CS 70 Discrete Mathematics and Probability Theory
Fall 2020 **DIS 00**

# 1 Implication

Which of the following implications are always true, regardless of $P$? Give a counterexample for each false assertion (i.e. come up with a statement $P(x,y)$ that would make the implication false).

(a) $\forall x \forall y P(x,y) \implies \forall y \forall x P(x,y)$.

(b) $\forall x \exists y P(x,y) \implies \exists y \forall x P(x,y)$.

(c) $\exists x \forall y P(x,y) \implies \forall y \exists x P(x,y)$.

# 2 Equivalences with Quantifiers

Evaluate whether the expressions on the left and right sides are equivalent in each part, and briefly justify your answers. For all $x \in \mathcal{U}$, if there exists a $y \in \mathcal{U}$ such that $Q(x,y) = T$ then $P(x) = T$

| | | |
|---|---|---|
| (a) | $\forall x \left((\exists y\, Q(x,y)) \Rightarrow P(x)\right)$ | $\forall x\, \exists y \left(Q(x,y) \Rightarrow P(x)\right)$ |
| (b) | $\neg \exists x\, \forall y \left(P(x,y) \Rightarrow \neg Q(x,y)\right)$ | $\forall x \left((\exists y\, P(x,y)) \wedge (\exists y\, Q(x,y))\right)$ |
| (c) | $\forall x\, \exists y \left(P(x) \Rightarrow Q(x,y)\right)$ | $\forall x \left(P(x) \Rightarrow (\exists y\, Q(x,y))\right)$ |

For all $x \in \mathcal{U}$, there exists $y \in \mathcal{U}$ such that if $Q(x,y) = T$ then $P(x) = T$

$\mathcal{U} = \mathbb{Z}$, $Q(x,y): x > y$
$P(x): x \in \mathbb{Z}$

# 3 XOR

$x=5$
$\rightarrow y=4$

The truth table of XOR (denoted by $\oplus$) is as follows.

| A | B | A $\oplus$ B |
|---|---|---|
| F | F | F |
| F | T | T |
| T | F | T |
| T | T | F |

1. Express XOR using only $(\wedge, \vee, \neg)$ and parentheses.

2. Does $(A \oplus B)$ imply $(A \vee B)$? Explain briefly.

3. Does $(A \vee B)$ imply $(A \oplus B)$? Explain briefly.

## 4 Truth Tables

Determine whether the following equivalences hold, by writing out truth tables. Clearly state whether or not each pair is equivalent.

(a) $P \wedge (Q \vee P) \equiv P \wedge Q$

(b) $(P \vee Q) \wedge R \equiv (P \wedge R) \vee (Q \wedge R)$

(c) $(P \wedge Q) \vee R \equiv (P \vee R) \wedge (Q \vee R)$