

1. Consider the matrix  $A = \begin{pmatrix} -2 & -1 \\ 5 & 2 \end{pmatrix}$ . Which of the following is true?

- (A)  $A$  has just one eigenvalue.
- (B)  $A$  has two distinct eigenvalues which are both real.
- (C)  $A$  has two distinct eigenvalues.

2. Consider the linear transformation  $d/dx : \mathcal{P}_3 \rightarrow \mathcal{P}_3$ . How many distinct eigenvalues does this linear transformation have?

(A) 1

(B) 2

(C) 3

(D) 4

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**Follow-up.** What is the characteristic polynomial of  $d/dx$ ?

3. True or False?

If  $\lambda$  is an eigenvalue of a square matrix  $A$ , then  $\lambda^2$  is an eigenvalue of  $A^2$ .

4. Calculate the eigenvalues, eigenspaces, geometric and algebraic multiplicities, and the characteristic polynomial of the map  $h : \mathbb{C}^2 \rightarrow \mathbb{C}^2$  given by

$$\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x + y \\ x + y \end{pmatrix}.$$

5. Calculate the eigenvalues, eigenspaces, geometric and algebraic multiplicities, and the characteristic polynomial of the map  $h : \mathbb{C}^3 \rightarrow \mathbb{C}^3$  given by

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} 2x + y - z \\ 3y - z \\ 2z \end{pmatrix}.$$