Worksheet W2Wed: Logarithms

Problem 1. Find solutions to the following equations.

(a) $\operatorname{Log}(z) = -i\pi/2$

(b) $\operatorname{Log}(z) = 3\pi i/2$

(c) $z^{1/2} = 1 - i$

(d) $z^{1/2} = -1 + i$

Problem 2. Recall that log (lower-case) denotes the multi-valued logarithm function. Find all values of the following.

(a) $\log(-i)$

(b) $\log(1 + i)$

(c) multi-valued $(-i)^i$, ie, $\exp(i \log(-i))$

(d) multi-valued $(1 + i)^{1+i}$, ie, $\exp((1 + i) \log(1 + i))$

Problem 3. What is the image of the annulus $1 < |z| < e$ under Log? How does this image change if you choose a different branch of the logarithm?

Problem 4. For which $z \in \mathbb{C} \setminus \{0\}$ is it true that $\operatorname{Arg}(\bar{z}) = -\operatorname{Arg}(z)$?

Problem 5. Suppose $c \in \mathbb{C} \setminus \{0\}$ and let $f(z) = z^c$ for all $z \neq 0$. Is it true that $f'(z) = cz^{c-1}$? If so, prove this carefully. If not, give a counterexample. Does anything change if a non-principal branch of the logarithm is used to define z^c?

Problem 6. (a) For what $z \in \mathbb{C}$ is it the case that $\operatorname{Log}(\exp(z)) = z$?

(b) For what $a, b \in \mathbb{C}$ is it the case that $\operatorname{Log} a^b = b \operatorname{Log} a$? (Find how a and b must be related geometrically in order for this equality to be true.)