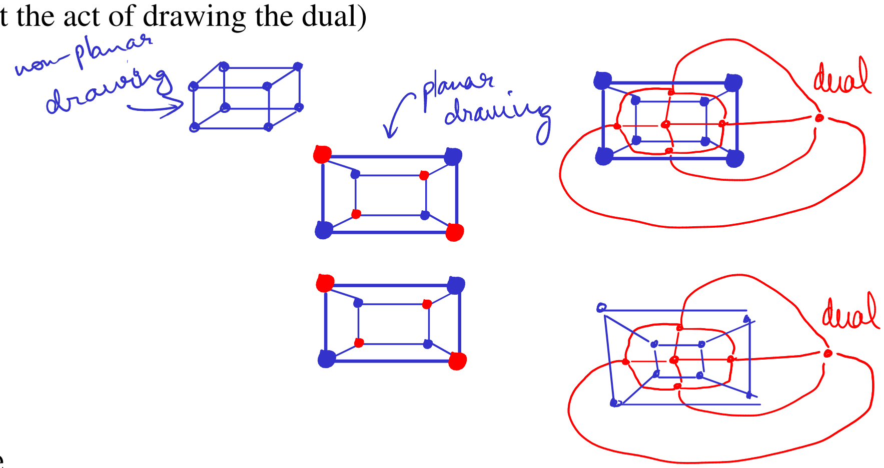
CS 70 Fall 2020 — Discrete Mathematics and Probability Theory — DIS 03A

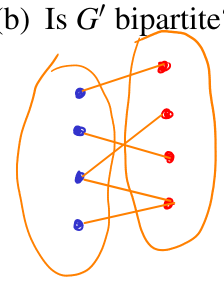
1 Cube Dual

We define a graph G by letting the vertices be the corners of a cube and having edges connecting adjacent corners. Define the *dual* of a planar graph G to be a graph G', constructed by replacing each face in G with a vertex, and an edge between every vertex in G' if the respective faces are adjacent in G.

(a) Draw a planar representation of G and the corresponding dual graph. Is the dual graph planar? (Hint: think about the act of drawing the dual)



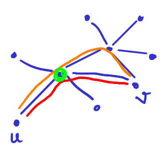
(b) Is G' bipartite?



2 True or False

(a) Any pair of vertices in a tree are connected by exactly one path.

True



0 path : Not tree

≥ 2 paths: Cycle, $\therefore$ not tree

construct a cycle

(b) Adding an edge between two vertices of a tree creates a new cycle.

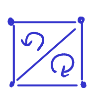
True

Name the new edge. $\{u, v\}$

$n-1$ add an edge : n verts & n edges

(c) Adding an edge in a connected graph creates exactly one new cycle.

False



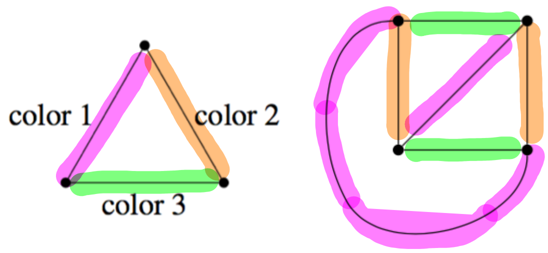
$\forall x\, P(x)$

$\neg(\forall x\, P(x)) \equiv \exists x\, \neg P(x)$

counter ex.

3 Edge Colorings

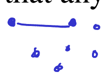
An edge coloring of a graph is an assignment of colors to edges in a graph where any two edges incident to the same vertex have different colors. An example is shown on the left.



(a) Show that the 4 vertex complete graph above can be 3 edge colored. (Use the numbers $1, 2, 3$ for colors. A figure is shown on the right.)

(b) Prove that any graph with maximum degree $d \geq 1$ can be edge colored with $2d - 1$ colors.

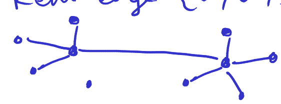
Base: $m = 1$



IS: $m = k+1$, max deg $d \geq 1$.

$d' \leq d$

Rem edge $\{u, v\}$.



$\deg u \leq d-1$

$\deg v \leq d-1$

$2(d-1) = 2d-2$

$2(5-1) = 8+1$

IH: $m = k$, max deg $d \geq 1$, $2d-1$

(c) Show that a $\boxed{\text{tree}}$ can be edge colored with d colors where d is the maximum degree of any vertex.

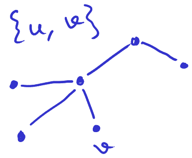
\# verts

Base: $n = 1$

IH: $n = k$, max deg is d, can color w/ d colors

IS: $n = k+1$, max deg is d

Remove leaf v, removed edge $\{u, v\}$



Before:
$\deg u = d$

After rem:
$\deg u = d-1$

1 more color for $\{u, v\}$

d colors.

$\deg u < d$

After $\deg u < d-1$

1 more color for $\{u, v\}$

$d-1$