1. True or False?

If $f : \mathbb{R} \to \mathbb{R}$ is differentiable and f'(a) = 0, then f has a local extremum at a.

True or False?

If $f,g:\mathbb{R}\to\mathbb{R}$ are functions such that f+g is differentiable at a point $a\in\mathbb{R}$, then f and g are also differentiable at a.

True or False?

The function $f: \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} \sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is differentiable.

4. True or False?

The function $f: \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} x \sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is differentiable.

5. True or False?

The function $f: \mathbb{R} \to \mathbb{R}$ given by

$$f(x) = \begin{cases} x^2 \sin(1/x) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is differentiable.