

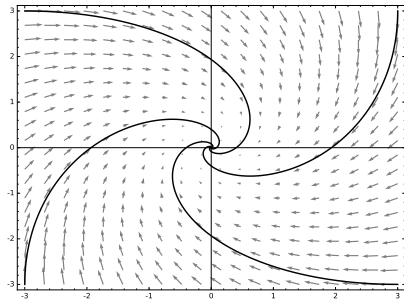
1. Suppose a 1 kg mass is attached to a wall by a spring of stiffness k and slides around on a surface with friction 2 kg/s, so that the equation of motion is governed by the ODE

$$x'' + 2x' + kx = 0.$$

For which of the following values of k is this system overdamped?

- (a) $k = 0.5 \text{ kg/s}^2$
- (b) $k = 1 \text{ kg/s}^2$
- (c) $k = 2 \text{ kg/s}^2$
- (d) None of the above

2. A is a matrix of real numbers and the phase portrait of a system $\vec{x}' = A\vec{x}$ is depicted to the right. Which of the following is a true statement about A ?



- (A) It has complex eigenvalues with positive real part.
- (B) It has complex eigenvalues with negative real part.
- (C) It has a deficient negative eigenvalue.
- (D) None of the above.

Consider the following matrix.

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

i is a deficient eigenvalue of A (with deficiency 1),
 $v_1 = (-i, 1, 0, 0)$ is an eigenvector, and $v_2 = (1, 0, 2, 2i)$ is a generalized eigenvector such that $(A - iI)v_2 = v_1$.

3. Which of the following is a solution to $\vec{x}' = A\vec{x}$?

- (A) $\vec{x}(t) = (\sin t, \cos t, 0, 0)$
- (B) $\vec{x}(t) = (\cos t + t \sin t, t \cos t, 2 \cos t, -2 \sin t)$
- (C) $\vec{x}(t) = (\sin t - t \cos t, t \sin t, 2 \sin t, 2 \cos t)$
- (D) More than one of the above

4. Which of the following is a basis for the solution space of the second order homogeneous linear ODE $x'' - x' - 6x = 0$?

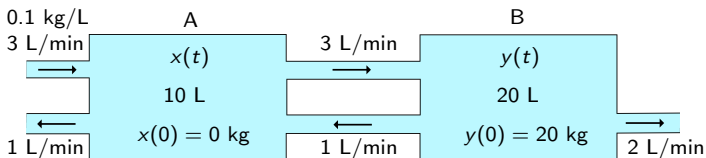
(A) e^{3t}, e^{-2t}

(B) $e^{3t}, -2e^{3t}$

(C) $e^{3t} - e^{-2t}, e^{3t} + e^{-2t}$

(D) More than one of the above

Salt water tanks! The input to tank A has a salt concentration of 0.1 kg/L.



5. If you were to solve the linear nonhomogeneous system

$$\begin{bmatrix} x \\ y \end{bmatrix}' = A \begin{bmatrix} x \\ y \end{bmatrix} + \vec{f}$$

using the method of undetermined coefficients, what form would you guess for the particular solution?

(A) \vec{a}

(B) $\vec{a}t + \vec{b}$

(C) Neither of the above

6. True or False?

Suppose the matrix

$$\begin{bmatrix} 0 & 1 \\ b & a \end{bmatrix}$$

has a repeated eigenvalue λ . Then $\lambda = a/2$.