Assignment 8

Sagnik Ghosh

May 2021

1 Introduction

In this week's assignment, we will explore the Digital Fourier Transform (DFT) using NumPy and Matplotlib. We will analyse the DFT of some known standard analog function by sampling them and plotting their transforms.

2 Random Noise

We find the transform and then the inverse transform of random noise and find the norm of their difference.

```
# Example 1
x=rand(100)
X=fft(x)
y=ifft(X)
c_[x,y]
print(abs(x-y).max())
```

```
3.421641777912936e-16
```

We notice that the difference is of the order of 10^{-16} which is due to the computer's limited precision.

3 Sinusoid

In this section, we plot the DFT of $sin(5t)$. We do this as follows:

```
# Spectrum of sin(5t) without wrapping
x=linspace(0,2*pi,128)
y=sin(5*x)
Y=fft(y)
figure()
subplot(2,1,1)
plot(abs(Y),lw=2)
```

```
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of $\sin(5t)$")
grid(True)
subplot(2,1,2)
plot(unwrap(angle(Y)),lw=2)
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$k$",size=16)
grid(True)
show()
```

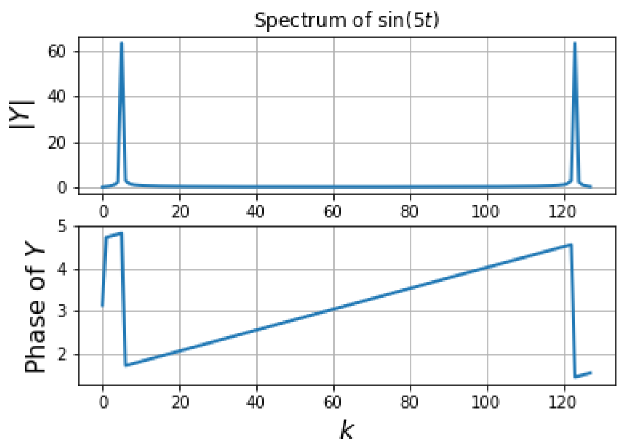

Figure 1: DFT of $sin(5t)$

We can notice a few inconsistencies in the above plot such as:

- Position of the spikes
- Magnitude of the spikes
- Frequency axis is not in place

The above inconsistencies can be fixed by using the $fftshift()$ function as follows,

```
# Spectrum of sin(5x) after fft shift
x=linspace(0,2*pi,129);x=x[:-1]
y=sin(5*x)
Y=fftshift(fft(y))/128.0
w=linspace(-64,63,128)
figure()
subplot(2,1,1)
plot(w,abs(Y),lw=2)
xlim([-10,10])
```

```
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of $\sin(5t)$")
grid(True)
subplot(2,1,2)
plot(w,angle(Y),'ro',lw=2)
ii=where(abs(Y)>1e-3)
plot(w[ii],angle(Y[ii]),'go',lw=2)
xlim([-10,10])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$k$",size=16)
grid(True)
show()
```

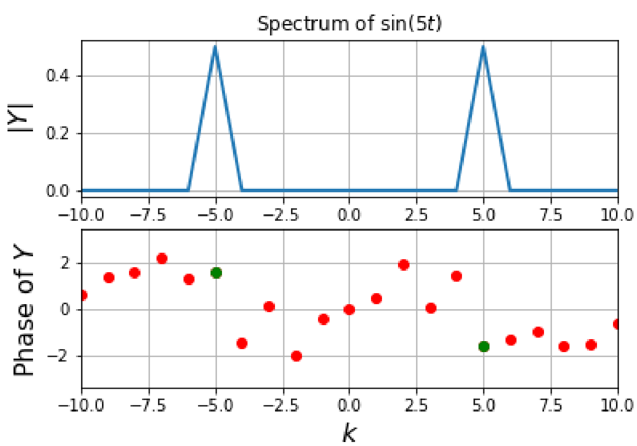

Figure 2: DFT of $sin(5t)$ after wrapping

4 Amplitude Modulation (AM)

In this section, we will analyse an Amplitude Modulated Signal of the form,

$$f(t) = (1 + 0.1\cos(t))\cos(10t)$$

We can expect one discernible main peak and two smaller peaks on either side of the main peak. We can find and plot the DFT of the above function as follows:

```
# AM Modulation
t=linspace(0,2*pi,129);t=t[:-1]
y=(1+0.1*cos(t))*cos(10*t)
Y=fftshift(fft(y))/128.0
w=linspace(-64,63,128)
figure()
subplot(2,1,1)
plot(w,abs(Y),lw=2)
xlim([-15,15])
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of $\left(1+0.1\cos\left(t\right)\right)\cos\left(10t\right)$")
grid(True)
subplot(2,1,2)
plot(w,angle(Y),'ro',lw=2)
xlim([-15,15])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$\omega$",size=16)
grid(True)
show()
```

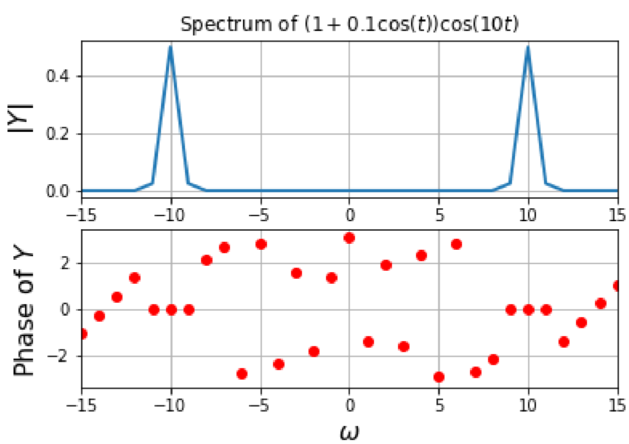

Figure 3: Spectrum of $(1 + 0.1\cos(t))\cos(10t)$

Note that we expected two side bands but we only got one broad main band. We guess that this happens due to a low frequency resolution. We can fix this as follows,

```
t=linspace(-4*pi,4*pi,513);t=t[:-1]
y=(1+0.1*cos(t))*cos(10*t)
Y=fftshift(fft(y))/512.0
w=linspace(-64,64,513);w=w[:-1]
figure()
subplot(2,1,1)
plot(w,abs(Y),lw=2)
xlim([-15,15])
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of $\left(1+0.1\cos\left(t\right)\right)\cos\left(10t\right)$")
grid(True)
subplot(2,1,2)
plot(w,angle(Y),'ro',lw=2)
xlim([-15,15])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$\omega$",size=16)
grid(True)
show()
```

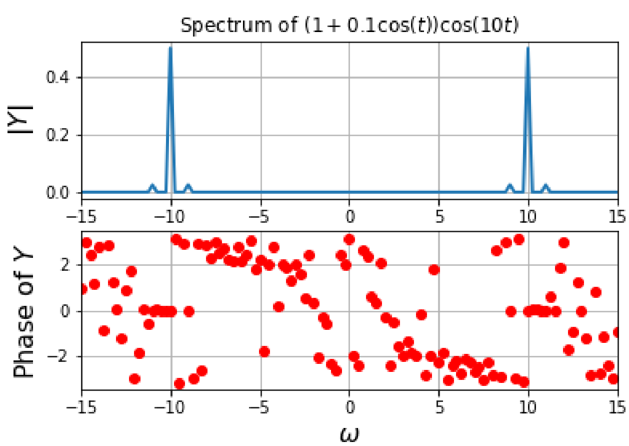

Figure 4: Spectrum of $(1 + 0.1\cos(t))\cos(10t)$

5 Spectra of $sin^3(t)$ and $cos^3(t)$

In this section, we will compute and plot the spectra of $sin^3(t)$ and $cos^3(t)$. Note that the above functions can be expanded as follows,

$$\sin^3 t = \frac{3\sin(t)-\sin(3t)}{4}$$
$$\cos^3 t = \frac{3\cos(t)+\cos(3t)}{4}$$

From the above identities, we can expect to have four peaks in the DFTs of the above functions with one pair having thrice the magnitude as the other pair.

DFT of $sin^3(t)$ can be plotted as follows,

```
# Spectrum of sin^3(t)
t=linspace(-4*pi,4*pi,513);t=t[:-1]
y=sin(t)**3
Y=fftshift(fft(y))/512.0
w=linspace(-64,64,513);w=w[:-1]
figure()
subplot(2,1,1)
plot(w,abs(Y),lw=2)
xlim([-15,15])
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of $sin^3(t)$")
grid(True)
subplot(2,1,2)
ii=where(abs(Y)>1e-3)
plot(w[ii],angle(Y[ii]),'go',lw=2)
xlim([-15,15])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$k$",size=16)
grid(True)
savefig('Figure5.png')
show()
```

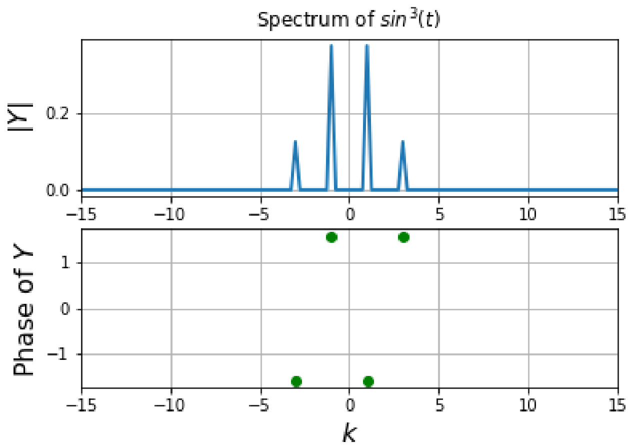

Figure 5: Spectrum of $sin^(t)$

The spectrum of $cos^3(t)$ can be plotted as follows:

```
# Spectrum of cos^3(t)
t=linspace(-4*pi,4*pi,513);t=t[:-1]
y=cos(t)**3
Y=fftshift(fft(y))/512.0
w=linspace(-64,64,513);w=w[:-1]
figure()
subplot(2,1,1)
plot(w,abs(Y),lw=2)
xlim([-15,15])
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of $cos^3(t)$")
grid(True)
subplot(2,1,2)
ii=where(abs(Y)>1e-3)
plot(w[ii],angle(Y[ii]),'go',lw=2)
xlim([-15,15])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$k$",size=16)
grid(True)
show()
```

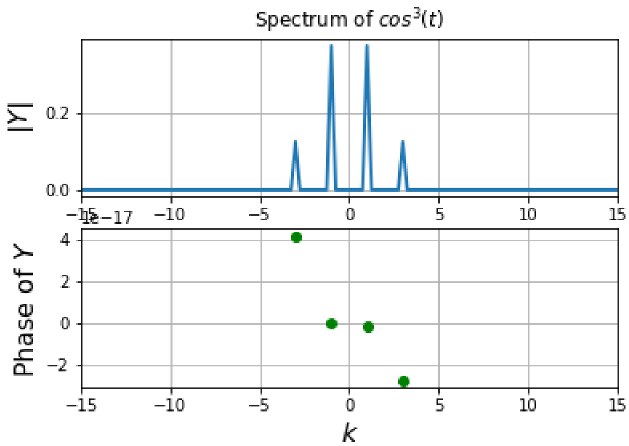

Figure 6: Spectrum of $cos^3(t)$

6 Frequency Modulation

In this section, we compute and plot the DFT of the following Frequency Modulated Waveform:

$$f(t) = \cos(20t + 5\cos(t))$$

This can be done as follows,

```
# Spectrum of FM
t=linspace(-4*pi,4*pi,513);t=t[:-1]
y=cos(20*t+5*cos(t))
Y=fftshift(fft(y))/512.0
w=linspace(-64,64,513);w=w[:-1]
figure()
subplot(2,1,1)
plot(w,abs(Y),lw=2)
xlim([-30,30])
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of Frequency Modulated function")
grid(True)
subplot(2,1,2)
ii=where(abs(Y)>1e-3)
plot(w[ii],angle(Y[ii]),'go',lw=2)
xlim([-30,30])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$k$",size=16)
grid(True)
show()
```

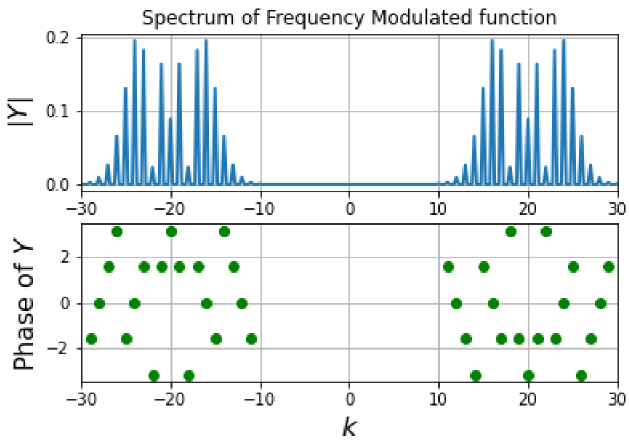

Figure 7:

We can notice that the number of peaks has increased and the height of the side peaks have increased.

7 Gaussian Case

In this section, we will compute the DFT of the Gaussian Distribution of the form:

$$f(t) = e^{-\frac{t^2}{2}}$$

The Continuous Time Fourier Transform of the above signal is as follows:

$$F(j\omega) = \frac{1}{\sqrt{2\pi}} e^{\frac{-\omega^2}{2}}$$

We can plot the estimated and the expected DFTs of the Gaussian Distribution as follows,

```
# Spectrum of Gaussian Estimated
t=linspace(-4*pi,4*pi,513);t=t[:-1]
y=exp(-(t*t)/2)
Y=fftshift(abs(fft(ifftshift(y))))/512.0
Y = Y*sqrt(2*pi)/max(Y)
w=linspace(-64,64,513);w=w[:-1]
figure()
subplot(2,1,1)
plot(w,abs(Y),lw=2)
xlim([-10,10])
ylabel(r"$|Y|$",size=16)
title(r"Spectrum of $exp(-t^2/2)$")
```

```
grid(True)
subplot(2,1,2)
ii=where(abs(Y)>1e-3)
plot(w[ii],angle(Y[ii]),'go',lw=2)
xlim([-10,10])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$k$",size=16)
grid(True)
show()

# Spectrum of Gaussian True
t=linspace(-4*pi,4*pi,513);t=t[:-1]
y=exp(-(t*t)/2)
Y=fftshift(abs(fft(y)))/512.0
Y = Y*sqrt(2*pi)/max(Y)
w=linspace(-64,64,513);w=w[:-1]
figure()
subplot(2,1,1)
plot(w,abs(Y),lw=2)
xlim([-10,10])
ylabel(r"$|Y|$",size=16)
title(r"True Spectrum of $exp(-t^2/2)$")
grid(True)
subplot(2,1,2)
ii=where(abs(Y)>1e-3)
plot(w[ii],angle(Y[ii]),'go',lw=2)
xlim([-10,10])
ylabel(r"Phase of $Y$",size=16)
xlabel(r"$k$",size=16)
grid(True)
show()
```

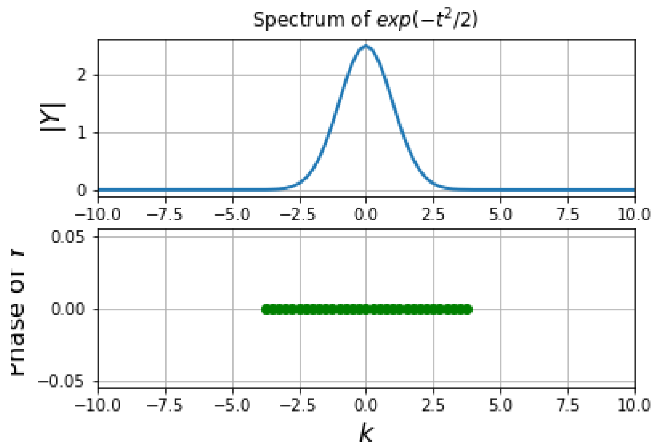

Figure 8:

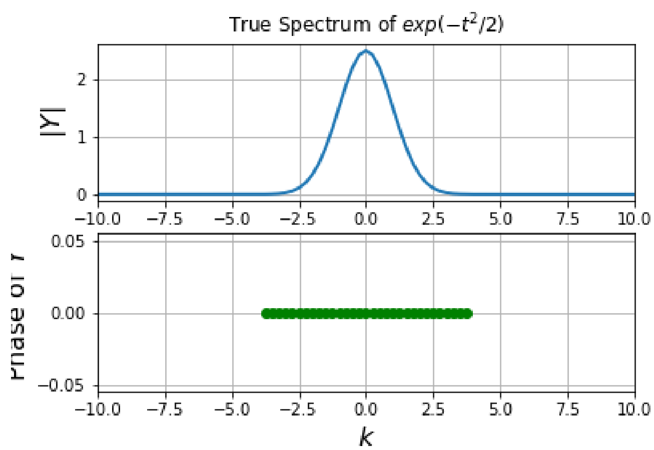

Figure 9:

8 Conclusion

- In the Random Noise Case, the difference between the original function and the inverse transform of its DFT was of the order of 10^{-16} because Python offers accuracy upto 16 digits.
- In the case of the sinusoid, when we plotted the DFT initially, we noticed some inconsistencies because the frequency axis was between 0 to 2π and

the magnitude of the peaks were also inconsistent. These issues were fixed by using fftshift() and dividing the transform by the number of samples.

- In the case of the AM wave, we expected one main peak and 2 sides peaks and instead we got one broad peak. This was fixed by increasing the number of samples.
- In the case of $sin^3(t)$ and $cos^3(t)$, we expected to have four peaks in the DFTs of the above functions with one pair having thrice the magnitude as the other pair and the plots were consistent with the above.
- In the case of the FM wave, we got an increased number of peaks and the heights of the side peaks increased.
- For the Gaussian case, we expected to get a Gaussian as the DFT and the plots were consistent with that.