1. If we rewrite the second order linear ODE

$$y'' - 3y' - 4y = 0$$

as a linear system $\dfrac{d\vec{y}}{dt} = A\vec{y}$ where $\vec{y} = \begin{bmatrix} y \\ y' \end{bmatrix}$, what is A?

(A) $\begin{bmatrix} 0 & 1 \\ 4 & 3 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 1 \\ 3 & 4 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & 4 \\ 1 & 3 \end{bmatrix}$

(D) None of the above

2. True or False?

The characteristic polynomial of $\begin{bmatrix} 0 & 1 \\ b & a \end{bmatrix}$ is

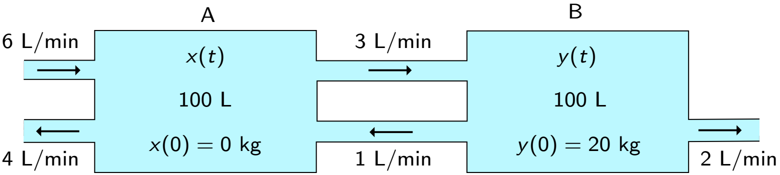
$$\lambda^2 - a\lambda - b.$$

3. True or False?

The characteristic polynomial of $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ c & b & a \end{bmatrix}$ is

$$-(\lambda^3 - a\lambda^2 - b\lambda - c).$$

4. Let x and y denote the quantities of salt in two interconnected 100 L tanks A and B, respectively, and suppose water flows between the two tanks as depicted below, where the input to tank A is pure water.

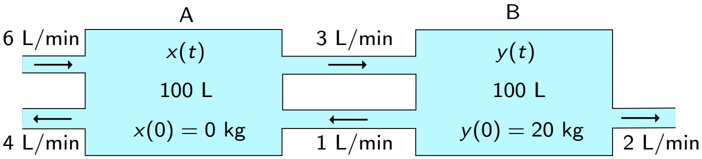


The system of ODEs that relates dx/dt and dy/dt to x and y is..

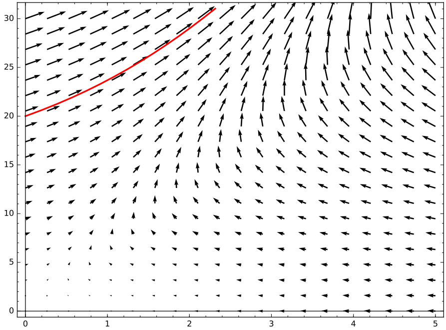
(A) linear and homogeneous.

(B) linear but not homogeneous.

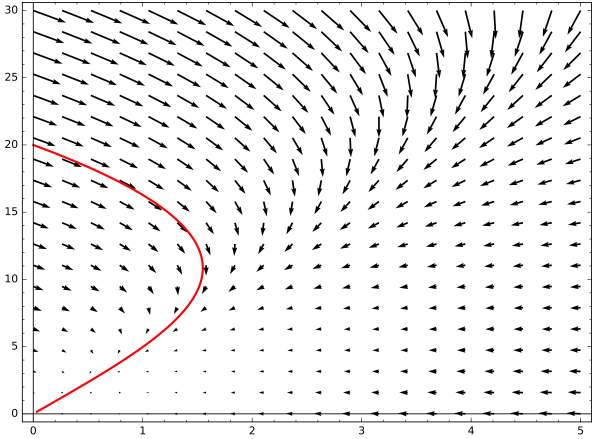
(C) nonlinear.



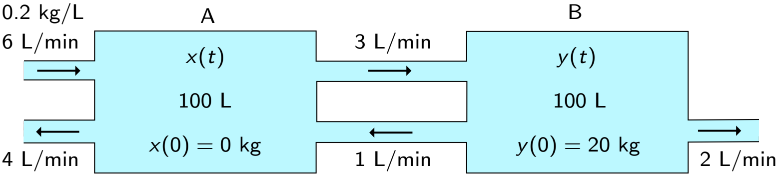
5. True or False? The system has the following phase portrait.



This is the real phase portrait.



6. Almost the same setup as the previous problem, except now the input to tank A has a salt concentration of 0.2 kg/L.



The system of ODEs that relates dx/dt and dy/dt to x and y is..

(A) linear and homogeneous.

(B) linear but not homogeneous.

(C) nonlinear.

If an object of mass $m > 0$ is attached to a wall by a spring of *stiffness* $k > 0$ and slides around on a frictionless surface, then a combination of Newton's Law and Hooke's Law says that the displacement x of the object is governed by the ODE $mx'' = -kx$.

7. If we rewrite this ODE as a first order linear system

$$\vec{x}' = A\vec{x} \text{ where } \vec{x} = \begin{bmatrix} x \\ x' \end{bmatrix},$$

what can we say about the eigenvalues of A?

(A) There is only one real eigenvalue.

(B) There are two distinct real eigenvalues.

(C) There are two distinct complex eigenvalues.