Select Reading Question Responses (3/4)

> Clarification on 3.13 D please

By definition, we have

$$P(A \mid B) = \frac{P(A \text{ and } B)}{P(B)}.$$

We know that $P(A \text{ and } B) = 0.1$ and $P(B) = 0.7$, so $P(A \mid B) = 0.1/0.7 \approx 0.143$.

> At first I thought it was strange that drawing an ace or a heart were independent because they seem connected in that if you draw one you won't get the other. But I guess that is the point– they have no overlap. No ability to be together in a single outcome.

There is an ace of hearts. In other words, there is one card in the deck that's both an ace and a heart. They're independent, if we let A represent drawing a heart and B represent drawing an ace, then $P(A \text{ and } B) = 1/52$ since there's just one ace of hearts in a deck, and $P(A)P(B) = (1/4)(1/13) = 1/52$ as well.

Actually, a piece of your original intuition is correct — two events that cannot both happen must be dependent! More precisely, if A and B are both possible but A and B is impossible, then A and B are dependent. For example, drawing a heart and drawing a spade are both possible (both have a $1/4$ chance). But drawing both is impossible: the joint probability is 0. If A represents drawing a heart and B represents drawing a spade, we have $P(A \text{ and } B) = 0$ but $P(A)P(B) = (1/4)(1/4) = 1/16 \neq 0$, so the two events are dependent.

> From my perspective it seems that Bayes' Theorem is just a faster way to calculate than the tree diagrams. However, I find the tree diagrams much easier to use and understand. Is this view incorrect? Do the two approaches accomplish different things and would I sometimes need to use Bayes' theorem instead of the tree diagrams?

There's nothing "incorrect" about this: it's perfectly fine if you prefer the tree diagrams. Anything you do with Bayes' theorem, you could theoretically do with tree diagrams as well — in fact, tree diagrams are really just a way of "visualizing" Bayes' theorem!

> Do tree diagrams always contain only two branches (primary and secondary)? Or is it possible that more complex tree diagrams with tertiary branches will come up?

You can have tree diagrams with an arbitrary amount of branching at each level, and also an arbitrary number of levels! For example, let's say I was doing a study of the political affiliations and opinions of many Americans. More specifically, let's say I ask everyone what political party they belong to (Democrat, Republican, Other), what they think about a having a Green New Deal (Support, Oppose, Unsure), and what they think about AOC (Like, Dislike, Unsure). In that case, I'd have a tree diagram with three levels, and ternary branching at each level.