Fall 15 Midterm 1

6. Simple proofs.

(a) Prove or disprove that for integers a, b, if $a + b \geq 1016$ that either a is at least 508 or b is at least 508.

(b) Prove or disprove that $\sqrt{8}$ is irrational.

(c) Let $R_0 = 0; R_1 = 2; R_n = 4R_{n-1} - 3R_{n-2}$ for $n \geq 2$.
Prove that $R_n = 3^n - 1$ for all $n \geq 0$.

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3. Its Own Inverse (5pts)

For $p > 1$, prove that $p - 1$ is always its own multiplicative inverse in mod p arithmetic.

6. Prove it by induction (10pts)

The j-th harmonic number is defined as

$$H_j = \sum_{i=1}^{j} \frac{1}{i} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{j}.$$

So $H_1 = 1, H_2 = 1.5, \ldots$.

Use induction to prove that for any positive integer n,

$$\sum_{j=1}^{n} H_j = H_1 + H_2 + \cdots + H_n = (n+1)H_n - n.$$

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2. [Proofs.] [20 pts]

A. (10 pts) Let D_n be the number of ways to tile a $2 \times n$ checkerboard with dominos, where a domino is a 1×2 piece. Prove that $D_n \leq 2^n$ for all positive integers n. (Find a recurrence relation for D_n. No need to give a proof. Then inductively prove the upper bound on D_n.)

Note that dominos can only be placed exactly aligned with checkerboard squares, and cannot be placed diagonally.

B. (10 pts) Show that $\forall$ odd $a \in N, a^2 = 1 \mod 8$.