$$n=4.$$
WTS:  $(1+\alpha)^{4} \ge 1+4\alpha$ 

$$(1+\alpha)^{4} = \sum_{k=0}^{4} {\binom{4}{k}}^{k}^{k} \alpha^{-k}$$

$$= {\binom{4}{0}}^{4} + {\binom{4}{1}}^{5} + {\binom{4}{2}}^{2} + {\binom{4}{3}}^{3} + {\binom{4}{4}}^{4}$$

$$= 4\alpha + 1$$