1. True or False?

The linear transformation $h : \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x + 2y \\ 2x + y \end{pmatrix}$$

is diagonalizable.

2. True or False?

Let V the vector space of all functions $\mathbb{R} \to \mathbb{R}$, and let $h : \mathbb{R}^2 \to V$ be the linear map such that

$$\begin{pmatrix} a \\ b \end{pmatrix} \mapsto a2^x + b3^x.$$

Then h is injective.

3. True or False?

If A is a matrix such that $A^3 = 0$, then A must have exactly one eigenvalue.

4. Let V be the null space of the map $h : \mathcal{P}_2 \to \mathbb{R}^3$ given by

$$a + bx + cx^2 \mapsto \begin{pmatrix} a+b \\ a+b \\ c \end{pmatrix}.$$

What is $\dim(V)$?

(A) 0

(B) 1

(C) 2

(D) 3

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$$a + bx + cx^2 \mapsto \begin{pmatrix} a+b \\ a+b \\ c \end{pmatrix}.$$

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Follow-up. What is $\dim(\mathscr{R}(h))$?

5. True or False?

The linear map $h : \mathbb{R}^3 \to \mathbb{R}^3$ given by

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} 2x + y \\ 3y + z \\ 2z \end{pmatrix}$$

is diagonalizable.

6. True or False?

Suppose A and B are both singular 2×2 matrices and $\lambda = 1$ is an eigenvalue for both A and B. Then A and B must be similar.

7. Suppose $B = \langle 3x, 2 + x^2, x^2 \rangle$ is a basis for $\mathcal{P}_2$, C is the standard basis for $\mathbb{R}^2$, and $h : \mathcal{P}_2 \to \mathbb{R}^2$ is the linear map such that

$$\mathrm{Rep}_{B,C}(h) = \begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & -1 \end{pmatrix}.$$

Which of the following is a basis for $\mathscr{N}(h)$?

(A) $\langle \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} \rangle$

(B) $\langle 2 + x + x^2 \rangle$

(C) $\langle 1 + x + 2x^2 \rangle$

(D) $\langle 2 + 6x + 2x^2 \rangle$

8. True or False?

The following two matrices are similar.

$$\begin{pmatrix} 1 & 3 \\ 1 & 3 \end{pmatrix} \qquad \begin{pmatrix} 0 & 0 \\ 0 & 4 \end{pmatrix}$$

9. Let

$$B = \langle \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} \rangle$$

and let V be the subspace of $\mathbb{R}^3$ spanned by B. For which of the following vectors $\vec{v}$ is $\mathrm{Rep}_B(\vec{v}) = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$?

(A) $(1, 1)$

(B) $(1, 1, 0)$

(C) $(1, 6, 1)$

(D) $(1, 3, 0)$

10. True or False?

Let $B = \langle 1, 1+x, 1+x+x^2 \rangle$ and let $h : \mathcal{P}_2 \to \mathcal{P}_2$ be the linear map such that

$$\mathrm{Rep}_{B,B} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}.$$

Then $4 + 3x + 2x^2$ is in $\mathscr{R}(h)$.

11. Let V be the set of all 3×3 upper triangular singular matrices. Which of the following is most accurate?

(A) V is a 6 dimensional vector space.

(B) V is a 5 dimensional vector space.

(C) V is a 4 dimensional vector space.

(D) V is not a vector space.

12. Let $V = \{p \in \mathcal{P}_3 : p(0) = p(1) = 0\}$. What is $\dim(V)$?

(A) 0

(B) 1

(C) 2

(D) 3

13. True or False?

If three polynomials in $\mathcal{P}_2$ all have degree 2, they must be linearly dependent.

14. True or False?

The map $h : \mathcal{P}_2 \to \mathcal{P}_2$ given by

$$a + bx + cx^2 \mapsto (a + b + c) + (a + b)x + ax^2$$

is an isomorphism.

15. True or False?

There exist bases B and C of $\mathbb{R}^2$ such that

$$\mathrm{Rep}_{B,C}(\mathrm{id}) = \begin{pmatrix} 1 & 3 \\ 2 & 6 \end{pmatrix}$$