

When finding the Grobner basis for some parametrized ideal, is it best to make as many of the g_i in G be in the first elimination ideal? Like in the reading on pg 135 only g_1 has t in it. Should this be the standard or is it variable depending on the ideal?

If $I \subseteq k[x_1, \dots, x_n]$ is an ideal and G is a Gröbner basis for I with respect to lex order where $x_1 > \dots > x_n$, the number of polynomials in $G_1 = G \cap k[x_2, \dots, x_n]$ can vary as I changes. It will not always be one less than the number of polynomials in G — not even if I is defined by “parametric” equations.

Also note that you don’t really have much control over this number, at least if you want G to be a reduced Gröbner basis. There is just one reduced Gröbner basis (with respect to a given monomial order), so the number of elements in both G and G_1 is completely determined by I .

For example, consider the following Sage calculations:

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In:      R.<t,x,y> = PolynomialRing(QQ, order='lex')
          I = Ideal(x-t^2,y-t^3)
          I.groebner_basis()
Out:     [t^2 - x, t*x - y, t*y - x^2, x^3 - y^2]
```

Notice that here, *three* polynomials of the reduced Gröbner basis G get excluded when passing to $G_1 = G \cap k[x, y]$.