1. True or False?

Consider the system

$$\frac{d\vec{x}}{dt} = A\vec{x}, \text{ where } A = \begin{bmatrix} -1 & 1 & 3 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}.$$

Then $\lim_{t\to\infty} \vec{x}(t) = 0$, regardless of the initial conditions.

2. Which of the following best describes the phase portrait of $\vec{x}' = \begin{bmatrix} 0 & 1 \\ -5 & 4 \end{bmatrix} \vec{x}$?

(A) Center point

(B) Spiral source

(C) Spiral sink

(D) None of the above

3. Which of the following functions are solutions to the ODE $x''' - 3x'' + 2x' = 0$?

(A) $x(t) = 2e^t$

(B) $x(t) = -3e^{2t}$

(C) $x(t) = 5$

(D) All of the above

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Follow-up. What is the general solution of this ODE?

4. Consider the same ODE $x''' - 3x'' + 2x' = 0$. Which of the following is a basis for the solution space?

(A) $1, e^t, e^{2t}$

(B) $1, 1 + e^t, 1 + e^t + e^{2t}$

(C) $1 + e^t + e^{2t}, e^t + e^{2t}, e^{2t}$

(D) All of the above

5. For which of the following ODEs is it the case that all three of the functions $x_1(t) = \cos(t)$, $x_2(t) = \sin(t)$ and $x_3(t) = e^t$ are solutions?

(A) $x''' + x'' + x' + x = 0$

(B) $x''' + x'' - x' - x = 0$

(C) $x''' - x'' + x' - x = 0$

(D) None of the above