p prime factor of b, ie, $p \mid b$

$$\Longrightarrow \quad p \mid b^2 \Longrightarrow p \mid a^2 \overset{EL}{\Longrightarrow} p \mid a$$

$\gcd(a,b)=1$ $\gcd(5,7)=1$

n integer, $\sqrt{n}=\frac{a}{b}$, where $\gcd(a,b)=1$.
we want to show that $b=1$.
Suppose for a contradiction that $b>1$. Then, by the fundamental thm of arithmetic, it must have a prime factor p.

$$p \mid b \Longrightarrow p \mid a.$$

$n=6$

LHS: $\gcd(1,6)+\gcd(2,6)+\gcd(3,6)+\gcd(4,6)+\gcd(5,6)+\gcd(6,6)$

1	2	3	2	1	6

RHS divisors $d \mid 6$	1	2	3	6
$6/d$	6	3	2	1
$\varphi(6/d)$	2	2	1	1

how many times d shows up on LHS

$$\sum_{d \mid 6} d \cdot \varphi\left(\frac{6}{d}\right) = \text{LHS}$$

divisor

# of times divisor shows up on LHS

Cnx: $\#\{k=1,\dots,n \mid \gcd(k,n)=d\} = \varphi(n/d)$

$$\frac{1}{p_1} + \cdots + \frac{1}{p_d} = \frac{p_2 \cdots p_d}{p_1 \cdots p_d} + \frac{}{p_1 \cdots p_d}$$

$$\frac{1}{p_2} \cdot \frac{c}{c} = \frac{c}{p_1 p_2 \cdots p_d}$$

$c = p_3 \cdots p_d \qquad p_2 c = p_2 p_3 \cdots p_d$

$$A = \frac{1}{p_1} + \cdots + \frac{1}{p_d}$$

$$A p_2 \cdots p_d = \frac{p_2 \cdots p_d}{p_1} + p_3 \cdots p_d + p_2 p_4 \cdots p_d + \cdots + p_2 p_3 \cdots p_{d-1}$$

$$\frac{p_2 \cdots p_d}{p_1} = \text{int.}$$

4.2(f) $\quad a \equiv b, \Rightarrow a^k \equiv b^k$ for any k.

$a = 76 \quad b = -1 \qquad k = 76^{76}$

$$\#\{k = 1, \ldots, n \mid \gcd(k, n) = d\}$$

$$\varphi(n/d) = \#\{\ell = 1, \ldots, n/d \mid \gcd(\ell, n/d) = 1\}$$

Fact: $\gcd(k,n) = d$ iff $\gcd(k/d, n/d) = 1$.

Thm 2.7 $\Rightarrow \underset{d}{\underset{\shortparallel}{\gcd(k,n)}} = d \cdot \underbrace{\gcd(k/d, n/d)}_{1}$ where $d = \gcd(k,n)$.

$9 \uparrow 3\cdot3$

$$8! = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

6: $2 \cdot 3 = 2a$ $\qquad$ 3: $3 = a$

n composite $\Longrightarrow$ $n = ab$ $\quad 1 < a \leq b < n$

Case 1: $a < b$ ...

Case 2: $a = b$ ...

If $n = a^2$, a shows up in $(n-1)!$

Need to show that $2a$ also shows up, ie,

$$2a \leq n-1$$