Day 22

1. Suppose a and b are nonidentity elements of different orders in a group G of order 35. Show that $\langle a, b \rangle = G$.

2. Is the permutation $\alpha = (1\ 3)(3\ 4\ 7)(2\ 4\ 3)(1\ 2)$ in S_7 even or odd?

3. Find all generators for the unique subgroup of Z_{20} that has order 4.

4. How many cosets of $\langle R_{90} \rangle$ are there in D_4? List the elements of each coset.

5. Calculate $9^{603} \bmod 7$.

6. Suppose $G = \langle g \rangle$ is a cyclic group of order 24. How many elements of g have order 3?

7. In S_4, let $H = \langle (1\ 2), (3\ 4) \rangle$. Show that Z_4 and H are *not* isomorphic.

8. Let G be a group. Show that G is cyclic of order n if and only if G is isomorphic to Z_n.

9. "$\mathrm{GL}(2, \mathbf{R})$ contains subgroups isomorphic to Z_n for all n."

This statement is...

(A) True.

(B) False.