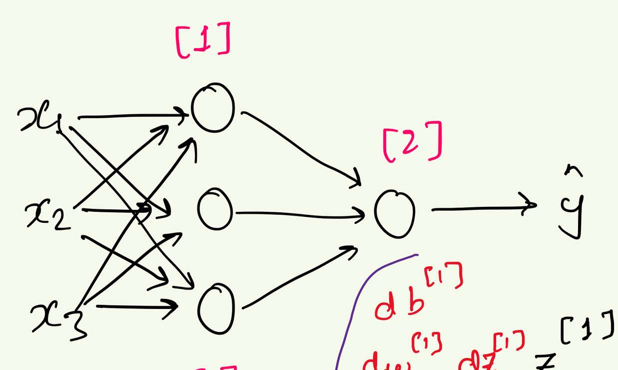
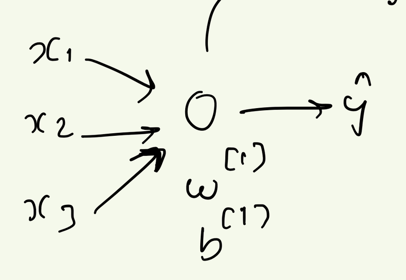
Week3

Saturday, May 2, 2020 11:40 AM

Shallow neural network :-





[ ] → layer

( ) → training example.

$w^{[1]}$, $b^{[1]}$

$db^{[1]}, dw^{[1]}, dz^{[1]}$ $\quad z^{[1]} = w^{[1]}x + b^{[1]}$

$da^{[1]}$ $\quad a^{[1]} = \sigma(z^{[1]})$

$dw^{[2]}, db^{[2]}, dz^{[2]}$ $\quad z^{[2]} = w^{[2]}.a^{[1]} + b^{[2]}$

$da^{[2]}$ $\quad a^{[2]} = \sigma(z^{[2]})$

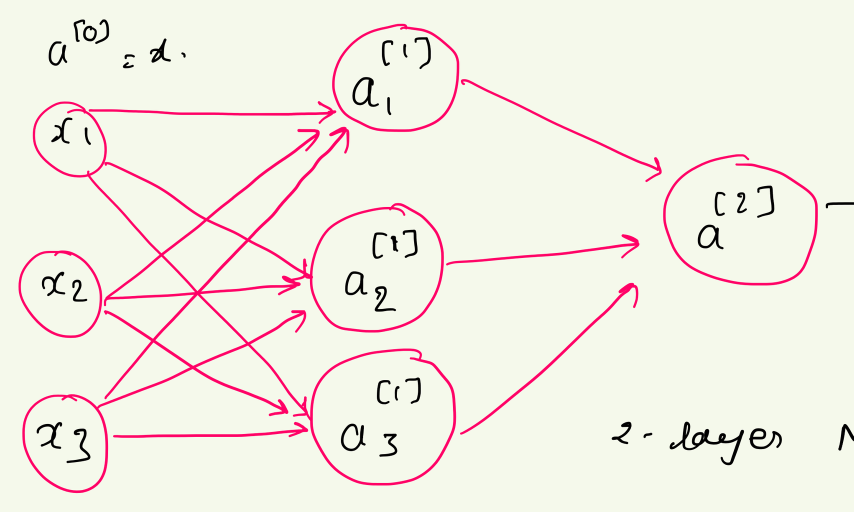
$\mathcal{L}(a^{[2]}, y)$

back propagation derivatives

** Consider $w_1^{[1]}$ as $w_1^{[1]T}$

$a^{[0]} = x.$

*



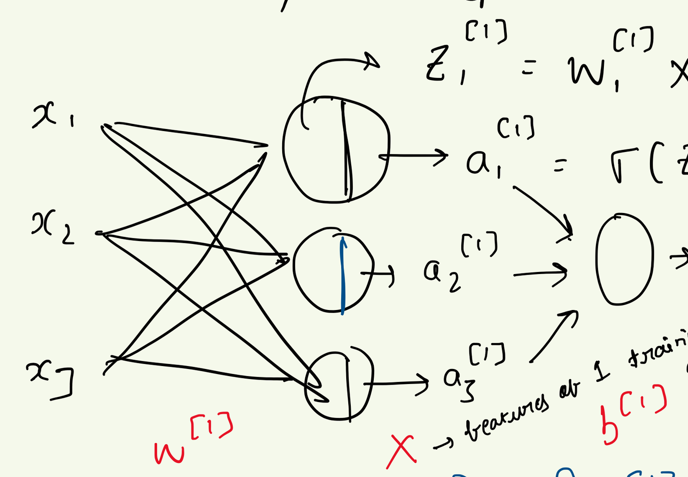
$\hat{y} = a^{[2]}$ ↳ prediction.

2-layer Neural n/w.

$$a^{[1]} = \begin{bmatrix} a_1^{[1]} \\ a_2^{[1]} \\ a_3^{[1]} \end{bmatrix}$$

for one image

Neural n/w representation :- for 1 training examples



$z_1^{[1]} = w_1^{[1]}x + b_1^{[1]}$

$a_1^{[1]} = \sigma(z_1^{[1]})$

$a_2^{[1]}$

$a_3^{[1]}$

Components of one layer

$$\underbrace{\begin{matrix} a_1^{[1]} & z_1^{[1]} & w_1^{[1]T} & b_1^{[1]} \\ a_2^{[1]} & z_2^{[1]} & w_2^{[1]T} & b_2^{[1]} \\ a_3^{[1]} & z_3^{[1]} & w_3^{[1]T} & b_3^{[1]} \end{matrix}}_{X}$$

X → features of 1 training example

→ final output

$$\overset{w^{[1]}}{\begin{bmatrix} w_1^{[1]T} \\ w_2^{[1]T} \\ w_3^{[1]T} \end{bmatrix}} \overset{X}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}} + \overset{b^{[1]}}{\begin{bmatrix} b_1^{[1]} \\ b_2^{[2]} \\ b_3^{[3]} \end{bmatrix}} = \begin{bmatrix} w_1^{[1]T}x_1 + b_1^{[1]} \\ w_2^{[1]T}x_2 + b_2^{[1]} \\ w_3^{[1]T}x_3 + b_3^{[1]} \end{bmatrix} = \overset{z^{[1]}}{\begin{bmatrix} z_1^{[1]} \\ z_2^{[1]} \\ z_3^{[1]} \end{bmatrix}} = \overset{a^{[1]}}{\begin{bmatrix} a_1^{[1]} \\ a_2^{[1]} \\ a_3^{[1]} \end{bmatrix}}$$

3*3 $\qquad$ 3*1

* w2 v3

~~Dimension of weight~~ matrix and $[x_1, x_2, x_3]$

logic of transpose

So, in short, equations :-

$$\underset{3*1}{z^{[1]}} = \boxed{\underset{3*3}{w^{[1]}}.\underset{3*1}{X}} + \underset{3*1}{b^{[1]}} \quad \text{\} for one layer.}$$

$$\underset{3*1}{a^{[1]}} = \sigma(\underset{3*1}{z^{[1]}})$$

$$\underset{1*1}{z^{[2]}} = \underset{1*3}{w^{[2]}}.\underset{3*1}{a^{[1]}} + \underset{1*1}{b^{[2]}}$$

$$\underset{1*1}{a^{[2]}} = \sigma(\underset{1*1}{z^{[2]}})$$

* Vectorization across multiple examples.

for $i = 1$ to m :-

$$z^{[1](i)} = w^{[1](i)}.x^{(i)} + b^{[1](i)}$$

$$a^{[1](i)} = \sigma(z^{[1](i)})$$

$$z^{[2](i)} = w^{[2](i)}a^{[1](i)} + b^{[2](i)}$$

$$a^{[2](i)} = \sigma(z^{[2](i)})$$

Visualizing the entire NN :-

layer 1

$$Z^{[1]} = w^{[1]}.X + b^{[1]}$$

$$A^{[1]} = \sigma(Z^{[1]})$$

layer 2

$$Z^{[2]} = w^{[2]}A^{[1]} + b^{[2]}$$

$$A^{[2]} = \sigma(Z^{[2]})$$

Matrix of matrices.

IMPORTANT

where,

1st ex. $\quad$ 2nd ex.

$$Z^{[1]} = [z^{[1](1)} \; z^{[1](2)} \; \dots \; z^{[1](m)}]$$

$$A^{[1]} = [A^{[1](1)} \; A^{[1](2)} \; \dots \; A^{[1](m)}]$$

↓ nodes $\quad$ examples →

node.

$$Z^{[2]} = [z^{[2](1)} \; z^{[2](2)} \; \dots \; z^{[2](m)}]$$

$$A^{[2]} = [A^{[2](1)} \; A^{[2](2)} \; \dots \; A^{[2](m)}]$$

* Gradient Descent :-

$$dz^{[2]} = A^{[2]} - Y$$

$$dw^{[2]} = \frac{1}{m} * dz^{[2]} * A^{[1]T}$$

$$db^{[2]} = \frac{1}{m} * \text{np.sum}(dz^{[2]}, \text{axis}=1, \text{keepdims}=\text{True})$$

$$dw^{[1]} = \frac{1}{m} * dz^{[1]} * X^T \qquad X^T = A^{[0]T}$$

$$db^{[1]} = \frac{1}{m} * \text{np.sum}(dz^{[1]}, \text{axis}=1, \text{keepdims}=\text{True})$$

* Initialize with small random weights. * → important

Large weights → large z → $z' \approx 0$, problem in backpropagation & gradient descent

* Computing Gradients

x, w, b → $\boxed{z = w^Tx + b}$ $\quad \boxed{a = \sigma(z)}$ $\quad \boxed{\mathcal{L}(a, y)}$

$\frac{d\mathcal{L}}{dw}$

$dw = dz \cdot x$

$db = dz.$

chain rule.

$dz = da * g'(z)$

↳ $\frac{d\mathcal{L}}{dz}$

$\frac{d\mathcal{L}}{da}$

$$da = -\frac{y}{a} + \frac{1-y}{1-a}$$

here $g(z)$ = sigmoid (z)

x, $w^{[1]}$, $b^{[1]}$ → $z^{[1]} = w^{[1]}x + b$ → $a^{[1]} = \sigma(z^{[1]})$ → $z^{[2]} = w^{[2]}.a^{[1]} + b^{[2]}$ → $a^{[2]} = \sigma(z^{[2]})$

$w^{[2]}$, $b^{[2]}$

$$dz^{[1]} = w^{[2]T}.dz^{[2]} * g^{[1]\prime}(z^{[1]})$$

$$dw^{[1]} = dz^{[1]}.x^T$$

$$db^{[1]} = dz^{[1]}.$$

$$dz^{[2]} = a^{[2]} - y$$

$$dw^{[2]} = dz^{[2]}.a^{[2]T}$$

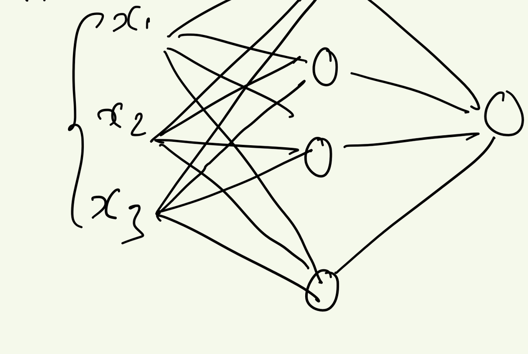
$$db^{[2]} = dz^{[2]}$$

$$w^{[2]} = n^{[2]} * n^{[1]}$$

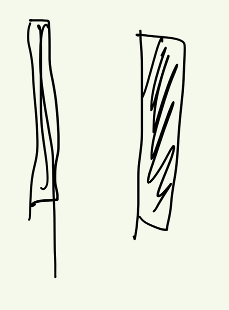
$dz^{[2]}, z^{[2]} = n^{[2]} * 1$ { if it's the last node }

$dz^{[1]}, z^{[1]} = n^{[1]} * 1.$ { if it's the first node }

↳ take care of rest



→ $x^{(1)}$ $x^{[2]}$ ... x^m



$n_x = n^{[0]}$ $\quad n^{[1]}$ $\quad n^{[2]}$

$n(x)$ → no. of features