Name:

Quiz 6

Part I (10 points). You will get 1 point for each correct answer, 0 points for each blank answer, and -1 point for each incorrect answer. The minimum possible score for this section is 0.

In all of the following, V is a finite dimensional complex vector space and $T \in \mathcal{L}(V)$. We write p_{char} and p_{min} for the characteristic and minimal polynomials of T, respectively.

(1) The degree of p_{min} is the number of distinct eigenvalues of T. T F

(2) If $\dim V = 3$ and T has eigenvalues 0, 2 and -2, then $T^3 = 4T$. T F

(3) If $\dim V = 5$ and $\operatorname{null} T \neq \operatorname{null} T^2$, then T has at most 3 distinct eigenvalues. T F

(4) T is not surjective if and only if $p_{\text{char}}(0) = 0$. T F

(5) If the matrix representation of T with respect to some basis is T F

$$M(T) = \begin{pmatrix} 7 & 1 & 0 & 0 & 0 \\ 0 & 7 & 1 & 0 & 0 \\ 0 & 0 & 7 & 0 & 0 \\ 0 & 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 0 & 3 \end{pmatrix}$$

then $p_{\text{char}} = p_{\text{min}}$.

(6) Suppose 2 is the only eigenvalue of T and that we know the following. T F

$$\begin{aligned} \dim \operatorname{null}(T - 2I) &= 2 \\ \dim \operatorname{null}(T - 2I)^2 &= 4 \\ \dim \operatorname{null}(T - 2I)^3 &= 6 \\ \dim \operatorname{null}(T - 2I)^4 &= 6 \end{aligned}$$

Then $p_{\text{min}}(z) = (z-2)^2$.

(7) If the matrix of T with respect to some basis is in Jordan form with two Jordan blocks of eigenvalue 0, one of size 2 and another one of size 3, then $\dim \operatorname{null} T^2 = \dim \operatorname{null} T + 2$. T F

(8) If $p_{\text{char}}(z) = (z-3)^2(z-2)^5$, it is possible to have $p_{\text{min}}(z) = (z-3)^4(z-2)$. T F

(9) If $T^2 = T$, then T is diagonalizable with all eigenvalues equal to 0 or 1. T F

(10) If 7 is an eigenvalue of T and $\operatorname{null}(T-7I)^3 = \operatorname{null}(T-7I)^4$, then 7 has multiplicity at most 3 as a root of p_{min}. T F

Part II (10 points).

(11) Suppose $T \in \mathcal{L}(\mathbf{C}^4)$ is given by $T(w,x,y,z) = (z,0,x,y)$. Let $v = (0,1,0,0)$ and note that $T^4 v = 0$ but $T^3 v \neq 0$. Calculate a basis for V with respect to which the matrix of T is in Jordan form.

(12) Does there exist an operator T on a finite dimensional complex vector space V such that $\dim \operatorname{null} T = 1$ and $\dim \operatorname{null} T^2 = 3$? If so, provide an example and prove that it is an example. Otherwise, prove that it is impossible.