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$$\operatorname{trace}\begin{pmatrix} 1 & 3 & -1 \\ 2 & 4 & 1 \\ 1 & -2 & 0 \end{pmatrix} = 5.$$

1. True or False?

The function $\operatorname{trace} : \mathcal{M}_{3\times 3} \to \mathbb{R}$ is linear.

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Follow-up. What is the dimension of the null space of this linear map?

2. True or False?

Consider the map $d/dx : \mathcal{P}_3 \to \mathcal{P}_3$, and let A be the matrix representation of this linear map with respect to the standard basis $\langle 1, x, x^2, x^3 \rangle$ of $\mathcal{P}_3$. Then $A^4 = 0$.

3. True or False?

Let B denote the standard basis of $\mathbb{R}^2$, $\pi_x : \mathbb{R}^2 \to \mathbb{R}^2$ is projection onto the x-axis and $\pi_y : \mathbb{R}^2 \to \mathbb{R}^2$ is projection onto the y-axis. Then

$$\mathrm{Rep}_{B,B}(\pi_x)\,\mathrm{Rep}_{B,B}(\pi_y) = 0.$$

4. True or False?

The only 2×2 matrices A such that $A^2 = I$ are the following:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

5. True or False?

The matrix

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

has no left inverse.