1.   True or False?

If λ is a repeated eigenvalue of a square matrix, it must be deficient.
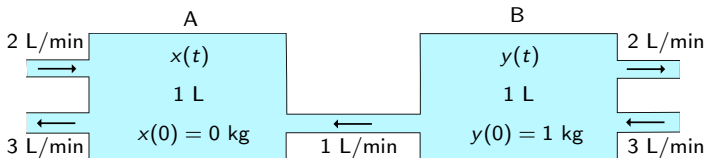
2. True or False?

If λ is a repeated eigenvalue of a 2×2 matrix of the form

$$A = \begin{bmatrix} 0 & 1 \\ b & a \end{bmatrix},$$

it must be deficient.

More salt-water tanks! Hooray! In the diagram below, the inputs to both tanks A and B are pure water.

2 L/min

A

$x(t)$

1 L

$x(0) = 0$ kg

3 L/min

1 L/min

B

$y(t)$

1 L

$y(0) = 1$ kg

2 L/min

3 L/min

3.   If A is the matrix such that $\dfrac{d}{dt}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = A \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}$, which of the following is true?

(A) A has a repeated deficient eigenvalue.

(B) A has a repeated complete eigenvalue.

(C) A has two distinct real eigenvalues.

(D) A has two distinct complex eigenvalues.

Here's a phase portrait for the previous problem.
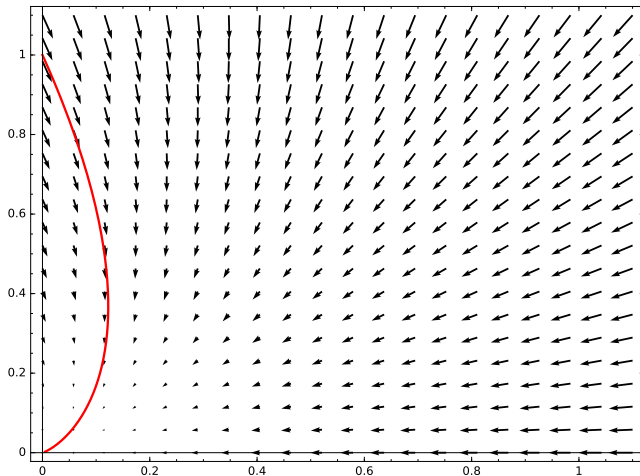
If an object of mass $m = 1$ kg is attached to a wall by a spring of stiffness $k = 20$ kg/s^2 and slides around on a surface with coefficient of friction $b = 4$ kg/s, the displacement x of the object from its equilibrium position is described by the ODE

$$x'' + 4x' + 20x = 0.$$

4.   True or False?

$\lim_{t \to \infty} x(t) = 0.$

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 0 \\ -1 & 0 & 3 \end{bmatrix}$$

The characteristic polynomial of A is $-(\lambda - 2)^3$. Two linearly independent eigenvectors corresponding to the eigenvalue $\lambda = 2$ are $\vec{v}_1 = (1, 0, 1)$ and $\vec{v}_2 = (0, 1, 0)$.

5. Which of the following statements is true?

(A) There exists a third eigenvector $\vec{v}_3$ corresponding to the eigenvalue $\lambda = 2$.

(B) There exists a generalized eigenvector $\vec{v}_3$ such that $(A - 2I)\vec{v}_3 = \vec{v}_1$.

(C) There exists a generalized eigenvector $\vec{v}_3$ such that $(A - 2I)\vec{v}_3 = \vec{v}_2$.

(D) None of the above.

6. For which matrix A does $\vec{x}' = A\vec{x}$ have the depicted phase portrait?

(A) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(D) None of the above