$P(k)$ = "computing $\gcd(s_{k+1}, s_k)$ requires $k$ divisions"

want to prove $P(k+1)$.

$$\underbrace{s_{k+2}}_{s_3} = 2\underbrace{s_{k+1}}_{s_2} + s_k$$ by definition of $s_{k+2}$ for all $k \geq 1$.

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$d = \gcd((n+1)!+1, n!+1)$.

$d \mid n$.

$d \mid n$ & $d \mid (n!+1)$.

$d \mid n \Rightarrow d \mid n!$

$\Rightarrow d \mid n!$ & $d \mid (n!+1)$

$\Rightarrow d \mid ((n!+1) - n!)$

$= d \mid 1$

$n \equiv 0 \pmod{d}$

$n! \equiv 0 \pmod{d}$

$n!+1 \equiv 1 \pmod{d}$

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$n=1$. I want to prove that computing $\gcd(s_2, s_1)$ requires 2 divisions.

$s_2 = 49 = 3 \cdot 14 + 7$.

$14 = 2 \cdot 7 + 0$

does require 2 divisions before I hit a remainder of 0.

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$n=1$. want to prove computing $\gcd(s_2, s_1)$ requires 1 division.

$s_2 = 2 \quad s_1 = 1$.

$2 = 2 \cdot 1 + 0$.

$\gcd(2,1) = 1$.

$[s_{k-2} < s_{k-1}]$ & $[s_{k-1} < s_k]$

$2s_{k-1} < 2s_k$.

$2s_{k-1} + s_{k-2} < 2s_k + s_{k-2} < 2s_k + s_{k-1}$

$s_k < s_{k+1}$

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$63n - 23x = -7$.

$\rightsquigarrow 63n + 23x' = -7$

$x' = -x$.

$n = 28 + 23t$.

$t = 0, 1, 2, \ldots, -1, \cancel{-2}$

$\boxed{t \geq -1.}$ [If $t \geq -1$, then $n \geq 0$.]

$x' = -77 - 63t. \rightsquigarrow x = 77 + 63t$. [If $t \geq -1$, then $x \geq 0$]

$\boxed{t \leq -2.}$

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$n = 1$ $\quad \frac{t_1}{1} \in \mathbb{Z}$.

$S = \{1, 2, \ldots\} = \{n \mid ?\}$

$n = 2$ $\quad \frac{t_1 + t_2}{2} \in \mathbb{Z}$.

$n = 3$ $\quad \frac{t_1 + t_2 + t_3}{3} \notin \mathbb{Z}$.

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$x = \begin{cases} 4k+3 \\ 4k-1. \end{cases}$

$a = 2n, \quad 2n+1$

$a^2 = 4k \quad 4k+1.$

$7q+3. \quad 7q+3$

$x \equiv 0 \pmod 4$

$x \equiv 3 \pmod 7$

$24 = 4 \cdot 6$
$= 7 \cdot 3 + 3$

$17 = 4 \cdot 4 + 1$
$= 7 \cdot 2 + 3$

$a = 7q \quad 7q+1 \quad 7q+2 \quad 7q+3 \quad \ldots \quad 7q+6.$

$a^2 = 49q$