Simulation of a Centralized Quantum Network for Multipartite Entanglement Distribution

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1 Introduction

Quantum networks enable the transmission and distribution of entangled quantum states across spatially separated nodes, providing the foundation for applications such as quantum cryptography, distributed quantum computing, and enhanced sensing capabilities. Multipartite entanglement, exemplified by GHZ states, is of particular importance due to its utility in multi-party quantum protocols.

This project builds upon the analysis presented by Avis et al. [1], which explores the performance of centralized quantum network nodes distributing GHZ states via teleportation-based protocols. Motivated by this work, a custom simulation framework was developed in Julia to model the underlying physical processes and to evaluate the network's behavior under realistic noise conditions.

The objectives of this project are as follows:

- Develop a quantum network model incorporating a centralized node responsible for distributing GHZ states.
- Accurately represent entanglement generation, BSM operations, and noise effects within the simulation.
- Analyze key performance indicators including fidelity, latency, and success probability across various parameter regimes.
- Compare and interpret simulation results within the context of existing theoretical models.

2 Model and Assumptions

2.1 Error Models

The simulation incorporates several realistic error mechanisms impacting entanglement distribution:

- **Entanglement Generation Probability:** Each attempt to generate entanglement succeeds with probability 0.9, modeling photon loss and channel imperfections.
- **Werner States:** Generated entangled pairs are modeled as Werner states with a mixing parameter of 0.9, representing a noisy mixture of maximally entangled and fully mixed states.
- **Dephasing Noise (T2):** Qubits experience T2 dephasing noise with a coherence time of 1.0 second, simulating loss of phase coherence in quantum memories.

2.2 Bell State Measurement Failure

Due to the Werner state mixture, Bell State Measurements may fail probabilistically if the measurement outcome corresponds to the fully mixed component. This phenomenon reflects the physical reality that noise degrades the success probability of entanglement swapping operations.

2.3 Network Assumptions

The following assumptions are made to streamline the simulation:

- Classical communication is instantaneous and error-free.
- Quantum gate operations (including teleportation and Bell State Measurements) are ideal and noiseless.
- Each link interface holds a single communication qubit, enabling parallel entanglement generation.
- No quantum error correction or entanglement purification is implemented.

3 Implementation Details:

3.1 Quantum Network Architecture

The architecture investigated in this study adopts a **centralized quantum network node** paradigm, where a single central node coordinates the distribution of multipartite entanglement to multiple end nodes. This design is inspired by the framework established in Avis et al. [**?**], featuring a star topology wherein each end node is linked individually to the central node via dedicated quantum channels.

3.1.1 Network Topology and Components

The network consists of the following primary elements:

- **Central Node:** Serves as the quantum repeater and entanglement distributor. It performs Bell State Measurements to facilitate entanglement swapping, enabling the creation of GHZ states shared among all end nodes.
- **End Nodes:** Connected individually to the central node, these nodes act as receivers of multipartite entangled states and may correspond to quantum processors or sensors in practical applications.

The star topology simplifies coordination by concentrating complex quantum operations at the central node, thereby minimizing the need for direct entanglement management between end nodes.

3.1.2 Entanglement Generation and Distribution

Entanglement generation along each channel is modeled as a probabilistic process, characterized by a Bernoulli distribution with success probability p dependent on physical parameters such as channel length and photon loss. The entangled pairs produced are represented as Werner states to incorporate realistic noise, rather than ideal Bell states.

The central node performs a sequence of Bell State Measurements to extend bipartite entanglement to a multipartite GHZ state through entanglement swapping. This approach is foundational to the operation of quantum repeaters and is essential for establishing long-range entanglement in scalable networks.

These noise sources collectively degrade entanglement fidelity and reduce the probability of successful entanglement distribution.

3.2 Relation to Reference Work and Distinctions

While this project reproduces several core results from Avis et al. [1], it contributes additional insights through comprehensive numerical simulations that explore a broader range of parameters, including multiple link distances and incorporate statistical variability across a large number of simulation runs.

Unlike the predominantly analytical approach of the paper, our work provides empirical data on latency and success rates under various noise conditions and network topologies. Furthermore, this simulation explicitly models the probabilistic failure of Bell State Measurements due to Werner state noise, which is not extensively discussed in the original publication.

3.3 Implementation Details

3.3.1 Simulation Framework

The simulation is implemented in Julia, leveraging its high-performance capabilities for scientific computation. The project is organized as follows:

- `src/CentralQNet.jl`: Core module controlling the quantum network simulation.
- `src/processes/bsm.jl`: Implements Bell State Measurement logic.
- `src/processes/link_gen.jl`: Handles entanglement generation attempts on quantum links.
- `src/processes/state_gen.jl`: Generates quantum states including GHZ and Werner states.
- `src/states/ghz.jl` and `werner.jl`: Define multipartite entangled states and their properties.
- `simulation/main.jl`: Executes simulation scenarios, varying parameters and collecting metrics.

3.3.2 Simulation Workflow

The simulation proceeds as follows:

1. Initialize network parameters, including number of nodes, link distances, and noise rates.
2. Probabilistically generate initial entangled pairs on each link.
3. Perform Bell State Measurements at the central node to create multipartite GHZ states.
4. Apply decoherence models to simulate noise effects on stored qubits.
5. Measure performance metrics such as success rate, fidelity, and latency.
6. Repeat the process for a large number of trials to gather statistical data.

3.3.3 Code Reproducibility

The codebase is thoroughly documented with inline comments and adheres to modular design principles. Random number generators can be seeded to ensure reproducibility of results. The repository is publicly accessible at here.

4 Evaluation Plan and Results

4.1 Simulation Scale and Setup

To obtain statistically meaningful results, the simulation was executed for one million (10^6) iterations. Given the complexity of the noise and error models, the total runtime was substantial.

4.2 Performance Metrics

The following average metrics were observed:

- **Average Time per Run:** Approximately 6.52×10^{-4} seconds (standard deviation: 2.68×10^{-4} seconds)
- **Average Fidelity:** Approximately 0.79 (standard deviation: 0.00058)
- **Success Rate:** Approximately 12.5%

4.3 Discussion of Results

The observed success probability reflects the combined impact of:

- Photon loss affecting entanglement generation attempts.
- Werner state noise reducing entanglement purity.
- Dephasing noise degrading coherence over time.
- Probabilistic failure of Bell State Measurements due to noisy states.

Notably, the fidelity remains relatively high when conditioned on successful runs, indicating that entanglement quality is preserved despite noise. These outcomes align well with theoretical predictions and underscore the challenges in achieving high-throughput entanglement distribution in realistic quantum networks.

4.4 Performance Across Link Distances

Table 1 summarizes the simulation results for three different quantum link distances: 25 km, 50 km, and 100 km. The attenuation length of the channel is set to $L_{\text{att}} = 22\,\text{km}$, which impacts the entanglement generation probability as a function of distance.

Observations:

- **Latency (Average Time)** increases significantly with distance, rising from approximately 2.9 ms at 25 km to over 120 ms at 100 km. This is expected due to the increased communication delay and repeated entanglement attempts required at longer distances.
- **Fidelity** decreases as link distance increases, dropping from about 0.78 at 25 km to 0.56 at 100 km. This degradation reflects the growing influence of noise and decoherence over longer channels.
- **Success Rate** remains relatively stable around 12.5% for 25 km and 50 km links but declines sharply to roughly 3% at 100 km. The steep drop highlights the difficulty in successfully generating entanglement across longer distances in the presence of attenuation and noise.

Table 1: Performance metrics for different link distances in the quantum network simulation.

Metric	**25 km**	**50 km**	**100 km**
Average Time (s)	0.00287	0.01783	0.12225
Std. Dev. Time (s)	0.00166	0.01101	0.04022
Average Fidelity	0.7843	0.7411	0.5591
Std. Dev. Fidelity	0.00527	0.02899	0.05141
Success Rate	0.1250	0.1248	0.0297

Interpretation: The results demonstrate the trade-off inherent in quantum network design between distance and performance. While shorter links provide higher fidelity and faster entanglement distribution, extending the reach of the network results in increased latency, reduced success rates, and lower-quality entangled states. The attenuation length $L_{\text{att}} = 22\,\text{km}$ serves as a benchmark for when performance degradation becomes pronounced.

These findings are consistent with theoretical models of quantum communication channels and reinforce the importance of quantum repeaters or alternative strategies to mitigate loss and noise in long-distance quantum networks.

4.5 Impact of Number of End Nodes

Table 2 presents simulation results comparing the performance metrics for networks with different numbers of end nodes ($n = 3$ and $n = 4$) at a fixed link distance of 50 km.

Table 2: Performance metrics for different numbers of end nodes at 50 km link length.

Metric	$n = 3$	$n = 4$	$n = 5$
Average Time (s)	0.01783	0.02018	0.02182
Std. Dev. Time (s)	0.01101	0.01124	0.01076
Average Fidelity	0.7411	0.6675	0.6029
Std. Dev. Fidelity	0.02899	0.03238	0.03240
Success Rate	0.1248	0.1249	0.0313

Interpretations: Increasing the number of end nodes from 3 to 5 results in a gradual increase in average latency, reflecting the additional entanglement generation and swapping operations required as more nodes participate in the network. Concurrently, average fidelity decreases significantly with each added node, indicating that increasing the number of nodes introduces more noise and decoherence pathways that degrade entanglement quality. While the success rate remains roughly constant between 3 and 4 nodes, it drops markedly when increasing to 5 nodes, suggesting that beyond a certain scale, the probability of successful entanglement distribution is negatively impacted by the added complexity and resource contention at the central node.

These findings highlight the trade-offs involved in scaling quantum networks and emphasize the importance of optimizing network size relative to performance requirements.

5 Discussion and Conclusion

The simulation results demonstrate that a centralized quantum network can successfully distribute GHZ states across multiple end nodes, but its performance is highly sensitive to both distance and network size. As link lengths increase from 25 km to 100 km, latency rises sharply and fidelity drops due to photon loss and dephasing, with success rates falling significantly at 100 km. Similarly, increasing the number of end nodes from 3 to 5 results in higher latency and progressively lower fidelity, with success rates remaining stable for up to 4 nodes but decreasing markedly at 5 nodes. These trends highlight coordination and memory limitations at the central node and reveal fundamental scalability challenges in star-topology quantum networks. They emphasize the need for improvements such as quantum repeaters, entanglement purification, and protocol parallelization. While the architecture performs adequately for small, short-distance networks, larger deployments will require more robust strategies to maintain efficiency and entanglement quality.

References

[1] Guus Avis, Filip Rozpedek, and Stephanie Wehner. Analysis of multipartite entanglement distribution using a central quantum-network node. *Phys. Rev. A*, 107:012609, Jan 2023.

Appendix: Code Access

The complete source code and simulation scripts are available at:
`https://github.com/sagnikpal2004/690qc-project`