Final Assignment

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1. Entanglement swapping

a. See the code

b. We can see that the final state is like the Initial state. So the Alice and Bob qubits Entanglement is verified. We can conclude that the Entanglement swapping is verified.

Code	Theoretical State
1 sim_ciruit() ✓ 0.0s Initial state Alice: [[1.+0.j] [0.+0.j]] Initial state Bob: [[1.+0.j] [0.+0.j]] Final state Alice: [[0. +0.00000000e+00j] [-0.70710678+1.73191211e-16j] [0.70710678-1.73191211e-16j] [0. +0.00000000e+00j]] Final state Bob: [[0. +0.00000000e+00j] [-0.70710678+1.73191211e-16j] [0.70710678-1.73191211e-16j] [0. +0.00000000e+00j]]	**From** **|00>** **TO** **½ (|01> + |10>)**

c. We can analyze that when we add depolarizing noise, the final state can swap.

Code		Theoretical State
------------------PROBA 0.0------ Initial state Alice: [[1.+0.j] [0.+0.j]] Initial state Bob: [[1.+0.j] [0.+0.j]] Final state Alice: [[0. +0.00000000e+00j] [-0.70710678+1.73191211e-16j] [0.70710678-1.73191211e-16j] [0. +0.00000000e+00j]] Final state Bob: [[0. +0.00000000e+00j] [-0.70710678+1.73191211e-16j] [0.70710678-1.73191211e-16j] [0. +0.00000000e+00j]]	------------------PROBA 0.1------ Initial state Alice: [[1.+0.j] [0.+0.j]] Initial state Bob: [[1.+0.j] [0.+0.j]] Final state Alice: [[0.00000000e+00+0.j] [-1.73191211e-16-0.70710678j] [-1.73191211e-16-0.70710678j] [0.00000000e+00+0.j]] Final state Bob: [[0.00000000e+00+0.j] [-1.73191211e-16-0.70710678j] [-1.73191211e-16-0.70710678j] [0.00000000e+00+0.j]]	**1/sqrt(2)*(|00> + |11>)** **TO** **1/sqrt(2)*(|01> + |10>)**

2. GHZ state distribution

a. See the code
b. Same. Here is the result of the initial and final state. We can see that the final state is matching with a GHZ state for 3 qubits:

Code	Theoretical State

```
1  sim_ciruit_GHZ()
✓ 0.0s

Initial state Charlie: [[1.+0.j]
 [0.+0.j]]
Final state Charlie: [[0.70710678+0.j]
 [0.        +0.j]
 [0.        +0.j]
 [0.        +0.j]
 [0.        +0.j]
 [0.        +0.j]
 [0.        +0.j]
 [0.70710678+0.j]]
```

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)\,.$$

c. When we add noise, the final state changes from (1/sqrt(2)) (|000> + |111>) to (1/sqrt(2)) (|010> + |101>) .

```
------------------PROBA 0.3-----------------
Initial state Charlie: [[1.+0.j]
 [0.+0.j]]
Final state Charlie: [[0.70710678+0.j]
 [0.        +0.j]
 [0.        +0.j]
 [0.        +0.j]
 [0.        +0.j]
 [0.        +0.j]
 [0.        +0.j]
 [0.70710678+0.j]]
```

```
------------------PROBA 0.4------------
Initial state Charlie: [[0.+0.j]
 [0.+1.j]]
Final state Charlie: [[0.+0.j        ]
 [0.+0.j        ]
 [0.+0.70710678j]
 [0.+0.j        ]
 [0.+0.j        ]
 [0.+0.70710678j]
 [0.+0.j        ]
 [0.+0.j        ]]
```

d. Below there is our simulation for N = 10

```
peekC: [None, None, None, None, None, None, None, None, None, None, Qubit('QS#28-10')]
peekN0: [Qubit('QS#28-0'), None, None, None, None, None, None, None, None, None]
peekN1: [Qubit('QS#28-1'), None, None, None, None, None, None, None, None, None]
peekN2: [Qubit('QS#28-2'), None, None, None, None, None, None, None, None, None]
peekN3: [Qubit('QS#28-3'), None, None, None, None, None, None, None, None, None]
peekN4: [Qubit('QS#28-4'), None, None, None, None, None, None, None, None, None]
peekN5: [Qubit('QS#28-5'), None, None, None, None, None, None, None, None, None]
peekN6: [Qubit('QS#28-6'), None, None, None, None, None, None, None, None, None]
peekN7: [Qubit('QS#28-7'), None, None, None, None, None, None, None, None, None]
peekN8: [Qubit('QS#28-8'), None, None, None, None, None, None, None, None, None]
peekN9: [Qubit('QS#28-9'), None, None, None, None, None, None, None, None, None]
Final state Charlie: [[0.70710678+0.j]
 [0.        +0.j]
 [0.        +0.j]
 ...
 [0.        +0.j]
 [0.        +0.j]
 [0.70710678+0.j]]
```

We can see that it is very powerful because there is an entanglement swapping with larger star networks.

3. Conclusion

Entanglement swapping is a fascinating quantum phenomenon that allows the establishment of entanglement between two particles that have never interacted directly. This process is crucial for quantum communication networks, particularly for extending the range of quantum key distribution (QKD) systems. By entangling particles across distant nodes, entanglement swapping paves the way for the realization of quantum repeaters and long-distance quantum networks.