$76^{76^{76}}$ remainder mod 7.

$76^{76}$

gcd(7, 76) = 1, so we can apply Fermat's thm.

$76^{7-1} \equiv 1 \pmod 7$

$76^{6} \equiv 1 \pmod 7$

$(76^{6})^{12} \equiv 1^{12} \equiv 1$

$76^{72} \equiv 1 \pmod 7$

$76^{72} \cdot 76^{4} \equiv 76^{4}$

$76^{76} \equiv 76^{4} \pmod 7$.

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If $a \equiv b \pmod n$, then $a^k \equiv b^k \pmod n$ for any $k$.

$76 \equiv 6 \mod 7$.

$76^2 \equiv 36 \mod 7$

$\equiv 1 \mod 7$. ?

$76^{76} \equiv 1 \mod 7$.

$76^{76} = 1 + 76$.

$76^{76^{76}} \overset{!}{\equiv} 6^{1+76}$

$76 \equiv 6 \rightsquigarrow 76^2 \equiv 6^2$

$\longrightarrow 76^{76} \equiv 6^{76}$

$76^{76} \equiv 6^{1}$

$76^2 \equiv 1^2$

$76^4 \equiv 1^4$

$76^8 \equiv 1^8$

$\vdots$

$76^{64} \equiv 1$

$76^{12} \equiv (7^2)^6 \equiv 1$

$76^{2k} = (76^2)^k \equiv 1^k = 1$

$76^{76}$ even

$\Rightarrow 76^{76^{76}} \equiv 1$

$$\frac{1}{p_1} + \underbrace{\frac{1}{p_2} + \cdots + \frac{1}{p_d}}_{\frac{k}{p_1}}$$

$p_1 = 2.$

$\frac{1}{2} + \cdots$

$\frac{1}{3} + \left[ a + \frac{2}{3} \right] = \frac{1}{3} + \left[ \frac{3a+2}{3} \right]$

$\vdots$

$$\frac{1}{p_1} + \frac{k}{p_1} = \frac{k+1}{p_1}$$

$\frac{1}{p_2} + \cdots + \frac{1}{p_d}$ can't equal $\frac{k}{p_1}$ for any $k$ ?

$$\frac{?}{p_2 \cdots p_d} = \frac{k}{p_1} \quad \overset{?}{\rightsquigarrow}$$

$$\frac{1}{p_1} + \cdots + \frac{1}{p_d} = \frac{?}{p_1 \cdots p_d} \rightsquigarrow$$

$$= \frac{p_2 \cdots p_d}{p_1 \cdots p_d} + \frac{p_1 p_3 \cdots p_d}{p_1 \cdots p_d} + \cdots + \frac{p_1 p_2 \cdots p_{d-1}}{p_1 \cdots p_d}$$

$$= \frac{p_2 \cdots p_d + p_1 p_3 \cdots p_d + \cdots + p_1 p_2 \cdots p_{d-1}}{p_1 \cdots p_d}$$

$5 \mid ( ?? + 15)$

$$= \frac{p_2 \cdots p_d + p_1 (p_3 \cdots p_d + \cdots + p_2 \cdots p_{d-1})}{p_1 \cdots p_d}$$

$$p_1 \mid \left[ (p_2 \cdots p_d) + p_1 (p_3 \cdots p_d + \cdots + p_2 \cdots p_{d-1}) \right]$$

$$\rightsquigarrow p_1 \mid p_2 \cdots p_d$$