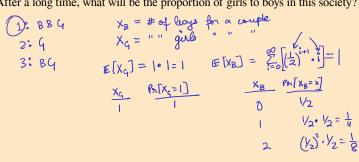
Every couple prefers to have a girl over a boy. There's a 50% chance that a child is a girl, 50% chance of being a boy. Children's genders are mutually indep.

Every couple will continue having children until they have one girl. After a long time, what will be the proportion of girls to boys in this society?



3red, 261, 2gg

 $\frac{\binom{3}{3}}{\binom{7}{3}} = \frac{10}{35} = \frac{2}{7}$

 $\frac{2010}{35} = \frac{4}{7}$

 $\frac{1.7}{35} = \frac{1}{7}$

How Many Marbles?

Leanne has 6 marbles, 2 red, 2 blue, and 2 green. She picks three marbles uniformly at random without replacement. Let X denote the number of blue marbles she draws.

(a) What is
$$\mathbb{P}[X=0]$$
, $\mathbb{P}[X=1]$, and $\mathbb{P}[X=2]$?

$$R_1[X=0] = \frac{\binom{4}{3}}{\binom{6}{3}} = \frac{4}{20} = \frac{1}{5}$$

$$\beta_n[X=1] = \frac{\binom{2}{1}\binom{4}{2}}{\binom{6}{3}} = \frac{2 \cdot 6}{20} = \frac{3}{5}$$

$$P_{\pi}[X=2] = \frac{\binom{2}{2}\binom{4}{1}}{|\Omega| = \binom{6}{3}} = \frac{1 \cdot 4}{20} = \frac{1}{5}$$

(a) What is $\mathbb{P}[X=0]$, $\mathbb{P}[X=1]$, and $\mathbb{P}[X=2]$?

$$\beta_n[X=1] = \frac{\binom{2}{1}\binom{4}{2}}{\binom{6}{3}} = \frac{2 \cdot 6}{20} = \frac{3}{5}$$

(b) What do your answers you computed in part (a) add up to?

(c) Compute $\mathbb{E}[X]$ from the definition of expectation.

$$\mathbb{E}[X] = \sum_{\substack{\text{proxible}\\\text{xc}}} \times \Pr[X = x] = 0 \cdot \frac{1}{5} + 1 \cdot \frac{3}{5} + 2 \cdot \frac{1}{5} = \boxed{1}$$

Suppose we define indicators
$$X_i$$
, $1 \le i \le 3$, where X_i is the indicator variable that equals 1 if the *i*th marble is a blue marble and 0 otherwise. Compute $\mathbb{E}[X]$ using linearity of expectation.

(e) Are the
$$X_i$$
 indicators independent? Does this affect your solution to part (d)?

$$X = \sum_{i=1}^{2} x_i$$

$$X_i = 1 \begin{cases} i^{th} \text{ morbile is allue} \end{cases}$$

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Solve each of the following problems using linearity of expectation. Explain your methods clearly.

(a) In an arcade, you play game A 10 times and game B 20 times. Each time you play game A, you win with probability 1/3 (independently of the other times), and if you win you get 3 tickets (redeemable for prizes), and if you lose you get 0 tickets. Game B is similar, but you win with probability 1/5, and if you win you get 4 tickets. What is the expected total number of tickets you receive?

(b) A monkey types at a 26-letter keyboard with one key corresponding to each of the lower-case English letters. Each keystroke is chosen independently and uniformly at random from the 26 possibilities. If the monkey types 1 million letters, what is the expected number of times the sequence "book" appears?

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(d)

3 Ball in Bins

You are throwing k balls into n bins. Let X_i be the number of balls thrown into bin i.

(a) What is $\mathbb{E}[X_i]$?

(b) What is the expected number of empty bins?

(c) Define a collision to occur when a ball lands in a nonempty bin (if there are n balls in a bin, count that as n-1 collisions). What is the expected number of collisions?

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