SP 19, Q1

**(t)** [3 pts] Let $f \in [0,1]$ be the *unknown*, fixed probability that a person in a certain population owns a dog (how cute!). We model $f$ with a hypothesis $h \in [0,1]$. Before we observe any data at all, we can't even guess what $f$ might be, so we set our prior probability for $f$ to be the uniform distribution, i.e., $P(f = h) = 1$ over $h \in [0,1]$. Now, we pick one person from the population, and it turns out that they have a cute little labradoodle named Dr. Frankenstein. Which of the following is true about the posterior probability that $f = h$ given this one sample point?

- ○ The posterior is uniform over $h \in [0,1]$.
- ○ The posterior increases nonlinearly over $h \in [0,1]$.
- ● The posterior increases linearly over $h \in [0,1]$.
- ○ The posterior is a delta function at 1.

$$P(f=h \mid \text{see one dog after 1 trial}) = \frac{P(\text{see one dog after 1 trial} \mid f=h)\underbrace{P(f=h)}_{=1 \text{ (uniform prior)}}}{P(\text{see one dog after 1 trial})} \leftarrow \text{just a scaling constant, no } f$$

$$\therefore P(f=h \mid \text{see one dog after 1 trial}) \propto P(\text{see one dog after 1 trial} \mid f=h)$$

$$= h^1(1-h)^0 \quad \because \text{\# of dogs after } n \text{ trials} \sim \text{Binomial}(n,h)$$

$$= \boxed{h}$$

note that each human can either be a dog owner or not, and each "trial" is us asking someone if they are a dog owner.
We think that the proportion of people who own dogs is $h$.
$h$ is a variable, and we want to find the value of $h$ that maximizes the likelihood of our observations.