

Day 11

1. What is the order of the subgroup $\left\langle \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \right\rangle$ of $GL(2, \mathbf{R})$?

(A) 2

(B) 4

(C) Infinite

(D) None of the above

2. On the set \mathbf{R} of real numbers, define a relation R where $a R b$ means that $a - b$ is an integer. The relation R is...

- (A) An equivalence relation.
- (B) Reflexive and symmetric, but not transitive.
- (C) Reflexive and transitive, but not symmetric.
- (D) None of the above.

Recall: injective = one-to-one, surjective = onto, bijective = one-to-one and onto

3. Consider the determinant function $\det : GL(2, \mathbf{R}) \rightarrow \mathbf{R}^*$.

This function is:

- (A) Neither injective nor surjective
- (B) Injective, but not surjective
- (C) Surjective, but not injective
- (D) Bijective

4. How many elements are in the set $\{x^2 \mid x \in D_4\}$?

(A) 1

(B) 2

(C) 4

(D) None of the above

5. The subset of rotations in D_n is...

(A) a subgroup of D_n .

(B) not a subgroup of D_n .

6. Inside $GL(2, \mathbf{R})$, consider the subset

$$H = \left\{ \begin{bmatrix} x & y \\ 0 & x^{-1} \end{bmatrix} \mid x \in \mathbf{R}^*, y \in \mathbf{R} \right\}.$$

Then H is...

- (A) a subgroup of $GL(2, \mathbf{R})$.
- (B) not a subgroup of $GL(2, \mathbf{R})$.

7. Which of the following is an *impossible* value for $\gcd(n, n + 2)$, no matter what integer n is chosen to be?

(A) 1

(B) 2

(C) 3

(D) None of the above (ie, all values above are possible)

8. $\gcd(x, 5) = \gcd(x - 5, 5)$ is true for...

- (A) All integers x .
- (B) Some integers x .
- (C) No integers x .

9. Suppose r_1, r_2, r_3 represent rotations from D_n and f_1, f_2, f_3 represent reflections in D_n . Then $r_1 r_2 f_1 r_3 f_2 f_3 r_3$ is a...

- (A) Rotation.
- (B) Reflection.
- (C) Can't say for sure either way.

10. Which of the following groups has the largest number of elements of finite order?

(A) \mathbf{R}

(B) \mathbf{R}^*

(C) $\text{GL}(2, \mathbf{R})$

(D) The group of all 2×2 matrices under addition