Let $B=\langle 1,1+x,1+x+x^2\rangle$ be a basis for \mathcal{P}_2 . Then the map $\operatorname{Rep}_B:\mathcal{P}_2\to\mathbb{R}^3$ given by

$$p\mapsto \mathsf{Rep}_B(p)$$

is an isomorphism.

The function $f: \mathbb{R}^2 \to \mathbb{R}^3$ given by

$$\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x \\ x+y \\ y \end{pmatrix}$$

is linear.

The function $f: \mathbb{R}^2 \to \mathbb{R}^3$ given by

$$\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x \\ x+y \\ y \end{pmatrix}$$

is linear.

Follow-up. What is $\mathcal{N}(f)$? What is $\mathcal{R}(f)$?

The function $f: \mathbb{R} \to \mathbb{R}$ given by f(x) = 2x - 1 is linear.

Every nonzero linear map $f: \mathbb{R}^2 \to \mathbb{R}^2$ is an isomorphism.

Every nonzero linear map $f : \mathbb{R} \to \mathbb{R}$ is an isomorphism.