Day 27

First Isomorphism Theorem

1. Let $\phi : Z_{10} \to Z_{10}$ be the homomorphism $\phi(x) = 2x$. What is $\ker \phi$? What is $\operatorname{im} \phi$?

2. Does there exist a surjective homomorphism $Z_{27} \oplus Z_3 \to Z_9 \oplus Z_9$?

3. Which U-group is the quotient $U(24)/U_{12}(24)$ isomorphic to?

This is an imprecise question without a straightforward "right answer," but it's still highly recommended!

4. Consider the surjective homomorphism $f : \mathbf{R} \oplus \mathbf{R} \to \mathbf{R}$ given by

$$f(x, y) = x + y.$$

Draw a picture of the induced isomorphism from $(\mathbf{R} \oplus \mathbf{R})/\ker f$ to $\mathbf{R}$.