Linear State Estimation with PMU

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November 17, 2020

Objectives

Objective: Develop a linear state estimation from real PMU data

1. π-model of a transmission line
2. Current flow equations
3. State estimation formulation
4. PMU data pre-processing
5. Estimating states
6. Calculate line power flow

π-model of a transmission line

The π-model of a transmission line is shown below,

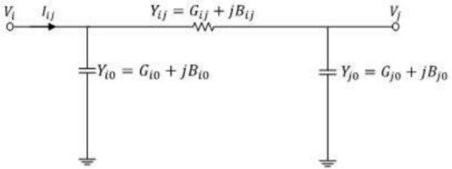

Figure: A π - model of the transmission line

Here, V_i and V_j are the voltage value on bus i and j, Y_{i0} and Y_{j0} are the transmission line charging admittance, and Y_{ij} is the transmission line admittance between bus i and bus j.

R.Liu, A. Srivastava, A. Askerman, D. Bakken and P. Panciatici, Decentralized State Estimation and Remedial Control Action for Minimum Wind Curtailment Using Distributed Computing Platform, IEEE Transactions on Industrial Applications, June 2017

Current flow equations

The current I_{ij} from bus i to bus j is,

$$I_{ij} = V_i Y_{i0} + (V_i - V_j) Y_{ij} \tag{1}$$

The rectangular counterparts are $V_i^R, V_i^I, V_j^R, V_j^I, I_{ij}^R$ and I_{ij}^I.
The current I_{ij} is written as,

$$I_{ij}^R + jI_{ij}^I = (V_i^R + jV_i^I)(G_{i0} + jB_{i0}) + (V_i^R + jV_i^I - V_2^R - jV_2^I)(G_{ij} + jB_{ij}) \tag{2}$$

Current flow equations

The real and imaginary parts of eq (1) are given as,

$$I_{ij}^R = (G_{i0} + G_{ij})V_i^R - G_{ij}V_j^R - (B_{i0} + B_{ij})V_i^I + B_{ij}V_j^I \tag{3}$$

$$I_{ij}^I = (B_{i0} + B_{ij})V_i^R - B_{ij}V_j^R + (G_{i0} + G_{ij})V_i^I - G_{ij}V_j^I \tag{4}$$

Linear State Estimation formulation

Let there be M nodes and N measurements. The measurements and the states can be expressed as,

$$z = Hx + e \tag{5}$$

1. $z \in \mathbb{R}^N$ is the vector of measurements
2. $H \in \mathbb{R}^{N \times 2M}$ is the Jacobian matrix
3. x is the $\mathbb{R}^{2M}$ dimensional system status which is the real and imaginary part of voltage values on each substation
4. $e \in \mathbb{R}^N$ is the noise vector of the measurements

Linear State Estimation formulation

The eq (5) can be expressed as,

$$\begin{bmatrix} V^R \\ V^I \\ I_{ij}^R \\ I_{ij}^I \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \\ H_{31} & H_{32} \\ H_{41} & H_{42} \end{bmatrix} \begin{bmatrix} V^R \\ V^I \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix} \tag{6}$$

1. V^R, V^I are state variables
2. H_{11}, H_{22} are the unit matrix
3. H_{12}, H_{21} are zero matrix
4. H_{31}, H_{42} correspond to the real parts in eq (3) and eq (4), and H_{32}, H_{41} correspond to the imaginary parts in eq (3) and eq (4).

Linear State Estimation formulation

The error free linear weighted least squares (WLS) method is used to solve the linear state estimation model with PMU data as [1],

$$J(x) = [z - Hx]^T [z - Hx] \tag{7}$$

The optimal solution is is obtained using the pseudo-inverse as,

$$\hat{x} = [H^T H]^{-1} H^T z \tag{8}$$

Example of Linear State Estimation - 2 bus system

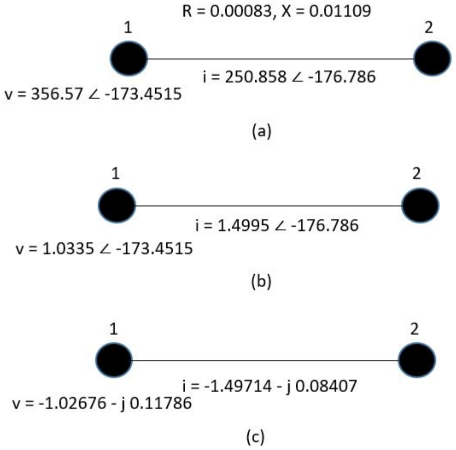

Figure: (a) Actual PMU measurements (b) measurements in p.u. and (c) measurements in rectangular coordinates

Building the Linear State Estimator

The state vector is written as,

$$\begin{bmatrix} V_1^R \\ V_1^I \\ V_2^R \\ V_2^I \end{bmatrix} \tag{9}$$

The measurement vector z is written as,

$$\begin{bmatrix} V_1^R \\ V_1^I \\ I_{12}^R \\ I_{12}^I \end{bmatrix} = \begin{bmatrix} -1.02676 \\ -0.11786 \\ -1.49714 \\ -0.08407 \end{bmatrix} \tag{10}$$

Building the Linear State Estimator

The H matrix is written as,

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ G_{12} & -G_{12} & -B_{12} & B_{12} \\ B_{12} & -B_{12} & G_{12} & -G_{12} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 6.711 & -6.711 & 89.669 & -89.669 \\ -89.669 & 89.669 & 6.711 & -6.711 \end{bmatrix} \tag{11}$$

Estimating States

The State Estimation formulation can be written as,

$$\begin{bmatrix} -1.02676 \\ -0.11786 \\ -1.49714 \\ -0.08407 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 6.711 & -6.711 & 89.669 & -89.669 \\ -89.669 & 89.669 & 6.711 & -6.711 \end{bmatrix} \begin{bmatrix} V_1^R \\ V_2^R \\ V_1^I \\ V_2^I \end{bmatrix} \tag{12}$$

The estimated states are,

$$\begin{bmatrix} \hat{V}_1^R \\ \hat{V}_1^I \\ \hat{V}_2^R \\ \hat{V}_2^I \end{bmatrix} = \begin{bmatrix} -1.0268 \\ -1.0264 \\ -0.11786 \\ -0.10119 \end{bmatrix} \tag{13}$$

The voltage phasor at Wachussets is $V_2 = 1.3014$ p.u, $\theta_2 = -174.37$.

Linear State Estimation of an Observable System

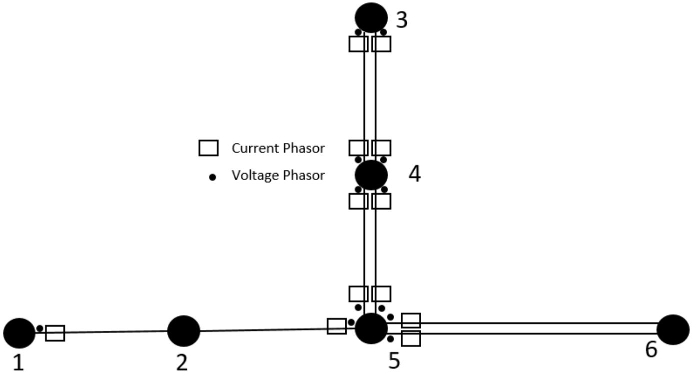

Figure: A simple 6-bus system with PMU measurements

PMU Voltages Phasors

Bus	Voltage	Angle	To	From
1	1.032082386	-171.9473	1	2
3	1.032745586	-173.4744	3	4
3	1.033538313	-173.4515	3	4
4	1.033062878	-174.3969	4	5
4	1.030525549	-174.8323	4	5
4	1.033062878	-174.3969	4	3
4	1.030525549	-174.8323	4	3
5	1.031191259	-174.4657	5	2
5	1.031286647	-174.6089	5	4
5	1.031413664	-174.7292	5	4
5	1.028527415	-174.7235	5	6
5	1.031033115	-174.4026	5	6

PMU Current Phasors

Bus	Current	Angle	To	From
1	2.298392152	-167.2296	1	2
5	2.946979914	8.1074	5	2
5	0.685683537	0.6532	5	4
5	0.689330431	-0.2406	5	4
5	3.275232614	-177.021	5	6
5	3.391944566	-177.9149	5	6
3	1.499020902	-176.7861	3	4
3	1.509962181	-176.2647	3	4
4	0.678389152	-168.8621	4	5
4	0.692977922	-170.2544	4	5
4	1.458900889	-0.0458	4	3
4	1.495373411	-0.6417	4	3

State Estimates

Bus	Measured		Estimated	
	Voltage	Angle	Voltage	Angle
1	1.032082386	-171.9473	**1.0297**	**-173.8168**
2	-	-	**1.0311**	**-175.5645**
3	1.033538313	-173.4515	**1.0311**	**-174.2107**
3	1.032745586	-173.4744	-	-
4	1.033062878	-174.3969	**1.0316**	**-174.2234**
4	1.030525549	-174.8323	-	-
4	1.033062878	-174.3969	-	-
4	1.030525549	-174.8323	-	-
5	1.031191259	-174.4657	**1.0319**	**-174.2248**
5	1.031286647	-174.6089	-	-
5	1.031413664	-174.7292	-	-
5	1.028527415	-174.7235	-	-
5	1.031033115	-174.4026	-	-
6	-	-	**1.0286**	**-175.7264**

State Estimation Residues under Ambient Conditions

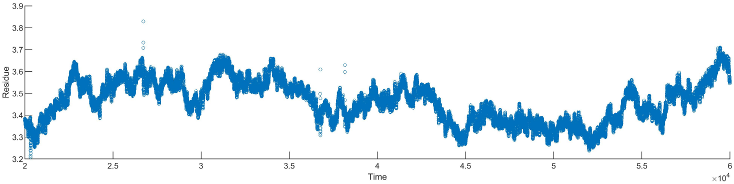

Figure: State Estimation Residues under Ambient Conditions

Questions

Please contact me if you have any questions:

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