1. True or False?

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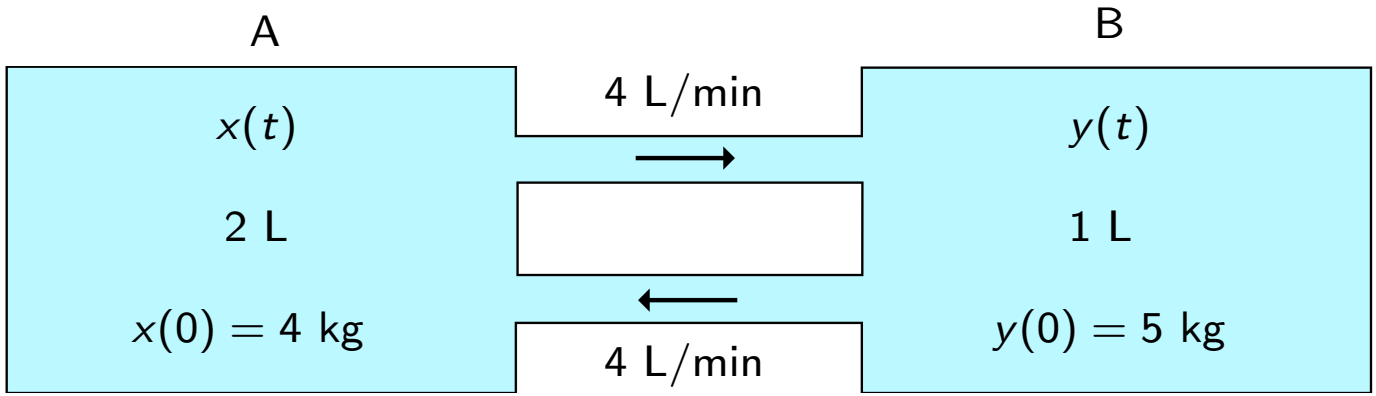
Follow-up. What does the phase portrait look like for various possiblities of a?

More interconnected salt-water tanks!



2. True or False?

$$\lim_{t\to\infty} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} 6 \\ 3 \end{bmatrix}$$

3. If we rewrite the linear system of ODEs

$$\begin{cases} x'' = t^2 x + y' + x' \\ y''' = tx' + \sin(t)y \end{cases}$$

as a first order system $\vec{x}\,' = A\vec{x}$ for a matrix A, what are the dimensions of the matrix A?

(A) 2×2

(B) 3×3

(C) 4×4

(D) 5×5

4. Consider the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.

Which of the following vectors is a generalized eigenvector of A but *not* an eigenvector?

(A) $\vec{e}_1 = (1, 0, 0)$

(B) $\vec{e}_2 = (0, 1, 0)$

(C) $\vec{e}_3 = (0, 0, 1)$

(D) None of the above

Consider the matrix $A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$.

Observe that $\lambda = 2$ is the only eigenvalue, that $\vec{e}_1$ is an eigenvector, that $(A - 2I)\vec{e}_2 = \vec{e}_1$ and that $(A - 2I)\vec{e}_3 = \vec{e}_2$.

5. Which of the following is *not* a solution of the system $\vec{x}' = A\vec{x}$?

(A) $\vec{x}(t) = e^{2t}(1, 0, 0)$

(B) $\vec{x}(t) = e^{2t}(t, 1, 0)$

(C) $\vec{x}(t) = e^{2t}(0, t, 1)$

(D) $\vec{x}(t) = e^{2t}(t^2/2, t, 1)$