

1 Continuous Joint Densities

The joint probability density function of two random variables X and Y is given by $f(x,y) = Cxy$ for $0 \leq x \leq 1, 0 \leq y \leq 2$, and 0 otherwise (for a constant C).

- (a) Find the constant C that ensures that $f(x,y)$ is indeed a probability density function.
- (b) Find $f_X(x)$, the marginal distribution of X .
- (c) Find the conditional distribution of Y given $X = x$.
- (d) Are X and Y independent?

2 Uniform Distribution

You have two fidget spinners, each having a circumference of 10. You mark one point on each spinner as a needle and place each of them at the center of a circle with values in the range $[0, 10)$ marked on the circumference. If you spin both (independently) and let X be the position of the first spinner's mark and Y be the position of the second spinner's mark, what is the probability that $X \geq 5$, given that $Y \geq X$?

3 Darts with Friends

Michelle and Alex are playing darts. Being the better player, Michelle's aim follows a uniform distribution over a circle of radius r around the center. Alex's aim follows a uniform distribution over a circle of radius $2r$ around the center.

- (a) Let the distance of Michelle's throw be denoted by the random variable X and let the distance of Alex's throw be denoted by the random variable Y .
- What's the cumulative distribution function of X ?
 - What's the cumulative distribution function of Y ?
 - What's the probability density function of X ?
 - What's the probability density function of Y ?
- (b) What's the probability that Michelle's throw is closer to the center than Alex's throw? What's the probability that Alex's throw is closer to the center?
- (c) What's the cumulative distribution function of $U = \min\{X, Y\}$?
- (d) What's the cumulative distribution function of $V = \max\{X, Y\}$?
- (e) What is the expectation of the absolute difference between Michelle's and Alex's distances from the center, that is, what is $\mathbb{E}[|X - Y|]$? [Hint: Use parts (c) and (d), together with the continuous version of the tail sum formula, which states that $\mathbb{E}[Z] = \int_0^\infty P(Z \geq z)dz$.]