

1 Probability Potpourri

Prove a brief justification for each part.

- (a) For two events A and B in any probability space, show that $\mathbb{P}(A \setminus B) \geq \mathbb{P}(A) - \mathbb{P}(B)$.
- (b) If $|\Omega| = n$, how many distinct events does the probability space have?
- (c) Suppose $\mathbb{P}(D \mid C) = \mathbb{P}(D \mid \overline{C})$, where \overline{C} is the complement of C . Prove that D is independent of C .

2 Aces

Consider a standard 52-card deck of cards:

- (a) Find the probability of getting an ace or a red card, when drawing a single card.
- (b) Find the probability of getting an ace or a spade, but not both, when drawing a single card.
- (c) Find the probability of getting the ace of diamonds when drawing a 5 card hand.
- (d) Find the probability of getting exactly 2 aces when drawing a 5 card hand.
- (e) Find the probability of getting at least 1 ace when drawing a 5 card hand.
- (f) Find the probability of getting at least 1 ace or at least 1 heart when drawing a 5 card hand.

3 Balls and Bins

Throw n balls into n labeled bins one at a time.

- (a) What is the probability that the first bin is empty?
- (b) What is the probability that the first k bins are empty?
- (c) Let A be the event that at least k bins are empty. Notice that there are $m = \binom{n}{k}$ sets of k bins out of the total n bins. If we assume A_i is the event that the i^{th} set of k bins is empty. Then we can write A as the union of A_i 's.

$$A = \bigcup_{i=1}^m A_i.$$

Write the union bound for the probability A .

- (d) Use the union bound to give an upper bound on the probability A from part (c).
- (e) What is the probability that the second bin is empty given that the first one is empty?
- (f) Are the events that "the first bin is empty" and "the first two bins are empty" independent?
- (g) Are the events that "the first bin is empty" and "the second bin is empty" independent?