Assignment 7

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1 Introduction

In this week's assignment, we will explore the scipy.signal library through analysing Linear Time Invariant Systems. We consider three systems:

- Forced Oscillatory System
- Coupled Differential Equations
- RLC Low Pass Filter

We find the rational transfer functions of the above systems, analyse them and plot them.

2 Question 1

In this question, we are considering a forced oscillatory system with input $f(t) = \cos(1.5t)e^{-0.5t}u_0(t)$ and the equation of system is as follows:

$$\ddot{x} + 2.25x = f(t)$$

The Laplace transform of the input is given by:

$$F(s) = \frac{s+0.5}{(s+0.5)^2+2.25}$$

On solving, we get the Laplace Transform of the output $x(t)$ as,

$$X(s) = \frac{s+0.5}{((s+0.5)^2+2.25)(s^2+2.25)}$$

We can visualise the output as follows,

```
# Returns the transfer function of Question 1 and 2
def transfer_func(freq, decay):
    denominator = np.polymul([1, 0, 2.25], [1, -2*decay, freq**2 + decay**2])
    numerator = np.poly1d([1, -decay])
    return sp.lti(numerator, denominator)

# Question 1
```

```
t, x = sp.impulse(transfer_func(1.5, -0.5), None, np.linspace(0, 50, 5001))
plt.plot(t, x)
plt.xlabel('t')
plt.ylabel('x')
plt.title('System Response with Decay = 0.5')
plt.savefig('Figure_1.png')
```

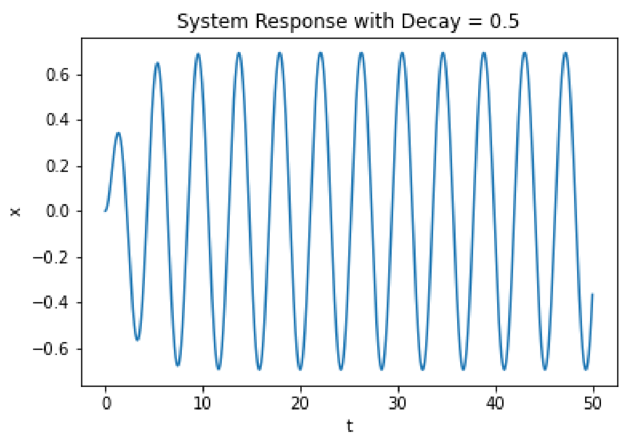

Figure 1: System Response with Decay = 0.5

3 Question 2

Let us analyse and plot the System Response for a smaller decay, i.e., decay = 0.05.

```
# Question 2
t, x = sp.impulse(transfer_func(1.5, -0.05), None, np.linspace(0, 50, 5001))
plt.plot(t, x)
plt.xlabel('t')
plt.ylabel('x')
plt.title('System Response with Decay = 0.05')
plt.savefig('Figure_2.png')
```

It can be noticed in the plot that amplitude varies and then reaches a steady value. This is because a smaller decay leads to the system reaching steady state/stabilising after a longer time compared the Question 1.

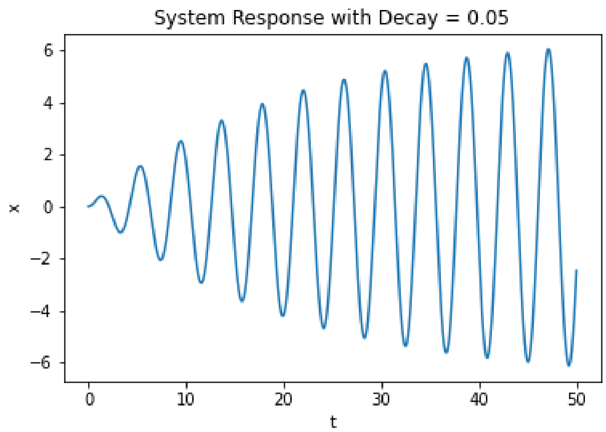

Figure 2: System Response with Decay = 0.05

4 Question 3

We will now analyse the system response for different values of frequency while keeping a constant decay. We will vary the frequency in the range 1.4 to 1.6.

```
# Question 3
frequencies = np.linspace(1.4, 1.6, 5)

for i in range(len(frequencies)):
    t, x = sp.impulse(transfer_func(frequencies[i], -0.05), None, np.linspace(0, 150, 5001))
    plt.plot(t, x)
    plt.xlabel('t')
    plt.ylabel('x')
    plt.title(f'System Response with Frequency = {frequencies[i]}')
    plt.show()
    plt.savefig(f'Figure_{i+3}')
```

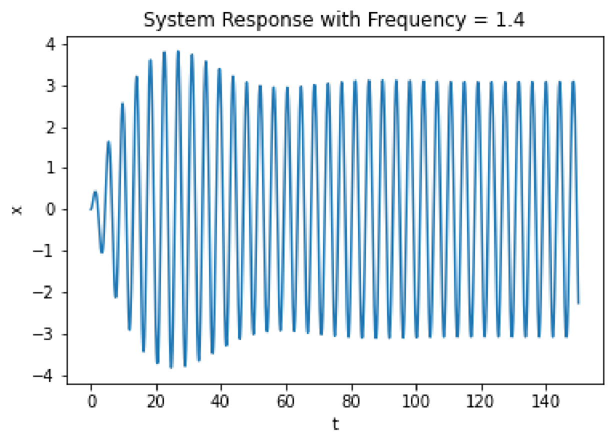

Figure 3: System Response with Frequency = 1.4

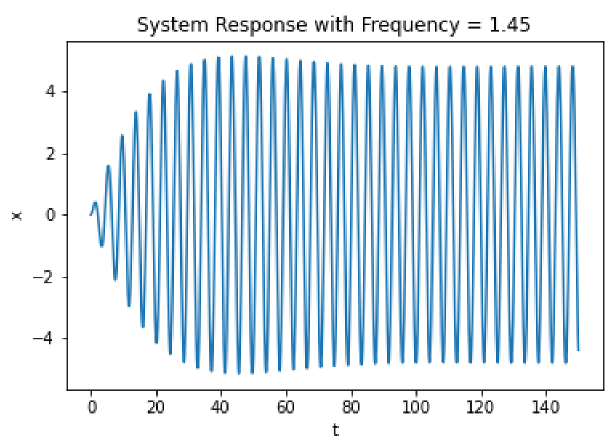

Figure 4: System Response with Frequency = 1.45

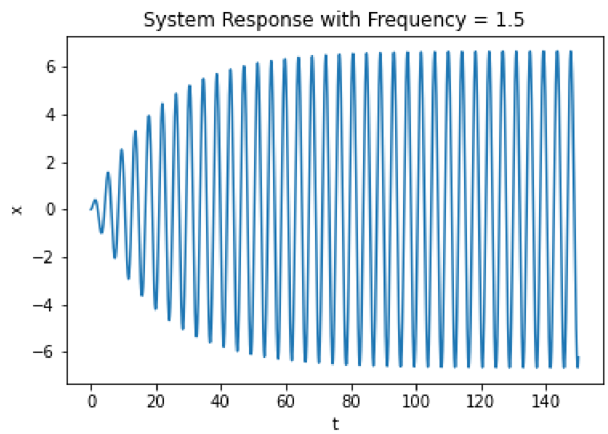

Figure 5: System Response with Frequency = 1.5

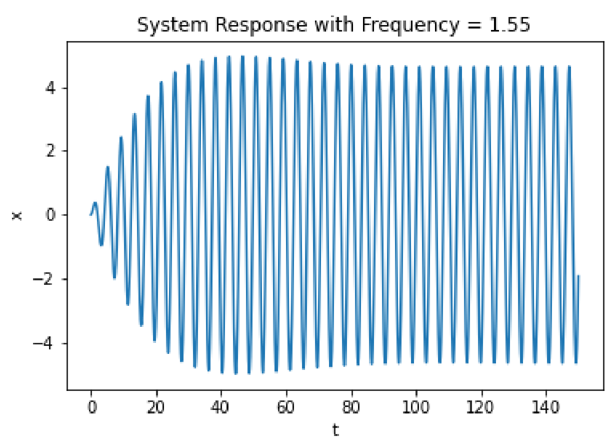

Figure 6: System Response with Frequency = 1.55

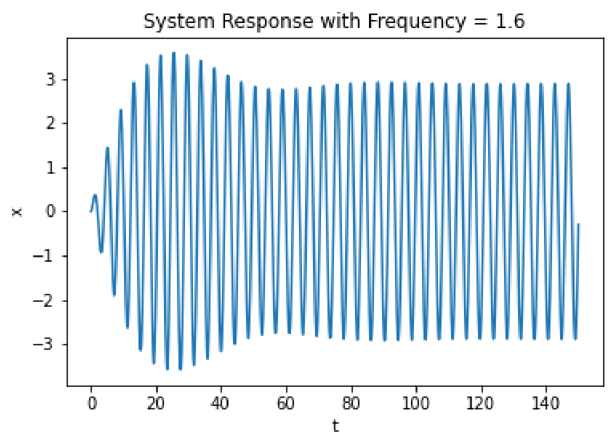

Figure 7: System Response with Frequency = 1.6

We notice that the amplitude is greatest for the case with Frequency = 1.5 because 1.5 is the natural frequency of the system.

5 Question 4

We now consider a coupled Differential system

$$\ddot{x} + (x - y) = 0$$

$$\ddot{y} + 2(y - x) = 0$$

with the initial conditions: $\dot{x}(0) = 0, \dot{y}(0) = 0, x(0) = 1, y(0) = 0$. Taking Laplace Transform and solving for $X(s)$ and $Y(s)$, We get:

$$X(s) = \frac{s^2 + 2}{s^3 + 3s}$$

$$Y(s) = \frac{2}{s^3 + 3s}$$

```
# Question 4

H_X = sp.lti([1,0,2],[1,0,3,0])
t,x = sp.impulse(H_X,None,np.linspace(0,50,5001))
plt.plot(t, x)

H_Y = sp.lti([2],[1,0,3,0])
t,y = sp.impulse(H_Y,None,np.linspace(0,50,5001))
plt.plot(t, y)
plt.legend(['X', 'Y'])
plt.title('System of Coupled Differential Equations')
plt.xlabel('t')
plt.ylabel('x,y')
plt.savefig('Figure_8.png')
```

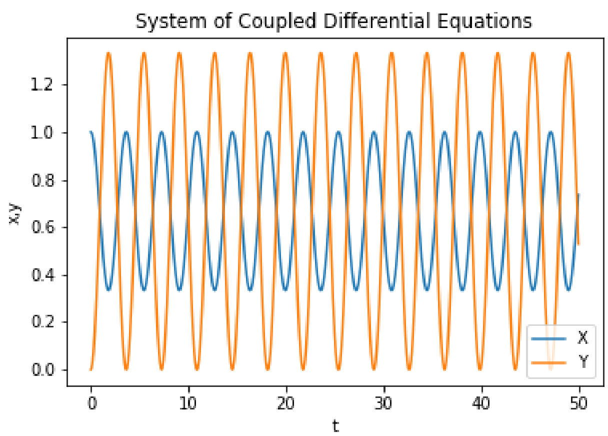

Figure 8: System of Coupled Differential Equations

6 Question 5

In this section, we will obtain and plot the magnitude and phase response of a Steady State Transfer Function of a Two Port Network. The transfer function of the given circuit is as follows:

$$H(s) = \frac{10^6}{s^2+100s+10^6}$$

```
# Question 5

# Magnitude plot
H = sp.lti([1],[1e-6*1e-6,100*1e-6,1]) # Transfer Function
w,S,phi=H.bode() # Generating magnitude and phase
plt.semilogx(w, S)
plt.title('Magintide Response')
plt.xlabel(r'$\omega$')
plt.ylabel(r'$|H(\omega)|$')
plt.savefig('Figure_9.png')
plt.show()

plt.semilogx(w, phi)
plt.title('Phase Response')
plt.xlabel(r'$\omega$')
plt.ylabel(r'$\angle(H(\omega))$')
plt.savefig('Figure_10.png')
plt.show()
```

It is clear from the transfer function that there are two left half plane poles and no zeroes. We can hence conclude that each pole contributes to a phase lag of 90^o and a slope change of -20dB/decade in the magnitude plot. The below plots are consistent with the above theory.

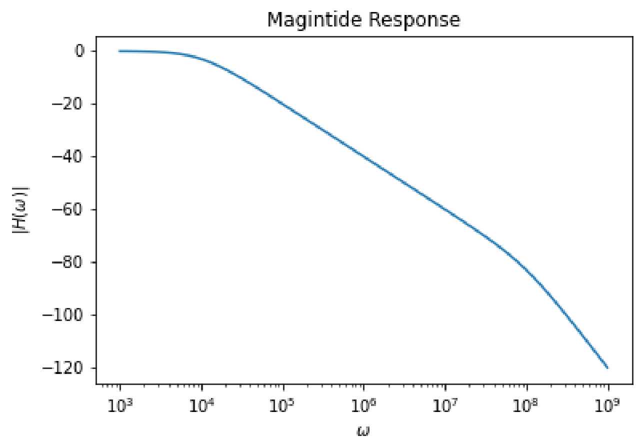

Figure 9: Magnitude Response

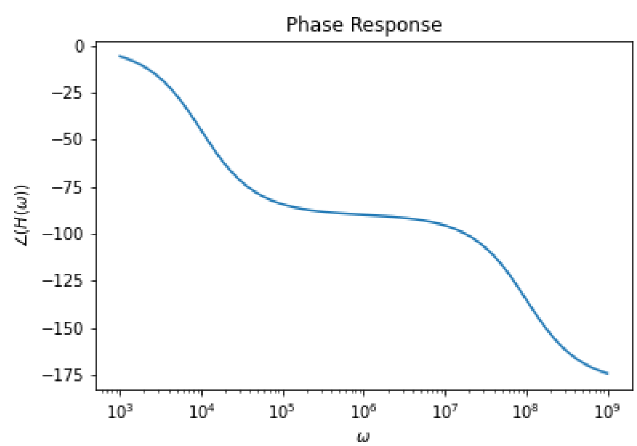

Figure 10: Phase Response

7 Question 6

In this section, we give an input to the system discussed in the previous section and analyse its output. The input is as follows:

$$v_i(t) = \cos\left(10^3 t\right) u(t) - \cos\left(10^6 t\right) u(t)$$

In the above we have two frequency components, one at a frequency of 10^3 and the other at a frequency of 10^6. Let us plot the output to analyse what happens to each of the frequency components.

Let us first plot the output in the range of 0 to $30\mu s$. This will help us notice the gain of the 10^6 frequency component of the input signal.

```
t = np.linspace(0,30*0.000001,1000)
vi = np.multiply(np.cos(1000*t)-np.cos(1000000*t),np.heaviside(t,0.5))
_,y,svec = sp.lsim(H,vi,t)

plt.plot(t, y)
plt.xlabel('t')
plt.ylabel(r'$v_o(t)$')
plt.savefig('Figure_11.png')
plt.show()
```

Next, we will plot the output in the range of 0 to $10ms$. This will help us notice the gain of the 10^3 frequency component.

```
t = np.linspace(0,10*0.001,100000)
vi = np.multiply(np.cos(1000*t)-np.cos(1000000*t),np.heaviside(t,0.5))
_,y2,svec = sp.lsim(H,vi,t)

plt.plot(t, y2)
plt.xlabel('t')
plt.ylabel(r'$v_o(t)$')
plt.savefig('Figure_12.png')
plt.show()
```

We can notice in the first plot that there is not much variation and hence we can conclude that the 10^6 frequency component has been attenuated significantly.

On the other hand, if we notice the second plot, there is a lot of variation and hence we can conclude that the 10^3 frequency component has not been attenuated.

The above observations can be explained if we take a closer look at the system we considered. The system we considered is an RLC Low Pass Filter and from the Bode Plot it is clear that the unity loop gain frequency is around 10^4 rad/s. Hence, there is unity gain for the 10^3 component whereas the 10^6 component has been attenuated significantly.

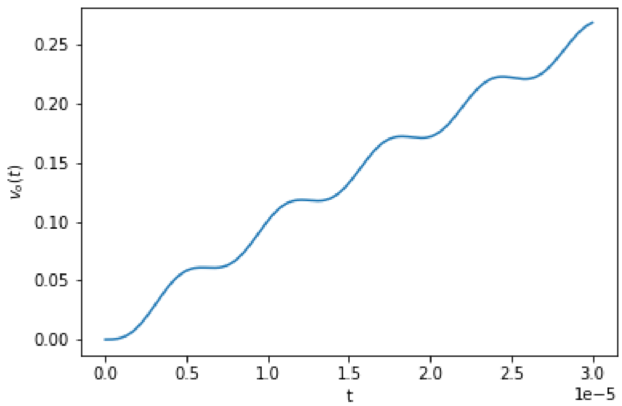

Figure 11: Output for $t < 30\mu s$

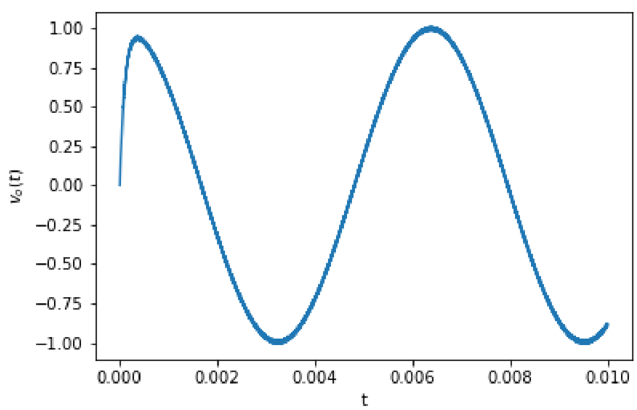

Figure 12: Output for $t < 10ms$

8 Conclusion

- In Question 1, we considered a Forced Oscillatory System and in Question 2 we considered the same system but with a different decay. We observed that a smaller decay results in a longer time to reach steady state.
- In Question 3, we considered the same system as in Question 2 but we varied the frequency in the range 1.4 to 1.6 and concluded that the largest amplitude was for frequency = 1.5 as that is the natural frequency of the system.
- In Question 4, we considered a system of coupled differential equation. We observed that both X and Y were completely out of phase.
- In Question 5, we considered an RLC circuit which behaves as a low pass filter and we plotted its Magnitude and Phase Response.
- In Question 6, we considered the same system as in Question 5, but we considered an input to the system with two frequency components. One of the frequency components was attenuated while the other was not, which was consistent with the fact that we considered a low pass filter.