# SRQR: Sets, Induction, Binomial Theorem (March 30, 2021)

> Section 1.1 mentions the second principle of finite induction and I was wondering if this was similar to the "minimal criminal" argument of induction?

> I am wondering the main difference between the first and the second principle of finite induction.

I like the phrase "minimal criminal" ☺. I learned that method under the name "minimal counterexample." In any case, the fact that that method of proof works is essentially due to the well-ordering principle.

The well-ordering principle, the first principle of induction (also just called "induction"), and the second principle of induction (also called "strong induction") are abstractly all equivalent to each other. Probably there are many situations where a proof written using one of these three principles can be re-formulated using another one. But sometimes one of those three is more convenient than the other two, so it's good to be familiar with all three.

In all three of these situations, you have some fixed integer $N$ (usually $N = 0$ or $N = 1$, but not always!) and some statements $P(n)$ for all $n \geqslant N$ that you'd like to prove to be true.

- When using the well-ordering principle, you'll assume that there exists a $n \geqslant N$ for which $P(n)$ is *not* true, and then find a contradiction.
- When using induction, you'll prove the base case $P(N)$. Then, for the inductive step, you'll prove $P(k+1)$ assuming you know $P(k)$.
- When using strong induction, there are actually two slight variants:
  - Variant 1: You'll prove the base case $P(N)$. Then, for the inductive step, you'll prove $P(k+1)$ assuming you know $P(k), P(k-1), \ldots, P(N)$ for all $k \geqslant N$.
  - Variant 2: You'll pick some appropriate positive integer $R$ and prove several base cases $P(N), P(N+1), \ldots, P(N+R)$. Then for the inductive step, you'll prove $P(k+1)$ assuming you know $P(k), P(k-1), \ldots, P(k-R)$ for all $k \geqslant N+R$.

The proof of formula (1) on pages 2–3 is an example of induction. The proof in example 1.1 of page 6 is an example of variant 2 of strong induction: here, $P(n)$ is the statement that $a_n < (7/4)^n$ and $R = 1$.

> How is the Lucas sequence different from the Fibonacci sequence?

The Lucas sequence as defined in example 1.1 is only different from the Fibonacci sequence in the choice of initial condition. It begins with $a_1 = 1$ and $a_2 = $, while the Fibonacci sequence usually begins $a_1 = a_2 = 1$. The recursive formula for higher terms is exactly the same!

> In example 1.1 (on page 6)... it lists four equations involving (7/4) and I'm lost as to how they jumped from line 3 to line 4.

In line three, we have something (namely, $(7/4)^{k-2}$) multiplied by 11/4. In line 4, we have the same thing multiplying by $(7/4)^2$. Notice that $(7/4)^2 = 3.0625 > 2.75 = 11/4$. That means that

$$(7/4)^{k-2}(11/4) < (7/4)^{k-2}(7/4)^2$$

which is precisely the same as what lines 3–4 in that displayed list of equations.