1. True or False?

Let V be the vector space whose elements are functions $f : \mathbb{R}^+ \to \mathbb{R}$, so that $f(x) = x$ and $g(x) = 1/x$ are both elements of V. Then $\{f, g\}$ is a linearly independent subset of V.

2. Let $B = \left\langle \begin{pmatrix} 1 \\ 1 \end{pmatrix}, v \right\rangle$. Which of the following vectors $v \in \mathbb{R}^2$ will make B a basis for $\mathbb{R}^2$?

(A) $v = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

(B) $v = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

(C) Both (A) and (B)

(D) Neither (A) nor (B)

3. Let

$$B = \left\langle \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\rangle$$

and let U be the span of B inside $\mathbb{R}^3$. Let v be the vector in U such that

$$\mathrm{Rep}_B(v) = \begin{pmatrix} 3 \\ -1 \end{pmatrix}.$$

Which of the following is v?

(A) $(3, -1, 0)$

(B) $(-1, 3, 2)$

(C) $(3, -1, 2)$

(D) $(3, 0, -1)$

4. True or False?

There exists a basis B of $\mathbb{R}^2$ such that

$$\mathrm{Rep}_B\left(\begin{pmatrix}1\\3\end{pmatrix}\right)=\begin{pmatrix}3\\1\end{pmatrix}.$$

5. True or False?

Let

$$U = \left\{ \begin{pmatrix} a+b \\ a+c \end{pmatrix} \in \mathbb{R}^2 : a, b, c \in \mathbb{R} \right\}.$$

What is $\dim(U)$?

(A) 3

(B) 2

(C) 1

(D) 0

6. What is $\dim(\mathcal{M}_{2\times 3})$?

(A) 2

(B) 3

(C) 5

(D) 6

7. Let $U = \{p \in \mathcal{P}_3 : p(7) = 0\}$. What is $\dim(U)$?

(A) 4

(B) 3

(C) 2

(D) 1

8. Let

$$U = \left\{ \begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \in \mathcal{M}_{2\times 2} : c - 2b = 0 \right\}.$$

What is $\dim(U)$?

(A) 4

(B) 3

(C) 2

(D) 1