8 Computability and Introduction to Probability

Computer Science Mentors 70

Oct 26 - Oct 30
Fall 2020

Prepared by: Aishani Sil, Austin Lei, Agnibho Roy, Debayan Bandyopadhyay, Abinav Routhu

Computability

1. Determine the computability of the following tasks. If it's not computable, write a reduction or self-reference proof. If it is, write the program. Hint: we can use the fact that TestHalt can't exist to prove other statements.

   (a) You want to determine whether a program P on input x prints "Hello World!" Is there a computer program that can perform this task? Justify your answer.

   (b) You want to determine whether a program P prints "Hello World!" before running the kth line of the program.

   (c) You want to determine whether a program P prints "Hello World!" in the first k steps of its execution. Is there a computer program that can perform this task?

2. Say that we have a program M that decides whether any input program halts as long as it prints out the string "ABC" as the first operation that it carries out. Can such a program exist? Prove your answer.

Introduction to Probability

1. **Rolling Die**
   Leanne rolls two fair six-sided die. For parts (c) through (e), imagine that Leanne wishes to compute the probability that the sum of the two die is 5.

   (a) How could you represent the sample space?

   (b) What is the size of the sample space?

   (c) How could we represent the event we are looking for?

   (d) What is the size of the event?

   (e) What is the probability that Leanne rolls a 5?

2. **Probably Poker**

   (a) What is the probability of drawing a hand with four of a kind (four cards of the same rank)?

   (b) What is the probability of drawing a straight (five cards in numerical order)?

   (c) What is the probability of drawing a flush (five cards all of the same suit)?

(d) What is the probability of drawing a straight flush (five cards in numerical order, all of the same suit)?

(e) What is the probability of drawing a hand with exactly one pair (two cards of matching rank)?

3. Oski is rolling 10 six-sided dice; what is the probability that he gets exactly two 4s?

4. Eric is looking at the weather forecast for the next few days. On Monday, there is a 30% chance it will rain, and a 70% chance it will be sunny. On Tuesday, there is a 20% chance it will rain, and a 80% chance it will be sunny. The weather on Monday does not affect the weather on Tuesday. What is the probability that there will be rain on at least one day?

5. For each permutation σ of 1 through n, let $\sigma(i)$ denote the value at position i. For example, if the permutation is $2, 4, 1, 3$ we have $\sigma(1) = 2$ and $\sigma(2) = 4$ (Source: CS 70 SP19 MT2)

(a) For a fixed $1 \leq k \leq n$, what is the probability that a permutation σ of 1 through n satisfies the property that $i < k, \sigma(i) < \sigma(k)$? Express your answers in terms of n and k.

(b) What is the probability that a permutation from 1 through n satisfies the property that for each i, $\sigma(\sigma(i)) = i$ and

$\sigma(i) \neq i$? (For example, the permutation $3, 4, 1, 2$ is such a permutation, since for example $\sigma(\sigma(1)) = \sigma(3) = 1$. You may assume n is even.

6. Eve is playing a game with Steve where she guesses the value (head or tail) of two coins that Steve has just flipped. She is told that one coin is facing heads. Given this information, what is the probability that both coins are facing heads?