$L(Y|X) = m^* x + b^*$

best linear predictor

$\hat{Y} = mX + b$

$$\min_{m,b} \mathbb{E}[(Y - \hat{Y})^2]$$

actual RV

$\mathbb{E}[Y|X]$ is linear in X

$$\mathbb{E}[Y|X] = \mathbb{E}[Y] + \frac{\text{cov}(X,Y)}{\text{Var}(X)}(X - \mathbb{E}[X]) = L(Y|X)$$

$$\mathbb{E}[Y|X=x] = \int_{-\infty}^{\infty} y\, f_{Y|X}(y|x)\, dy$$

$$= \int_0^2 y\, dy = \left(\frac{y^2}{2}\right|_0^2 = 2$$

$$f_X(x) = \begin{cases} 1/2 & 0 \le x \le 2 \\ 0 & \text{o/w} \end{cases}$$

$$f_{X,Y}(x,y) = \begin{cases} 1/2 \\ 1/2 \end{cases}$$

$$f_{Y|X}(y|x) = \begin{cases} 1 & 0 \le x \le 1 \\ 0 & \text{o/w} \end{cases}$$

$$f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)}$$

$$= \frac{1/2}{1/2}$$

$$= 1$$

$$\mathbb{E}[Z] = \int_{-\infty}^{\infty} z\, f_Z(z)\, dz$$

$$\mathbb{E}[\underbrace{\mathbb{E}[X]}] = \mathbb{E}[X]$$

$\mathbb{E}[\text{number}] = \text{number}$

$$\mathbb{E}[Y \boxed{\mathbb{E}[X]}]$$

$$= \mathbb{E}[Y]\mathbb{E}[X]$$