

Final Sp15 Q2

(d) X and Y are independent random variables modulo n . You don't know the distribution of X , but you know that Y is uniformly distributed. What can you say about the distribution of $Z = X + Y \bmod n$? Justify your answer.

(e) X and Y are independent random variables with normal distribution with mean m_1 and m_2 respectively, and variance σ_1^2 and σ_2^2 respectively. Describe the distribution of $Z = X + Y$ (including mean and variance).

What if we are not told that X and Y are independent?

Final Fa18 Q1h

True False

- ☐ ☐ For dependent random variables X, Y and constants a, b , it is possible that $\mathbb{E}[aX + bY] \neq a\mathbb{E}[X] + b\mathbb{E}[Y]$.
- ☐ ☐ $\mathbb{E}[XY] = \mathbb{E}[X]\mathbb{E}[Y]$ if and only if X and Y are independent.
- ☐ ☐ Consider two random variables X and Y with ranges \mathcal{A}_X and \mathcal{A}_Y , respectively. If there exist $a \in \mathcal{A}_X$ and $b \in \mathcal{A}_Y$ such that $\mathbb{P}[X = a, Y = b] = \mathbb{P}[X = a]\mathbb{P}[Y = b]$, then X and Y are independent.

Q2

- (m) Let A and B denote two events such that $A \subset B$. Suppose $\mathbb{P}[A] = a$ and $\mathbb{P}[B] = b$, and let I_A and I_B denote the indicator random variables for A and B , respectively. Find $\text{Cov}(I_A, I_B)$.

