| | |
|---|---|
| 10 | 3+7 |
| 11 | X |
| 12 | 3·4 |
| 13 | 2·3+7 |
| 14 | 2·7 |
| 15 | 3·5 |
| 16 | 3·3+7 |
| 17 | 1·3+2·7 |
| 18 | 3·6 |
| 19 | 4·3+7. |
| 20 | 2·3 2·7 |

$$n = \begin{cases} 3\cdot\frac{n}{3} & \text{if remainder when you divide } n \text{ by } 3 \text{ is } 0 \\ \boxed{\phantom{xx}} & \ldots 1 \\ \boxed{\phantom{xx}} & \ldots 2. \end{cases}$$

$\binom{0}{0}$ 1

$\binom{1}{0}$ 1 1 $\binom{1}{1}$

$\binom{2}{0}$ 1 2 $\binom{2}{1}$ 1 $\binom{2}{2}$

1 3 3 1

1 4 6 4 1

2.3.12. $\gcd(a, a+n) \mid n.$ ← prove this (I)

then, use this to deduce that $\gcd(a,a+1)=1.$ (II)

(II) $\gcd(a,a+1) \mid 1$ by the result applied in the case $n=1$.
By thm 2.2(b), $\gcd(a,a+1) = \pm 1$. But gcds are defined to be positive, so $\gcd(a,a+1)=1$.

(I)

$(\underbrace{ax+(a+n)y})k = n$

thm 2.3 tells us that gcd's can be written in this form.

→ corollary says that anything that's written in this form is div. by gcd.

$n = a\overset{-1}{x} + (a+n)\overset{1}{y} = \cancel{(-a+a)}+n$

$n = \underbrace{-a+ay}_{+1} + \underbrace{ny}_{+1}$

Notice that $n = -1 \cdot a + 1 \cdot (a+n)$, ie, it is a linear combination of $a$ & $a+n$. Since $\gcd(a, a+n)$ divides every linear combination of $a$ & $a+n$ by corollary to thm 2.3, we conclude that $\gcd(a, a+n) \mid n$.