1 Cube Dual

We define a graph G by letting the vertices be the corners of a cube and having edges connecting adjacent corners. Define the *dual* of a planar graph G to be a graph G', constructed by replacing each face in G with a vertex, and an edge between every vertex in G' if the respective faces are adjacent in G.

(a) Draw a planar representation of G and the corresponding dual graph. Is the dual graph planar? (Hint: think about the act of drawing the dual)

(b) Is G' bipartite?

2 True or False

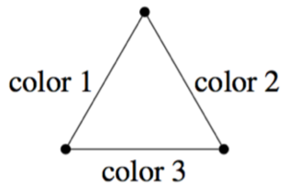
(a) Any pair of vertices in a tree are connected by exactly one path.

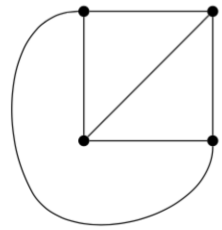
(b) Adding an edge between two vertices of a tree creates a new cycle.

(c) Adding an edge in a connected graph creates exactly one new cycle.

3 Edge Colorings

An edge coloring of a graph is an assignment of colors to edges in a graph where any two edges incident to the same vertex have different colors. An example is shown on the left.





(a) Show that the 4 vertex complete graph above can be 3 edge colored. (Use the numbers $1, 2, 3$ for colors. A figure is shown on the right.)

(b) Prove that any graph with maximum degree $d \geq 1$ can be edge colored with $2d - 1$ colors.

(c) Show that a tree can be edge colored with d colors where d is the maximum degree of any vertex.