# ZOMBIE PROJECT

## 1. Data Generation

There's a zombie outbreak! In a small town of 50 inhabitants, one person has been infected by a zombie virus. Form groups of 3 and decide on some infection rules and movement rules. How many humans/cells can one zombie infect? How many cells can a human move in a single time step? How many cells can a zombie move in a single time step? How do you deal with the edge of the grid? To simplify things, let's assume that humans and zombies do not reproduce or die, so that the total number of zombies and humans combined is always 50, and also that humans cannot recover from zombiehood. Aside from that, feel free to decide on rules however you like, and to clearly record all of the rules you're using to generate your data.

Let $z(t)$ and $h(t)$ denote the numbers of zombies and humans at time $t$, respectively. We know that $z(0) = 1$ and $h(0) = 49$. Set up a spreadsheet[1] like the table below.

| $t$ | $z(t)$ | $h(t)$ | $\Delta z/\Delta t$ | $\Delta h/\Delta t$ | | |
|---|---|---|---|---|---|---|
| 0 | 1 | 49 | | | | |
| 1 | | | | | | |
| 2 | | | | | | |
| 3 | | | | | | |

Let one person in the group place 1 zombie on one of the hex grids using a marker of one color, and let 1 person place 49 humans on the other hex grid using a marker of a different color. Overlap the grids, and use the infection rules you decided on to figure out which humans got infected. Record the number of zombies $z(1)$. Then use your movement rules to move the humans and zombies around, and repeat the process to record $z(2), z(3), \ldots$.

Keep going until you can see that $z(t)$ will not change anymore. When you finish, use the spreadsheet to figure out $\Delta z/\Delta t$ and $\Delta h/\Delta t$ for each time step. As you proceed with the modeling, you might also decide to add more columns that calculate other things.

Generate a line chart plotting $z(t)$ against $t$.

**Submission.** By 11:59pm on second Wednesday, one person in each group should send me an email telling me everyone who's in your zombie project group and attaching a PDF of the line chart of $z(t)$ your group generated from the data.

## 2. Modeling

**Set up a differential equation.** Start by making lots and lots of plots! Maybe make a line chart plotting $h(t)$ against $t$, or $\Delta z/\Delta t$ against $t$, or $\Delta z/\Delta t$ against $z$, or $(\Delta z/\Delta t)/z$ against $h$, or... The possibilities are endless!

[1] Google Sheets, LibreOffice Calc, Microsoft Excel, ...

Use your plots to generate a differential equation relating $dz/dt$ to $z$. For example, if you happened to find that the line chart plotting $\Delta z/\Delta t$ against $z$ looks kind of like the line $y = 3x$, this suggests that $z(t)$ is governed by the differential equation

$$\frac{dz}{dt} = 3z(t).$$

Note that differential equations like this will have many different solutions; to single one out, you have to specify some *initial conditions*. For us, we know that $z(0) = 1$, and we can use this as our initial condition.

**Solve the differential equation.** We don't know how to solve differential equations yet. We'll discuss some easy ones a bit later in the course; to solve harder ones, you'll have to wait till you take MA315 (ordinary differential equations).

But that's okay, you can just get a computer to figure out a solution for you! For example, if you type in

`z'(t) = 3*z(t) and z(0) = 1`

into Wolfram Alpha, you'll find that the solution to the differential equation $dz/dt = 3z(t)$ subject to the initial condition $z(0) = 1$ is the function $z(t) = e^{3t}$.

You can also use the open-source mathematical computing software Sage to solve differential equations (and also to plot the solutions, and also to plot your data). Alternatively, you could also decide to use Mathematica, which isn't free, but you'll have access to a license for Mathematica during your time at CC.

**Analyze the solution.** Generate a plot of the solution to the differential equation you've found, subject to the initial condition $z(0) = 1$, and compare it to the plot of the original data.

- Describe the plot of the solution to the differential equation. On what intervals is it increasing/decreasing? What is its concavity? Points of inflection? Does these mathematical properties of the solution make sense to you, knowing that the solution is supposed to model the zombie population?
- Are there certain time intervals where the model describes the actual data well? Can you explain why the model works well using the differential equation?
- Are there certain time intervals where the model describes the actual data very poorly? Can you explain why the model doesn't work well using the differential equation?

**Repeat.** Go back and adjust your differential equation to try and correct its deficiencies, and then analyze your new model! Repeat until you've found something you're reasonably happy with.

**Submission.** By 11:59pm on the third Monday, one person in each group should me an email attaching a PDF that describes at least two differential equations that your group used to try and model your data. For each differential equation, there should be a graph of the solution to your differential equations plotted on the same graph as your data,[2] so that it's clearly visible how well the model works. Also include some discussion about each of your models, answering the questions described above, and also comparing the models you came up with.

[2] I highly recommend you try to figure out how to get a computer to do this; it's good to have practice learning how to get computers to do things for you. If no one in your group can figure it out, you can do it by hand and scan it in, but make sure it's a clean plot.