

Worksheet 1: Sets

Problem 1. Each of the following describes a finite set using set-builder notation. List all of the elements of each of these sets.

- (a) $\{3x + 2 : x \in \mathbb{Z}, 0 \leq x \leq 5\}$ (c) $\{x \in \mathbb{R} : \cos x = 1, |x| \leq 10\}$
(b) $\{x \in \mathbb{R} : x^3 + 5x^2 = -6x\}$ (d) $\{6a + 2b : a, b \in \mathbb{Z}, |a|, |b| \leq 4\}$

Problem 2. Identify the pattern and describe each of the following sets using set-builder notation.

- (a) $\{0, 4, 16, 36, 64, 100, \dots\}$ (c) $\{\dots, -\frac{3}{2}, -\frac{3}{4}, 0, \frac{3}{4}, \frac{3}{2}, \frac{9}{4}, 3, \frac{15}{4}, \frac{9}{2}, \dots\}$
(b) $\{\dots, \frac{1}{27}, \frac{1}{9}, \frac{1}{3}, 1, 3, 9, 27, \dots\}$ (d) $\{4, 9, 25, 49, 121, 169, \dots\}$

Problem 3. Draw a picture of each of the following sets in the x - y plane.

- (a) $\{(x, y) : x \in [0, 1], y \in [1, 2]\}$ (d) $\{(x, x + y) : x \in \mathbb{R}, y \in \mathbb{Z}\}$
(b) $\{(x, x^2) : x \in \mathbb{R}\}$ (e) $[1, 2] \times \{1, 2, 3\}$
(c) $\{(x, y) : x, y \in \mathbb{R}, x^2 + y^2 \leq 1\}$ (f) $\mathbb{N} \times \mathbb{Z}$

Problem 4. Draw a picture of each of the following in \mathbb{R}^3 .

- (a) $[0, 1] \times [0, 1] \times [0, 1]$ (b) $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\} \times [0, 1]$

Problem 5. Let $X = [-1, 3] \times [0, 2]$ and $Y = [0, 3] \times [1, 4]$ inside \mathbb{R}^2 . Draw a picture of the following sets.

- (a) $X \cup Y$ (d) $Y - X$
(b) $X \cap Y$ (e) \bar{X}
(c) $X - Y$ (f) $\bar{X} \cup Y$

Problem 6. Suppose A , B , and C are in a universal set U . Draw Venn diagrams for the following.

- (a) $\overline{A \cup B}$ (c) $(A - B) \cup (B \cap C)$
(b) $(B \cap C) - A$ (d) $\overline{A \cap B \cap C}$

Problem 7. Draw a picture of each of the following sets inside \mathbb{R}^2 .

- (a) $\bigcup_{a \in \mathbb{R}} \{a\} \times [0, 1]$ (c) $\bigcap_{n \in \mathbb{N}} \mathbb{Z} \times \{\dots, -2n, -n, 0, n, 2n, 3n, \dots\}$
(b) $\bigcap_{n \in \mathbb{N}} [-n, n] \times [-n, n]$ (d) $\bigcup_{x \in [0, 1]} [x, 1] \times [0, x^2]$

Problem 8. Let $S = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}$. Find all subsets of S .

Problem 9. Determine whether or not each of the following sets has a smallest element. If it does, say what the smallest element is.

- (a) $\{4x + 6y \in \mathbb{N} : x, y \in \mathbb{Z}\}$ (b) $\{4x + 6y \in \mathbb{Z} : x, y \in \mathbb{Z}\}$

Problem 10. Some words can be used to describe words. For example, the word *verb* describes words like *breathe* and *grow*. The word *noun* describes words like *tree* and *absurdity*, and in fact, it also describes *verb* and *noun*. The word *pentasyllable* describes words like *imagination* and also *pentasyllable* itself. Let us say that a word is a *heteroseme* if it does not describe itself. For example, we've just seen that *verb* is a heteroseme while *noun* and *pentasyllable* are not. Is *heteroseme* a heteroseme?