Simplify claim:

Every $n \geq 12$ can be written in the form $3x+7y$ for $x, y \geq 0$.

[$P(n) =$ " n can be written ... "]

Base cases: $P(12)$ $P(13)$ $P(14)$

$$12 = 3\cdot 4 + 7\cdot 0$$
$$13 = 3\cdot 2 + 7\cdot 1$$
$$14 = 3\cdot 0 + 7\cdot 2$$

Inductive step: Assume know $P(1), \ldots, P(k)$.
want to prove $P(k+1)$.

Since $P(k-2)$ is true, there exist $x, y \geq 0$
such that $k-2 = 3x+7y$.

$$k+1 = (k-2)+3 = 3x+7y+3$$
$$= 3(x+1)+7y$$

$\rightarrow t_n = \frac{n(n+1)}{2}$

$$t_1 + \cdots + t_n = \sum_{k=1}^{n} \frac{k(k+1)}{2} = nx$$

1.1 #1 (a)

1.1 #1 (c)

$$\sum_{k=1}^{n} k(k+1) = \frac{n(n+1)(n+2)}{3}$$

$$= \frac{n(n+1)(n+2)}{6} = nx$$

$$\frac{(n+1)(n+2)}{6} = x$$

$n=1$. $\frac{2\cdot 3}{6} = 1$.

$n=2$ $\frac{3\cdot 4}{6} = 2$

$n=3$ $\frac{4\cdot 5}{6} \notin \mathbb{Z}$

$P(n) =$ " $(1+a)^n \geq 1+na$ "

$P(n)$ for all $n \geq 0$.

$P(0)$ ...

Assume $P(k)$. Want to prove $P(k+1)$.

know: $(1+a)^k \geq 1+ka$

$\cdot(1+a)$

want: $(1+a)^{k+1} \geq 1+(k+1)a$

$$(1+a)^{k+1} = (1+a)^k(1+a) \geqslant (1+ka)(1+a)$$

$$= 1 + (k+1)a + ka^2$$

$$\geqslant 1 + (k+1)a \quad (\text{since } ka^2 \geqslant 0)$$

$$(1+a)^{k+1} \geqslant (1+ka)(1+a) \geqslant 1 + (k+1)a$$

$$\sum_{k=1}^{n} \frac{k(k+1)}{2}$$

sum of triangulars

$$\sum_{k=1}^{n} k(k+1)$$

LHS of 2.1#1(c).

$$(1+a)^n = \sum_{k=0}^{n} \binom{n}{k} 1^{n-k} a^k$$

$$= \sum_{k=0}^{n} \binom{n}{k} a^k$$

$$= \binom{n}{0}a^0 + \binom{n}{1}a^1 + \binom{n}{2}a^2 + \cdots + \binom{n}{n}a^n$$

$$= 1 + \binom{n}{1}a^1 + \binom{n}{2}a^2 + \cdots + \binom{n}{n-1}a^{n-1} + 1$$

want: $(1+a)^n \geqslant 1 + na$

$$= 1 + na + \binom{n}{2}a^2 + \cdots + \binom{n}{n-1}a^{n-1} + 1$$

$$\geqslant 1 + na.$$

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n = # in each pile

63 piles

$63n$ in piles

7 not in piles

$\Rightarrow 63n + 7$ total bananas

23 travellers

$\frac{63n+7}{23}$ bananas/traveller