n = # of bananas in pile.

63 piles, 7 extra bananas.

distributed among

$63x - 23y = -7$

1. Find gcd using Euclidean alg.
2. write gcd as linear combination of 63 & -23.
3. ...

---

a	a^2	remainder div by 7
0	0	0
1	1	1
2	4	4
3	9	2
4	16	2
5	25	4
6	36	1
7	49	0
8	64	1
9	81	4
		2
		2
		4
		1
		⋮

[If I know remainder of a when divide by 7, I can predict remainder of a^2.]

$a = 7q \rightsquigarrow a^2 = (7q)^2 = 49q^2 = 7(7q^2)$ rem 0.

$= 7q+1$ ....

$= 7q+2$ ...

$= 7q+3 \rightsquigarrow a^2 = (7q+3)^2 = 49q^2 + 42q + 9 = (49q^2 + 42q + 7) + 2$
$= 7(7q^2 + 6q + 1) + 2$

$=$ ⋮

$= 7q+6$

$P(n) =$ " $\gcd(S_{n+1}, S_n)$ needs n^2 division "

1.1 #1 (a) $1+2+\cdots+n = \frac{n(n+1)}{2} = t_n$.

(c) $1\cdot 2 + 2\cdot 3 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}$

$\sum_{k=0}^{n} k(k+1)$

$t_1 + \cdots + t_n = \sum_{k=0}^{n} \frac{k(k+1)}{2}$

① Find formula for $\frac{t_1+\cdots+t_n}{n}$ in terms of n.

② Try out some values of n (start from $n=0\ldots$)

find a pattern among n for which $\frac{t_1+\cdots+t_n}{n}$ is an integer.

③ $\{n \geq 1 \mid n \mid (t_1+\cdots+t_n)\} = \{n \geq 1 \mid (\cdots\cdots)\}$

---

n not a multiple of 3.

$n = 3k+1$

$\frac{(3k+2)(3k+3)}{6}$ ← mult. 3

3
6
9
12
15
18
⋮

$S_1 = 1$ $S_2 = 2$ $S_3 = 5$ $S_4 = 12$ $S_5 = 29$. ...

$S_6 = 2\cdot 29 + 12 = 70$

$\gcd(S_{n+1}, S_n)$ for $n=4$.

$\gcd(29, 12)$

$29 = 2\cdot 12 + 5$
$12 = 2\cdot 5 + 2$
$5 = 2\cdot 2 + 1$
$2 = 2\cdot 1 + 0$

4

$\gcd(S_{n+1}, S_n)$ for $n=5$.

$\gcd(70, 29)$.

$70 = 2\cdot 29 + 12$
$29 = 2\cdot 12 + 5$
$12 = 2\cdot 5 + 2$
$5 = 2\cdot 2 + 1$
$2 = 2\cdot 1 + 0$

5

"Computing $\gcd(S_{n+1}, S_n)$ requires n divisions and $\gcd(S_{n+1}, S_n) = 1$." $= P(n)$

prove by induction!

Assume $P(k)$.

$\gcd(S_{k+2}, S_{k+1})$

$$S_{k+2} = \underbrace{2}\ S_{k+1} + \underbrace{S_k}$$

why is this actually remainder? to know that, need to know

$0 \leq S_k < S_{k+1}$

The next division is

$S_{k+1} = \underline{\quad}\ S_k + \underline{\quad}$

which is exactly what you would do to compute $\gcd(S_{k+1}, S_k)$.

$$\boxed{\begin{array}{l} Q(n) = \text{" } S_n < S_{n+1} \text{ "} \\ \text{prove this by induction.} \end{array}}$$

$$t_n = \frac{n(n+1)}{2}$$

1.1 #1 (c).

$$1 \cdot 2 + 2 \cdot 3 + \cdots + n(n+1) = \frac{n(n+1)(n+2)}{3}.$$

use this to find a closed form formula for

$t_1 + \cdots + t_n$

analyze $\dfrac{t_1 + \cdots + t_n}{n}$ and see when it's an integer.