1. Consider the linear map $h : \mathbb{R}^2 \to \mathbb{R}^2$ which reflects vectors across the y-axis. Which of the following is the matrix representation of this map with respect to the standard basis on both the domain and the codomain?

(A) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

(B) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

(C) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

(D) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

2. Consider the linear map $h : \mathbb{R}^3 \to \mathbb{R}^3$ which rotates vectors counterclockwise around the z-axis by 90°. Which of the following is the matrix representation of this map with respect to the standard basis on both the domain and the codomain?

(A) $\begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(B) $\begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(C) $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$

3. Consider the unique linear map $h : \mathbb{R}^2 \to \mathbb{R}^2$ which has the property that

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} 2 \\ 0 \end{pmatrix} \text{ and } \begin{pmatrix} 1 \\ 0 \end{pmatrix} \mapsto \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

Which of the following is the representation of this map with respect to the standard basis on both the domain and the codomain?

(A) $\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

(B) $\begin{pmatrix} 2 & -1 \\ 0 & 0 \end{pmatrix}$

(C) $\begin{pmatrix} -1 & 3 \\ 0 & 0 \end{pmatrix}$

4. Suppose a matrix A represents a linear map $\mathbb{R}^2 \to \mathcal{M}_{2\times 2}$. How many rows and columns does the matrix A have?

(A) 2 rows, 2 columns

(B) 2 rows, 4 columns

(C) 4 rows, 2 columns

(D) 4 rows, 4 columns

5. True or False?

Suppose $B = \langle 1, 1+x, 1+x+x^2 \rangle$ is a basis for $\mathcal{P}_2$ and $h : \mathcal{P}_2 \to \mathcal{P}_2$ is the linear map such that

$$\mathrm{Rep}_{B,B}(h) = \begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 3 \\ 1 & 4 & 3 \end{pmatrix}.$$

Then h is an isomorphism.

6. True or False?

Let $B = \langle \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \rangle$, and let $h : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear map such that

$$\mathrm{Rep}_{B,B}(h) = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}.$$

Then h is given by $\begin{pmatrix} x \\ y \end{pmatrix} \mapsto \begin{pmatrix} x + 2y \\ 3x + 4y \end{pmatrix}$.

7. True or False?

Let $h : \mathbb{R}^3 \to \mathcal{P}_2$ be the linear map whose representation with respect to the standard basis on $\mathbb{R}^3$ and the basis $\langle 1, 1+x^2, x\rangle$ on $\mathcal{P}_2$ is the following matrix.

$$\begin{pmatrix} 1 & 3 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

Then $1+2x$ is in $\mathscr{R}(h)$.