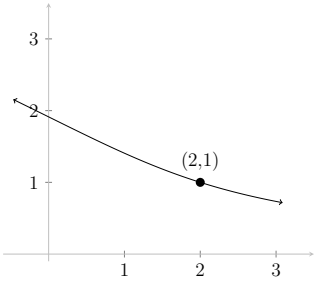
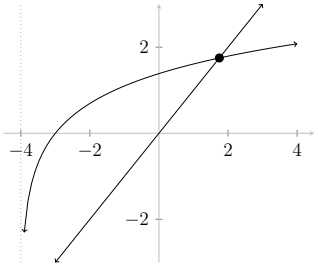
WORKSHEET: LINEAR APPROXIMATION

1. Find an approximations of the stated quantities, using linearizations and/or Newton's method. If you use Newton's method, use at least 2 steps.
   a. Approximate $\sqrt{26}$.
   b. Approximate $1/\sqrt{101}$.
   c. Approximate $\ln(0.97)$.
   d. The point $(2, 1)$ is on the curve $y^3 + 3xy = 7$, depicted below. Approximate the y-coordinate of the point on the curve where $x = 2.1$.



   e. Approximate the solution to $x^4 - 6x^2 + x + 5 = 0$ that is closest to $x_0 = 2$.
   f. Approximate the unique positive solution to the equation $\ln(x + 4) = x$.



   g. Approximate the coordinates of point P on the graph of $f(x) = \cos(x)$ such that the tangent line at P passes through the origin.
2. The radius of a spherical ball is measured at $r = 25$ cm. Estimate the maximum error in the volume and the surface area of the ball if r is accurate to within 0.5 cm.
3. If you deposit P dollars in a retirement fund every year for N years with the intention of then withdrawing Q dollars per year for M years, you must earn interest at a rate $r > 0$ satisfying

$$P(b^N - 1) = Q(1 - b^{-M})$$

   where $b = 1 + r$. Assume \$2000 is deposited each year for 30 years and the goal is to withdraw \$10,000 per year for 25 years. Use Newton's method to compute b, and then find r.