$$\Pr\left[\bigcup_{i=1}^{n} A_i\right] \le \sum_{i=1}^{n} \Pr[A_i]$$

$O(n)$

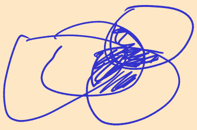
Time Series

AR

MA

SARMA

$$\Pr\left[\bigcup_{i=1}^{n} A_i\right] = $$



$$\sum \Pr[A_i]$$
$$- \sum\sum \Pr[A_i \cap A_j]$$
$$+ \sum\sum\sum \Pr[A_i \cap A_j \cap A_k]$$

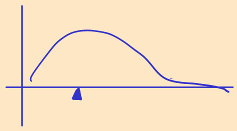
$$\bigcup_{i=1}^{4} A_i = P(A_1)+P(A_2)+P(A_3)+P(A_4) \leftarrow \binom{4}{1}$$

$2^4 - 1$

$$\left(\begin{aligned} &- P(A_1 \cap A_2) - P(A_1 \cap A_3) \\ &- P(A_2 \cap A_3) - P(A_1 \cap A_4) \\ &- P(A_2 \cap A_4) - P(A_3 \cap A_4) \end{aligned}\right) \leftarrow \binom{4}{2}$$

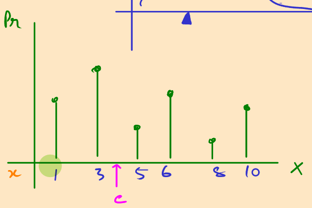
$$\left(+ P(A_1 \cap A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_4)\right) \leftarrow \binom{4}{3}$$
$$+ \qquad\qquad +$$

$$- P(A_1 \cap A_2 \cap A_3 \cap A_4) \leftarrow \binom{4}{4}$$



<u>Markov's Inequality</u> $X \ge 0$

$$c \Pr[X \ge c] \le \mathbb{E}[X]$$



$$\mathbb{E}[X] = \sum_{x} x \Pr[X = x]$$

$$\ge \sum_{x \ge c} x \Pr[X = x]$$ nonneg x

$\Pr[X \ge c]$

$$\ge \sum_{x \ge c} c \Pr[X = x]$$

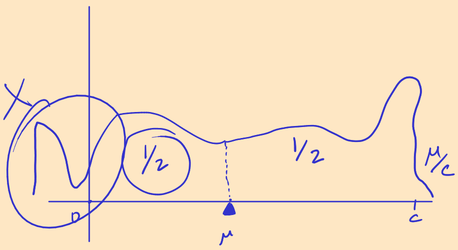
$$= c \sum_{x \ge c} \Pr[X = x]$$

$$= c \Pr[X \ge c]$$ ✓

Remove
$$\left(\begin{aligned} &1 \Pr[X=1] \\ &+ 3\Pr[X=3] \end{aligned}\right)$$
$$+ \cancel{5}\,4 \Pr[X=5]$$
$$+ \cancel{6}\,4 \Pr[X=6]$$
$$+ \cancel{8}\,4 \Pr[X=8]$$
$$+ \cancel{10}\,4 \Pr[X=10]$$

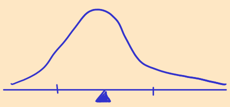
$c = 4$

$$\underbrace{0.5 + 0.01}{} + 0.7 + 3 + 2.1$$
$$\ge \quad 0.7 + 3 + 2.1$$



<u>Chebyshev's Inequality</u> X

$$\Pr[|X - \mu| \ge c] \le \frac{Var(X)}{c^2}$$



$$Y = (X - \mu)^2 \ge 0$$

$$\Pr[Y \ge c^2] \le \frac{\mathbb{E}[Y]}{c^2} \leftarrow \text{markov}$$

$$\Rightarrow \Pr[|X - \mu| \ge c] \le \frac{Var(X)}{c^2}$$

1 Probabilistic Bounds

$X \leq 10$

A random variable X has variance $\text{Var}(X) = 9$ and expectation $\mathbb{E}[X] = 2$. Furthermore, the value of X is never greater than 10. Given this information, provide either a proof or a counterexample for the following statements.

(a) $\mathbb{E}[X^2] = 13$.

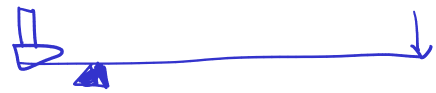
$$Var(X) = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$$
$$\Rightarrow 9 = \mathbb{E}[X^2] - 4$$
$$\Rightarrow \mathbb{E}[X^2] = 13.$$

(b) $\mathbb{P}[X = 2] > 0$.

$Pr[X = \mathbb{E}[X]] > 0$

$\boxed{\text{False}}$

X	Pr
-1	$1/2$
5	$1/2$

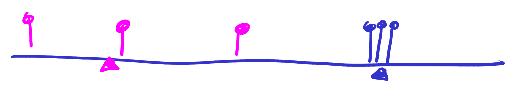


$X > 2$	$X < 2$
$X = 2$	$X = 2$

(c) $\mathbb{P}[X \geq 2] = \mathbb{P}[X \leq 2]$.

$Pr[X \geq 0.9] = 0.9$
$\neq$
$Pr[X \leq 0.9] = 0.1$

X	Pr
-7	0.1
3	0.9



(d) $\mathbb{P}[X \leq 1] \leq 8/9$.

$X \leq 10$

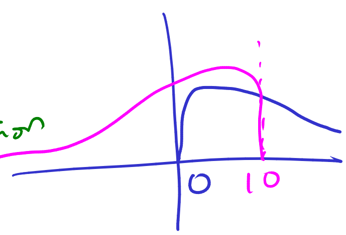
$-1(X - 10)$

$Z = 10 - X$

$Z \geq 0$ ← Markov precondition

$\underbrace{Pr[Z \geq 9] \leq \frac{8}{9}}$

$\underbrace{\Rightarrow Pr[X \leq 1] \leq \frac{8}{9}}$



X	$X - 10$
5	-5
3	-7

$\underline{\mathbb{E}[Z]}$
$= 10 - \mathbb{E}[X] = 8$
$10 - 2 = 8$

$10 - X \geq -1 + 10$
$\Leftrightarrow 10 - X \geq 9$

(e) $\mathbb{P}[X \geq 6] \leq 9/16$.

$5 \leq 7$
$-5 \geq -7$

$Pr[X \leq 1]$
$= Pr[-X \geq -1]$
$= Pr[10 - X \geq 9]$

2 Working with the Law of Large Numbers

(a) A fair coin is tossed multiple times and you win a prize if there are more than 60% heads. Which number of tosses would you prefer: 10 tosses or 100 tosses? Explain.

(b) A fair coin is tossed multiple times and you win a prize if there are more than 40% heads. Which number of tosses would you prefer: 10 tosses or 100 tosses? Explain.

(c) A fair coin is tossed multiple times and you win a prize if there are between 40% and 60% heads. Which number of tosses would you prefer: 10 tosses or 100 tosses? Explain.

(d) A fair coin is tossed multiple times and you win a prize if there are exactly 50% heads. Which number of tosses would you prefer: 10 tosses or 100 tosses? Explain.

3 Continuous Computations

Let X be a continuous random variable whose PDF is cx^3 (for some constant c) in the range $0 \le x \le 1$, and is 0 outside this range.

(a) Find c.

(b) Find the CDF of X.

(c) Find $\mathbb{E}(X)$.