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A Bernoulli's Law Lab in a Bottle

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Bernoulli's law is a fundamental relationship in fluid dynamics that is covered in most introductory physics courses. Basically a statement of conservation of energy for an open fluid system, Bernoulli's law is often used in labs and examples to analyze the lift force on a fixed wing or the forces on objects in a fluid flow.[1-4] Although these are legitimate uses of the law, they do not introduce students to the way in which several concepts, such as Bernoulli's law and the conservation of mass, are used in combination to study the dynamics of fluid systems. Thus, to give students an easily understandable introduction to this method of analysis, we present a laboratory experience in which the drain time of water flowing out of an inverted soda bottle is measured and calculated for a set of easily interchangeable exit holes.

Fig. 1. Setup for experiment. (*Photo by Bruce Chakrin.)*

Experiment

The basic arrangement for the lab, shown in Fig. 1, includes a plastic 2-L bottle with the bottom cut off, a set of bottle caps with different sized holes drilled through their center, and a standard motion sensor positioned directly above the inverted bottle on the same post. A small plastic pail is located directly below the bottle to catch the water as it drains out of the system, and a pitcher or beaker is needed to fill the bottle when ready to take data. The exact distance between the open part of the bottle at the top of the apparatus and the sensor is not critical but needs to be large enough so that water can be poured into the inverted bottle.

For each laboratory setup, a set of four bottle caps with holes drilled in their centers is provided. After filing the holes to get rid of the large fragments left by the drill bits, the hole diameters can be measured by the students with a set of calipers. For the data presented in this article, the set of caps have hole diameters of 7.2 mm, 8.8 mm, 9.8 mm, and 12.1 mm, which we labeled caps 1–4, respectively. The height to which students are to fill the inverted bottle with water should be indicated with a line marked on the bottle. We marked our water level 20 cm above the exit hole in the cap. This provided enough water so the drain times were less than a minute but long enough so that they were easy to measure.

When ready to take data, students measure and

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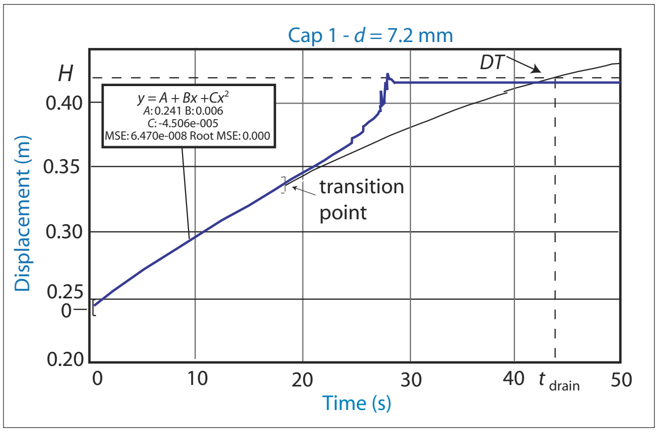

Fig. 2. Sample of data taken by the motion sensor as it monitors the displacement of the top of the water drained through a cap with a 7.2-mm hole.

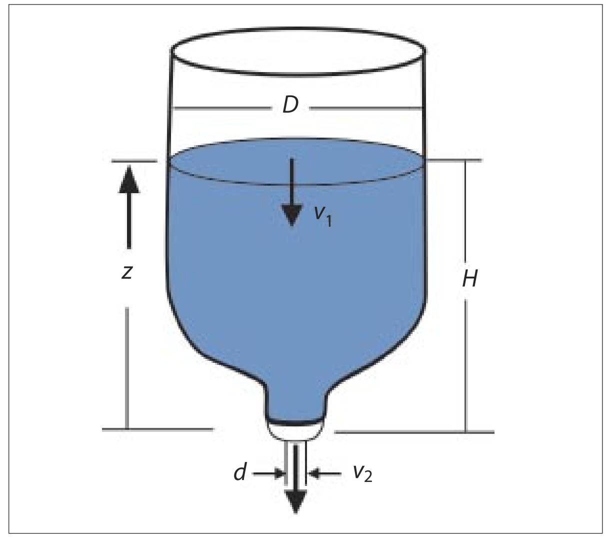

Fig. 3. Diagram of the experimental setup.

record the diameter of the hole in the cap and then screw the cap onto the bottle. They then cover the hole tightly with their thumb and fill the bottle with water up to the marked line. Students then initiate the computer collection of data by the motion sensor, which measures the location of the top of the water with ultrasonic pulses. The coordination between the time of release and the time at which data begin to be collected is not difficult since the PASCO MS-0001 motion sensor, which costs approximately $79, makes a series of rapid "clicks" that can be heard just before it begins to take data. Figure 2 is a plot of the displacement of the top surface of the water as a function of time using a cap with a 7.2-mm hole. As can be seen, the sensor should continue to collect data for some time after the water has drained from the bottle. The usable data will be easily recognizable on the graph.

The first step in the analysis of the data is to fit a quadratic curve to the part of the data that represents the water moving through the part of the bottle that has a constant diameter. The transition point in the data, which marks the time where the bottle begins to taper down to the cap, is labeled in Fig. 2. A quadratic curve is fit to the segment of the data starting from the beginning of the data acquisition to this point of transition, indicated with square brackets in Fig. 2. In our case we fit the curve to the data with the built-in curve fitting features included with Vernier's Logger Pro software, but a quadratic curve could also be fit with many other programs such as Excel. This curve fit is done so that a simpler analysis can be used as if the bottle were a cylinder with a hole in the bottom instead of considering the shape of the bottle. For the data displayed in Fig. 2, the drain time (t_{drain}) was measured to be 43.6 s by finding the time at which the quadratic curve intersects the data at the location of the top of the water when it reaches the cap. An arrow labeled with the letters *DT* (for Drain Time) points to this intersection point on Fig. 2, which also has the displacement *H* labeled. It should be noted that the motion sensor only records a relative distance away so that even though it is pointing down, it records the location of the top surface of the water as a positive displacement.

Analysis

To find an expression for the time it takes water to drain out of this system, students use Bernoulli's law in tandem with the conservation of mass and some basic equations of kinematics. For an incompressible and nonviscous fluid, Bernoulli's law is often expressed as

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g z_1 = P_2 + \tfrac{1}{2}\rho v_2^2 = \rho g z_2 = \text{Constant}, \quad (1)$$

where g is the acceleration of gravity, P_n is the internal pressure of the fluid at the point n in the system, ρ is the density of the fluid, v_n is the speed of the fluid at point n, and z_n is the height of the fluid at point n from a set origin. For the system analyzed in this lab, depicted in Fig. 3, the origin is set at the location of the hole in the cap, and up is the positive direction. Also, it must be mentioned that although there are frictional forces between the bottle and the flowing water, the boundary layer where these forces are significant is small enough to be neglected in this experiment. It should be made clear to students that if the viscosity of the fluid or the geometry of the system were changed, these expressions may not provide an accurate representation of the system, and a much more complicated analysis, which takes this boundary layer into account, is required.[5]

For the flow of an incompressible fluid, the conservation of mass, often referred to as the continuity equation, states that the rate of change of mass for a system ($\Delta m_1/\Delta t$) must equal the net flow of mass ($\Delta m_2/\Delta t$) in or out of a system. For the situation analyzed in this lab, the volume is the product of the surface area (A) of the water at points 1 and 2 and a distance in the direction of flow, downward. The continuity equation for our situation and in many common applications is expressed as:

$$\rho A_1 v_1 = \rho A_2 v_2 , \tag{2}$$

with v_1 and v_2 the velocity of the water at points 1 and 2, respectively. This in fact is the form of the continuity equation, which is commonly presented in some introductory physics texts.[6]

Since the water is exposed to the atmosphere at the top surface and at the hole in the cap (designated points 1 and 2), the pressure at these points is the atmospheric pressure and thus equal to each other. Under the conditions of this experiment, the density of water is constant throughout the fluid. The areas of the circular bottle and holes in the caps are given by $(\pi/4)(D)^2$ and $(\pi/4)(d)^2$, respectively. Solving Eq. (2) for v_2 and substituting the result into Bernoulli's equation, the expression for the velocity of the top of the water as it drains out the bottom is given as

$$(v_1)[1 - (D^4/d^4)]^{1/2} = [2g\,(z_2 - z_1)]^{1/2}. \tag{3}$$

If the class performing this lab uses calculus, set $z_2 = 0$, $v_1 = -(dz_1/dt)$, separate the variables, and integrate both sides to get the expression for the drain time (t_d) of the bottle, which is the time it takes the top surface of the water to move from z_1 to z_2. It should be noted that to get Eq. (7) for the drain time, the limits of integration for z and t should be set to H to 0 and 0 to t_d, respectively.[7] As an additional exercise, the time derivative of both sides of Eq. (3) could be taken to show that the nonviscous theory predicts a constant acceleration.

If the class performing the lab is algebra-based, both sides of the Eq. (3) are squared to give

$$(v_1)^2[1 - (D^4/d^4)] = 2g\,(z_2 - z_1). \tag{4}$$

Next, assuming a constant acceleration for the top surface of the water, which is consistent with the nonviscous theory used, the basic kinematic equations

$$v_1{}^2 - v_{1o}{}^2 = 2\,a\,d \tag{5}$$

and

$$x = x_o + v_{1o}t + (1/2)a\,t^2 \tag{6}$$

are used with the top surface of the water initially at rest, so v_{1o} is set to zero in both expressions. With these initial conditions Eqs. (5) and (6) provide substitution for $v_1{}^2$ and $(z_2 - z_1)$ in Eq. (4) so that it can then be rearranged as an equation of the drain time (t_d) of the bottle. From Eq. (5), since the distance traveled by the top surface of the water d is set equal to H, the expression ($2\,a\,H$) can be substituted for $v_1{}^2$ in Eq. (4). Since both the acceleration (a) and displacement (d) are in the same direction, the sign of the expression is positive. From Eq. (6), since the initial and final location of the top surface of the water is $x_o = z_1$ and $x = z_2$, respectively, then $(x - x_o) = (z_1 - z_2)$ so that $(z_2 - z_1)$ in Eq. (4) can be substituted with $-(1/2)a(t_d)^2$. Using the negative sign in front of $(1/2)a(t_d)^2$ to switch the order of the expression in the square parentheses, deleting the acceleration (a) from both sides of Eq. (4), and solving for (t_d) results in the expression for the drain time,

$$t_d = \sqrt{(2H/g)[(D^4/d^4) - 1]}. \tag{7}$$

With Eq. (7) a drain time of the bottle can be calculated for the different bottle cap hole diameters, *d*. For our conceptual physics class, we provide Eq. (7) and an explanation of its derivation. For the algebra-based physics class, we provide Eqs. (1), (2), (5), and (6), and a set of instructions, and for our calculus-based class, we would provide Eqs. (1) and (2), and instructions, so that they can each derive the drain time expression for themselves.

Data were taken using the procedure described in this paper with caps 1-4. The drain times were calculated using Eq. (7) with the measured hole diameter listed above, a bottle diameter *D* of 10.5 cm and a height *H* of 20 cm. The comparison of the measured and calculated drain times given in Table I demonstrates good agreement between the experiment and the calculations. The percent difference between the calculated and measured values is less than 6% for all four caps. These small percent differences seem to be independent of the size of the hole in the cap. We thus attribute the difference to procedural errors such as not synchronizing the release of the water flow with the start of data collection, errors in measuring the starting height of the water level, and choosing an inappropriate portion of the data to fit with the curve. Also, the good agreement between the experiment and calculations helps demonstrate that the assumption of nonviscous fluids flowing at a constant acceleration is valid for this experiment.

During the development of this lab we tried different sized holes and learned that for holes much larger than 12.1 mm in diameter, the time was so short that the uncertainty in the start time played a larger role in the analysis, which led to some larger inconsistencies in the measurements. We also learned that for holes much smaller than the 7.2-mm diameter, the drain time was too long to be practical in a lab.

Table I. Measured and calculated drain times.

Cap Number	Exit Hole Diameter (mm)	Measured Drain Time (s)	Calculated Drain Time (s)	% difference
1	7.2	43.6	43.0	1.4
2	8.8	27.8	28.8	3.5
3	9.8	23.4	23.2	0.9
4	12.1	14.3	15.2	5.9

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