Week 7 Day 2

Review

1. The matrix A below is row equivalent to the matrix B. Find bases for the null space, column space, and row space of A.

$$A = \begin{bmatrix} 1 & 3 & 9 & -7 \\ 0 & 1 & 4 & -3 \\ 2 & 1 & -2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & -3 & 2 \\ 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

2. Consider the subspace $U = \{p \in \mathbb{P}_3 : p(-1) = 0\}$ of $\mathbb{P}_3$. What is $\dim(U)$?

(A) 1

(B) 2

(C) 3

(D) None of the above

3. Let $\mathscr{B} = \{(1,0,0),(0,1,0)\}$. Which of the following is true about $\mathscr{B}$?

(A) $\mathscr{B}$ is a basis for $\mathbb{R}^3$.

(B) $\mathscr{B}$ is a basis for a subspace of $\mathbb{R}^3$.

(C) $\mathscr{B}$ is a basis for $\mathbb{R}^2$.

(D) None of the above OR more than one of the above.

The *trace* of a 2×2 matrix is defined to be the sum of its diagonal entries:

$$\operatorname{tr} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = a + d.$$

4. (A) True or (B) False? The set

$$S = \{A \in M_{2\times 2} : \operatorname{tr}(A) = 0\}$$

is a subspace of the vector space $M_{2\times 2}$ of all 2×2 matrices.

Follow-up. If it is a subspace, what is $\dim(S)$?

5. The set $\mathscr{B} = \{1+t, 1+2t, 1+t^2\}$ is a basis for $\mathbb{P}_2$. For which polynomial p is it the case that $[p]_{\mathscr{B}} = (2, 1-1)$?

(A) $p(t) = 2 + t - t^2$

(B) $p(t) = -1 + t + 2t^2$

(C) $p(t) = 2 + 4t - t^2$

(D) None of the above

Follow-up. How would you go about verifying that $\mathscr{B}$ is a basis for $\mathbb{P}_2$, if you weren't given this information?

6. (A) True or (B) False? The set

$$S = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} : x, y \text{ are integers} \right\}$$

is a subspace of $\mathbb{R}^2$.

7. Let $T: \mathbb{R}^n \to \mathbb{R}^n$ be a linear map with standard matrix A. Which of the following statements are equivalent to the statement "A is invertible"?

(A) T is one-to-one.

(B) A has rank n.

(C) A has nullity 0.

(D) None of the above OR more than one of the above.