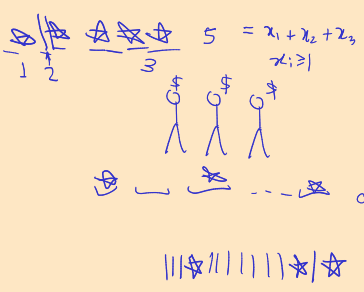
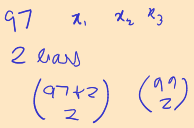
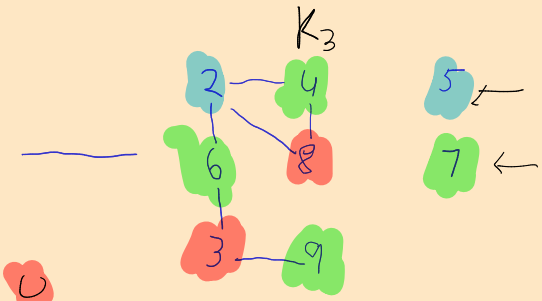
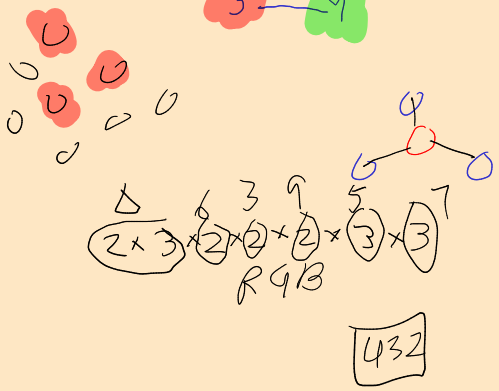

5 = x1 + x2 + x3
xi ≥ 1
1
2
3


97 x1 x2 x3
2 bars
(97+2 choose 2)
(99 choose 2)

99 bars
3 stars
(stars + bars choose bars)
2 bars
100 stars


(n choose k)


K3
2
4
5
6
8
7
3
9

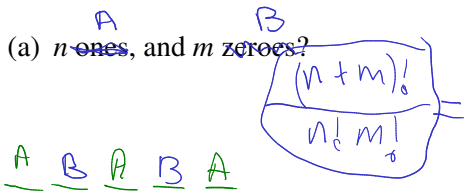

2 × 3 × 2 × 2 × 2 × 3 × 3
RGB
432

CS 70 Discrete Mathematics and Probability Theory
Fall 2021 DIS 6A

1 Strings

What is the number of strings you can construct given:

(a) n ~~ones~~, and m ~~zeroes~~?



$\binom{n+m}{n} = \binom{n+m}{m}$

$n=2 \quad m=1$

AAB ABA, BAA

$A_1A_2B_1$ $A_1A_2B_1$ $A_2A_1B_1$

$A_1B_2A_3B_1A_2$
$A_3B_1A_2B_2A_1$

(b) n_1 A's, n_2 B's and n_3 C's?

$$\frac{(n_1+n_2+n_3)!}{n_1!\,n_2!\,n_3!}$$

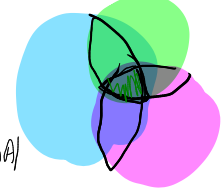
(c) $n_1, n_2, \ldots, n_k$ respectively of k different letters?

$$\frac{(n_1+n_2+\cdots+n_k)!}{n_1!\,n_2!\cdots n_k!}$$

2 The Count

(a) How many of the first 100 positive integers are divisible by 2, 3, or 5?

$A = \{2, 4, 6, 8, 10, \ldots\}$
$B = \{3, 6, 9, 12, 15, \ldots\}$
$C = \{5, 10, 15, 20, \ldots\}$



$|A|+|B|+|C|$
$-|A\cap B| - |B\cap C| - |C\cap A|$
$+|A\cap B\cap C|$

$\frac{100}{2} + \lfloor\frac{100}{3}\rfloor + \lfloor\frac{100}{5}\rfloor - \lfloor\frac{100}{6}\rfloor - \lfloor\frac{100}{10}\rfloor - \lfloor\frac{100}{15}\rfloor + \lfloor\frac{100}{30}\rfloor = 74$

$100 // 3$

(b) The Count is trying to choose his new 7-digit phone number. Since he is picky about his numbers, he wants it to have the property that the digits are non-increasing when read from left to right. For example, 9973220 is a valid phone number, but 9876545 is not. How many choices for a new phone number does he have?

$\binom{16}{9} = \binom{7+b}{7} = \binom{7+b}{b}$

Bars: between digits
stars:

9 8 7 6 5 4 3 2 1 0

8775430

9 8 7 6 5 4 3 2 1 0

(c) Now instead of non-increasing, they must be strictly decreasing. So 9983220 is no longer valid, while 9753210 is valid. How many choices for a new phone number does he have now?

6 _ _ _ _ _ _ 1

7 5

$\frac{10!}{3!}$

1 3 4 8 7 6 5 2 0 9

9754321

$\binom{10}{3} = \binom{10}{7}$

7654320

7654302

3 Digits

(a) How many 7-digit numbers have no two adjacent digits equal?

(b) How many 5-digit palindromes are there? (A palindrome is a number that reads the same way forwards and backwards. For example, 27872 and 48484 are palindromes, but 28389 and 12541 are not.)

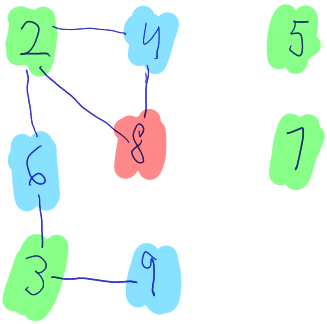
4 Divisor Graph Colorings

Define G where we have $V = \{2,3,4,5,6,7,8,9\}$, and we add an edge between vertex i and vertex j if i divides j, or j divides i.

(a) Draw G.

(b) Explain why we cannot vertex-color G with only 2 colors.

(c) How many ways can we vertex-color G with 3 colors?



$(3 \times 2 \times 1) \times 2 \times 2 \times 2 \times 3 \times 3$

$= 432$