Disc Joint prob mass func (pmf)

$Pr[X=x \text{ and } Y=y]$

LOTUS

$E[g(X,Y)]$

$= \sum_{x,y} g(x,y) Pr[X=x \wedge Y=y]$

Cont. Joint prob density func (pdf)

$Pr[X \in [x_1, x_2] \text{ and } Y \in [y_1, y_2]]$

$= \int_{y_1}^{y_2} \int_{x_1}^{x_2} f_{X,Y}(x,y)\, dx\, dy$

MMSE

Y X

$\hat{Y} = g(x) = \begin{cases} 1 & \text{if } x=1 \\ 0 & \text{if } x=0 \end{cases}$

$\hat{Y}$ \ Y	-1	1
-1	0	$1/2$
1	$1/2$	0

$\min_{f(x)=\hat{Y}} E[(Y-\hat{Y})^2]$

$E[(Y-\hat{Y})^2] = \frac{1}{2}(2) + \frac{1}{2}(2)$

$= 2$

$E[Y|X] = E[Y] + \frac{Cov(X,Y)}{Var(X)}(X - E[X])$

$= f(X)$

LLSE

linear LSE $g(x) = mx + b$ ~~ax^2+bx+c~~

$b = E[Y] - \frac{Cov(X,Y)}{Var(X)} E[X]$

$m = \frac{Cov(X,Y)}{Var(X)}$

$\min_{m,b} E[(Y - \underbrace{(mX+b)}_{\hat{Y}})^2]$