1. True or False?

Suppose $-4, 2, 17$ are eigenvalues of $T \in \mathcal{L}(\mathbf{R}^3)$. Then there exists $v \in \mathbf{R}^3$ such that $Tv = (11, 1, -5) + 9v$.

2. True or False?

The "forward shift" operator $T \in \mathcal{L}(\mathbf{C}^\infty)$ given by

$$T(z_1, z_2, \dots) = (0, z_1, z_2, \dots)$$

has no eigenvalues.

3. True or False?

There exists a finite dimensional vector space V and an operator $T \in \mathcal{L}(V)$ such that every nonzero $v \in V$ is an eigenvector of T.

4. True or False?

Suppose $T \in \mathcal{L}(V)$ and u, v are eigenvectors of T such that $u + v$ is also an eigenvector. Then u and v are eigenvectors corresponding to the same eigenvalue.

5. True or False?

If V is finite dimensional and U is a subspace such that U is invariant under T for every $T \in \mathcal{L}(V)$, then $U = \{0\}$ or $U = V$.