$n = 9$ $\quad (n-1)! = 8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

$\uparrow$ $\quad$ $2a$ $\quad$ a

$3 \cdot 3$

n composite $\Longrightarrow$ $n = ab$ where $1 < a \leq b < n$.

case 1. $a < b$ ...

case 2. $a = b$ ... [important that $n \neq 4$]

$n \neq 4.$

$n \geq 5.$

$a = \sqrt{n} \geq \sqrt{5} > 2$

$a \geq 3$

$n = a^2$

$2a \leq n-1$

— $2a \leq a^2 - 1$ for all $a \geq 3$.

could do this by induction.

$a > 2.$

$2 < a$

$2a < a \cdot a$

— since $a > 2$,

$2a < a \cdot a = a^2 = n$

$2a \leq n-1.$

— suppose that $2a \geq n$

$2 \geq \frac{n}{a} = a$

contradiction!

---

a \equiv b \pmod{n}

$n = 6$

LHS: $\gcd(1,6) + \gcd(2,6) + \gcd(3,6) + \gcd(4,6) + \gcd(5,6) + \gcd(6,6).$

$1 \quad 2 \quad 3 \quad 2 \quad 1 \quad 6$

RHS $\quad d = 1, 2, 3, 6$

$\frac{n}{d} = 6, 3, 2, 1$

$\varphi\left(\frac{n}{d}\right) = 2, 2, 1, 1$