Week 8 Day 2

Eigenstuff of the Derivative

Make sure you know your neighbors' names. Then take 2 minutes to discuss:

Let $T\colon \mathbb{P}_2 \to \mathbb{P}_2$ be the linear map that takes a polynomial p to its derivative $T(p) = p'$. What are the eigenvalues of T? Is T diagonalizable?

Remark. We've only defined eigenvalues and diagonalizability for *matrices*, not for linear maps, so you'll have to discuss how to even make sense of the questions first!

Inner Product Spaces

1. If $\mathbf{u} = (3, 4)$, what is $\|\mathbf{u}\|$?

(A) 3

(B) 4

(C) 5

(D) None of the above

2. On $\mathbb{R}^2$, define an inner product by setting

$$\left\langle \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}, \begin{bmatrix} x_2 \\ y_2 \end{bmatrix} \right\rangle = 2x_1x_2 + y_1y_2.$$

If $\mathbf{u} = (3, 4)$, what is $\|\mathbf{u}\|$?

(A) 3

(B) 4

(C) 5

(D) None of the above

3. For $p, q \in \mathbb{P}_2$, define an inner product by

$$\langle p, q \rangle = p(-1)q(-1) + p(0)q(0) + p(1)q(1).$$

What is $\langle 2 + t, -1 + t^2 \rangle$?

(A) 1

(B) 2

(C) 3

(D) None of the above

4. For $p, q \in \mathbb{P}_2$, define an inner product by

$$\langle p, q \rangle = p(-1)q(-1) + p(0)q(0) + p(1)q(1).$$

What is $\|2 + t - t^2\|$?

(A) 2

(B) 4

(C) 8

(D) None of the above

5. (A) True or (B) False? Suppose V is an inner product space. For any scalar c and vector v, we have $\|cv\| = c\|v\|$.