Week 9 Day 1

Favorite Vector Space

Make sure you know your neighbors' names. Then take 2 minutes to discuss:

If you had to choose a favorite vector space, which would you choose? Why? Does your answer change if you weren't allowed to choose $\mathbb{R}^n$ for any n?

Orthogonality

1. Which of the following vectors in $\mathbb{R}^2$ is orthogonal to $(1, 1)$?

(A) $(-1, 1)$

(B) $(1, -1)$

(C) $(-1, -1)$

(D) None of the above OR more than one of the above.

2. On $\mathbb{P}_2$, consider the inner product

$$\langle p, q \rangle = p(-1)q(-1) + p(0)q(0) + p(1)q(1).$$

Which of the following polynomials q is orthogonal to $p(t) = t^2$?

(A) $q(t) = 1$

(B) $q(t) = t$

(C) $q(t) = 1 - t^2$

(D) None of the above OR more than one of the above

3. Which of the following is an orthogonal basis for $\mathbb{R}^3$?

(A) $(1, 0, 0), (0, 1, 0), (0, 0, 1)$

(B) $(1, 1, 1), (-1, 1, 0), (-1, 0, 1)$

(C) $(1, 1, 1), (-1, 1, 0), (-1, -1, 2)$

(D) None of the above OR more than one of the above.

4. $\mathscr{B} = \{(1,1,1),(-1,1,0),(-1,-1,2)\}$ is an orthogonal basis for $\mathbb{R}^3$. For $\mathbf{v} = (1,0,0)$, find $[\mathbf{v}]_{\mathscr{B}}$.

5. On $\mathbb{P}_2$, consider the inner product

$$\langle p, q \rangle = p(-1)q(-1) + p(0)q(0) + p(1)q(1).$$

Is $\mathscr{B} = \{1 - t^2, t, t^2\}$ an orthogonal basis for $\mathbb{P}_2$? What is $[1 + t + t^2]_{\mathscr{B}}$?