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A MODEL STYDY OF FREE VORTEX FLOW

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ABSTRACT

Vortex is the swirling motion of fluid around a central region. Pumps used in different industrial applications draw water from water reservoirs. The existence of free surface vortices has been a significant focus of study for hydraulic engineers. This paper presents the study of formation and progression of free vortex flow. An experimental model of water reservoir is setup and free vortex flow generated above suction orifice at fixed submergence depth and discharge. Flow visualization is used to examine the vortex formation and its structure. During experiment and data analysis, different flow behavior of evolved vortex and how different parameters affected the development and strength of vortex are closely explored. Variations in output parameters, tangential velocity, surface profile geometry, pressure head due to change in input parameters such as orifice diameter, flow discharge are investigated. Results estimate that free surface vortex Profile is characterized by central air core surrounded with free vortex region. Vortex profile pattern remains same for increased suction orifice diameter but vortex core radius expands. With decrement in relative submergence, strength of vortex increases. Pressure head increases with reduction of vortex radius. Tangential velocity is found to be inversely proportional to radius of free vortex and it is similar to that of the Rankine vortex.

Keywords: free vortex, Rankine vortex, vortex profiles, vertical vortex, flow visualization

1. INTRODUCTION

A Flow pattern in which the streamlines are concentric circles and fluid particles do not rotate about their own axis as they revolve around vortex center is called free or irrotational vortex.

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Vortices have their origin in swirling flow, which occurs due to their location relative to surrounding boundaries. Vortex formation is observed in various natural phenomena such as Tropical cyclones, tornados, swirling flow around rocks in rivers, smoke rings, and water flow in bath tub as well as in numerous engineering applications, few of them are trailing vortices on sides of aircraft wings, Air vortex cannon, vortices around main rotor of helicopter and vortex structures at rotating ship propellers. In Free vortex, fluid rotates about a Centre under the effect of gravity only, without supplying any form of energy. The formation of vortices between the pump suction intake and the suction tank fluid surface causes air to enter the pump suction. This mixture of air and water in the pump reduces the pump capacity. The formation of such vortices must therefore be avoided. It has been found out in studies that even 1% air volume fraction can cause significant reduction in efficiency of centrifugal pump and 10% air entrainment by volume can lead to serious damage of pump. Hydraulic pumps and turbines are designed for uniform fluid flow and air induced vortex flow will result in off design operation. Studies suggest that vortex formation depends upon submergence height, viscosity, velocity with which fluid is leaving the tank, density of fluid and geometry of inlet. The free surface vortex has been studied by many researchers in the past, and is still an important subject in the field of hydraulic engineering. According to Kelvin theorem for a non-viscous fluid, circulation around a close contour is independent of time so for potential fluid flow, circulation is conserved around a moving loop (Meyer, 1982, 71) [1]**.** Vortex dynamics originated when Hurmann von Helmholtz [2] introduced concepts of vortex line, vortex filament and derived vorticity equation for ideal incompressible fluid. Magneto hydrodynamics was discovered by Batchelor [3].A simple model of bath tub vortex was presented by Feynman et al. [4] for ideal fluids. A classic mathematical model was proposed by Rankine [5] in which vortex motion had an inner core of solid body rotation surrounded by outer zone of irrotational motion & he derived different equations for tangential velocity of two regions. Andersen et. al. [6] presented detailed characteristics of bathtub vortex model. Odgard [7] derived an analytical equation for critical submergence & studied various factors affecting it. Ojha et al [8] investigated factors responsible for decay of strength of vortex downstream of submerged vane. Zheng et al. [9] carried out numerical simulation of the free surface vortex in a barrel with a bottom drain hole by giving certain tangential inflow velocity. Zhao et al. [10] investigated free surface vortex in a barrel with a bottom drain hole by giving certain tangential inflow velocity numerically. Shibata et al. [11]

proposed a method to predict the submerged vortex in pump sumps. In recent years other researchers [12-15] have also studied characteristics of free surface vortex motion and parameters affecting it. In the current work, the main purpose of study is to understand the formation and evolution of free surface vortex and its characteristics and so experimentally investigated herein. Experimental analysis is established to examine effect of changing outlet area over vortex profile geometry, tangential velocity, vortex strength, pressure head distribution keeping submergence height same.

2. ANALYTICAL APPROACH

An equation for the vorticity in incompressible flow is obtained by applying the curl operation to the Navier-Stokes equation,

$$\partial_t \omega + (u \cdot \nabla)\omega = (\omega \cdot \nabla)u + \nu \nabla^2 \omega \quad \text{...(1)}$$

Where quantity *w* is called vorticity and corresponds to rotation of the fluid whereas *u* is velocity. Flow without vorticity is called irrotational flow. It is unrealizable to get a closed-form solution for the vertical vortex flow, which could satisfy completely the continuity equation and the momentum equations. The Rankine vortex is one of the exact solutions of the Navier-Stokes equations. Its tangential velocity varies as,

$$V_\theta = \frac{\Gamma r}{2\pi r_m^2}, r \le r_m \quad \& \quad V_\theta = \frac{\Gamma}{2\pi r}, r > r_m \quad \text{...(2)}$$

Where V_θ is tangential velocity, Γ is circulation, r radius and r_m is the core radius. Circulation is an important parameter to measure the vortex strength.

Expression for pressure distribution is,

$$dp = \rho \frac{V^2}{r} dr - \rho g dz \quad \text{...(3)}$$

Where p is pressure, ρ density, V velocity, r radius, g acceleration due to gravity & z is height from the datum.

3. PHYSICAL MODEL

Figure 1 shows photograph of experimental setup on which experiments are performed. It includes a transparent acrylic cylinder mounted on PVC base plate fixed with leveling screws and a hydraulic bench. Whole assembly is manufactured from corrosion resistant high graded plastic material. Water enters or exits through four orifices given in the side bottom of the cylinder. An overflow duct controls water level from getting too high and submergence height can be adjusted with it.

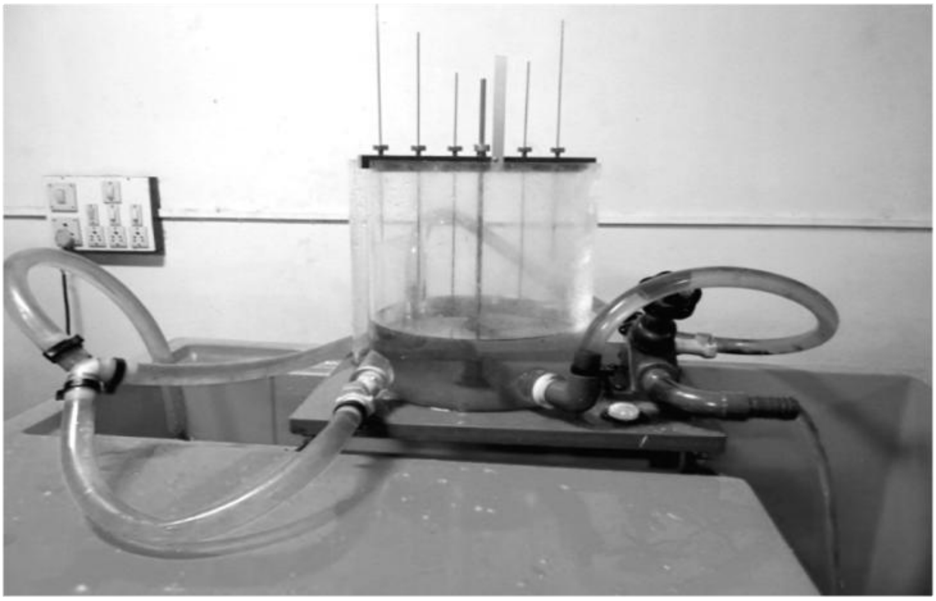


Figure 1. Experimental set-up

Cylinder is having a hole in the center of the base through which fluid leaves the vessel. Interchangeable push-in orifices of different diameters can be plugged in center hole to produce free surface vortex. The whole assembly is placed above the channel on the hydraulic bench. Hydraulic bench is comprised of a volumetric tank, connection hoses, sump tank, drain valve & a self-priming centrifugal pump. Water is drawn from volumetric tank by pump and supplied to cylinder via three way tubing assembly. Inlet flow rate is regulated by a flow control valve. To minimize turbulent flow, a stilling baffle is used and a glass tube with scale indicates water level. Water flows out of cylinder from exit hole to sump tank via channel. Free vortex measurements were taken with a depth gauge and pitot tube. Two orifices in the side of cylinder, angled at 15^0 are used to provide initial rotation to flow. Water entry was made through these orifices and by varying central outlet orifice diameter, different observations were recorded. Following are the specifications of experimental setup:

Pump: centrifugal type, Motor: 0.37 kW, Sump tank volumetric capacity: 250 litre

Cylindrical vessel diameter: 245 mm, Volumetric tank capacity: 40 litre.

Submergence level: 18 cm, Container diameter: 24 cm

4. EXPERIMENTAL RESULTS & ANALYSIS

We modeled flow such that the outflow through suction orifice is driven by gravity. On setting pump switched on, water started filling in the cylinder through the tubes. The surface fell rapidly towards the base center and sucked air to form the air core flow. Air core extended to the center orifice and arrived at the intake to form the air entrainment vortex thus a swirling flow began occurring and an intense surface depression is generated above central drain-orifice. At last the swirling flow gradually attained stable profile after some time. Submergence height i.e. depth of fluid exiting point below free water surface is the key responsible factor for vortex generation. Submergence level depends on number of variables such as water flow rate, viscosity, surface tension, geometry of inlet and outlet points. Submergence level was kept fixed by regulating water flow rate in the container. When inflow rate in cylinder was increased by varying pump discharge, and outflow rate was constant so submergence level raised and soon after that vortex disappeared. We kept Submergence height 18 cm from vessel base and different observations were recorded by changing outflow rate due to change in area of outlet. Experiments were conducted with three orifices having different outlet diameters of 16mm, 24 mm and 30mm.

Figure 2 shows visualization of structure of the free vortex profile generated. Flow field obtained is locally irrotational but streamlines are circular about center of the orifice. Profile has rotational symmetry and consist an inner air core surrounded by line vortex region which is independent of height. Free surface vortex adjacent to Inner air core region was found to be height dependent and at various height values of vortex, measurements of radius were observed and this ratio of variation in vortex profile radius with height from base are tabulated in Table 1 and plotted in figure 3(a) for three different outlet orifices, here 1, 2, 3 denotes 16mm, 24mm and 30 mm diameter orifices.

Table 1. Height variation with radial distance

Z_1(m)	R_1(m)	Z_2(m)	R_2(m)	Z_3(m)	R_3(m)
0.164	0.027	0.163	0.032	0.162	0.035
0.159	0.025	0.16	0.029	0.155	0.032
0.153	0.023	0.156	0.026	0.15	0.03
0.144	0.021	0.15	0.024	0.143	0.027
0.135	0.019	0.142	0.022	0.133	0.024
0.125	0.017	0.13	0.019	0.12	0.022
0.115	0.014	0.115	0.017	0.11	0.021

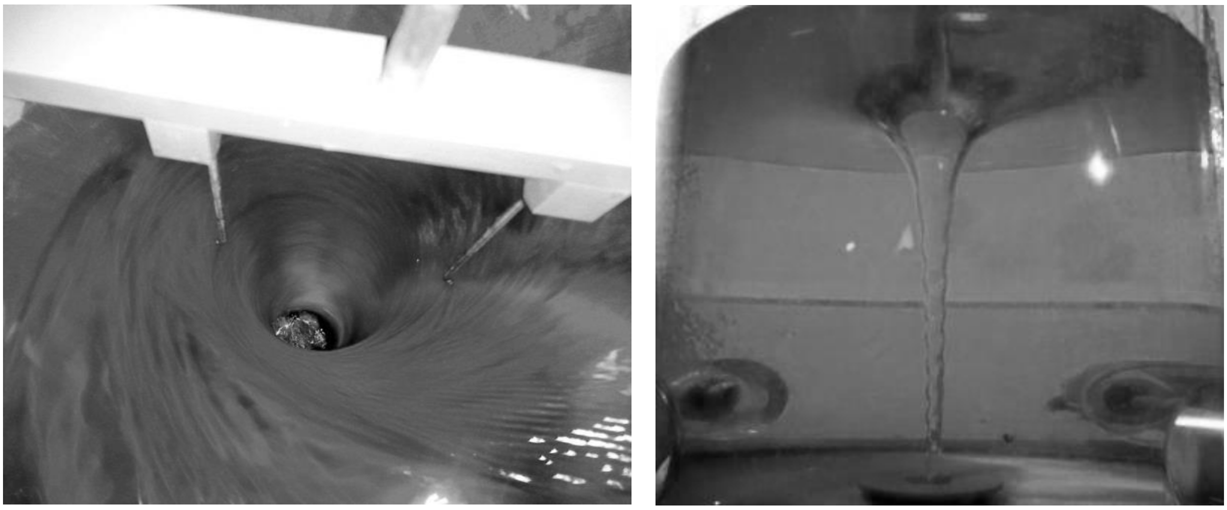


Figure 2. Flow visualization of Air entraining free Vortex

Figure 3(b) is depicting free vortex surface profile of inner air core region surrounded with free vortex region is established which took shape above 16 mm diameter size drain and it can be clearly seen that with increasing height, vortex profile diameter increases. Figure 4(a) is showing comparison between vortex profiles obtained for 16mm, 24 mm and 30 mm outlet diameter outlets. For larger suction diameter, expanded central vortex profile for same values of height accomplished though pattern of profile structure was similar.

Observations of Pressure head obtained in pitot tube at different radius of rotating air core and line water vortex for 30 mm outlet diameter orifice are tabulated in Table 2, pressure head increased with reduction of vortex radius. Tangential velocity in the vortex core was calculated and plotted for 30 mm diameter outlet orifice in figure 4(b). Tangential velocity was found to be inversely proportional to radius of free vortex and velocity in air core region is approximated from Rankine vortex model. Circulation of free vortex is calculated for 30 mm diameter orifice and found to be least variable for different vortex radial positions and that is shown in figure 5(b). For three orifices, vortex strength and stream function are calculated at different values of relative submergence, i.e. Ratio of submergence height and suction orifice diameter and shown in table 3 and graphically represented in figure 5(a). Both shows increment with increasing orifice diameter so with increase in outlet area, stronger vortex formed.

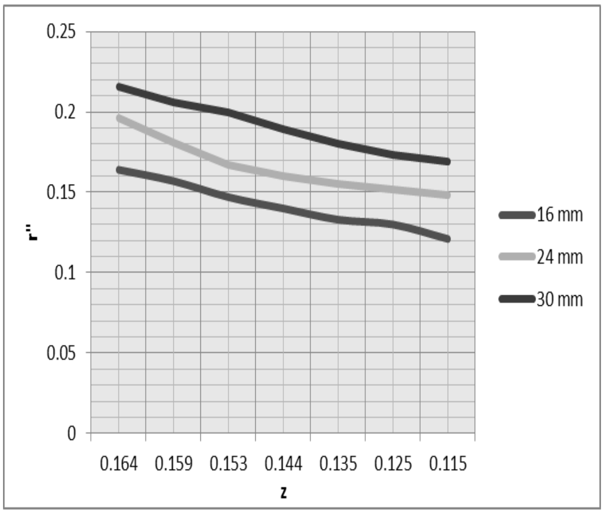


Figure 3(a). variation of free vortex radius with free surface height

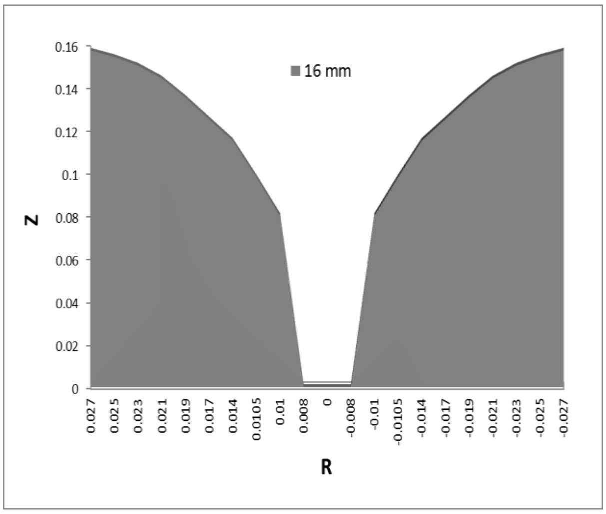


Figure 3(b). surface profile for 16 mm orifice

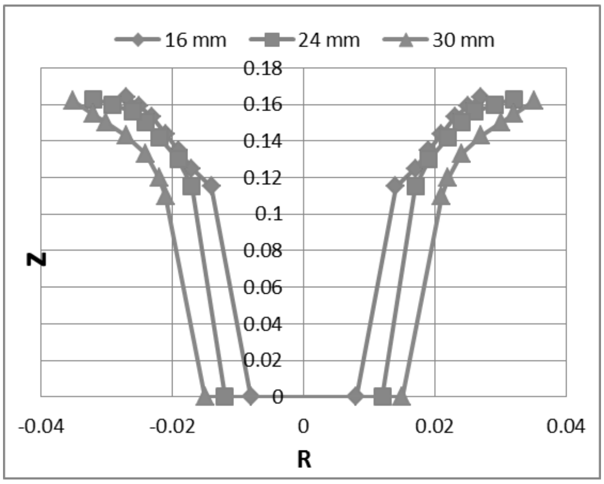


Figure 4(a). comparison of profiles for different orifices

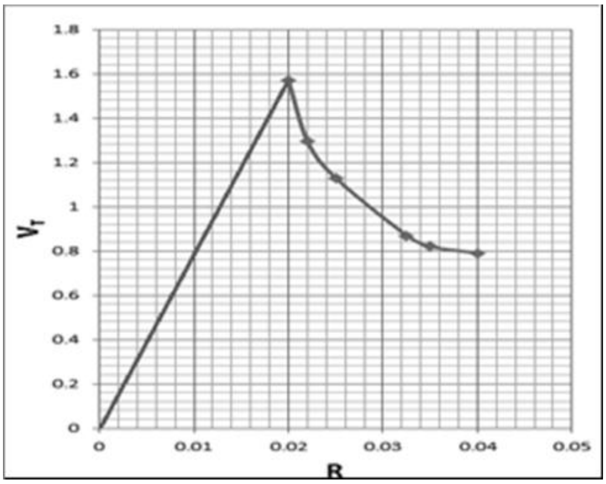


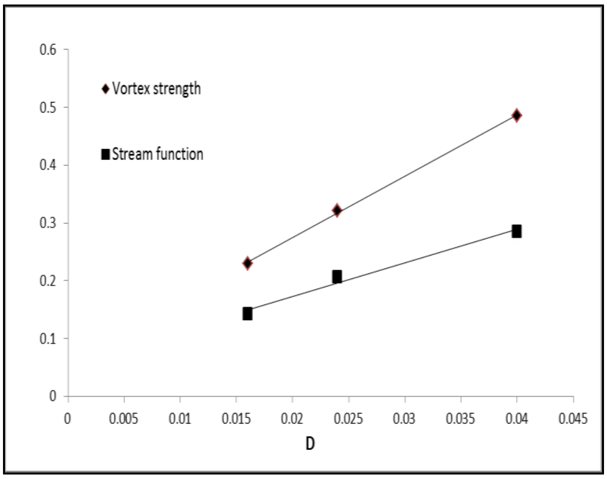
Figure 4(b). Tangential velocity distribution

Table 2. Head difference in Pitot tube against vortex radius

R_3(m)	0.0325	0.025	0.022	0.02
H(m)	0.039	0.065	0.086	0.125

Table 3: Vortex strength Γ and stream function ψ for free vortex

Relative submergence (H/D)	Γ(m^2/s)	ψ
6	0.485	0.285
7.5	0.322	0.207
11.25	0.233	0.142



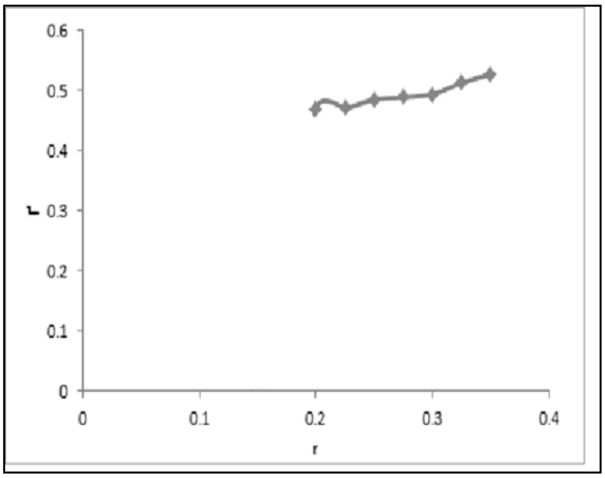


Figure 5(a). vortex strength & stream function values variation with Orifice diameter

5(b). circulation along vortex radius for 30 mm outlet orifice.

5. CONCLUSIONS

In this work, the formation, strength and pattern of free surface vortex are studied on a reduced scale model of water reservoir. The impacts of affecting factors on the formation are analyzed herein. The present limited experimental study leads to the following conclusions. During experiment, an intense free vortex and a surface depression formed above the drain orifice. It consist an air core region surrounded by free vortex. Free vortex surface extended to outlet drain orifice. In result analysis it is found out that Free surface vortex adjacent to Inner air core region was height dependent, vortex radius expanded as height increased and on comparing this radius- height ratio for different outlet areas, it is established that with increment in outlet area, higher values of vortex radius obtained for same values of height though profile pattern was invariable. Pitot tube observations suggested that pressure head increased with reducing free vortex radius. Tangential velocity was found to be inversely proportional to radius of free vortex. Circulation along free vortex relatively remained of constant value. With increase in outlet area, strength and stream function value enhanced.

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