Cat qubits

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1 Introduction

When manipulating quantum bits in order to make computation, quantum phenomenas like decoherence are the main causes of errors. This is due to the interaction of a supposed isolated quantum system with its noisy environment. With the multiple efforts made to isolate a quantum system as much as possible, the lifetime of a superconducting qubit has been increased from a few nano-seconds to 100µs-1ms.

In order to increase this lifetime, the principal tool used is Quantum Error Correction and is a way to encode a logical qubit, itself composed by many physical qubits. Nevertheless, build error correction for a large number of logical qubits requires to implement way more physical qubits. That is when Alice & Bob come out whith their cat qubits that could perform error correction with less more qubits to implement and exponentially reducing bit-flip errors.

We will see how these cat qubits formally work and then we will have a look at Alice & Bob technology.

2 Cat qubits for error correction

2.1 Error correction

Due to decoherence and noisy environments, many errors happens when manipulating qubits. A qubit is subject to both bit-flips and phase-flips, in which a qubit randomly flips between super-positions.

A bit-flip error is the flip of a bit from 0 to 1 or 1 to 0. A phase-flip error is when a positive qubit becomes negative and vice-versa.

To deal with these errors, Quantum Error Correction has been the inevitable tool to try to erase those errors. With the design of a logical qubit composed by many physical qubits, it is possible to increase the lifetime of a system without it enters in a decoherent state.

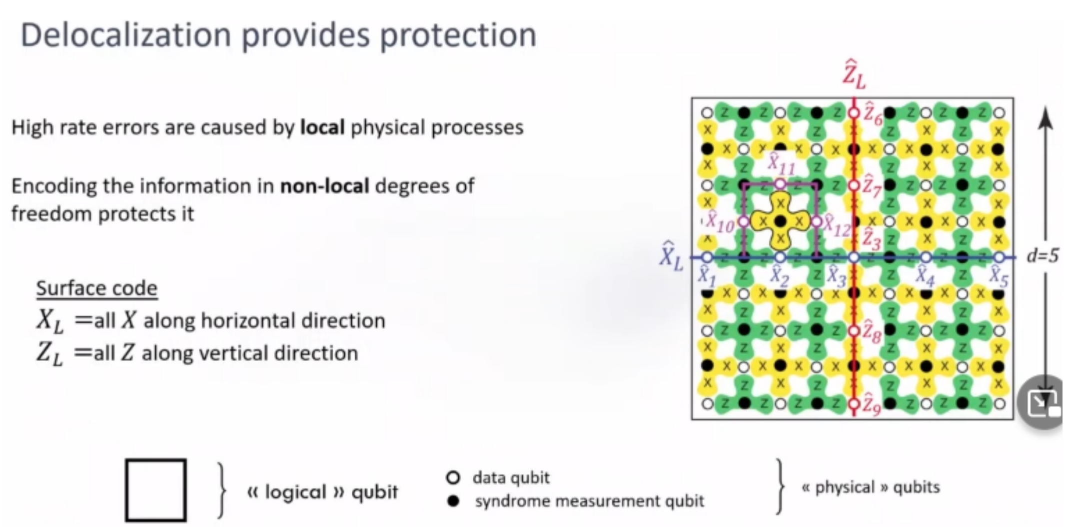

Figure 1: Example of a surface code.

The figure shown above is an example of a surface code used to reduce quantum errors. In this 2D surface are encoded logical qubits represented by a square. These logical qubit are formed by a layout of physical qubits. In this code, we can see that one logical qubit is formed by 8 physical qubits. At a larger scale, this would be unfeasible. That is why cat qubits seem to be a great opportunity to go over this challenge.

2.2 Cat qubits

A cat qubit is an ideal qubit without an error. It is based of the quantum state of the Schrödinger's cat. The concept of Schrödinger's cat is a hypothetical scenario that is commonly used to explain the superposition principle in quantum mechanics. In this scenario, a cat is confined within a sealed box along with a radioactive substance. According to the principle of superposition, the cat is considered to be in a state of being both alive and dead simultaneously, until the box is opened and observed. Upon observation, the cat's state would collapse into a single state - either alive or dead. Similarly, qubits, which are quantum bits, also operate under the superposition principle and can exist in both states of 0 and 1 at the same time.

In order to solve the problem of errors that occurs during quantum computation, Yale physicists have developed error-correcting cat qubit. While multiple qubits are typically required to maintain one effective qubit, a single cat qubit can prevent phase flips independently. This is because the cat qubit is able to encode an effective qubit into a superpostion of two states within a single electronic circuit. In this specific case, the electronic circuit is a superconducting microwave resonator, where the two states of the cat qubit correspond to the oscillations to the resonator.

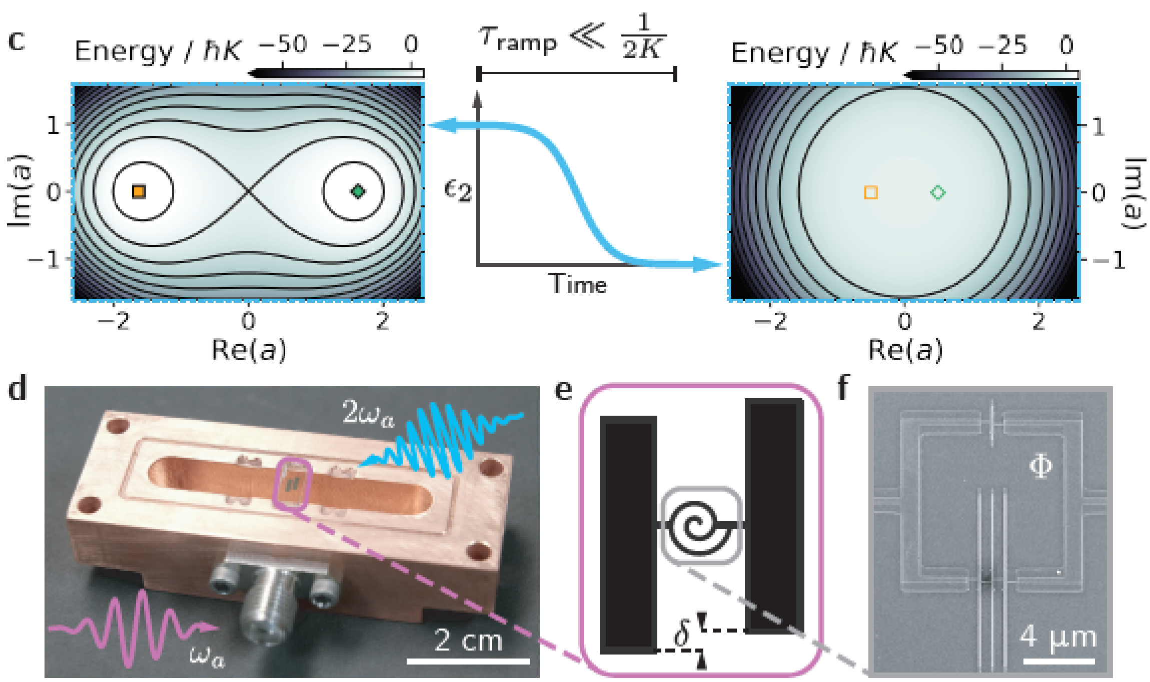

Figure 2: Qubit encoding, stabilization and implementation

2.2.1 fault-tolerant operations

Figure 3 shows a schematic diagram of how to realize a universal fault-tolerant quantum computation using cat qubits. In this scheme, the fundamental operations are performed on the cat qubits in a bias-preserving way. This means that the superposition of the cat qubits is preserved, even when operations are performed on them.

The left part of the diagram represents the fundamental operations performed on the cat qubits. These operations are the building blocks for the fault-tolerant logic operations that are performed on

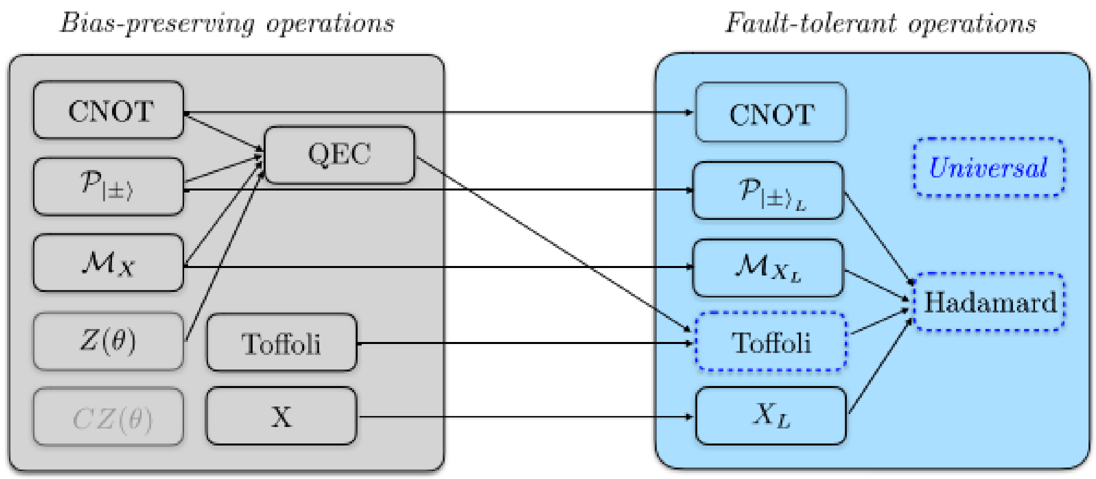

Figure 3: fault-tolerant operations

the repeating cat qubits, which are shown on the right side of the diagram. The arrows indicate how the fault-tolerant logic operations are constructed from the fundamental operations.

In other words, the fault-tolerant logic operations are constructed by combining and repeating the fundamental operations in a way that preserves the superposition of the cat qubits. This approach to fault-tolerant quantum computing is promising because it can protect against errors in qubits and gates, which is essential for the scalability of quantum computers.

The process of building fault-tolerant gates using repeated cat qubits involves several steps. The first step is to identify the set of fundamental bias-preserving operations for cat qubits, which include operations such as P, ic, MX, X, Z, $CNOT$ and $Toffoli$. These operations are used to construct fault-tolerant coded operations at the repeating code level, denoted SL, which includes P^L, ic^L, MX^L, XL, $CNOT$, and $Toffoli$.

A notable advantage of this construction is that the logical CNOT gate can be implemented transversely using the fundamental CNOT gate, simplifying the hardware required for implementation. In addition, the universality of SL is established because it contains the Toffoli gate in the computational basis, while the state preparation and measurement are performed in the dual basis.

The final step is to demonstrate the ability to construct a logical Hadamard gate, which is a one-qubit basis change operation, using gates from the SL set. This achievement establishes the universality of fault-tolerant gate construction.

3 Alice & Bob cat qubit

Alice & Bob's mantra is to "Think inside the box". Using cat qubits, they indeed deal with error correction from the inside.

Actually using bosonic error correction, the most important point to make the error correction effective is to encode the quantum information in a space that contains one quantum system. Thus, they decided to encode the information into harmonic oscillators because they are continuous variable systems that provides an infinite-dimensional Hilbert space. They have been focussing on this infinite Hilbert space of such systems and encode information in such a way that errors become tractable. But also on the phase space of the Harmonic oscillator to encode information in a non-local manner in this space. Thus, they are capable of delocalizing quantum information over many more states, obtaining the same level of error protection and with less physical systems.

In a quantum manner, they use a microwave resonator that exchanges photon pairs with its environment. The two stationnary states they manage are the coherent states $|\alpha\rangle$ and $|-\alpha\rangle$

Concretely they built a simple dynamic that stabilizes two coherent states of amplitude α and in opposite phases in a harmonic oscillator. It exponentially reduces bit-flip error (i.e. from $+\alpha$ to $-\alpha$) while only linearly increasing phase-flip (i.e. 1 photon loss probability). In order to correct the

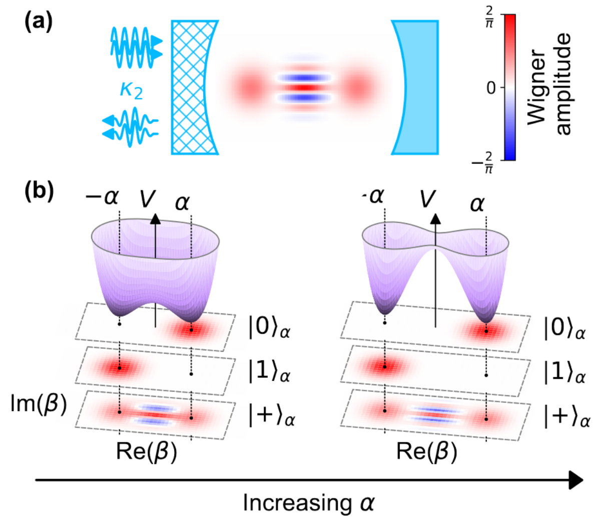

Figure 4: Alice & Bob cat qubit encoding in a harmonic oscillator. (a).The exchange of photon between the resonnator and its environment. (b).The encoding of information according to the amplitude α

remaining type of error, they ensure that a linear repetition code can be used to reach a significantly low error rate.

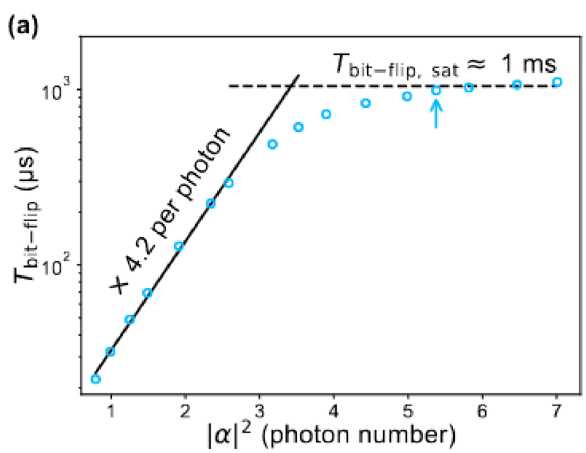

Figure 5: Exponential increasing of the bit-flip time according to the cat size $|\alpha|^2$

4 Conclusion

Due to their potential to solve errors that occurs during quantum computational, the potential of cat qubits makes them a significant breakthrough in quantum computing. Although the realization of cat qubits are complicated because of the sensibility of quantum systems to the environment, some companies like Alice & Bob have shown that the approach of using cat qubit is promising for their development. With continuous improvements in the fabrication of quantum systems, it is possible that cat qubits may become a key component in the creation of quantum computers capable of solving complex problems without computational error.