1. True or False?

$\mathbb{R}^2$ is isomorphic to

$$U = \left\{ \begin{pmatrix} a \\ a+b \\ b \end{pmatrix} \in \mathbb{R}^3 : a, b \in \mathbb{R} \right\}.$$

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Follow-up. If you think $\mathbb{R}^2$ and U are isomorphic, find an isomorphism $f : \mathbb{R}^2 \to U$ and then find its inverse.

2. True or False?

The function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = 2x - 1$ is linear.

3. True or False?

The map $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^3$ is an isomorphism.

4. True or False?

The map $f : \mathbb{R}^2 \to \mathbb{R}^3$ given by

$$\begin{pmatrix} a \\ b \end{pmatrix} \mapsto \begin{pmatrix} a \\ a+b \\ 2a-b \end{pmatrix}$$

is an isomorphism.

4. True or False?

The map $f : \mathbb{R}^2 \to \mathbb{R}^3$ given by

$$\begin{pmatrix} a \\ b \end{pmatrix} \mapsto \begin{pmatrix} a \\ a+b \\ 2a-b \end{pmatrix}$$

is an isomorphism.

Follow-up. Is it injective? Is it surjective?

5. True or False?

The map $f : \mathcal{P}_2 \to \mathcal{P}_2$ given by

$$f(ax^2 + bx + c) = bx^2 - (a + c)x + a$$

is an isomorphism.

5. True or False?

The map $f : \mathcal{P}_2 \to \mathcal{P}_2$ given by

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is an isomorphism.

Follow-up. If you think it is an isomorphism, what is the inverse of f?