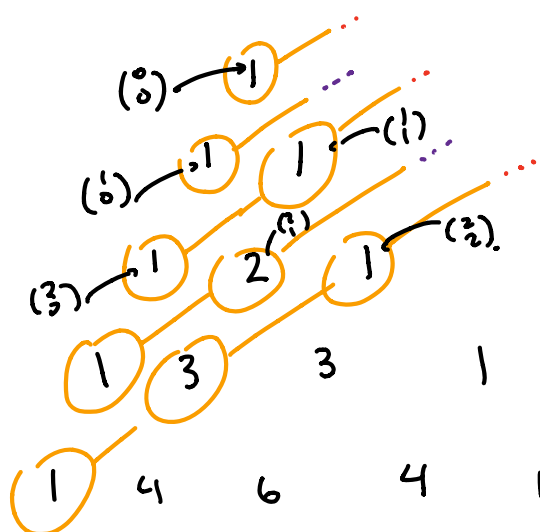


10	$3+7$
11	\times
12	$3 \cdot 4$
13	$2 \cdot 3 + 7$
14	$2 \cdot 7$
15	$3 \cdot 5$
16	$3 \cdot 3 + 7$
17	$1 \cdot 3 + 2 \cdot 7$
18	$3 \cdot 6$
19	$4 \cdot 3 + 7$
20	$2 \cdot 3 \cdot 2 \cdot 7$

$$n = \begin{cases} 3 \cdot \frac{n}{3} & \text{if remainder when } n \text{ is divided by 3 is 0} \\ \boxed{} & \dots 1 \\ \boxed{} & \dots 2 \end{cases}$$



2.3.12. $\gcd(a, a+n) \mid n$. \downarrow prove thm (I)
 then, use this to deduce that $\gcd(a, a+1)=1$. (II)

(II) $\gcd(a, a+1) \mid 1$ by the result applied in the case $n=1$.
 By thm 2.2(b), $\gcd(a, a+1) = \pm 1$. But gcds are defined to be positive, so $\gcd(a, a+1) = 1$.

(I)

$$(\underbrace{ax + (a+n)y}_{\uparrow})k = n$$

thm 2.3 tells us that gcd's can be written in this form.

→ corollary says that anything that's written in this form is div. by gcd.

$$n = \overset{-1}{\downarrow} ax + (a+n) \overset{1}{\downarrow} y = \cancel{(-a+a)} + n$$

$$n = \underbrace{-a + ay}^{+1} + \underbrace{ny}^{-1}$$

Notice that $n = -1 \cdot a + 1 \cdot (a+n)$, ie, it is a linear combination of a & $a+n$. Since $\gcd(a, a+n)$ divides every linear combination of a & $a+n$ by corollary to thm 2.3, we conclude that $\gcd(a, a+n) \mid n$.