# The Visible Hand: Price Discrimination under Heterogeneous Precision*

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**Abstract**

Technological innovations have allowed some sellers to collect detailed information about buyers. We study these changes in a standard search-theoretic model of imperfect competition, featuring buyers with heterogeneous private valuations for quality, and introduce sellers who observe valuation signals of heterogeneous precision. Signals induce third-degree price discrimination, and their precision largely dictates whether they are used to increase trade versus increase markups - impacting aggregate surplus and its distribution. When buyers' valuations are more heterogeneous, imprecisely informed sellers prioritize high-markups despite limiting trade, and precision relaxes this tension, allowing them to pursue high-markups when it is least obstructive, and encouraging them to extend low-markup offers that increase trade upon signals indicative of low valuation - increasing aggregate surplus and benefiting (hurting) buyers with a low (high) valuation. However, when valuations are more homogeneous, imprecisely informed sellers prioritize trade, and precision can encourage them to extend high-markup offers that limit trade upon signals indicative of high valuation, hurting all buyers and even decreasing aggregate surplus. In either case, additional precision makes sellers more profitable, but its effect on competitors can be positive or negative. Competitors suffer (benefit) the most when laggards (leaders) gain precision.

**Keywords:** Asymmetric Information, Beliefs, Data, Imperfect Competition, Mechanism Design, Pricing, Search.

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# 1 Introduction

**Matt Murray (Wall Street Journal, Editor in Chief):** *The perception of a lot of people is that you've morphed...In a lot of ways, you're thought of as a data company more than a retail company.*
**Jeff Wilke (CEO, Amazon Worldwide Consumer):** *There was a corner pharmacy where I grew up. The pharmacist had been there forever. When you walked in, he knew what you liked to buy...That's the same thing we're doing. Our main purpose in storing your purchases is so that we can recommend something that you might want to buy the next time.*

-"Amazon's Defense of Private Brands" (*WSJ*, 10/24/19).

Amazon's well-documented harvesting and leveraging of consumer data exemplifies a sweeping transformation in how contemporary firms operate. A surge in the availability of data and in the power of analytical methods that uncover its insights[1] now helps firms better discern consumer preferences and tailor offers[2]. These methods have spread across industries, but diffusion within each has been heterogeneous. However, despite the importance of this pipeline, its impact on markets is not well understood.

There are several natural questions about this technological change. First, whether it benefits us in aggregate and as consumers is of immediate importance. In this vein, policymakers have raised concerns about potentially detrimental effects on competition (FTC (2012, 2013, 2014), CEA (2015), UK Competition and Markets Authority (2021)) and proactively moved towards containing them with wide-ranging measures, such as the EU's General Data Protection Regulation (European Commission (2012)). Second, how do firms benefit from investments in predictive technologies? The growing analytics gap[3] between Amazons of the world and more traditional businesses suggests that investment is profitable, but in the right context. So, what is that context? And, how does this type of investment affect competitors? To address these questions, we leverage a standard search-theoretic framework of imperfect competition, featuring buyers with heterogeneous private valuations for quality, and introduce sellers with buyer valuation signals of heterogeneous precision. In particular, sellers are differentiated by an ex-ante characteristic, their predictive skill, which is reflected in the precision of their signals. We characterize equilibria analytically, linking properties of information with properties of offers, and study the comparative statics of both precision and competition, documenting their effects on social surplus, in aggregate terms, as well as its distribution between and within buyers and sellers.

Precision fundamentally determines the impact of information on trade and the distribution of its gains. It shapes sellers' trade-off between profiting through higher sales versus higher markups, so sellers with different levels of precision have different offer strategies. In particular, sellers with more precise information are more willing to extend low-markup offers that increase sales upon signals indicative of low buyer valuations, and high-markup offers that decrease sales upon signals indicative of high buyer valuations - with opposite effects on trade efficiency. Nevertheless, precision always improves the efficiency of a chosen offer strategy, because it allows sellers to extend high-markup offers

[1] Since 2018, the McKinsey Global Institute has conducted a yearly survey on the "State of AI", "representing the full range of regions, industries, company sizes, functional specialties, and tenures". Approximately half of all firms consistently report the adoption of AI in at least one business function, while the average number of functions has doubled since the first survey.

[2] Mikians et al. (2012, 2013), Hannak et al. (2014), Chen et al. (2015) document the pervasiveness of these methods among sellers, both large and small, while industry surveys by Deloitte (2018) and McKinsey (2023) echo these points, respectively finding widespread use of AI for personalization and that "Marketing and Sales" along with "Product/Service Development" are the most common business functions for AI applications.

[3] In McKinsey's surveys, high-performing organizations are more than three times as likely to report that their data and analytics contributed at least 20% of earnings before interest and taxes.

when these are least obstructive to trade (when buyers have a high willingness to pay) and low-markup offers when these are most needed to trade (when buyers have a low willingness to pay). The net of these effects is such that precision can increase or decrease trade efficiency. When buyers' valuations are more heterogeneous, imprecisely informed sellers pursue inefficient high-markup strategies, and precision relaxes this motive; however, when buyers' valuations are more homogeneous, imprecisely informed sellers pursue efficient high-sale strategies, and precision can weaken this motive. Beyond its aggregate impact, precision is inherently redistributive, shifting not only the share of surplus between buyers and sellers, but also its distribution within each side of the market. On the demand side, high-valuation buyers - the principal targets of high markups - broadly suffer from precision, whereas low-valuation buyers - the principal victims of rationed trade - broadly benefit from it. On the supply side, additional precision improves the profitability of sellers, but its impact on competing sellers can be positive or negative. Precision relaxes competition for high-valuation buyers but intensifies competition for low-valuation ones, and either effect can dominate. Generally, we find that precision growth among laggards is associated with negative profit externalities, whereas precision growth among leaders is associated with positive profit externalities, allowing all sellers to benefit.

We confirm the traditional effects of competition, but also find a new - concerning - one. Characteristically, competition increases trade efficiency and buyer surplus. However, competition and imprecision both decrease the sensitivity of sellers' offers to their signals, even becoming signal insensitive. This complementarity results in low-precision sellers being particularly prone to forego using their predictive skill, and implies that competition can exacerbate the documented disparity in firm usage of these technologies.

Before proceeding with more detailed results, it is helpful to describe the components of our model. On one side of the market, there is a unit mass of buyers with low or high private valuations for the quality of a good. On the other side, there is a unit mass of sellers who produce the good operating a common technology with a convex cost for quality. We introduce imperfect competition by matching buyers and sellers in the style of Burdett and Judd (1983): each buyer matches with either one or two sellers who make simultaneous take-it-or-leave-it offers, without any information about the number of competitors in the match, beyond the commonly known matching protocol. In this reduced-form formulation, we capture the full range of competitive intensity, from monopoly to perfect competition, by varying the probability of matching with only one seller. Our innovation is (a) introducing additional seller information about buyers' valuations with flexible precision, and (b) allowing this ex-ante characteristic to be heterogeneous among sellers. We model this by assuming that the population distribution of buyer valuations is common knowledge but that each seller observes an informative signal about the matched buyer's valuation. Signals are essentially pairwise independent conditionally on the buyer's valuation, so they are only informative about the type of competing sellers through the signal's information about the buyer's valuation. For our purposes, a discretization with two relative levels of precision suffices: less precisely informed *amateurs* and more precisely informed *sharks*. This allows us to capture the full range of seller valuation information, from perfect[4] to fully imperfect (only prior information), by varying the absolute level of amateur and shark precision[5].

We introduce most of the core ideas in a simpler environment, where sellers offer an identical good of exogenously specified quality (a commodity), so that predictive skill only orients pricing, and then proceed to the general environment, where each seller can also choose the quality of the goods that it offers, so that predictive skill orients pricing and production. We find that information imbues

[4]Where $3^{rd}$ degree price discrimination becomes $1^{st}$ degree price discrimination.

[5]We could also study the effects of precision by varying the proportion of amateur and shark sellers, but the analysis would be analogous and less direct.

equilibrium offers with several ordering properties across both environments. Recalling the timing of this incomplete information game, sellers match with buyers, observe a valuation-relevant signal, and update their valuation beliefs. At this interim stage, a seller's type is summarized by its posterior probability that the matched buyer is of low-valuation. In equilibrium, we find that high-markup offers are more profitable in matches with high-valuation buyers, whereas high-sale offers, which share more of the gains from trade with buyers, are more profitable in matches with low-valuation buyers. Therefore, sellers of larger type, who place more weight on being in a low-valuation match, extend more attractive and efficient offers. And, precision is intimately connected with sellers' types. For one, precision makes it more likely that a seller observes a low (high) signal in a low- (high-) valuation match, and is of relatively high (low) type. But, precision also makes posterior valuation beliefs more extreme, because more precise signals trigger larger updates. Since sellers' prioritization of sales versus markups is dictated by their type, sharks who observe high signals make the least attractive/highest markup/most inefficient offers, while sharks who observe low signals make the most attractive/lowest markup/most efficient offers - amateur offers are intermediate in these aspects. Sharks are, therefore, more profitable because (1) they are more likely to observe the signal that orients toward the right profitability lever (sales versus markups), and (2) they exercise each lever more aggressively.

Nevertheless, when signals can inform production, there are significant differences. Equilibria change in form as sellers augment signal valuation information with that which can be extracted through screening menus, made up of the price-quality pairs for each type of buyer; in other words, they perform second- and third- degree price discrimination concurrently. The enhanced ability of sellers to extract buyer information rents changes how reliant they are on predictive skill. In particular, we find that under sufficient competition, amateur sellers ignore their signals, simply choosing to extend maximally efficient offers to all buyers, all the time. Efficiency is also measured differently. When demand information only informs pricing, efficiency of trade is measured at the extensive margin (whether a buyer finds an acceptable offer), but when demand information also informs production, trade efficiency is measured at intensive margin (how much quality a buyer obtains), since buyers always accept some seller's offer. Lastly, we learn that trends in precision provide an alternative explanation for trends in markups (De Loecker 2020), as precision leaders can have high markups (from trade with high-valuation buyers), high sales (from sales to low-valuation buyers), and high costs (from trading more quality with low-valuation buyers).

*Literature Review* - Our approach extends a tradition that leverages the canonical search-theoretic framework of Burdett and Judd (1983), which teaches us that imperfect information about the level of competition can generate price dispersion, as sellers choose mixed strategies. Garrett[6] et al. (2019) introduce buyers with heterogeneous valuations for a vertically differentiated good to this setting, which induces sellers to mix, but over screening menus. We advance this literature by enhancing sellers' information: our sellers (a) observe informative signals about matched buyers' valuations, and (b) the signal precision is heterogeneous. By introducing additional information, we can flexibly characterize the impact of precision on prices, products, and welfare; whereas, by accounting for its heterogeneity, we can characterize the impact of trends at different points in the precision distribution.

Generally, we contribute to the extensive literature on price discrimination, and particularly to the branches that study the welfare ramifications of second- and third-degree price discrimination. The latter's tradition extends back to Pigou (1920), while the former has been an active field since the seminal work of Mussa and Rosen (1978). A salient theme has been the double-edged nature of price discrimination: potentially increasing efficiency but also redistributing surplus. Our concern for the

[6]Contemporaneously, Lester et al. (2019) study the flipside of Garret et al.'s setting - a lemons problem with an analogous matching mechanism where privately informed sellers obtain bids from uninformed buyers.

principal factors that determine the sign of these effects aligns with research that has analyzed the influence of market structure[7] and demand characteristics[8]. Our contribution to this branch is two-fold. First, we bridge second- and third- degree price discrimination, as sellers practice both concurrently when quality is endogenous, which allows us to obtain the insight that less precisely informed sellers can choose to exclusively practice second-degree price discrimination. And second, we link the nature of price discrimination to novel factors. In detail, by having our sellers learn structurally, about individual buyers' preferences (rather than about aggregate reduced-form parameters), we are able to connect preference heterogeneity to the effects of price discrimination, whereas by modeling sellers' information in a rich fashion, we are able to do the same for the level and distribution of precision.

Our work also forms part of a growing literature that investigates the microfoundations and macroeconomic implications of the analytics pipeline. At the data collection stage, research[9] has focused on the incentives of buyers to disclose information about their preferences - trading off the desirability of product offers with their price or privacy costs. At the insight extraction stage, the role of intermediaries and their use of information design (repackaging data) to achieve various objectives, including aggregate and distributional welfare objectives[10], has been a main concern. These works are complementary to ours, since they investigate the process of generating predictive precision, meanwhile, we study its implications in diverse settings. In this vein, the link between data and market structure has been studied by a multidisciplinary literature that incorporates frameworks from macroeconomics, industrial organization, and finance[11]. We share some of the pro-welfare and pro-consumer effects of information and competition, but explicitly modeling the documented heterogeneity on both sides of the market allows us to parse their composition and pin-point the factors that reverse them, while our concern for the use of information to price discriminate allows us to derive new relations between precision and offer dispersion/order that can inform empirical work.

The paper proceeds as follows: in **??**, we introduce the general elements of the environment, in Section 3, we study economies where quality is exogenous, including their comparative statics, and in Section 4, we endogenize quality. Primarily technical aspects are developed in the Appendix, but we discuss them in varying degrees of detail in the main text where they reinforce the exposition.

## 2 Environment

We study a two-sided market that features heterogeneity on both the buyer and seller sides. A unit mass of buyers have single-unit demands and heterogeneous tastes for the quality of a traded good.

**Assumption 2.1** (Additively Separable Utility)**.** *A buyer with a marginal value for quality $\theta_i$ obtains utility $u(q, x; \theta_i) = \theta_i q - x$ from consuming a good of quality $q$ at a price of $x$.*

The utility of not trading is normalized to zero.

Sellers can produce a good of quality $q$ using a common cost function for quality $\phi(q)$. Upon a successful transaction with a buyer for a contract $(q, x)$ - a good of quality $q$ and price $x$ - sellers receive profits $x - \phi(q)$. The market between buyers and sellers follows an exogenously defined matching process. Following the style of Burdett and Judd (1983), a buyer matches with one or two randomly

[7] See Holmes (1989), Armstrong and Vickers (2001), Rochet and Stole (1997, 2002), Stole (2007), Vives (2011), and Rhodes and Zhou (2022).

[8] See Robinson (1933), Schmalensee (1981), Varian (1985), and Aguirre et al. (2010)

[9] See Braghieri (2019), Ichihashi (2020), Bonatti and Cisternas (2020), Hidir and Vellodi (2021), and Ali et al. (2023).

[10] See Bergemann et al. (2015, 2018), Elliot et al. (2021), Guo et al. (2022), Yang (2022), Haghpanah and Siegel (2023), Galperti et al. (2023), and Ichihashi and Smolin (2023).

[11] See Begenau at al. (2018), Agrawal et al. (2018, 2019), Kehoe et al. (2020), Farboodi and Veldkamp (2022), and Eeckhout and Veldkamp (2022)

and independently drawn sellers with the probability of a single match given by $\tilde{\rho} \in (0, 1)$. Because we assume the number of matches is only privately known by the buyer, sellers face uncertainty as to whether they are competing against a peer for a buyer's purchase within a given match. As such, matched sellers assign not competing a probability,

$$\rho = P(\text{single match}|\text{matched}) = \frac{\tilde{\rho}}{\tilde{\rho} + 2(1 - \tilde{\rho})} \tag{2.1}$$

where $\rho$ governs the level of competition, ranging from the extremes of perfect competition ($\rho = 0$) to monopoly ($\rho = 1$).

While the ex-ante distribution of buyer valuations is common knowledge, sellers also observe a private signal about the valuation of each buyer with whom they match. Our principal innovation is to introduce heterogeneity amongst sellers in the precision of these signals[12]. We consider the simplest discretization of precision, where the sellers are either less precisely informed *amateurs* or more precisely informed *sharks*. Their respective signal precisions are denoted

$$\alpha_e = P^e(\text{signal} = i|\text{matched buyer's valuation} = \theta_i) \quad i \in \{l, h\} \tag{2.2}$$

$$0.5 \leq \alpha_a < \alpha_s < 1 \tag{2.3}$$

In this sense, the two relative precision categories of amateur and shark will remain fixed throughout the paper, even if the absolute level of precision that they represent differs. The mass of amateurs (sharks) is $\mu(e) = \gamma$ ($\mu(s) = 1 - \gamma$).

Agents do not make choices before matching, so our setting is that of an incomplete information game that is solved at the interim stage, after matches have formed and sellers have observed their signals. A seller's type is then its posterior belief of being in a low match, $p_l^{e,j} = P^e(\theta = \theta_l|j)$,

$$p_l^{e,j} = \frac{P^e(j|\theta_l)p(\theta_l)}{P^e(j|\theta_l)p(\theta_l) + P^e(j|\theta_h)p(\theta_h)} = \frac{\alpha_e p(\theta_l)}{\alpha_e p(\theta_l) + (1 - \alpha_e)p(\theta_h)} \tag{2.4}$$

Having introduced the environment, we will first analyze the role of information in a simple setting where sellers produce a good of exogenously fixed quality, such as a commodity, so optimal offers take the form of a single quoted price per unit $x$. Subsequently, we will endogenize quality, allowing sellers to choose the quality of the goods that they offer. This will allow us to analyze the connection between precision and fit of the products available to buyers, but result in a more complex set of optimal offers that take the form of screening menus composed of quality-price contracts $((q_l, x_l), (q_h, x_h))$.

# 3 Exogenous Quality Setting

For the remainder of this section, we assume that all sellers produce a homogeneous good of exogenously fixed quality for free.

**Assumption 3.1.** *Exogenous Quality Setting Assumptions The traded good has quality $q = 1$ and a cost of production, $\phi(1) = 0$.*

This means that there are gains from trade with both types of buyers, and that each buyer simply chooses the lowest price offer it obtains, provided it is below her valuation $\theta_i$. The exclusion of product design (quality) choices will allow us to isolate the role of the pricing channel, but still characterize

[12] In practice, precision can be improved by acquiring more/better data or by improving the methods with which data are analyzed. Heterogeneity in precision is, therefore, natural when we consider the variability in firms' data and analytical resources.

some of the main relationships between precision, the properties of offers, and welfare that extend to settings where quality is endogenous.

We find that sellers who are more convinced that they are matched with high-valuation buyers offer higher prices, often beyond the willingness to pay of low-valuation buyers, and thus forego trade in some matches. Inversely, sellers who are more convinced that they are matched with low-valuation buyers offer lower prices, often substantially below the willingness to pay of high-valuation buyers. Both types of mispricing lower profitability, but pricing out low-valuation buyers also decreases the efficiency of trade. More precise valuation information makes any given (offer) strategy more profitable and efficient because it reduces these instances of mispricing; however, it can also induce sellers to adjust their strategies. Precision gives sellers the confidence to offer larger and more frequent discounts upon signals indicative of low valuation, but also to offer larger and more frequent price hikes upon signals indicative of high valuation. While the former increases trade, the latter limits it. Naturally, any adjustment in strategy is towards greater profitability, and we will find that, in equilibrium, additional precision always benefits sellers who upskill, but its externalities on other agents (buyers or competitors) can be positive or negative, as these are always the net of the direct effect on any given seller strategy and the equilibrium effect that induces sellers to choose different strategies.

Generally, we find that high-valuation buyers and competitors suffer from a seller's gain in precision, while low-valuation buyers and trade efficiency benefit, but each of these trends can be reversed. Before characterizing the determinants of distributional and aggregate effects, we will formally introduce the seller's problem and structure of equilibria.

## 3.1 Seller Problem and Equilibrium Concept

The expected profits of a type $p_l^{e,j}$ seller from a price offer $x$ are the product of its profits per sale, $x$, times its expected sales, $P^e(\text{sale at price } x|j)$. As in private value auctions, sellers know the value of having an offer accepted at the time of submission, the profits per sale $x$, but must infer the probability of its acceptance. We call the *sales* associated with an offer exactly this probability, which reflects the uncertainty a seller has about four variables: (1) the buyer's valuation, (2) whether the buyer is matched with another seller, (3) the type of such a competitor, and (4) the offer that this type of competitor would make. A type $p_l^{e,j}$ seller's optimal prices are therefore,

$$x^{e,j}(P^e(\cdot|j)) = \underset{x\in\mathbb{R}}{\operatorname{argmax}}\, P^e(\text{sale at price } x|j)x \tag{3.1}$$

and its strategy is a cumulative distribution function $F^{e,j}(x)$ over prices. This type's expected sales can then be decomposed as,

$$\begin{aligned} P^e(\text{sale at price } x|j) &= \rho P^e(\text{sale at price } x|\text{monopoly match}, j) \\ &+ (1-\rho)\Bigg(P^e(\theta_l|\text{competitive match}, j)P^e(\text{sale at price } x|\theta_l, \text{competitive match}, j) \\ &+ P^e(\theta_h|\text{competitive match}, j)P^e(\text{sale at price } x\theta_h, \text{competitive match}, j)\Bigg) \\ &= \rho(p_l^{e,j}\mathbb{1}(x\leq\theta_l) + p_h^{e,j}) + (1-\rho)\Bigg(p_l^{e,j}(1-F(x|\theta_l))\mathbb{1}(x\leq\theta_l) + p_h^{e,j}(1-F(x|\theta_h))\Bigg) \end{aligned} \tag{3.2}$$

where $F(x|\theta_i)$ is the seller's expected distribution of competitor price offers matches with $\theta_i$ valuation buyers. This decomposition highlights and quantifies the distinct sources of uncertainty facing sellers - the buyer's valuation ($p_i^{e,j}$ terms), the existence of competitors ($\rho$ terms), and their competing offers ($F(x|\theta)$ terms). This last term encapsulates uncertainty about the type of competitors and their chosen price, emerging naturally as an average of each seller type's strategy,

$$\begin{aligned}F(x|\theta) &= \sum_{\substack{e\in\{a,s\}\\ j\in\{l,h\}}} P(e \text{ competitor})P(e \text{ seller observes } j|\theta)P(e \text{ seller offer price below } x|\text{signal } j)\\ &= \sum_{\substack{e\in\{a,s\}\\ j\in\{l,h\}}} \mu(e)P^e(j|\theta)F^{e,j}(x) \end{aligned} \tag{3.3}$$

where the weights originate from the commonly known matching protocol and signal structure: $\mu(e)$ is the mass of $\alpha_e$ precision sellers in the population, $P^e(j|\theta)$ is the probability that they observe a $j$ value signal in a match with a $\theta$ valuation buyer, and $F^{e,j}(x)$ is the cumulative distribution of prices they would offer upon observing $j$.

**Definition 3.1** (Bayes-Nash Equilibrium)**.** *A Bayes-Nash equilibrium of an economy with exogenous quality is a vector of seller strategies, $\{F^{e,j}\}_{\substack{e\in\{a,s\}\\ j\in\{l,h\}}}$, for each type of seller, satisfying* $\text{supp}(F^{e,j}) \subseteq x^{e,j}(P^e(\cdot|j))$ *for all $(e,j) \in \{a,s\} \times \{l,h\}$, when their expected sales are given by* (3.2)-(3.3).

These equilibria are quite tractable. In particular, strategies and offer distributions are continuous, almost everywhere differentiable, and strictly increasing on at most two convex interval(s). And, their qualitative are intuitive but also revealing, such as the fact that higher prices are more profitable in high matches and are offered by sellers whose posteriors place more weight on being matched with a high-valuation buyer - crisply illustrating the role of information. The proofs of these points are constructive, heavily reliant on economic reasoning, and relatively brief, so it will be insightful to go through them in the main text.

## 3.2 Equilibrium

### 3.2.1 Corner Cases: Monopoly and Perfect Competition

We first analyze the two corner cases of monopoly ($\rho = 1$) and perfect competition ($\rho = 0$) to fix ideas. These easily yield key insights that extend to economies where competition is imperfect ($\rho \in (0,1)$) while also motivating their study, as the limitations of the analysis in the corners will be apparent.

**Monopoly -** A monopolist's pricing choice involves the traditional trade-off between sales and profits per sale. Matched buyers have no alternative offers; they either accept the seller's offer to buy the good for a price of $x$ and obtain utility $\theta_i - x$, or they reject it and obtain zero utility. Optimal take-it-or-leave-it offers must, therefore, give zero surplus to the lowest (valuation) buyer who would accept it, meaning that a seller's optimal offer must be equal to some buyer's valuation. A monopolist sets a sales maximizing price of $x = \theta_l$, and trades with every buyer, or a profit-per-sale maximizing price of $x = \theta_h$, and only trades with high-valuation buyers depending on which has greater expected revenues (recall that costs are zero),

$$\theta_l \lesseqgtr (1 - p_l^{e,j})\theta_h \iff p_l^{e,j}\theta_l \lesseqgtr (1 - p_l^{e,j})(\theta_h - \theta_l)$$

Which of the two prices the monopolist prefers is determined by her valuation beliefs, summarized by $p_l^{e,j}$. If she is sufficiently convinced that the buyer's valuation is low (high $p_l^{e,j}$), the low price $x = \theta_l$ is optimal, since the expected value of its additional sales, $p_l^{e,j}\theta_l$, is greater than the expected cost of its discount, $(1 - p_l^{e,j})(\theta_h - \theta_l)$; otherwise, if she is sufficiently convinced that the buyer's valuation is high, she expects extracting their information rents to be the most profitable action and chooses a price of $x = \theta_h$. The threshold probability $p_l^*$ dictating this choice is given by,

**Proposition 3.1** (Monopoly). *If $\rho = 1$, a seller of type $p_l^{e,j}$ offers a price*[13]*,*

$$x^*(p_l^{e,j}) = \begin{cases} \theta_h & \textit{if } p_l^{e,j} < p_l^* \\ \theta_l & \textit{otherwise} \end{cases} \tag{3.4}$$

*for the threshold probability,*

$$p_l^* = \frac{\theta_h - \theta_l}{\theta_h} \tag{3.5}$$

Our first general takeaway is that precision increases price dispersion, as posteriors of sellers with more precise information (larger $\alpha_e$) are more dispersed ($p_l^{e,l} - p_l^{e,h}$ increases), and posterior dispersion makes it more likely that upon a low versus high signal the seller is above versus below the threshold $p_l^*$, respectively. Said differently, precision dictates whether sellers choose to practice third-degree price discrimination, a salient point that will reappear when we endogenize quality choices. A monopolist who does not price discriminate sets either a uniformly low or high price, so a precision-induced switch to price discrimination can have very different impacts on trade efficiency and buyer welfare, depending on which of the two signal-invariant prices would have been chosen under less precise signals.

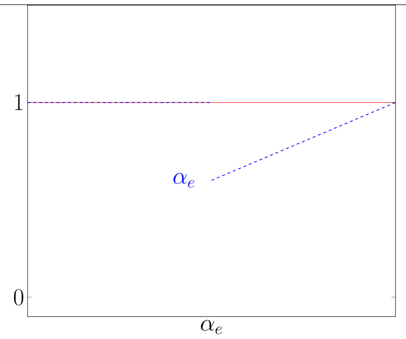
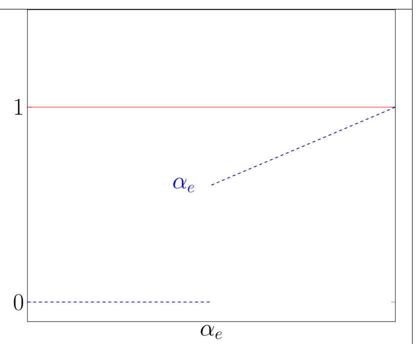
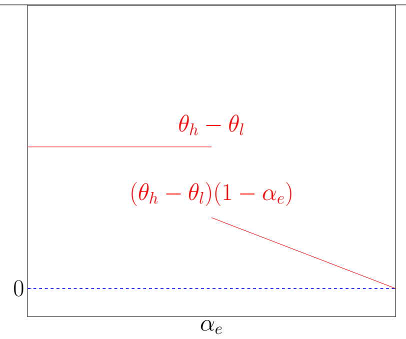
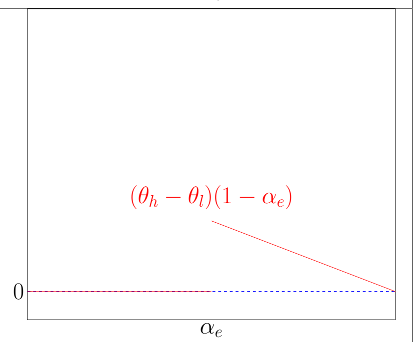
| Valuation Heterogeneity $\theta_h - \theta_l$ / Y-axis Variable | **Low** | **High** |
|---|---|---|
| Probability of Trade: $\mathbb{P}(x \leq \theta_i \mid \theta_i)$ |  |  |
| Buyer Surplus: $\mathbb{E}[\,u_i\,]$ |  |  |

**Table 1:** Consider a setting where every seller has precision $\alpha_e$ and buyers always match with a single seller ($\tilde{\rho} = \rho = 1$). Low (dashed) versus High valuation buyer outcomes.

When buyers' valuations are sufficiently heterogeneous, imprecisely informed sellers face severe adverse selection, so they set a uniformly high price ($\theta_h$) and only trade with high-valuation buyers. They only have an incentive to offer a low price ($\theta_l$) upon a low signal, when signals are sufficiently precise, so precision is crucial for efficiency in these settings. The switch to price discrimination not

[13]We break profitability ties in favor of trade, but that is immaterial.

only increases trade with low-valuation buyers, the gains of which are entirely captured by sellers, but it also allows high-valuation buyers to obtain some information rents from instances of mispricing (with probability $1-\alpha_e$ in each match). In this sense, additional precision can make all agents weakly better off.

However, when buyers' valuation are relatively homogeneous, imprecisely informed sellers face little adverse selection, so they set a uniformly low price ($\theta_l$) and trade with both types of buyer. Analogously, they only have an incentive to offer a high price ($\theta_h$) upon a high signal, when signals are sufficiently precise, but the switch to price discrimination opens the door to mispricing. Low price offers to high-valuation buyers simply redistribute, transfering seller surplus to high-valuation buyers, but high price offers to low-valuation buyers are inefficient, decreasing aggregate surplus.

This role of preference heterogeneity in the relation between precision and efficiency as well as the distribution of surplus generalizes, but in economies where competition is imperfect ($\rho \in (0,1)$), we will be able to analyze the impact of precision on other sellers - its profit externalities - and on the nontrivial surplus (greater than zero) of low-valuation buyers.

**Perfect Competition** - At the other end of perfect competition ($\rho = 0$), where sellers engage in Bertrand price competition and the unique equilibrium outcome is them offering the good at a price equal to its cost and buyers capturing all gains from trade.

**Proposition 3.2** (Perfect Competition). *If $\rho = 0$, the unique Bayes-Nash equilibrium is for almost every seller to price the good at cost, $x = 0$, with probability 1.*

This is, therefore, the wrong setting to study the precision of sellers' information about buyers' valuations for a vertically differentiated good because it is irrelevant. Nevertheless, this level of competition highlights its classic pro-efficiency and pro-consumer surplus effects, and these extend to economies with imperfect competition.

### 3.2.2 Structure

By (3.3), equilibrium distributions of offers in a low or high match - $F(x|\theta_l)$ and $F(x|\theta_h)$, respectively - are averages of each type of seller's strategy, weighted by the mass of sellers with the respective type in matches with buyers of the respective valuation. Since buyers of either valuation have a strictly positive probability of matching with any type of seller, these distributions share identical supports - their differences lie in how mass is allocated within these.

**Proposition 3.3** (Identical Supports). *Equilibrium price distributions $(F(x|\theta_l), F(x|\theta_h))$ of competitor price offers in low and high matches have identical supports,*

$$\text{supp}(F(x|\theta_l)) = \text{supp}(F(x|\theta_h)) \tag{3.6}$$

Further, so long as some buyers obtain one offer but others obtain two, the equilibrium distributions - both aggregate price distributions and, by extension, seller strategies - are atomless.

**Proposition 3.4** (Continuous Distributions). *Equilibrium distributions $F(\cdot\,|\,\theta_i)$ and $F^{e,j}(\cdot)$ are atomless.*

Sellers always have a strictly positive probability of competing against a peer of identical type, so a price greater than its unit cost (0) cannot be an atom, because deviating with an infinitesimal discounts would sacrifice negligible profits per sale in exchange for a discrete increase in sales. Additionally, any price offer equal to the unit cost is strictly dominated by some more expensive one, because the latter would generate some profits in each sale and at least generate sales in noncompetitive matches. This rules out an atom at unit cost and implies that equilibrium distributions of offers, $F(x|\theta)$, are continuous.

Since a loss of sales is the only deterrent to a price increase, and sales are only lost by becoming more expensive than another seller's offer or a buyer's valuation, offer distributions must be locally increasing at any price in their support that is not equal to some buyer's marginal valuation.

**Proposition 3.5** (Strictly Increasing Distributions). *Given prices $x, x' \in \operatorname{supp}(F(x|\theta))$ such that $x < x' < \theta_l$ or $\theta_l < x < x' < \theta_h$, $F(x|\theta)$ is strictly increasing on the interval $[x, x']$ for $\theta \in \{\theta_l, \theta_h\}$.*

An immediate implication is that the price offers to any buyer are distributed in at most two disjoint intervals: one formed by prices weakly below the low-valuation, $x \leq \theta_l$, and another formed by prices strictly above it, $\theta_l < x$. As such, we only need to find the endpoints of these intervals to fully characterize the support of prices - below the lowest price the distributions take value zero and above the highest they take value 1, naturally.

We start with the suprema of low-trade-permitting and overall prices. Since offer distributions are atomless (Proposition 3.4), a seller who offers the highest overall price expects to only successfully sell in a monopoly match, so it bids accordingly, as per Equation (3.4). Comparing across sellers types, the one most willing to offer a high price at the cost of ceding all potential sales outside of monopoly matches is exactly the type of seller who places the greatest posterior weight of being matched with a high-valuation buyer. The link between precision and posteriors,

$$p_l^{s,h} < p_l^{a,h} < p_l < p_l^{a,l} < p_l^{s,l} \tag{3.7}$$

therefore, makes sharks who observe high signals the principal candidates for offering the highest equilibrium price, and this price is equal to high-valuation buyers' valuation, whenever these types of sellers expect such an offer to be optimal in monopoly matches.

**Proposition 3.6** (Highest Price). *The highest overall equilibrium price,*

$$\overline{x} = \begin{cases} \theta_h & \text{if } p_l^{s,h} \leq p_l^* \\ \theta_l & \text{otherwise} \end{cases} \tag{3.8}$$

Similarly, a seller who offers the highest low-trade-permitting price only expects to sell to a low-valuation buyer without any lower offers from competing sellers, so it chooses the highest price that every such buyer would accept, mainly $x = \theta_l$.

**Proposition 3.7** (Form of Supports). *For $\theta_i \in \{\theta_l, \theta_h\}$,*

$$\operatorname{supp}(F(x|\theta_i)) = \begin{cases} [\underline{x}, \theta_l] \cup [\hat{x}, \theta_h] & \text{if } p_h^{s,h}\theta_h > \theta_l \\ [\underline{x}, \theta_l] & \text{otherwise} \end{cases} \tag{3.9}$$

*where $\theta_l < \hat{x}$ is the lowest equilibrium price offer that only allows trade with high-valuation buyers.*

It is conceptually important to highlight that price offers above low-valuation buyers' valuation are separating: they are only ever accepted by high-valuation buyers and always rejected by low ones. As such, they are chosen to be optimal in high matches; in other words, sellers offering such prices anticipate that they will only ever be accepted by high-valuation buyers and optimize conditional on that outcome. By removing considerations about being in a low or high match and focusing entirely on high trade, every type of seller faces the same incentives when choosing separating prices, so they all agree on the optimal set of separating prices. Some types just expect that such prices sacrifice too many sales in expectation, and instead choose pooling offers that also allow them to trade with low-valuation buyers.

Since every type of seller has the same criteria for optimal separating prices, the equation that pins down their equilibrium distribution is independent of sellers' type and follows from the necessary equilibrium condition of maintaining equal expected profits in high matches. We have argued that the highest price is either $\theta_l$ or $\theta_h$, and only the latter is separating, so separating offers should be as profitable as an offer of $\theta_h$,

$$\rho\theta_h = (\rho + (1-\rho)(1 - F(x|\theta_h)))\, x \tag{3.10}$$

where we implicitly apply the convexity of the separating support (Proposition 3.7).

**Proposition 3.8** (Separating Prices in High Matches). *If* (3.2) *holds, the equilibrium distribution of offers in a high match is,*

$$F(x|\theta_h) = 1 - \frac{\rho}{1-\rho}\frac{\theta_h - x}{x} \quad \forall \hat{x} \le x \le \theta_h \tag{3.11}$$

An offer of $\theta_h$ maximizes (minimizes) high (low) match profitability,

$$\Pi(x = \theta_h|\theta = \theta_l) = 0$$
$$\Pi(x = \theta_h|\theta = \theta_h) = \theta_h$$

and separating prices preserve these points, so they can only be offered by sellers who place sufficient posterior probability on high matches. As such, there exists a threshold posterior $\hat{p}_l \le p_l^*$ satisfying,

$$\hat{p}_h \Pi(x = \theta_h|\theta_h) = \sup_{x' \le \theta_l} \left(\hat{p}_l \Pi(x = x'|\theta_l) + \hat{p}_h \Pi(x = x'|\theta_h)\right) \tag{3.12}$$

that dictates which types of sellers make separating offers - if any.

**Proposition 3.9** (Threshold Posterior). *Given any equilibrium, there exists a threshold posterior,*

$$\hat{p}_l = \sup_{x' \le \theta_l} \frac{\Pi(x = \theta_h|\theta_h) - \Pi(x = x'|\theta_l)}{\Pi(x = \theta_h|\theta_h) + \Pi(x = x'|\theta_l) - \Pi(x = x'|\theta_h)} \tag{3.13}$$

*such that,*

- ***Separating Types:*** *Sellers of lower type, $p_l^{e,j} < \hat{p}_l$, only offer separating prices.*
- ***Dual Type:*** *Sellers of equal type, $p_l^{e,j} = \hat{p}_l$, offer separating and pooling prices.*
- ***Pooling Types:*** *Sellers of larger type, $\hat{p}_l < p_l^{e,j}$, only offer pooling prices.*

This coarse grouping of prices is a manifestation of an ordering property, linking the indirect utility that sellers offer buyers with their posterior valuation assessments, which is characteristic of economies where sellers have predictive skill - heterogeneous or not. In particular, we will see that not only in the current setting where quality is exogenous, but also later in the general setting where quality is endogenous, the offers of sellers who place more posterior weight on low matches will be weakly more attractive to buyers of either valuation, and this relationship will often be strict. In the current setting with exogenous quality, when any separating prices are offered, pooling prices must be weakly decreasing in the type of the seller who offers them, and the ordering must be strict when only sellers who observe low signals offer them. Separating prices don't have to satisfy ordering relations with respect to the seller's type but it will be without loss of generality to assume that they are strictly ordered as well. Ordered equilibria are tractable, intuitive, provide a crisp analytical characterization of the equilibrium structure that predictive skill generates, and allow us to study the comparative statics unambiguously (there is a unique ordered equilibrium). We will present the reasoning that underlies orderedness, deriving core insights about the importance of precision and its heterogeneity in the process.

Consider the type of seller who offers the highest equilibrium pooling price $x = \theta_l$. This offer is only accepted in monopoly matches or in competitive matches where a separating price - all of which are higher - is offered, but since these prices are offered by sellers with low enough posterior $p_l$ beliefs (below $\hat{p}_l$ in Proposition 3.9), they are more likely in matches with high-valuation buyers[14]. As such, the more a particular seller expects to face high-valuation buyers, the more it would expect to win in such matches offering the highest pooling price; in other words, a seller's expected sales from the highest pooling price decrease in its type $p_l^{e,j}$, and strictly so if any separating prices are offered in equilibrium. This is sufficient to uniquely identify the type of seller who offers the highest pooling price in the latter equilibria.

**Proposition 3.10** (Lowest Pooling Seller Type)**.** *If separating prices are offered in equilibrium, the highest pooling price $x = \theta_l$ is only offered by (a) sellers of the threshold type $p_l^{e,j} = \hat{p}_l$ or (b) sellers of the lowest type above this threshold $p^{e,j} > \hat{p}_l$, in which case sellers of types $\geq p_l^*$ only offer separating prices.*

However, when all equilibrium offers are pooling, there is no reason why offers should necessarily be ordered, and indeed there exists a type-invariant equilibrium where every type of seller has the same mixed strategy, so precision does not play a role. We solve for this invariant equilibrium in the Appendix, but focus in the main text on economies where predictive skill is not superfluous.

**Assumption 3.2** (Some Separating Offers)**.** *Shark precision is such that some of their offers, upon observing a high signal, are separating ($p_l^{e,h} < \hat{p}_l$).*

A sufficient for the existence of an ordered separating equilibrium is that sellers who observe high signals offer separating prices.

**Assumption 3.3** (All Pooling Offers made by $l$ Signal Sellers)**.** *All sellers who observe high signals only offer prices $x > \theta_l$.*

In our comparative static analysis, we will construct an ordered equilibrium, and if some high signal sellers offer pooling prices, verify that no deviations exist.

We can obtain the distribution of offers over the remaining pooling offers in economies that satisfy Assumption 3.2 and Proposition 3.11 through an inductive argument that extends the previous expected sales logic. In particular, given an $x < \theta_l$ and the inductive assumption that there exists a $p_l'$ such that equilibrium prices $x < x' \leq \theta_l$ are offered only by sellers of type $p_l^{e,l} \leq p_l'$, then the foregone profits of a seller who considers a discount are decreasing in its type, since $F(x|\theta_l) \leq F(x|\theta_h)$, but the gain in sales that it expects are increasing in its type, as this makes the expected mass of $p_l^{e,l}$ sellers who offer these prices larger.

[14]Which are more likely produce the high signal that gives rise to low posterior $p_l$s among sellers

**Proposition 3.11** (Pooling Price Ordering). *If* (3.2) *and hold, for any pair of equilibrium prices* $x \in \text{supp}(F^{e,j}), x' \in \text{supp}(F^{e',j'})$,

$$x < x' \quad \iff \quad p_l^{e,j} \leq p_l^{e',j'} \tag{3.14}$$

It is also without loss to assume that this strict ordering extends to separating prices, since all sellers agree on the optimal separating prices and the equilibrium distribution of offers in high matches is uniquely determined over these separating prices by (3.11).

**Lemma 3.1** (Separating Price Ordering). *Given an equilibrium distribution of competitor prices* $(F(x|\theta_l), F(x|\theta_h))$*, there exists a set of optimal seller strategies* $\{F^{e,j}(x)\}_{\substack{e\in\{a,s\},\\ j\in\{l,h\}}}$

- *Equation* (3.3) *is satisfied.*
- *Separating Price Ordering: Given any two types* $p_l^{e,j} < p_l^{e',j'} \leq \hat{p}_l$,

$$[\max(\hat{x}, \underline{x}^{e,j}), \overline{x}^{e,j}] = [\hat{x}, \theta_h] \cap \text{supp}(F^{e,j}(x)) < \text{supp}(F^{e',j'}(x)) = [\overline{x}^{e,j}, \overline{x}^{e',j'}]$$

  *with* $\hat{p}_l$ *satisfying* (3.13).

Recapping, so long as any separating prices are offered in equilibrium and only sellers who observe high signals offer separating prices, there exists an equilibrium where prices are strictly ordered by the seller's type: if both prices being compared are separating the ordering property is imposed without loss, if one price pools and the other separates ordering is necessary, whereas if both prices pool then ordering is also necessary under Proposition 3.11.

The ordering of each seller type's strategy implies that both the equilibrium distributions of competitor prices in either type of match and the profitability of prices in either type of match both have ordering properties. As for the distributions, note that the average mass of sellers from each type in a match depends on the buyer's valuation - in high (low) matches, more amateurs and sharks observe a signal of $h$ ($l$) - so, when higher types offer low prices, the equilibrium distribution of offers in high matches first-order stochastic dominates that in low matches (by (3.3)).

**Proposition 3.12** (Competitor Bid Ordering). *If the supports of seller types' price distributions are ordered as per,*

$$\text{supp}(F^{e,l}) \leq \text{supp}(F^{e,h}) \quad \textit{for } e \in \{a, s\} \tag{3.15}$$

*the distribution of prices in high matches first-order stochastic dominates that in low matches,*

$$F(x|\theta_h) < F(x|\theta_l) \quad \forall x \in \text{supp}(F(y|\theta)) \tag{3.16}$$

For the profitability ordering, note that separating offers are equally profitable in either type of match, but the equilibrium profitability of pooling offers is such that the types who offer them are indifferent. Necessarily then, for any two prices $x, x' \in \text{supp}(F^{e,j})$ offered by sellers of type $p_l^{e,j}$,

$$p_h^{e,j}\Pi(x|\theta_h) + p_l^{e,j}\Pi_h(x|\theta_l) = p_h^{e,j}\Pi(x'|\theta_h) + p_l^{e,j}\Pi_h(x'|\theta_l) \tag{3.17}$$

$$\frac{p_l^{e,j}}{p_h^{e,j}} = \frac{\Pi(x|\theta_h) - \Pi(x'|\theta_h)}{\Pi(x'|\theta_l) - \Pi(x|\theta_l)} \tag{3.18}$$

so cheaper pooling offers are more (less) profitable in low (high) matches. Of course, any trade-off between high and low match profits that makes a seller of type $p_l^{e,j}$ indifferent also incentivizes sellers of lower (higher) type to bid above (below), and this is why offer-to-profitability monotonicity and offer-to-seller-type monotonicity are two sides of the same coin.

**Lemma 3.2** (Offer Profitability Ordering)**.** *In an ordered equilibrium,*

- *All separating offers yield the identical average profits in low (high) matches.*
- *The average profits in low (high) matches of pooling offers increase as these become cheaper.*

This relation will be important in unpacking the profit externalities of precision because it influences the types of offers that other sellers make and thus the kinds of ex-post regret that they face.

Lastly, we note that, when an ordered equilibrium exists, it is unique. This follows by induction, since the optimal offers of a seller are determined by the mass of bids above, which shape its expected sales, and these are ordered by the offering seller's type. The familiar base case is that of sellers with the lowest type, sharks who observe $h$. No type of seller bids above them, so their strategy is uniquely determined. Inductively, given a seller type $p_l^{e',j'}$ and uniquely determined strategies of lower type sellers ($p_l^{e,j} < p_l^{e',j'}$) who offer higher prices, the strategy of a $p_l^{e',j'}$ type seller is also uniquely determined. We carry out this explicit process and derive sellers' strategies $\{F^{e,j}\}_{\substack{e\in\{a,s\}\\ j\in\{l,h\}}}$ in the Appendix.

**Proposition 3.13** (Unique Ordered Equilibrium)**.** *There is a unique ordered equilibrium.*

## 3.3 Over- and Under-bidding

Sellers' offers are optimal under their interim expected sales assessments. However, since their information is imperfect, their offers in each match are not necessarily the most profitable ones ex-post. These offers are the ones that maximize profits when sales are conditioned on all information about the match - the buyer's valuation, the existence of competing sellers, and the type of any competitors.

Information about the number of competitors and their type complements that of the buyer's valuation, so we can generalize Lemma 3.2 to situations where an individual seller might have varying degrees of additional information. In particular, we consider three levels of nested information where the seller knows (1) the buyer's valuation, the presence of a competing seller, and its type, (2) the buyer's valuation and the presence of a competing seller, and (3) only the buyer's valuation.

**Lemma 3.3** (Optimal Prices Conditionally on Match Type)**.** *In the ordered equilibrium, conditionally on the buyer's valuation, the number of competitors, the type of the competitor,*

- *If the buyer's valuation is high, the optimal competitive price is* $X^*(\theta_h, \textit{competitive}, p_l^{e,j} \textit{ type competitor}) = \underline{x}^{e,j}$.
- *If the buyer's valuation is low, the optimal competitive price is* $X^*(\theta_l, \textit{competitive}, p_l^{e,j} \textit{ type competitor}) = \min(\underline{x}^{e,j}, \theta_l)$.

*whereas, conditionally on only the buyer's valuation and the number of competitors,*

- *If the buyer's valuation is high, the optimal price is* $X^*(\theta_h, \textit{competitive}) = \hat{x}$.
- *If the buyer's valuation is low, the optimal price is* $X^*(\theta_l, \textit{competitive}) = \underline{x}$.

*and, conditionally on only the buyer's valuation,*

- *If the buyer's valuation is high, the set of optimal price offers is* $X^*(\theta_h) = [\hat{x}, \theta_h]$.

- *If the buyer's valuation is low, the optimal price is $X^*(\theta_l) = \underline{x}$.*

*Whereas, if the seller knew that it has no competitors and the buyer's valuation, then its optimal price offer is the valuation.*

Since equilibrium distributions of prices are atomless (by Proposition 3.4), almost every offer that sellers make is suboptimal under the perfect information posterior. Naturally, if they sell, their offer in that particular match was too low (over-bid buyer surplus), and inversely if they do not (under-bid buyer surplus), but much of this regret is unavoidable. Indeed, no seller has information about the number of competitors, and even knowing a competitor's type, there would remain uncertainty about its offer, since equilibrium strategies are mixed. However, a part of regret is linked to the buyer's valuation, as winning (losing) makes having been in a high (low) match more likely, and sellers with more predictive skill have more precise valuation information. The right measure of ex-post regret, therefore, compares a seller's offer with the optimal one it would have made if it had known the buyer's valuation. Under this metric, sharks' more precise information brings their posterior closer on average to the one conditioned on the buyer's valuation and allows them to obtain higher profits/lower regret.

To connect with our analysis of economies with endogenous quality choices, it is helpful to take a closer look at the connection between an offer's type (separating vs. pooling) and its vulnerability to each type of regret (under- versus over-bidding regret). Under-bidding affects both separating and pooling offers, as a loss indicates having been in a low match and offered too high a price ($x > X^*(\theta_l)$). However, over-bidding only affects pooling offers, as a win indicates having been in a high match and separating offers are optimal in these, but pooling offers are not ($x < X^*(\theta_h)$). Separating offers, therefore, allow sellers to minimize their over-bidding and will be the preferred mechanism in economies with endogenous quality choices, where sellers can screen even more profitably by offering menus of price-quality contracts $((q_l, x_l), (q_h, x_h))$.

## 3.4 Comparative Statics

With a clear understanding of the structure and analytical form of ordered equilibria, we study the effect of competition and precision on prices, trade efficiency, and the level as well as the distribution of aggregate surplus between and within buyers and sellers.

In the case of buyer welfare, we examine the average utility that buyers of each valuation obtain,

$$\mathscr{W}_b(\theta) = \rho E[u(1, \min(\theta, x_i); \theta)] + (1-\rho) E[\max_{i \in \{1,2\}} u(1, \min(\theta, x_i); \theta)] \tag{3.19}$$

where $x_i$ are iid draws from the conditional equilibrium marginals $F_i(x_i|\theta_i)$. Seller profits from low- and high-valuation buyer matches depend on (1) the average number of matches they are in, (2) their sales per match, and (3) their profits per sale. Since sellers with equal precision are otherwise identical, we track average profits in low and high matches conditioning on this ex-ante attribute,

$$\mathscr{M} = \tilde{\rho} + 2(1-\tilde{\rho}) \tag{3.20}$$

$$\Pi^e = \Big[ \underbrace{\mathscr{M}}_{\text{number of matches}} \underbrace{P^e(\text{sale at price } x, j)}_{\text{probability of selling}} \underbrace{x}_{\text{profits-per-sale}} \Big| e \Big] \tag{3.21}$$

where we average over the probabilities that the seller observes each signal $j \in \{l, h\}$ in a match with a buyer of each valuation $\theta_i \in \{\theta_l, \theta_h\}$, and the prices $x \sim F^{e,j}$ when they are of either interim type $p_l^{e,j}$. Finally, because inefficient trade occurs solely from low-valuation buyers being priced out, we track efficiency by the probability low-valuation buyers trade in any given match,

$$\mathcal{Q} = \quad \rho F(x \leq \theta_l|\theta_l) + (1-\rho)P_{x_1,x_2\sim F(\cdot|\theta_l)}(\min(x_1,x_2) \leq \theta_l|\theta_l) \tag{3.22}$$

Competition promotes greater efficiency and buyer surplus. Sellers, anticipating "more" competitors in the average match, offer more attractive terms to buyers to sustain optimal sales. As sellers decrease prices across the board, low-valuation buyers have a higher probability of receiving an acceptable offer, thus boosting trade efficiency and the aggregate surplus, all of which is captured by buyers.

The effects of variation in precision are more nuanced, depending on both the level of adverse selection and competition. For buyers, the low typically benefit from increased precision, while the high are harmed by it. This difference is rooted in the point that buyers obtain more attractive offers when sellers observe low signals, and precision reduces incidents of misclassification - benefiting low-valuation buyers, but hurting the high. However, these general trends can be reversed. The combination of low adverse selection and low competition can make precision harmful for all buyers, because sales are relatively inelastic in these economies, so precision weakly incentivizes sales upon a low signal but strongly incentivizes extraction upon a high one. And inversely, the combination of high adverse selection and low competition make precision beneficial for all buyers, because low signals then give sellers the strong incentive needed to break away from extraction (given the inelasticity of sales and high level of adverse selection). Trade efficiency tracks low-valuation buyers' utility, so these points extend to its relation with precision with the caveat that the link between low levels of adverse selection and the inefficiency of precision is even more resilient - present even under high levels of competition.

For sellers, precision is always individually beneficial, but typically detrimental to competitors. As in the case of buyer surplus and efficiency, this latter relation is not ubiquitous: the profit externalities of precision can be positive and allow all sellers to benefit. Economies with low adverse selection and low competition or high adverse selection and high competition are prime candidates for precision to benefit all sellers, since imprecisely informed sellers offer low prices in both, and precision strongly improves their extraction, ultimately relaxing the competition that peers expect from these sellers who observe high signals.

### 3.4.1 Competition

When buyers obtain more offers, lower $\rho$, sellers expect them to have more alternatives, particularly more alternatives below any price. Given a fixed set of strategies, this reduces the sales that sellers expect at any price. They respond by lowering prices to restore sales, and since their equilibrium strategies are mixed, this is done by shifting mass towards lower prices. As such, the general equilibrium effect on sellers' strategies reinforces the partial equilibrium effect on the number of offers, both improving buyers' offers.

We take a closer look at this dynamic in the more relevant set of equilibria - those in which separating prices are offered, so that predictive skill has a role. In the ordered equilibrium, the highest overall price $\overline{x}$ does not depend on the level of competition - only on the posterior of sharks that observe a high signal (by (3.6)). However, the probability of prices below depends closely on competition. Inspecting (A.4), we note that the strategy of these sharks increases strictly (in the first-order stochastic dominance ordering) with competition,

$$F^{s,h}(x;\rho_2) < F^{s,h}(x;\rho_1) \quad \rho_1 < \rho_2 \ , \ \forall x < \overline{x}$$

In other words, when there is more competition ($\rho$ increases), they place more mass on lower prices. Induction then allows us to extend this ordering claim to other types of sellers. The support of their prices does not just extend downwards, however, but instead shifts as their most expensive offer also becomes cheaper[15]

As we have argued, offers help buyers in two ways. Fixing seller strategies, the average minimum price offer decreases in the average number of offers. Indirectly, seller strategies decrease and shift mass towards lower prices, compounding the drop in the average minimum price offer. This benefits all buyers. High-valuation buyers obtain more surplus per trade, whereas low-valuation buyers obtain more surplus per trade and possibly more trades. This latter efficiency gain occurs whenever some pooling and separating prices are offered, as competition shifts some of the mass from separating prices to pooling ones. Therefore, efficiency is weakly (but often strictly) increasing in the level of competition.

However, sellers' average profits drop, in matches with either type of buyer, and the reasoning is straightforward. Recall that the sales a matched seller expects from a price $x$ are,

$$\begin{aligned}P^e(\text{sale at a price } x|j) &= \rho(p_h^{e,j} + p_l^{e,j}\mathbb{1}(x \le \theta_l)) \\ &+ (1-\rho)\sum_{\substack{j'\in\{l,h\},\\ e'\in\{a,s\}}} \mu(e')\left(p_l^{e,j}P^{e'}(j'|\theta_l)\mathbb{1}(x\le\theta_l) + p_h^{e,j}P^{e'}(j'|\theta_h)\right)(1-F^{e',j'}(x))\mathbb{1}(p_l^{e',j'} \le p_l^{e,j})\end{aligned}$$

and these decrease because competitive matches because more likely ($\rho$ and $1-\rho$ terms) and because the distribution of competitor bids shifts mass towards lower prices ($F(x|\theta)$ terms). Therefore, any offer is less profitable in any match,

$$\Pi^{e,j}(x';\rho) = P^e(\text{sale at a price } x'|j;\rho)x' \tag{3.23}$$

$$\Pi^{e,j}(x;\rho_1) < \Pi^{e,j}(x;\rho_2) \le \Pi^{e,j}(x';\rho_2) \quad x \in \text{supp}(F^{e,j}(x;\rho_1))\ ,\ x' \in \text{supp}(F^{e,j}(x;\rho_2)) \tag{3.24}$$

On the one hand, when matches are more competitive, they are less profitable for sellers. On the other hand, competition allows them to match with more buyers, and this is particularly beneficial for sellers whose offers have a higher probability of being chosen, mainly types who offer lower prices. Nevertheless, ex-ante profits involve computing the product of the average number of matches, $M(\tilde{\rho}) = \tilde{\rho} + 2(1-\tilde{\rho})$, with a matched seller's expected sales per match, and the conditioning on matching in these places the number of matches in the denominator throughout, via $\rho = \frac{\tilde{\rho}}{M(\tilde{\rho})}$ and $1-\rho = \frac{2(1-\tilde{\rho})}{M(\tilde{\rho})}$, so the average number of matches term cancel,

$$\begin{aligned}&M(\rho)\Pi^{e,j}(\rho) \\ &= E_{x\sim F^{e,j}}\Bigg[\tilde{\rho}(p_h^{e,j} + p_l^{e,j}\mathbb{1}(x\le\theta_l)) \\ &+ 2*(1-\tilde{\rho})\sum_{\substack{j'\in\{l,h\},\\ e'\in\{a,s\}}} \mu(e')\left(p_l^{e,j}P^{e'}(j'|\theta_l)\mathbb{1}(x\le\theta_l) + p_h^{e,j}P^{e'}(j'|\theta_h)\right)(1-F^{e',j'}(x))\mathbb{1}(p_l^{e',j'} \le p_l^{e,j})\Bigg]\end{aligned}$$

and average ex-ante profits satisfy an analogous set of inequalities as profits per match at any price

[15] In the base case of sharks who observe $h$, we establish the first order stochastic dominance ordering and that $\underline{x}^{s,h}$ is strictly decreasing in $1-\rho$. This allows us to assume by induction that $\underline{x}_l^{e,j}$ is strictly decreasing in $1-\rho$. Since the highest price offered by a type $p_l^{e,j}$ seller, $\overline{x}_l^{e,j}$, is weakly decreasing in the lowest price offered by the adjacent type below, the inductive step implies $\overline{x}_l^{e,j}$ is weakly decreasing in $1-\rho$. We pair this point with the fact that $P^e(\text{sale at price } \overline{x}_l^{e,j}|\theta_h)$ and $P^e(\text{sale at price } \overline{x}_l^{e,j}|j)$ are strictly decreasing in $1-\rho$ (by (A.6) and (A.11)) to establish that $F^{e,j}(x;\rho_2)$ strictly first-order stochastic dominates $F^{e,j}(x;\rho_1)$, for $\rho_1 < \rho_2$, and that $\underline{x}_l^{e,j}$ is strictly decreasing in $1-\rho$.

(3.24),

$$M(\rho_1)\Pi^{e,j}(\rho_1) < M(\rho_2)\Pi^{e,j}(\rho_2) \tag{3.25}$$

by the same reasoning as before[16]. The additional sales that some types achieve are not insufficient compensation for the lower prices it takes to get them, so the profits of every type of seller decrease.

**Theorem 3.1** ($\rho$ Comparative Statics)**.** *Consider two economies that differ only in the level of competition $\rho_1 < \rho_2$ with the corresponding ordered equilibria,*

$$\{F(x|\theta;\rho_i), F^{e,j}(x;\rho_i)\}_{\substack{\theta\in\{\theta_l,\theta_h\}\\ e\in\{a,s\}\\ j\in\{l,h\}}}$$

*As competition increases,*

- ***Distributions Strictly Increasing:*** *$F(x|\theta;\rho_2) < F(x|\theta;\rho_1)$ and $F^{e,j}(x;\rho_2) < F^{e,j}(x;\rho_1)$ for all $(e,j,x) \neq (s,h,\overline{x})$.*
- ***Supports Increasing:*** *$\underline{x}^{e,j}(\rho_1) < \underline{x}^{e,j}(\rho_2)$ and $\overline{x}^{e,j}(\rho_1) \leq \overline{x}^{e,j}(\rho_2)$ for all $e,j$.*
- ***Buyer Surplus Strictly Increasing:*** *$\mathscr{W}_b(\theta;\rho_2) < \mathscr{W}_b(\theta;\rho_1) \forall\theta$.*
- ***Trade Efficiency Increasing:*** *$\mathcal{Q}(\rho_2) \leq \mathcal{Q}(\rho_1)$.*
- ***Profits Strictly Decreasing:*** *$\Pi^e(\rho_1) < \Pi^e(\rho_2) \quad \forall e$.*

We illustrate these effects in Figure 1 and Figure 2.

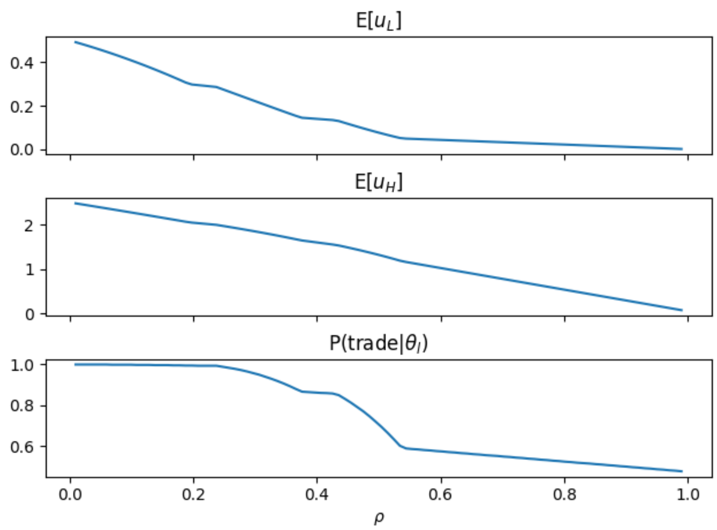



**Figure 1: Buyer Surplus and Efficiency Progression** The common parameters are $[\theta_l, \theta_h, p_l] = [1, 3, 0.5]$ for buyers, $[\alpha_a, \alpha_s] = [0.55, 0.95]$ for sellers' precision, and $\gamma = 0.5$ for the proportion of amateurs.

[16] $\tilde{\rho}$ decreases if and only if $\rho$ does, so $F^{e,j}(x;\rho)$ increases if and only if $\tilde{\rho}$ decreases. As such, the terms inside the expectation, once we cancel the average number of matches, are pointwise (in $x$) increasing in $\rho$.

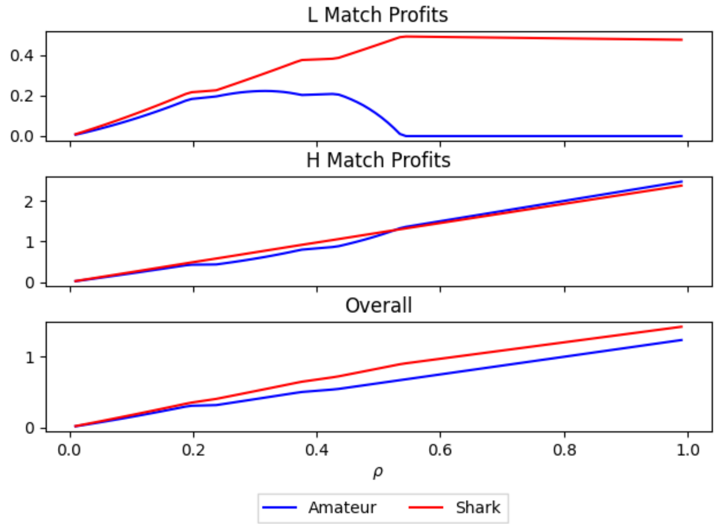



**Figure 2: Profitability Progression** The common parameters are $[\theta_l, \theta_h, p(\theta_l)] = [1, 3, 0.5]$ for buyers, $[\alpha_a, \alpha_s] = [0.55, 0.95]$ for sellers' precision, and $p(a) = 0.5$ for the proportion of amateurs.

#### 3.4.2 Precision

A larger number of offers is the traditional channel that heightens competition; however, it is not the only one. The precision of a seller's demand information also affects the distribution of prices that they extend to buyers and so the degree of competition that peer sellers expect.

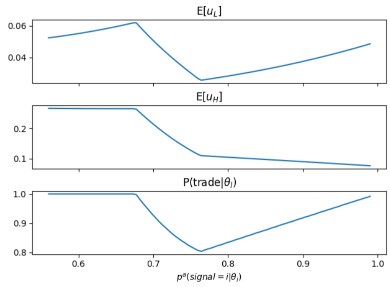
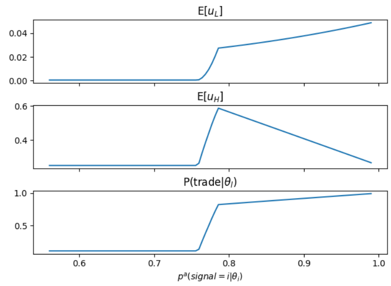
Precision of information about buyers' valuations shapes a seller's distribution of posteriors, hence types, in two ways. First, it changes how often the seller's signal correctly classifies the buyer; in other words, how often the seller's type is relatively low or high in matches with a buyer of each valuation - the *classification effect*. Second, it changes how extreme its posterior valuation beliefs are upon each signal; in other words, how large its type is upon each signal - the high and low signal *type effect*. Jointly, these increase the precision of its expected sales, which are central to the profitability of its offers. The closer a seller's expected sales are to the perfect valuation information forecast $P(\text{sale at price } x|\theta)$, the more precise her sales vs. markup trade-off. In particular, a seller with greater confidence about being in a high match expects more sales at any price (in the ordered equilibrium), so it offers higher prices - and vice versa for a seller with greater confidence about being in a low match. Said differently, precision makes the offers of sellers who observe high (low) signals less (more) competitive. The strategic response of other sellers to this, the *competition effect*, is why changes in the precision of a given class of sellers' signals can also produce changes in the offers of sellers with differing precision. These three effects of precision do not always share the same sign and prevent blanket statements about the impact of precision on trade efficiency and welfare. However, the model allows us to categorize its aggregate impact along intuitive structural dimensions - the level of competition, adverse selection, and precision - and to concisely explain the mechanisms behind it.

The advancement of shark and amateur predictive skill is quite different. Practically, the former represents the adoption of firms like Amazon and OpenAI of the latest methodologies in artificial intelligence and big data, whereas the latter represents the diffusion of best practices in data analytics among other firms. Theoretically, the former (latter) pushes the right (left) tail of the precision distribution. And economically, the orderedness of prices by sellers' type means that their strategic implications differ. We will study these two types of technological progress by varying the precision of the signals that *all*[17] sharks or amateurs observe.

In general, precision improves the efficiency of trade, benefits (hurt) low (high) buyers, and increases (decrease) the profits of sellers who (do not) obtain it. Therefore, the expected effects are also the generally correct ones. The exceptions are far from exceptional, however, so understanding them will be important.

**Buyer Surplus and Efficiency** The existence of pooling prices does not just benefit high-valuation buyers, who can then obtain information rents, but also low-valuation buyers, as competition among pooling offers intensifies in the mass and the type of sellers offering them. Therefore, buyers benefit from precision to the degree that it drives sellers to offer lower pooling prices, and economies with high adverse selection are prime candidates for this effect. In these economies, competition would be the only incentive for an uninformed seller to trade with low-valuation buyers; however, as the precision of its information increases, low signals become a sufficient incentive for low trade. Low-valuation buyers are able to obtain more attractive offers from these sellers, because of the classification effect (more observe low signals in low matches), low signal type effect (higher type upon a low signal), and low signal competition effects (stronger competition that these sellers who observe low signals exert on any higher precision sellers who also observe low signals). High-valuation buyers can benefit as well when the low signal type and classification effects are strong enough, as sellers still misclassify them at times, but when precision is high enough, the classification effect, which hurts them, is dominant. The principal difference in economies with low adverse selection is that an uninformed seller would always offer pooling prices, preferring to trade with all buyers, so precision, which encourages higher separating prices upon a high signal, hurts high-valuation buyers and can only benefit the low under enough precision so that the classification and competition effects are strong enough.

[17] The "effect" of changing an individual seller's precision of information will be obvious once we understand the effects of changing that of an entire class of sellers, where effect is in quotes because although discussion about the impact on an individual seller's actions is well-defined, there is no aggregate equilibrium impact to perturbations of measure zero agents.

<table>
<tr><td>Valuation Het. / Competition</td><td>Low<br>$[\theta_l, \theta_h] = [1, 1.25]$</td><td>High<br>$[\theta_l, \theta_h] = [1, 3]$</td></tr>
<tr><td>Low<br>$\rho = 0.9$</td><td>
</td><td>
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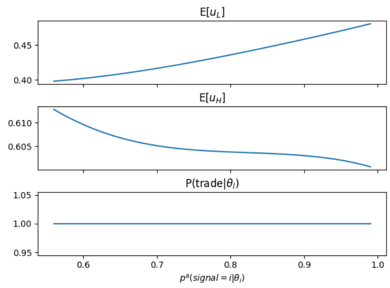
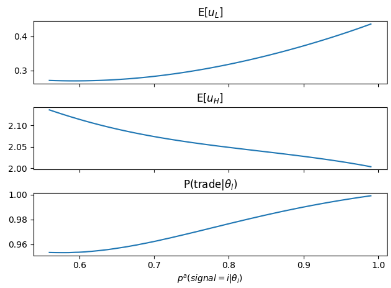
<tr><td>High<br>$\rho = 0.2$</td><td>
</td><td>
</td></tr>
</table>

**Table 2: Buyer Surplus and Efficiency Effects of Precision** Almost all sellers are amateurs (90%) and we increase the precision of their signals along the respective x-axes. Each figure is representative of the effect that precision has on buyer surplus and trade at different levels of competition and adverse selection. The common parameters are $p(\theta_l) = 0.5$ for the mass of low-valuation buyers, and $\alpha_s = 0.999$ for shark precision.

Efficiency increases with precision under high adverse selection, whereas under low adverse selection it is initially decreasing, but then recovers. A simple application of the product rule shows us why. Consider an economy with homogeneous precision, where sellers start to offer separating prices upon a high signal but continue to pool upon a low one,

$$P(\text{trade}|\theta_l; \alpha) = \alpha \underbrace{P(\text{pooling offer}|l; \alpha)}_{=1} + (1-\alpha)P(\text{pooling offer}|h; \alpha)$$

$$\frac{\partial}{\partial \alpha} P(\text{trade}|\theta_l; \alpha) = 1 - P(\text{pooling offer}|h; \alpha) + (1-\alpha) \underbrace{\frac{\partial}{\partial \alpha} P(\text{pooling offer}|h; \alpha)}_{<0}$$

when precision is still too low, the increase in separating offers upon high signals is dominant, whereas if precision is higher, the decrease in misclassification is.

**Profits** Sellers benefit individually from more precise information, as it allows them to make more profitable offers on average - closer to the optimal prices conditionally on the matched buyer's valuation. However, since precision affects the offers of a given seller, it also affects the profitability of other sellers. When we consider a change in the precision of an entire class of sellers (amateurs or sharks), the net response of the average amateur or shark profit is the average of their individual benefit + profit externalities. The individual benefit is straightforward, but the profit externality brings together several components that require closer attention.

The profit externalities of precision are clearest among sellers whose precision remains constant. First, consider the peer effect of amateur precision on sharks. If sharks observe a high signal, they always offer higher prices than amateurs, whereas if they observe a low one, they always offer lower prices than amateurs; as such, sharks' expected sales are invariant in amateur precision. In this sense, shark over-/under- bidding and the profitability of their high signal offers are all invariant in amateur precision. Amateur precision uniquely affects sharks only through the markups that they obtain upon a low signal, which occur primarily in low matches. Amateur precision increases the mass of amateurs that observe low signals in low matches and makes them keener on offering low prices upon it, so more precisely informed amateurs tend to compete more aggressively in matches with low-valuation buyers, lowering sharks' profits. However, in economies with low adverse selection and low amateur precision, the stronger incentive to offer high prices upon a high signal can dominate and make amateurs compete less aggressively, even in matches with low-valuation buyers, increasing sharks' profits.

The fundamental difference with shark precision is that the precision of leaders determines the type of equilibrium. This is why, when their information is imprecise, low adverse selection equilibria feature only pooling offers that maximize trade, and sellers' strategies are symmetric. Once sharks' information becomes sufficiently precise, they start making separating offers upon high signal; in other words, they start using their predictive skill. This turns on the profit externalities of precision, which are at first discretely negative, as a switch to an ordered equilibrium is a switch to an equilibrium where sellers over-bid (buyer surplus) in high matches and under-bid in low matches - both of which decrease profitability. Nevertheless, in separating equilibria, the qualitative effects of shark precision on amateurs are qualitatively analogous to amateurs' on sharks - beneficial/costly on net under similar combinations of competition, adverse selection, and precision - even if the channels that bring it about differ. Amateurs offer lower prices than sharks who observe high signals and high prices than sharks who observe low ones, and we recall that the profitability of separating offers is invariant in sellers' type (all agree on the optimal separating offers), so the principal benefit that amateurs obtain from shark precision is an increase in the mass who offer separating prices during high-valuation matches, whereas the principal cost is an increase in the mass who undercut them in low-valuation matches.

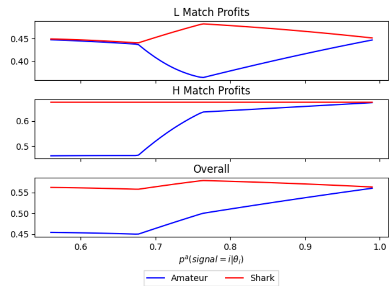
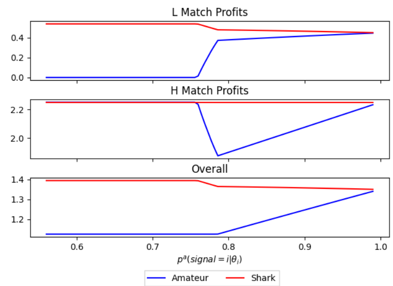
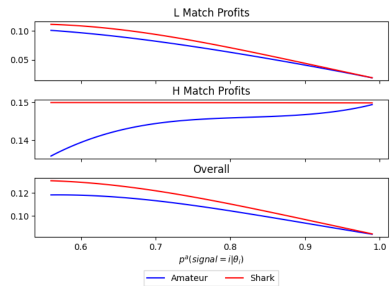
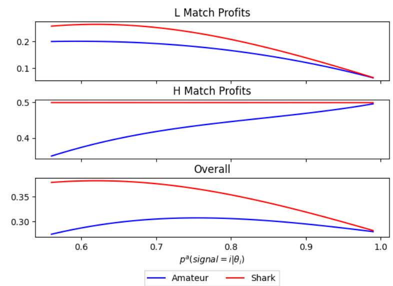
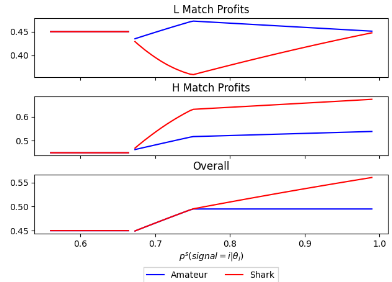
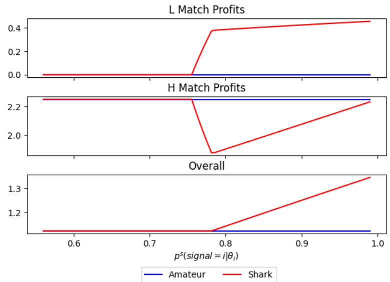
| Valuation Het. / Competition | **Low** $[\theta_l, \theta_h] = [1, 1.25]$ | **High** $[\theta_l, \theta_h] = [1, 3]$ |
|---|---|---|
| **Low** $\rho = 0.9$ |  |  |
| **High** $\rho = 0.2$ |  |  |

**Table 3: Profits Effects of Amateur Precision** A supermajority of sellers are amateurs (90%) and we increase the precision of their signals along the respective x-axes. Each figure is representative of the effect that precision has on their and sharks' profits in low matches, high matches, and the average match. The common parameters are $p(\theta_l) = 0.5$ for the mass of low-valuation buyers, and $\alpha_s = 0.999$ for shark precision.

<table>
<tr><td>Valuation Het.<br>Competition</td><td>Low<br>$[\theta_l, \theta_h] = [1, 1.25]$</td><td>High<br>$[\theta_l, \theta_h] = [1, 3]$</td></tr>
<tr><td>Low<br>$\rho = 0.9$</td><td>
</td><td>
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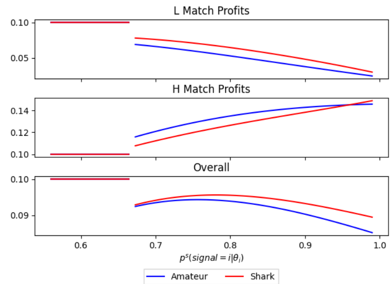
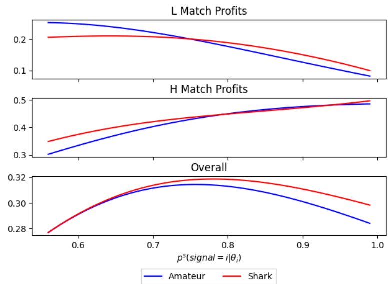
<tr><td>High<br>$\rho = 0.2$</td><td>
</td><td>
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</table>

**Table 4: Profit Effects of Shark Precision** A supermajority of sellers are sharks (99%) and we increase the precision of their signals along the respective x-axes. Each figure is representative of the effect that precision has on their and amateurs' profits in low matches, high matches, and the average match. The common parameters are $p(\theta_l) = 0.5$ for the mass of low-valuation buyers, and $\alpha_a = 0.55$ for shark precision.

# 4 Endogenous Quality Setting

Although pricing is one of the main use cases of demand information, another that is at least as important and particularly prevalent in sellers' recent data analytics applications is production - the problem of choosing what to offer to each buyer. Abstracting away from this problem has been convenient analytically and even practically negligible in situations where buyers perceive goods as highly substitutable or sellers' production is inflexible. However, to study settings where product differentiation and design are significant aspects, we will incorporate product choice by allowing sellers to pick the quality of the goods that they offer buyers. Although many of the core insights from exogenous product choice economies will generalize, some in exact form and others with close analogs, the significant differences lend nuance to the effects of precision on welfare and efficiency.

## 4.1 Seller Problem and Equilibrium Concept

The interesting cases of endogenous quality are those where the efficient quality of trade with low- and high-valuation buyers is different. This requires some cost convexity. We will work with a piecewise-linear cost function that kinks at the efficient qualities of trade with low- and high-valuation buyers.

**Assumption 4.1** (Piecewise Linear Costs)**.** *The cost function $\phi(\cdot)$ is piecewise linear, convex, strictly* *increasing with*

$$\phi(q) = \begin{cases} \kappa_l q & q \leq q_l^* \\ \kappa_l q_l^* + \kappa_m q & q_l^* < q \leq q_h^* \\ \kappa_l q_l^* + \kappa_m (q_h^* - q_l^*) + \kappa_h q & q_h^* < q \end{cases}$$

*where* $0 < \kappa_l < \theta_l$*,* $\theta_l < \kappa_m < \theta_h$*, and* $\theta_h < \kappa_h$ *and the efficient qualities of trade with a buyer of each valuation are given by,*

$$q_i^* = \underset{q}{\operatorname{argmax}}\, \theta_i q - \phi(q) \tag{4.1}$$

Piecewise linearity makes the marginal cost of quality revisions locally constant, lending considerable tractability[18]. Since sellers can choose quality and its valuation among buyers is heterogeneous, they have the ability to improve their screening of buyers by going beyond single quality-price offers and instead offering each buyer a menu $((q_l, x_l), (q_h, x_h))$ composed of a pair of quality-price contracts, where the contract $(q_i, x_i)$ is intended for a buyer of valuation $\theta_i$. When both contracts are identical, the menu is equivalent to a single contract offer, but we will see that such a menu is never optimal for sellers, the first sign that the exogenous product choice assumption imposes serious economic limitations.

There are three components to the profits that a matched seller expects from a menu: (1) the profits per sale from each contract, (2) the probability that a buyer of a given valuation chooses each contract, and (3) the probabilities that the buyer has each valuation. Profits per sale from a contract, $\pi(q_i, x_i) = x_i - \phi(q_i)$, are simply the difference between the lump sum price $x_i$ and the seller's cost of producing the quality $q_i$. The probability that a buyer of a given valuation chooses a contract is the probability that it has no better offers. By the Revelation Principle, it is sufficient to restrict attention to individually rational and incentive-compatible menus, so the probability that a buyer of valuation $\theta_i$ chooses the contract $(q_j, x_j)$ is zero if $i \neq j$, and otherwise equal to the sum of the probability that the seller's offer is the only one[19] ($\rho$) plus the probability that it has an inferior offer from another seller. To compute the probability that the seller's offer beats that of a competitor, we define the marginal distribution $F_i(u_i|\theta_i)$ over indirect utilities offered to $\theta_i$ valuation buyers by sellers of each type via the joint distribution of utilities in such matches,

$$F(u_l, u_h|\theta_i) = \sum_{\substack{e\in\{a,s\} \\ j\in\{l,h\}}} \mu(e) P^e(j|\theta_i) F^{e,j}(u_l, u_h) \tag{4.2}$$

where $\mu(e)$ is the mass of sellers with precision $\alpha_e$, $P^e(j|\theta_i)$ is the proportion of them that would observe $j$ signals when matched with a $\theta_i$ valuation buyer (and so that would be of type $p_l^{e,j}$), and $F^{e,j}(u_l, u_h)$ is the joint distribution over indirect utility offers implied the menus that sellers of type $p_l^{e,j}$ mix over. The marginal distributions $F_i(u_i|\theta_i)$ follow directly from this joint distribution. Therefore, the probability that a $\theta_i$ valuation buyer chooses a contract $(q_i, x_i)$ contract is $\Psi_i(u_i; F) = \rho + (1 - \rho) F_i(u_i|\theta_i)$, where $u_i = u(q_i, x_i; \theta_i)$ is the indirect utility it offers $\theta_i$ valuation buyers. Lastly, a matched seller's probability that the buyer has low and high-valuation is given by its type $p_l^{e,j}$ and the respective complementary probability $1 - p_l^{e,j}$. Since each contract $(q_i, x_i)$ determines the seller's expected profits in a match with each kind of buyer, the expected profits from the menu are therefore

[18] Our main results are not dependent on this restriction.

[19] The probability that the seller is a monopolist given that it is matched.

given by the average of these,

$$\begin{aligned}\Pi^{e,j}(q_l, x_l, q_h, x_h) &= p_l^{e,j}\Psi_l(u(q_l, x_l; \theta_l); F)\pi(q_l, x_l) + p_h^{e,j}\Psi_h(u(q_h, x_h; \theta_h); F)\pi(q_h, x_h) \\ &= \sum_{i=l,h} p_i^{e,j}\Psi_i(u(q_i, x_i; \theta_i); F)\pi(q_i, x_i)\end{aligned}$$

weighted by the probabilities that the buyer is of low or high-valuation.

## 4.2 Seller Strategies

We so far have implicitly allowed for both pooling and separating offers; however, as our notation suggests, only separating menus are offered in equilibrium.

**Corollary 4.1** (No Pooling in Equilibrium)**.** *Equilibrium menus separate buyers of each valuation.*

Cost convexity is at the core of this result. It is the reason why a pooling contract can always be improved through a separating revision that adds quality to the high contract, at a price (above marginal costs) only high-valuation buyers are willing to pay, and reduces the quality of the low, in exchange for a discount only low-valuation buyers are interested in. We provide a simple graphical representation of this procedure in Figure 3. Beyond its economic relevance, this result is analytically convenient because it allows us to avoid both the classical threat to existence of equilibrium (a la Rothschild Stiglitz (1976)) and a more complicated diversity of offers (as in Lester et al. (2019)).

Beyond separating buyers, equilibrium menus have quite a bit of additional structure. In particular, the quality-price terms of each contract featured in a menu are closely linked to the utility they allow their targeted buyer to obtain. The forward direction is obvious since incentive constraints and individual rationality of sellers menus imply buyers will always choose their intended contract from $((q_l, x_l), (q_h, x_h))$,

$$\begin{aligned}(IC_i) &: \quad u(q_i, x_i; \theta_i) \geq u(q_{\neg i}, x_{\neg i}; \theta_i) \ \forall i \in \{l, h\} \\ (IR_i) &: \quad u(q_i, x_i; \theta_i) \geq 0\end{aligned}$$

so the indirect utility offered to buyers of each respective valuation by a menu is simply that offered by their respective contracts,

$$\begin{aligned}u_l &= \theta_l q_l - x_l \\ u_h &= \theta_h q_h - x_h\end{aligned}$$

The backward direction follows from conditions that profit maximality imposes on optimal offers, and they imply the existence of a bijection between the indirect utilities $(u_l, u_h)$ and quality-price terms $((q_l, x_l), (q_h, x_h))$ of contracts in any *equilibrium* menu.

**Theorem 4.1** (Converting to Indirect Utilities)**.** *Consider an equilibrium menu $((q_l, x_l), (q_h, x_h))$ with associated indirect utilities $(u_l, u_h)$. Qualities are then given by,*

$$q_l(u_l, u_h) = \begin{cases} \frac{u_h - u_l}{\Delta\theta} & u_h - u_l < q_l^*\Delta\theta \\ q_l^* & u_h - u_l \geq q_l^*\Delta\theta \end{cases} \qquad q_h(u_l, u_h) = \begin{cases} \frac{u_h - u_l}{\Delta\theta} & u_h - u_l > q_h^*\Delta\theta \\ q_h^* & u_h - u_l \leq q_h^*\Delta\theta \end{cases} \tag{4.3}$$

*and prices by $x_i = \theta_i q_i - u_i$.*

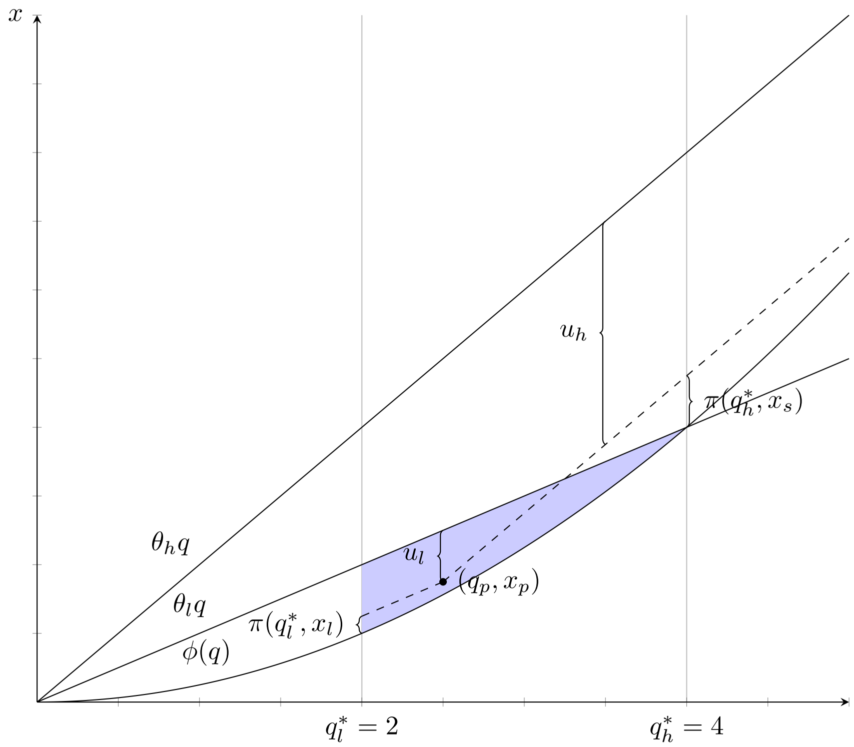



**Figure 3:** Example with $\theta_h = 4$ , $\theta_l = 2$ , $\phi(q) = \frac{1}{2}x^2$. Contracts are depicted as points in the $(q, x)$ space. Implied utilities are given by the vertical distance of a contract's y-axis coordinate to the zero utility indifference curves of the respective $\theta_i$ valuation buyers, while profits per sale are given by the vertical distance to the seller's cost function $\phi(q)$. The blue region represents the set of pooling contracts that are individually rational for buyers of either valuation, imply non-negative profits, and are not dominated by another pooling contract.

A candidate pooling offer $(q_p, x_p)$ lies on dashed iso-utility lines for buyers with low and high-valuation; we follow these to the right and left, respectively, until reaching the efficient qualities of trade with each type of buyer at prices $x_i = \theta_j q_i^* - u(q_p, x_p; \theta_i)$. This alternative separating bid $((q_l = q_l^*, x_l = x_p - \theta_l(q_p - q_l^*), (q_h = q_h^*, x_h = x_p + \theta(q_h^* - q_p))$ remains incentive compatible and strictly dominates the pooling offer: the same utility to buyers of either valuation (hence the same probability of winning) but strictly higher profits per sale.

Therefore, when an incentive constraint binds at an optimal menu, the difference in utilities $u_h - u_l$ uniquely pins down the qualities $q_l, q_h$ offered in each contract, while the level of these utilities then uniquely determines their respective prices. When both incentive constraints are slack, however, it is optimal to offer efficient qualities to each type of buyer, which is why we will refer to these menus as *dually efficient* from now on, and the prices follow uniquely in the same fashion.

This result connecting the quality-price form of menus to their associated indirect utilities is standard in mechanism design - referred to as the *parametric-utility approach* (Rochet and Stole (2006)) - and originates from the fact that seller surplus (profits per sale) equals the gains from trade $S_i(q) = \theta_i q - \phi(q)$ net of buyer surplus,

$$\pi(q_i, x_h) = x_i - \phi(q_i) = (x_i - \theta_i q_i) + (\theta_i q_i - \phi(q_i)) = S_i(q_i) - u(q_i, x_i; \theta_i)$$

Profits then increase both from minimizing buyer surplus *and* maximizing trade efficiency. As such, consider the optimal way to offer a pair of utilities $(u_l, u_h)$. If it is possible to do so with a dually efficient menu that respects incentive constraints, then it is optimal to do so and unlock a larger social surplus. If it is not possible and a buyer's incentive constraint would be violated by such an offer, then a problematic $IC_h$ ($IC_l$) constraint is corrected most profitably by under-providing (over-providing) quality in the low (high) contract. In these cases, a valuation $\theta_i$ buyer - whose incentive constraint binds at the optimal menu that offers $(u_l, u_h)$ - is indifferent between their contract and the one intended for the opposite $\theta_{\neg i}$ valuation buyer, $u_h - u_l = q_{\neg i}\Delta\theta$.

Since optimal offers can be expressed in terms of indirect utilities, it is natural to recast the strategy of each type of seller $p_l^{e,j}$ as a distribution $F^{e,j}(u_l, u_h)$ over pairs of utility offers $(u_l, u_h)$. In this notation, the profits that a type $p_l^{e,j}$ seller expects from an offer $(u_l, u_h)$ is,

$$\begin{aligned}
\Pi_i(u_l, u_h; F) &= \Psi_i(u_i; F)\pi_i(u_l, u_h) \\
\Pi^{e,j}(u_l, u_h; F) &= \sum_{i=l,h} p_i^{e,j} \Pi_i(u_l, u_h; F)
\end{aligned}$$

when the distribution of utilities is contested matches is $F$. Therefore, the *level* of indirect utilities $(u_l, u_h)$ determines both the probability of winning the contested offers $F_i(u_i)$ and the part of the buyer surplus of the profits per sale $\pi(u_l, u_h) = S(u_l, u_h) - u_i$, while the *difference* in generosity towards a high and low-valuation buyer, $u_h - u_l$, determines the part of the social surplus of the profits per sale, through the efficiency of trade with them. Given the sample space $\Omega = [0, S_l^*] \times [0, S_h^*]$, Borel $\sigma$-algebra, and set of countably additive probability measures $\mathscr{P}$ over it, this concise specification of the seller's problem allows us to introduce the intuitive equilibrium concept.

**Definition 4.1** (Bayes-Nash Equilibrium). *An equilibrium is a probability measure $F \in \mathscr{P}$ over indirect utility offers $(u_l, u_h)$, and seller type $\{p_l^{e,j}\}_{\substack{e\in\{a,s\}\\ j\in\{l,h\}}}$ mixed strategies $F^{e,j}$ such that,*

- *Rational Expectations:* $F(u_l, u_h) = \sum_{i\in\{l,h\}} p(\theta_i) F(u_l, u_h|\theta_i)$ *where each* $F(\cdot|\theta_i)$ *satisfies* (4.2).
- *Seller Optimality:* $\text{supp}(F^{e,j}) \subseteq \text{argmax}\, \Pi^{e,j}(u_l, u_h; F)$ *for all* $(e, j) \in \{a, s\} \times \{l, h\}$.

From here on, the dependence of various functions on equilibrium terms will be implied, so notation such as $(\cdot; F)$ will be suppressed and used only where it is important to avoid ambiguity.

### 4.2.1 Corner Cases: Monopoly and Perfect Competition

We start building intuition as in exogenous quality economies, by first analyze the equilibria of corner economies where sellers always bid alone ($\rho = 1$) or against another seller ($\rho = 0$). The screening problem of a monopolist is well understood from Mussa Rosen (1978). The only trade-off for this seller is between the efficiency of trade with low-valuation buyers and the share of efficient social surplus $S_h^*$ that it captures in trade with the high-valuation buyer. A type $p_l^{e,j}$ monopolist thus solves

$$\max_{u_l, u_h \geq 0} p_l^{e,j}(S_l(u_l, u_h) - u_l) + p_h^{e,j}(S_h(u_l, u_h) - u_h)$$

where incentive compatibility is implicit in the functional form of social surplus terms. We can ignore menus with (1) $u_h - u_l > q_l^* \Delta\theta$ or (2) $u_l > 0$, as they would not entail greater efficiency of trade and strictly lower profits per sale from buyers of at least one valuation than the offer $(u_l, u_h) = (0, q_l^* \Delta\theta)$. The incentive constraint of high-valuation buyers, therefore, binds at an optimal menu, and the optimal quality in the low contract, for a type $p_l^{e,j}$ seller with strictly convex costs, is determined by the marginal condition,

$$\underbrace{p_l^{e,j} \cdot \frac{\theta_l - \phi'(q_l(0, u_h^{e,j}))}{\Delta\theta}}_{\text{efficiency gains}} - \underbrace{p_h^{e,j}}_{\text{rent losses}} = 0 \tag{4.4}$$

The left-hand term is the marginal gain in efficiency made possible by increasing the indirect utility offered to high-valuation buyers. This relaxes their incentive constraint and allows the seller to provide a more efficient quality to low-valuation buyers, increasing the social surplus from trade with them and thus increasing the profitability of each low sale. The right-hand term represents the rents surrendered by making a more generous offer to high-valuation buyers - featuring the same quality at a strictly lower price. Monopolists weigh these marginal effects by their type, the conditional probability $p_l^{e,j}$ of being in a low match.

**Lemma 4.1** (Monopoly). *If $\rho = 1$, a seller of type $p_l^{e,j}$ with strictly convex costs offers a menu $(u_l^{e,j}, u_h^{e,j})$ with $u_l^{e,j} = 0$ and $u_h^{e,j}$ satisfying Equation* (4.4).

Consider two monopolists of types $p_l^{e,j} < p_l^{e',j}$ and note that (4.4) necessarily implies the latter is more generous, $u_h^{e,j} < u_h^{e',j}$. This means that monopolists' bids are monotonically ranked (*ordered*) by type, a property that we saw in exogenous quality economies where some rationed offers were extended, and that will also generalize to endogenous quality economies featuring interior levels of competition ($\rho \in (0, 1)$). We can also easily characterize the impact of precision on all agents in the monopoly setting,

- **Buyers:** high-valuation buyers prefer to face a seller with less precision, whereas low-valuation buyers are indifferent.
- **Sellers:** Precision strictly increases ex-ante profits.
- **Social Planner:** Precision strictly improves the efficiency of trade: high-valuation buyers always receive efficient offers, but that of the low-valuation is increasing in precision.

These relations extend to interior levels of competition with an essential caveat: competitive pressure allows low-valuation buyers to benefit from precision and partake in some of the additional social surplus it generates.

The other corner where sellers are guaranteed to bid against another seller in every match ($\rho = 0$) is also well understood, as each match again becomes a setting of Bertrand price competition. Therefore, sellers make dually efficient offers, and buyers capture the entire social surplus.

**Theorem 4.2** (Perfect Competition). *If $\rho = 0$, the unique Bayes-Nash equilibrium involves almost every seller offering $((q_l, x_l), (q_h, x_h)) = ((q_l^*, \phi(q_l^*)), (q_h^*, \phi(q_h^*)))$ with probability 1.*

On one hand, this implies the negative point that perfect competition is the wrong setting for the study of predictive skill about buyer's willingness to pay for vertically differentiated goods, mainly because it is irrelevant. On the other hand, it implies the positive point that predictive skill is not valuable if sellers cannot use it to improve the profitability of their offers and that the equilibrium structure has a large impact on this. In interior economies, competition will also promote efficiency and consumer surplus, but it will differentially shape the returns to precision for amateurs and sharks because the latter's offers will often be dually efficient and thus unimprovable at the margin by more precise demand information. Lastly, we highlight that a strategic dependence between sellers' bids and those of peers requires some degree of competition, so interior levels, where competition is not so strong as to render predictive skill moot, will allow us to study the externalities of precision on other sellers. We will find that sellers become more profitable through their own predictive skill and even sometimes that of their peers.

## 4.3 Equilibrium Structure

In this section, we discuss some general properties satisfied by the menus of a candidate equilibrium. Each has intuitive appeal, either from an economic standpoint or because of the mathematical tractability that they impart. By leveraging these in conjunction with the optimality conditions of sellers' problems, we can solve for this equilibrium analytically and obtain a complete characterization. In the Appendix, we show that these properties hold in any equilibrium where at least some offer rations low-valuation buyers.

### 4.3.1 Claims about Equilibrium Structure

There are three main areas that benefit from additional structure: (1) the distributions over indirect utilities offered by sellers in low and high matches, $F_i$, (2) the relationship of incentive compatibility constraints to the generosity (indirect utility) of menus, and (3) the relationship between generosity of a menu with the type $p_l^{e,j}$ of the seller who offers it.

The equilibrium distributions of indirect utilities $F_i$ offered in each match are weighted averages of each seller type's mixed strategy, and it inherits their properties.

**Claim 4.1.** *The equilibrium marginal distributions over indirect utilities $F_i$ for $i \in \{l, h\}$,*

1. *are atomless.*
2. *have a connected support of low utility offerings $\Upsilon_l = [\underline{u}_l, \overline{u}_l]$.*
3. *have a support of high utility offerings $\Upsilon_h = \cup_{\substack{e\in\{a,s\},\\ j\in\{l,h\}}} [\underline{u}_h^{e,j}, \overline{u}_h^{e,j}]$, made up of the contiguous bids by type $p_l^{e,j}$ sellers.*
4. *are continuously differentiable with densities $f_i$ in the interior and one-sided derivatives at the boundaries.*

By avoiding atoms and gaps in the supports of each seller type's mixtures, erratic behavior is curtailed. It is never optimal to bunch up and compete at a single point in the space of utility offers, as one might see in the corner cases of monopoly and Bertrand, nor do we see discontinuous jumps in generosity among sellers of the same type. While continuous densities allow us to consider marginal incentives, which helps convey economic intuition and solve for the equilibrium as the solution of a standard differential system.

As in the closely related work of Lester et al. (2019) and Garret et al. (2019), a property called *ordering*, which refers to an equilibrium where ranking menus by the utility offered to high or low-valuation buyers is identical, lends a great degree of tractability. This property turns out to be necessary in any equilibrium where low-valuation buyers' incentive constraint is slack in every offered menu, and a sufficient condition for this is the assumption that sellers cannot profitably trade the efficient high quality with low-valuation buyers.

**Assumption 4.2** (No $IC_l$ Cost Condition)**.** *The piecewise linear cost function $\phi(\cdot)$ is such that,*

$$\theta_l q_h^* \leq \kappa_m (q_h^* - q_l^*) + \kappa_l q_l^* \tag{4.5}$$

**Claim 4.2.** *low-valuation buyer's incentive constraint never binds at equilibrium menus.*

Eliminating the possibility of a binding low-valuation buyer constraint preserves efficiency at the top, so high-valuation buyers always obtain their efficient quality ($q_h^*$), but low-valuation buyers are rationed ($q_l < q_l^*$) whenever the offer intended for them is part of a menu in which high-valuation buyers' incentive constraint binds, as in the monopoly setting.

We will focus on equilibria in which offers are (a) ordered by their generosity, but also (b) monotone in the seller's type, meaning that generosity increases with the seller's posterior probability of being matched with a low-valuation buyer $p_l^{e,j}$, as in the ordered equilibria of economies where quality is exogenous.

**Claim 4.3.** *Given two equilibrium menus $(u_l, u_h)$ and $(\tilde{u}_l, \tilde{u}_h)$ offered by sellers of respective types $p_l$ and $\tilde{p}_l$ with $u_i > \tilde{u}_i$ for some $i \in \{l, h\}$,*

1. *It is also true that $u_{\neg i} \geq \tilde{u}_{\neg i}$ and strictly so if both menus are offered by sellers of the same type.*
2. *The gap between low and high utilities $u_h - u_l$ increases strictly with generosity.*
3. *Each support $\Upsilon_i = [\underline{u}_i, \overline{u}_i]$ is such that $\overline{u}_i \leq S_i^*$. Further, there exists a $u_i^{de} \in [\underline{u}_i, \overline{u}_i]$ such that all $u_i < u_i^{de}$ are in $IC_h$ binding menus and all $u_i \geq u_i^{de}$ are dually efficient.*

Due to Theorem 4.1, the efficiency of any menu is directly pinned down by the difference in its respective utility offers $u_h - u_l$, and since low-valuation buyers' incentive constraint does not bind in the menus of these equilibria, a larger utility difference only improves efficiency - weakly raising the quality of low trade towards the optimal, $q_l^*$. Orderedness in generosity then follows from the complementarity between relaxing the incentive compatibility constraint of a high-valuation buyer (through a more generous $u_h$ term) - which permits more profitable sales to low-valuation buyers - and increasing low sales (through a more generous $u_l$ term). In equilibrium, the first effect dominates - sellers face stronger incentives to increase the generosity of their high vs low offer - so that the difference in utilities $u_h - u_l$ and, consequently, efficiency is weakly increasing in generosity. This upward efficiency progression then also generates a natural grouping of menus, with the least generous menus also being the least efficient (constrained by high-valuation buyer incentive compatibility), but above a generosity threshold, becoming dually efficient (unconstrained by either buyer's incentive compatibility). The monotonic relation between a seller's type and generosity, on the other hand, is due to the link between generosity and profitability of equilibrium contracts in a match with each kind of buyer.

**Claim 4.4.** *Given two equilibrium menus $(u_l, u_h)$ and $(\tilde{u}_l, \tilde{u}_h)$ offered by sellers of respective types $p_l$ and $\tilde{p}_l$ with $u_i > \tilde{u}_i$ for some $i \in \{l, h\}$,*

1. *Profitability conditionally on matching with a buyer of low (high) valuation is increasing (decreasing) in generosity,*

$$\Pi_l(u_l, u_h) > \Pi_l(\tilde{u}_l, \tilde{u}_h) \text{ and } \Pi_h(u_l, u_h) > \Pi_h(\tilde{u}_l, \tilde{u}_h)$$

2. *Generosity is increasing in the seller's conviction of facing a low-valuation buyer,* $p_l > \tilde{p}_l$*, and strictly so if high-valuation buyers' incentive constraint binds at* $(u_l, u_h)$*.*

Equilibrium profits conditionally on matching with a low (high) buyer provide the incentives that order the offers of sellers with different interim assessments (types). In particular, the profitability of menus in low (high) matches will increase (decrease) with their generosity. The origin of this relation is clear when we consider the first order condition satisfied by the bids of sellers offering menus at which high-valuation buyers' incentive constraint binds,

$$\begin{aligned} 0 &= p_h^{e,j} \frac{\partial \Pi_h}{\partial u_h}(u_h, u_l) + p_l^{e,j} \underbrace{\frac{\partial \Pi_l}{\partial u_h}(u_h, u_l)}_{\geq 0} \\ 0 &= p_l^{e,j} \frac{\partial \Pi_l}{\partial u_l}(u_h, u_l) \end{aligned}$$

When high-valuation buyers' incentive constraint binds at a menu, high generosity relaxes this constraint and makes low trade more profitable, so the term $\frac{\partial \Pi_l}{\partial u_h}(u_h, u_l)$ is strictly positive. As such, the profits from high sales of constrained menus ($\Pi_h$) are locally decreasing in generosity towards high-valuation buyers ($u_h$) but invariant in that towards the low ($u_l$), while their profits from low sales ($\Pi_l$) are increasing generosity towards high-valuation buyers and at a local maximum with respect to low-valuation buyer generosity. When we consider the set of constrained menus, the least generous ones are offered by the sellers who are more relatively more concerned about the profitability in high matches, mainly those sellers with the lowest type $p_l^{e,j}$. The efficiency-generosity relation places these constrained menus below any dually efficient ones, so sellers who are sufficiently concerned about low match profits offer the latter (dually efficient) menus. However, when menus are dual efficient, incentive constraints are slack, and changing the generosity of the contract offered to a buyer of either valuation does not impact the profitability of the paired contract, offered to a buyer of the other valuation (by Theorem 4.1). The term $\frac{\partial \Pi_l}{\partial u_h}(u_h, u_l)$ is, therefore, equal to zero at dually efficient offers, so these satisfy the pair of equations,

$$\begin{aligned} 0 &= \frac{\partial \Pi_h}{\partial u_h}(u_h, u_l) \\ 0 &= \frac{\partial \Pi_l}{\partial u_l}(u_h, u_l) \end{aligned}$$

which implies that every dually efficient menu has identical profits in a match with a $\theta_i$ buyer, and so that all sellers - irrespective of their type - are indifferent between these menus. The most we can necessarily say about the relation between a seller's type and dually efficient menus is that the type must be high enough for the seller to offer any dually efficient menus. Any additional relation between sellers' type and the generosity of dually efficient menus is not just unnecessary but unappealing because it would allow precision to shape the share of surplus that buyers of each valuation obtain, purely due to the equilibrium coordination role of sellers' signals, rather than their informative content about buyers' valuations. This is an important difference with economies where quality is exogenous, since the generosity (as reflected by their price) of offers in these, including of efficient ones that allow trade with low-valuation buyers, is strictly monotone in the seller's type when the equilibrium features some constrained (inefficient) offers.

We close by connecting these points about profitability, generosity, and a seller's type to the benefits of precision. In our setting, with two information structures (levels of precision), each of which features a random variable (signal) that can take two possible values (low or high), there are four types of sellers at the interim stage, corresponding to each precision and signal combination. Naturally, sellers with the highest precision have the greatest conviction about their signals, and receiving a high (low) signal leads any seller to revise their prior down (up), so types are ranked as per,

$$p_l^{s,h} < p_l^{a,h} < p_l^{a,l} < p_l^{s,l}$$

So that predictive skill has a role, consider an equilibrium where some inefficient menus are offered. The least generous menus, towards a buyer of either valuation, are offered by sharks who observe high signals, while the most generous are offered by sharks who observe low signals. Amateur offers have intermediate generosity and only overlap with those of sharks if they are dually efficient. As such, in low (high) matches, a greater share of sharks (than amateurs) observe signals that orient their offers to the top (bottom) of the generosity distribution, where the most profitable low (high) match menus are found. This makes sharks more profitable in the average match than amateurs - the benefit of predictive skill.

## 4.4 Comparative Statics

We will perform an analogous comparative static analysis to Section 3.4, relating points of commonality and departure, thereby tracing out where product design makes its mark. The aggregate statistics for welfare and trade efficiency are similar after adjusting for the fact that we express strategies in the space of indirect utility offers in these settings with endogenous quality choices. Recycling notation, the welfare of each type of buyer follows as,

$$\mathscr{W}_b(\theta) = \rho E[u_i] + (1-\rho)E[\max(u_i, \tilde{u}_i)] \tag{4.6}$$

where $u_i$ are iid draws from the conditional equilibrium marginals $F_i(u_i|\theta_i)$. Seller profits are given by,

$$\Pi^e = E\Big[ \underbrace{\mathscr{M}}_{\text{number of matches}} \underbrace{\Psi_i(u_i)}_{\text{probability of selling}} \underbrace{S_i(u_l, u_h) - u_i}_{\text{profits-per-sale}} \Big| e \Big] \tag{4.7}$$

where we also average over the probabilities that the seller observes each signal $j \in \{l, h\}$ in a match with a buyer of each valuation $\theta_i \in \{\theta_l, \theta_h\}$, and that it offers the menus $(u_l, u_h) \sim F^{e,j}$ when they of either type $p_l^{e,j}$. Lastly, in the case of efficiency, high-valuation buyers always receive efficient offers, but the low may get rationed, so we measure the efficiency of trade by the average quality offered to the latter,

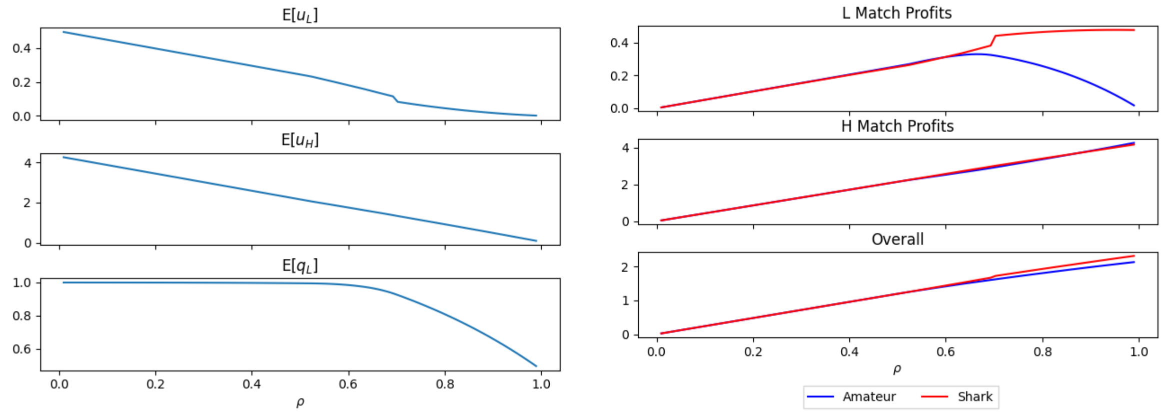
$$\rho E[q_l|\theta_l] + (1-\rho)E[\max(q_l, q_l')|\theta_l] \tag{4.8}$$

where we rely on the equilibrium property that generosity and efficiency are positively correlated, so buyers always select an offer with the greatest efficiency available.

Flexible production introduces changes that are profound, yet easy to overlook because they can be drowned out by the overarching point that the qualitative effects of competition and precision on efficiency and welfare aggregates do not change - directionally similar, under similar environments. However, closer inspection reveals the fundamental differences that information-driven production decisions introduce to welfare analysis. For one, it changes where we should look to track efficiency. Whereas pricing uniquely impacted trade efficiency at the extensive margin - determining whether a buyer trades or not by finding any acceptable prices - when production was inflexible (exogenous), removing this barrier allows information to shape the level of trade at the intensive margin - determining the quality of trade a buyer obtains. Stronger still, the optimality of screening menus results in almost every offer allowing *some* low trade, and so that *all* action is at the intensive margin - a theoretical insight that informs empirical analyses. Further, precision heterogeneity becomes a central issue, as it is potentially intrinsic to these settings, and it is only by including it in our model that we find the novel reason why. In particular, we find that competition affects amateurs and sharks differently, making the former's (latter's) offers less (more) sensitive to additional precision; therefore, competition neutralizes amateur predictive skill - making it less hurtful for other sellers - and amplifies shark predictive skill - making it more beneficial for all sellers.

### 4.4.1 Competition

The aggregate effects of competitive are similar when we allow information to also orient production - increasing trade efficiency, buyers' share of surplus, and decreasing sellers' profits - but the manner in which these arise differs. Mechanically, this set of results is linked by the fact that competition increases the sales gains from generosity, so sellers offer more generous menus, and complementarity between the utility offers to each type of buyer both links their joint progression and that of the low-quality good. Buyers almost always trade in these settings, so efficiency increases because of action at the intensive margin (volume of trade). This is a significant difference from the usual case in the literature, and also in our model when quality is exogenous, where pricing uniquely shapes efficiency - either pricing buyers out or not - and action is at the extensive margin (probability of trade).




**(a) Buyer Surplus and Efficiency** **(b) Profitability**

**Figure 4:** The common parameters are $[\theta_l, \theta_h, p_l] = [1, 3, 0.5]$ for buyers, $[\kappa_l, \kappa_m, q_l^*, q_h^*] = [0.5, 1, 2, 5]$ for sellers' cost functions, $[\alpha_a, \alpha_s] = [0.55, 0.95]$ for sellers' information technologies, and $p(a) = 0.5$ for proportion of amateurs.

### 4.4.2 Precision

The aggregate qualitative effects of precision are also similar. In fact, the comparison of figures under exogenous, Table 2 - Table 4, and endogenous quality, Table 5-Table 7, reveals a striking resemblance down to the very marginal effects of precision (curvature). But, upon closer inspection, we do notice some significant differences.

When it comes to buyers, the principal difference of endogenizing quality - beyond a direct change in the variety of goods - is that when sellers cannot screen, they agree - irrespective of their type - on the optimal inefficient (separating) offers, whereas when sellers can screen, they agree - irrespective of their type - on the optimal (dually) efficient offers. Under sufficient competition, amateurs offer efficient offers in both types of economies; so, in the former, precision still influences their preferred bids, but in the latter, it does not. This is why in the lower left-hand figure of Table 2 their precision has aggregate effects, but in Table 5 it does not.

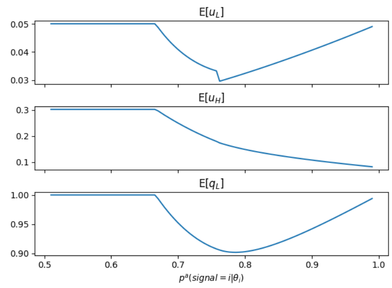
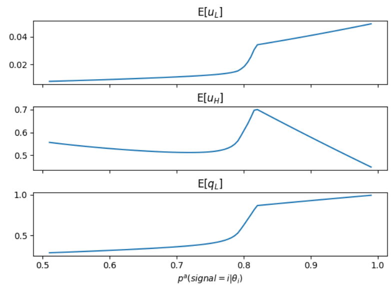
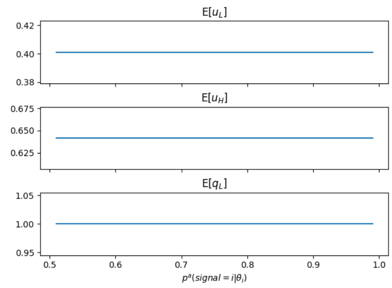
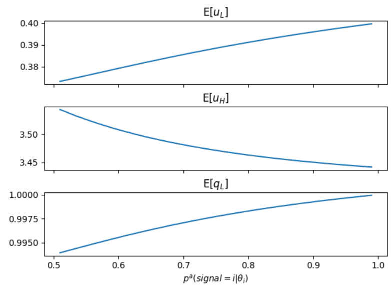
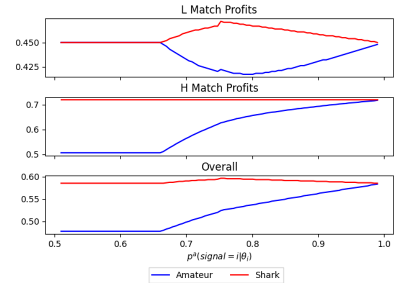
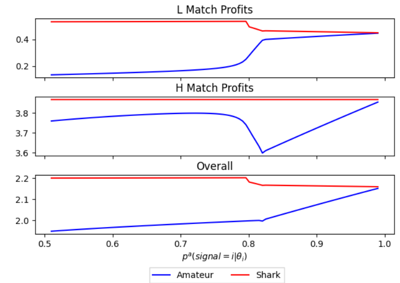
| Competition \ Adverse Selection | **Low**<br>$[\theta_l, \theta_h] = [1, 1.25]$ | **High**<br>$[\theta_l, \theta_h] = [1, 3]$ |
|---|---|---|
| **Low**<br>$\rho = 0.9$ |  |  |
| **High**<br>$\rho = 0.2$ |  |  |

**Table 5: Utility and Efficiency Effects of Precision:** Almost all sellers are amateurs (90%), and we increase the precision of their signals along the respective x-axes. Each figure is representative of the effect that precision has on buyer surplus and trade at different levels of competition and adverse selection. The common parameters are $p(\theta_l) = 0.5$ for the mass of low-valuation buyers, $[\kappa_l, \kappa_m, q_l^*, q_h^*] = [0.5, 1.2, 2, 1, 2]$ for sellers' cost functions, and $\alpha_s = 0.999$ for shark precision.

When it comes to sellers, the flip side of competition making amateur offers less sensitive to their signals is that amateur precision becomes less threatening to sharks. Concretely, we see this in the lower rows of Table 3 and Table 6, where amateur precision is less damaging to shark profits. Whereas the flip side of sellers not agreeing on the inefficient offers under endogenous quality choices is that shark precision more strongly relaxes competition during high matches, because it doesn't just change the mass of them that prefer to extend inefficient offers (as is also the case under exogenous quality), but also the terms of those inefficient offers - becoming stringier as their confidence in a high match increases. As a result, if we compare Table 4 and Table 7, it is evident that shark precision is more beneficial for amateurs when it can also inform production decisions.

<table>
<tr><th>Adverse Selection / Competition</th><th>Low<br>$[\theta_l, \theta_h] = [1, 1.25]$</th><th>High<br>$[\theta_l, \theta_h] = [1, 3]$</th></tr>
<tr><td>Low<br>$\rho = 0.9$</td><td>
</td><td>
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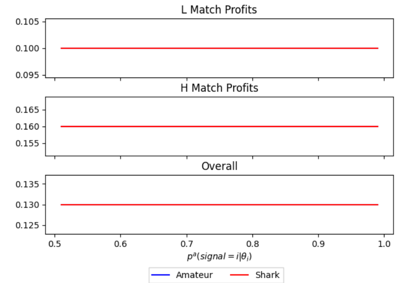
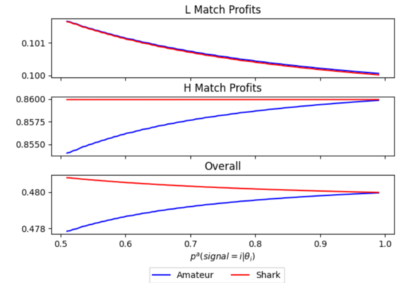
<tr><td>High<br>$\rho = 0.2$</td><td>
</td><td>
</td></tr>
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**Table 6: Profit Effects of Amateur Precision:** A supermajority of sellers are amateurs (90%) and we increase the precision of their signals along the respective x-axes. Each figure is representative of the effect that precision has on their and sharks' profits in low matches, high matches, and the average match. The common parameters are $p(\theta_l) = 0.5$ for the mass of low-valuation buyers, $[\kappa_l, \kappa_m, q_l^*, q_h^*] = [0.5, 1.2, 2, 1, 2]$ for sellers' cost functions, and $\alpha_s = 0.999$ for shark precision.

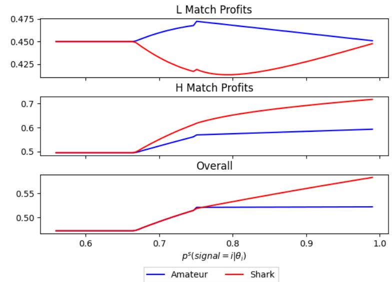
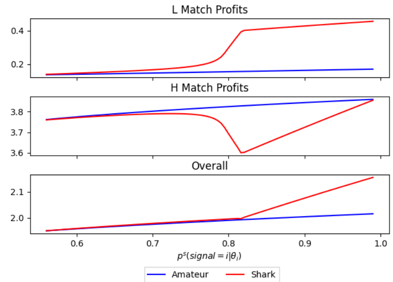
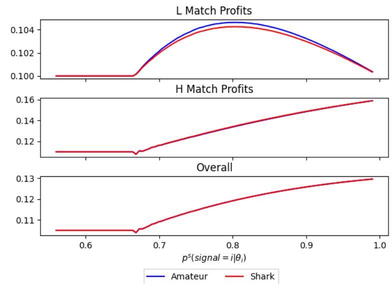
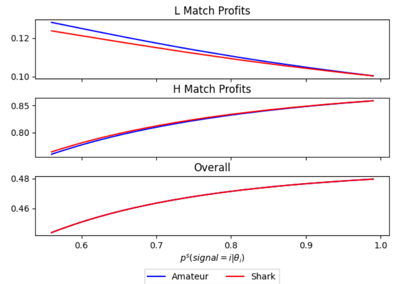
| Competition \ Adverse Selection | **Low** $[\theta_l, \theta_h] = [1, 1.25]$ | **High** $[\theta_l, \theta_h] = [1, 3]$ |
|---|---|---|
| **Low** $\rho = 0.9$ |  |  |
| **High** $\rho = 0.2$ |  |  |

**Table 7: Profit Effects of Shark Precision:** A supermajority of sellers are sharks (90%) and we increase the precision of their signals along the respective x-axes. Each figure is representative of the effect that precision has on their and amateurs' profits in low matches, high matches, and the average match. The common parameters are $p(\theta_l) = 0.5$ for the mass of low-valuation buyers, $[\kappa_l, \kappa_m, q_l^*, q_h^*] = [0.5, 1.2, 2, 1, 2]$ for sellers' cost functions, and $\alpha_a = 0.55$ for amateur precision.

## 5 Conclusion

A race in analytics is currently taking place, where firms' performance is intrinsically linked to their ability to source and analyze data. Although it may be too early to tell, an emerging literature[20] argues that many of the technologies that are being deployed - machine learning, big data, data mining, natural language processing, etc. - are indeed general-purpose technologies (GPT) i.e. "widely used, capable of ongoing technical improvement, and enabling innovation in application sectors" (Bresnahan et al. (1995), Bresnahan (2010)). Since the consequences are likely to be deep, pervasive, and persistent, research on this analytics pipeline is fundamental and urgent.

Motivated by the size of the retail sector and its intense reliance on data to optimize product and pricing decisions, we answer a "crucial" question posed by Bergemann and Bonatti (2019) in their review of markets for information: "what are the implications of acquiring an advantage in a downstream market by means of better data (e.g. improvements in the predictive power of an algorithm)?" The problem is addressed within a model that features sellers with predictive skill that has strategic value, as they face adverse selection from buyers and imperfect competition from other sellers. Aligning with empirical documentation of broad endemic differences in firms' use of predictive technologies, as well as forward-looking policy concerns about these disparities, we allow the precision of firms' predictions to be heterogeneous. We find predictive precision to be generically efficiency-enhancing but redistributive. On the demand side, it tends to benefit (hurt) low- (high-) valuation buyers. On the supply side, unsurprisingly, firms use more precise forecasts to obtain more profitable

[20] Brynjolfsson, Rock and Syverson (2019), Cockburn et al. (2019), Trajtenberg (2019), Goldfarb (2019)

sales, but, more interestingly, their predictive skill can also benefit competitors. This suggests the need for nuanced analysis of distributional impacts, in contrast to fears of endemic harm that have tinged public discussion.

Several modeling compromises have left open avenues for future research with connections to concurrent work. For one, predictive skill is exogenous in our model, despite the indications that there are potentially very interesting forces at work in its initial acquisition. Second, once an initial level of predictive skill has been acquired, our reduced-form model of precision cannot consider the important feedback loop between data and the strategic aspects that it endows sellers with. These choices constrain our model to be static in its most natural interpretation, inhibiting us from exploring essential dynamics, such as learning within and across each side of the market, market structure, and prices as well as production. A formal characterization of these would be very helpful in understanding the drivers of observed asymmetries. Lastly, the naivete of buyers with respect to sellers' information could also benefit from relaxation, given its importance to outcomes, even within the present setting, as well as its role in a host of results emerging from the literature on consumer privacy. Much like these agents, we are using a model to understand a complicated problem; however, structural concerns have led us to sacrifice additional complexity for the sake of intelligibility. Our hope is that this work contributes to an ongoing discussion that advances the precision of our understanding.

# Appendix A Exogenous Quality Setting: Seller Strategies

## A.1 Economies with Separating and Pooling Offers

Starting with the strategy of seller types that offer the (weakly) highest prices (sharks who observe high signals), the supremum of their distribution's support is as specified by Proposition 3.6. If their highest price is $\theta_l$, we focus on the symmetric equilibrium where all sellers have the same mixture to avoid introducing a purely coordination role for predictive skill. When every type of seller has the same strategy, however, the distribution of competitor offers in both low and high matches is precisely equal to it, so a single equation a every price offer pins down this distribution $F(x)$; mainly, that which guarantees that every offer yields the same profits as the highest one,

$$\rho\theta = (\rho + (1-\rho)(1-F(x)))x \tag{A.1}$$

**Proposition A.1** (All Pooling Symmetric Equilibrium)**.** *If the highest equilibrium is pooling, then the distribution of competition offers in low and high matches, as well as, each type of seller's strategy is given by the distribution,*

$$F(x) = 1 - \frac{\rho}{1-\rho}(\theta - x) \tag{A.2}$$

However, when the highest equilibrium price is separating, we study the unique ordered equilibrium, without loss of generality, and since seller strategies are ordered in these equilibria, we solve for each of these differing strategies sequentially, starting with that of sharks who observe high signals. In the interest of clarity, we will first derive strategies assuming that no seller type offers both separating and pooling prices; then we will show how to modify the solution when the assumption is violated. Since all prices of sharks who observe high signals are separating in these economies, their lowest price offer is also separating and given by,

$$\begin{aligned} \rho\theta_h &= (\rho + (1-\rho)\mu(s)P^s(h|\theta_h))\,\underline{x}^{s,h} \\ \implies \underline{x}^{s,h} &= \frac{\rho}{\rho + (1-\rho)\mu(s)P^s(h|\theta_h)}\theta_h \end{aligned} \tag{A.3}$$

where $\mu(s)P^s(h|\theta_h)$ is the mass of peer sharks who observe $h$ that a seller expects to face in competitive matches if the buyer is of high-valuation - it would always lose against sellers of higher type, as they offer lower prices in the ordered equilibrium. The allocation of mass between the upper and lower bound is then uniquely determined by (3.3) and (3.11),

$$F^{s,h}(x) = \begin{cases} 1 & \forall\ x \in (\theta_h, \infty) \\ 1 - \frac{\rho}{(1-\rho)P(p_l^{s,j}|j,\theta_h)}\frac{\theta_h - x}{x} & \forall\ x \in [\underline{x}^{s,h}, \theta_h] \\ 0 & \forall\ x \in (-\infty, \underline{x}^{s,h}) \end{cases} \tag{A.4}$$

$$P(p_l^{s,j}|\theta_h) = \mu(s)P^s(h|\theta_h) \tag{A.5}$$

By induction, the strategy of any other type of seller $p_l^{e,j}$ who only makes separating offers,

$$F^{e,j}(x) = \begin{cases} 1 & \forall\ x \in (\overline{x}^{e,j}, \infty) \\ \left(1 - \frac{P^e(\overline{x}^{e,j}\ \text{sale}|\theta_h)}{(1-\rho)P^e(p_l^{e,j}|\theta_h)} \frac{\overline{x}^{e,j}-x}{x}\right) & \forall\ x \in [\underline{x}^{e,j}, \overline{x}^{e,j}] \\ 0 & \forall\ x \in (-\infty, \underline{x}^{e,j}) \end{cases} \tag{A.6}$$

$$P(p_l^{e,j}|\theta_h) = \mu(e)P^e(j|\theta_h) \tag{A.7}$$

$$P(x\ \text{sale}|\theta_h) = \rho + (1-\rho) \sum_{\substack{e' \in \{a,s\} \\ j' \in \{l,h\}}} \mu(e')P^{e'}(j'|\theta_h)(1 - F^{e',j'}(x)))\mathbb{1}(p_l^{e',j'} \leq p_l^{e,j}) \tag{A.8}$$

$$\overline{x}^{e,j} = \underline{x}^{e,j,-} \tag{A.9}$$

$$\underline{x}^{e,j} = \frac{P^e(\overline{x}^{e,j}\ \text{sale}|j,\theta_h)}{P^e(\underline{x}^{e,j}\ \text{sale}|j,\theta_h)}\overline{x}^{e,j} \tag{A.10}$$

where $p_l^{e,j,-}$ is the closest type from below. The terms $(1 - F^{e',j'}(x)))\mathbb{1}(p_l^{e',j'} \leq p_l^{e,j})$ in (A.8) are uniquely determined by the inductive assumption, if they are the strategies of lower type sellers $(p_l^{e',j'} < p_l^{e,j})$, or otherwise equal to zero, if are the strategies of higher type sellers $(p_l^{e,j} < p_l^{e',j'})$. As such, the strategy of the $p_l^{e,j}$ type seller, $F^{e,j}(x)$, is also uniquely determined.

Continuing with sellers who offer pooling prices. The highest such price is $\overline{x}^{e,j} = \theta_l$ (Proposition 3.7), and the form of upper and lower bounds as well as strategies of the types of sellers who make pooling offers are analogous to those of types who make separating offers, after adjusting the type's expected sales to include its posterior probability of being in a low match,

$$F^{e,j}(x) = \begin{cases} 1 & \forall\ x \in (\overline{x}^{e,j}, \infty) \\ 1 - \frac{P^e(\overline{x}^{e,j}\ \text{sale}|j)}{(1-\rho)P^e(p_l^{e,j}|j)} \frac{\overline{x}^{e,j}-x}{x} & \forall\ x \in [\underline{x}^{e,j}, \overline{x}^{e,j}] \\ 0 & \forall\ x \in (-\infty, \underline{x}^{e,j}) \end{cases} \tag{A.11}$$

$$P^e(p_l^{e,j}|j) = p_l^{e,j}\mu(e)P^e(j|\theta_l) + p_h^{e,j}\mu(e)P^e(j|\theta_h) \tag{A.12}$$

$$P^e(x\ \text{sale}|j) = \rho + (1-\rho) \sum_{\substack{j' \in \{l,h\}, \\ e' \in \{a,s\}}} \mu(e') \left(p_l^{e,j}P^{e'}(j'|\theta_l) + p_h^{e,j}P^{e'}(j'|\theta_h)\right)(1 - F^{e',j'}(x))\mathbb{1}(p_l^{e',j'} \leq p_l^{e,j}) \tag{A.13}$$

$$\overline{x}^{e,j} = \underline{x}^{e,j,-} \tag{A.14}$$

$$\underline{x}^{e,j} = \frac{P^e(\overline{x}^{e,j}\ \text{sale}|j)}{P^e(\underline{x}^{e,j}\ \text{sale}|j)}\overline{x}^{e,j} \tag{A.15}$$

It is understood that $x^{e,j,-} = \theta_l$ for the type of seller that offers the highest pooling price.

## A.2 Seller Strategies with Disjoint Support

We can verify if a seller type who offers separating prices also offers any pooling ones, by checking if offering the highest possible pooling price ($\theta_l$), and beating all sellers of equal or lower type in contested matches, it could make strictly greater profits than those of monopoly offers,

$$p_j^{e,h}\rho\theta_h < \left(\rho + (1-\rho) \sum_{p_l^{e',j'} \leq p^{e,j}} (p_l^{e,j}P(p_l^{e',j'}|\theta_l) + p_h^{e,j}P(p_l^{e',j'}|\theta_h))\right)\theta_l$$

$$\frac{\rho}{(1-\rho)\sum_{p_l^{e',j'} \leq p^{e,j}} (p_l^{e,j}P(p_l^{e',j'}|\theta_l) + p_h^{e,j}P(p_l^{e',j'}|\theta_h))} < \frac{\theta_l}{p_j^{e,h}\theta_h - \theta_l} \tag{Discontinuity Condition}$$

When this condition holds, this type's separating distribution continues as specified by (A.6), up to its lowest separating offer $\hat{x}_l^{e,j}$ given by,

$$\begin{aligned} & p_h^{e,j}\left(\rho+(1-\rho)\sum_{p_l^{e',j'}<p^{e,j}} P(p_l^{e',j'}|\theta_h)\right)\overline{x}^{e,j} \\ &= \left(\rho+(1-\rho)\sum_{p_l^{e',j'}\leq p^{e,j}} (p_l^{e,j}P(p_l^{e',j'}|\theta_l)+p_h^{e,j}P(p_l^{e',j'}|\theta_h))(1-F^{e',j'}(\hat{x}^{e,j}))\right)\theta_l \end{aligned} \tag{A.16}$$

since no other type offers prices in the gap $[\theta_l, \hat{x}_l^{e,j}]$, nor are there any point masses in equilibrium mixtures, whereby $F^{e,j}(\theta_l) = F^{e,j}(\hat{x}^{s,h})$. As such, we can solve for $\hat{x}^{e,j}$ explicitly by substituting the form of the distribution over separating offers (A.6),

$$\begin{aligned} & \hat{x}^{e,j} \\ &= \frac{\overline{x}^{e,j}}{1+\frac{p_h^{e,j}\left(\rho+(1-\rho)\sum_{p_l^{e',j'}<p^{e,j}} P(p_l^{e',j'}|\theta_h)\right)\frac{\overline{x}^{e,j}}{\theta_l}-\left(\rho+(1-\rho)\sum_{p_l^{e',j'}<p^{e,j}}(p_l^{e,j}P(p_l^{e',j'}|\theta_l)+p_h^{e,j}P(p_l^{e',j'}|\theta_h))\right)}{(p_l^{e,j}P(p_l^{e,j}|\theta_l)+p_h^{e,j}P(p_l^{e,j}|\theta_h))\frac{P^e(\overline{x}^{e,j}\text{ sales}|j)}{P(p_l^{e,j}|\theta_h)}}} \end{aligned} \tag{A.17}$$

The mixture continues downwards over pooling prices in the standard manner, as per (A.11), until the lowest overall price,

$$\begin{aligned} & p_h^{e,j}\left(\rho+(1-\rho)\sum_{p_l^{e',j'}<p^{e,j}} P(p_l^{e',j'}|\theta_h)\right)\overline{x}^{e,j} \\ &= \left(\rho+(1-\rho)\sum_{p_l^{e',j'}\leq p^{e,j}} (p_l^{e,j}P(p_l^{e',j'}|\theta_l)+p_h^{e,j}P(p_l^{e',j'}|\theta_h))\right)\underline{x}^{e,j} \\ \implies \underline{x}^{s,h} &= \frac{p_h^{e,j}\left(\rho+(1-\rho)\sum_{p_l^{e',j'}<p^{e,j}} P(p_l^{e',j'}|\theta_h)\right)}{\rho+(1-\rho)\sum_{p_l^{e',j'}\leq p^{e,j}}(p_l^{e,j}P(p_l^{e',j'}|\theta_l)+p_h^{e,j}P(p_l^{e',j'}|\theta_h))} \end{aligned} \tag{A.18}$$

# Appendix B Endogenous Quality Setting: Equilibrium Properties

In this section, we will discuss the technical aspects of the candidate equilibrium introduced in Section 4.3 and explain why many of its distinguishing properties hold in all equilibria. An even stronger result will follow that the ordered symmetric equilibrium is unique, which allows us to affirm the genericity of the comparative static analysis in Section 4.4.

## B.1 Equilibrium Distributions and Orderedness

We begin by recalling the concept of an ordered equilibrium, which connects the level of indirect utility offered to low and high-valuation buyers in a menu. Put simply, in an ordered equilibrium, sellers who offer more indirect utility in the contract intended for a high-valuation buyer must do the same in the contract intended for a low valuation one.

**Definition B.1** (Orderedness). *An equilibrium is said to be weakly-ordered if, for any two equilibrium menus* $(u_l, u_h)$ *and* $(u_l', u_h')$,

$$(u_h - u_h')(u_l - u_l') \geq 0$$

*When the inequality holds strictly in almost every*[21] *comparison, we refer to the equilibrium as ordered.*

These two properties have also been referred to as rank preserving and strictly rank preserving in related work (including Lester et al. (2019)). Like them, we find that orderedness is necessary holds whenever a buyer's incentive compatibility constraint binds at one of the menus that is being compared.

**Lemma B.1** (Ordered Equilibrium). *Almost every equilibrium menu* $(u_l, u_h)$ *featuring a binding incentive compatibility constraint is ordered when compared to another equilibrium menu* $(u_l{}', u_h{}')$. *That is,*

$$(u_h - u_h{}')(u_l - u_l{}') > 0$$

The economic rationale underlying this complementarity in indirect utilities is familiar from Garrett et al. (2019) and intuitively explained in Section 4.3. Consider a seller who increases the indirect utility it offers to high-valuation buyers $u_h$. If their incentive constraint binds, this relaxes it and allows for an increase in the quality provided to buyers of low valuation in the paired contract $(q_l, x_l)$, as per $q_l = \frac{u_h - u_l}{\Delta\theta}$. The seller can then offer low-valuation buyers the same utility $u_l$, while obtaining strictly larger profits in each sale to them; however, when low sales are more profitable, the seller also bids more aggressively for them, and this is done by making them more appealing through a low utility increase ($\uparrow u_l$). The channel that creates this complementarity is, therefore, the connection between the efficiency of the contracts and the utility they offer to both buyers. Since this link is missing among dually efficient offers, at which incentive compatibility constraints are slack, they do not need to be ordered.

Orderedness simplifies the equilibrium structure substantially, and heterogeneity in sellers' interim beliefs does not alter the fundamental complementarity between providing additional utility to low and high-valuation buyers, but rather how interested sellers are in forfeiting high sale profitability for low one. Therefore, the heterogeneity of the posteriors *moderates* the joint progression of $u_l$ and $u_h$, but does not change the correlation between these.

**Theorem B.1** (Type Monotonicity). *Let* $(u_l, u_h)$ *and* $(u_l', u_h')$ *be two equilibrium menus sharing a common binding incentive compatibility constraint with* $u_i < u_i{}'$. *Then,*

1. *High sale profits per-match are decreasing in generosity,* $\Pi_h(u_l, u_h) > \Pi_h(u_l', u_h')$, *while low ones increase,* $(u_l, u_h) < \Pi_l(u_l', u_h^j{}')$.
2. *If the menus are offered by sellers of respective types* $p_l \neq p_l'$, *then* $p_l < p_l'$

Consider the menu $(u_l(p), u_h(p))$ occupying the $p^{th}$ generosity-percentile in the equilibrium distribution. Then, $u_h(p)$ increases enough for profits from high sales $\Psi_h(p)(S_h^* - u_h(p))$ to decrease, but $u_l(p)$ grows passively enough to not undo the additional profitability of profits from low sales. The relationship between low/high trade profits per-match and generosity creates an equilibrium structure where seller types comparatively more interested in profits from high sales make offers that are less generous towards buyers with either valuation than those made by seller types comparatively more interested in profits from low sales. In particular, sellers relatively more convinced that they face high-valuation buyers will aim to depress bids as much as possible so as to extract these buyers'

[21] Up to a measure zero set of menus.

(information) rents, whereas sellers who are relatively more convinced that they face low-valuation buyers give greater consideration to capturing profitable trade with them and cede additional rents to both low and high-valuation buyers to do so.

Efficiency gains thus far have been described as taking place within low trade, implicitly treating the incentive constraint of high-valuation buyers as the only relevant one. In fact, this is a necessary property of equilibria in any economy where sellers' costs satisfy Assumption 4.2. Furthermore, in these, offers are grouped in two sets of menus. The most rationed menus, at which high-valuation buyers' incentive constraints binds, are also the least generous, and then any that are dually efficient are also more generous towards low and high-valuation buyers.

**Lemma B.2** (Stacking)**.** *The menus offered in an equilibrium where firms' costs satisfy Assumption 4.2 are partitioned into separate incentive compatibility regions such that*

1. *low-valuation buyers' incentive constraint does not bind in at any menu.*
2. *If some dually efficient menus are offered, there exists a $u_i^{de}$ such that all utilities $u_i < u_i^{de}$ are offered in menus at which high-valuation buyers' incentive constraint binds, whereas all utilities $u_i \geq u_i^{de}$ are offered in dually efficient menus.*

Consider the logic that drives this, from the least generous bid to the most generous. The least generous menu is offered by a seller who only expects to sell if it is in a lone match, so it offers exactly the menu it'd choose if it was a monopolist with the same assessment of the buyer's probable valuation, and a monopolist would never offer a menu at which low-valuation buyers' incentive constraint binds - featuring $u_h - u_l > q_h^* \Delta\theta$ - since they could strictly increase profits from high sales by offering fewer rents to high-valuation buyers. Competition seller types who make more generous offers drives the efficiency of these alongside their generosity (through $u_h - u_l$ growth). When there is sufficient upward pressure on generosity/efficiency, such that the utility gap reaches $u_h - u_l = q_l^* \Delta\theta$, menus become dual efficient. Without an efficiency benefit to high rent concession ($\uparrow u_h$) or efficiency loss to low rent extraction ($\downarrow u_l$) among dually efficient offers, the growth of $u_h - u_l$ slows so that low-valuation buyers' incentive constraint never binds.

The results we have covered so far do not rely on the differentiability in any way, but it is a convenient feature to convey intuition and maintain tractability. It is even better to be able to work with continuously differentiable conditional distributions $F_i(u_i|\theta_i)$ over the utilities offered by the average seller to each kind of buyer. Fortunately, equilibria also have these properties.

**Lemma B.3** (Equilibrium Distributions)**.** *Equilibrium distributions $F_i(u_i|\theta_i)$ for $i \in \{l, h\}$.*

1. *Do not have atoms in their supports $\Upsilon_i$.*
2. *Have a convex, connected low support $\Upsilon_l = [\underline{u}_l, \overline{u}_l]$. The high support $\Upsilon_h$ is the union of at most two convex sets disjoint sets, composed of the high utilities offered in menus where high-valuation buyers' incentive constraint bids and is slack, respectively. Furthermore, suprema over utilities always satisfy $\overline{u}_i \leq S_i^*$.*
3. *Are continuously differentiable on the interior of their supports with one-sided derivatives at the boundaries.*

Atoms make it possible for sellers to obtain discrete increases in sales in exchange for infinitesimal discounts in profits per sale, so it is clear that these cannot exist. Given that low-valuation buyers' incentive constraint does not bind in equilibrium, gaps in the low support would allow the sellers offering a menu with implied utility at the top of the gap to increase their profitability in low matches by decreasing the rents that the menu offers to low-valuation buyers, which would achieve identical low sales but strictly higher profits in each one. The logic for the convexity claim among high offers depends on whether the point $u_h$ that we are considering is such that menus that offer it are dually efficient or constrained. In the former case, high-valuation buyers' incentive constraint is slack, so a gap below any such utility would allow a seller that offers it to become more profitable by lowering these high rents, which would preserve high sales, increase high profits per sale, and not affect the efficinecy/profitability of its low-valuation buyer sales. In the latter case, a constrained menu is ordered when compared with any other, so a gap on the high side is either accompanied by one on the low side (which we have ruled out), an atom at its low utility offer (which we have ruled out), or a situation there are two constrained menus (with the same low utility term, but one has the high utility term at the top of the gap and the other the one at the bottom) and low sales vary locally in low generosity in such a way that it is not preferrable to alter the low offer whether the menu is more or less profitable in each low sale, which is not possible. Lastly, the differentiability claims follow because sellers' indifference must be maintained by the probability of winning in combination with profits per sale, and additively separable utilities allowed us to rewrite profits per sale as $S_i(u_l, u_h) - u_i$, which is smooth in marginal changes to either utility, so differentiability of equilibrium distributions becomes necessary to rule out infinitesimal deviations.

## B.2 Equilibrium System of Equations Derivation

We now briefly derive the system of equations that allow us to obtain the candidate equilibrium's analytical closed form. Recall that the problem of a type $p_l^{e,j}$ seller is to offer a menu $(u_l, u_h)$ that maximizes her expected profits,

$$\Pi^{e,j}(u_l, u_h) = \sum_{i=l,h} p_i^{e,j}\Psi_i(u_i)(S_i(u_l, u_h) - u_i)$$

subject to the constraint $u_h \geq u_l \geq 0$. The first-order conditions of this problem highlight the interdependence between the optimal amount of utility extended to each type of buyer, as well as the role of posteriors in determining the relative importance of various trade-offs. Based on the fact that only high-valuation buyers' incentive constraint can bind in the candidate equilibrium, the seller's optimality conditions are

$$\frac{\partial}{\partial u_l}: \quad \underbrace{p_l^{e,j}(1-\rho)f_l(u_l|\theta_l)(S_l(u_l, u_h) - u_l)}_{\text{sales gains}} - \underbrace{p_l^{e,j}\Psi_l(u_l)}_{\text{rent losses}} + \underbrace{p_l^{e,j}\Psi_l(u_l)\frac{\partial S_l}{\partial u_l}(u_l, u_h)}_{\text{efficiency losses}} = 0 \tag{B.1}$$

$$\frac{\partial}{\partial u_h}: \quad \underbrace{p_h^{e,j}(1-\rho)f_h(u_{h|\theta_h})(S_h^* - u_h)}_{\text{sales gains}} - \underbrace{p_h^{e,j}\Psi_h(u_h)}_{\text{rent losses}} + \underbrace{p_l^{e,j}\Psi_l(u_l)\frac{\partial S_l}{\partial u_h}(u_l, u_h)}_{\text{efficiency gains}} = 0 \tag{B.2}$$

Similar terms appear in both equations. The first two capture a typical trade-off between expected sales versus rents per-sale. By increasing indirect utility $u_i$, a seller makes her offer more attractive to $\theta_i$ valuation buyers, thus increasing the probability of selling to them by the mass of equilibrium menus that it would be preferred over in contested matches, mainly $p_i^{e,j}(1-\rho)f_i(u_i|\theta_i)$, whereas the cost of surrendering said rents is directly proportional to the likelihood of trading $p_i^{e,j}\Psi_i(u_i)$ with buyers of this valuation. The third term determines the efficiency effect of an increase in generosity towards $\theta_i$ valuation buyers ($u_i$), and it stems from the point that univariate changes in generosity $u_i$ alter the difference in offered utilities $u_h - u_l$, which drives efficiency. When high-valuation buyers' incentive constraint binds at a menu, generosity towards low-valuation buyers ($u_l$ increases) requires further rationing ($q_l$ decrease), thereby reducing the gains of trade with them $S_l(u_l, u_h)$ and, by extension, the profitability of their purchases; the opposite holding for generosity towards high-valuation buyers. In other words, generosity in the low (high) offer has an efficiency cost (benefit), when high-valuation buyers' incentive constraint is locally binding. Whereas if low-valuation buyer's incentive constraint is slack, the efficiency term disappears and the only consideration for the seller is the aforementioned trade-off between the from sales and rents given to buyers of the same valuation.

The implicit objects of immediate interest are the marginal conditional utility distributions $F_i(u_i|\theta_i)$, which shape the nature of competition. Marginals have densities that measure the mass at points on the supports of some seller types' mixed strategy. These supports are atomless, convex, and monotone in the seller's type (overlapping only among any dually efficient bids). Locally, each conditional marginal density $f_i(u_i|\theta_i)$, therefore, corresponds to a weighted density of each seller type's conditional marginal density. In particular, if the utility is offer in an inefficient menu, then these is a unique seller type $p_{;}^{e,j}$ that offers it and the conditional marginal density is given by,

$$f_i(u_i|\theta_i) = \mu(e)P^e(j|\theta_i)f_i^{e,j}(u_i|\theta_i)$$

For utilities that are offered in dually efficient menus, the conditional marginal density still takes a weighted average form but there is a much simpler way to solve for the utility offers, so we will not use that relation.

To obtain the distribution over utilities in inefficient menus, we will further rewrite the first-order conditions of sellers offering these by applying additional equilibrium properties. In particular, we recall that menus are ordered, so the particular ones of seller types $p_l^{e,j}$ who make inefficient offers are as well, and can be written as functions $(u_l(Q), u_h(Q))$ of the menu's generosity quantile $Q$ in seller type $p_l^{e,j}$'s mixed strategy. This allows us to apply the inverse function theorem and link the conditional marginal densities $f_i(u_i|\theta_i)$ to the progression of utilities,

$$f_i(u_i|\theta_i) = \frac{\mu(e)P^e(j|\theta_i)}{\dot{u}_i^{e,j}(Q)} \tag{B.3}$$

so the conditional marginal distributions take the form,

$$F_i(u_i|\theta_i) = \mu(e)P^e(j|\theta_i)u_i^{e,j,(-1)}(u_i) + \sum_{\substack{e',j' \\ \text{s.t. } p^{e',j'} < p^{e,j}}} \mu(e')P^{e'}(j'|\theta_i) \tag{B.4}$$

where $u_i^{e,j,(-1)}(\cdot)$ is understood to be the inverse of the strictly monotone functions $u_i^{e,j}(Q)$. And, the $Q^{th}$ quantile menu from a $p_l^{e,j}$ type obtains average sales per match,

$$\Psi_i^{e,j}(Q) = \rho + (1-\rho)F_i(u_i^{e,j}(0)|\theta_i) + (1-\rho)\mu(e)P^e(j|h)Q$$

in matches with $\theta_i$ valuation buyers.

Substituting (B.3) and (B.4) into the first-order conditions produces a standard system of ordinary differential equations that pins down the indirect utilities offered in constrained equilibrium menus. Piecewise linear costs make marginal efficiency effects locally constant, which decouples these equations and allows us to obtain analytical solutions: the equation that drives high utility $u_h^{e,j}(Q)$ is independent, under piecewise linear costs, and we can then substitute its solution into the equation driving the progression of low utility $u_l^{e,j}(Q)$. Specifically, note the marginal efficiency term becomes,

$$\frac{\partial S_l}{\partial u_h}(u_l, u_h) = (\theta_l - \kappa_l)\frac{\partial q_l}{\partial u_h} = \frac{\theta_l - \kappa_l}{\Delta\theta}$$

So, the differential system governing the progression of utilities in inefficient menus offered by a type $p_l^{e,j}$ seller is,

$$\dot{u}_l^{e,j}(Q)\left[-\frac{\theta_l - \kappa_l}{\Delta\theta} - 1\right]\Psi_l^{e,j}(Q) + (1-\rho)\mu(e)P^e(j|l)(S_l(u_l^{e,j}(Q), u_h^{e,j}(Q)) - u_l^{e,j}(Q)) = 0 \tag{B.5}$$

$$\dot{u}_h^{e,j}(Q)\left[\frac{p_l^{e,j}}{p_h^{e,j}}\frac{\theta_l - \kappa_l}{\Delta\theta}\Psi_l^{e,j}(Q) - \Psi_h^{e,j}(Q)\right] + (1-\rho)\mu(e)P^e(j|h)(S_h^* - u_h^{e,j}(Q)) = 0 \tag{B.6}$$

If offers become dually efficient, either among the offer seller type or because this seller type prefers to jump right to making dually efficient offers when it transitions from those of the adjacent seller type below, we will solve for the remaining utility offers with a different equation. As for the exact form of the utilities that solve the system (B.5)-(B.6), we define $\Xi^{e,j} = \frac{p_h^{e,j}P(j|h)}{p_h^{e,j}P(j|h) - p_l^{e,j}P(j|l)\frac{\theta_l - \kappa_l}{\Delta\theta}}$ to tighten the expressions and write the high utility term as,

$$u_h^{e,j}(Q) = S_h^* - C_h^{e,j}\left(p_h^{e,j}\Psi_h^{e,j}(Q) - p_l^{e,j}\Psi_l^{e,j}(Q)\frac{\theta_l - \kappa_l}{\Delta\theta}\right)^{-\Xi^{e,j}} \tag{B.7}$$

with,

$$C_h^{e,j} = \frac{S_h^* - u_h^{e,j}(0)}{\left(p_h^{e,j}\Psi_h^{e,j}(0) - p_l^{e,j}\Psi_l^{e,j}(0)\frac{\theta_l - \kappa_l}{\Delta\theta}\right)^{-\Xi^{e,j}}} \tag{B.8}$$

which simplifies to,

$$u_h^{e,j}(Q) = S_h^* - (S_h^* - u_h^{e,j}(0))\left(\frac{p_h^{e,j}\Psi_h^{e,j}(0) - p_l^{e,j}\Psi_l^{e,j}(0)\frac{\theta_l - \kappa_l}{\Delta\theta}}{p_h^{e,j}\Psi_h^{e,j}(Q) - p_l^{e,j}\Psi_l^{e,j}(Q)\frac{\theta_l - \kappa_l}{\Delta\theta}}\right)^{\Xi^{e,j}} \tag{B.9}$$

Substituting this explicit form of high utility into the differential equation for low utility, we then obtain its functional form,

$$u_l^{e,j}(Q) = \frac{\Psi_l^{e,j}(0)}{\Psi_l^{e,j}(Q)}\left(C_l^{e,j} + \frac{(1-\rho)\mu(e)P^e(j|l)}{\Psi_l^{e,j}(0)}\frac{\theta_l - \kappa_l}{\theta_h - \kappa_l}\left(\frac{C_h^{e,j}}{-p_h^{e,j}P^e(j|h)}\right.\right. \tag{B.10}$$

$$\left.\left. * \left(\left(p_h^{e,j}\Psi_h^{e,j}(Q) - p_l^{e,j}\Psi_l^{e,j}(Q)\frac{\theta_l - \kappa_l}{\Delta\theta}\right)^{1-\Xi^{e,j}} - \left(p_h^{e,j}\Psi_h^{e,j}(0) - p_l^{e,j}\Psi_l^{e,j}(0)\frac{\theta_l - \kappa_l}{\Delta\theta}\right)^{1-\Xi^{e,j}}\right)\right)\right) \tag{B.11}$$

with $C_l^{e,j} = u_l^{e,j}(0)$. This heavy expression is not very informative, but we can derive an implicit form of the low utility term - as a function of both its generosity quantile $Q$ and the high utility it is paired with $u_h^{e,j}(\cdot)$ - that is quite helpful. To do this, we simplify (B.6) and rewrite it as,

$$\Psi_l^{e,j}(Q)\dot{u}_l^{e,j}(Q) + \psi_l^{e,j}(Q)u_l^{e,j}(Q) = \psi_l^{e,j}(Q)\frac{\theta_l - \kappa_l}{\theta_h - \kappa_l}u_h^{e,j}(Q)$$

$$\implies \frac{\mathrm{d}}{\mathrm{d}Q}\left(\Psi_l^{e,j}(Q)u_l^{e,j}(Q)\right) = \psi_l^{e,j}(Q)\frac{\theta_l - \kappa_l}{\theta_h - \kappa_l}u_h^{e,j}(Q)$$

so that,

$$u_l^{e,j}(Q) = u_l^{e,j}(0)\frac{\Psi_l^{e,j}(0)}{\Psi_l^{e,j}(Q)} + \frac{\theta_l - \kappa_l}{\theta_h - \kappa_l}\int_0^Q \frac{\psi_l^{e,j}(x)}{\Psi_l^{e,j}(Q)}u_h^{e,j}(x)\,\mathrm{d}x \tag{B.12}$$

This expression highlights the fact that the low offer is a conditional expectation of the high offers made in less generous menus. This characterization provides an immediate proof for the point that the difference in utilities, $u_h^{e,j}(Q) - u_l^{e,j}(Q)$, and hence efficiency, increases with respect to generosity, and allows us to more easily think about the response of buyer welfare to parameter perturbations, by focusing on the response of high-valuation buyer surplus and then averaging these to get that of low-valuation buyers.

The initial condition of adjacent types $p_l^{e,j} < p_l^{e',j'}$ depends on whether (a) high-valuation buyers' incentive constraint binds at most generous menu $(u_l^{e,j}(1), u_h^{e,j}(1))$ of the lower type $p_l^{e,j}$ and (b) when that happens, whether the next type of seller would prefer for their least generous bid to also inefficient or dually efficient. This decision is determined by the efficiency gain from additional high rents on the profitability of the least generous low offer,

$$\begin{aligned} &-p_h^{e',j'}\Psi_h^{e',j'}(0) + p_l^{e',j'}\Psi_l^{e',j'}(0)(\theta_l - \kappa_l)\frac{\partial q_l}{\partial u_h} \\ &= -p_h^{e',j'}\Psi_h^{e',j'}(0) + p_l^{e',j'}\Psi_l^{e',j'}(0)\frac{\theta_l - \kappa_l}{\Delta\theta} \end{aligned}$$

When this is $> 0$, sellers of the higher type $p_l^{e',j'}$ prefer for their lowest bid to be dually efficient, and there is a discontinuous jump in generosity between $u_h^{e,j}(1) < u_h^{e',j'}(0)$. When the condition is $< 0$, instead, sellers of the higher type prefer for their least generous offers to also be inefficient, and there is no discountinuity between $u_h^{e,j}(1) < u_h^{e',j'}(0)$. In either case, the progression of the low utility terms is continuous and the higher seller type's least generous low utility offer is exactly the most generous one of the lower seller type, $u_l^{e,j}(1) = u_l^{e',j'}(0)$.

We will prove that the difference in utilities $u_h^{e,j}(Q) - u_l^{e,j}(Q)$ increases in the seller's quantile and hence the efficiency of the offer, so if these reach the point $u_h^{e,j}(Q) - u_l^{e,j}(Q) = q_l^*\Delta\theta$ such that the menu becomes dually efficient, then we solve for the dually efficient menus with a simpler equation. In particular, given the utility pair $(u_l^{de}(0), u_h^{de}(0))$ at which menus become dually efficient and the mass of sellers that makes constrained offers in $\theta_i$ matches $F_i(u_i^{de})$, then the equations that low and high offers must satisfy for dually efficient bids to have the necessary property of being equally profitable in low (high) matches is,

$$(\rho+(1-\rho)F_i(u_i^{de}(0)|\theta_i))(S_i^* - u_i^{de}(0))) = (\rho+(1-\rho)(1-F_i(u_i^{de}(0)|\theta_i))Q)(S_i^* - \underline{u}_i^{de}(Q)) \quad \text{for } i \in \{l, h\} \tag{B.13}$$

where $Q$ is the quantile among dually efficient menus of the utility $u^{de}(Q)$, and $1 - F_i(u_i^{de}(0)|\theta_i)$ is the probability that a seller makes a dually efficient offer in a $\theta_i$ match. By construction, these indirect utility functions have all the properties stipulated earlier: strictly increasing in generosity, monotone in seller type, identically ranked by their low and high utility offerings, and jointly forming a bottom pair of intervals comprised of utilities offered in menus where high-valuation buyers' incentive constraint binds, potentially followed above by another pair of intervals comprised of utilities offered in dually efficient menus.

# Appendix C Endogenous Quality: Equilibrium Property Proofs

*Proof of Theorem 4.1.* Per sale profits from $\theta_i$ valuation buyer intended contracts take the form:

$$\pi(q_i, x_i) = x_i - \phi(q_i) = (\theta_i q_i - \phi(q_i)) + (x_i - \theta_i q_i) = S_i(q_i) - u_i \tag{C.1}$$

and $S_i(q_i)$ is concave, reaching a maximum at $q_i^*$. Due to concavity, any incentive-compatible menu featuring a low quality $q_l > q_l^*$ is strictly dominated by one with a revised low contract of $(q_l^*, u_l - \theta_l(q_l - q_l^*))$, while any incentive-compatible menu featuring a high quality $q_h < q_h^*$ is strictly dominated by one with a revised high contract of $(q_h^*, u_h + \theta_h(q_h^* - q_h))$. In other words, the optimal incentive-compatible menus feature $q_l \leq q_l^*$ and $q_h \geq q_h^*$.

Furthermore, $IC_i$ must bind[22] if the $\theta_{\neg i}$ intended contract is not efficient; else, the revised contract $(\tilde{q}_{\neg i}, \tilde{x}_{\neg i})$ featuring[23] $\tilde{q}_{\neg i} = \frac{u_h - u_l}{\Delta\theta}$ and $\tilde{x}_{\neg i} = \theta_{\neg i}\tilde{q}_{\neg i} - u_{\neg i}$ could be paired with the former $\theta_i$ contract for a strictly dominant menu - same expected $\theta_{\neg i}$ sales (by preserving the utility that a $\theta_{\neg i}$ valuation buyer obtains), strictly higher profits-per-sale in $\theta_{\neg i}$ matches (by preserving $u_{\neg i}$ and increasing the gains from trade (see (C.1))), and maintaining the incentive compatibility of $\theta_i$ buyers ($\theta_i\tilde{q}_{\neg i} - \tilde{x}_{\neg i} = u_i$). Inversely, since only one constraint $IC$ can bind in a given menu, profit maximality implies that any menu featuring an inefficient $q_{\neg i}$ offer, also features efficient $q_i = q_i^*$ quality provision.

Incentive compatibility bounds the qualities offered in each contract by the suggested ratio: $q_l \leq \frac{u_h - u_l}{\Delta\theta}$ (to satisfy $IC_h$) and $q_h \geq \frac{u_h - u_l}{\Delta\theta}$ (to satisfy $IC_l$). We have shown that $IC_i$ binds when $q_{\neg i} \neq q_i^*$ though, so equality of the respective bound yields the form of the inefficient quality $q_{\neg i} = \frac{u_h - u_l}{\Delta\theta}$. Lastly, any menu featuring efficient quality provision in both contracts must feature utility offerings satisfying $q_l^*\Delta\theta \leq u_h - u_l \leq q_h^*\Delta\theta$.

We conclude that: (1) menus featuring inefficient low valuation buyer provision are $IC_h$ binding, feature $q_l = \frac{u_h - u_l}{\Delta\theta} < q_l^*$, and are paired with efficient high-valuation buyer contracts, (2) menus featuring inefficient high-valuation buyer provision are $IC_l$ binding, feature $\frac{u_h - u_l}{\Delta\theta} = q_h < q_h^*$, and are paired with efficient low valuation buyer contracts, and (3) doubly efficient menus correspond to those for which $q_l^*\Delta\theta \leq u_h - u_l \leq q_h^*\Delta\theta$ almost all of which have locally slack $IC$ constraints with the exception of boundary ones satisfying $q_i^*\Delta\theta = \frac{u_h - u_l}{\Delta\theta}$ at which $IC_{\neg i}$ binds. □

*Proof of Theorem 4.2.* Recall that seller types with degenerate beliefs have measure zero, so the following arguments apply to almost every bid offered in a match, in particular those that would be offered by types with nondegenerate conditional beliefs.

Consider the essential infimum and supremum, $\underline{u}_i$ and $\overline{u}_i$, respectively, on the equilibrium indirect utility offered to $\theta_i$ valuation buyers. By Theorem 4.1, equilibrium menus are separating and these must correspond to the bounds on utilities extended in $\theta_i$ intended contracts, with profits $S_i(u_l, u_h) - u_i$. If $\overline{u}_i > S_i^*$, then a discrete mass of menus would feature contracts with a non-zero probability of being accepted and entail negative profits per sale. As such, $\overline{u}_i \leq S_i^*$ and we will argue that the lower bound $\underline{u}_i$ is exactly $S_i^*$.

Suppose that $\underline{u}_i < S_i^*$. By Theorem 4.1, optimal contracts with $u_i < S_i^*$ entail strictly positive[24] profits per sale if $q_i > 0$ and zero profits per sale if $q_i = 0$. Further, there are only two possibilities in a neighborhood of $\underline{u}_i$: either $\underline{u}_i$ is an atom, or half-open sets $[\underline{u}_i, \underline{u}_i + \delta)$ are assigned arbitrarily small mass, as $\delta \searrow 0$, by the equilibrium distribution of indirect utility offerings by competitors in $\theta_i$ matches $F_i(u_i|\theta_i)$.

Recall that every contract makes nonnegative profits per sale, since not trading (offering zero quality) is always an option. As such, if there were an atom at $\underline{u}_i$, the menu implied by pairing a slightly more generous $\theta_{\neg j}$ offering $u_{\neg j} + 2\varepsilon$ and $\underline{u}_i + \varepsilon$ (for $\varepsilon > 0$ small) would be strictly dominant, entailing (at worst) an arbitrarily small decrease in $\theta_{\neg j}$ per sale profits, a discrete increase in $\theta_i$ expected sales, and thus a discrete increase in $\theta_i$ profits per match. Whereas if there were no atom at

[22]It is elementary to check that only one $IC$ constraint can bind in a menu.

[23]Where $\tilde{q}_{\neg i} < q_{\neg i}$ if $\neg i = l$ and $>$ if $\neg i = h$.

[24]If $q_i = q_i^*$, the claim follows. If $q_i \neq q_i^*$ and $u_i = S_i(q_i)$, then lowering $u_i$ by a small $\varepsilon < u_i - \underline{u}_i$ would allow the seller to still win matches with some probability $u_i$ and make strictly positive profits in these, as the revision would both increase the efficiency and lower the generosity of these sales.

the lower bound, contracts offering $u_i$ arbitrarily close have $\theta_i$ match profits arbitrarily close to zero (they almost surely compete against a more generous seller); so a discrete mass of these is strictly dominated by a menu of the form given in the atom case.

We conclude that almost every seller selects a pair of contracts that extend implied utilities $(S_l^*, S_h^*)$. By Theorem 4.1, the unique optimal menu that satisfies these conditions is $((q_l^*, \phi(q_l^*)), (q_h^*, \phi(q_h^*)))$. □

*No Atoms.* Toward a contradiction, suppose that $F_h$ had an atom at $u_h$ and let $(u_l, u_h)$ be an equilibrium menu featuring this high bid generosity.

We begin by showing that $S_h(u_l, u_h) - u_h > 0$ in any equilibrium offer $(u_l, u_h)$. Suppose not. Then we must have $S_l(u_l, u_h) - u_l \leq 0$, otherwise offering a pooling menu of only the low valuation buyer's contract would strictly increase the seller's expected profits - strictly positive per-sale profits from $\theta_h$ sales and strictly positive probability of being accepted ($\rho > 0$), even by low-valuation buyers. But then expected profits $\Pi \leq 0$, which contradicts seller optimization: the seller can always offer the menu $((0,0), (q_h^*, \theta_h q_h^*))$ and obtain strictly positive expected profits. With this fact, we can rule out an atom at any $u_h$ in $\text{supp}(\Psi_h)$.

In particular, note that at any $u_h$ with discrete mass and for any type of seller $p_l$,

$$
\begin{aligned}
&\lim_{\varepsilon \searrow 0} \Pi(u_l + \varepsilon, u_h + \varepsilon) - \Pi(u_l, u_h) \\
&= \lim_{\varepsilon \searrow 0} \left\{ \sum_{k=l,h} p_k \Psi_k(u_k + \varepsilon)(S_k(u_l + \varepsilon, u_h + \varepsilon) - u_k - \varepsilon) - \sum_{k=l,h} p_k \Psi_k(u_k)(S_k(u_l, u_h) - u_k) \right\} \\
&= \lim_{\varepsilon \searrow 0} \big((1-\rho)(F_h(u_h + \varepsilon) - F_h(u_h))\big)(S_h(u_l, u_h) - u_h) \\
&> 0 \qquad \text{(C.2)}
\end{aligned}
$$

and so $(u_l, u_h)$ would be strictly dominated by $(u_l + \varepsilon, u_h + \varepsilon)$, for some $\varepsilon > 0$.

Furthermore, $S_l(u_l, u_h) - u_l \geq 0$ in equilibrium; otherwise, this menu would be strictly dominated by one with the paired offers $(q_l, x_l) = (0,0)$ and $(q_h, x_h) = (q_h^*, \theta_h q_h^* - u_h)$, which maintain expected high match profits and strictly those of low ones (these buyers either select a no-loss contract, $(0,0)$, or one with strictly positive profits per sale, $(q_h, x_h)$).

We close by ruling out atoms among low bids. Suppose that $\text{supp}(\Psi_l)$ had an atom at $u_l$ and let $(u_l, u_h)$ be an equilibrium menu featuring this low bid generosity. Inequalities (C.2) rule out $S_l(u_l, u_h) - u_l > 0$, so the only possible menu with such a low offering must be one that makes zero profits in low matches $S_l(u_l, u_h) - u_l = 0$. Suppose that $u_l > 0$. If $IC_l$ is slack, then menu with slightly less low generosity $(u_l - \varepsilon, u_h)$ is strictly dominant (low efficiency nondecreasing hence positive profits per low sale, some low sales since $\rho > 0$, high trade profitability not affected), whereas if $IC_l$ binds, low-valuation buyers obtain utility $u_l > 0$ from the high contract, which has strictly positive profits per sale, so the seller could just pool all buyers on this contract and makes strictly positive profits in low matches as well. Lastly, if $u_l = 0$ and $S_l(u_l, u_h) - u_l = 0$, it follows (by Theorem 4.1) that $u_h = 0$, so a strictly positive mass of such $(u_l, u_h)$ menus would give rise to an atom in $\text{supp}(\Psi_h)$. □

*Weak-Orderedness.* Consider two equilibrium menus $(u_l, u_h)$ and $(\tilde{u}_l, \tilde{u}_h)$, offered by sellers of (possibly equal) respective types $p_l$ and $\tilde{p}_l$, which violate weak-orderedness; without loss, suppose that this takes place via $\tilde{u}_h > u_h$ and $u_l > \tilde{u}_l$. We proceed case-by-case, depending on the incentive compatibility constraint that binds at $(u_l, u_h)$.

Suppose that $IC_h$ binds at $(u_l, u_h)$. Then,

$$\begin{aligned}
\Psi_l(u_l) &> \Psi_l(\tilde{u}_l) \\
S_l(u_l, \tilde{u}_h) - S_l(u_l, u_h) &\geq S_l(\tilde{u}_l, \tilde{u}_h) - S_l(\tilde{u}_l, u_h) \geq 0 \\
S_l(u_l, \tilde{u}_h) - S_l(u_l, u_h) &> 0
\end{aligned}$$

where the second set of inequalities follows by the convexity of costs, Theorem 4.1,

$$\begin{aligned}
S_l(u_l, u_h) &= \theta_l q_l(u_l, u_h) - \phi\left(q_l(u_l, u_h)\right) \\
q_l(u_l, u_h) &= \begin{cases} \frac{u_h - u_l}{\Delta\theta} & \text{if } u_h - u_l < q_l^* \Delta\theta \\ q_l^* & \text{otherwise} \end{cases}
\end{aligned}$$

while the third is due to the fact that $IC_h$ binds at $(u_l, u_h)$ (by assumption), so the efficiency of low trade achieved by $(u_l, \tilde{u}_h)$ must be strictly greater. Thus,

$$\Psi_l(u_l)\left(S_l(u_l, \tilde{u}_h) - S_l(u_l, u_h)\right) - \Psi_l(\tilde{u}_l)\left(S_l(\tilde{u}_l, \tilde{u}_h) - S_l(\tilde{u}_l, u_h)\right) > 0$$

and,

$$\Psi_l(u_l)\left(S_l(u_l, \tilde{u}_h) - u_l\right) - \Psi_l(\tilde{u}_l)\left(S_l(\tilde{u}_l, \tilde{u}_h) - \tilde{u}_l\right) > \Psi_l(u_l)\left(S_l(u_l, u_h) - u_l\right) - \Psi_l(\tilde{u}_l)\left(S_l(\tilde{u}_l, u_h) - \tilde{u}_l\right) \quad \text{(C.3)}$$

When facing high-valuation buyers, Theorem 4.1 indicates that,

$$\begin{aligned}
S_h(u_l, u_h) &= \theta_h q_h(u_l, u_h) - \phi\left(q_h(u_l, u_h)\right) \\
q_h(u_l, u_h) &= \begin{cases} \frac{u_h - u_l}{\Delta\theta} & \text{if } u_h - u_l > q_h^* \Delta\theta \\ q_h^* & \text{otherwise} \end{cases}
\end{aligned}$$

so high contracts are decreasing in efficiency with respect to the difference in utility that a menu offers to high vs low valuation buyers. But the weak-orderedness violation implies $\tilde{u}_h - u_l < \tilde{u}_h - \tilde{u}_l$ and $u_h - u_l < u_h - \tilde{u}_l$, so that cost convexity also implies,

$$S_h(u_l, \tilde{u}_h) - S_h(\tilde{u}_l, \tilde{u}_h) \geq S_h(u_l, u_h) - S_h(\tilde{u}_l, u_h) \geq 0$$

which is equivalent to,

$$\Psi_h(\tilde{u}_h)\left((S_h(u_l, \tilde{u}_h) - \tilde{u}_h) - (S_h(\tilde{u}_l, \tilde{u}_h) - \tilde{u}_h)\right) \geq \Psi_h(u_h)\left((S_h(u_l, u_h) - u_h) - (S_h(\tilde{u}_l, u_h) - u_h)\right) \geq 0 \quad \text{(C.4)}$$

Jointly (C.3) and (C.4) yield,

$$\begin{aligned}
& p_l\left[\Psi_l(u_l)\left(S_l(u_l, \tilde{u}_h) - u_l\right) - \Psi_l(\tilde{u}_l)\left(S_l(\tilde{u}_l, \tilde{u}_h) - \tilde{u}_l\right)\right] \\
&+ p_h\left[\Psi_h(\tilde{u}_h)\left(S_h(u_l, \tilde{u}_h) - \tilde{u}_h\right) - \Psi_h(\tilde{u}_h)\left(S_h(\tilde{u}_l, \tilde{u}_h) - \tilde{u}_h\right)\right] \\
&> p_l\left[\Psi_l(u_l)\left(S_l(u_l, u_h) - u_l\right) - \Psi_l(\tilde{u}_l)\left(S_l(\tilde{u}_l, u_h) - \tilde{u}_l\right)\right] \\
&+ p_h\left[\Psi_h(u_h)\left(S_h(u_l, u_h) - u_h\right) - \Psi_h(u_h)\left(S_h(\tilde{u}_l, u_h) - u_h\right)\right] \\
&\geq 0
\end{aligned}$$

where the last inequality comes from the optimality of $(u_l, u_h)$ for a $p_l$ type seller. Necessarily then, at least one of the terms in brackets at the topmost expression must be $> 0$ and we know, by (C.4), that the second term is $\geq 0$. If the first was $> 0$ or if it were $= 0$ (and so the second term in brackets was $> 0$), then any seller - including a $\tilde{p}_l$ type - would strictly prefer $(u_l, \tilde{u}_h)$ over $(\tilde{u}_l, \tilde{u}_h)$. The only alternative then is that this first bracket term is $< 0$ and so that the second bracket term is $> 0$. Given this fact, we can take advantage of (C.3), to note that the first term in brackets of the bottom expression is also $< 0$ and the second $> 0$. The latter strict inequality can only hold when $IC_l$ binds at both $(\tilde{u}_l, u_h)$ and, since $\tilde{u}_h > u_h$, also at $(\tilde{u}_l, \tilde{u}_h)$.

However, for $(u_l, u_h)$ to be preferred by a type $p_l$ seller over $(u_l, \tilde{u}_h)$, which is strictly more profitable in low matches (since $(u_l, u_h)$ is $IC_h$ binding), the chosen menu must be strictly more profitable in the high. But then,

$$\begin{aligned} &\Psi_h(u_h)\left(S_h(\tilde{u}_l, u_h) - u_h\right) - \Psi_h(\tilde{u}_h)\left(S_h(\tilde{u}_l, \tilde{u}_h) - \tilde{u}_h\right) \\ &\geq \Psi_h(u_h)\left(S_h(u_l, u_h) - u_h\right) - \Psi_h(\tilde{u}_h)\left(S_h(u_l, \tilde{u}_h) - \tilde{u}_h\right) \\ &> 0 \end{aligned} \tag{C.5}$$

and $(\tilde{u}_l, u_h)$ is strictly better than $(\tilde{u}_l, \tilde{u}_h)$ in high matches and identical in the low (same sales, same profits per-sale) - making it strictly preferable for a $\tilde{p}_l$ type.

These arguments are sufficient to establish that[25] for a given optimal menu $(u_l, u_h)$ at which $IC_h$ binds, then any other equilibrium menu $(\tilde{u}_l, \tilde{u}_h)$ with $\tilde{u}_i > u_i$ - offered by any type of seller - must have $\tilde{u}_{\neg i} \geq u_{\neg i}$.

The only comparisons left to consider are those between two $IC_l$ binding menus and those between one at which $IC_l$ binds one with another that is dually efficient; any comparison where $IC_h$ binds in a menu is covered by the previous reasoning. But, if $IC_l$ is to bind in either of the potentially non-weakly-ordered menus, giving rise to $\tilde{u}_h > u_h$ and $u_l > \tilde{u}_l$, then it must bind at the menu with the largest utility difference, $(\tilde{u}_l, \tilde{u}_h)$. Suppose that this is so and consider the alternative bid $(\tilde{u}_l, u_h)$.

By the fact that $(u_l, u_h)$ is either dually efficient or $IC_l$ binding, the same must be true for $(\tilde{u}_l, u_h)$ and $(u_l, \tilde{u}_h)$, which have strictly larger differences in offered utilities, so $S_l^* = S_l(u_l, u_h) = S_l(\tilde{u}_l, u_h) = S_l(\tilde{u}_l, \tilde{u}_h) = S_l(u_l, \tilde{u}_h)$ and,

$$\begin{aligned} &\Psi_l(\tilde{u}_l)\left(S_l(\tilde{u}_l, u_h) - \tilde{u}_l\right) - \Psi_l(\tilde{u}_l)\left(S_l(\tilde{u}_l, \tilde{u}_h) - \tilde{u}_l\right) \\ &= \Psi_l(u_l)\left(S_l(u_l, u_h) - u_l\right) - \Psi_l(u_l)\left(S_l(u_l, \tilde{u}_h) - u_l\right) \\ &= 0 \end{aligned} \tag{C.6}$$

Given that $IC_l$ binds at $(\tilde{u}_l, \tilde{u}_h)$ however, $S_h(u_l, \tilde{u}_h) - S_h(\tilde{u}_l, \tilde{u}_h) > S_h(u_l, u_h) - S_h(\tilde{u}_l, u_h)$ and,

$$\begin{aligned} &\Psi_h(u_h)\left(S_h(\tilde{u}_l, u_h) - u_h\right) - \Psi_h(\tilde{u}_h)\left(S_h(\tilde{u}_l, \tilde{u}_h) - \tilde{u}_h\right) \\ &> \Psi_h(u_h)\left(S_h(u_l, u_h) - u_h\right) - \Psi_h(\tilde{u}_h)\left(S_h(u_l, \tilde{u}_h) - \tilde{u}_h\right) \\ &\geq 0 \end{aligned} \tag{C.7}$$

where the weak inequality is due to the choice of $(u_l, u_h)$ over $(u_l, \tilde{u}_h)$ by the $p_l$ type seller, while the strict inequality follows from the $IC_l$ constraint binding in all four of menus under consideration. Jointly (C.6) and (C.7) then imply that $(\tilde{u}_l, \tilde{u}_h)$ is strictly dominated by $(\tilde{u}_l, u_h)$ for any seller type however, since the latter is equally profitable in low matches and strictly better in the high. $\square$

*Ordered, Support Convexity, and Profit Ranking.* We will now strengthen the claim that equilibrium

[25] Note that the case of $IC_h$ binding at $(u_l, u_h)$ and $u_h > \tilde{u}_h$ but $\tilde{u}_l > u_l$ is subsumed in one we've established, because $IC_h$ must also bind at $(\tilde{u}_l, \tilde{u}_h)$ in these other inequalities.

menus are weakly ordered to one of (strict) orderedness. The proof is broken up into 8 steps which will show the monotonicity of profits per low and high match with respect to generosity as well as convex supports for the probability of winning distributions $\Psi_j$.

**Step 1 (Conditional Expected Sales):** *Every type of seller expects the same sales, $\Psi_i(u_i)$ from an indirect utility offer of $u_i$.*

The buyer's type determines the distribution of signals observed by sellers with each precision, $\alpha_e$, and so the distribution of seller types,

$$P(p_l = p|\theta_i) = \sum_{e,\,j} \mathbb{1}\left(p = P^e(\theta_l|j)\right) P^e(j|\theta_i)\mu(e)$$

A seller's mixing distribution, $P((u_l, u_h)|p_l)$, is determined by her type. As such, every seller's expected distribution of competitor bids,

$$F(\tilde{u}_i \leq u_i|\theta_i) = \sum_{p_l} P(\tilde{u}_i \leq u_i|p)P(p_l|\theta_i)$$

is identical when the buyer's type is $\theta_i$.$\square$

**Step 2 (No $IC_l$):** *Under Assumption 4.2, $IC_l$ binding menus are not offered in equilibrium.*

The profits per sale high sale of any menu $(u_l, u_h)$ are given by,

$$S_h(u_l, u_h) - u_h \leq S_h^* - u_h$$

so $u_h \leq S_h^*$, as losses are strictly increasing in $u_h$ for larger values. The gap between utilities of any menu is therefore bounded by,

$$\begin{aligned} u_h - u_l \leq u_h \leq S_h^* &= (\theta_h - \kappa_m)(q_h^* - q_l^*) + (\theta_h - \kappa_l)q_l^* \\ &\leq (\theta_h - \theta_l)(q_h^* - q_l^*) + (\theta_h - \theta_l)q_l^* \\ &= \Delta\theta q_h^* \end{aligned}$$

**Step 3 (Dually Efficient Characterization):** *If $\Delta\theta q_l^* < \Delta u < \Delta\theta q_h^*$, then sufficiently small changes $u_h$ or $u_l$ leave expected profits per low and high match unchanged. Further, the existence of a menu $(\overline{u}_l^{de}, \overline{u}_h^{de})$ with $\Delta\theta q_l^* < \overline{u}_h^{de} - \overline{u}_l^{de} \leq \Delta\theta q_h^*$ implies the existence of another menu $(\underline{u}_l^{de}, \underline{u}_h^{de})$ with $\underline{u}_{h,}^{de} - \underline{u}_l^{de} = \Delta\theta q_l^*$ and convex regions $[\underline{u}_i^{de}, \overline{u}_i^{de}]$ for $i \in \{l, h\}$ made up of offerings from dually efficient menus that satisfy the strict inequalities. Lastly, if $\Delta\overline{u}^{de} = \Delta\theta q_h^*$, then there exists a $\delta > 0$, such that a bid of $(u_l - \delta, u_h - \delta)$ has identical profits in low and high matches.*

Since lowering either offering does not affect the efficiency of bids when the inequalities are strict, a gap below either $u_i$ would allow a strict increase in profitability from undercutting the original utility by some $\epsilon > 0$. Given that there aren't gaps under either coordinate then, there can't be $IC_h$ binding menus immediately below - by weak-orderedness and these menus' strictly smaller $\Delta u$ respectively - so, all coordinates $u_i' \in [u_i - \delta, u_i]$ for $\delta > 0$ small and $j \in \{l, h\}$ belong to dually efficient menus, and consequently, $\Psi_i(u_i')(S_i^* - u_i') = \Psi_i(u_i)(S_i^* - u_i)$ (otherwise, some seller would be able to deviate to a bid with higher expected profits per-match without affecting the efficiency of the paired contract). This establishes the first sentence's claim.

The preceding logic implies that dually efficient menus satisfying the pair of strict inequalities have other dually efficient menus immediately below them in generosity and that the profitability of offering these menus, conditionally on matching with either type of buyer, is identical. Consider the infimum $u_i$ coordinates of the contiguous intervals made up by these dually efficient menus and let us refer to it as $\underline{u}_i^{de}$ for $i \in \{l, h\}$. Note that $\Delta \underline{u}^{de} = \Delta\theta q_l^*$, since otherwise necessarily there'd be a gap below one of the $\underline{u}_i^{de}$ coordinates (no sufficiently close dually efficient menus with coordinates below, whereas coordinates from $IC_h$ binding menus would create gaps), which would allow an analogous deviation[26] as in the previous paragraph for those seller types offering dually-efficient menus with $u_i$'s close to $\underline{u}_i^{de}$. This establishes the second sentence.

For the case that $\Delta\overline{u}^{de} = \Delta\theta q_h^*$, it is sufficient to observe that all entries immediately below the menu $(\overline{u}_l^{de}, \overline{u}_h^{de})$ and sufficiently close must also be dually efficient to avoid creating a gap. By continuing down distribution in either coordinate one must eventually reach a dually efficient menu with $\Delta u < \Delta\theta q_h^*$; else, there'd either be a gap (allowing the familiar deviation) if one encountered an $IC_h$ menu before reaching a dually efficient one that satisfied the pair of strict inequalities, or it'd hold that $\underline{u}_l^{de} = 0$ with a paired high utility of $\tilde{u}_h = \underline{u}_l^{de} + \Delta\theta q_h^*$. In this last case, the lack of atoms, weak-orderedness (ruling out a non-zero mass of $IC_h$ binding ones having entries below $\tilde{u}_h$), and lack of dually efficient bids with $\Delta u < \Delta\theta q_h^*$ would imply that there is a gap below $\tilde{u}_h$ and so that this menu's profitability could be strictly improved by lowering its high offering (more profits per high sale without decreasing the efficiency of low sales).

To prove the statement in the last sentence, consider the supremum over dually efficient bids $u_i$ with $u_i < \overline{u}_j^{de}$ that belong to a menu with $\Delta u < \Delta\theta q_h^*$, and refer to it as $u_j'$. If $u_j' = \overline{u}_j^{de}$, we are done by the previous claims. If $u_j' < \overline{u}_j^{de}$, then all menus with bids in $[u_j', \overline{u}_j^{de}]$ for either $j \in \{l, h\}$ must have $\Delta u = \Delta\theta q_h^*$ and it is sufficient to establish the invariance of profits in this region, as the previous arguments take over for dually efficient menus with bids below. Note that the existence of a high offering $u_h$ that allows greater expected profits per high match than neighboring ones above would allow these sellers above to strictly increase their expected profits per high match (while leaving low match ones unperturbed), by instead choosing $u_h$ as the high pairing; by extension of this local argument, expected high match profits are maintained by menus satisfying $\Delta\theta q_h^*$ in the interval $[u_h', \overline{u}_h^{de}]$. The invariance of expected low match profits for menus with bids in $[u_l', \overline{u}_l^{de}]$ follows by a similar logic: the existence of a point $u_l$ in this interval allowing larger expected low match profits would allow sellers above to shift their high and low offering by the same amount so as to obtain the superior expected profits per low match while preserving the same expected profits per high match.

**Step 4 (Convex $\Psi_l$ Support):** *There are no gaps in* $\mathrm{supp}(\Psi_l)$.

Since $IC_l$ never binds in an equilibrium bid by Step 2, gaps in $\mathrm{supp}\,\Psi_l$ would imply the existence of a deviation for sellers bidding menus featuring a $u_l$ offer close to the top of said gap. Such a seller could decrease its low offer to some value in the gap, so as to obtain

[26] Profits conditionally on buyer type are continuous with respect to generosity due to the lack of atoms in the bid distribution, while the continuity of profits per sale follows from the efficiency formulation $S_i(u_l, u_h) - u_i$.

a discrete increase in profits per low sale, while sacrificing an arbitrarily small number of low sales (sales continuous in generosity) and maintaining expected profits per high match, thereby strictly increasing expected profits. The only non-obvious case is if $\Delta u = \Delta\theta q_h^*$, but then we know from Step 3 that all the $u_h'$ immediately below $u_h$ preserve expected profits per high match, so a revision of $u_l$ and $u_h$ by the same $\delta$ downward would do what is described.

**Step 5 (Ordereness Violations):** *The only possible violation of strict-orderedness between two equilibrium menus $(u_l, u_h)$ and $(u_l', u_h')$, where $IC_h$ binds in at least one of them, is if $u_l = u_l'$, $u_h' > u_h$, and $p_l' > p_l$ for the respective seller types that offer these.*

Given two equilibrium menus $(u_l, u_h)$ and $(u_l', u_h')$, such that $IC_h$ binds in at least one, these must be weakly ordered, so the violation of strict ordering must involve either $u_l = u_l'$ and $u_h > u_h'$ or $u_h = u_h'$ and $u_l > u_l'$. However, the second case cannot be.

To see this point, note that if $u_h = u_h'$ and $u_l > u_l'$, $IC_h$ must bind at the menu with the smaller generosity difference, $(u_l, u_h)$. As such, any other bid $(\tilde{u}_l, \tilde{u}_h)$ with $\tilde{u}_l \in [u_l', u_l]$ is weakly ordered with respect to this menu, so $\tilde{u}_h \leq u_h$. And if $IC_h$ bound at $(\tilde{u}_l, \tilde{u}_h)$, then it too would be weakly ordered with respect to $(u_l', u_h')$, so that $u_h' \leq \tilde{u}_h$. Jointly, these statements imply that $u_h = \tilde{u}_h = u_h'$ for any $IC_h$ binding bid with a low offering in interval $[u_l', u_l]$ and thus that a nonzero mass of such menus gives rise to an atom at $u_h$ (contradicting the lack of atoms). We only have the mass of dually efficient menus left to fill the intervals below $u_l$. Weak ordering requires these to satisfy $\tilde{u}_h \leq u_h$ and dual efficiency $\Delta\theta q_l^* \leq \Delta\tilde{u}$, so that,

$$\tilde{u}_l \leq \tilde{u}_h - \Delta\theta q_l^* \leq u_h - \Delta\theta q_l^* < u_l$$

Implying the existence of a $\delta > 0$, for which $\Psi_l([u_l - \delta, u_l]) = 0$ (contradicting Step 4).

If $u_l' = u_l$ and $u_h' > u_h$ instead, $IC_h$ must bind at $(u_l, u_h)$; therefore, $(u_l', u_h')$ is strictly more profitable in low matches (more efficient, same low generosity, same sales). The menu $(u_l, u_h)$ is offered in equilibrium though, so it cannot be strictly dominated and must be superior to $(u_l', u_h')$ in high matches. This difference in profits conditionally on a buyer type can only be optimal for two sellers if they differ in seller type, $p_l' \neq p_l$, with the one who places more weight on low matches $p_l' > p_l$ offering the menu $(u_l', u_h')$ that is more profitable in them.

**Step 6 (Profit Ranking):** *Given two menus $(u_l, u_h)$ and $(u_l', u_h')$ with $u_l < u_l'$, it must be that the expected profits per high match satisfy $\Psi_h(u_h')(S_h^* - u_h') \leq \Psi_h(u_h)(S_h^* - u_h)$ and inversely for low match profits, $\Psi_l(u_l')(S_l(u_l', u_h') - u_l') \geq \Psi_l(u_l)(S_l(u_l, u_h) - u_l)$, with both inequalities either strict or equal. In particular, if the inequalities are strict, the type $p_l'$ of the seller who bids $(u_l', u_h')$ must be strictly greater than that of the seller who bids $(u_l, u_h)$.*

Consider menus $(u_l, u_h)$ and $(u_l', u_h')$ with $u_l < u_l'$ close. If $\Delta\theta q_l^* \leq \Delta u' \leq \Delta\theta q_h^*$, any $(u_l, u_h)$ with $u_l$ sufficiently close must be dually efficient and maintain expected profits per high and low match by Step 3. Whereas if $\Delta u' < q_l^* \Delta\theta$, for $u_l$ close: (a) $\Pi_h(u_l', u_h') = \Pi_h(u_l, u_h')$, (b) to rule out a deviation toward the more efficient bid $(u_l, u_h')$ by the type bidding $(u_l, u_h)$, necessarily $\Pi_h(u_l, u_h') < \Pi_h(u_l, u_h)$, and (c) to rule out a deviation, by the type offering $(u_l', u_h')$, towards the more high match profitable bid $(u_l, u_h)$, necessarily

$\Pi_l(u'_h, u'_l) > \Pi_l(u_h, u_l)$. The menu with the more generous low offer is therefore more profitable in low matches and must be offered by the seller of type $p_l$, which places more weight on low matches. This local reasoning around any point supp$(\Psi_l)$ yields the global claim.

□

*Continuous Differentiable Distributions.* We will show that the functions $\Psi_h(\cdot)$ and $\Psi_l(\cdot)$ are continuously differentiable. Since these functions are given by the composition of $F_h$ and $F_l$ with a continuous mononote function, this will prove the continuous differentiability of the equilibrium offer distributions.

We present the case of $\Psi_h$ ($\Psi_l$'s is analogous). Let $u_h$ be a utility level offered in the interior of the support of $F_h$ and $u_l$ be its paired low utility offering such that the menu $(u_l, u_h)$ is optimal for some seller of type $p_l$. We proceed to bound the difference $\Psi_h(u_h + \varepsilon) - \Psi_h(u_h)$ from above and below.

For any $\varepsilon \in \mathbb{R}$, the expected profit to this type of seller from the deviation menu $(u_l, u_h + \varepsilon)$ can be decomposed as:

$$\begin{aligned}
&p_l \Psi_l(u_l)(S_l(u_l, u_h + \varepsilon) - u_l) + p_h \Psi_h(u_h + \varepsilon)(S_h(u_l, u_h + \varepsilon) - u_h - \varepsilon) \\
&= p_l \Psi_l(u_l)(S_l(u_l, u_h) - u_l) + p_h \Psi_h(u_h)(S_h(u_l, u_h) - u_h) \\
&+ p_l \Psi_l(u_l)(S_l(u_l, u_h + \varepsilon) - S_l(u_l, u_h)) + p_h \Psi_h(u_h)(S_h(u_l, u_h + \varepsilon) - \varepsilon - S_h(u_l, u_h)) \\
&+ p_h(\Psi_h(u_h + \varepsilon) - \Psi_h(u_h))(S_h(u_l, u_h + \varepsilon) - u_h - \varepsilon)
\end{aligned}$$

and by optimality, we must have $\Pi(u_l, u_h; p_l) \geq \Pi(u_l, u_h + \varepsilon; p_l)$, which implies the following inequality,

$$\begin{aligned}
&p_h(\Psi_h(u_h + \varepsilon) - \Psi_h(u_h))(S_h(u_l, u_h + \varepsilon) - u_h - \varepsilon) \\
&\leq p_l \Psi_l(u_l)(S_l(u_l, u_h) - S_l(u_l, u_h + \varepsilon)) + p_h \Psi_h(u_h)(S_h(u_l, u_h) - S_h(u_l, u_h + \varepsilon) + \varepsilon)
\end{aligned} \tag{C.8}$$

Similarly, for any $\varepsilon \in \mathbb{R}$ such that $u_h + \varepsilon$ is in the interior of the support of $F_h$, there exists $u_{l,\varepsilon}$ for which $(u_{l,\varepsilon}, u_h + \varepsilon)$ is the optimal bid of some seller with type $\tilde{p}_l$. And, decomposing the profits from this bid to said seller as above:

$$\begin{aligned}
&\tilde{p}_l \Psi_l(u_{l,\varepsilon})(S_l(u_{l,\varepsilon}, u_h + \varepsilon) - u_{l,\varepsilon}) + \tilde{p}_h \Psi_h(u_h + \varepsilon)(S_h(u_{l,\varepsilon}, u_h + \varepsilon) - u_h - \varepsilon) \\
&= \tilde{p}_l \Psi_l(u_{l,\varepsilon})(S_l(u_{l,\varepsilon}, u_h) - u_{l,\varepsilon}) + \tilde{p}_h \Psi_h(u_h)(S_h(u_{l,\varepsilon}, u_h) - u_h) \\
&+ \tilde{p}_l \Psi_l(u_{l,\varepsilon})(S_l(u_{l,\varepsilon}, u_h + \varepsilon) - S_l(u_{l,\varepsilon}, u_h)) + \tilde{p}_h \Psi_h(u_h)(S_h(u_{l,\varepsilon}, u_h + \varepsilon) - \varepsilon - S_h(u_{l,\varepsilon}, u_h)) \\
&+ \tilde{p}_h(\Psi_h(u_h + \varepsilon) - \Psi_h(u_h))(S_h(u_{l,\varepsilon}, u_h + \varepsilon) - u_h - \varepsilon)
\end{aligned}$$

Again, since here $(u_{l,\varepsilon}, u_h + \varepsilon)$ is optimal for a seller of type $\tilde{p}_l$, we must have $\Pi(u_{l,\varepsilon}, u_h + \varepsilon; \tilde{p}_l) \geq \Pi(u_{l,\varepsilon}, u_h; \tilde{p}_l)$, which implies the following inequality,

$$\begin{aligned}
&\tilde{p}_h(\Psi_h(u_h + \varepsilon) - \Psi_h(u_h))(S_h(u_{l,\varepsilon}, u_h + \varepsilon) - u_h - \varepsilon) \\
&\geq \tilde{p}_l \Psi_l(u_{l,\varepsilon})(S_l(u_{l,\varepsilon}, u_h) - S_l(u_{l,\varepsilon}, u_h + \varepsilon)) + \tilde{p}_h \Psi_h(u_h)(S_h(u_{l,\varepsilon}, u_h) - S_h(u_{l,\varepsilon}, u_h + \varepsilon) + \varepsilon)
\end{aligned} \tag{C.9}$$

So, by (C.8) and (C.9), we can form a squeezed inequality for the derivative of $\Psi_h(\cdot)$,

$$\frac{\frac{\tilde{p}_l}{\tilde{p}_h}\Psi_l(u_{l,\varepsilon})(S_l(u_{l,\varepsilon},u_h)-S_l(u_{l,\varepsilon},u_h+\varepsilon))+\Psi_h(u_h)(S_h(u_{l,\varepsilon},u_h)-S_h(u_{l,\varepsilon},u_h+\varepsilon)+\varepsilon)}{\varepsilon(S_h(u_{l,\varepsilon},u_h+\varepsilon)-u_h-\varepsilon))}$$
$$\leq \frac{\Psi_h(u_h+\varepsilon)-\Psi_h(u_h)}{\varepsilon} \leq$$
$$\frac{\frac{p_l}{p_h}\Psi_l(u_l)(S_l(u_l,u_h)-S_l(u_l,u_h+\varepsilon))+\Psi_h(u_h)(S_h(u_l,u_h)-S_h(u_l,u_h+\varepsilon)+\varepsilon)}{\varepsilon(S_h(u_l,u_h+\varepsilon)-u_h-\varepsilon)}$$

Considering the right-hand derivative, for $\varepsilon \searrow 0$ small $S_h(u_l,u_h+\varepsilon)-u_h-\varepsilon > 0$, since $(u_l,u_h)$ is an equilibrium menu and (as we've argued in Section C in the proof ruling out atoms) $S_h(u_l,u_h)-u_h > 0$ for all such menus. Once the nonzero denominator is established, we can recall that $q_i(u_l,u_h)$ (specified in Theorem 4.1) is everywhere left as well as right differentiable in each variable and that $S_i(u_l,u_h) = \theta_i q_i(u_l,u_h) - \phi(q_i(u_l,u_h))$, so taking the limit of the right-hand expression, we obtain:

$$\frac{-\frac{p_l}{p_h}\Psi_l(u_l)\frac{\partial S_l}{\partial_+ u_h}(u_l,u_h)+\Psi_h(u_h)\left(1-\frac{\partial S_h}{\partial_+ u_h}(u_l,u_h)\right)}{S_h(u_l,u_h)-u_h}$$

The left-hand side expression of the inequality is similar to the right, with the exception that $u_{l,\varepsilon}$ is substituted in for $u_l$ and the weighting probabilities are $(\tilde{p}_l,\tilde{p}_h)$ instead of $(p_l,p_h)$, thus requiring a different argument. If $u_h - u_l > q_l^*\Delta\theta$, we establish in the orderedness proof that offerings locally around both $u_l$ and $u_h$ are those of dually efficient bids and that they preserve expected profits from both low and high matches - $\Pi_i(u_{l,\varepsilon},u_h+\varepsilon) = \Pi_i(u_{l,\varepsilon},u_h)$ for $i \in \{l,h\}$. So, if a seller of type $\tilde{p}_l$ finds $(u_{l,\varepsilon},u_h+\varepsilon)$ optimal, then so would a seller of type $p_l$, yielding a limit of the left-hand side expression as above (replacing $\tilde{p}_l$ with $p_l$ for $\varepsilon$ small). On the other hand, if $u_h - u_l < q_l^*\Delta\theta$, $IC_h$ binds locally for all menus with $\varepsilon > 0$ small, so the strict ordering of profits by type requires $\tilde{p}_l \searrow p_l$. Lastly, if $u_h - u_l = q_l^*\Delta\theta$, then we must be able to select a subsequence with $u_h - u_{l,\varepsilon_{\tilde{n}}} > u_h - u_l$ or $< u_h - u_l$. In the former (latter) case, $IC_h$ is slack (binds) at the menus of the subsequence, so the argument from the $u_h - u_l > q_l^*\Delta\theta$ ($u_h - u_l < q_l^*\Delta\theta$) case applies, and this is sufficient to obtain a convergent left-hand inequality.

Therefore, the limit as $\varepsilon \to 0$ of the left-hand and right-hand inequalities is identical, thus establishing the right-hand derivative claim:

$$\frac{\mathrm{d}\Psi_h}{\mathrm{d}_+ u_h}(u_h) = \frac{-\frac{p_l}{p_h}\Psi_l(u_l)\frac{\partial S_l(u_l,u_h)}{\partial_+ u_h}+\Psi_h(u_h)(1-\frac{\partial S_h(u_l,u_h)}{\partial_+ u_h})}{S_h(u_l,u_h)-u_h}$$

The left-hand derivative argument is analogous except that we consider the menu $(u_l, u_h - \varepsilon)$.

As stated at the start, the case of $\Psi_l(\cdot)$ is analogous, ultimately yielding,

$$\frac{\mathrm{d}\Psi_l}{\mathrm{d}u_l}(u_l) = \frac{-\frac{p_h}{p_l}\Psi_h(u_h)\frac{\partial S_h(u_l,u_h)}{\partial u_l}+\Psi_l(u_l)(1-\frac{\partial S_l(u_l,u_h)}{\partial u_l})}{S_l(u_l,u_h)-u_l}$$

Under strictly convex costs, at the sole[27] points of possible non-differentiability where $u_h - u_l = \Delta\theta q_i^*$,

$$\frac{\partial S_i(u_l,u_h)}{\partial_+ u_i}(u_l,u_h) = \frac{\partial S_i(u_l,u_h)}{\partial_- u_i}(u_l,u_h) = \frac{\theta_i - \phi'(q_i^*)}{\Delta\theta} = 0$$

hence the stronger differentiability claim for $\Psi_i$. If costs instead take a piecewise form, unless $u_h - u_l = \Delta\theta q_i^*$, then $S_i(u_l,u_h)$ is locally linear (or constant) in each variable, so that the left and right

[27] For a given equilibrium distribution.

derivatives are also equal. Since $S_i(u_l, u_h) - u_i$ are continuous in $(u_l, u_h)$ and $\Psi_i(\cdot)$ are continuous by the lack of atoms, the distributions $\Psi_i$ are also continuously differentiable on the interior of the supports with the exception of points $u_i$ corresponding to the (no more than two) bids $(u_l, u_h)$ at which $u_h - u_l = \Delta\theta q_i^*$. $\square$

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