

# Parametrized Quantum Circuits

## Tomography and Compression

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# One-shot introduction to Quantum Computing

Terms and conditions apply!

# One Qubit!

- ▶ We shall deal with qubits (two-level systems)
- ▶ Can be represented as a complex linear combination of two orthogonal basis vectors

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$$

such that

$$|\alpha|^2 + |\beta|^2 = 1$$

- ▶ Measurement destroys quantum information!

# Multiple qubits!

- ▶ Basis states become tensor products of the single qubit basis states.
- ▶ For two-qubit states,  $|00\rangle = |0\rangle \otimes |0\rangle$ ,  $|01\rangle$ ,  $|10\rangle$ ,  $|11\rangle$
- ▶ Example:

$$\frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

- ▶ Entanglement: states that are **not** product states

# Evolution, Gates!

- ▶ Unitary matrices!
- ▶ Therefore, linear, reversible (will be used soon!)
- ▶ Examples:

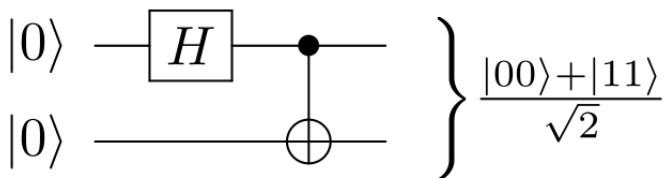
$$H|0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \quad H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$$

$$X|0\rangle = |1\rangle \quad X|1\rangle = |0\rangle$$

$$Z|0\rangle = |0\rangle \quad Z|1\rangle = (-1)|0\rangle$$

- ▶ Multi-qubit gates: CNOT, Flips second qubit if first qubit is 1, does nothing if not.
- ▶ You get circuits when gates act one after the other.

## An example of a circuit<sup>1</sup>



$$|00\rangle \rightarrow \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \otimes |0\rangle \rightarrow \frac{1}{\sqrt{2}} (|00\rangle + |10\rangle) \rightarrow \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$$

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<sup>1</sup>Wikipedia: [https://commons.wikimedia.org/wiki/File:The\\_Hadamard-CNOT\\_transform\\_on\\_the\\_zero-state.png](https://commons.wikimedia.org/wiki/File:The_Hadamard-CNOT_transform_on_the_zero-state.png)

# Parametrized Gates

- ▶ Remember title of the talk?
- ▶ **Parametrized** quantum circuits.
- ▶ X-Rotation, Z-Rotation

$$RX(\theta) = \begin{bmatrix} \cos(\theta/2) & -i \sin(\theta/2) \\ -i \sin(\theta/2) & \cos(\theta/2) \end{bmatrix}$$

$$RZ(\theta) = \begin{bmatrix} \exp(-i\theta/2) & 0 \\ 0 & \exp(-i\theta/2) \end{bmatrix}$$

- ▶  $RX(\pi) = -iX$ ,  $RZ(\pi) = -iZ$

# Why do people care?

- ▶ Several speed-ups (\*) over classical algorithms are already known
- ▶ **polynomial 'time'** prime factoring (Shor's algorithm):
  - ▶ Hardness assumption of prime factoring is the basis for RSA cryptosystem
- ▶ **sub**-linear search (Grover's algorithm)
  - ▶ Unstructured search has  $\Theta(n)$  complexity; Grover search has  $O(\sqrt{n})$  complexity



## Half-shot introduction to QML

# Many flavors

- ▶ Speeding up classical machine learning algorithms using quantum algorithms for linear algebra: Harrow-Hassidim-Lloyd for solving linear equations, singular value estimation etc.
- ▶ Problems that are classically hard, eg Traveling Salesman, QAOA (quantum approximate optimization algorithm) by Farhi-Goldstone-Gutman
- ▶ Solving quantum problems, eg quantum state tomography
- ▶ Running famous quantum algorithms on near-term quantum hardware
  - ▶ Decoherence, low depth and other issues.

# Parametrized quantum algorithms

**Key idea:** Let's use a parametrized circuit, optimize the parameters, and use the trained circuit to solve the problem.

Typically lower depth circuits for many important problems.

# Computing gradients

- ▶ In many cases, the **same** quantum circuit can be used to compute the function **and** the gradient of the function
- ▶ Uses the **parameter shift rule**
- ▶ This is a generalization of...

$$f(x) = \sin(x) \Rightarrow f'(x) = \frac{1}{2} \sin\left(x + \frac{\pi}{2}\right) - \frac{1}{2} \sin\left(x - \frac{\pi}{2}\right)$$

Parametrized quantum algorithm for quantum  
state tomography

# What is quantum state tomography?

Given several copies of a quantum state, learn (a classical description of) the state.

$$|\psi\rangle = \alpha |0\rangle + \beta |1\rangle$$

We want to learn  $\alpha$  and  $\beta$ .

Metric is **fidelity**, in this case the **inner product** between the target and the reconstruction. We want this to be **close to 1**.

## A very simple idea<sup>2</sup>

- ▶ Learn the parameters of a circuit that maps the state to a **known** state
- ▶ Then, use the reverse of the circuit to find what the original state was

$$U|\psi\rangle = |0\rangle \Rightarrow |\psi\rangle = U^\dagger |0\rangle$$

- ▶ Single-Shot Measurement Learning

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<sup>2</sup>Sang Min Lee, Jinhyoung Lee, and Jeongho Bang. “Learning unknown pure quantum states”. In: *Phys. Rev. A* 98 (5 Nov. 2018), p. 052302.

# Single-Shot Measurement Learning

Run the circuit, measure the obtained state, then update the parameters

Parameter update rule is given by

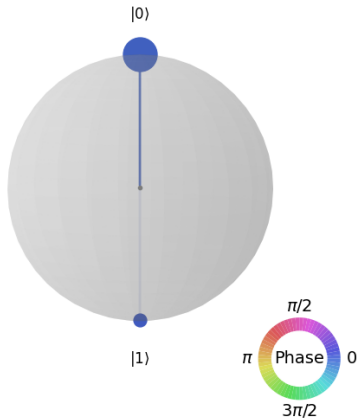
$$M_S^{(n+1)}, \mathbf{p}^{(n+1)} = \begin{cases} M_S^{(n)} + 1, \mathbf{p}^{(n)} & \text{if SUCCESS} \\ 0, \mathbf{p}^{(n)} + \alpha(M_S^{(n)} + 1)^{-\beta} \mathbf{r} & \text{if FAILURE} \end{cases}$$

Keep doing this until  $M_S^{(n)} = M_{\text{stopping}}$

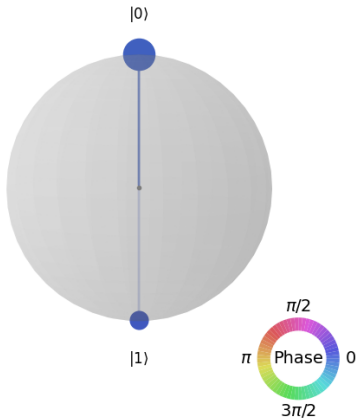
**Gradient free!**



# Simulation results (target state)



# Simulation results (obtained state)



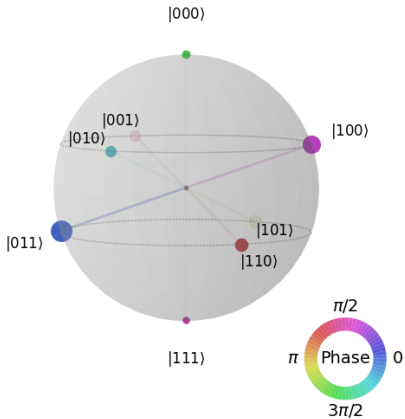
## Simulation results

- ▶ Fidelity was 0.97
- ▶ **Doesn't work** for multiple qubits, unless it's product state

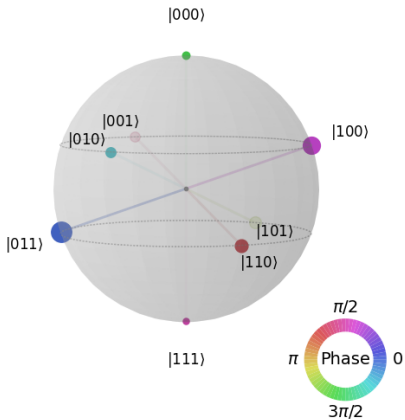
## Different approach

- ▶ We can use gradient-based methods directly, using the parameter shift rule to compute gradients.
- ▶ Can use any of your favourite optimizers - gradient descent, Adagrad, Adam...

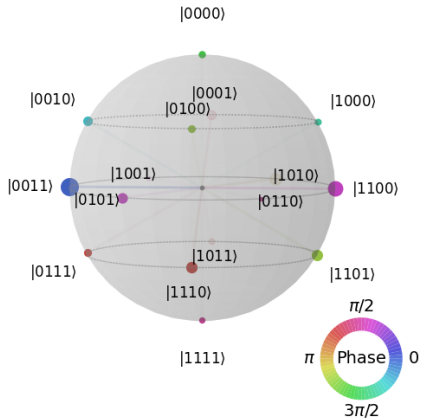
## Simulation results 3 qubits (target state)



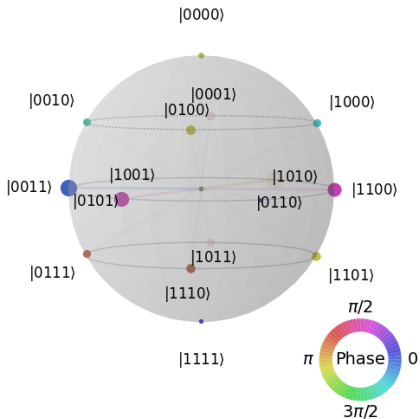
## Simulation results 3 qubits (obtained state) [Fidelity - 0.999]



## Simulation results 4 qubits (target state)



## Simulation results 4 qubits (obtained state) [Fidelity - 0.867]





## Further comments

- ▶ Fidelity starts going down for more qubits
- ▶ Most likely reason - barren plateaus
- ▶ To try: other non-gradient based optimization methods, eg particle swarm optimization

Thanks!