1. Consider the "forward shift" operator $T : \mathbf{F}^\infty \to \mathbf{F}^\infty$ given by

$$T(x_0, x_1, x_2, \dots) = (0, x_0, x_1, x_2, \dots).$$

(A) T is injective and surjective.

(B) T is injective but not surjective.

(C) T is surjective but not injective.

(D) T is neither surjective nor injective.

2. Consider the differentiation operator $D : \mathcal{P}(\mathbf{R}) \to \mathcal{P}(\mathbf{R})$, given by $D(f) = f'$.

(A) D is injective and surjective.

(B) D is injective but not surjective.

(C) D is surjective but not injective.

(D) D is neither surjective nor injective.

3. Suppose V is a finite dimensional vector space and $S, T \in \mathcal{L}(V)$ are such that ST is invertible. Then...

(A) Both S and T must be invertlbe.

(B) S must be invertible, but T need not be.

(C) T must be invertlbe, but S need not be.

(D) Neither S nor T need be invertible.