Week 10 Day 1

Reminders

- Today: Review.
- Wednesday and Friday: No class.
  - Instead, I'll have office hours 10am–1pm (at the usual location, ie, UCSD Town Square*).
- Saturday: Final exam 3–6pm.

Review

1. (A) True or (B) False? The following matrix is diagonalizable.

$$\begin{bmatrix} -1 & 0 & 2 \\ -1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

2. What is the area of the parallelogram with one vertex at the origin and adjacent vertices at $(2, 1)$ and $(1, -2)$?

(A) 3

(B) 4

(C) 5

(D) None of the above

3. What is the distance from $(3, 3, 4)$ to (the closest point on) the plane $U = \text{span}\{(1, 0, 0), (1, 1, 0)\}$?

(A) 3

(B) 4

(C) 5

(D) None of the above

4. Let $W = \text{span}\{(1,1,1),(1,-1,0)\}$. Find a basis for $W^{\perp}$.

5. Which of the following is true?

(A) Any matrix with characteristic polynomial $(\lambda - 2)^2$ must be diagonalizable.

(B) Any matrix with characteristic polynomial $(\lambda - 1)(\lambda - 2)$ must be diagonalizable.

(C) Both of the above.

(D) None of the above.

6. On $\mathbb{P}_1$, consider the inner product

$$\langle p, q \rangle = p(0)q(0) + p(1)q(1).$$

Let $U = \text{span}\{1\}$ inside $\mathbb{P}_2$. What is the projection of $1 + t$ onto U?

(A) 0

(B) 1

(C) 2

(D) None of the above

7. (A) True or (B) False? The following matrix is diagonalizable.

$$\begin{bmatrix} -1 & 0 & 2 \\ -3 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

8. Suppose $\mathscr{B}$ is a basis for a vector space V. Is it possible for there to be two *distinct* vectors v, w in V which have the same coordinate vector with respect to $\mathscr{B}$?

(A) Yes

(B) No

9. For a polynomial $p(t)$, let $p'(t)$ be its derivative. Consider the subset $S = \{p \mid p'(0) = 0\}$ of $\mathbb{P}_2$. Which of the following is true about S?

(A) It is not a subspace of $\mathbb{P}_2$.

(B) It is a subspace, and it is 2-dimensional.

(C) It is a subspace, and it is 1-dimensional.

(D) None of the above.

10. On $\mathbb{P}_1$, consider the inner product

$$\langle p, q \rangle = p(0)q(0) + p(1)q(1).$$

Let $U = \text{span}\{1\}$ inside $\mathbb{P}_2$. Find a basis for $U^{\perp}$.

11. (A) True or (B) False? If the columns of a matrix A form an orthogonal set of vectors, then $A^T A = I$.

12. (A) True or (B) False? If U is a subspace of an inner product space V and v is a vector that is in both U and $U^{\perp}$, then $v = 0$.

13. (A) True or (B) False? If U is a subspace of an inner product space V and v is a vector that is not in U, then $\mathrm{proj}_U(v)$ must be nonzero.