Derivation and Ambiguity

Course Name: Compiler Design
Course Code: CSE331
Level:3, Term:3
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Derivation

A derivation is basically a sequence of production rules, in order to get the input string. During parsing, we take two decisions for some sentential form of input:

- Deciding the non-terminal which is to be replaced.
- Deciding the production rule, by which, the non-terminal will be replaced.
- To decide which non-terminal to be replaced with production rule, we can have two options.

Derivation Types

- **Left-most Derivation**

If the sentential form of an input is scanned and replaced from left to right, it is called left-most derivation. The sentential form derived by the left-most derivation is called the left-sentential form.

- **Right-most Derivation**

If we scan and replace the input with production rules, from right to left, it is known as right-most derivation. The sentential form derived from the right-most derivation is called the right-sentential form.

Example

- Production rules:

E → E + E

E → E * E

E → id

Input string: id + id * id

The left-most derivation is:	The right-most derivation is:
E → E * E E → E + E * E E → id + E * E E → id + id * E E → id + id * id	E → E + E E → E + E * E E → E + E * id E → E + id * id E → id + id * id

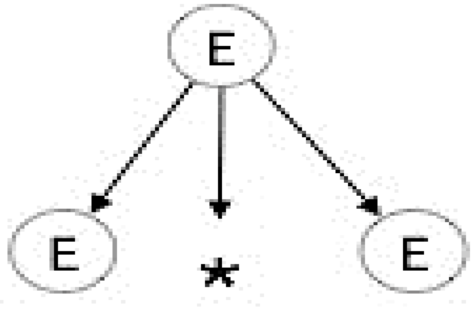
Parse Tree

- A parse tree is a graphical depiction of a derivation.
- It is convenient to see how strings are derived from the start symbol.
- The start symbol of the derivation becomes the root of the parse tree.

Constructing the Parse Tree

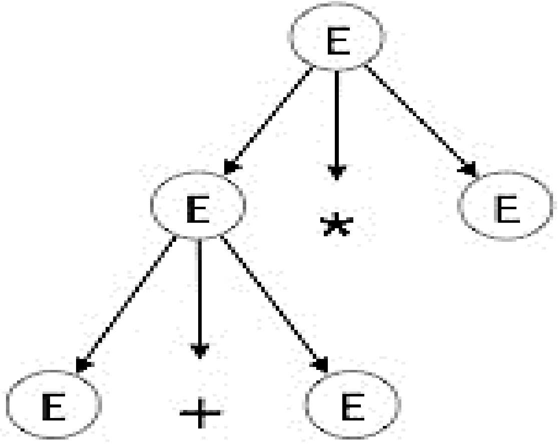
- Step 1:

E → E * E



Step 2:

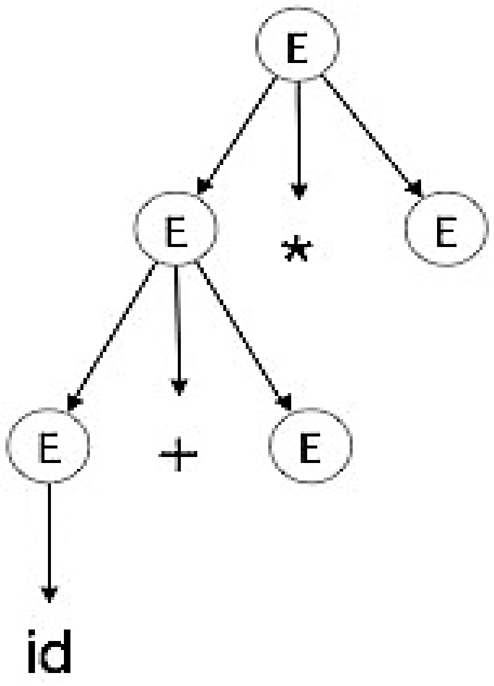
E → E + E * E



Constructing the Parse Tree …

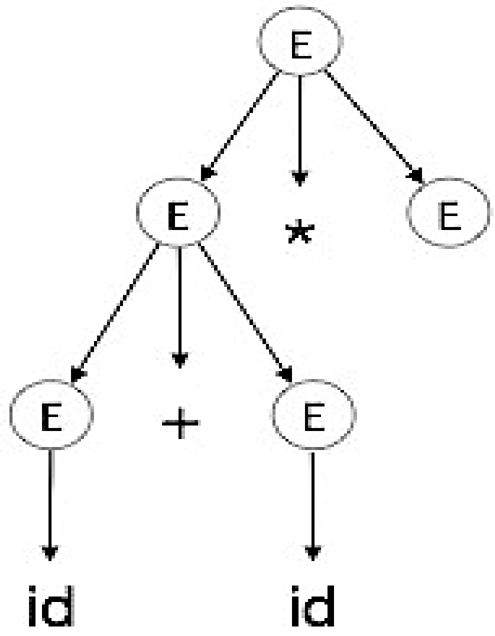
- Step 3:

E → id + E * E



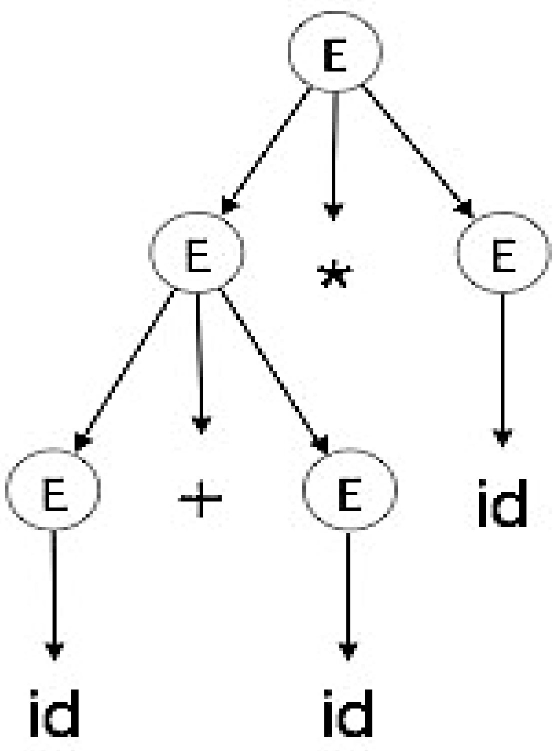
Step 4:

E → id + id * E



Constructing the Parse Tree …

Step 5:
E → id + id * id



- In a parse tree:

-All leaf nodes are terminals.

-All interior nodes are non-terminals.

-In-order traversal gives original input string

Ambiguity

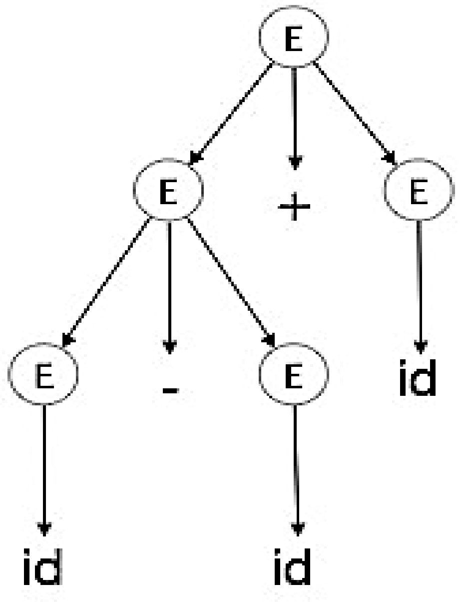
- A grammar G is said to be ambiguous if it has more than one parse tree (left or right derivation) for at least one string.

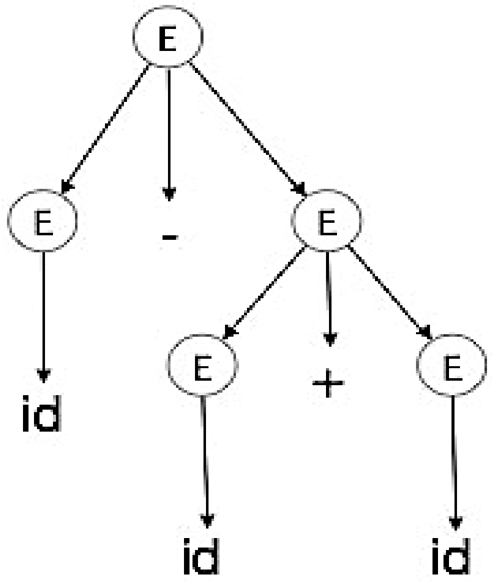
Example: E → E + E

E → E – E

E → id

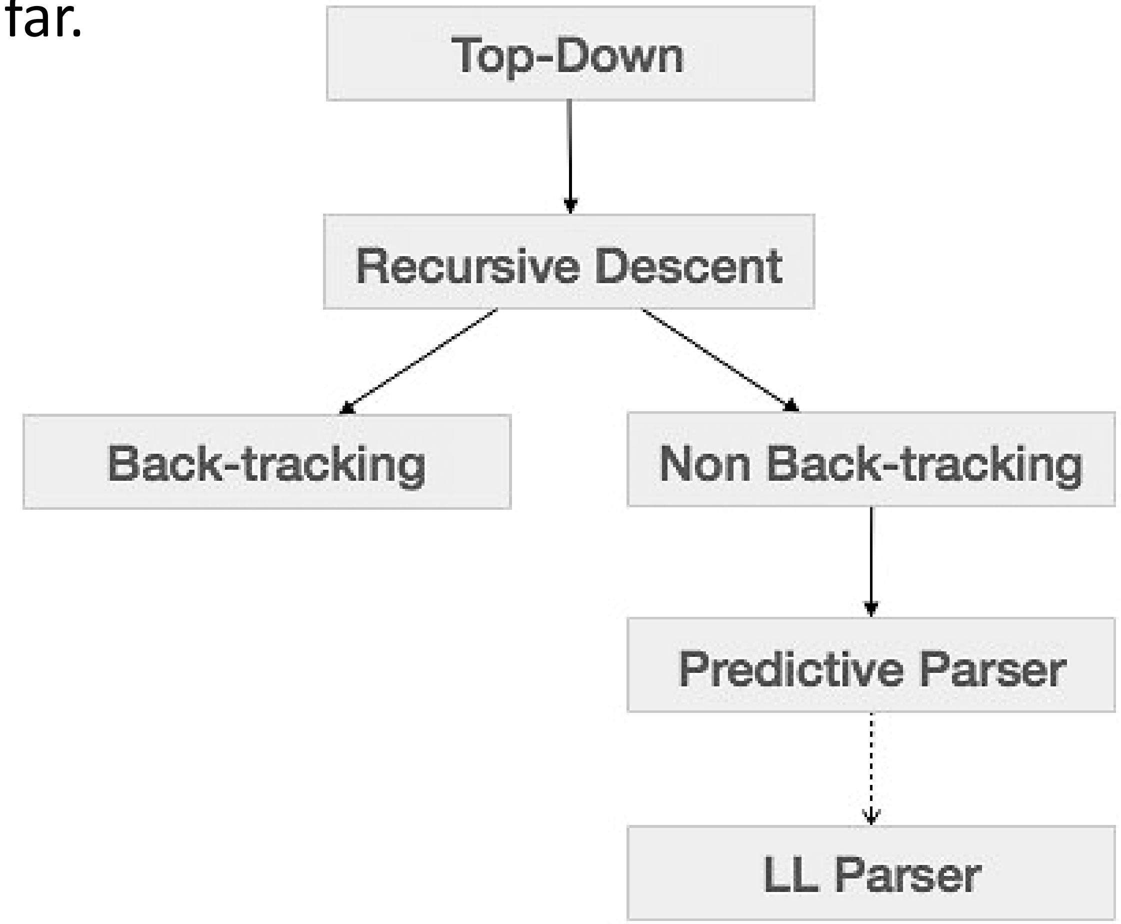
For the string id + id – id, the above grammar generates two parse trees:





Top-Down Parsing

- Parsing that we have seen so far.



Top-Down Parsing

- Parsing is the process of determining if a string of tokens can be generated by a grammar.
- For any context-free grammar there is a parser that takes at most $O(n^3)$ time to parse a string of n tokens.
- ***Top-down parsers*** build parse trees from the *top (root)* to the *bottom (leaves)*.
- Two top-down parsing are to be discussed:

  o ***Recursive Descent Parsing***

  o ***Predictive Parsing*** An efficient non-backtracking parsing called for LL(1) grammars.

Example of Top-Down Parsing

Consider the grammar

$S \rightarrow cAd$

$A \rightarrow ab \mid a$

Input string w = cad

To construct a parse tree for this string using top-down approach, initially create a tree consisting of a single node labeled S.

Example of Top-Down Parsing

$S \rightarrow cAd$
$A \rightarrow ab \mid a$
Input string $w = cad$

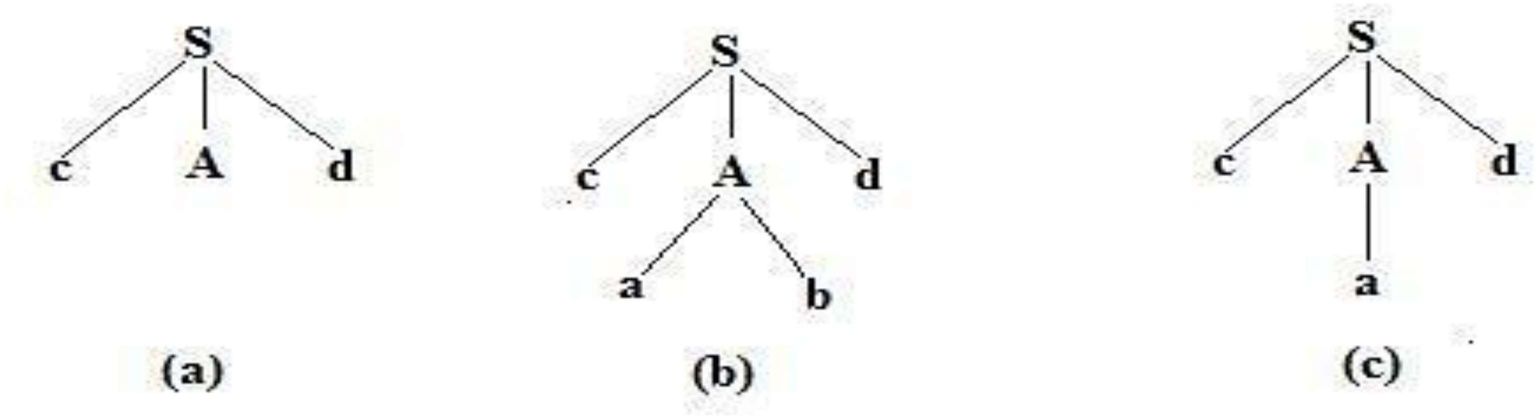


Fig 2.5 Steps in top-down parse

Procedure of Top-down Parsing

- An input pointer points to c, the first symbol of w.
- Then use the first production for S to expand the tree and obtain the tree.
- The leftmost leaf, labeled c, matches the first symbol of w.
- Next input pointer to a, the second symbol of w.
- Consider the next leaf, labeled A.
- Expand A using the first alternative for A to obtain the tree.

Procedure of Top-down Parsing

- Now have a match for the second input symbol. Then advance to the next input pointer d, the third input symbol and compare d against the next leaf, labeled b. Since b does not match d, report failure and go back to A to see whether there is another alternative. (Backtracking takes place).
- If there is another alternative for A, substitute and compare the input symbol with leaf.
- Repeat the step for all the alternatives of A to find a match using backtracking. If match found, then the string is accepted by the grammar. Else report failure.
- A left-recursive grammar can cause a recursive-descent parser, even one with backtracking, to go into an infinite loop.
- As discussed above, an easy way to implement a recursive descent parsing with backtracking is to create a procedure for each non-terminals.

Transition Diagram for Predictive Parsers

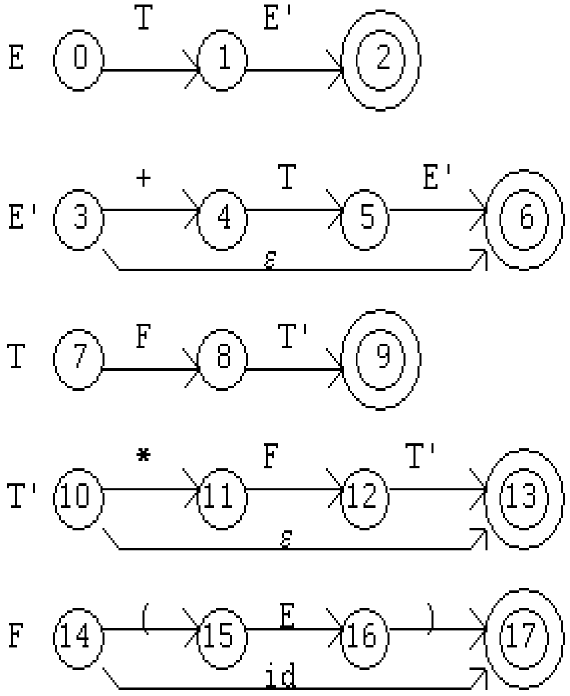
$$E \rightarrow T\,E'$$

$$E' \rightarrow +\,T\,E' \mid \varepsilon.$$

$$T \rightarrow F\,T'$$

$$T' \rightarrow *\,F\,T' \mid \varepsilon.$$

$$F \rightarrow (E) \mid \mathbf{id}$$

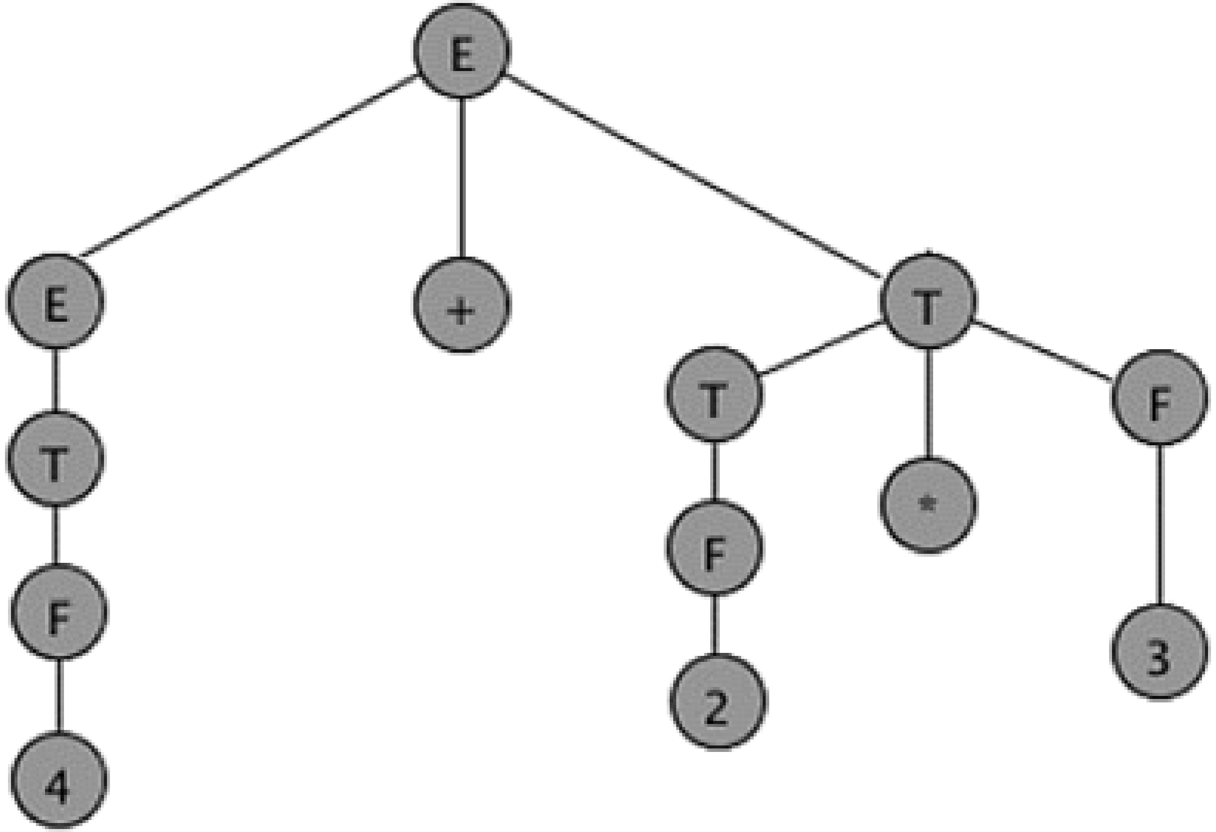


Example: Build the parse tree for the arithmetic expression 4 + 2 * 3 using the expression grammar:

$$E \rightarrow E + T \mid E - T \mid T$$
$$T \rightarrow T * F \mid F$$
$$F \rightarrow a \mid (E)$$

where a represents an operand of some type, be it a number or variable. The trees are grouped by height.

$E \rightarrow E + T \mid E - T \mid T$
$T \rightarrow T * F \mid F$
$F \rightarrow a \mid (E)$



Grammer:

$E \rightarrow E+E \mid a$

Derivation:

$$
\begin{aligned}
E &\Rightarrow E+\check{E} \\
&\Rightarrow \check{E}+E+E \\
&\Rightarrow a+\check{E}+E \\
&\Rightarrow a+a+\check{E} \\
&\Rightarrow a+a+a
\end{aligned}
$$

Example: Build the parse tree for the arithmetic expression "a+a+a" using the expression grammar:

$$E \rightarrow E+E \mid a$$

AMBIGUOUS

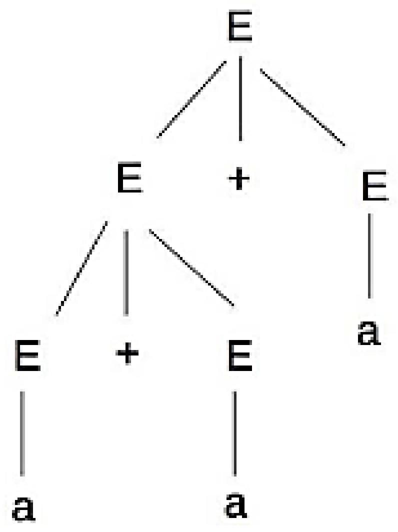
Consider again, the grammar specifying only addition in expression:

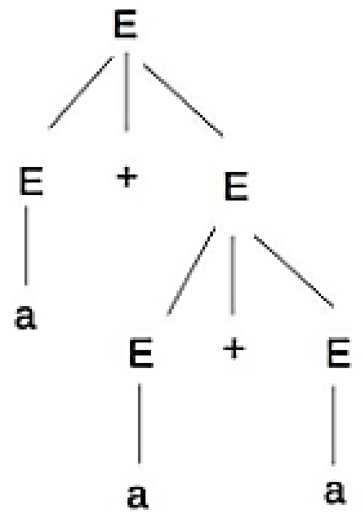
$E \rightarrow E+E \mid a$

Right Most Derivation(LMD):

Left Most Derivation(LMD):

$E \Rightarrow$ Ě+E
$\Rightarrow$ Ě+E+E
$\Rightarrow$ a+Ě+E
$\Rightarrow$ a+a+Ě
$\Rightarrow$ a+a+a





$E \Rightarrow$ E+Ě
$\Rightarrow$ E+E+Ě
$\Rightarrow$ E+Ě+a
$\Rightarrow$ Ě+a+a
$\Rightarrow$ a+a+a

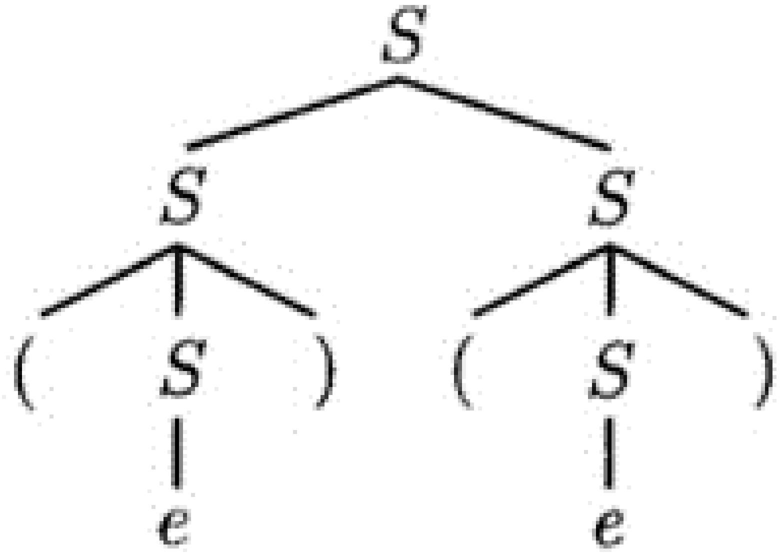
NOT AMBIGUOUS

UNAMBIGUOUS

The grammar is: S → S S | (S) | ε

Consider the string "()()"

S ⇒ Š S
⇒ (S̆) S
⇒ ( ) Š
⇒ ( ) (S̆)
⇒ ( ) ( )

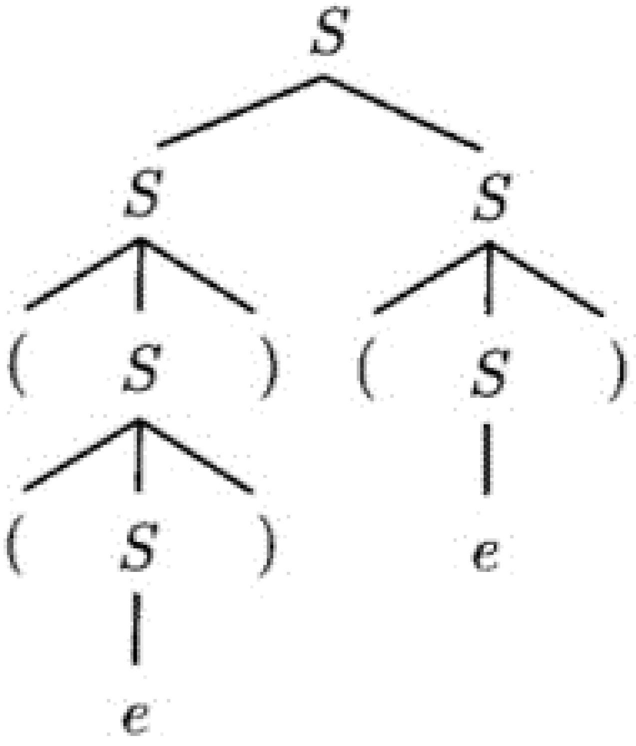


S ⇒ S Š
⇒ S̆ (S)
⇒ (S) (Š)
⇒ (Š) ( )
⇒ ( ) ( )

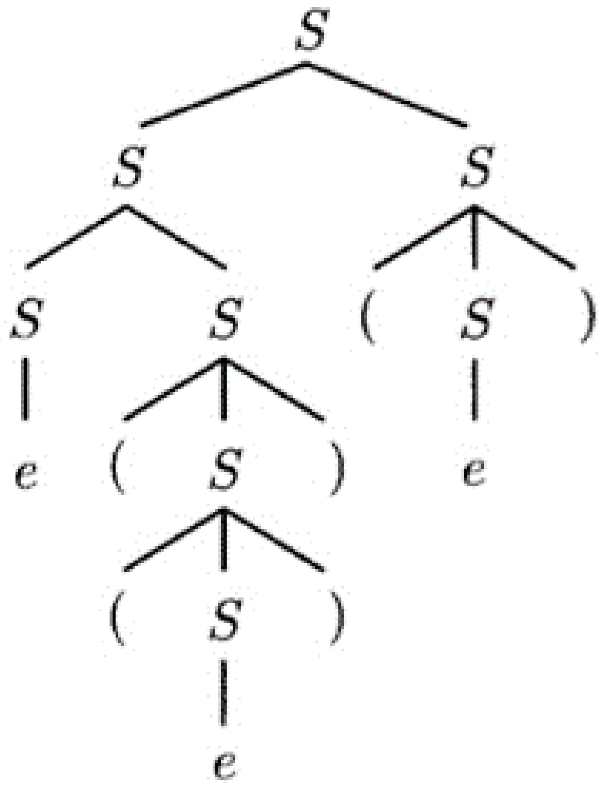
The grammar is: S ⇾ S S | (S) | ε
Consider the string "(())()"

AMBIGUOUS

S ⇒ ŠS
⇒ (Š)S
⇒ ((Š))S
⇒ (())Š
⇒ (())(Š)
⇒ (())()



S ⇒ ŠS
⇒ SŠS
⇒ S(Š)S
⇒ S((Š))S
⇒ S(())Š
⇒ S(())(Š)
⇒ Š(())()
⇒ (())()



THANK YOU