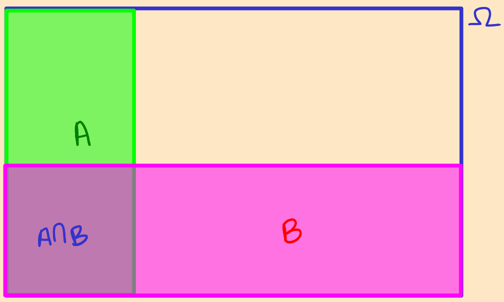
$P(A|B) = \frac{1}{2}$

A get heads from a coin toss

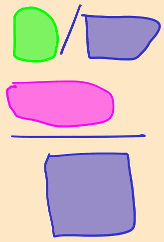
B choose a fair coin

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$



$P(A) =$

$P(B) =$



$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$$

$$P(\Omega) = 1$$

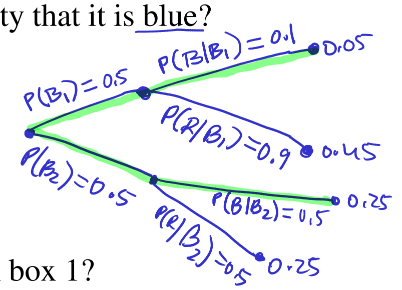
$$P(A|\Omega) = \frac{P(A \cap \Omega)}{P(\Omega)} = P(A)$$

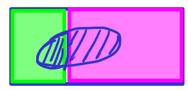
1 Box of Marbles

You are given two boxes: one of them containing 900 red marbles and 100 blue marbles, the other one contains 500 red marbles and 500 blue marbles.

(a) If we pick one of the boxes randomly, and pick a marble what is the probability that it is blue?

$$P(B) = P(B_1 \cap B) + P(B_2 \cap B) = P(B|B_1)P(B_1) + P(B|B_2)P(B_2)$$
$$= 0.05 + 0.25 = 0.3$$





B_1, B_2 mut excl., coll. exh.

(b) If we see that the marble is blue, what is the probability that it is chosen from box 1?

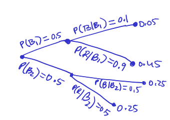
$P(B) = 0.3$

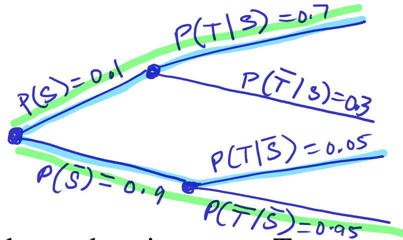
$$P(B_1|B) = \frac{P(B|B_1)P(B_1)}{P(B)} = \frac{0.1 \times 0.5}{0.3} = \frac{1}{6}$$

(c) Suppose we pick one marble from box 1 and without looking at its color we put it aside. Then we pick another marble from box 1. What is the probability that the second marble is blue?

B_1 B_2 R_1 R_2

$$P(B_2) = P(B_2 \cap R_1) + P(B_2 \cap B_1)$$
$$= P(B_2|R_1)P(R_1) + P(B_2|B_1)P(B_1)$$
$$= \frac{100}{999} \cdot \frac{9}{10} + \frac{99}{999} \cdot \frac{1}{10}$$
$$= \frac{1}{10}$$





2 Duelling Meteorologists

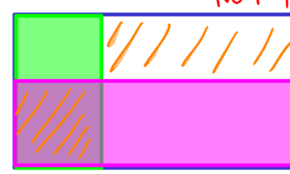
Tom is a meteorologist in New York. On days when it snows, Tom correctly predicts the snow 70% of the time. When it doesn't snow, he correctly predicts no snow 95% of the time. In New York, it snows on 10% of all days.

(a) If Tom says that it is going to snow, what is the probability it will actually snow?

$$P(S|T) = \frac{P(S\cap T)}{P(T)} = \frac{0.07}{0.07+0.045} = \frac{14}{23}$$

(b) Let A be the event that, on a given day, Tom predicts the weather correctly. What is $\mathbb{P}(A)$?

NOT TO SCALE



$$P(A) = P(T\cap S) + P(\bar{T}\cap\bar{S})$$

$$= P(S|T)P(T) + P(\bar{S}|\bar{T})P(\bar{T}) = \frac{37}{40}$$

(c) Tom's friend Jerry is a meteorologist in Alaska. Jerry claims that she is a better meteorologist than Tom even though her overall accuracy is lower. After looking at their records, you determine that Jerry is indeed better than Tom at predicting snow on snowy days and sun on sunny day. Give an instance of the situation described above. *Hint: what is the weather like in Alaska?*

$$P(T|S) < P(J|S)$$

$$P(\bar{T}|\bar{S}) < P(\bar{J}|\bar{S})$$

$$P(T\text{ correct}) = P(T\cap S) + P(\bar{T}\cap\bar{S}) \boxed{>} P(J\cap S) + P(\bar{J}\cap\bar{S}) = P(J\text{ correct})$$

$$\| $$

$$\frac{37}{40} = 0.925$$

Simpson's Paradox

$P(S)$

$P(T|S)=0.7$

$P(\bar{T}|\bar{S})=0.95$

$P(S_{NY})=0.1$

$P(J|S_A)=0.8$

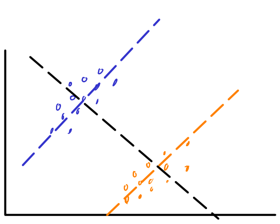
$P(\bar{J}|\bar{S}_A)=1.0$

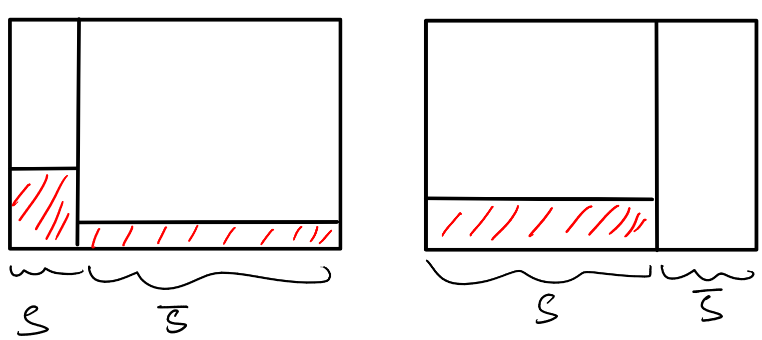
$P(S_A)=0.8$

$0.8\times0.8 + 1.0\times0.2$

$= 0.64+0.2$

$= 0.84$





3 Binary Conditional Probabilities

Let us consider a sample space $\Omega = \{\omega_1, \ldots, \omega_N\}$ of size $N > 2$, and two probability functions $\mathbb{P}_1$ and $\mathbb{P}_2$ on it. That is, we have two probability spaces: $(\Omega, \mathbb{P}_1)$ and $(\Omega, \mathbb{P}_2)$.

If for every subset $A \subset \Omega$ of size $|A| = 2$ and every outcome $\omega \in \Omega$ it is true that $\mathbb{P}_1(\omega \mid A) = \mathbb{P}_2(\omega \mid A)$, then is it necessarily true that $\mathbb{P}_1(\omega) = \mathbb{P}_2(\omega)$ for all $\omega \in \Omega$? That is, if $\mathbb{P}_1$ and $\mathbb{P}_2$ are equal conditional on events of size 2, are they equal unconditionally? (*Hint*: Remember that probabilities must add up to 1.)