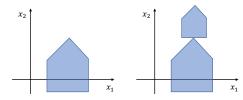
## 1 Decision Space

Let's further the intuition about how we can compose arbitrarily complex decision boundaries with a neural network. Consider the images below. For each one, build a network of units with a single output that fires if the input is in the shaded area.



**Take-away:** MLPs can capture any classification boundary. MLPs are universal classifiers. Note that we haven't said anything yet about their ability to generalize.

## 2 Backrop in Practice: Staged Computation

For the function f(x, y, z) = (x + y)z:

- (a) Decompose f into two simpler functions.
- (b) Draw the network that represents the computation of f.
- (c) Write the forward pass and backward pass (backpropagation) in the network.
- (d) Update your network drawing with the intermediate values in the forward and backward pass. Use the inputs x = -2, y = 5, and z = -4.

## 3 Backpropagation Practice

- (a) Chain rule of multiple variables: Assume that you have a function given by  $f(x_1, x_2, ..., x_n)$ , and that  $g_i(w) = x_i$  for a scalar variable w. How would you compute  $\frac{d}{dw} f(g_1(w), g_2(w), ..., g_n(w))$ ? What is its computation graph?
- (b) Let  $Z = XW + \mathbf{1}b$ , where  $Z \in \mathbb{R}^{d_n \times d_{out}}$ ,  $X \in \mathbb{R}^{d_n \times d_{in}}$ ,  $W \in \mathbb{R}^{d_{in} \times d_{out}}$ , b is a row vector in  $\mathbb{R}^{d_{out}}$ , and  $\mathbf{1}$  is a column vector in  $\mathbb{R}^{d_{in}}$ . Given  $\frac{\partial L}{\partial Z} \in \mathbb{R}^{d_n \times d_{out}}$ , where l is a scalar loss, calculate  $\frac{\partial L}{\partial W}$  and  $\frac{\partial L}{\partial b}$ .

## 4 Model Intuition

- (a) What can go wrong if you just initialize all the weights in a neural network to exactly zero? What about to the same nonzero value?
- (b) Adding nodes in the hidden layer gives the neural network more approximation ability, because you are adding more parameters. How many weight parameters are there in a neural network with architecture specified by  $d = \left[d^{(0)}, d^{(1)}, ..., d^{(N)}\right]$ , a vector giving the number of nodes in each of the N layers? Evaluate your formula for a 2 hidden layer network with 10 nodes in each hidden layer, an input of size 8, and an output of size 3.
- (c) Consider the two networks in the image below, where the added layer in going from Network A to Network B has 10 units with linear activation. Give one advantage of Network A over Network B, and one advantage of Network B over Network A.

