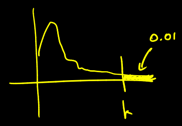
$X \sim \mathcal{D}$? $\quad E[X] \quad Pr[X > k] \le \frac{E[X]}{k}$

$X \ge 0$



$E[X] = \frac{np}{n} = p$

$X_i = \begin{cases} 1 & \text{if } i^{th} \text{ person is ANTI VAX wp } p \\ 0 & \text{o/w} \end{cases}$

$X_i \sim Bern(p)$

iid X_i

$Var\left(\frac{\sum_{i=1}^{n} X_i}{n}\right)$

$X = \frac{\sum_{i=1}^{n} X_i}{n}$

$n = 10000$

$\sum_{i=1}^{n} X_i = 20$

$Y = \sum_{i=1}^{n} X_i$ Binom

$Var(Y) = np(1-p) = \frac{1}{n^2} Var\left(\sum_{i=1}^{n} X_i\right)$ Bern

$X \ge 0$ ✓

$= \frac{1}{n^2} \sum_{i=1}^{n} Var(X_i)$

$Pr[X > k] \le 0.01$

$\Rightarrow \frac{E[X]}{k} \le 0.01$

$= \frac{1}{n^2} \cdot n Var(X_i)$

$= \frac{1}{n^2} \cdot n \cdot p(1-p)$

$\Rightarrow \frac{\frac{20}{10000}}{k} \le 0.01$

$= \frac{p(1-p)}{n}$

$\Rightarrow k \ge \frac{20}{100} = 0.2$

Find 95% CI

⇓

Find the WIDTH of CI 2ε

such that

$1 - Pr[|X - E[X]| \le \varepsilon] \le 1 - 0.95$

$Pr[|X - E[X]| \ge \varepsilon] \le 0.05$

$\le \frac{Var(X)}{\varepsilon^2} \le 0.05$

$\Rightarrow \frac{\frac{p(1-p)}{n}}{\varepsilon^2} \le 0.05$

$\Rightarrow \varepsilon \ge \sqrt{\frac{20\, p(1-p)}{n}}$

$= \sqrt{\frac{20\, p(1-p)}{10000}}$

$\le \sqrt{\frac{1}{2000}}$

$X \sim Expo(\lambda)$

$Var(X) = \frac{1}{\lambda^2}$

$\lambda \ge 10$

$\Rightarrow Var(X) \le \frac{1}{100}$

$Pr[|X - E[X]| \ge \varepsilon] \le \frac{Var(X)}{\varepsilon^2} = \frac{\sigma^2}{\varepsilon^2}$

estimate ± margin of err

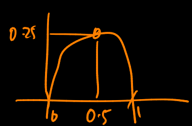
$\frac{20}{10000} \pm \sqrt{\frac{20\, p(1-p)}{10000}}$

$\left(\frac{20}{10000} - \sqrt{\frac{20\, p(1-p)}{10000}},\ \frac{20}{10000} + \sqrt{\frac{20\, p(1-p)}{10000}}\right)$

$Pr[|X - E[X]| \ge \varepsilon\sigma] \le \frac{Var(X)}{\varepsilon^2\sigma^2} = \frac{\sigma^2}{\varepsilon^2\sigma^2} = \frac{1}{\varepsilon^2}$

estimate ± margin of err

$p(1-p)$



Chebyshev: $\sum_{i=1}^{n} X_i \sim Binom(n, p)$

$X_i \sim Bern(p)$

$\frac{d}{dp}(p - p^2) = 1 - 2p = 0$

$\Rightarrow p = \frac{1}{2}$

$X_i \sim Poisson(\lambda)$

$E\left[\frac{\sum_{i=1}^{n} X_i}{n}\right] = \frac{n\lambda}{n} = \lambda, \quad Var = \frac{\lambda}{n}$

CLT: $\frac{\overbrace{\left(\sum_{i=1}^{n} X_i\right)}^{RV} - \overbrace{np}^{mean}}{\underbrace{\sqrt{Var\left(\sum_{i=1}^{n} X_i\right)}}_{Var}} \overset{approx}{\sim} \mathcal{N}(0,1)$

X_i

$\sum_{i=1}^{n} X_i$

$\frac{X - E[X]}{\sqrt{Var(X)}} \overset{approx}{\sim} \mathcal{N}(0,1)$

$X = \sum_{i=1}^{n} X_i$, iid X_i

approx is good

⇓

CLT

$\hat{\mu}$

$\frac{\left(\frac{\sum_{i=1}^{n} X_i}{n}\right) - p}{\sqrt{Var\left(\frac{\sum_{i=1}^{n} X_i}{n}\right)}}$

$\|$

$\sqrt{\frac{1}{n} Var(\sum X_i)}$

$\frac{\hat{\mu} - \mu}{\sigma/\sqrt{n}} \overset{approx}{\sim} \mathcal{N}(0,1)$

$Pr[|X - E[X]| \le \varepsilon] \ge 0.95$

~~$Pr[|X - E[X]| \ge \varepsilon] \le 0.05$~~

$$\hat{\mu} = \frac{\sum_{i=1}^{n} X_i}{n}$$

$$Pr[|\hat{\mu} - \mu| \le \varepsilon] \ge 0.95$$

$$\Rightarrow Pr\left[\underbrace{\frac{|\hat{\mu} - \mu|}{\sigma/\sqrt{n}}}_{\mathcal{N}(0,1)} \le \frac{\varepsilon}{\sigma/\sqrt{n}}\right] \ge 0.95$$

$$\Rightarrow Pr\left[|Z| \le \frac{\varepsilon}{\sigma/\sqrt{n}}\right]$$

$$= Pr\left[-\frac{\varepsilon}{\sigma/\sqrt{n}} \le Z \le \frac{\varepsilon}{\sigma/\sqrt{n}}\right]$$

$$= \Phi\left(\frac{\varepsilon}{\sigma/\sqrt{n}}\right) - \Phi\left(\frac{-\varepsilon}{\sigma\sqrt{n}}\right)$$

$$= \Phi\left(\frac{\varepsilon}{\sigma/\sqrt{n}}\right) - \left(1 - \Phi\left(\frac{\varepsilon}{\sigma/\sqrt{n}}\right)\right)$$

$$= 2\Phi\left(\frac{\varepsilon}{\sigma/\sqrt{n}}\right) - 1 \le 0.95$$

$$\Rightarrow \Phi\left(\frac{\varepsilon}{\sigma/\sqrt{n}}\right) \le 0.975$$

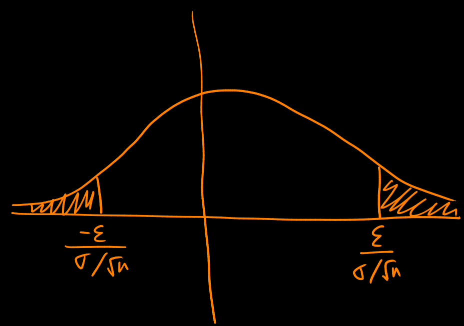
$$\Rightarrow \frac{\varepsilon}{\sigma/\sqrt{n}} \le 1.96$$

$$\Rightarrow \varepsilon \le 1.96 \frac{\sigma}{\sqrt{n}} = 1.96 \frac{0.5}{\sqrt{10000}} = \frac{0.98}{100}$$

given OR given

$\Phi(x)$

cdf of $\mathcal{N}(0,1)$



$X_i \sim Bern(p)$

$\sigma = \sqrt{p(1-p)} \le \sqrt{0.25} = 0.5$

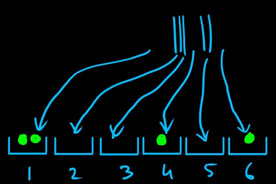
$X_i \sim Pois(\lambda)$

$\sigma = \lambda \le 10$

$X_i \sim Expo(\lambda)$

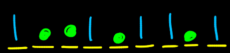
$\sigma = \frac{1}{\lambda^2} \le \frac{1}{100}$ if $\lambda \ge 10$

$$\frac{20}{10000}$$



6^4

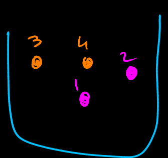
$bins^{balls}$



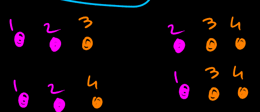
bins−1 + balls

$\binom{bins-1+balls}{balls}$

$\binom{bins-1+balls}{bins-1}$



$\binom{4}{3} = 4$



$$\hat{\mu} = \frac{\sum_{i=1}^{n} X_i}{n}$$

Binom

Bern

??

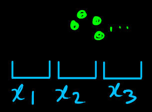
$\sigma = \sqrt{Var(X_i)}$

$\sqrt{n}\,\sigma = \sqrt{Var(\quad)}$

$$\frac{\sigma}{\sqrt{n}} = \sqrt{Var(\hat{\mu})}$$

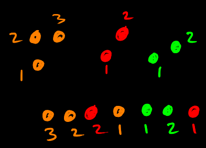
$x_1 + x_2 + x_3 = 100$

$x_1, x_2, x_3 \geq 0, \in \mathbb{Z}$

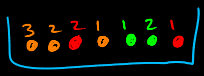


How many solutions?

$\binom{100-1+3}{3} = \binom{102}{3}$



$\frac{7!}{3!2!2!}$



Dist

Pick 3 w/ repl $\rightsquigarrow 7^3$

Pick 3 w/o repl $\rightsquigarrow 7\cdot 6\cdot 5 = \frac{7!}{(7-3)!}$