Week 4 Day 2

Fixed vs Growth Mindsets

Make sure you know your neighbors' names. Then take about 2 minutes to discuss what you know about the phrases "fixed mindset" and "growth mindset." If you haven't heard them before, there's brief summary below. Do you feel like you have a growth mindset towards math? What are some non-math areas of your life that you have a growth mindset in?

A person has fixed mindset in an area if they believe their intelligence or abilities in that area are fixed traits that cannot change. Characteristics of having a fixed mindset include: avoiding challenges, giving up easily, ignoring useful criticism, feeling threatened by others, etc. In contrast, a growth mindset is the belief that your intelligence and abilities in that area can grow with time, effort, and persistence. Characteristics of a having a growth mindset include: embracing challenges, persisting through obstacles, learning from feedback, feeling inspired by others' successes, etc.

Review

1. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the function that "projects vertically onto the line $y = x$." In other words, for any point $\mathbf{v}$ in $\mathbb{R}^2$, the output $T(\mathbf{v})$ is obtained by moving $\mathbf{v}$ either up or down until it lands on the line $y = x$. Then...

(A) T is not linear.

(B) T is linear.

Follow-up. If you think T is not linear, can you justify why? If you think T is linear, can you write down the standard matrix of the transformation?

2. Suppose A is a 3×4 matrix. Which of the following statements is the "odd one out"?

(A) Every row of A has a pivot.

(B) The linear transformation $T\colon \mathbb{R}^4 \to \mathbb{R}^3$ defined by $T(\mathbf{x}) = A\mathbf{x}$ is onto.

(C) The columns of A are linearly independent.

(D) The equation $A\mathbf{x} = \mathbf{b}$ is consistent for all $\mathbf{b}$ in $\mathbb{R}^3$.

3. Which of the following matrices has linearly independent columns?

(A) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

(D) None of the above OR more than one of the above.

4. Suppose A and AB are the matrices below.

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \quad AB = \begin{bmatrix} 1 & 2 \\ 0 & -1 \end{bmatrix}$$

Does there actually exist a matrix B for which this is possible? If not, explain why not. If there does, come up with as much information about B as possible.

5. Let $T: \mathbb{R}^3 \to \mathbb{R}^4$ be the linear map given by

$$T(x, y, z) = (x + y + z, y + z, x - y - z, x).$$

This map is...

(A) Both one-to-one and onto

(B) One-to-one but not onto

(C) Onto but not one-to-one

(D) Neither one-to-one nor onto

6. (A) True or (B) False? If A is a matrix and there exists a vector $\mathbf{b}_0$ in $\mathbb{R}^n$ such that $A\mathbf{x} = \mathbf{b}_0$ is consistent, then $A\mathbf{x} = \mathbf{b}$ is consistent for *every* $\mathbf{b}$ in $\mathbb{R}^n$.

7. The size of the standard matrix of a linear map $T : \mathbb{R}^5 \to \mathbb{R}^2$...

(A) is necessarily 2×5.

(B) is necessarily 5×2.

(C) is necessarily something else.

(D) can't be determined for sure without knowing a formula or other description for T.