> If the ordered set F has elements that are not of, for example, lex order, do we need to reorder the elements in F?

Do not re-order the list F of divisors! That can change the answer. You can re-order the terms within each polynomial (whether that's f or each polynomial that occurs in F) so that they're in descending order with respect to whatever order you're using (eg, lex), but you cannot re-order the overall list.

> If we divide by f_1 and get to a point where we cannot divide by f_1, then we move on to dividing by f_2 and reach a point where we cannot divide by f_2 anymore, is it always the case that we are finished at that point? In other words, if the remainder after dividing by f_2 is divisible by f_1, are we allowed to go back and divide by f_1 again?

It's not just that you're *allowed* to, but you *must*! You're not done dividing until no term of your dividend is divisible by the leading term of any of your divisors.