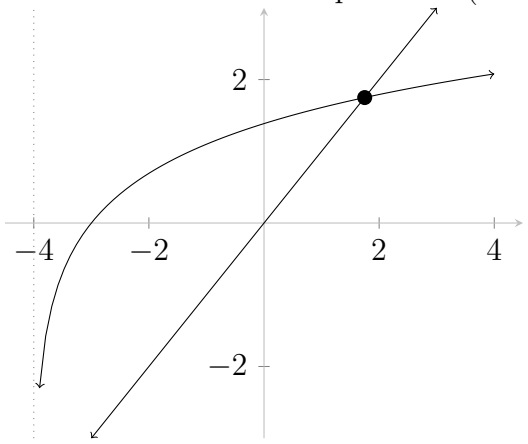
WORKSHEET: LINEAR APPROXIMATION

1. Use at least 2 steps of Newton's Method to approximate the following.
   (a) Approximate the solution to $x^4 - 6x^2 + x + 5 = 0$ that is closest to $x_0 = 2$.
   (b) Approximate the unique positive solution to the equation $\ln(x + 4) = x$.



2. The radius of a spherical ball is measured at $r = 25$ cm. Estimate the maximum error in the volume and the surface area of the ball if r is accurate to within 0.5 cm.
3. If you deposit P dollars in a retirement fund every year for N years with the intention of then withdrawing Q dollars per year for M years, you must earn interest at a rate $r > 0$ satisfying

$$P(b^N - 1) = Q(1 - b^{-M})$$

   where $b = 1 + r$. Assume \$2000 is deposited each year for 30 years and the goal is to withdraw \$10,000 per year for 25 years. Use Newton's method to compute b, and then find r.
4. (Challenging) Approximate the coordinates of point P on the graph of $f(x) = \cos(x)$ such that the tangent line at P passes through the origin.