1. True or False?

Suppose V is a finite dimensional inner product space. Every self-adjoint $T \in \mathcal{L}(V)$ has a cube root.

2. True or False?

Suppose V is a finite dimensional complex inner product space and $T \in \mathcal{L}(V)$ is a normal operator with eigenvalues -1 and 1. Then $T^2 = I$.

3. True or False?

Suppose V is a finite dimensional complex inner product space and $T \in \mathcal{L}(V)$ is a normal operator such that

$$T^3 + 2T^2 + T = 0.$$

Then $p_{\min}(z) = z^2 + z$.

3. True or False?

Suppose V is a finite dimensional complex inner product space and $T \in \mathcal{L}(V)$ is a normal operator such that

$$T^3 + 2T^2 + T = 0.$$

Then $p_{\min}(z) = z^2 + z$.

If $\dim V = 3$, is it true that $p_{\mathrm{char}}(z) = z^3 + 2z^2 + z$?

4. True or False?

Suppose $T \in \mathcal{L}(\mathbf{R}^4)$. Suppose further that e_1, e_2, e_3 is an orthonormal list of eigenvectors of T, and that v is an eigenvector such that

$$\langle v, e_1 \rangle^2 + \langle v, e_2 \rangle^2 + \langle v, e_3 \rangle^2 = 0.$$

Then T is self-adjoint.