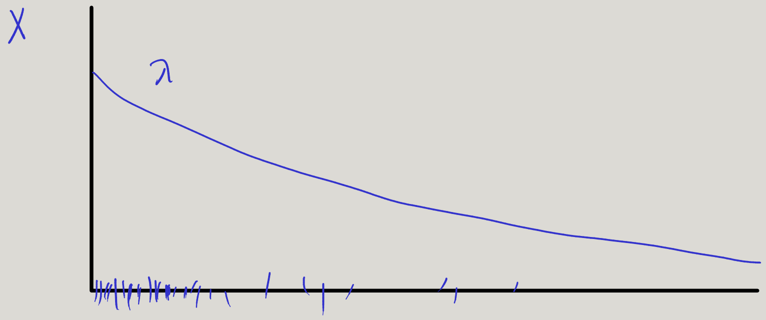
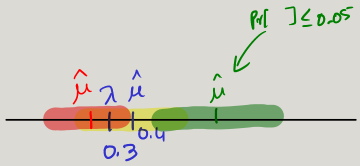


$$\left\{ \begin{array}{l} X_1 \\ X_2 \\ \vdots \\ X_n \end{array} \right. \qquad \text{Expo}(\underline{?})$$



$$A_n = \frac{X_1 + \cdots + X_n}{n}$$

$$\hat{\lambda} = \frac{1}{A_n} = \frac{1}{\hat{\mu}}$$

$X \sim \text{Expo}(\lambda)$

$$E[X] = 1/\lambda \implies \lambda = \frac{1}{E[X]} \approx \hat{\mu}$$

$$E\left[\frac{1}{A_n}\right] = E[1/\hat{\mu}] = \lambda \qquad \hat{\lambda} = \frac{1}{\hat{\mu}}$$

$$E[A_n]$$

$$= E\left[\frac{1}{n}\sum_{i=1}^{n} X_i\right]$$

$$= \frac{1}{n}\sum_{i=1}^{n} E[X_i] = \frac{1}{n} \cdot n \frac{1}{\lambda} = \frac{1}{\lambda} = \mu$$

$$\text{Var}(A_n) = \frac{1}{n^2} \cdot n \text{Var}(X_1) = \frac{1}{n\lambda^2}$$

$$\frac{A_n - \mu}{\sqrt{1/n\lambda^2}} \overset{\text{approx}}{\sim} \mathcal{N}(0,1)$$

$$\Pr[|A_n - \mu| \le 0.1\mu] \ge 0.95$$

$$\implies \Pr\left[\frac{|A_n - \mu|}{\sqrt{1/n\lambda^2}} \le 0.1\mu\right] \ge 0.95$$

$m = 12$

$ax \bmod m$

$\underbrace{a^{-1}}_{\text{const}} \underbrace{a x}_{\text{var}} \equiv a^{-1} 5 \mod 7$

$3x = 12k + 5$

$3x \equiv 5 \mod 12 \qquad \implies \underbrace{3x - 12k}_{} = 5$

$6x \equiv 5 \mod 12$

$0, 6, 0, 6, 0, 6, \qquad \underbrace{0, 3, 6, 9}, \underbrace{0, 3, 6, 9}, \underbrace{0, 3, 6, 9}$

$0, 6, 0, 6, 0, 6$

$X_i \sim \text{Bern}(p)$

$X_i^2 \sim \text{Bern}(p)$

X_i what values?	what prob?
0	$1-p$
1	p

$X_i X_j \sim \text{Bern}(\Pr[X_i X_j = 1] = p^3)$

X_i^2	
0^2	$1-p$
1^2	p

$\Pr[X_i = 1 \cap X_j = 1]$

$= \Pr[X_i = 1 \mid X_j = 1] \Pr[X_j = 1]$

$= p \cdot p^2 = p^3$

$X_j \backslash X_i$	0	1
0	0	0
1	0	1

$$\left(\sum X_i\right)^2 = \sum X_i^2 + \sum_{1 \le i \ne j \le n} X_i X_j$$

$$\text{Var}(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$$

$$\mathbb{E}[X^2] = \mathbb{E}\left[\left(\sum_{i=1}^n X_i\right)^2\right] = \sum_{i=1}^n \mathbb{E}[X_i^2] + \sum_{1 \le i \ne j \le n} \mathbb{E}[X_i X_j] = np - 2(n-1)b + (n-1)(n-2)c$$

$n(n-1) - 2(n-1) = (n-2)(n-1)$

$X \sim \text{Pois}(\lambda)$

W is waiting time

$\# \text{ of arrivals} = X \sim \text{Pois}(\underset{\text{spam}}{1} + \underset{\text{normal}}{\lambda})$

$$\{W \le t\} \Leftrightarrow \{X = 1,\ \text{Pois}((1+\lambda)t)\}$$

$$\Pr[W \le t] = \Pr[\text{Pois}((1+\lambda)t) > 0]$$

$$= 1 -$$