

2015

November

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FRIDAY

Week - 48

$$X^n = (X_1, \dots, X_n) \text{ n iid } \text{Bern}(1/2)$$

$$Z^n \sim n \text{ iid } \text{Bern}(\alpha) . \quad 0 \leq \alpha \leq 1/2$$

$$Y^n = X^n \oplus Z^n$$

$$\Omega = \{0, 1\}$$

$$\Omega_n = \{0, 1\}^n$$

$$b : \Omega_n \rightarrow \Omega$$

$$b^{-1}(0) = \{x^n \in \Omega_n \mid b(x^n) > 0\}.$$

Lexicographical ordering . \prec_L

$$x^k \prec_L \tilde{x}^k$$

Notes

iff $x_j \prec \tilde{x}_j$ for some j and $x_i = \tilde{x}_i \forall i \neq j$

January					February					March					April					May					June					
M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S			
1	2	3	4				1	30	31		1			1	2	3	4	5		1	2	3	1	2	3	4	5	6	7	
5	6	7	8	9	10	11	2	3	4	5	6	7	8	2	3	4	5	6	7	8	9	10	11	12	13	14				
12	13	14	15	16	17	18	9	10	11	12	13	14	15	9	10	11	12	13	14	15	16	17	18	19	20	21				
19	20	21	22	23	24	25	16	17	18	19	20	21	22	16	17	18	19	20	21	22	23	24	25	26	18	19	20	21	22	
26	27	28	29	30	31		23	24	25	26	27	28	29	23	24	25	26	27	28	29	27	28	29	30	31	25	26	27	28	29

Look about you. Take hold of the things that are here. Let them talk to you. You learn to talk to them. — George Washington Carver

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SATURDAY 28

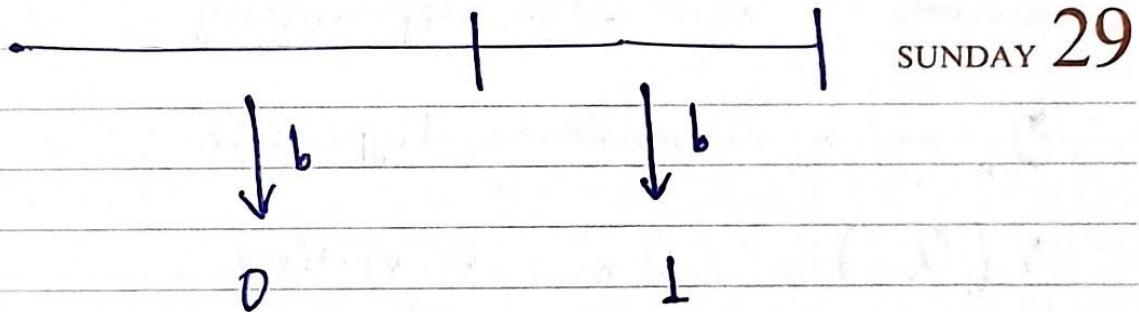
$L_k(M)$ = initial segment of size M in the lex ordering on $\{0, 1\}^k$.

$$L_3(4) = \{000, 001, 010, 011\}$$

(essentially converting $[M] \rightarrow$ binary).

b is lex iff

$$b^{-1}(0) = L_n(|b^{-1}(0)|)$$



Notes

July					August					September					October					November					December																
M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S														
1	2	3	4	5			31		1	2	1	2	3	4	5	6	1	2	3	4	30		1	1	2	3	4	5	6												
6	7	8	9	10	11	12	3	4	5	6	7	8	9	10	11	12	13	5	6	7	8	9	10	11	2	3	4	5	6	7	8	9	10	11	12	13					
13	14	15	16	17	18	19	10	11	12	13	14	15	16	17	18	19	20	12	13	14	15	16	17	18	9	10	11	12	13	14	15	14	15	16	17	18	19	20			
20	21	22	23	24	25	26	17	18	19	20	21	22	23	21	22	23	24	25	26	27	19	20	21	22	23	24	25	16	17	18	19	20	21	22	21	22	23	24	25	26	27
27	28	29	30	31			24	25	26	27	28	29	30	28	29	30			26	27	28	29	30	31		23	24	25	26	27	28	29	28	29	30	31					

As soils are depleted, human health, vitality and intelligence go with them. ~ Louis Bromfield

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MONDAY

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2015
NovemberConjecture 2

for any given n and fixed bias $\text{IP} [b(x^n) = 0]$
 satisfying $H [\text{IP} [b(x^n) = 0]] \geq 1 - H(\alpha)$,
the conditional entropy $H(b(x^n) | y^n)$ is minimized
when b is lex.

→ structure

Conjecture 3 $b : \mathcal{S}_n \rightarrow \mathcal{S}$ is lex then

$$H(b(x^n) | y^n) \geq H(b(x^n)) - H(\alpha).$$

→ inequality

Conjecture 2 and edge-isoperimetry $Q_n \Leftarrow n\text{-dimensional hypercube}$ $V(Q_n) = \mathcal{S}_n \Leftarrow \text{vertices}$. $S \subseteq V(Q_n)$

Notes $\partial S \Leftarrow$ number of edges that need to be deleted
 to disconnect S from any vertex not in S .

January	February	March	April	May	June																
M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	
1 2 3 4		1 30 31	1	1 2 3 4 5	1 2 3 4 5 6 7	1 2 3 4 5	1 2 3	1 2 3 4 5 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7	1 2 3 4 5 6 7
5 6 7 8 9 10 11	2 3 4 5 6 7 8	2 3 4 5 6 7 8	6 7 8 9 10 11 12	4 5 6 7 8 9 10	11 12 13 14 15 16 17	15 16 17 18 19 20 21	22 23 24 25 26 27 28	18 19 20 21 22 23 24	20 21 22 23 24 25 26	11 12 13 14 15 16 17	15 16 17 18 19 20 21	22 23 24 25 26 27 28	1 2 3 4 5 6 7	8 9 10 11 12 13 14	15 16 17 18 19 20 21	22 23 24 25 26 27 28	29 30				
12 13 14 15 16 17 18	9 10 11 12 13 14 15	9 10 11 12 13 14 15	13 14 15 16 17 18 19	11 12 13 14 15 16 17	15 16 17 18 19 20 21	18 19 20 21 22 23 24	25 26 27 28 29 30	27 28 29 30	20 21 22 23 24 25 26	18 19 20 21 22 23 24	11 12 13 14 15 16 17	15 16 17 18 19 20 21	22 23 24 25 26 27 28	1 2 3 4 5 6 7	8 9 10 11 12 13 14	15 16 17 18 19 20 21	22 23 24 25 26 27 28				
19 20 21 22 23 24 25	16 17 18 19 20 21 22	16 17 18 19 20 21 22	20 21 22 23 24 25 26	18 19 20 21 22 23 24	25 26 27 28 29 30	29 30															
26 27 28 29 30 31	23 24 25 26 27 28	23 24 25 26 27 28 29	27 28 29 30																		

A study has shown that there are possibly over 30 million species of insects dwelling in the canopies of tropical forests.

Theorem 2 Harper's edge. Isoperimetric Inequality.

$$S \subseteq V(Q_n) \quad |S| = k$$

$$\Rightarrow |\partial(S)| \geq |\partial(L_n(k))|$$

Proof uses compression operators

$$\mathcal{I} \subseteq [n] \quad |\mathcal{I}| = k$$

$$\mathcal{I} = \{i_1, \dots, i_k\} \text{ a lexicographic order}$$

for $B \subseteq \Omega_n$ and

$$x^n \text{ s.t. } x_i = 0 \forall i \in \mathcal{I}$$

the \mathcal{I} section of B at x^n is

$$B_{\mathcal{I}}(x^n) = \left\{ z^k; y^n \in B, y_i = \begin{cases} z_j & \text{if } i = i_j \in \mathcal{I} \\ x_i & \text{otherwise} \end{cases} \right\}$$

Notes

July	August	September	October	November	December								
M	T	W	T	F	S	S	M	T	W	T	F	S	S
1 2 3 4 5	31	1 2 3 4 5 6	1 2 3 4 5 6 7	1 2 3 4 5 6 7 8	1 2 3 4 5 6 7 8 9	1 2 3 4 5 6 7 8 9 10 11	30	1	1 2 3 4 5 6	1 2 3 4 5 6 7 8	7 8 9 10 11 12 13	14 15 16 17 18 19 20	21 22 23 24 25 26
6 7 8 9 10 11 12	3 4 5 6 7 8 9	7 8 9 10 11 12 13	5 6 7 8 9 10 11 12 13	2 3 4 5 6 7 8 9 10 11 12 13	9 10 11 12 13 14 15 16 17 18	9 10 11 12 13 14 15 16 17 18	16 17 18 19 20 21 22	21 22 23 24 25 26 27	28 29 30	24 25 26 27 28 29 30	28 29 30 31	28 29 30 31	28 29 30 31
13 14 15 16 17 18 19	10 11 12 13 14 15 16	14 15 16 17 18 19 20	12 13 14 15 16 17 18 19 20	19 20 21 22 23 24 25	16 17 18 19 20 21 22	23 24 25 26 27 28 29	21 22 23 24 25 26 27	28 29 30 31	28 29 30 31	28 29 30 31	28 29 30 31	28 29 30 31	28 29 30 31
20 21 22 23 24 25 26	17 18 19 20 21 22 23	21 22 23 24 25 26 27	19 20 21 22 23 24 25	16 17 18 19 20 21 22	23 24 25 26 27 28 29	28 29 30 31	21 22 23 24 25 26 27	28 29 30 31	28 29 30 31	28 29 30 31	28 29 30 31	28 29 30 31	28 29 30 31
27 28 29 30 31	24 25 26 27 28 29 30	28 29 30	26 27 28 29 30 31	23 24 25 26 27 28 29	28 29 30 31	28 29 30 31	23 24 25 26 27 28 29	28 29 30 31	28 29 30 31	28 29 30 31	28 29 30 31	28 29 30 31	28 29 30 31

"What you do speaks so loudly that I cannot hear what you say" — Ralph Waldo Emerson"

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WEDNESDAY

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2015
DecemberExample

$$B = \{000, \underset{\sim}{001}, 011, \underset{\sim}{101}\}$$

$$B(001) = \{0, 1\}$$

 $\{1\}$
 $\mathcal{X} = \{1\}$

$$x_i = 0 \text{ when } i=1 \\ \text{satisfied}$$

$$y_i = \begin{cases} z_j & \text{if } i = i_j \in \mathcal{X} \\ x_i & \text{otherwise} \end{cases}$$

0 1

$$B_{\{2\}}(100) = \{\emptyset\}$$

$$B_{\{1, 2\}}(000) = \{00\}$$

Notes

$$B_{\{1, 2\}}(001) = \{00, 01, 10\}$$

January	February	March	April	May	June														
M	T	W	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	
1 2 3 4		1 30 31	1	1 2 3 4 5		1 2 3	1 2 3 4 5 6 7						1 2 3	1 2 3 4 5 6 7					
5 6 7 8 9 10 11	2 3 4 5 6 7 8	2 3 4 5 6 7 8	6 7 8 9 10 11 12	4 5 6 7 8 9 10		8 9 10 11 12 13 14	8 9 10 11 12 13 14						15 16 17 18 19 20 21						
12 13 14 15 16 17 18	9 10 11 12 13 14 15	9 10 11 12 13 14 15	13 14 15 16 17 18 19	11 12 13 14 15 16 17		15 16 17 18 19 20 21	15 16 17 18 19 20 21						22 23 24 25 26 27 28						
19 20 21 22 23 24 25	16 17 18 19 20 21 22	16 17 18 19 20 21 22	20 21 22 23 24 25 26	18 19 20 21 22 23 24		22 23 24 25 26 27 28	22 23 24 25 26 27 28						29 30						
26 27 28 29 30 31	23 24 25 26 27 28	23 24 25 26 27 28 29	27 28 29 30	25 26 27 28 29 30 31		29 30													

As an algorithm what it states is the following.

$$B_{\mathcal{I}}(x)$$

the coordinates of x that appear in \mathcal{I} are all zero.

$\Rightarrow \mathcal{I}$ is a set of indices.

\Rightarrow match whatever is not in \mathcal{I} with all the members of B .

\Rightarrow for members of B that have such a match, pick up what they have in the indices in \mathcal{I} .

\mathcal{I} compression of B

$$= (C_{\mathcal{I}}(B))_{\mathcal{I}}(x^n) = L_K(|B_{\mathcal{I}}(x^n)|) \forall x^n$$

Notes

July					August					September					October					November					December								
M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S						
1	2	3	4	5			31		1	2	3	4	5	6		1	2	3	4	30		1	2	3	4	5	6						
6	7	8	9	10	11	12		3	4	5	6	7	8	9	7	8	9	10	11	12	2	3	4	5	6	7	8	9	10	11	12	13	
13	14	15	16	17	18	19		10	11	12	13	14	15	16	14	15	16	17	18	19	10	11	12	13	14	15	14	15	16	17	18	19	20
20	21	22	23	24	25	26		17	18	19	20	21	22	23	21	22	23	24	25	26	19	20	21	22	23	24	21	22	23	24	25	26	27
27	28	29	30	31				24	25	26	27	28	29	30	28	29	30	26	27	28	29	31	23	24	25	26	27	28	29	28	29	30	31

$c_{\mathcal{I}}$ replaces each \mathcal{I} -section of B with an initial segment of the lex order.

B is \mathcal{I} compressed if $c_{\mathcal{I}}(B) = B$.

$c_{\mathcal{I}}(B)$ is always \mathcal{I} compressed.

Example $B = \{000, 001, 011, 101\}$

$$c_{\{\mathcal{I}\}}(B) = \underbrace{1}_{K} \underbrace{1}_{L} \underbrace{1}_{X^B} \underbrace{1}_{x^{n^n}} \underbrace{1}_{Y},$$

~~FB is doing this~~

Algorithmically,

- match indices in \mathcal{I} with the vectors in B
- wherever they match, replace with lex order

January					February					March					April					May					June				
M	T	W	T	F	S	M	T	W	T	F	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S				
1	2					1	2	3	4	5	6	7	8	9	10	11	12	13	1	2	3	4	5	6	7				
8	9	10	11			2	3	4	5	6	7	8	9	10	11	12	13	14	1	2	3	4	5	6	7				
15	16	17	18			9	10	11	12	13	14	15	16	17	18	19	20	21	8	9	10	11	12	13	14				
22	23	24	25			16	17	18	19	20	21	22	23	24	25	26	27	28	11	12	13	14	15	16	17				
29	30	31				23	24	25	26	27	28	29	30	31	25	26	27	28	29	30	31	29	30						
5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31			
12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	15	16	17	18	19	20	21	22	23	
19	20	21	22	23	24	25	26	27	28	29	30	31	23	24	25	26	27	28	29	30	31	22	23	24	25	26	27	28	29

If you want to make your dreams come true, the first thing you have to do is wake up. — J.M. Power

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SATURDAY

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~~$$X = \{1\}$$~~

$$B_X(000) = \{0\} \rightsquigarrow \{0\}$$

$$B_X(001) = \{0, 1\} \rightsquigarrow \{0, 1\}$$

$$B_X(010) = \{\phi\} \rightsquigarrow \{\phi\}$$

$$B_X(011) = \{0\} \rightsquigarrow \{0\}$$

$$C_{\{1\}}(B) = \{000, 001, 011, 101\}$$

SUNDAY 6

~~$$X = \{2, 3\}$$~~

$$B_X(000) = \{\cancel{00}, 01, 11\}$$

$$\rightsquigarrow \{00, 01, 10\}$$

$$B_X(100) = \{01\} \rightsquigarrow \{00\}$$

Notes

$$(C_X(B)) = \{000, 001, 010, 100\}$$

July					August					September					October					November					December				
M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S		
1	2	3	4	5	31		1	2	1	2	3	4	5	6	1	2	3	4	30		1	1	2	3	4	5	6		
6	7	8	9	10	11	12	3	4	5	6	7	8	9	10	11	12	13	14	15	2	3	4	5	6	7	8			
13	14	15	16	17	18	19	10	11	12	13	14	15	16	17	18	19	20	21	22	9	10	11	12	13	14	15			
20	21	22	23	24	25	26	17	18	19	20	21	22	23	24	25	26	27	19	20	21	22	16	17	18	19	20			
27	28	29	30	31			21	22	23	24	25	26	27	28	29	30	31	26	27	28	29	30	23	24	25	26	27		
							24	25	26	27	28	29	30	28	29	30	31		26	27	28	29	30	31	28	29	30	31	

By working faithfully eight hours a day you may eventually get to be boss and work twelve hours a day. — Robert Frost

$$C_{\{1,3\}}(\{000, 001, 011, 101\})$$

$$= \{000, 001, 100, 010\}$$

Algorithmically, collect all members of B that
~~remain~~ are the same at points outside I ,
replace points in I by lex orders

$$A = \{ 000, 010, 011, 111 \}$$

$$C_{\{2\}}(A) = \{00011010, 1011100\}.$$

$$\{ \quad 000, 010, 001, 101 \quad \}$$

$$C_{\{3\}}(A) = \{000, 001, 010, 100\}$$

Notes

January					February					March					April					May					June													
M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S				
1	2	3	4				1	2	3	4	5	6	7	8	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21			
5	6	7	8	9	10	11	2	3	4	5	6	7	8	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21					
12	13	14	15	16	17	18	9	10	11	12	13	14	15	9	10	11	12	13	14	15	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28		
19	20	21	22	23	24	25	16	17	18	19	20	21	22	16	17	18	19	20	21	22	20	21	22	23	24	25	26	18	19	20	21	22	23	24	25	26	27	28
26	27	28	29	30	31		23	24	25	26	27	28	29	23	24	25	26	27	28	29	27	28	29	30	25	26	27	28	29	30	31	29	30					

ve learned that no matter what happens, or how bad it seems today, life does go on,
and it will be better tomorrow. ~ Maya Angelou

Observations

$$\circ |C_{\mathcal{I}}(B)| = |B|$$

$\circ B$ is \mathcal{I} compressed

$\Rightarrow B$ is \mathcal{I} compressed $\wedge J \subset \mathcal{I}$.

Theorem 3

$$b: \mathcal{P}_n \rightarrow \mathcal{P}$$

$\mathcal{I} \subseteq \{1, \dots, n\}$ satisfy $|\mathcal{I}| = 2$

$$\text{If } \hat{b}: \mathcal{P}_n \rightarrow \mathcal{P}$$

$$\text{s.t. } \hat{b}^{-1}(0) = C_{\mathcal{I}}(b^{-1}(0))$$

Then

$$I(\hat{b}(x^n); y^n) \geq I(b(x^n); y^n)$$

Notes

July					August					September					October					November					December				
M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S		
1	2	3	4	5	31		1	2	1	2	3	4	5	6		1	2	3	4	30		1	1	2	3	4	5	6	
6	7	8	9	10	11	12	3	4	5	6	7	8	9	10	11	12	13	5	6	7	8	9	10	11	12	13	14		
13	14	15	16	17	18	19	10	11	12	13	14	15	16	17	18	19	20	12	13	14	15	16	17	18	19	20	21		
20	21	22	23	24	25	26	17	18	19	20	21	22	23	24	25	26	27	19	20	21	22	23	24	25	26	27	28		
27	28	29	30		24	25	26	27	28	29	30	28	29	30		26	27	28	29	31	23	24	25	26	27	28	29	30	31

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WEDNESDAY

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If c_I changes an element of $b^{-1}(0)$, it moves it lower in lexicographical order.

If we apply them for different $|I| = 2$

\Rightarrow terminates at \hat{b} which is I -compressed

$\forall I$ with $|I| \leq 2$.

Corollary $S_n = \{ b : \mathbb{S}_n \rightarrow \mathbb{S} \text{ for which } b^{-1}(0) \text{ is } I \text{-compressed } \forall I \text{ s.t. } |I| \leq 2 \}$

In maximizing $I(b(x^n); y)$ we can stick to $b \in S_n$.

Allows enumeration of functions in S_n .

Notes and evaluating $I(b(x^n); y^n)$ $b \in S_n$.

January					February					March					April					May					June					
M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S			
1	2	3	4				1	30	31		1			1	2	3	4	5		1	2	3	1	2	3	4	5	6	7	
5	6	7	8	9	10	11	2	3	4	5	6	7	8	2	3	4	5	6	7	8	9	10	11	12	13	14				
12	13	14	15	16	17	18	9	10	11	12	13	14	15	9	10	11	12	13	14	15	16	17	15	16	17	18	19	20	21	
19	20	21	22	23	24	25	16	17	18	19	20	21	22	16	17	18	19	20	21	22	23	24	18	19	20	21	22	23	24	
26	27	28	29	30	31		23	24	25	26	27	28	29	23	24	25	26	27	28	29	30	25	26	27	28	29	30	31	29	30

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THURSDAY 10

Gives intuition for ~~then~~ conjecture ②.

because applying I -compression to $b^{-1}(0)$ yields a function \hat{b} that is

i) closer to being lex.

given
ii) $|I| \leq 2$ satisfies

$$H(\hat{b}(x^n) | y^n) \leq H(b(x^n) | y^n)$$

$|I| = n$ proves the conjecture.

But,

counterexamples where compression increases

$H(b(x^n) | y^n)$ for $|I| > 2$ but

reduces $H(b(x^n) | y^n)$ for $|I| = n$.

Notes

July					August					September					October					November					December				
M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S		
1	2	3	4	5			31	1	2	1	2	3	4	5	6		1	2	3	4	30		1	1	2	3	4	5	6
6	7	8	9	10	11	12		3	4	5	6	7	8	9	10	11	12	13	5	6	7	8	9	10	11	12	13		
13	14	15	16	17	18	19		10	11	12	13	14	15	16	17	18	19	20	12	13	14	15	16	17	18	19	20		
20	21	22	23	24	25	26		17	18	19	20	21	22	23	24	25	26	27	19	20	21	22	23	24	25	16	17	18	
27	28	29	30	31				24	25	26	27	28	29	30	28	29	30	31	26	27	28	29	30	31	23	24	25	26	27

Don't let life discourage you; everyone who got where he is had to begin where he was. ~ Richard L. Evans

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Rem 1

Threshold function and local maximum in terms of mutual information provided about y^n .

Rem 2 $b: \mathcal{S}_n \rightarrow \mathcal{S}$ is monotone if

$b(x^n) \leq b(\tilde{x}^n)$ when $x_i \leq \tilde{x}_i$ for $1 \leq i \leq n$.

monotone boolean functions are \mathcal{I} -compressed &

\mathcal{I} with $|\mathcal{I}| = 1$,

or $1 \rightarrow I(b(x^n); y^n)$ is maximized by a monotone function.

Th 3 \Rightarrow we may consider only regular functions.

Notes

January					February					March					April					May					June						
M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S				
1	2	3	4				1	30	31		1			1	2	3	4	5			1	2	3	1	2	3	4	5	6	7	
5	6	7	8	9	10	11	2	3	4	5	6	7	8	2	3	4	5	6	7	8	9	10	11	12	13	14					
12	13	14	15	16	17	18	9	10	11	12	13	14	15	9	10	11	12	13	14	15	16	17	18	19	20	21					
19	20	21	22	23	24	25	16	17	18	19	20	21	22	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30			
26	27	28	29	30	31		23	24	25	26	27	28	29	23	24	25	26	27	28	29	30	31	25	26	27	28	29	30	31	29	30

In three words I can sum up everything I've learned about life: It goes on. — Robert Frost

2015
December

Week - 50

SATURDAY 12

Proof of theorem 1

$X^n \leftarrow$ iid Bern(1/2).

$Y^n \leftarrow X^n + Z^n \leftarrow \text{Bsc}(\alpha)$.

$b : \{0,1\}^n \rightarrow \{0,1\}$

with $\text{IP}[b(X^n) = 0] = 1/2$

we
have

$$\sum_{i=1}^n I(b(X^n); Y_i) \leq 1 - H(\alpha).$$

lemma $r^2 \leq 1$

$$g \in [0, 1]$$

SUNDAY 13

an optimal solution for the non-convex
program

Notes

July					August					September					October					November					December												
M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S										
1	2	3	4	5			31	1	2	1	2	3	4	5	6		1	2	3	4	30		1	1	2	3	4	5	6								
6	7	8	9	10	11	12		3	4	5	6	7	8	9		7	8	9	10	11	12	13	5	6	7	8	9	10	11	12	13						
13	14	15	16	17	18	19		10	11	12	13	14	15	16		14	15	16	17	18	19	20	12	13	14	15	16	17	18	19	20						
20	21	22	23	24	25	26		17	18	19	20	21	22	23		21	22	23	24	25	26	27	19	20	21	22	23	24	25	26	27						
27	28	29	30	31				17	18	19	20	21	22	23		28	29	30					26	27	28	29	30	31	23	24	25	26	27	28	29	30	31

You gain strength, courage and confidence by every experience in which you stop to look fear in the face. — Eleanor Roosevelt

14 MONDAY

Week - 51

2015
December

$$\min \sum_{i=1}^n H\left(\frac{1 + px_i}{2}\right)$$

$$\text{s.t. } \sum_{i=1}^n n_i^2 \leq r_0^2$$

is given by $n_i = p$ and $x_i = 0$ for $i = 2, \dots, n$

Proof Taylor expand H around $1/2$

$$\sum_{i=1}^n H\left(1 + \frac{px_i}{2}\right)$$

$$= n - \log e \sum_{k=1}^{\infty} \sum_{i=1}^n \frac{(px_i)^{2k}}{(2k-1) 2^k}$$

for fixed k , the sum

$$\sum_{i=1}^n x_i^{2k} \text{ is convex in } x_i^2$$

Notes

January	February	March	April	May	June
M T W T F S S	M T W T F S S	M T W T F S S	M T W T F S S	M T W T F S S	M T W T F S S
1 2 3 4	1 30 31	1	1 2 3 4 5	1 2 3	1 2 3 4 5 6
5 6 7 8 9 10 11	2 3 4 5 6 7 8	2 3 4 5 6 7 8	6 7 8 9 10 11 12	4 5 6 7 8 9 10	8 9 10 11 12 13 14
12 13 14 15 16 17 18	9 10 11 12 13 14 15	9 10 11 12 13 14 15	13 14 15 16 17 18 19	11 12 13 14 15 16 17	15 16 17 18 19 20 21
19 20 21 22 23 24 25	16 17 18 19 20 21 22	16 17 18 19 20 21 22	20 21 22 23 24 25 26	18 19 20 21 22 23 24	22 23 24 25 26 27 28
26 27 28 29 30 31	23 24 25 26 27 28	23 24 25 26 27 28 29	27 28 29 30	25 26 27 28 29 30 31	29 30

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Week - 51

TUESDAY 15

and maximized subject to $\sum_{i=1}^n x_i^2 \leq r^2$
when $x_1 = r$ and $x_i = 0$ for $i = 2, \dots, n$.

Proof of Theorem 1

x^n take values in $\{-1, 1\}^n$
 y^n

$$b : \{-1, 1\}^n \rightarrow \{-1, 1\}$$

Any boolean function b can be written as

$$b(x^n) = \sum_{S \subseteq [n]} \hat{b}(S) \prod_S (x^n)$$

$$\hat{b}(S) = \mathbb{E}_{x^n} b(x^n) \prod_S (x^n)$$

Notes

$$S = \emptyset \Rightarrow \prod_S (x^n) = 1.$$

July	August	September	October	November	December
M T W T F S	M T W T F S	M T W T F S	M T W T F S	M T W T F S	M T W T F S
1 2 3 4 5	31	1 2	1 2 3 4 5 6	1 2 3 4 30	1 2 3 4 5 6
6 7 8 9 10 11 12	3 4 5 6 7 8 9	7 8 9 10 11 12 13	5 6 7 8 9 10 11	2 3 4 5 6 7 8	7 8 9 10 11 12 13
13 14 15 16 17 18 19	10 11 12 13 14 15 16	14 15 16 17 18 19 20	12 13 14 15 16 17 18	9 10 11 12 13 14 15	14 15 16 17 18 19 20
20 21 22 23 24 25 26	17 18 19 20 21 22 23	21 22 23 24 25 26 27	19 20 21 22 23 24 25	16 17 18 19 20 21 22	21 22 23 24 25 26 27
27 28 29 30 31	24 25 26 27 28 29 30	28 29 30	26 27 28 29 30 31	23 24 25 26 27 28 29	28 29 30 31

Do first things first, and second things not at all. — Peter Drucker

16 WEDNESDAY

Week - 51

2015
December

① For all x^n , $\prod_S (x^n)^2 = 1$.

$$\mathbb{E} \prod_S (x^n)^2 = 1$$

② If $S \neq T$ and $S \neq \emptyset$ $T \neq \emptyset$ then

$\prod_S (x^n)$ and $\prod_T (x^n)$ are independent
and uniformly distributed on $\{-1, 1\}$

$$\mathbb{E} \prod_S (x^n) \prod_T (x^n) = 0$$

If $\emptyset = S \neq T$ then

$$\begin{aligned} & \mathbb{E} \prod_S (x^n) \prod_T (x^n) \\ &= \mathbb{E} \prod_T (x^n) = 0 \end{aligned}$$

Notes

January	February	March	April	May	June														
M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S
1 2 3 4		1 30 31		1	1 2 3 4 5				1 2 3	1 2 3 4 5 6 7									
5 6 7 8 9 10 11	2 3 4 5 6 7 8	2 3 4 5 6 7 8	6 7 8 9 10 11 12	4 5 6 7 8 9 10	8 9 10 11 12 13 14														
12 13 14 15 16 17 18	9 10 11 12 13 14 15	9 10 11 12 13 14 15	13 14 15 16 17 18 19	11 12 13 14 15 16 17	15 16 17 18 19 20 21														
19 20 21 22 23 24 25	16 17 18 19 20 21 22	16 17 18 19 20 21 22	20 21 22 23 24 25 26	18 19 20 21 22 23 24	22 23 24 25 26 27 28														
26 27 28 29 30 31	23 24 25 26 27 28	23 24 25 26 27 28 29	27 28 29 30	25 26 27 28 29 30 31	29 30														

The only people who find what they are looking for in life are the fault finders. ~ Foster's Law

③ Parseval's theorem

$$1 = \mathbb{E} [b(x^n)]^2$$

$$= \sum_{s \in [n]} \hat{b}(s)^2.$$

$$\mathbb{E} [b(x^n)^2]$$

$$= \mathbb{E} \left[\left(\sum_s \hat{b}(s) \pi_s(x^n) \right) \left(\sum_T \hat{b}(T) \pi_T(x^n) \right) \right]$$

$$= \mathbb{E} \sum_s \sum_T \hat{b}(s) \hat{b}(T) \mathbb{E} \left[\pi_s(x^n) \pi_T(x^n) \right]$$

$$= \sum_s \hat{b}(s)^2$$

Notes ④ $\hat{b}(\phi) = \mathbb{E}[b(x^n)] = 0$

since $b(x^n)$ is equiprobable.

July					August					September					October					November					December				
M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S		
1	2	3	4	5	31		1	2	1	2	3	4	5	6	1	2	3	4	30		1	1	2	3	4	5	6		
6	7	8	9	10	11	12	3	4	5	6	7	8	9	10	11	12	13	5	6	7	8	9	10	11	12	13			
13	14	15	16	17	18	19	10	11	12	13	14	15	16	17	18	19	20	12	13	14	15	16	17	18	19	20			
20	21	22	23	24	25	26	17	18	19	20	21	22	23	24	25	26	27	19	20	21	22	23	24	25	26	27			
27	28	29	30	31			24	25	26	27	28	29	30	28	29	30	31		26	27	28	29	23	24	25	26	27		

18 FRIDAY

Week - 51

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$$\text{IP} [b(x^n) = 1 \mid y_i = y_i] \xrightarrow{\text{easy to see on expanding this}}$$

$$= \frac{1 + \mathbb{E} [b(x^n) \mid y_i = y_i]}{2}$$

$$= 1 + \mathbb{E} \left[\sum_s \hat{b}(s) \pi_s(x^n) \mid y_i = y_i \right]$$

$$= 1 + \sum_s \hat{b}(s) \mathbb{E} \left[\pi_s(x^n) \mid y_i = y_i \right]$$

fix rest
and flip one
value

2

≈ 0 unless $s = \emptyset$ or

$s = \{i\}$ since $\{x_j\}_{j \in s \setminus \{i\}}$
are independent of y_i

Notes

$$= \frac{1 + \hat{b}(\emptyset) + \sum_i y_i \hat{b}(\{i\})}{2}$$

January					February					March					April					May					June				
M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S		
1	2	3	4				1	30	31		1			1	2	3	4	5		1	2	3	1	2	3	4	5	6	7
5	6	7	8	9	10	11	2	3	4	5	6	7	8	2	3	4	5	6	7	8	9	10	11	12	13	14			
12	13	14	15	16	17	18	9	10	11	12	13	14	15	9	10	11	12	13	14	15	16	17	18	19	20	21			
19	20	21	22	23	24	25	16	17	18	19	20	21	22	16	17	18	19	20	21	22	23	24	25	26	27	28			
26	27	28	29	30	31		23	24	25	26	27	28	29	23	24	25	26	27	28	29	30	25	26	27	28	29	30	29	30

I am an optimist. It does not seem too much use being anything else. — Winston Churchill

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Week - 51

SATURDAY 19

$$= \frac{1 + p y_i \hat{b}(\{\cdot\})}{2} \quad \checkmark$$

$$\rho \stackrel{\triangle}{=} \mathbb{E}[x_i | y_i = 1] = 1 - 2\alpha$$

$$\sum_{i=1}^n I(b(x^n); y_i)$$

$$= n H(b(x^n)) - \sum_{i=1}^n H(b(x^n) | y_i)$$

equiprobable

$$= n - \sum_{i=1}^n H(b(x^n) | y_i)$$

SUNDAY 20

minimize subject to $\mathbb{P}[b(x^n) = 1] = \frac{1}{2}$

$$\sum_{i=1}^n H(b(x^n) | y_i)$$

$$= \sum_{i=1}^n \frac{1}{2} \left[H(b(x^n) | y_i = 1) \right]$$

Notes

July							August							September							October							November							December						
M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S	M	T	W	T	F	S	S							
1	2	3	4	5	31		1	2	1	2	3	4	5	6		1	2	3	4	30		1	2	3	4	5	6		1	2	3	4	5	6							
6	7	8	9	10	11	12	3	4	5	6	7	8	9	10	11	12	13	5	6	7	8	9	10	11	2	3	4	5	6	7	8										
13	14	15	16	17	18	19	10	11	12	13	14	15	16	17	18	19	20	12	13	14	15	16	17	18	9	10	11	12	13	14	15										
20	21	22	23	24	25	26	17	18	19	20	21	22	23	24	25	26	27	19	20	21	22	23	24	25	16	17	18	19	20	21	22										
27	28	29	30	31			24	25	26	27	28	29	30		28	29	30	31	26	27	28	29	30	31	23	24	25	26	27	28	29	30	31	30	31						