Discrete Time Crystal: The Vanilla Unitary as a Brickwall Circuit

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1. Model

The model we will be concerned with is the brick-wall circuit from Ipolitti et al [**Khemani2021DTCinNISQ**, **ippolitti2022time**]. The General Floquet Model considered here, that exhibits a Discrete Time Crystal in 1D is an disordered Ising model with periodic $\pi/2$ Kicks about the x-axis. The system is probed only at stroboscopic times, $t = n\tau$, and $\tau = 1$

$$U_F = e^{-ig\sum_i X_i} e^{-i\tau(\sum_i J_i Z_i Z_{i+1} + H_{int})} e^{-i\tau(\sum_i h_i Z_i)} \tag{1}$$

Here we have adopted the standard information theoretic notation (i.e have denoted Pauli operator $\sigma_i^Z = Z_i$ etc.) The H_{int} denotes some generic interaction such as fields or coupling,

$$H_{int} = \frac{\theta}{2}\sum_i [X_i X_{i+1} + Y_i Y_{i+1}] (\text{XY Coupling})$$

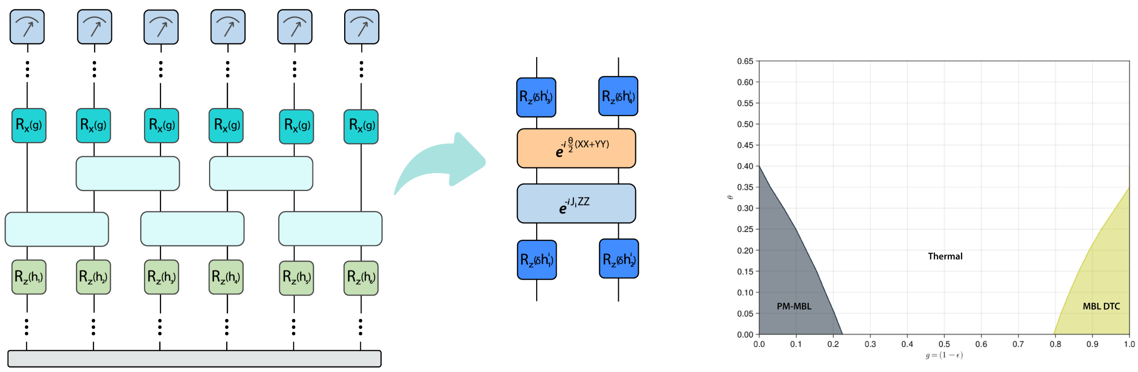

Figure 1. Schematic representation of the circuit and the associated phase diagram. The circuit exhibit three phases. For smaller values of θ and ϵ close to 1, that is g close to 0, it can support a MBL phase which is paramagnetic in nature. For small θ and ϵ close to 0, that is g close to 1, it supports a DTC MBL phase with a period multiplicity of 2. In the middle region the system tend to scramble. The phase boundary was extracted from results of [**Khemani2021DTCinNISQ**], and was obtained from finite size scaling of the disorder averaged level spacing ratio.

Among these parameters J_i, h_i are disordered and are uniformly sampled from $[0, \frac{\pi}{2}]$ to ensure localisation and thereby to prevent the system from heating up to triviality from the kicks [**Khemani2015phasestructre**]. In absence of interaction and with perfect kicks it is trivial to see that operators of the form $\langle Z(0), Z(n)\rangle$ breaks the discrete time symmetry and exhibits a period doubling. The non-triviality of the DTC as a *phase* comes from the fact that this feature persists strongly even in the presence of interactions as well as with imperfect kicks (i.e $g = \frac{1}{2} = (\pi - \epsilon)$ with finite but small ϵ).

In a quantum device this is realised as a brick-wall circuit.

We also add further small uncertainties in the realisations of the two body gates

$$\theta = [\bar{\theta} - \frac{\Delta\theta}{2}, \bar{\theta} + \frac{\Delta\theta}{2}] \tag{2}$$

where $\Delta\theta = \frac{\pi}{50}$ and random single body Z gates, $RZ(\Delta h)$, to both the legs before and after the application of the two body gates. $\Delta h = \frac{\pi}{50}$.

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