Name:

Quiz 5

Part I (10 points). You will get 1 point for each correct answer, 0 points for each blank answer, and -1 point for each incorrect answer. The minimum possible score for this section is 0.

(1) There exists a unique polynomial of degree at most 3 whose graph passes through the points $(0, 1)$, $(1, -1)$, $(2, -1)$, and $(3, 2)$. T F

(2) Suppose $T \in \mathcal{L}(\mathbf{F}^5)$ is not surjective and $\dim E(2, T) = 4$. Then T is diagonalizable. T F

(3) Suppose $T \in \mathcal{L}(\mathbf{F}^3)$ has eigenvectors u, v, w with the property that none of these three vectors is a scalar multiple of either of the others. Then T is diagonalizable. T F

(4) Suppose $T \in \mathcal{L}(V)$, U is an invariant subspace, and W is a subspace such that $V = U \oplus W$. Then W is invariant. T F

(5) Suppose $T \in \mathcal{L}(\mathbf{F}^5)$ and $\dim E(4, T) = 4$. Then either $T - 2I$ or $T + 2I$ is invertible. T F

(6) Suppose $T \in \mathcal{L}(\mathbf{F}^3)$ has two distinct invariant subspaces of dimension 2. Then there exists an invariant subspace of dimension 1. T F

(7) Suppose $T \in \mathcal{L}(V)$ has no eigenvalues. Then T^2 has no eigenvalues. T F

(8) Suppose $T \in \mathcal{L}(\mathbf{F}^3)$ and u, v, w are linearly independent eigenvectors. If $u + v, v + w, w$ are also eigenvectors, then T is a scalar multiple of the identity. T F

(9) Suppose $T \in \mathcal{L}(\mathbf{F}^6)$ and U is a 5 dimensional invariant subspace such that the restriction operator $T|_U$ is diagonalizable. Then T is diagonalizable. T F

(10) Suppose 2 is an eigenvalue of $T \in \mathcal{L}(\mathbf{F}^5)$ such that the eigenspace $U = E(2, T)$ has dimension 2 and the quotient operator T/U has eigenvalues 4, 5, 6. Then T is diagonalizable. T F

Part II (10 points).

(11) Let $T \in \mathcal{L}(\mathbf{F}^2)$ be given by $T(x, y) = (x + y, 3y)$. Is T diagonalizable? If so, find a basis of $\mathbf{F}^2$ such that $M(T)$ is diagonal. Otherwise, explain why no such basis exists.

(12) Let V be a finite dimensional vector space over $\mathbf{C}$ and suppose $T \in \mathcal{L}(V)$ is an operator such that 4 is an eigenvalue of T^2. Show that

$$\dim \operatorname{null}(T - 2I) + \dim \operatorname{null}(T + 2I) > 0.$$