1 Zerg Player

A Zerg player wants to produce an army to fight against Protoss in early game, and he wants to have a small army which consumes exactly 10 supply. And he has the following choices:

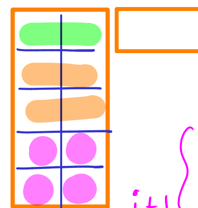
$\binom{\overset{n}{5}+\overset{k}{3}-1}{}$ $\cancel{\binom{7}{3}}$ $\binom{6}{2}$

Z
- Zerglings: consumes 1 supply

H + R
- Hydralisk: consumes 2 supply
- Roach: consumes 2 supply

$i = H+R$
$10-2i = Z$
$H, R \geq 0$

$i=3$
$i+1$



H	R
0	i
1	$i-1$
2	$i-2$
⋮	⋮
i	0

$i+1$

$\sum_{i=0}^{5}(i+1) = 15 = \binom{6}{2}$

How many different compositions can the player's army have? Note that Zerglings are indistinguishable, as are Hydralisks and Roachs.

2 Strings

What is the number of strings you can construct given:

$= \binom{n_1+n_2+n_3}{n_1, n_2, n_3}$

(a) n ones, and m zeroes? $\frac{(n+m)!}{n!\,m!} = \binom{n+m}{n} = \binom{n+m}{m}$

$m=n=2$
1100
1010
1001
0110
0101
0011

$n+m$ symbols
$n+m=4$

AABBB
AAB BB
BAB A B

$1_2\, 1_1\, 0_1\, 0_2$
$4\times3\times2\times1 = 4!$
$\frac{(m+n)!}{n!\,m!}$

(b) n_1 A's, n_2 B's and n_3 C's? $\frac{(n_1+n_2+n_3)!}{n_1!\,n_2!\,n_3!}$

$A_1B_1A_2B_2C_1\, 1_2\, 1_1\, 0_1\, 0_2$
$A_2B_1A_1B_2C_1\, 1_1\, 1_2\, 0_1\, 0_2$
$2!$

(c) $n_1, n_2, \ldots, n_k$ respectively of k different letters? $\frac{(n_1+n_2+\cdots+n_k)!}{n_1!\,n_2!\cdots n_k!}$

3 Counting Game

RPG games are all about explore different mazes. Here is a weird maze: there are 2^n rooms, where each room is the vertex on a the n-dimensional hypercube, labeled by a n bit binary string.

For each room, there are n different doors, each door corresponding to an edge on the hypercube. If you are at room i, and choose door j, then you will go to room $i \oplus 2^j$ (flips the $j+1$-th bit in number i).

(a) How many different shortest path are there from room 0 to room $2^n - 1$?

$n!$

$\underbrace{00000}_{n}$ $\underbrace{11111}_{n}$

0001
0000

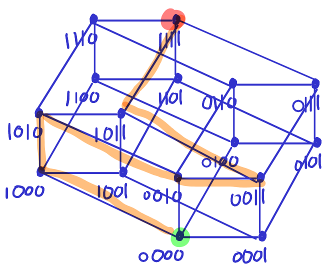
(# of bits to flip)!

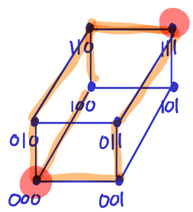
(b) How many different paths of $n+2$ steps are there to go from room 0 to room $2^n - 1$?

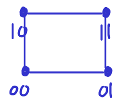
(c) If $n = 8$, how many different shortest pathes are there from room 0 to room 63 that pass through 3 and 19?

0000 At step i, $n-i$ zeros

↳ 0100 → 0110 → 0111 → 1111







0000 4−0 ✓

↳ 1000 → 1010 4−2 ✓ 1011 → 1111

4−1 ✓ 0010 → 0011

4−3

4−3+2

000

111

n choices of bits to flip

$n-1$ choices of bits to flip

$n-2$ ''

$n-3$ ''

⋮

1

(b) Let's say mistake at step i,

make mistake for $1 \le i \le n$

$\underbrace{n-i+2}$ zeros

$$\sum_{i=1}^{n} \binom{n}{i} i! \, i \, \underbrace{(n-i+1)!} \; ?$$