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Quiz 1

Part I (10 points). You will get 1 point for each correct answer, 0 points for each blank answer, and -1 point for each incorrect answer. The minimum possible score for this section is 0.

(1) If V is a vector space over $\mathbf{R}$, and we have $u, v \in V$ and a nonzero $\lambda \in \mathbf{R}$ such that $\lambda u = \lambda v$, then $u = v$. **T** F

(2) The set $\{(a, b, c) \in \mathbf{R}^3 : a + 2b + c = 4\}$ is a subspace of $\mathbf{R}^3$. T **F**

(3) The set $\{(a, b, c) \in \mathbf{R}^3 : a^2 = b^2\}$ is a subspace of $\mathbf{R}^3$. T **F**

(4) Suppose U is a subspace of a vector space V such that $U + W = \{0\}$ for some subspace W. Then $U = \{0\}$. **T** F

(5) The set of functions $f : \mathbf{R} \to \mathbf{R}$ such that $f(0) = b$ is a subspace of $\mathbf{R}^{\mathbf{R}}$ if and only if $b = 0$. **T** F

(6) The set of functions $f : \mathbf{R} \to \mathbf{R}$ such that $f(a) = 0$ is a subspace of $\mathbf{R}^{\mathbf{R}}$ if and only if $a = 0$. T **F**

(7) Let $U = \{(x, y, x + y, x - y) \in \mathbf{R}^4 : x, y \in \mathbf{R}\}$. Then U is a subspace of $\mathbf{R}^4$, and there exist subspaces W_1 and W_2, neither of which equals $\{0\}$, such that $\mathbf{R}^4 = U \oplus W_1 \oplus W_2$. **T** F

(8) Let E be the subset of even[1] functions in $\mathbf{R}^{\mathbf{R}}$ and B the subset of bounded[2] functions. Then E and B are subspaces of $\mathbf{R}^{\mathbf{R}}$, and the sum $E + B$ is direct. T **F**

(9) Let $V = \{(a, b) : a, b \in \mathbf{R}\}$. Define addition on V coordinate-wise, and define a scalar multiplication operation @ by the formula T **F**

$$\lambda @ (a, b) = (\lambda a, 0)$$

for all $(a, b) \in V$ and $\lambda \in \mathbf{R}$. Then V, equipped with these operations, is a vector space over $\mathbf{R}$.

(10) Let $V = \{a \in \mathbf{R} : a > 0\}$ be the set of positive real numbers. Define an addition operation # and a scalar multiplication operation @ on V by the formulas **T** F

$$a \# b = ab \text{ and } \lambda @ a = a^{\lambda}$$

where $a, b \in V$ and $\lambda \in \mathbf{R}$. Then V, equipped with these two operations, is a vector space over $\mathbf{R}$.

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[1] A function $f : \mathbf{R} \to \mathbf{R}$ is said to be *even* if $f(x) = f(-x)$ for all $x \in \mathbf{R}$.

[2] A function $f : \mathbf{R} \to \mathbf{R}$ is said to be *bounded* if there exists some C such that $|f(x)| \leq C$ for all $x \in \mathbf{R}$.

Part II (10 points). Consider the following subspaces of $\mathbf{F}^3$.

$$U = \{(x, 2x, y) \in \mathbf{F}^3 : x, y \in \mathbf{F}\} \text{ and } W = \{(x, y, 0) \in \mathbf{F}^3 : x, y \in \mathbf{F}\}$$

(11) Prove that $U + W = \mathbf{F}^3$.

Any element $(x, y, z) \in \mathbf{F}^3$ can be written as a sum

$$(x, y, z) = (0, 0, z) + (x, y, 0),$$

where clearly $(0, 0, z) \in U$ and $(x, y, 0) \in W$.

(12) Is it true that $U \oplus W = \mathbf{F}^3$? Prove your assertion.

No, it is not true. Observe that

$$U \cap W = \{(x, 2x, 0) \in \mathbf{F}^3 : x \in \mathbf{F}\} \neq \{0\}.$$

Alternatively, note that

$$0 = (1, 2, 0) + (-1, -2, 0)$$

where $(1, 2, 0) \in U$ and $(-1, -2, 0) \in W$, so 0 can be written in a nontrivial way as a sum of an element of U and an element of W.