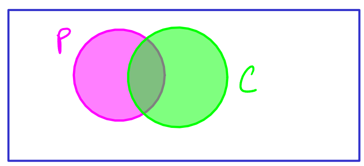
CS 70 Discrete Mathematics and Probability Theory
Fall 2020 **DIS 9A**

1 Venn Diagram

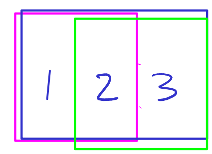
Out of 1,000 computer science students, 400 belong to a club (and may work part time), ~~500~~ 800 work part time (and may belong to a club), ~~and 50 belong to a club and work part time~~.

(a) Suppose we choose a student uniformly at random. Let C be the event that the student belongs to a club and P the event that the student works part time. Draw a picture of the sample space Ω and the events C and P.



$\{1, 2\}$

$\{2, 3\}$



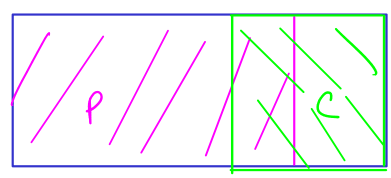
(b) What is the probability that the student belongs to a club?

0.4

$$\frac{|C|}{|\Omega|} = \frac{400}{1000}$$

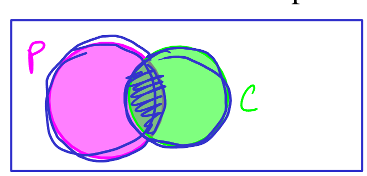
(c) What is the probability that the student works part time?

$$\frac{|P|}{|\Omega|} = \frac{500}{1000} = 0.5$$



(d) What is the probability that the student belongs to a club AND works part time?

$$\frac{|P \cap C|}{|\Omega|} = \frac{50}{1000} = \frac{1}{20}$$



lowest $= 0$

highest $= \frac{400}{1000}$

$|\Omega| = 1000$

$|P| = 800$,

$|C| = 400$

0.2

0.4

(e) What is the probability that the student belongs to a club OR works part time?

0.85

$$|P \cup C| = |P| + |C| - |P \cap C| = 400 + 500 - 50 = 850$$

2 Flippin' Coins

Suppose we have an unbiased coin, with outcomes H and T, with probability of heads $\mathbb{P}[H] = 1/2$ and probability of tails also $\mathbb{P}[T] = 1/2$. Suppose we perform an experiment in which we toss the coin 3 times. An outcome of this experiment is (X_1, X_2, X_3), where $X_i \in \{H, T\}$.

(a) What is the *sample space* for our experiment?

$\Omega = \{(X_1, X_2, X_3) : X_i \in \{H,T\}, 1 \le i \le 3\}$, $\Omega = \left\{ \begin{matrix} (H,H,H), (H,H,T), (T,H,H), (T,H,T), \\ (H,T,H), (H,T,T), (T,T,H), (T,T,T) \end{matrix} \right\}$

(b) Which of the following are examples of *events*? Select all that apply.

- $\{(H,H,T),(H,H),(T)\}$ $\not\subseteq \Omega$
- $\{(T,H,H),(H,T,H),(H,H,T),(H,H,H)\}$
- $\{(T,T,T)\}$
- $\{(T,T,T),(H,H,H)\}$
- $\{(T,H,T),(H,H,T)\}$

(c) What is the complement of the event $\{(H,H,H),(H,H,T),(H,T,H),(H,T,T),(T,T,T)\}$? $= E$

$\overline{E} = E^c = \Omega \setminus E$ $\left\{ \begin{matrix} (H,H,H), (H,H,T), (T,H,H), (T,H,T), \\ (H,T,H), (H,T,T), (T,T,H), (T,T,T) \end{matrix} \right\}$

(d) Let A be the event that our outcome has 0 heads. Let B be the event that our outcome has exactly 2 heads. What is $A \cup B$?

$\{ \underbrace{(T,T,T)}_{A}, \underbrace{(T,H,H), (H,H,T), (H,T,H)}_{B} \}$

(e) What is the probability of the outcome (H,H,T)?

$A = \{(H,H,T)\}$ $|A| = 1$ $P(A) = \frac{|A|}{|\Omega|} = \frac{1}{8}$

(f) What is the probability of the event that our outcome has exactly two heads?

$A = \{(T,H,H), (H,H,T), (H,T,H)\}$

$P(A) = \frac{|A|}{|\Omega|} = \frac{3}{8}$

(g) What is the probability of the event that our outcome has at least one head?

$\frac{7}{8}$

$A =$ roll ≥ 1 heads $= \Omega \setminus \{(T,T,T)\}$

$\overline{A} =$ roll 0 heads $= \{(T,T,T)\}$

$P(A) = 1 - P(\overline{A}) = 1 - \frac{1}{8} = \frac{7}{8}$ ✓

3 Counting & Probability

Consider the equation $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 70$, where each x_i is a non-negative integer. We choose one of these solutions uniformly at random.

Ω = set of all sol^{n}s

(a) What is the size of the sample space?

$$|\Omega| = \binom{70+6-1}{6-1} = \binom{75}{5} = \binom{75}{70}$$

☆ | | ☆ | ☆ . . .

70 ☆ , 6−1 bars

(b) What is the probability that both $x_1 \geq 30$ and $x_2 \geq 30$?

$A \cap B = x_1 \geq 30,\ x_2 \geq 30$

$$|A \cap B| = \binom{10+6-1}{6-1} = \binom{15}{5}$$

10 remaining ☆ , 6−1 bars

$$P(A \cap B) = \frac{\binom{15}{5}}{\binom{75}{5}}$$

(c) What it the probability that either $\underbrace{x_1 \geq 30}_{A}$ or $\underbrace{x_2 \geq 30}_{B}$?

40+6−1   40+6−1

$$|A \cup B| = |A| + |B| - |A \cap B| = \binom{45}{5} + \binom{45}{5} - \binom{15}{5}$$

6−1

$$P(A \cup B) = \frac{\binom{45}{5} + \binom{45}{5} - \binom{15}{5}}{\binom{75}{5}}$$