HI and MI Regions (% of GDP)

Looking at Japan alone, the effects are similar to those in the HI region, as shown in Figure 15. With more catching up, Japan's current account position is higher by more than 1 percentage point and the year of turning into a net borrower is postponed to 2048. On the other hand, under less catching up, Japan becomes a net lender in 2040.

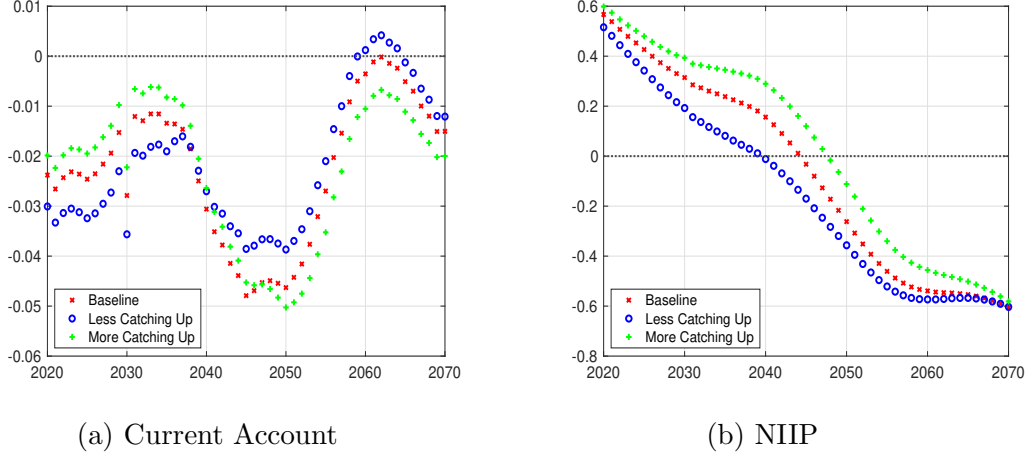
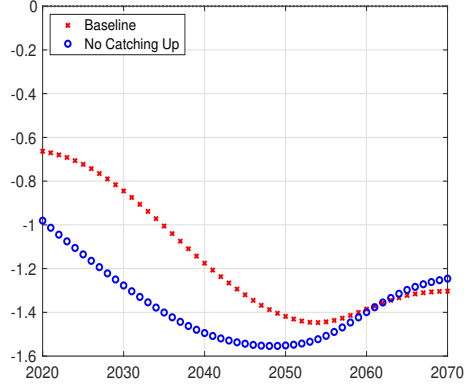


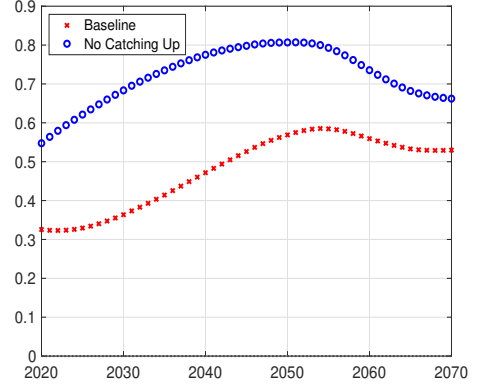
Figure 15: Current Account and NIIP in Japan (% of GDP)

Note that the quantitative difference in the current account positions (of all regions) between more or less catching up is not as large as that between the baseline case and the full convergence of survival probabilities case. The latter is admittedly an extreme assumption whereas the former is less so. The living standards as measured by GDP per capita relative to that in the HI region is 28% in the MI region in 2015 (our calibration). This rises to 38% in 2045 under the baseline assumption on the convergence of TFP growth rates. With more catching up, the convergence takes 10 years longer, allowing for another decade of faster growth in the MI region, raising their living standards to 42% compared to the less catching up, which raises GDP per capita in the MI region to only 34% of the HI region by 2045.

**No TFP Level Convergence At All:** We now compute the transition assuming that all three regions retain their relative TFP levels and therefore living standards as of 2015. Any changes in their NIIPs relative to those in the baseline case would come directly from differential aging which impacts the regional capital labor ratios differently and thereby leading to different capital movements across borders to equalize the returns in the world capital market.



(a) NIIP in HI



(b) NIIP in MI

Figure 16: NIIP in the HI and MI Regions (% of GDP)

According to Figure 16, the ‘counterfactually’ low TFP levels in the MI region would necessitate a massive capital outflow from MI to HI relative to the baseline transition. We mentioned before that comparing the NIIPs in the data to those in the model is problematic for the HI and MI regions since the data counterparts include capital flows into/from countries in the same region. For Japan, we are able to make a cleaner comparison and Figure 17 indicates that assumptions regarding alternative future paths of TFP make a significant difference in NIIP with respect to the rest of the world. In particular, Japan’s NIIP turns negative much earlier relative to the baseline case as Japan’s TFP remains much higher than that in the MI region due to ‘no convergence’ which necessitates larger capital inflows to Japan to equalize the returns across regions in the world financial market.

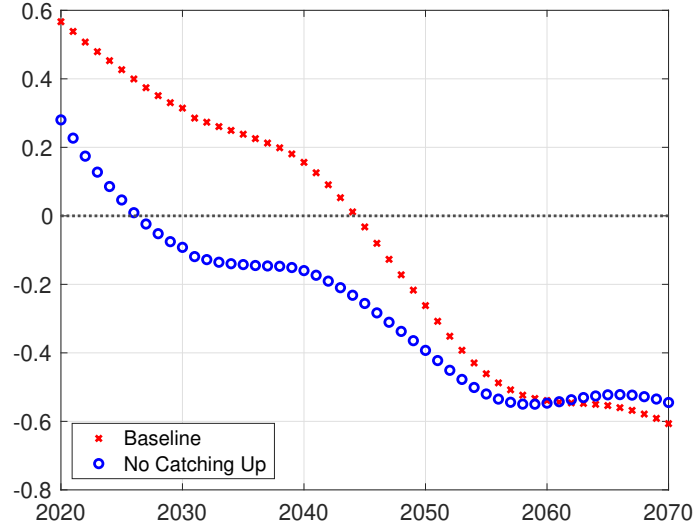


Figure 17: NIIP in Japan (% of GDP)

The year of becoming a net borrower is 2027 with massive capital inflows from the MI region under this calculation compared to 2045 under the baseline assumptions. Again, this reflects the quantitatively powerful role played by TFP in shaping capital flows across borders.

## 7.4 Demographics vs TFP

It is not possible to conduct a truly counterfactual experiment to isolate the separate effects of demographics and TFP on future capital flows. The demographic projections come from the official sources and the TFP paths are determined by our assumptions on the extent and timing of catching up of living standards. Nevertheless, we present a few snapshots of some of the figures displayed in the previous subsections to get an idea about the relative strengths of differential aging versus TFP levels on future capital flows.

Table 6 shows the values of net foreign asset positions or NIIPs in 2030, 2040, and 2050 for the three regions: HI, MI and Japan. In addition to our baseline results, the table reports the ‘MI High Longevity’ case in which we assume that the conditional survival probabilities of individuals in the MI region increase very rapidly so that their longevity is as high as that in the HI region and Japan by 2045. As we mentioned before, this is against the projections by the United Nations and perhaps an extreme assumption. Table 6 also shows the NIIPs for the case of zero convergence of the MI living standards in which the typical MI individual remains at the 28% of the GDP per capita of the HI region individual forever. This is arguably also not realistic but these two rather extreme assumptions help us understand the relative strengths of demographics versus TFP in



shaping international capital flows.<sup>12</sup>

Table 6: NIIPs in the Three Regions (% of GDP)

	2030	2040	2050
Baseline			
HI	−84%	−118%	−142%
MI	36%	47%	56%
Japan	31%	16%	−26%
MI High Longevity			
HI	−119%	−169%	−208%
MI	53%	69%	83%
Japan	−4%	−41%	−101%
MI No TFP Convergence			
HI	−128%	−150%	−155%
MI	68%	78%	81%
Japan	−9%	−16%	−39%

According to the quantitative findings in Table 6, alternative projections of aging and TFP in the MI region relative to the HI region and Japan produce somewhat similar changes in international capital flows. For the MI region, the range of NIIPs from 2030 to 2050 changes from 36%-56% in the baseline case to 53%-83% and 68%-81% for the ‘aging fast’ and ‘stagnant per capita GDP’ cases, respectively.

Although this is not quite an apples-to-apples comparison, our numerical results suggest that future TFP paths have as important quantitative effects on NIIPs as future demographic realizations.

## 7.5 Fiscal Policy and Capital Flows

**Fiscal Adjustments in the Three Regions:** We now look at the magnitude of tax adjustments in these three regions to maintain fiscal sustainability in the future. In our baseline results, we assumed that the lump-sum tax is used to satisfy government budgets, holding the initial debt to output ratios and other tax rates constant. Figure 18 shows the time paths of lump-sum taxes that are required to satisfy the government budget constraint in these regions in the closed economy transitions.

Aging demographics will raise expenditures for old-age transfers and by more in economies that have more generous benefits. At the same time, a decline in the labor force would reduce tax revenues. As shown in the figure, Japan will experience a

<sup>12</sup>Table 6 uses some of the numerical results in Figures 9, 11, 12, 16, and 17.

rapid increase in the fiscal burden during the next decades. The MI region has a lower replacement rate of social security and will experience a milder increase initially but tax will continue to rise as dependency ratios increase rapidly.<sup>13</sup>

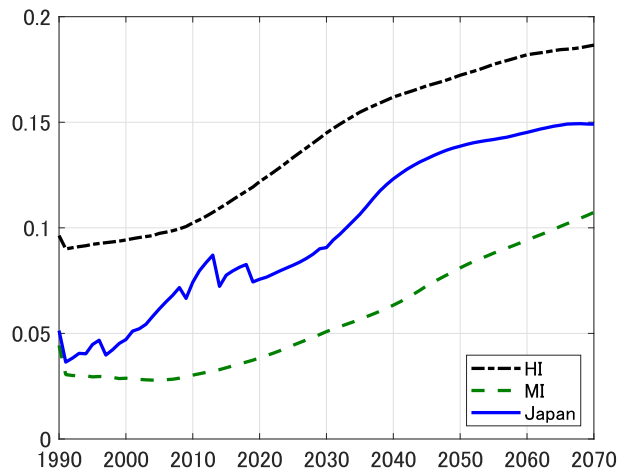


Figure 18: Total Lump-sum Taxes in Closed Economy (% of GDP)

## 7.6 More Generous Pensions in the MI Region

The social security in the MI region is less comprehensive than in the HI region or Japan and the replacement rate of public pensions is lower. In this subsection we simulate a transition assuming that the MI region starts to provide more generous retirement benefits. More precisely, we let the replacement rate of the MI region, which stands at 26.8% in the baseline, increase to the level of HI, 47.8%, gradually over the next 20 years.

Under this alternative assumption, households in the MI region will have lower retirement saving motives and the resulting decline in their capital stock raises equilibrium open economy interest rates. Given higher interest rates within the MI region (in addition to the rest of the world), capital will flow less towards other regions.

As shown in Figure 19, NIIP in the HI region will be less negative and in the MI less positive. Net external wealth in Japan will turn negative by about ten years later than in the baseline model and the level of external wealth will be less negative thereafter as shown in Figure 20.

<sup>13</sup>The figure for lump-sum taxes in the open economy is not included but the paths are very similar to those in the closed economy.

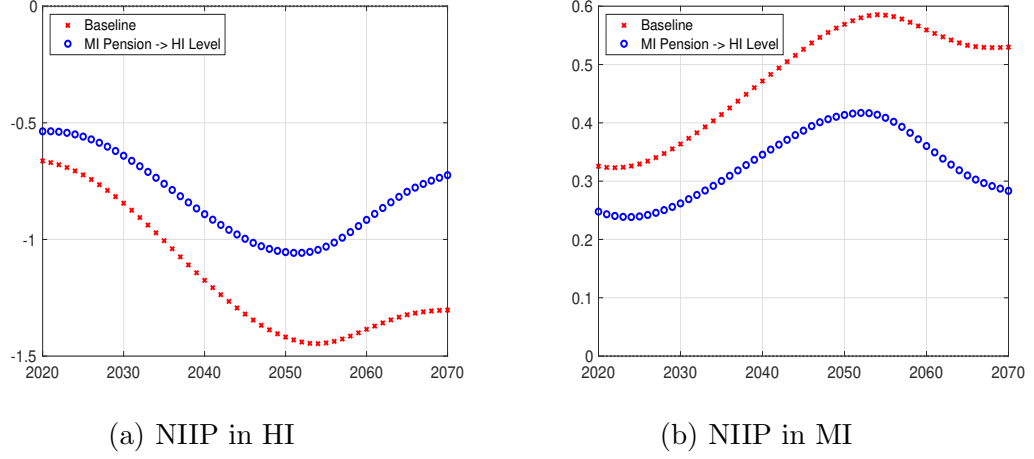


Figure 19: NIIP in the HI and MI Regions (% of GDP)

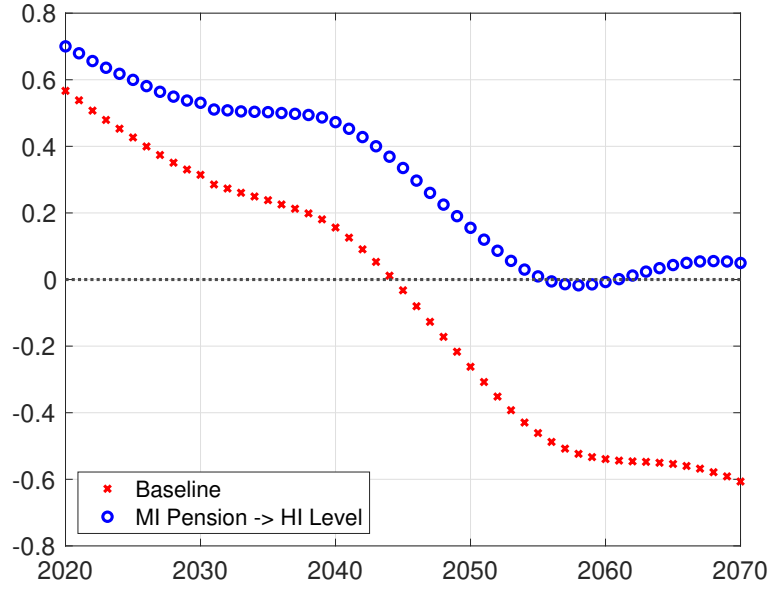


Figure 20: NIIP in Japan (% of GDP)

Table 7 reports a few snapshots of Figures 19 and 20. As pensions become more generous in the MI region, the negative impact on private saving leads to a reduction in capital accumulation. This reduction in the capital labor ratio in turn leads to a higher return to capital in the closed economy and requires a slowdown of capital flowing out of the MI region to equalize the regional returns in the world capital market. As a result,

the net lender position of the MI region shrinks just as the net borrower position of the HI region. Japan enjoys a larger net lender position compared to the baseline case.

Table 7: NIIPs (% of GDP) in the Three Regions

	2030	2040	2050
Baseline			
HI	−84%	−118%	−142%
MI	36%	47%	56%
Japan	31%	16%	−26%
MI High Pensions			
HI	−64%	−89%	−105%
MI	26%	35%	41%
Japan	53%	47%	16%

## 7.7 Sensitivity to Transaction Costs

In the baseline simulation, the parameter  $\phi$  that represents ‘transaction costs of capital’ is set to 0.41 and we assume that the cost will fall gradually to zero after 2020. The symmetric transaction cost is a reduced form device which approximates a cost that lenders in the international capital markets incur in investing in less developed financial markets or an extra benefit that lenders in less developed markets derive from investing in ‘safer’ assets in high income economies. It is of course an open question how large such costs are or how they will evolve over time in the future. In this section we characterize equilibrium transitions under alternative scenarios about paths of transaction costs, assuming that they will fall but only by about one-half, to 0.20 in the long run, or they do not change and remain at 0.41.

In these new scenarios, households in the MI region will perceive investment in Japan (and HI) as providing the additional benefit longer and continue to lend additional amounts. Hence, as shown in Figure 21, the HI region will receive a greater amount of capital flow, thus experiencing larger current account deficits, and will have more negative net external wealth. The magnitude of the net external wealth differs but the direction of capital flows remains unchanged under alternative scenarios. Note that the effect on the magnitude of capital flows occurs in the distant future, starting just before 2050.

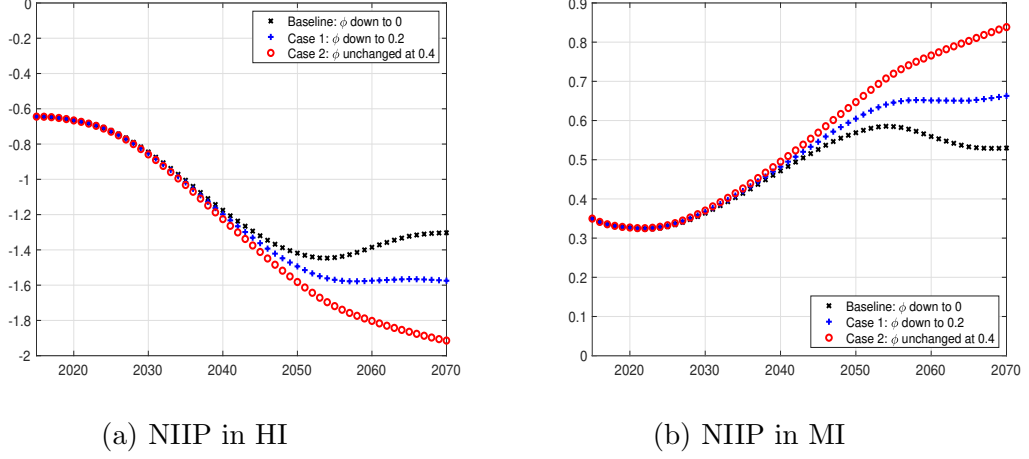


Figure 21: Alternative Capital Adjustment Costs and NIIP (% of GDP)

The effect of alternative transaction costs on Japan's NIIP is similar to that on HI. There is hardly any effect over the next few decades and the year Japan becomes a net borrower, but after 2050 the nondeclining appetite for Japan's domestic assets leads to a larger capital inflow.<sup>14</sup>

## 8 Conclusion

With improvements in longevity in all countries and declines in fertility in most regions, populations have been aging. The timing and the severity of these demographic trends are different across advanced and emerging economies. With existing differences in social security programs in place, the differential aging will impact saving and investment decisions and therefore capital flows in coming decades.

This paper develops a general equilibrium model populated with overlapping generations of individuals located in three regions that demonstrate large differences in their demographic trends and generosity of social security programs. Labor is immobile and there are segmented labor markets with regional wage rates. Capital, on the other hand, is allowed to move across borders with the return to capital determined in an imperfect world capital market subject to a symmetric transaction cost. Two of the three regions consist of a large number of High Income (HI) and Middle Income (MI) countries and the third region is Japan. This specification allows us to isolate a country that has experienced aging earliest among the advanced economies and where the aging is most severe. As a result, Japan serves as a laboratory case for our model's mechanisms in determining the future path of capital flows.

<sup>14</sup>The NIIP figure for Japan is not included as it follows the same path as the baseline case with more negative values for alternative non-vanishing values of the transaction cost.

The demographic trends of countries in the HI and MI regions in the model are taken from United Nations' population projections whereas we rely on the Japanese government's projections for our third region. After calibrating the model to regional macroeconomic and fiscal indicators, we calculate equilibrium transition paths from the 1990s to a future balanced growth path. Our main quantitative finding is that the MI region is always a net lender to the rest of the world (RoW) while the HI region is a net borrower. The faster rise in the capital labor ratio in the HI region and Japan due to faster aging would lower the closed-economy returns to capital. On the other hand, higher TFP levels in the HI region and Japan would make these regions more attractive to investors. When capital is allowed to cross borders, albeit with a transaction cost, the MI region becomes a net lender. Japan starts as a net lender to the RoW but quickly starts to experience current account deficits, eventually becoming a net borrower by 2045.

The main numerical results and additional calculations using alternative demographic projections and TFP paths suggest that future TFP realizations are just as important to determine future capital flows as the differential aging across regions.

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## A Computation of the Equilibrium

We describe the computation of the equilibrium in an open economy, where the labor income tax rate in each period is adjusted to achieve the government budget balance. All the other policy variables either remain fixed throughout the transition or move deterministically.

**Step 1:** Compute the initial and final steady-states of the model. Let the transition between the two steady states take  $T$  periods, long enough so the economy converges smoothly to the final steady state.

**Step 2:** Guess three  $T$ -dimensional vectors for the world interest rate, the lump-sum tax, and accidental bequests in each region. The first and last entry of these vectors are the initial and final stationary equilibrium values computed in Step 1.

Given the path for the interest rates, using the property of the constant returns to scale technology and the optimization conditions for the firm, sequences of wages in each region can be derived. the problem of the households can be solved in each region.

**Step 3:** Given prices and the sequence of tax rates and accidental bequests obtained in Step 2, solve households' problem. Recall the budget constraint of the household at time  $t$ :

$$(1 + \tau_{c,t}^r) c_{j,t}^r + a_{j+1,t+1}^j = y_{j,t}^r + [1 + (1 - \tau_{a,t}^r) r_t^r] (a_{j,t}^r + b_t^r) + p_{j,t}^r - \tau_{ls,t}^r.$$

Denote net-of-tax gross interest rate as

$$R_t^r \equiv 1 + (1 - \tau_{a,t}^r) r_t^r.$$

From the first order condition with respect to asset holdings next period, we obtain

$$\frac{c_{j+1,t+1}}{c_{j,t}} = \left[ \beta s_{j+1,t+1}^r \frac{1 + \tau_{c,t}^r}{1 + \tau_{c,t+1}^r} R_{t+1}^r \right]^{\frac{1}{\theta}} \equiv g_{j+1,t+1}^c, \quad (16)$$

which is the optimal growth rate of consumption between age  $j$  and  $j + 1$  and between time  $t$  and  $t + 1$ . Iterating backward over (16), we obtain that

$$c_{j+1,t+j} = c_{1,t} \prod_{k=1}^j g_{k+1,t+k}^c.$$

The discounted present value of the total (gross of taxes) lifetime consumption expenditures of the household of age 1 at time  $t$  is given as

$$\bar{c}_{1,t} = c_{1,t} \left[ (1 + \tau_{c,t}^r) + \sum_{j=1}^{J-1} (1 + \tau_{c,t+j}^r) \prod_{k=1}^j \frac{s_{k+1,t+k}^r}{R_{t+k}^r} g_{k+1,t+k}^c \right]. \quad (17)$$

In general, the discounted present value of the total (gross of taxes) lifetime consumption expenditures of the household of age  $j^*$  at time  $t$  is

$$\bar{c}_{j^*,t} = c_{j^*,t} \left[ (1 + \tau_{c,t}^r) + \sum_{i=j^*}^{J-1} (1 + \tau_{c,t+(i-j^*+1)}^r) \prod_{k=j^*}^i \frac{s_{k+1,t+(k-j^*+1)}^r}{R_{t+(k-j^*+1)}^r} g_{k+1,t+(k-j^*+1)}^c \right]. \quad (18)$$

Define a variable  $x_{j,t}^r$ , as a sum of earnings, pensions and a bequest transfer net of taxes:

$$x_{j,t}^r = y_{j,t}^r + p_{j,t}^r + R_t^r b_t^r \quad (19)$$

The discounted present value of the total (net of taxes) lifetime earnings and bequest transfers of a household of age 1 at time  $t$  is:

$$\bar{x}_{1,t} = x_{1,t}^r + \sum_{j=1}^{J-1} \left( \prod_{k=1}^j \frac{s_{k+1,t+k}^r}{R_{t+k}^r} \right) x_{j+1,t+j}^r, \quad (20)$$

where we are implicitly imposing the initial condition  $a_1 = 0$ . The discounted present value of the total (net of taxes) lifetime earnings of a household of age  $i^*$  at time  $t$  is:

$$\bar{x}_{j^*,t} = x_{j^*,t}^r + \sum_{j=j^*}^{J-1} \left( \prod_{k=j^*}^j \frac{s_{k+1,t+(k-j^*+1)}^r}{R_{t+(k-j^*+1)}^r} \right) x_{j+1,t+(j-j^*+1)}^r + R_t^r a_{j^*,t}. \quad (21)$$

Since individual optimization requires  $\bar{c}_{j^*,t} = \bar{x}_{j^*,t}$  for each age  $j^*$  and time  $t$ , from (18) and (21), we obtain  $c_{j^*,t}$  as

$$c_{j^*,t} = \frac{\bar{x}_{j^*,t}}{\left[ (1 + \tau_{c,t}^r) + \sum_{i=j^*}^{J-1} (1 + \tau_{c,t+(i-j^*+1)}^r) \prod_{k=j^*}^i \frac{s_{k+1,t+(k-j^*+1)}^r}{R_{t+(k-j^*+1)}^r} g_{k+1,t+(k-j^*+1)}^c \right]}.$$

Note that  $a_{j^*,t}$  in equation (21) is computed residually from  $c_{j^*-1,t-1}$  and the budget constraint (3) :

$$a_{j^*,t} = x_{j^*-1,t-1}^r + R_{t-1}^r a_{j^*-1,t-1} - (1 + \tau_{c,t-1}^r) c_{j^*-1,t-1}.$$

**Step 4:** Aggregating asset holdings of all age groups and using equation (10) for each region, we obtain the implied sequence for external wealth of the region  $X_t^r$ . Together with the world capital market clearing condition (8), we arrive at a new guess for the sequence of the world interest rate. We use the government budget constraints (6) in each region with price sequences to update our guess for the tax rate. From the life-cycle asset decisions, we compute the accidental bequests left by the deceased in each region to update the guess for the bequests in the next iteration. If convergence is not reached, we restart from Step 3 with the new vector of guesses.

## B Summary of calibration targets

This section provides further details on the data used in the model calibration. Table B.1 reports a summary by region of the indicators used as targets in calibrating the model. Table B.2 presents a description of the macro data used and a short explanation of how calibration targets were computed; the sample period over which the statistics are calculated is also reported.

Table B.1: Summary of calibration targets

Parameter	Description	Target	Period	Value by region		
				Japan	HI <sup>c</sup>	MI <sup>c</sup>
<i>Preferences and endowments</i>						
$\beta$	Subj. discount factor	Interest rate, %	2015	4		
<i>Production technology</i>						
$Z_0^r$	TFP level (initial)	GDP per capita level, \$ PPP	2015	40,763	45,373	12,696
$\lambda_t^r$	TFP growth rate	GDP per capita growth, %	1990-2015	1.1	1.4	3.9
<i>Government</i>						
$D_t/Y_t$	Debt to GDP ratio	General gov. net debt to GDP, %	1990-2015	100 <sup>a</sup>	50.9	30.7
$G_t/Y_t$	Gov. purch. to GDP ratio	General gov. total expenditure to GDP, %	1990-2015	34.9	41.5	25.2
$\kappa_t^r$	Pension replacement rate	Net replacement rate, coverage adjusted	2014	38.5	47.8	26.8
$\tau_w^r$	Labor income tax	Avg. eff. labor income tax, %	2000-2014	29.8	32.8	17.0
$\tau_a^r$	Capital income tax	Avg. eff. capital income tax, %	2000-2014	34.7	34.1	18.8
$\tau_c^r$	Consumption tax	Avg. eff. consumption tax, %	2000-2014	3 – 8 <sup>b</sup>	10.9	12.7

Notes: <sup>a,b</sup> See text for details on how targets were set; <sup>c</sup> High Income (HI): United States, Canada, Europe (UE 27, UK) Australia, New Zealand; Middle Income (MI): China, Hong-Kong, Taiwan, South Korea, Singapore, Thailand, Indonesia, Malaysia, Philippines, Vietnam, India, Saudi Arabia, United Arab Emirates, Turkey, Russia and South Africa.

Table B.2: Dataset description

Indicator	Period	Description	Sources
GDP per-capita (PPP, current international \$)	2015	Gross domestic product divided by midyear population and converted to international dollars using purchasing power parity exchange rates. The target indicator by region is computed as a cross-country weighted mean, using GDP at current PPPs for weighting.	World Bank World Development Indicators (2017)
GDP per-capita growth (%)	1990-2015	The annual growth rate of GDP per-capita is calculated from GDP per-capita measured in constant local currency. The target indicator by region is a time average of the cross-country weighted mean. GDP at constant 2010 US\$ is used for weighting.	World Bank World Development Indicators (2017)
General government total expenditure (% GDP)	1990-2015	Total spending of general government includes expenditures incurred by the central, state and local government, and social security funds. Total expenditure comprises current outlays, including interest payments on government debt and social transfers, and net investment in non-financial assets. The target total expenditure to GDP ratio by region is a time average of the cross-country weighted mean. GDP at constant 2010 US\$ is used for within-region weighting.	IMF World Economic Outlook (2017)
General government net debt (% GDP)	1990-2015	Net debt of general government is given by gross debt minus financial assets corresponding to debt instruments (monetary gold and SDRs, currency and deposits, etc.). The target net debt to GDP ratio by region is a time average of the cross-country weighted mean. GDP at constant 2010 US\$ is used for within-region weighting.	IMF World Economic Outlook (2017); Ministry of Finance for Japan
Labor income tax rate (%)	2000-2014	Average effective tax rate computed following the method by Mendoza, Razin and Tesar (1994). Tax revenue data and national account aggregates available from various sources. The target indicator by region is a time average of the cross-country weighted mean. GDP at constant 2010 US\$ is used for within-region weighting.	OECD Revenue Statistics (2017), OECD National Accounts Statistics (2017), UN National Accounts Statistics (2017)
Capital income tax rate (%)	2000-2014	Average effective tax rate computed following the method by Mendoza, Razin and Tesar (1994). Tax revenue data and national account aggregates available from various sources. The target indicator by region is a time average of the cross-country weighted mean. GDP at constant 2010 US\$ is used for within-region weighting.	OECD Revenue Statistics (2017), OECD National Accounts Statistics (2017), UN National Accounts Statistics (2017)

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Indicator	Period	Description	Sources
Consumption tax rate (%)	2000-2014	Average effective tax rate computed following the method by Mendoza, Razin and Tesar (1994). Tax revenue data and national account aggregates available from various sources. The target indicator by region is a time average of the cross-country weighted mean. GDP at constant 2010 US\$ is used for within-region weighting. For Japan, actual statutory tax rates are used, ranging from 3% in 1990 to 10% in 2019.	OECD Revenue Statistics (2017), OECD National Accounts Statistics (2017), UN National Accounts Statistics (2017); Ministry of Finance for Japan
Net replacement rate, coverage adjusted (%)	2014	The net replacement rate (NRR) is given by net pension entitlements divided by net pre-retirement lifetime earnings, thus accounting for individual income tax and social contributions (for a mean male earner). The adjustment for coverage is done by multiplying the NRR by active coverage (defined as the number of contributors to the social security system divided by labor force). GDP at current PPPs is used for within-region weighting.	OECD Pensions at a Glance (2014), World Bank Pensions Database (2014)
Pension retirement age	2013	It is the statutory retirement age (mean of females and males if they differ) at which people eligible to old-age pension start receiving benefits. Total population is used for within-region weighting.	World Bank Pensions Database (2014)
Total population by age groups	1990-2100	Historical data and projections (medium variant). Total population is used for within region weighting.	UN World Population Prospects (Rev. 2017); National Institute of Population and Social Security Research (IPSS) (2017) for Japan
Age-specific fertility rate	1990-2100	The age-specific fertility rate is the number of births to women in a particular age group divided by the number of women in that group. Historical data and projections (medium variant). Total population is used for within-region weighting.	UN World Population Prospects (Rev. 2017); National Institute of Population and Social Security Research (IPSS) (2017) for Japan
Current account (% GDP)	2000-2015	It shows the flows of goods, services, primary income, and secondary income between residents and non-residents.	World Bank World Development Indicators (2017); Ministry of Finance for Japan
Net External Wealth (% GDP)	1996-2019	It is the net international investment position (NIIP) of an economy, measuring at a point in time the difference between the value of financial assets of residents that are claims on non-residents (or are gold bullion held as reserve assets) and the liabilities of residents to non-residents.	IMF BoP and IIP Statistics (2020), World Bank World Development Indicators (2017); Bank of Japan
Capital-output ratio	1990-2017	Capital stock at current PPPs (in mil. 2011US\$) divided by output-side real GDP at current PPPs; GDP at current PPPs is used for within region weighting.	Penn World Table (rel.9.1); World Bank World Development Indicators (2017)