1. True or False?

For any 3×3 matrix of constants A, the nonhomogeneous linear system

$$\vec{x}' = A\vec{x} + \begin{bmatrix} e^t \\ \sin(t) \\ t^2 + 1 \end{bmatrix}$$

has a solution.

2. True or False?

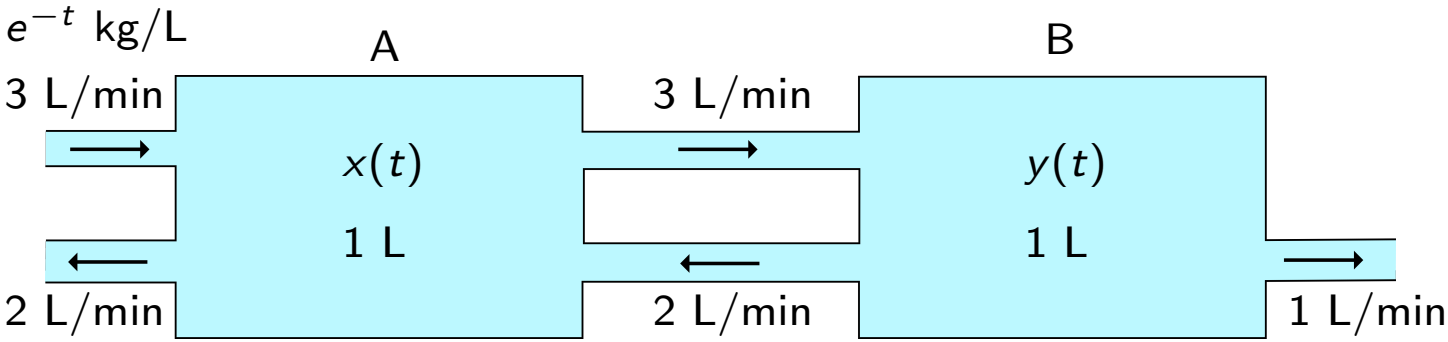
Suppose A is a 2×2 matrix of constants, and that $\vec{x}_1$ and $\vec{x}_2$ are functions satisfying

$$\vec{x}_1\,' = A\vec{x}_1 + \begin{bmatrix} \sin(t) \\ 0 \end{bmatrix} \quad \text{and} \quad \vec{x}_2\,' = A\vec{x}_2 + \begin{bmatrix} 0 \\ \cos(t) \end{bmatrix}.$$

Then $\vec{x} = \vec{x}_1 + \vec{x}_2$ must be a solution for the system

$$\vec{x}\,' = A\vec{x} + \begin{bmatrix} \sin(t) \\ \cos(t) \end{bmatrix}.$$

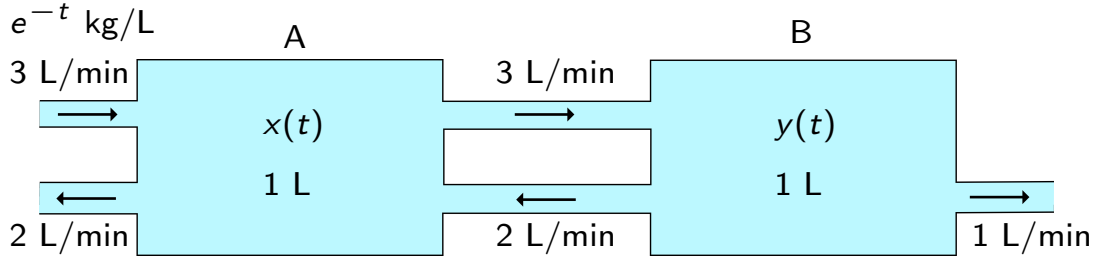
Salt water tanks! Let $x(t)$ and $y(t)$ denote the quantities of salt, in kg, in the two tanks A and B depicted below. Suppose that the input to tank A has a salt concentration of $f(t) = e^{-t}$ kg/L at time t.



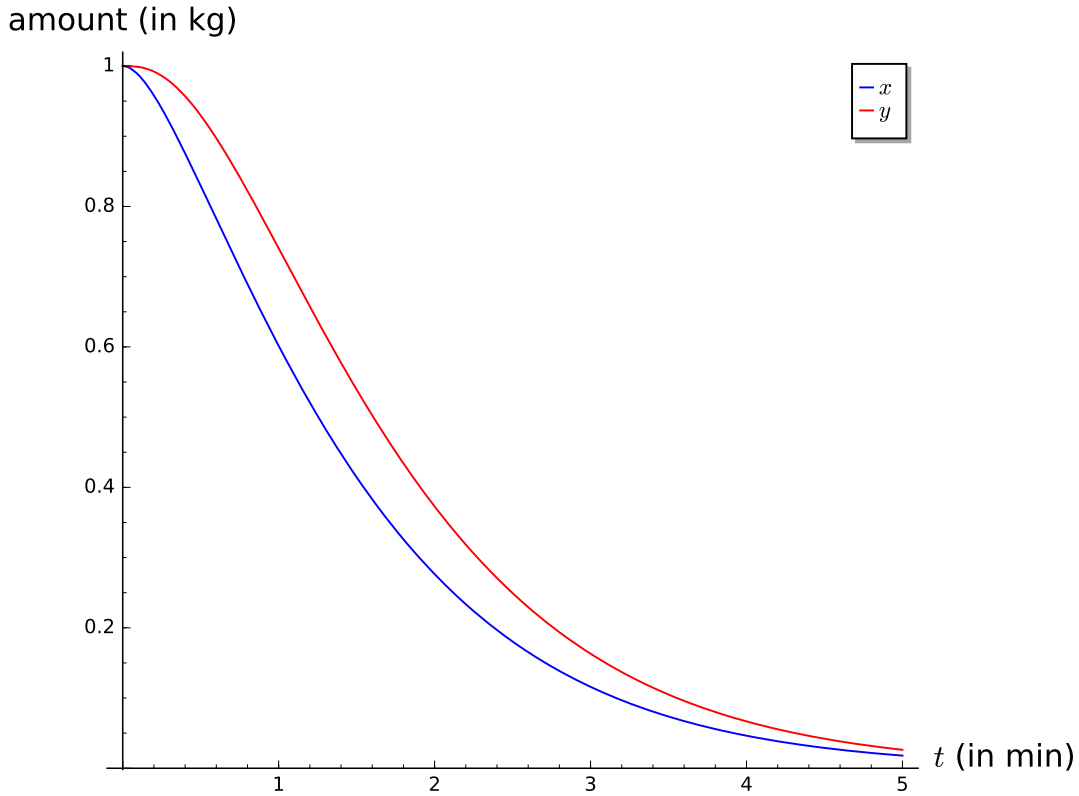
3. True or False?

$$\lim_{t\to\infty} \begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = 0.$$

(Don't start calculating...)



Here's what actually happens, if $x(0) = 1$ and $y(0) = 1$.



4. Suppose A is a 2×2 matrix of real numbers all of whose entries are positive, and that the functions x and y are both always positive. If x and y satisfy the linear system

$$\frac{d}{dt}\begin{bmatrix} x \\ y \end{bmatrix} = A\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e^{2t} \\ 0 \end{bmatrix},$$

which of the following functions could x be?

(A) $x(t) = t^2 + 1$

(B) $x(t) = t^4 + t^2 + 1$

(C) $x(t) = e^t$

(D) None of the above