Chapter 4

Signal Generators

Square Wave Generator

A square wave is a *periodic* wave that has two distinct and fixed levels, and oscillates between these two values. They do not necessarily have to be of opposite polarity. The amount of time allocated for the two levels may also vary. A few examples are as follows:

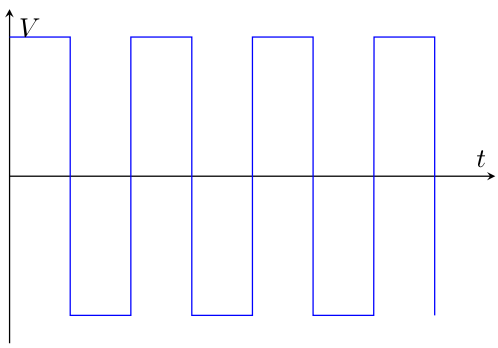

Figure 4.1: A symmetric square wave

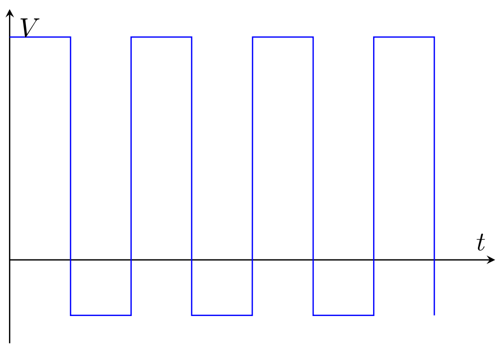

Figure 4.2: A square wave with offset

Now, to produce such a signal, we first need two distinct voltage levels. Can you recall any such components that only outputs two voltages? Yes, it's the Op Amp, when acting as a comparator or Schmitt Trigger. Which one should we use? Well, as we can see in the above figures, we need switching between the voltage levels at multiple instances, which means varying thresholds. So, the comparator is not of much use here. We will need to use the Schmitt Trigger, but with some modifications. One of the terminal voltage needs to oscillate between the thresholds, so that we get switching between V_H & V_L at the output. Let's see the circuit without further trailers.

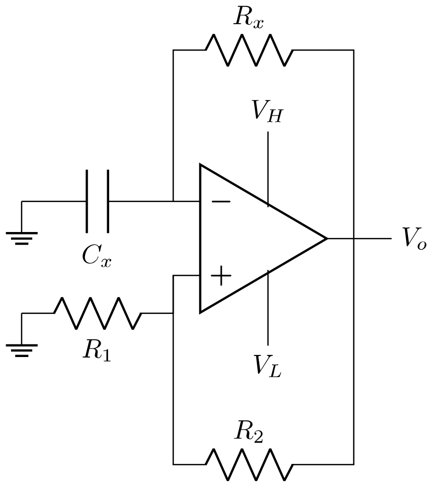

Figure 4.3: A Square Wave Generator

Although the circuit in figure 4.3 seem to have both feedback networks, the output behaviour is dictated by the positive feedback. Hence, it is effectively a *non-inverting Schmitt Trigger*. The input voltage of it is the voltage across the capacitor C_x. Let's now see in details how does this produce a square wave.

Analysis of Operation: Since no current is drawn at any terminals of the Op Amp, we can isolate the negative feedback network and perform a separate analysis. It is an *RC circuit* as shown below:

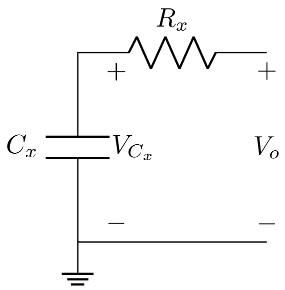

Figure 4.4: RC circuit of the negative feedback network

Since the output voltage, V_o has only two possible distinct DC values (V_H & V_L), the RC circuit in figure 4.4 will always have a DC voltage applied across it, irrespective of the value of V_o.

Now recall from CSE250, for a simple series RC circuit, if a DC voltage of V_s is applied, the voltage response of the capacitor is as follows:

$$V_c(t) = V_s + [V_c(t_i) - V_s]e^{-\frac{t-t_i}{\tau}}$$

For our circuit in figure 4.4, we can write,

$$V_{C_x} = V_o + [V_{C_x}(t_i) - V_o]e^{-\frac{t-t_i}{\tau}} \tag{4.1}$$

Here, $\tau = R_x C_x$

We have already said that due to positive feedback, the output voltage can only have the values V_H & V_L. And since it's an inverting Schmitt Trigger, assuming V_o to be V_H initially, it will switch from V_H to V_L, when the inverting terminal voltage rises above V_{TH}. Let the switching instant is T_1. Also, we may assume the initial voltage across the capacitor to be V_{TL}. Since this is the highest possible value up to which the output would stay V_L. So, basically we are considering the starting point of our analysis the moment when the output has just switched from V_L to V_H because the capacitor has reached V_{TL}. Assuming $t_i = 0$, these informations can be put together in equation 4.1 in the following manner:

$$V_{TH} = V_H + [V_{TL} - V_H]e^{-\frac{T_1 - 0}{R_x C_x}}$$

$$\Rightarrow e^{-\frac{T_1}{R_x C_x}} = \frac{V_{TH} - V_H}{V_{TL} - V_H} \Rightarrow -\frac{T_1}{R_x C_x} = ln(\frac{V_{TH} - V_H}{V_{TL} - V_H})$$

$$\Rightarrow T_1 = R_x C_x ln(\frac{V_{TL} - V_H}{V_{TH} - V_H}) = R_x C_x ln(\frac{V_{TL} - V_H}{V_{TH} - V_H}) \tag{4.2}$$

Let's get a visual representation of what has happened till now.

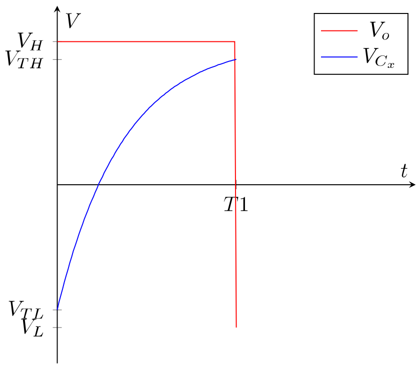

Figure 4.5: Response of Square Wave Generator(half period)

Does figure 4.5 ring any bell? The output voltage has covered the half period of a square wave. At T_1, V_o has switched to V_L since, the inverting terminal input, V_{C_x} has reached V_{TH}. Let's complete our analysis for the second half.

The next switching will occur when V_{C_x} reaches V_{TL} again. Then the output will switch back to V_H. Let this happens at $t = T_1 + T_2$. Using equation 4.1, we can write,

$$V_{TL} = V_L + [V_{TH} - V_L]e^{-\frac{T_1 + T_2 - T_1}{R_x C_x}}$$

$$\Rightarrow e^{-\frac{T_2}{R_x C_x}} = \frac{V_{TL} - V_L}{V_{TH} - V_L} \Rightarrow -\frac{T_2}{R_x C_x} = ln(\frac{V_{TL} - V_L}{V_{TH} - V_L})$$

$$\Rightarrow T_2 = R_x C_x ln(\frac{V_{TH} - V_L}{V_{TL} - V_L}) \tag{4.3}$$

Now let's complete our square wave:

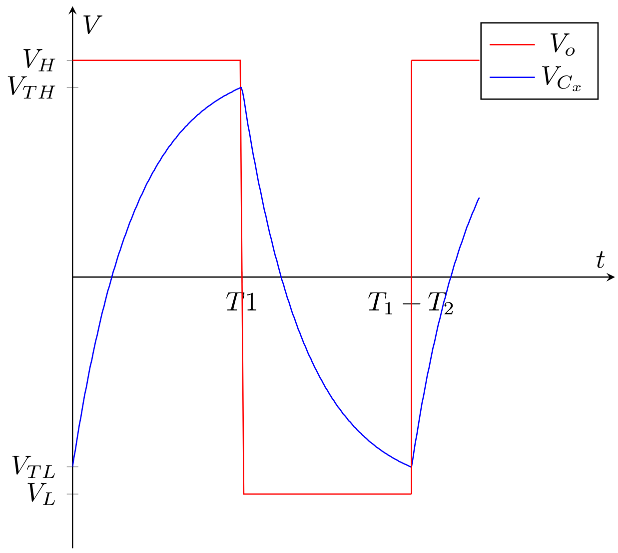

Figure 4.6: Response of Square Wave Generator

Thus, the capacitor voltage V_{C_x} oscillates between V_{TL} & V_{TH} and the output voltage changes accordingly between V_L & V_H. The result is the square wave with high peak of V_H and low peak of V_L.
Some important parameters & facts of Square Wave Generator:

1. **Time Period:** The time period of the generated square wave is $T = T_1 + T_2$.

2. **Duty Cycle:** Duty cycle for the square wave is considered to be the fraction of the period it attains the high peak. Thus,

$$D = \frac{T_1}{T_1 + T_2} \times 100 \tag{4.4}$$

3. From the default circuit in figure 4.2, it is an inverting Schmitt Trigger without any applied reference voltage. Thus, its thresholds will be, $V_{TH} = \frac{R_1}{R_1+R_2}V_H$ & $V_{TL} = \frac{R_1}{R_1+R_2}V_L$.

4. To get a **50%** duty cycle, we need $T_1 = T_2$. For that to happen,

$$R_xC_xln\left(\frac{V_{TL} - V_H}{V_{TH} - V_H}\right) = R_xC_xln\left(\frac{V_{TH} - V_L}{V_{TL} - V_L}\right)$$

$$\Rightarrow \frac{V_{TL} - V_H}{V_{TH} - V_H} = \frac{V_{TH} - V_L}{V_{TL} - V_L} \quad \Rightarrow \quad \frac{\left(\frac{R_1}{R_1+R_2}V_L\right) - V_H}{\left(\frac{R_1}{R_1+R_2}V_H\right) - V_H} = \frac{\left(\frac{R_1}{R_1+R_2}V_H\right) - V_L}{\left(\frac{R_1}{R_1+R_2}V_L\right) - V_L}$$

$$\Rightarrow \frac{\frac{R_1}{R_1 + R_2}V_L - V_H}{V_H} = \frac{\frac{R_1}{R_1 + R_2}V_H - V_L}{V_L}$$

$$\Rightarrow \frac{R_1}{R_1 + R_2}(\frac{V_L}{V_H} - \frac{V_H}{V_L}) = 0$$

$$\Rightarrow |V_H| = |V_L|$$

Triangular Wave Generator

Like the square wave, a triangular wave also has two peaks. However, it switches between these two peaks with a finite slope, unlike the vertical switching of the square wave.

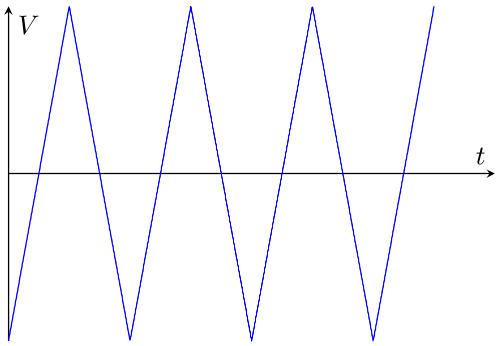

Figure 4.7: A symmetric triangular wave

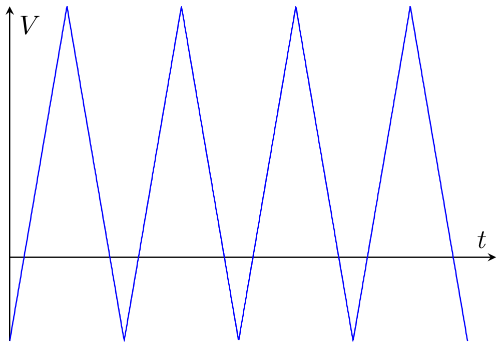

Figure 4.8: A triangular wave with offset

So, how do we generate such signals? The straight forward answer is by integrating a square wave. How? Well, we know that the integral of a constant value is a straight line with a slope proportional to that value. Sounds tough to understand? Just have a look below:

$$\int c\,dx = cx$$

Since the square wave is made up of two constant peaks, they intigrate to straight lines proportional to their peaks. So, do we simply pass the output of our previously studied square wave generator to an integrator? Well, we can, but there is a better method. Let's look at the better circuit, the triangular wave generator: