Week 5 Day 3

Pick A Word

Make sure you know your neighbors' names. Then pick one of the following linear algebra words, and take 2 minutes to summarize to each other what you understand about that word:

Vector Space, Subspace, Basis, Dimension

Coordinates

1. Let U be the span of $\mathscr{B} = \{(1,1,0), (0,1,1)\}$. Which of the following is true about the vector $\mathbf{v} = (3,8,5)$?

(A) It is not in U.

(B) It is in U, and its $\mathscr{B}$-coordinate vector is $(3,8,5)$.

(C) It is in U, and its $\mathscr{B}$-coordinate vector is $(3,5)$.

(D) None of the above.

2. (A) True or (B) False? $\{1, 2t, -2+4t^2, -12t+8t^3\}$ is a basis for $\mathbb{P}_3$.

3. Suppose U is a 3-dimensional subspace of $\mathbb{R}^4$ and the vectors $\mathbf{u}_1 = (1, 1, 0, 0)$ and $\mathbf{u}_2 = (0, 1, 1, 0)$ are in U. If this is all of the information you're given about U, which of the following is guaranteed to be true?

(A) $\mathbf{v} = (1, 2, 1, 0)$ must be in U.

(B) $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{w}\}$ must be a basis for U, where $\mathbf{w} = (0, 0, 0, 1)$.

(C) $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{x}\}$ must be a basis for U, where $\mathbf{x} = (0, 0, 1, 1)$.

(D) None of the above OR more than one of the above.

4. Let $T : \mathbb{P}_3 \to \mathbb{R}^2$ be the linear map given by

$$T(p) = \begin{bmatrix} p(0) \\ p(1) \end{bmatrix}.$$

Find a basis for the null space of A. Find a basis for the column space of A.

5. (A) True or (B) False? In the vector space of all real-valued functions on $\mathbb{R}$, the functions $\sin(t)$ and $\cos(t)$ are linearly independent.

6. Inside the vector space of all real-valued functions on $\mathbb{R}$, consider the set $\mathcal{B} = \{\sin^2(x), \cos^2(x)\}$. This set is linearly independent (why?), so it is a basis for $U = \operatorname{span} \mathscr{B}$. Which of the following is true?

(A) The constant function 1 is not in U.

(B) The constant function 1 is in U, and $[1]_{\mathcal{B}} = (1, 1)$.

(C) The constant function 1 is in U, and $[1]_{\mathcal{B}} = (0, 0)$.

(D) None of the above.