Goals for Today

- Review Time-Varying LQR
- Motivating Vehicle Dynamics
- Coordinate System and Rigid Body Motion
- Longitudinal Vehicle Motion
- Lateral Vehicle Motion

Review Time-Varying LQR

1. Select a trajectory to be followed
2. Linearize about the trajectory
3. Apply Riccati Equation

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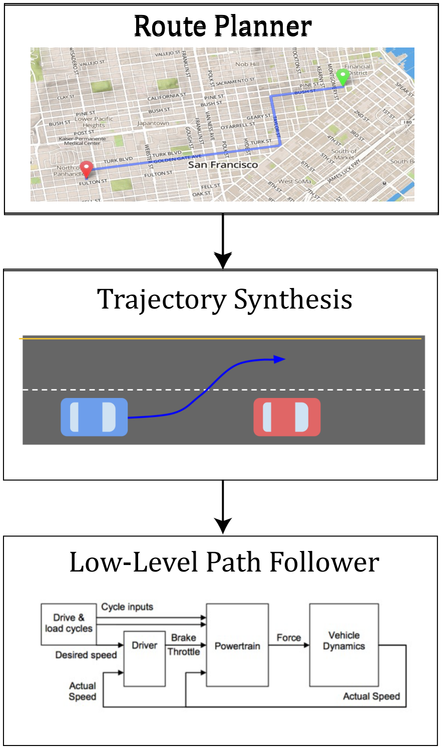
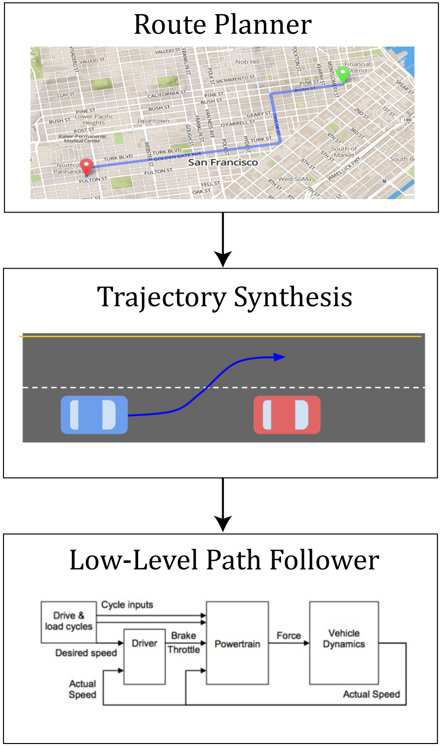
Review of Vehicle Dynamics

- Complete graduate level treatment is in ME542 (Vehicle Dynamics)
- Our goal:
  - Introduce basic concepts and terminology
  - Develop topics, and models, we will subsequently need for control system design
  - Longitudinal and Lateral Dynamics
  - Use textbook entitled Automotive Control Systems by Ulsoy, Peng, and Cakmakci

AV Control Currently



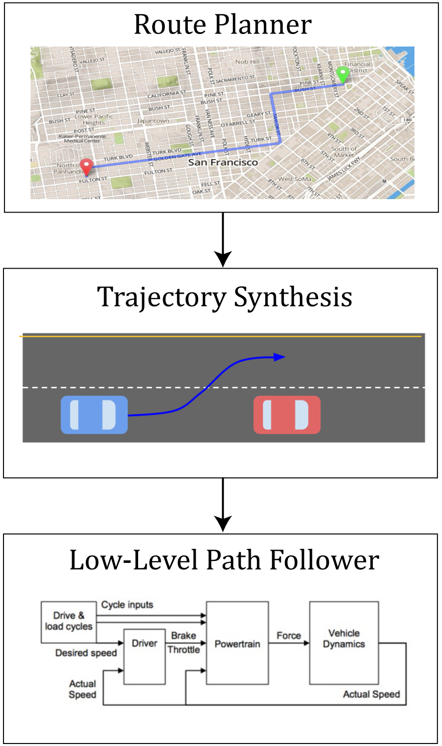
Strategies to Ignore Dynamics



$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} v\cos(\theta) \\ v\sin(\theta) \\ \dot{\theta} \end{bmatrix}$$

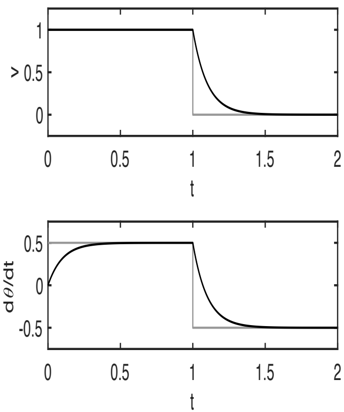
- Sample from control space to generate paths
- Sample from (x,y) space and then use pure pursuit

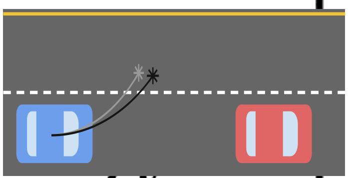
Real-World Has Dynamics



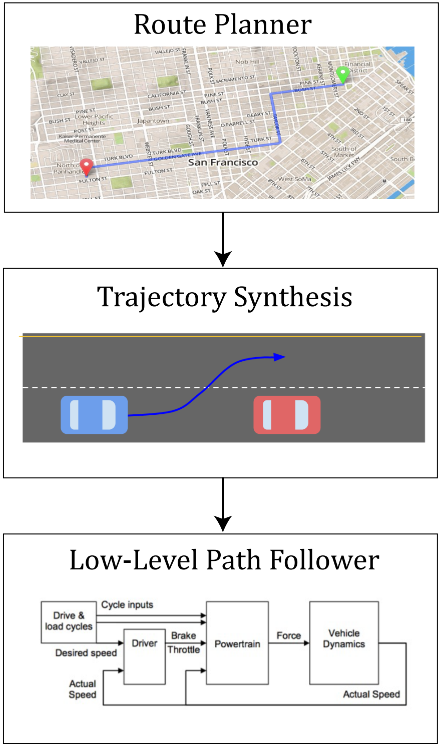
- Kinematic Trajectory Synthesis
- Dynamic Following

$$\begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} \quad \begin{bmatrix} v\cos(\theta) \\ v\sin(\theta) \\ \dot{\theta} \\ \alpha \end{bmatrix}$$





Don’t Drive Aggressively or in Adverse Weather



Uber’s autonomous cars drove 20,354 miles and had to be taken over at every mile, according to documents

A first look at Uber’s progress.

BY JOHANA BHUIYAN | @JMBOOYAH | MAR 16, 2017, 6:14PM EDT

A Google self-driving car caused a crash for the first time

A bad assumption led to a minor fender-bender

by Chris Ziegler | Feb 29, 2016, 1:50pm EST

Google self-driving car gets pulled over for driving too slowly

Automated vehicle caused traffic jam while travelling at just 24mph in a 35mph zone, causing police to pull the car over

Google's self-driving car in broadside collision after other car jumps red light

Autonomous Lexus SUV could not prevent accident that caved in front and rear passenger-side doors, setting off airbags and forcing it to be towed away

Is That Enough?

1. Is it feasible for AV to drive much slower than traffic on suburban/urban roads?

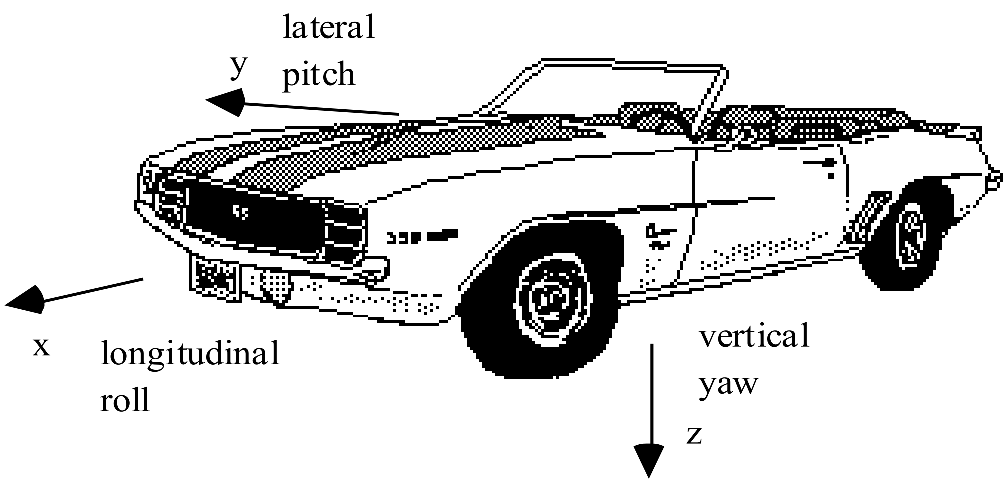
2. Is it feasible for AV to not operate in adverse weather conditions?

Outline

- Review Time-Varying LQR
- Motivating Vehicle Dynamics
- Coordinate System and Rigid Body Motion
  1. Vehicle coordinate system
  2. Rigid Body Dynamics
- Longitudinal Vehicle Motion
- Lateral Vehicle Motion

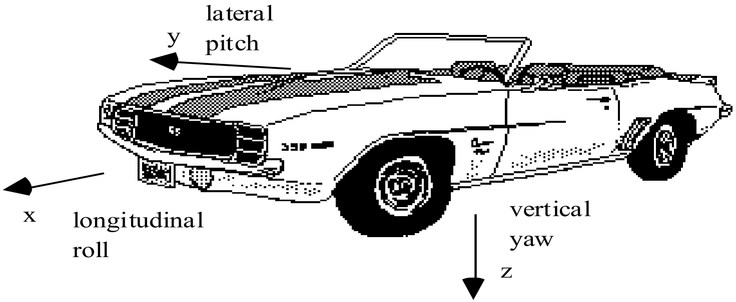
Vehicle Coordinate System

- The Society of Automotive Engineers (SAE)introduced standard coordinates and notations for describing vehicle dynamics
- Vehicle-fixed coordinate system:



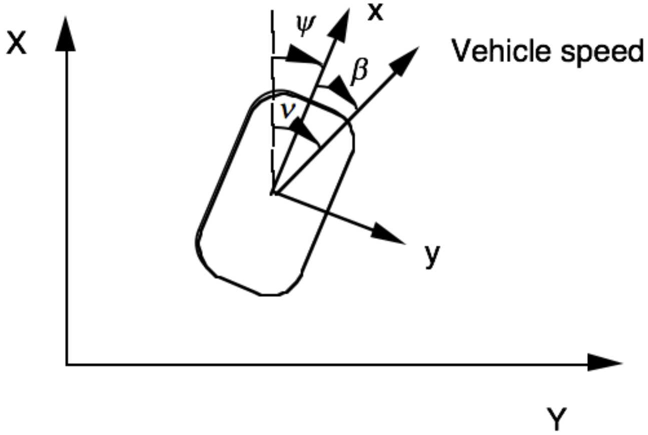
SAE Vehicle - Coordinate System

Axis	Translational Velocity	Angular Displacement	Angular Velocity	Force	Moment
x	u (forward)	ϕ	p or $\dot{\phi}$ (roll)	F_x	M_x
y	v (lateral)	θ	q or $\dot{\theta}$ (pitch)	F_y	M_y
z	w (vertical)	ψ	r or $\dot{\psi}$ (yaw)	F_z	M_z



Earth Fixed Coordinate + Side Slip

OXYZ fixed on Earth (does not turn with vehicle)



Course angle $\nu = \psi + \beta$

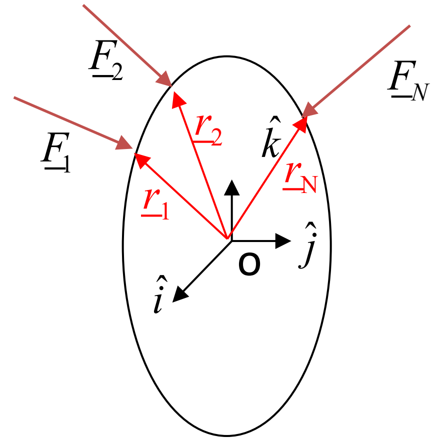
Heading angle

Side slip (angle)

Newton/Euler Equations

Translation $\sum_i^N \underline{F}_i = \underline{F} = m\frac{d\underline{V}}{dt}$

Rotation $\sum_i^N \underline{r}_i \times \underline{F}_i = \underline{M}_o = \frac{d\underline{H}}{dt}$



Where $\underline{H}$ is the *angular momentum* of the rigid body:

$$\underline{H} = \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix} = \begin{bmatrix} I_{xx} & -I_{yx} & -I_{zx} \\ -I_{xy} & I_{yy} & -I_{zy} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$I_{xx} = \int_V (y^2 + z^2)\rho \cdot dV$$

$$I_{xy} = \int_V (xy)\rho \cdot dV$$

Vehicle Rigid Body Motion

Translational motion: $\sum F = ma$

$$\sum F = ma = m\frac{d}{dt}\left(u\hat{i} + v\hat{j} + w\hat{k}\right)$$

$$= m \cdot \left(\dot{u}\hat{i} + \dot{v}\hat{j} + \dot{w}\hat{k} + u\dot{\hat{i}} + v\dot{\hat{j}} + w\dot{\hat{k}}\right)$$

$$= m\left(\frac{d}{dt}\begin{bmatrix} u \\ v \\ w \end{bmatrix} + \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \begin{bmatrix} u \\ v \\ w \end{bmatrix}\right)$$

$$\sum F_x = m\left(\dot{u} + qw - rv\right)$$

$$\sum F_y = m\left(\dot{v} + ru - pw\right)$$

$$\sum F_z = m\left(\dot{w} + pv - qu\right)$$

Vehicle Rigid Body Motion

Rotational motion: $M = \dot{H}$

$$\sum M = \frac{d}{dt}\begin{bmatrix} I_{xx}p - I_{xy}q - I_{xz}r \\ -I_{xy}p + I_{yy}q - I_{yz}r \\ -I_{xz}p - I_{yz}q + I_{zz}r \end{bmatrix} + \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \begin{bmatrix} I_{xx}p - I_{xy}q - I_{xz}r \\ -I_{xy}p + I_{yy}q - I_{yz}r \\ -I_{xz}p - I_{yz}q + I_{zz}r \end{bmatrix}$$

$$\sum M_x = I_{xx}\dot{p} - I_{xy}\dot{q} - I_{xz}\dot{r} - I_{xz}pq - I_{yz}q^2 + I_{zz}rq + I_{xy}pr - I_{yy}qr + I_{yz}r^2$$

$$\sum M_y = -I_{xy}\dot{p} + I_{yy}\dot{q} - I_{yz}\dot{r} + I_{xx}pr - I_{xy}qr - I_{xz}r^2 + I_{xz}p^2 + I_{yz}qp - I_{zz}rp$$

$$\sum M_z = -I_{xz}\dot{p} - I_{yz}\dot{q} + I_{zz}\dot{r} - I_{xy}p^2 + I_{yy}qp - I_{yz}rp - I_{xx}pq + I_{xy}q^2 + I_{xz}rq$$

Simplified Vehicle Rigid Body Motion

- Assume
  - vehicle is symmetric in the xz plane ($I_{xy} = I_{yz} = 0$)
  - *p,q,r,v*, and *w* are small, i.e., their products are negligible.
  - $u = u_o + u'$, and u' is small compared with u_o.

$$\sum F_x = m\dot{u}$$

$$\sum F_y = m\left(\dot{v} + ru_0\right)$$

$$\sum F_z = m\left(\dot{w} - qu_0\right)$$

$$\sum M_x = I_{xx}\dot{p} - I_{xz}\dot{r}$$

$$\sum M_y = I_{yy}\dot{q}$$

$$\sum M_z = -I_{xz}\dot{p} + I_{zz}\dot{r}$$

Linear!

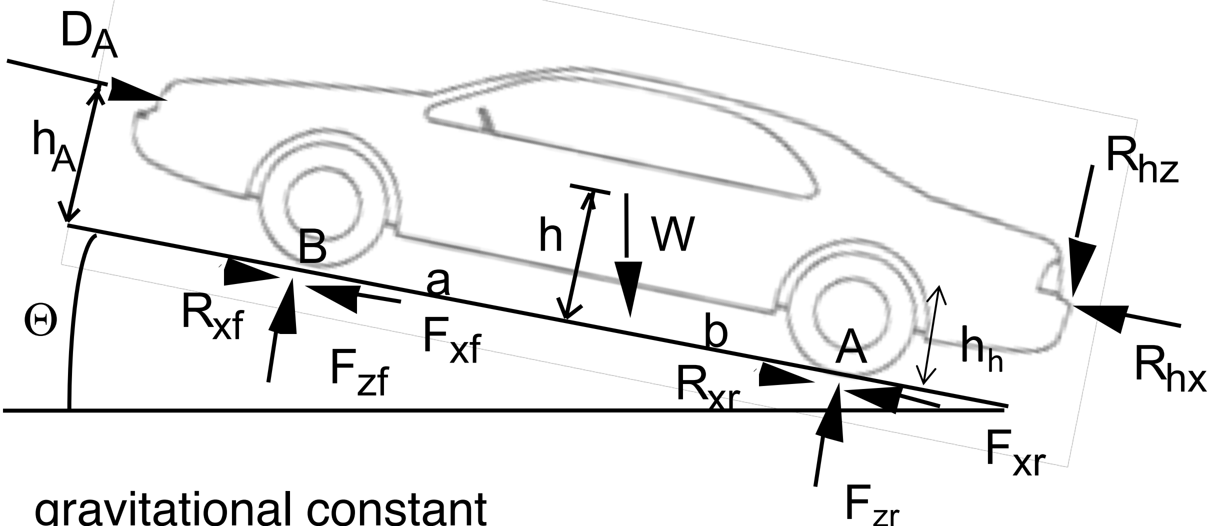
Naturally groups into 3 sets:

1. Longitudinal
2. Lateral/yaw/roll
3. Vertical/pitch

Outline

- Review Time-Varying LQR
- Motivating Vehicle Dynamics
- Coordinate System and Rigid Body Motion
- Longitudinal Vehicle Motion
  1. Force Balance
  2. Longitudinal Slip and Tire Friction
- Lateral Vehicle Motion
- Vertical Vehicle Motion

Longitudinal Vehicle Motion



g: gravitational constant

D_A : aerodynamic drag force

R_h : drawbar force

W : weight of the vehicle (= mg)

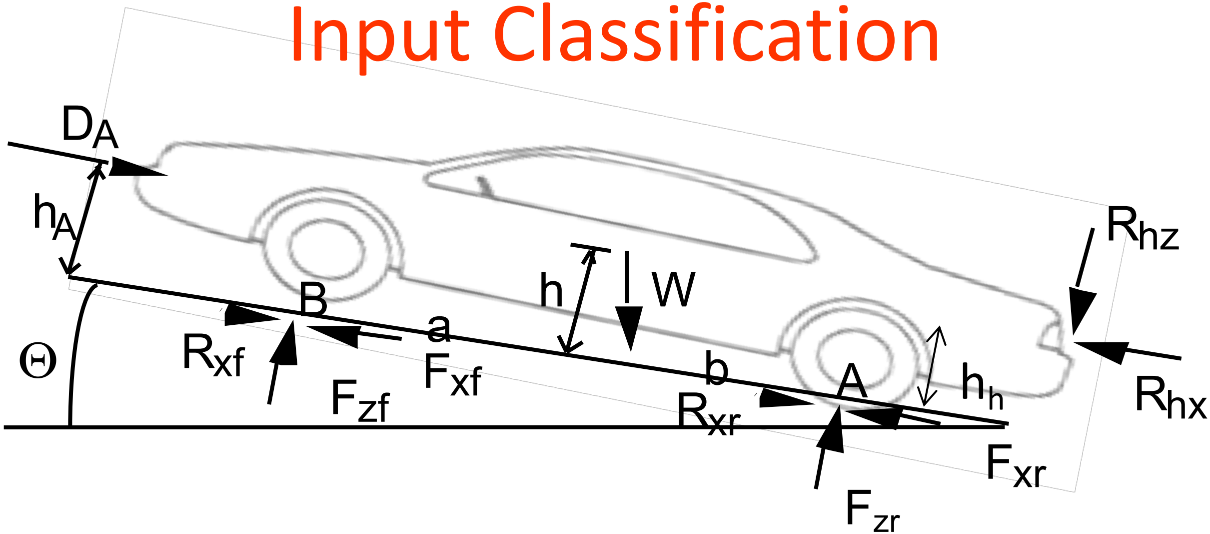
F_x : tractive force

F_z : tire normal force

R_x : rolling resistance force

ma_x: inertial force

Input Classification



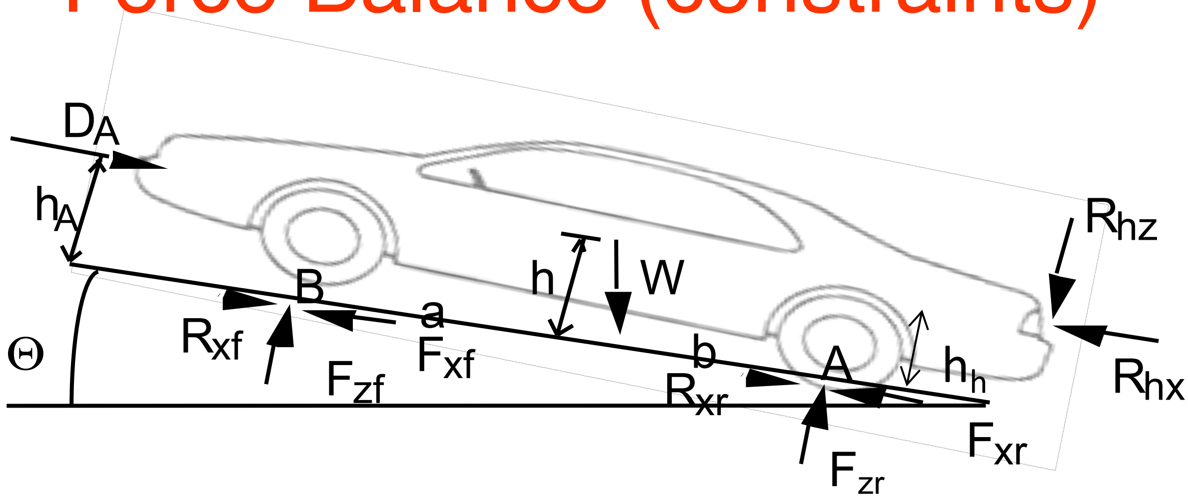
Controlled Inputs

Tractive forces (2)

Uncontrolled Inputs

Aerodynamic drag(1)
Rolling resistance(2)
Hitch forces(2)
Normal forces(2)
Gravity force(1)

Force Balance (constraints)



Force balance in the vertical (*z*) direction:

$$0 = W\cos\Theta - F_{zf} - F_{zr} + R_{hz}$$

Force balance in the longitudinal (*x*) direction:

$$ma_x = \frac{W}{g}a_x = F_{xr} + F_{xf} - W\sin(\Theta) - R_{xr} - R_{xf} - D_A + R_{hx}$$

Moment balance (about point A or point B)

Example 4.1
Aerodynamic Drag and Rolling Resistance

- Assumptions: no gradient, no hitch force, neutral gear (i.e., no tractive forces)
- Perform two coast-down tests:

	High speed test	Low speed test
Initial speed	V_{i1}	V_{i2}
Final speed	V_{f1}	V_{f2}
Time duration	t_1	t_2
Average speed	$V_1 = \dfrac{V_{i1} + V_{f1}}{2}$	$V_2 = \dfrac{V_{i2} + V_{f2}}{2}$
Average deceleration	$a_1 = \dfrac{V_{i1} - V_{f1}}{t_1}$	$a_2 = \dfrac{V_{i2} - V_{f2}}{t_2}$

Ex. 4.1 (cont.)

$$ma_x = \frac{W}{g}a_x = \cancel{F_{xr}} + \cancel{F_{xf}} - \cancel{W\sin(\Theta)} - R_{xr} - R_{xf} - D_A + \cancel{R_{hx}}$$

No traction

Flat road

No hitch force

$$D_{A1} + R_x = 0.5\rho C_d A V_1^2 + fmg = ma_1$$

$$D_{A2} + R_x = 0.5\rho C_d A V_2^2 + fmg = ma_2$$

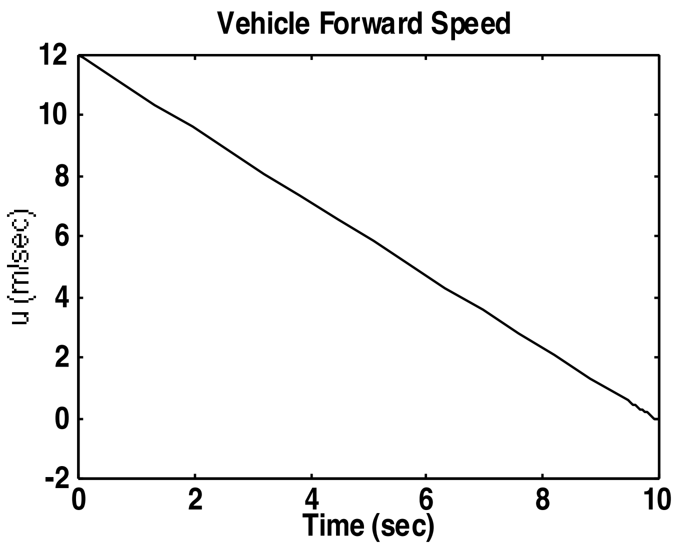
$$C_d = \frac{m(a_1 - a_2)}{0.5\rho A(V_1^2 - V_2^2)}$$

$$f = \frac{a_1 V_2^2 - a_2 V_1^2}{g(V_2^2 - V_1^2)}$$

Example 4.3
Vehicle Longitudinal Dynamics Simulation
(No Tire Dynamics)

$$ma_x = \frac{W}{g}a_x = F_{xr} + F_{xf} - W\sin(\Theta) - R_{xr} - R_{xf} - D_A + \cancel{R_{hx}}$$

```
% Ex4_3.m
% Init. time, final tim
% values of the varia
  ti=0.0; tf=10.0; ui =
% Tol and trace are u
% by the integration
  tol = 1.0E-4; trace
% Perform integratio
% the results in x
  [t,u] = ode23('Ex4_
% Plot the results
plot(t,u,'r')
title('Vehicle Forwa
xlabel('Time (sec)')
ylabel('u (m/sec)'); grid;
```

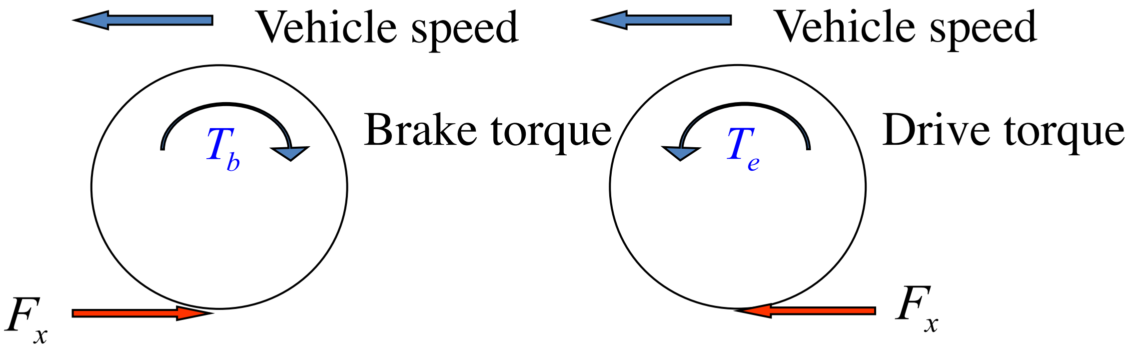



```
a(t,u);
g;
=0.02;
2; uw=0.0;
sin(Theta) ...
w)^2)];
```

Longitudinal Dynamics

- Mainly influenced by power generation (engine) and transfer (transmission, tires) devices
- When road friction is high, tire is mainly linear and can be ignored (part of vehicle inertia)
- When road friction is low, tire motion may be decoupled from vehicle motion, and a tire force model becomes important.
- Radius of wheel: r_w
- Tire rotational speed: ω

Longitudinal Dynamics



$$\dot{\omega} = \frac{T_e - T_b - r_w F_x - f_w F_z - b_w \omega}{J}$$

Longitudinal Dynamics

$$\dot{\omega} = \frac{T_e - T_b - r_w F_x - f_w F_z - b_w \omega}{J}$$

ω	wheel rotational velocity	f_w	dry friction at wheel
T_e	engine torque	b_w	viscous friction at wheel
T_b	braking torque	J	wheel inertia

Longitudinal slip

Longitudinal slip

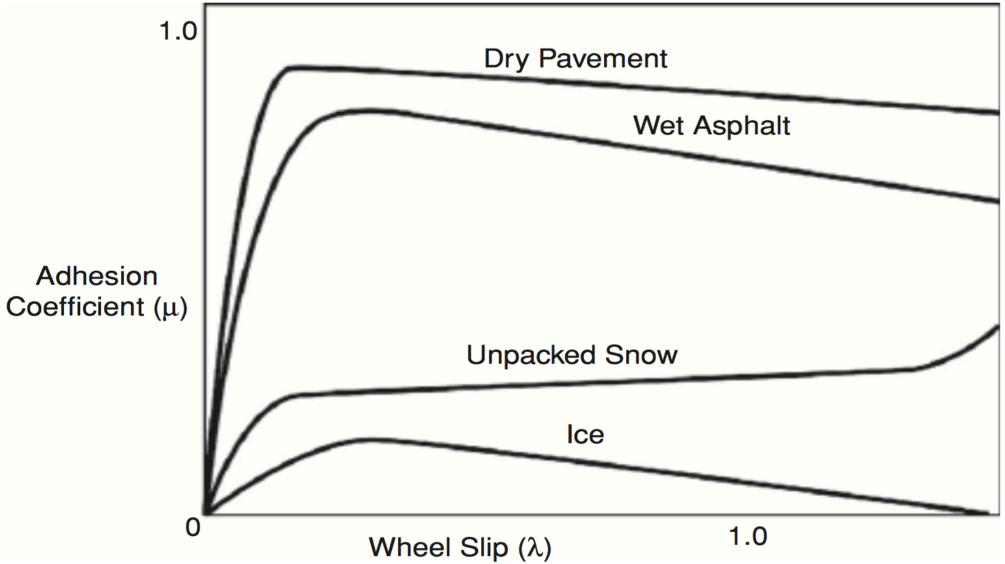
Driven (accelerating) case

$$\lambda \equiv \frac{\omega r_w - u}{\omega r_w}$$

Braking case

$$\lambda \equiv \frac{\omega r_w - u}{u}$$

$$\mu = \frac{F_x}{F_z}$$



Complete Longitudinal Model

$$\dot{u} = \frac{-0.5\rho C_d A(u+u_w)^2 - fmg\cos\Theta - mg\sin\Theta + N_w F_x}{m}$$

$$\dot{\omega} = \frac{T_e - T_b - r_w F_x - f_w F_z - b_w \omega}{J}$$

$$\dot{u} = \frac{-0.5\rho C_d A(u+u_w)^2 - fmg\cos\Theta - mg\sin\Theta + N_w F_z \mu(\lambda)}{m}$$

$$\dot{\omega} = \frac{T_e - T_b - r_w F_z \mu(\lambda) - f_w F_z - b_w \omega}{J}$$

Modeling Tire Friction

- Tire forces and moments are highly nonlinear and difficult to model.
- Thus, models such as the Brush or Pacejka “Magic Formula” models, have been developed for use in simulation studies.
- We will discuss one of these soon!

Outline

- Review Time-Varying LQR
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- Lateral Vehicle Motion
  1. 2 DOF Transient Model
  2. Nonlinear Tire Model

Vehicle Lateral Models

$$\sum F_x = m\dot{u}$$

Lateral

$$\sum F_y = m\left(\dot{v} + r u_0\right)$$

$$\sum F_z = m\left(\dot{w} - q u_0\right)$$

Roll

$$\sum M_x = I_{xx}\dot{p} - I_{xz}\dot{r}$$

$$\sum M_y = I_{yy}\dot{q}$$

Yaw

$$\sum M_z = -I_{xz}\dot{p} + I_{zz}\dot{r}$$

<u>Two popular types of models:</u>

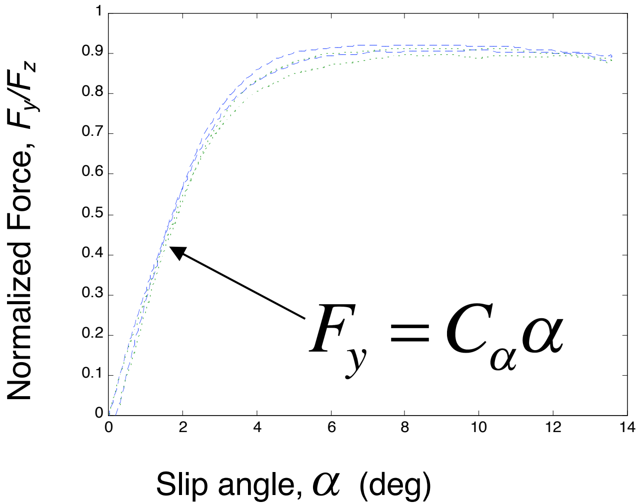
Lateral/Yaw (2DOF Handling, Lateral/Yaw)

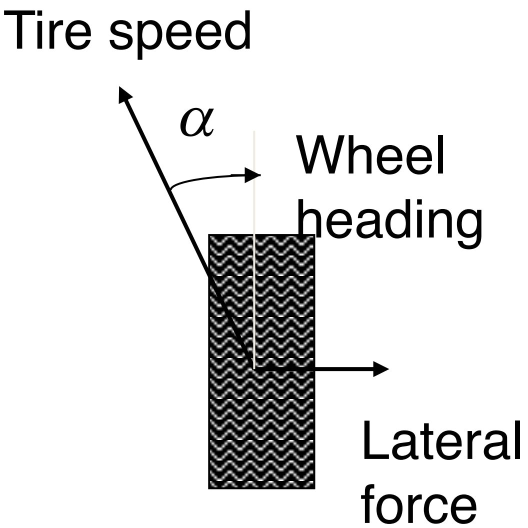
Lateral/Yaw/Roll (3DOF Lateral/Yaw/Roll)

Vehicle Lateral Dynamics

Lateral and yaw (i.e., need to find equations for $\dot{v}$ and $\dot{r}$)

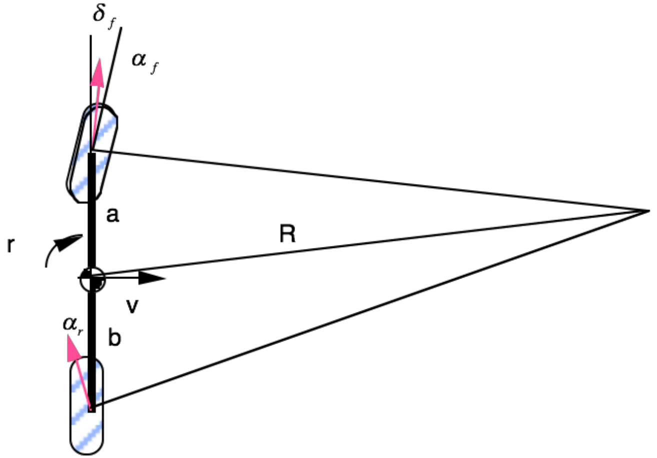
Critical step is to link input (steering) with tire lateral forces — which are the real inputs to lateral dynamics.





The 2 DOF "Bicycle" Model

Relationship between (front axle) steering angle and slip angles:



u: forward speed
v: lateral speed
r: yaw rate
R: Turning radius (u/r)
L=a+b: wheel base

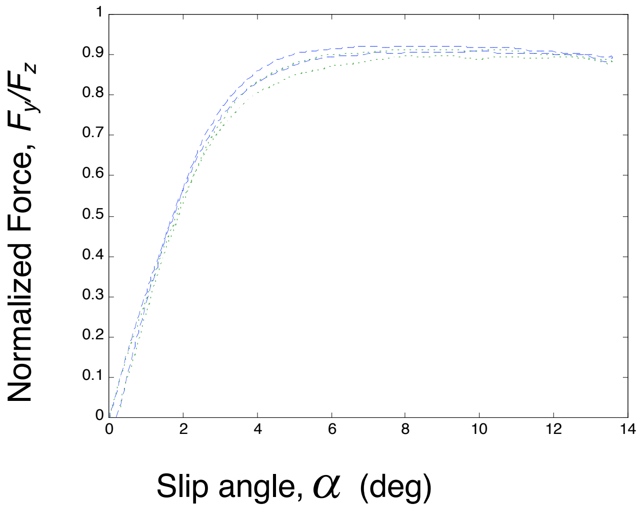
$$\delta_f = \frac{L}{R} + \alpha_f - \alpha_r$$

$$\alpha_f = \delta_f - \tan^{-1}\left(\frac{v+ar}{u}\right) \approx \delta_f - \frac{v+ar}{u}$$

$$\alpha_r = \delta_r - \tan^{-1}\left(\frac{v-br}{u}\right) \approx -\frac{v-br}{u}$$

Ackermann angle

Tire Lateral Forces



Tire lateral forces:

- depend on normal force
- are a nonlinear function of tire slip, and
- depend on tire design (e.g., radial, bias-ply) and inflation pressure.

Small slip angle:

$$F_y = C_\alpha \alpha$$

2DOF Vehicle Transient Model

$$\sum F_x = m\dot{u}$$

$$\sum F_y = m(\dot{v} + ru_o)$$

$$\sum F_z = m(\dot{w} - qu_o)$$

$$\sum M_x = I_{xx}\dot{p} - I_{xz}\dot{r}$$

$$\sum M_y = I_{yy}\dot{q}$$

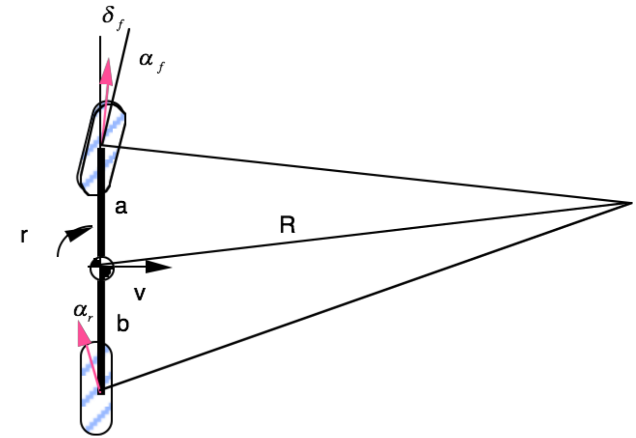
$$\sum M_z = -I_{xz}\dot{p} + I_{zz}\dot{r}$$

Figure out the tire forces and corresponding yaw moment

Keep the model linear

- small angle assumption
- linear tire approximation

2DOF Vehicle Transient Model



Slip angles:

$$\alpha_f = \delta_f - \tan^{-1}\left(\frac{v+ar}{u}\right) \approx \delta_f - \frac{v+ar}{u}$$

$$\alpha_r = \delta_r - \tan^{-1}\left(\frac{v-br}{u}\right) \approx -\frac{v-br}{u}$$

Lateral tire force: $F_y = C_\alpha \alpha$

$\sum F_y = m(\dot{v} + u_0 r):$

$$C_{\alpha f}\left[\delta_f - \frac{v+ar}{u_0}\right] + C_{\alpha r}\left[-\frac{v-br}{u_0}\right] = m(\dot{v} + u_0 r)$$

$\sum M_z = I_z \dot{r}:$

$$C_{\alpha f}\left[\delta_f - \frac{v+ar}{u_0}\right]a + C_{\alpha r}\left[-\frac{v-br}{u_0}\right]b = I_z \dot{r}$$

State Space Form of the 2DOF Model

$$\frac{d}{dt}\begin{bmatrix} v \\ r \end{bmatrix} = \begin{bmatrix} \dfrac{-(C_{\alpha f}+C_{\alpha r})}{mu_o} & \dfrac{bC_{\alpha r}-aC_{\alpha f}}{mu_o} - u_o \\ \dfrac{bC_{\alpha r}-aC_{\alpha f}}{I_z u_o} & \dfrac{-(a^2C_{\alpha f}+b^2C_{\alpha r})}{I_z u_o} \end{bmatrix}\begin{bmatrix} v \\ r \end{bmatrix} + \begin{bmatrix} \dfrac{C_{\alpha f}}{m} \\ \dfrac{aC_{\alpha f}}{I_z} \end{bmatrix}\delta_f$$

Or equivalently, if β is the sideslip angle at the CG:

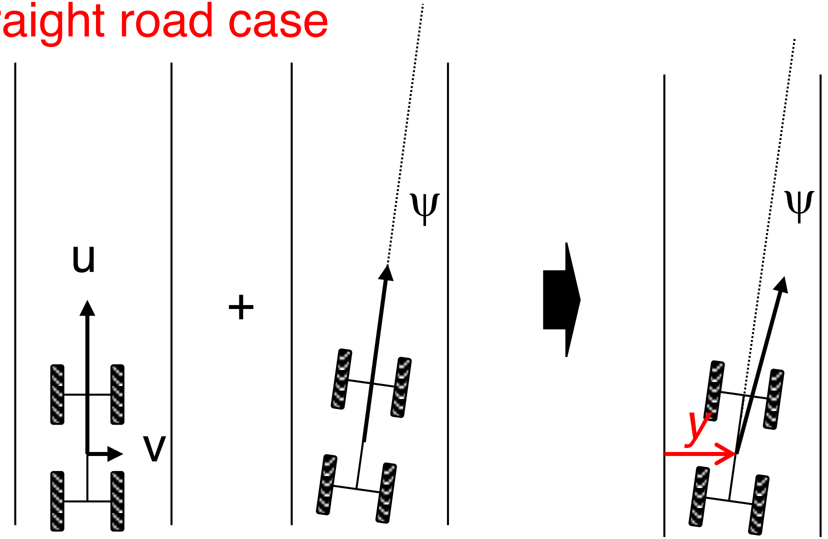
$$\tan\beta = \frac{v}{u_0} \quad \text{or} \quad v \approx \beta u_0$$

$$\begin{bmatrix} \dot{\beta} \\ \dot{r} \end{bmatrix} = \begin{bmatrix} -\left(\dfrac{C_{\alpha f}+C_{\alpha r}}{mu_o}\right) & -\left(\dfrac{aC_{\alpha f}-bC_{\alpha r}}{mu_o^2}\right)-1 \\ -\left(\dfrac{aC_{\alpha f}-bC_{\alpha r}}{I_z}\right) & -\left(\dfrac{C_{\alpha f}a^2+C_{\alpha r}b^2}{I_z u_o}\right) \end{bmatrix}\begin{bmatrix} \beta \\ r \end{bmatrix} + \begin{bmatrix} \dfrac{C_{\alpha f}}{mu_o} \\ \dfrac{aC_{\alpha f}}{I_z} \end{bmatrix}\delta_f$$

Bicycle Model State Equation (Road Following)

Augment the handling model with the dynamics of the "displacement variables" (both lateral displacement and yaw angle)

Straight road case

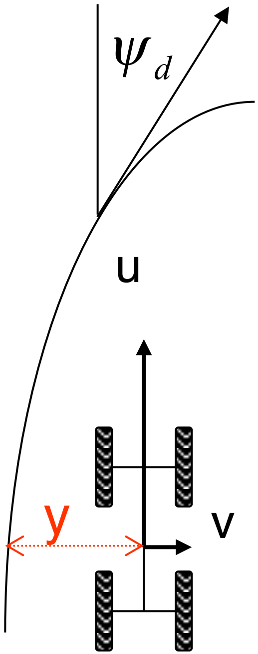


$$\dot{y} = v + u\psi$$

$$\dot{\psi} = r$$

Road Input to Vehicle Dynamics

Curved road case: need to define a road “disturbance” input



Road radius of curvature = R

Desired yaw rate $r_d = \dfrac{u}{R}$ $\quad \int r_d = \psi_d$

$$\boxed{\begin{aligned} \dot{y} &= v + u\psi \\ \dot{\psi} &= r \end{aligned}}$$

$\Rightarrow$

$$\boxed{\begin{aligned} \dot{y} &= v + u(\psi - \psi_d) \\ \frac{d}{dt}(\psi - \psi_d) &= r - r_d \end{aligned}}$$

Road Following Model

- Standard "bicycle model" only has 2 states (*v* and *r*).
- When we need to simulate vehicle road following behavior, 2 extra states (y, ψ) are included:

$$\frac{d}{dt}\begin{bmatrix} y \\ \dot{y} \\ \psi - \psi_d \\ r \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -(\frac{C_{\alpha f} + C_{\alpha r}}{m u_o}) & \frac{C_{\alpha f} + C_{\alpha r}}{m} & \frac{-aC_{\alpha f} + bC_{\alpha r}}{m u_o} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{-aC_{\alpha f} + bC_{\alpha r}}{I_z u_o} & \frac{aC_{\alpha f} - bC_{\alpha r}}{I_z} & -(\frac{C_{\alpha f} a^2 + C_{\alpha r} b^2}{I_z u_o}) \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \\ \psi - \psi_d \\ r \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{C_{\alpha f}}{m} \\ 0 \\ \frac{aC_{\alpha f}}{I_z} \end{bmatrix} \delta_f + \begin{bmatrix} 0 \\ -u_o \\ -1 \\ 0 \end{bmatrix} r_d$$

- When the road is curved, the orientation (yaw angle) of the road is denoted as ψ_d, and its rate of change as r_d
- For a straight road, set $\psi_d = r_d = 0$.

Bicycle 2DoF Model Equation

$$\frac{d}{dt}\begin{bmatrix} v \\ r \end{bmatrix} = \begin{bmatrix} \dfrac{-(C_{\alpha f}+C_{\alpha r})}{mu} & \dfrac{bC_{\alpha r}-aC_{\alpha f}}{mu}-u \\ \dfrac{bC_{\alpha r}-aC_{\alpha f}}{I_z u} & \dfrac{-(a^2C_{\alpha f}+b^2C_{\alpha r})}{I_z u} \end{bmatrix}\begin{bmatrix} v \\ r \end{bmatrix} + \begin{bmatrix} \dfrac{C_{\alpha f}}{m} \\ \dfrac{aC_{\alpha f}}{I_z} \end{bmatrix}\delta_f$$

$$\Downarrow$$

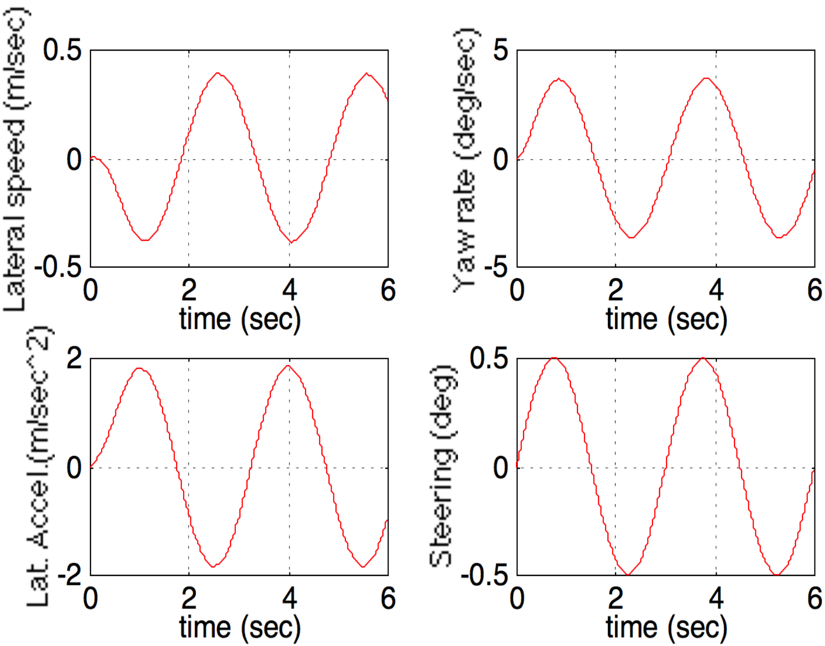
$$\frac{d}{dt}\begin{bmatrix} y \\ v \\ \psi-\psi_d \\ r \end{bmatrix} = \begin{bmatrix} 0 & 1 & u & 0 \\ 0 & -\dfrac{C_{\alpha f}+C_{\alpha r}}{mu} & 0 & \dfrac{bC_{\alpha r}-aC_{\alpha f}}{mu}-u \\ 0 & 0 & 0 & 1 \\ 0 & \dfrac{bC_{\alpha r}-aC_{\alpha f}}{I_z u} & 0 & -\dfrac{a^2C_{\alpha f}+b^2C_{\alpha r}}{I_z u} \end{bmatrix}\begin{bmatrix} y \\ v \\ \psi-\psi_d \\ r \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{C_{\alpha f}}{m} \\ 0 \\ \dfrac{aC_{\alpha f}}{I_z} \end{bmatrix}\delta_f + \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix} r_d$$

Ex. 4.6 Vehicle Handling Simulations

```
% Ex4_6a.m
% 2DOF model vehicle parameters
a   =  1.14;                % distance c.g. to front axle (m)
L   =  2.54;                % wheel base (m)
m   =  1500;                % mass (kg)
Iz  =  2420.0;   % yaw moment of inertia (kg-m^2)
Caf =  44000*2;             % cornering stiffness--front axle (N/rad)
Car =  47000*2;             % cornering stiffness-- rear axle (N/rad)
b=L-a;    g=9.81;
Kus = m*b/(L*Caf) - m*a/(L*Car);  % (rad/(m/sec^2))
u=20.0;                     % forward speed in m/sec
A=[-(Caf+Car)/(m*u),  (b*Car-a*Caf)/(m*u)-u
   (b*Car-a*Caf)/(Iz*u), -(a^2*Caf+b^2*Car)/(Iz*u)];
B=[Caf/m;      a*Caf/Iz];
C_lat = [1 0]; D_lat = 0;                % Lateral speed
C_yaw = [0 1]; D_yaw = 0;     % Yaw rate
C_acc=A(1,:) + u*[0,1];
D_acc = B(1);               % Lateral acceleration
C = [C_lat; C_yaw; C_acc];
D = [D_lat; D_yaw; D_acc];

t=[0:0.01:6];
U=0.5*pi/180*sin(1/3*2*pi*t);
```

```
Y=lsim(A,B,C,D,U,t);


Subplot, subplot(221)
plot(t,Y(:,1),'r'); grid
xlabel('time (sec)')
ylabel('Lateral speed (m/sec)')

subplot(222)
plot(t,Y(:,2)*180/pi,'r'); grid
xlabel('time (sec)')
ylabel('Yaw rate (deg/sec)')

subplot(223)
plot(t,Y(:,3),'r'); grid
xlabel('time (sec)')
ylabel('Lat. Accel.(m/sec^2)')

subplot(224)
plot(t,U*180/pi,'r'); grid
xlabel('time (sec)')
ylabel('Steering (deg)')
```

Ex. 4.6 Vehicle Handling Simulations (cont.)




```
Subplot, subplot(221)
plot(t,Y(:,1),'r'); grid
xlabel('time (sec)')
ylabel('Lateral speed (m/sec)')

subplot(222)
plot(t,Y(:,2)*180/pi,'r'); grid
xlabel('time (sec)')
ylabel('Yaw rate (deg/sec)')

subplot(223)
plot(t,Y(:,3),'r'); grid
xlabel('time (sec)')
ylabel('Lat. Accel.(m/sec^2)')

subplot(224)
plot(t,U*180/pi,'r'); grid
xlabel('time (sec)')
ylabel('Steering (deg)')
```

Quality of Approximations: Magic Formula

$$F_x = D_x \sin(C_x \tan^{-1}(B_x \phi_x)) + S_{vx}$$
$$F_y = D_y \sin(C_y \tan^{-1}(B_y \phi_y)) + S_{vy}$$
$$\phi_x = (1 - E_x)(\lambda + S_{hx}) + \frac{E_x}{B_x} \tan^{-1}(B_x(\lambda + S_{hx}))$$
$$\phi_y = (1 - E_y)(\alpha + S_{hy}) + \frac{E_y}{B_y} \tan^{-1}(B_y(\alpha + S_{hy}))$$

B=stiffness factor (B_x for longitudinal, B_y for lateral)

C=shape factor

D=Peak factor (determines peak magnitude)

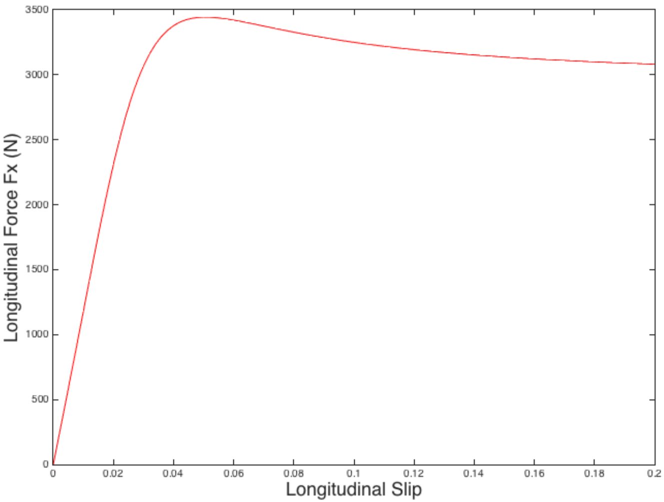
E=curvature factor

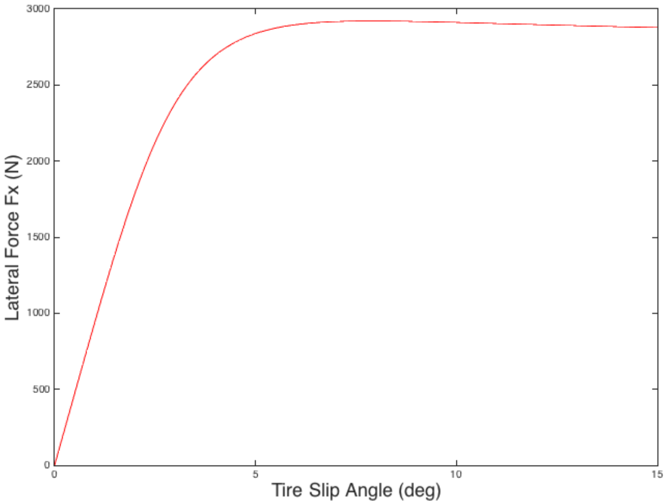
S_h=horizontal offset

S_v=vertical offset

MAKE SURE SLIP ANGLE, α ,IS GIVEN IN DEGREES!

Longitudinal and Lateral Forces





check magicformulaslip.m

Note this is per TIRE!

Combined Longitudinal/Lateral/Yaw Nonlinear Model

$$\dot{x} = u\cos\psi - v\sin\psi$$

$$\dot{u} = \frac{fW + N_w F_x}{m}$$

$$\dot{\omega} = \frac{T - r_w F_x - f_w F_z - b_w \omega}{J}$$

$$\dot{y} = u\sin\psi + v\cos\psi$$

$$\dot{v} = \frac{F_{yf} + F_{yr}}{m} - ur$$

$$\dot{\psi} = r$$

$$\dot{r} = \frac{F_{yf}a + F_{yr}b}{I_z}$$

- Inputs are: T and δ_f
- Use Magic Formula for F_x, F_{yf}, F_{yr}
- $\alpha_f = \delta_f - \tan^{-1}(\frac{v+ar}{u})$
- $\alpha_r = \tan^{-1}(\frac{v-br}{u})$

Simulate Nonlinear Model

- Ex 1: dynamicbicycle_fastcontrol.m
- Ex 2: dynamicbicycle_lanecontrol.m

Summary

- Coordinate System and Rigid Body Motion
- Longitudinal Vehicle Motion
- Lateral Vehicle Motion.