1. Let $B = \langle 1, x, x^2 \rangle$ and $C = \langle 1, 1 + x, 1 + x + x^2 \rangle$ be bases for $\mathcal{P}_2$. Which of the following matrices changes representations with respect to B into representations with respect to C?

(A) $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

(B) $\begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{pmatrix}$

(C) Neither of the above.

2. Consider the matrix

$$A = \begin{pmatrix} 3 & 1 & 4 \\ 2 & -1 & 1 \\ 0 & 0 & 4 \end{pmatrix}.$$

For which bases B and C of $\mathcal{P}_2$ is $A = \mathrm{Rep}_{B,C}(\mathrm{id})$?

(A) $B = \langle 3 + 2x, 1 - x, 4 + x + 4x^2 \rangle,\ C = \langle 1, x, x^2 \rangle$

(B) $B = \langle 5 + 2x, -x, 9 + 5x + 4x^2 \rangle,\ C = \langle 1, 1 + x, 1 + x + x^2 \rangle$

(C) Both (A) and (B)

(D) Neither (A) nor (B)

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(C) Both (A) and (B)

(D) Neither (A) nor (B)

Follow-up. If $C = \langle 1 + 2x, 1 - 2x, x^2 \rangle$, what must B be so that $A = \mathrm{Rep}_{B,C}(\mathrm{id})$?

3. True or False?

There exists a matrix A such that

$$A\begin{pmatrix}1\\3\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix} \quad \text{and} \quad A\begin{pmatrix}2\\6\end{pmatrix}=\begin{pmatrix}-1\\-1\end{pmatrix}.$$

4. True or False?

If a square matrix A is similar to another square matrix B, then A^2 is also similar to B^2.

5. True or False?

The following two matrices are similar.

$$\begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} \quad \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}$$

6. True or False?

The following two matrices are similar.

$$\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \quad \begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}$$