1, 2, 3, 4, 5, 6

Expected val = average

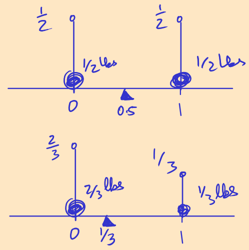
H T

\$0 , \$1

Fair coin

reward / toss = 0.5

$\frac{1}{2}\cdot 1 + \frac{1}{2}\cdot 0 = \frac{1}{2}$



p- coin

$p\cdot 0 + (1-p)\cdot 1 = 1-p$

1 Pullout Balls

Suppose you have a bag containing six balls numbered $1,2,3,4,5,6$.

(a) You perform the following experiment: pull out a single ball and record its number. What is the expected value of the number that you record?

X	Pr[X]
1	1/6
2	"
3	"
4	"
5	"
6	"

$$E[X] = \sum_{i=1}^{6} i \cdot Pr[i] = \frac{1}{6}\cdot(1+\cdots+6) = 3.5$$

(b) You repeat the experiment from part (a), except this time you pull out two balls together and record the product of their numbers. What is the expected value of the total that you record?

$$E[X] = \frac{\sum_{i=1}^{5}\left(i \cdot \sum_{j=i+1}^{6} j\right)}{\binom{6}{2}} = \frac{35}{3}$$

$\binom{6}{2} = 15$

12 13 14 15 16
23 24 25 26
34 35 36
45 46
56

X: 2, 3, 4, 5, 6, 6, 8

2 How Many Queens?

You shuffle a standard 52-card deck, before drawing the first three cards from the top of the pile. Let X denote the number of queens you draw.

(a) What is $\mathbb{P}(X=0)$, $\mathbb{P}(X=1)$, $\mathbb{P}(X=2)$ and $\mathbb{P}(X=3)$?

$X = $ # of queens

$$Pr[X=0] = \frac{48}{52}\cdot\frac{47}{51}\cdot\frac{46}{50} = \binom{4}{0}\binom{48}{3}\Big/\binom{52}{3} = \frac{4324}{5525}$$

$$Pr[X=1] = \frac{4}{52}\cdot\frac{48}{51}\cdot\frac{47}{50} + \frac{48}{52}\cdot\frac{4}{51}\cdot\frac{47}{50} + \frac{48}{52}\cdot\frac{47}{51}\cdot\frac{4}{50} = \underbrace{\binom{4}{1}\binom{48}{2}}_{\text{\# fav outcomes}}\Big/\underbrace{\binom{52}{3}}_{\text{\# total outcomes}} = \frac{1128}{5525}$$

$$Pr[X=2] = \binom{4}{2}\binom{48}{1}\Big/\binom{52}{3} = \frac{72}{5525}$$

$$Pr[X=3] = \binom{4}{3}\binom{48}{0}\Big/\binom{52}{3} = \frac{1}{5525}$$

(b) What do your answers you computed in part a add up to?

1 All possibilities!

(c) Compute $\mathbb{E}(X)$ from the definition of expectation.

$$E[X] = 0\cdot\frac{4324}{5525} + 1\cdot\frac{1128}{5525} + 2\cdot\frac{72}{5525} + 3\cdot\frac{1}{5525} = \frac{3}{13}$$

$X_1 = 1,\ X_2 = 0,\ X_3 = 1$

o o o o o o o o o o o o

Q A Q

(d) ~~Are the X_i indicators independent~~? Are the events "Queen on 1st card" and "Queen on 2nd card" indep? "Queen on 3rd card"

$$Pr[\text{Queen on } 1^{st} \text{ card}] = Pr[\text{Queen on } 2^{nd} \text{ card}] = \frac{1}{13}$$

$$Pr[\text{Queen on } 1^{st} \text{ card} \cap \text{Queen on } 2^{nd} \text{ card}] = \frac{4}{52}\cdot\frac{3}{51} = \frac{1}{221} \neq \frac{1}{13}\cdot\frac{1}{13} = \frac{1}{169}$$

$$E[X] = E[X_1+X_2+X_3] = E[X_1] + E[X_2] + E[X_3] = \frac{1}{13}+\frac{1}{13}+\frac{1}{13} = \frac{3}{13}$$

$$X_i = \begin{cases} 1 & \text{if } i^{th} \text{ card is Q} \\ 0 & \text{o/w} \end{cases}, \quad \therefore X = X_1 + X_2 + X_3$$

3 Head Count

Consider a coin with $\mathbb{P}(\text{Heads}) = 2/5$. Suppose you flip the coin 20 times, and define X to be the number of heads.

n coin flips, p prob of H, # of H

(a) Name the distribution of X and what its parameters are.

$X \sim \underline{\text{Binomial}}\,(n = 20, p = 2/5)$ $\quad X_i \sim \text{Bernoulli}(2/5)$, $X = \sum_{i=1}^{20} X_i$

(b) What is $\mathbb{P}(X = 7)$?

$Pr[X = 7] = \binom{20}{7}\left(\frac{2}{5}\right)^7\left(\frac{3}{5}\right)^{13}$ $\quad Pr[X = x] = \binom{n}{x} p^x (1-p)^{n-x}$

(c) What is $\mathbb{P}(X \geq 1)$? Hint: You should be able to do this without a summation.

$Pr[X \geq 1] = 1 - Pr[X = 0] = 1 - \left(1 - \frac{2}{5}\right)^{20}$

(d) What is $\underline{\mathbb{P}(12 \leq X \leq 14)}$?

$Pr[12 \leq X \leq 14] = \sum_{x=12}^{14} \binom{20}{x}\left(\frac{2}{5}\right)^x\left(1 - \frac{2}{5}\right)^{20-x}$