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(B) 1

(C) 2

(D) 3 or more

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Follow-up. Draw a phase diagram.

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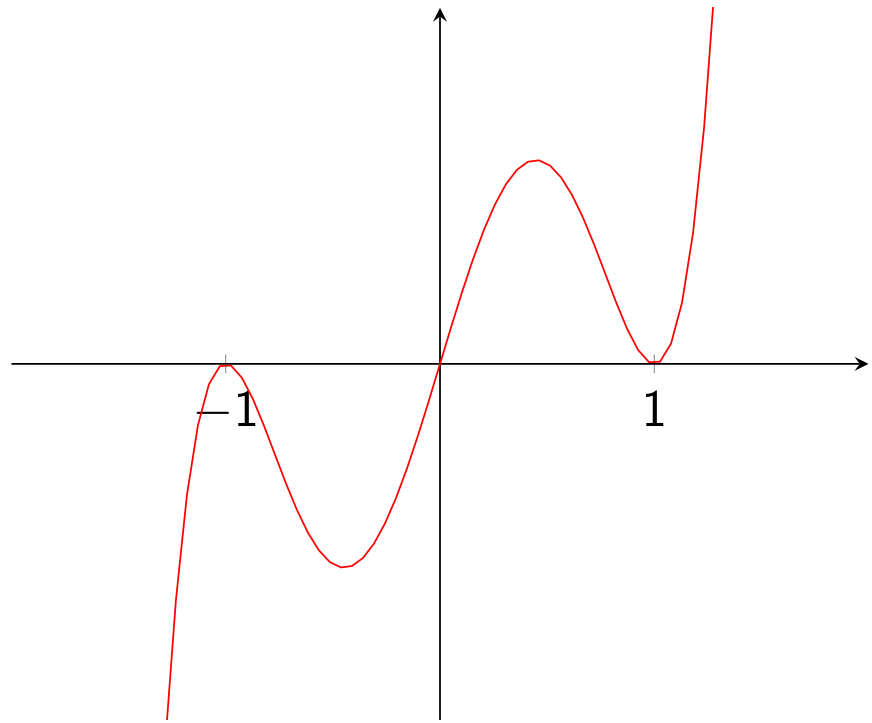

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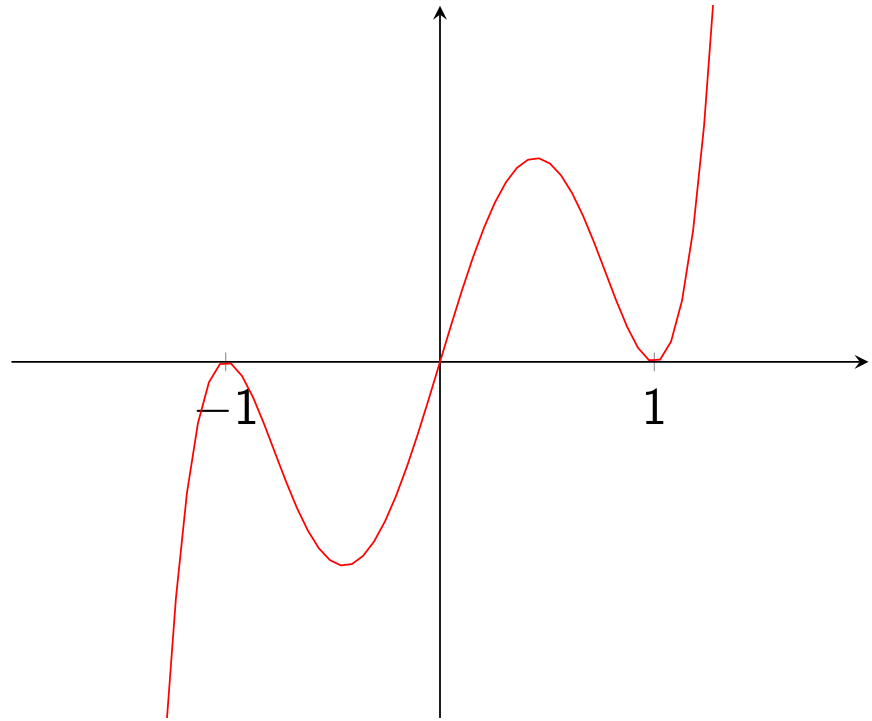

Follow-up. Draw a phase diagram.

The downwards velocity v of an object of mass m in free fall, subject to air resistance, satisfies the differential equation

$$\frac{dv}{dt} = g - \frac{bv}{m},$$

where $g = 9.8$ m/s^2 is gravitational acceleration and $b > 0$ is a constant (with units kg/s) that depends on the density of air and the shape of the object.

3. True or False?

$\lim_{t\to\infty} v = \frac{gm}{b}$, regardless of the initial velocity of the object.

4. True or False?

The initial value problem

$$y' = y^{1/3}, \quad y(0) = 0$$

has a unique solution.[1]

[1] This is not a great problem. A better formulation of the intended statement would be... There exists a unique continuous function f on the interval $[0, \infty)$ such that $f(0) = 0$ and $f'(x) = f(x)^{1/3}$ for all $x > 0$.

5. True or False?

Let p be the discontinuous function

$$p(x) = \begin{cases} 1 & \text{if } x < 2 \\ 3 & \text{if } x \geq 2 \end{cases}$$

There is a continuous function y, defined on all real numbers and differentiable everywhere except possibly at $x = 2$, such that

$$y' + p(x)y = x \text{ and } y(0) = 1.$$