ML - Multiple Linear Regression Model (Part II).

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In [1]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import seaborn as sns
import os
from sklearn.linear_model import LinearRegression
from sklearn.model_selection import train_test_split
```

In [2]:
```
os.chdir(r'D:\Sagun Shakya\Python\Data Sets')
```

In [3]:
```
land = pd.read_csv('real_estate.csv')
land.head()
```

Out[3]:

	price	size	year
0	234314.144	643.09	2015
1	228581.528	656.22	2009
2	281626.336	487.29	2018
3	401255.608	1504.75	2015
4	458674.256	1275.46	2009

In [4]:
```
land.isnull().sum()
```

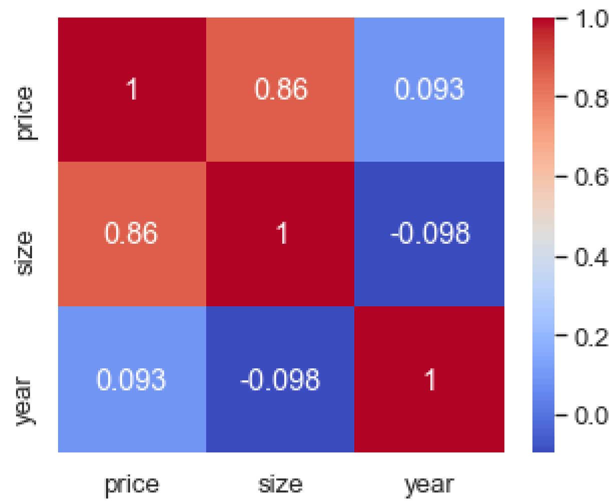
Out[4]:
```
price    0
size     0
year     0
dtype: int64
```

In [5]:
```
land.describe()
```

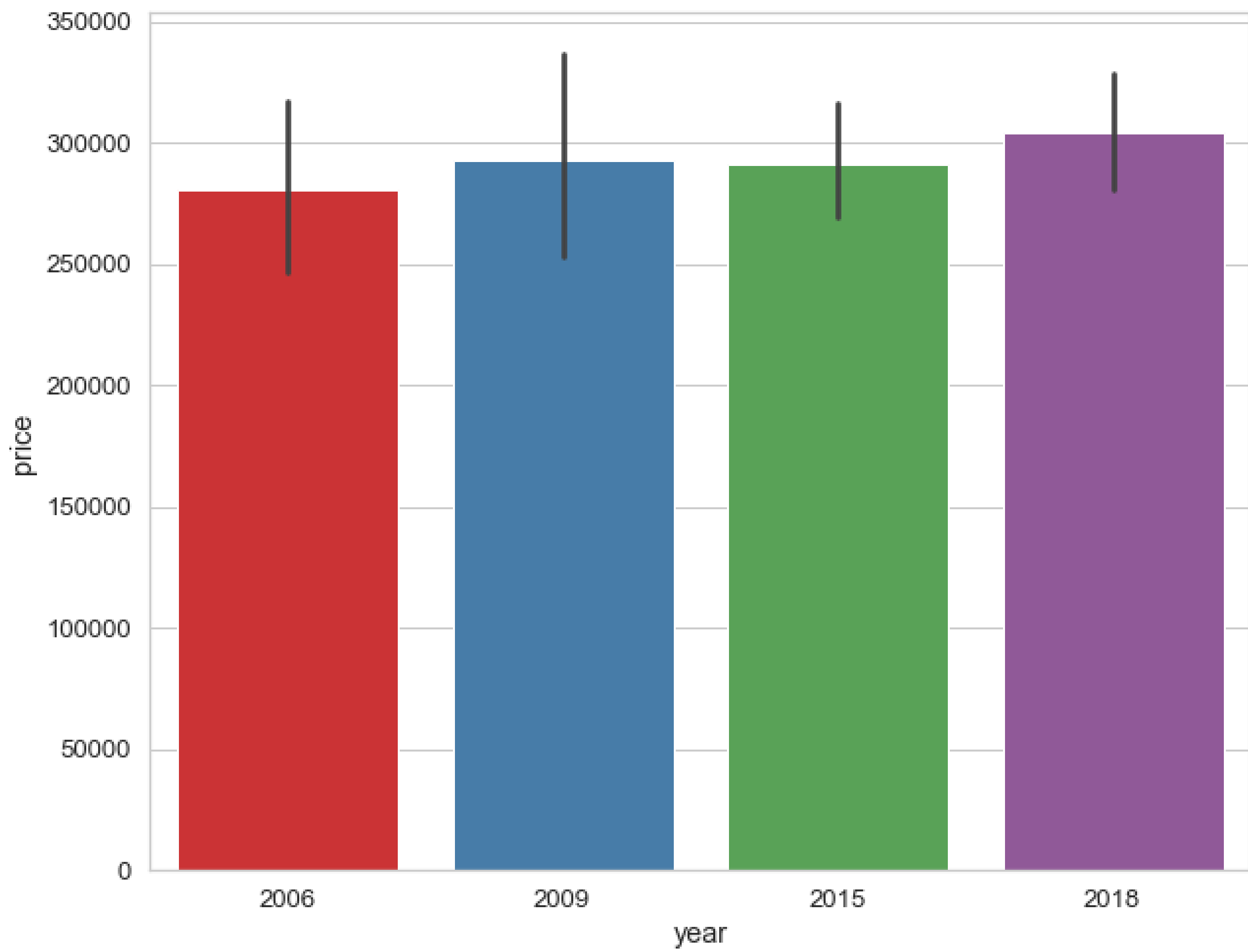
Out[5]:

	price	size	year
count	100.000000	100.000000	100.000000
mean	292289.470160	853.024200	2012.600000
std	77051.727525	297.941951	4.729021
min	154282.128000	479.750000	2006.000000
25%	234280.148000	643.330000	2009.000000
50%	280590.716000	696.405000	2015.000000
75%	335723.696000	1029.322500	2018.000000
max	500681.128000	1842.510000	2018.000000

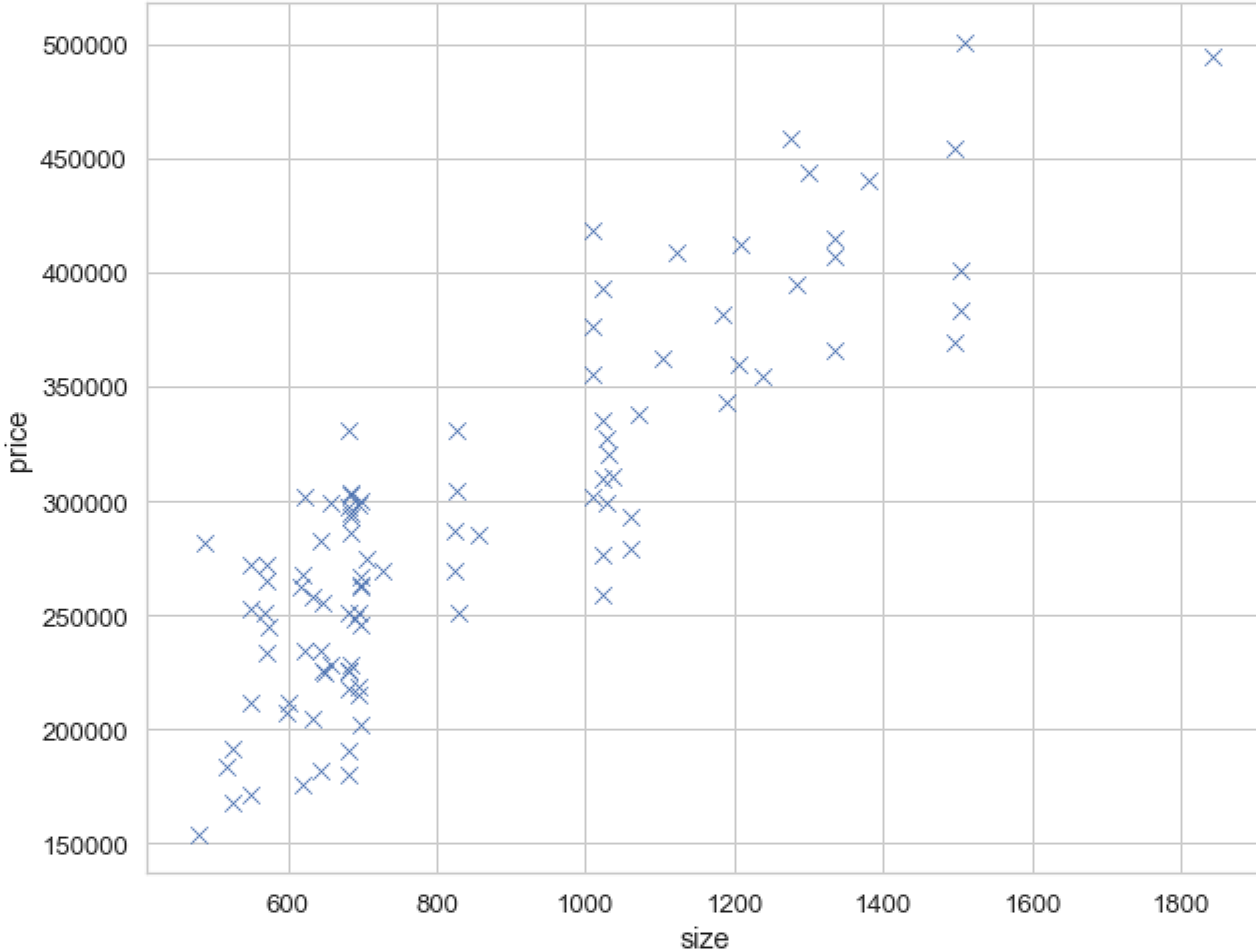
In [6]:
```
col_vals = land.corr().columns.values

plt.figure(figsize = (5,4))
sns.set(style = 'whitegrid', font_scale = 1.2)

sns.heatmap(data=land.corr(), xticklabels=col_vals, yticklabels=col_vals, cmap = 'coolwarm', annot = True)
plt.show()
```



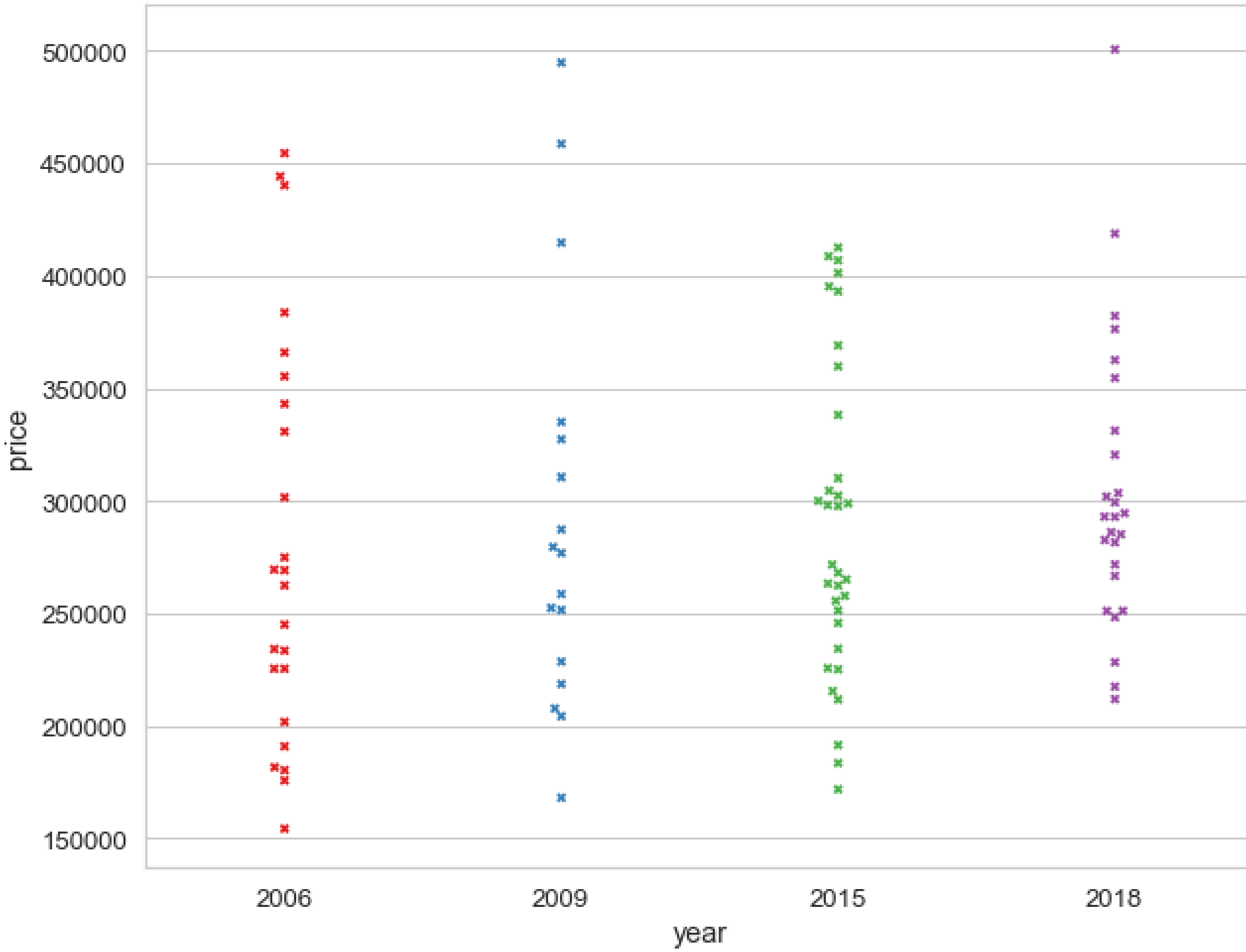
In [7]:
```
plt.figure(figsize = (10,8))
sns.set(style = 'whitegrid', font_scale = 1.2)

sns.barplot(x= land.year, y = land.price, palette = 'Set1')
plt.show()
```



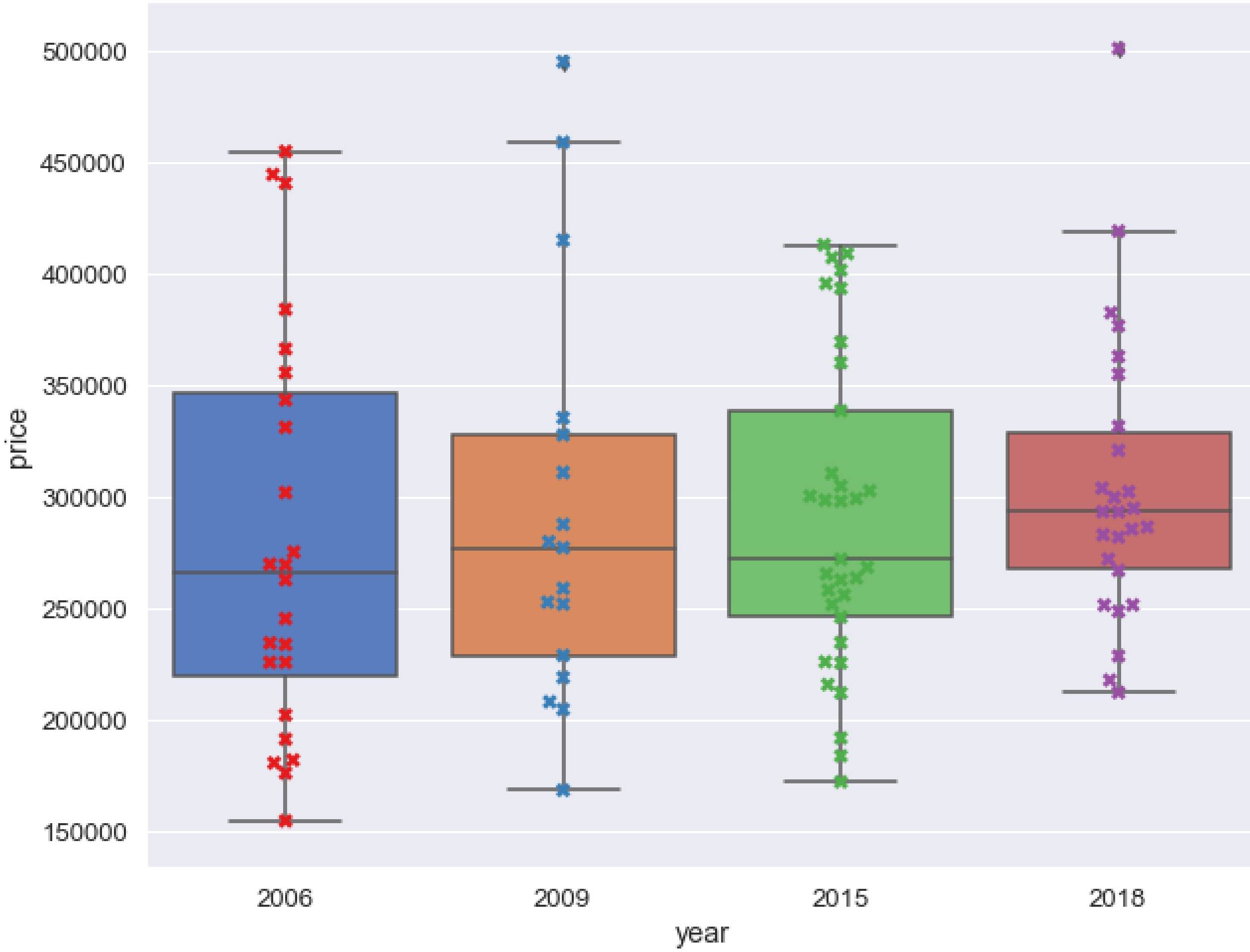
In [8]:
```
plt.figure(figsize = (10,8))
sns.set(style = 'whitegrid', font_scale = 1.2)

sns.scatterplot(x= land['size'], y = land.price, marker = 'x', s = 70)
plt.show()
```



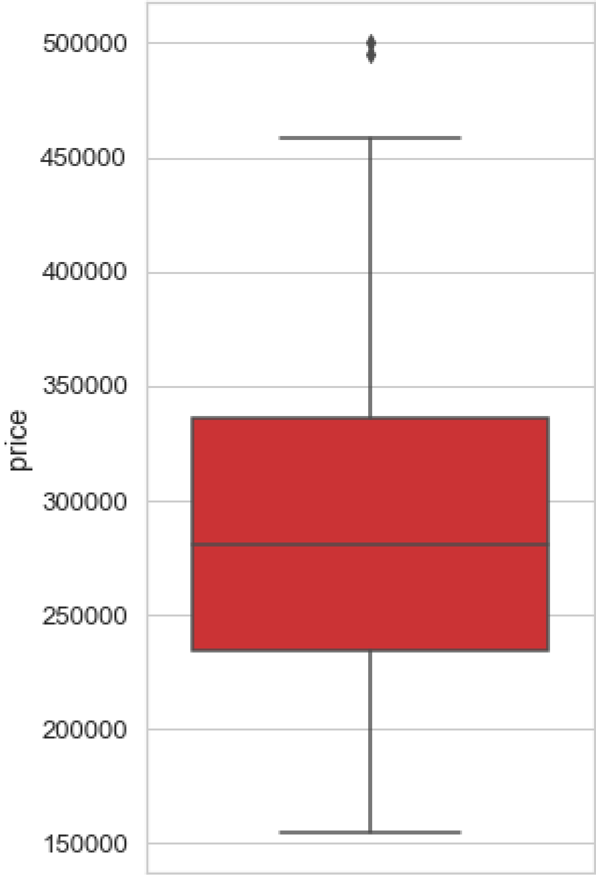
In [9]:
```
plt.figure(figsize = (10,8))
sns.set(style = 'whitegrid', font_scale = 1.2)

sns.swarmplot(x= land.year, y = land.price, palette = 'Set1', marker = 'X')
plt.show()
```



In [10]:
```
plt.figure(figsize = (10,8))
sns.set(style = 'darkgrid', font_scale = 1.2)

sns.boxplot(y = land['price'], x = land['year'], palette = 'muted')
sns.swarmplot(x= land.year, y = land.price, palette = 'Set1', marker = 'X', s = 7.5)
plt.show()
```



In [11]:
```
plt.figure(figsize = (4,8))
sns.set(style = 'whitegrid', font_scale = 1.2)

sns.boxplot(y = land.price, palette = 'Set1')
plt.show()
```



Detection and removal of outliers:

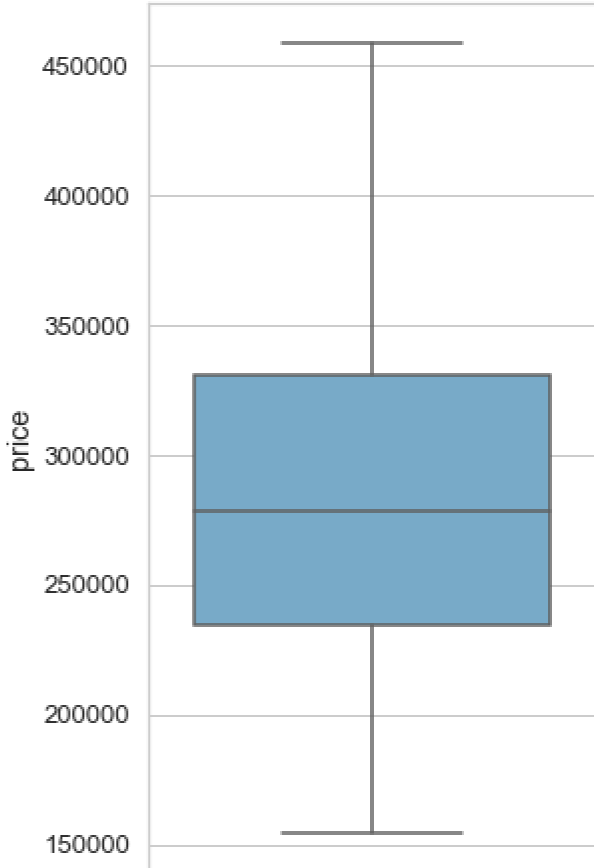
In [12]:
```
q1 = land['price'].quantile(1/4)
q3 = land['price'].quantile(3/4)
IQR = q3 - q1
lowerLimit = q1 - 1.5 * IQR
upperLimit = q3 + 1.5 * IQR
```

In [13]:
```
pricing = land['price']

# Only those values lying outside the interval are labelled as index_outliers.
# And, their index values are taken.

index_outliers = pricing[ pricing.between(lowerLimit,upperLimit) == False].index.values
print(index_outliers)
```

```
[11 55]
```

In [14]:
```
land.drop(index_outliers, axis = 0, inplace = True)
```

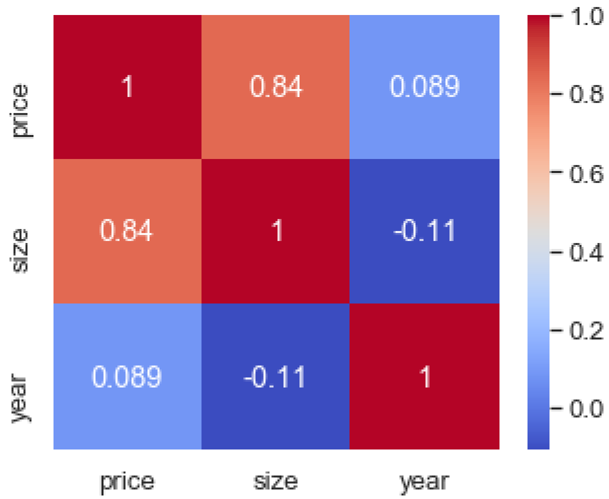
In [15]:
```
plt.figure(figsize = (4,8))
sns.set(style = 'whitegrid', font_scale = 1.2)

sns.boxplot(y = land.price, palette = 'Blues')
plt.show()
```



As we can see, there are no outliers left.

Checking for correlation once again after removing the outliers:

In [16]:
```
col_vals = land.corr().columns.values

plt.figure(figsize = (5,4))
sns.set(style = 'whitegrid', font_scale = 1.2)

sns.heatmap(data=land.corr(), xticklabels=col_vals, yticklabels=col_vals, cmap = 'coolwarm', annot = True)
plt.show()
```



Multiple Regression Analysis:

Case I: Using all of the data as a training set.

In [17]:
```
Y = land['price']
X = land[['size', 'year']]
```

In [18]:
```python
model = LinearRegression()

model.fit(X, Y)
```

Out[18]:
```
LinearRegression(copy_X=True, fit_intercept=True, n_jobs=None, normalize=False)
```

In [28]:
```python
# Checking the weights and intercepts values.
print('Weight values: ', model.coef_.round(4), '\nIntercept: ', model.intercept_.round(4))
```

```
Weight values:  [ 225.1564 2772.4462]
Intercept:  -5479961.3056
```

In [20]:
```python
model.predict([[1500,2015], [1500,2019], [1500,2020]]).round(2)
```

Out[20]:
```
array([444252.45, 455342.23, 458114.68])
```

Testing the significance of the results.

First we take a different approach to view the statistical summary of the test i.e statsmodel.

- Note: sklearn is helpful for finding indivisual scores.

In [27]:
```python
import statsmodels.api as sm
```

In [33]:

```
Y = land['price']
X = land[['size', 'year']]
X = sm.add_constant(X)

model1 = sm.OLS(Y,X).fit()
model1.summary()
```

Out[33]:

OLS Regression Results

Dep. Variable:	price	**R-squared:**	0.742
Model:	OLS	**Adj. R-squared:**	0.737
Method:	Least Squares	**F-statistic:**	136.8
Date:	Sat, 18 Jan 2020	**Prob (F-statistic):**	1.06e-28
Time:	15:25:11	**Log-Likelihood:**	-1168.1
No. Observations:	98	**AIC:**	2342.
Df Residuals:	95	**BIC:**	2350.
Df Model:	2		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	-5.48e+06	1.6e+06	-3.417	0.001	-8.66e+06	-2.3e+06
size	225.1564	13.684	16.454	0.000	197.990	252.323
year	2772.4462	796.144	3.482	0.001	1191.901	4352.992

Omnibus:	9.601	**Durbin-Watson:**	2.223
Prob(Omnibus):	0.008	**Jarque-Bera (JB):**	3.633
Skew:	0.119	**Prob(JB):**	0.163
Kurtosis:	2.087	**Cond. No.**	9.39e+05

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 9.39e+05. This might indicate that there are
strong multicollinearity or other numerical problems.

Remarks:

- Prob(F - statistic) = 1.06e-28 is a very lower value as compared to α = 0.05. So, we reject the null hypothesis that all of the weights/coefficients are equal to zero.
  - H_o : All of the weights/coefficients are equal to zero.
  - H_1 : At least on of the weights/coefficients is not equal to zero.
- Degrees of freedom: f ~ F(2,95).
- The indivisual p - values i.e P>|t| is also very lower than α = 0.05. This means that, indivisually speaking, the features are quite significant to determine the response variable.

Below is a very important trick to store the summary objects in a dataframe.

In [36]:

```
import statsmodels.api as sm

mymodel = sm.OLS(Y,X)
results = mymodel.fit()           # 'results' store the summary object.
```

Using dir() method to get all the atrributes of the object results.

In [46]:
```python
def dump(obj):
  for attr in dir(obj):
    print("obj.%s = %r" % (attr, getattr(obj, attr)))

print(dump(results))
```

```
obj.HC0_se = const    1.608260e+06
size     1.518223e+01
year     7.987765e+02
dtype: float64
obj.HC1_se = const    1.633456e+06
size     1.542009e+01
year     8.112908e+02
dtype: float64
obj.HC2_se = const    1.638910e+06
size     1.560235e+01
year     8.139886e+02
dtype: float64
obj.HC3_se = const    1.670347e+06
size     1.603842e+01
year     8.295908e+02
dtype: float64
obj._HCCM = <bound method RegressionResults._HCCM of <statsmodels.regression.linear_model.OLSResults object at 0x00000
21EC20BF2B0>>
obj.__class__ = <class 'statsmodels.regression.linear_model.RegressionResultsWrapper'>
```

Estimated sum of squares:

In [48]:
```python
results.ess
```

Out[48]:
```
372352442855.9922
```

F Statistic P - value and f - value:

In [57]:
```python
results.f_pvalue,results.fvalue
```

Out[57]:
```
(1.0630201397628987e-28, 136.83530657343707)
```

Fitted values:

In [58]:
```
results.fittedvalues.head()
```

Out[58]:
```
0    251313.672930
1    237635.299155
2    224551.643862
3    445321.940026
4    377061.150786
dtype: float64
```

P - values:

In [37]:
```
results.pvalues
```

Out[37]:
```
const    9.325403e-04
size     1.471793e-29
year     7.530334e-04
dtype: float64
```

Coefficients:

In [38]:
```
results.params
```

Out[38]:
```
const   -5.479961e+06
size     2.251564e+02
year     2.772446e+03
dtype: float64
```

Confidence Interval:

In [40]:
```
results.conf_int()
```

Out[40]:

	0	1
const	-8.663478e+06	-2.296445e+06
size	1.979897e+02	2.523231e+02
year	1.191901e+03	4.352992e+03

In [41]:
```
# Lower limit.
results.conf_int()[0]
```

Out[41]:
```
const   -8.663478e+06
size     1.979897e+02
year     1.191901e+03
Name: 0, dtype: float64
```

In [42]:
```
# Upper limit.
results.conf_int()[1]
```

Out[42]:
```
const   -2.296445e+06
size     2.523231e+02
year     4.352992e+03
Name: 1, dtype: float64
```

In [59]:
```
model1.summary()
```

Out[59]:

OLS Regression Results

Dep. Variable:	price	**R-squared:**	0.742
Model:	OLS	**Adj. R-squared:**	0.737
Method:	Least Squares	**F-statistic:**	136.8
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Time:	16:04:29	**Log-Likelihood:**	-1168.1
No. Observations:	98	**AIC:**	2342.
Df Residuals:	95	**BIC:**	2350.
Df Model:	2		
Covariance Type:	nonrobust		

	coef	**std err**	**t**	**P>|t|**	**[0.025**	**0.975]**
const	-5.48e+06	1.6e+06	-3.417	0.001	-8.66e+06	-2.3e+06
size	225.1564	13.684	16.454	0.000	197.990	252.323
year	2772.4462	796.144	3.482	0.001	1191.901	4352.992

Omnibus:	9.601	**Durbin-Watson:**	2.223
Prob(Omnibus):	0.008	**Jarque-Bera (JB):**	3.633
Skew:	0.119	**Prob(JB):**	0.163
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Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 9.39e+05. This might indicate that there are
strong multicollinearity or other numerical problems.

In [107]:
```python
# Creating a function that converts the summary object into a dataframe.

def global_summary_to_dataframe(results):
    myData = dict()

    # Number of observations:
    myData['num_observations'] = results.nobs
    myData['num_residuals'] = results.df_resid
    myData['degrees_of_freedom'] = results.df_model

    # Coefficient of determination:
    myData['r_squared'] = results.rsquared
    myData['r_squared_adjusted'] = results.rsquared_adj

    # Sum of squares:
    myData['explained_sumOFsquares'] = results.ess
    myData['residual_sumOFsquares'] = results.ssr
    myData['total_sumOFsquares'] = results.ess + results.ssr

    # Mean sum of squares:
    myData['mse_model'] = results.mse_model
    myData['mse_residue'] = results.mse_resid
    myData['mse_total'] = results.mse_total

    # F - statistic:
    myData['f_pvalue'] = results.f_pvalue
    myData['f_value'] = results.fvalue

    #Log - likelihood functional value.
    myData['log_likelihood'] = results.llf

    temp = pd.DataFrame(myData, index = np.arange(1))
    temp1 = temp.transpose()
    temp1.columns = ['Values']
    temp1.style.format("{:.1f}")
    return temp1

Data = global_summary_to_dataframe(results)
Data
```

Out[107]:

	Values
num_observations	9.800000e+01
num_residuals	9.500000e+01
degrees_of_freedom	2.000000e+00
r_squared	7.423174e-01
r_squared_adjusted	7.368925e-01
explained_sumOFsquares	3.723524e+11
residual_sumOFsquares	1.292557e+11
total_sumOFsquares	5.016081e+11
mse_model	1.861762e+11
mse_residue	1.360586e+09
mse_total	5.171218e+09
f_pvalue	1.063020e-28
f_value	1.368353e+02
log_likelihood	-1.168060e+03

In [95]:
```
type(float(results.ess))
```

Out[95]:
```
float
```

Case II: Using 75% of the data as a training set.

In [21]:
```
x_train, x_test, y_train, y_test = train_test_split(X, Y, test_size = 0.25, random_state = 42)
```

In [22]:
```
newModel = LinearRegression()

newModel.fit(x_train, y_train)
```

Out[22]:
```
LinearRegression(copy_X=True, fit_intercept=True, n_jobs=None, normalize=False)
```

In [23]:
```
# Checking the weights and intercepts values.
print('Weight values: ', newModel.coef_, '\nIntercept: ', newModel.intercept_)
```

```
Weight values:  [ 218.71705612 2648.08659193]
Intercept:  -5225830.271680301
```

In [24]:
```
# Prediction.
y_predicted = newModel.predict(x_test)
```

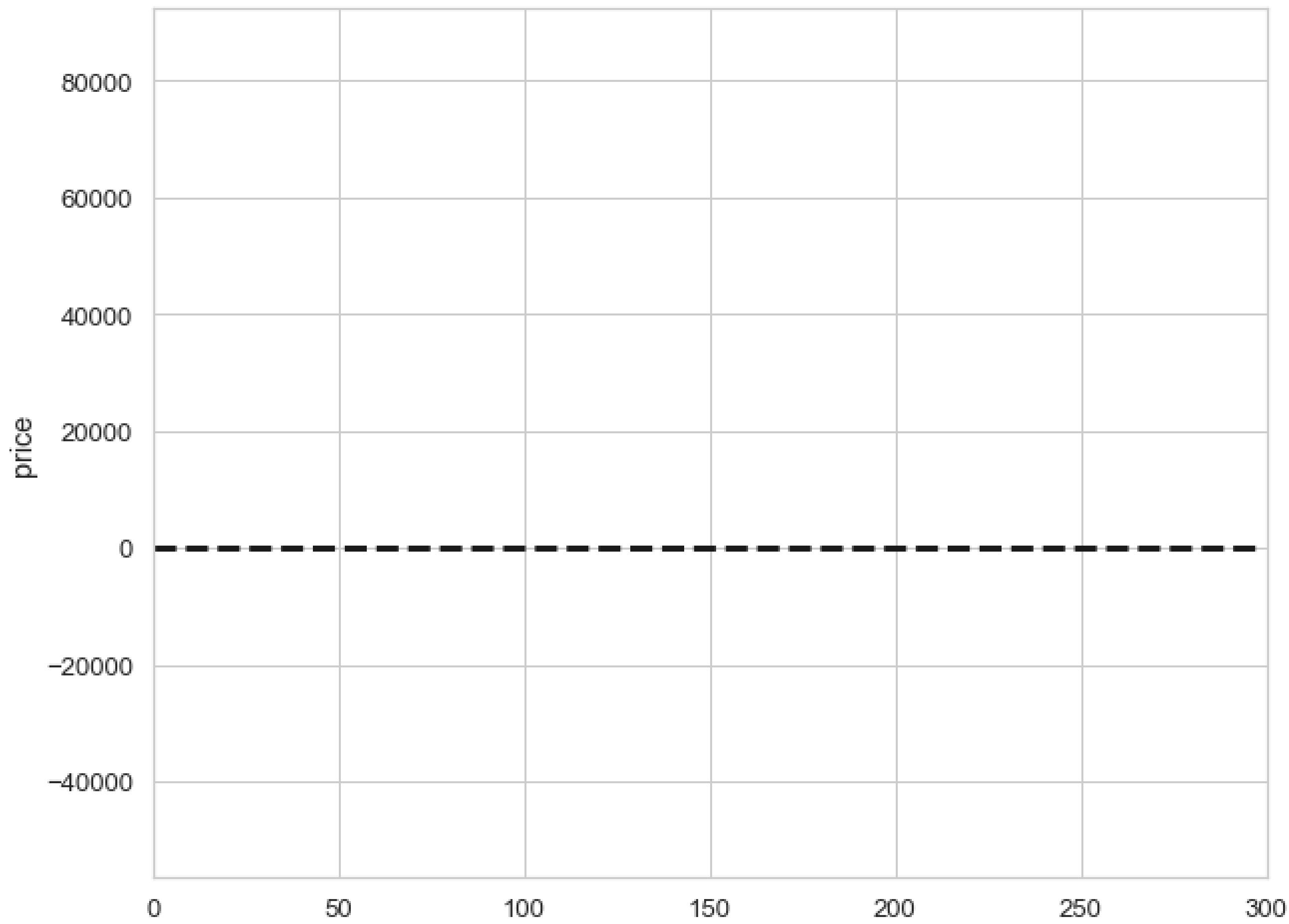
Residuals.

In [25]:
```
error = y_test - y_predicted
print(error)
```

```
64    42403.005318
41   -49762.145537
96   -10558.922201
19   -43380.136425
83    24020.321551
85    37505.148284
66    48279.579387
43    28716.531932
10   -46539.413127
0    -16404.818673
32   -17204.371356
77   -12157.920292
48     9274.752182
27     4997.668380
45    37269.335829
4     85533.708102
23   -39983.270605
13    80004.348284
92    36203.299131
75    18226.961542
50   -26027.384891
72    45335.778380
70   -40817.955002
16   -20703.746762
40   -23536.394553
Name: price, dtype: float64
```

In [26]:
```
plt.figure( figsize = (10,8) )
sns.set(style = 'whitegrid', font_scale = 1.2)

sns.scatterplot(x = y_predicted, y = error, color = '#658cb7'
               )
plt.plot(np.arange(300), np.zeros(300), 'k--', linewidth = 3)
plt.xlim(0,300)
plt.show()
```



The End.