1. True or False?

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What if the word "injective" is replaced with the word "surjective" in the above statement?

2. Regard $\mathbf{C}$ as a real vector space and $\mathbf{R}$ as a subspace of $\mathbf{C}$. Which of the following is a basis for the quotient $\mathbf{C}/\mathbf{R}$?

(A) $1, i$

(B) $1 + \mathbf{R}, i + \mathbf{R}$

(C) $573.1224 + \mathbf{R}$

(D) None of the above.

3. True or False?

Suppose $T \in \mathcal{L}(\mathbf{C}^3)$ is surjective. Then $\mathbf{C}^3/\operatorname{null} T = \{0\}$.

4. Suppose V is an n-dimensional vector space and $\phi \in \mathcal{L}(V, \mathbf{F})$ is nonzero. Then $\dim(V/(\operatorname{null}\phi))$ is...

(A) 1

(B) $n-1$

(C) n

(D) None of the above.

5. True or False?

Suppose V is a vector space and U is a subspace such that $v_1 + U, v_2 + U$ is a basis for V/U. Then

$$V = U \oplus \operatorname{span}(v_1, v_2).$$

6. True or False?

Suppose $U = \{(x, x, y) : x, y \in \mathbf{C}\} \subseteq \mathbf{C}^3$. Then

$$(i, 1+i, 1) + U = (1, 2, 17) + U.$$

7. True or False?

If V and W are finite dimensional, then $\mathcal{L}(V, W)$ is isomorphic to $\mathcal{L}(W', V')$.

8. True or False?

Suppose U and W are subspaces of a vector space V. Then

$$(U + W)^0 = U^0 \cap W^0.$$

9. Suppose V is a vector space, $T \in \mathcal{L}(V)$ and v_1, v_2, v_3 is a basis of V such that

$$M(T) = \begin{pmatrix} 0 & 2 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Let $U = \operatorname{span}(v_1)$ and observe that $v_2 + U, v_3 + U$ is a basis for U. Let $\pi : V \to V/U$ be the quotient map given by $\pi(v) = v + U$. What is $M(\phi \circ T)$ with respect to the basis v_1, v_2, v_3 for V and $v_2 + U, v_3 + U$ for V/U?