1.6 Momentum

FOCUS POINTS

★ Define momentum, impulse and resultant force and use the correct equations to calculate them.
★ Solve simple one-dimensional problems using the principle of the conservation of momentum.

When a tennis ball is struck by a racket or a gas molecule rebounds from the side of its container, their behaviour can be understood by introducing the concept of momentum. Momentum is defined as the product of mass and velocity. In a collision, momentum is conserved unless there are external forces acting such as friction. You can demonstrate conservation of momentum with a Newton's cradle (Figure 1.7.10, p. 66); the last ball in the line moves off with the same velocity as the first. Collisions generally occur over a very short interval of time; the shorter the time interval the greater the force on the bodies involved in the collision. Crumple zones at the front and rear of a car help to prolong the collision time and reduce the force of an impact.

Momentum is a useful quantity to consider when bodies are involved in collisions and explosions. It is defined as the mass of the body multiplied by its velocity and is measured in kilogram metre per second (kg m/s) or newton second (N s).

$$\text{momentum} = \text{mass} \times \text{velocity}$$

In symbols, momentum $p = mv$ and the change in momentum

$$\Delta p = \Delta(mv)$$

A 2 kg mass moving at 10 m/s has momentum 20 kg m/s, the same as the momentum of a 5 kg mass moving at 4 m/s.

Key definition

Momentum mass × velocity

Practical work

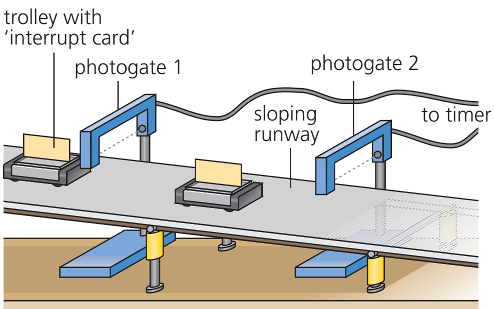
Collisions and momentum

Safety

- Take care when rolling the trolley down the ramp. Ensure it is clear at the bottom of the ramp and use a side barrier to prevent the trolley from falling onto the floor.

Figure 1.6.1 shows an arrangement which can be used to find the velocity of a trolley before and after a collision. If a trolley of length l takes time t to pass through a photogate, then its velocity = distance/time = l/t.

Two photogates are needed, placed each side of the collision point, to find the velocities before and after the collision. Set them up so that they will record the time taken for the passage of a trolley.



▲ **Figure 1.6.1**

A tickertape timer or motion sensor, placed at the top end of the runway, could be used instead of the photogates if preferred.

Attach a strip of Velcro to each trolley so that they adhere to each other on collision and compensate the runway for friction (see Topic 1.5.1). Place one trolley at rest halfway down the runway and another at the top; give the top trolley a push. It will move forwards with uniform velocity and should hit the second trolley so that they travel on as one. Using the times recorded by the photogate timer, calculate the velocity of the moving trolley before the collision and the common velocity of both trolleys after the collision.

Repeat the experiment with another trolley stacked on top of the one to be pushed so that two are moving before the collision and three after.

Copy and complete the tables of results.

Before collision (m_2 at rest)		
Mass m_1 (no. of trolleys)	Velocity v/m/s	Momentum m_1v
1		
2		

After collision (m_1 and m_2 together)		
Mass $m_1 + m_2$ (no. of trolleys)	Velocity v_1/m/s	Momentum $(m_1 + m_2)v_1$
2		
3		

1 Do the results suggest any connection between the momentum before the collision and after it in each case?

2 Why is it necessary to tilt the runway slightly before taking measurements?

3 a Calculate the momentum of a 2 kg trolley moving with a velocity of
  i 0.2 m/s
  ii 0.8 m/s
  iii 5 cm/s.

  b Calculate the momentum of a trolley moving at 3 m/s and having a mass of
  i 200 g
  ii 500 g
  iii 1 kg.

Conservation of momentum

When two or more bodies act on one another, as in a collision, the total momentum of the bodies remains constant, provided no external forces act (e.g. friction).

This statement is called the **principle of conservation of momentum.** Experiments like those in the *Practical work* section show that it is true for all types of collisions.

Worked example

Suppose a truck of mass 60 kg moving with velocity 3 m/s collides and couples with a stationary truck of mass 30 kg (Figure 1.6.2a). The two move off together with the same velocity v which we can find as follows (Figure 1.6.2b).

Total momentum before collision is

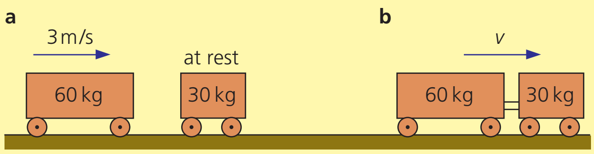
$$(60\,\text{kg} \times 3\,\text{m/s}) + (30\,\text{kg} \times 0\,\text{m/s}) = 180\,\text{kg m/s}$$

Total momentum after collision is

$$(60\,\text{kg} + 30\,\text{kg}) \times v = 90\,\text{kg} \times v$$

Since momentum is not lost

$$90\,\text{kg} \times v = 180\,\text{kg m/s or } v = 2\,\text{m/s}$$



▲ **Figure 1.6.2**

Now put this into practice

1 A trolley of mass 3 kg moving with velocity 5 m/s collides and couples with a stationary trolley of mass 2 kg and the two move off together with the same velocity v. Assuming momentum is not lost in the collision, calculate the value of v.

2 A trolley of mass 5 kg moving with velocity 5 m/s collides with a stationary trolley of mass 2 kg. The 5 kg trolley stops and the 2 kg trolley moves off with velocity v. Assuming momentum is not lost in the collision, calculate the value of v.

Explosions

Momentum, like velocity, is a vector since it has both magnitude and direction. Vectors cannot be added by ordinary addition unless they act in the same direction. If they act in exactly opposite directions, such as east and west, the smaller subtracts from the greater, or if they are the same, they cancel out.

Momentum is conserved in an explosion such as occurs when a rifle is fired. Before firing, the total momentum is zero since both rifle and bullet are at rest. During the firing the rifle and bullet receive *equal* but *opposite* amounts of momentum so that the *total* momentum after firing is zero. For example, if a rifle fires a bullet of mass 0.01 kg with a velocity of 300 m/s,

$$\text{forward momentum of bullet} = 0.01\,\text{kg} \times 300\,\text{m/s} = 3\,\text{kg m/s}$$

$$\therefore \quad \text{backward momentum of rifle} = 3\,\text{kg m/s}$$

If the rifle has mass m, it recoils (kicks back) with a velocity v such that

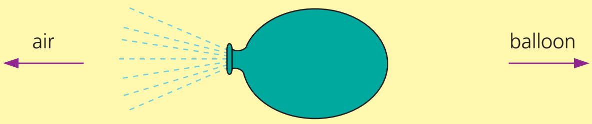
$$mv = 3\,\text{kg m/s}$$

Taking $m = 6$ kg gives $v = 3/6$ m/s $= 0.5$ m/s.

Rockets and jets

If you release an inflated balloon with its neck open, it flies off in the opposite direction to that of the escaping air. In Figure 1.6.3 the air has momentum to the left and the balloon moves to the right with equal momentum.

This is the principle of rockets and jet engines. In both, a high-velocity stream of hot gas is produced by burning fuel and leaves the exhaust with large momentum. The rocket or jet engine itself acquires an equal forward momentum. Space rockets carry their own oxygen supply; jet engines use the surrounding air.



▲ **Figure 1.6.3** A deflating balloon demonstrates the principle of a rocket or a jet engine.

Test yourself

1. What is the momentum in kg m/s of a 10 kg truck travelling at
   a. 5 m/s
   b. 20 cm/s
   c. 36 km/h?
2. A ball X of mass 1 kg travelling at 2 m/s has a head-on collision with an identical ball Y at rest. X stops and Y moves off. What is Y's velocity?
3. A boy with mass 50 kg running at 5 m/s jumps on to a 20 kg trolley travelling in the same direction at 1.5 m/s. What is their common velocity?
4. A girl of mass 50 kg jumps out of a rowing boat of mass 300 kg on to the bank, with a horizontal velocity of 3 m/s. With what velocity does the boat begin to move backwards?

Force and momentum

If a steady force F acting on an object of mass m increases its velocity from u to v in time Δt, the acceleration a is given by

$$a = (v - u)/\Delta t$$

Substituting for a in $F = ma$,

$$F = \frac{m(v-u)}{\Delta t} = \frac{mv - mu}{\Delta t}$$

We also have

$$\text{impulse} = F\Delta t = mv - mu = \Delta(mv)$$

where mv is the final momentum, mu the initial momentum and $F\Delta t$ is called the **impulse**.

Since $\quad F\Delta t = mv - mu = \Delta(mv)$

We can write $\quad F = \dfrac{\Delta(mv)}{\Delta t} = \dfrac{\Delta p}{\Delta t}$

and define the **resultant force** F as the change in momentum per unit time.

This is another version of Newton's second law. For some problems it is more useful than $F = ma$.

Key definitions

Impulse force × time for which force acts

Resultant force the rate of change in momentum per unit time

Sport: impulse and collision time

The good cricketer or tennis player 'follows through' with the bat or racket when striking the ball (Figure 1.6.4a). The force applied then acts for a longer time, the impulse is greater and so also is the gain of momentum (and velocity) of the ball.

When we want to stop a moving ball such as a cricket ball, however, its momentum has to be reduced to zero. An impulse is then required in the form of an opposing force acting for a certain time. While any number of combinations of force and time will give a particular impulse, the 'sting' can be removed from the catch by drawing back the hands as the ball is caught (Figure 1.6.4b). A smaller average force is then applied for a longer time.

▲ **Figure 1.6.4a** This cricketer is 'following through' after hitting the ball.

▲ **Figure 1.6.4b** Cricketer drawing back the hands to catch the ball

The use of sand gives a softer landing for long-jumpers (Figure 1.6.5), as a smaller stopping force is applied over a longer time. In a car crash the car's momentum is reduced to zero in a very short time. If the time of impact can be extended by using **crumple zones** (see Figure 1.7.11, p. 66) and extensible seat belts, the average force needed to stop the car is reduced so the injury to passengers should also be less.

▲ **Figure 1.6.5** Sand reduces the athlete's momentum more gently.

Test yourself

5 A force of 5 N is applied to a cricket ball for 0.02 s. Calculate
   a the impulse on the ball
   b the change in momentum of the ball.

6 In a collision, a car of mass 1000 kg travelling at 24 m/s comes to rest in 1.2 s. Calculate
   a the change in momentum of the car
   b the steady stopping force applied to the car.

Revision checklist

After studying Topic 1.6 you should know and understand the following:

✔ the relationship between force and rate of change of momentum and use it to solve problems.

After studying Topic 1.6 you should be able to:

✔ define momentum and apply the principle of conservation of momentum to solve problems

✔ recall that in a collision, impulse = $F\Delta t$ and use the definition to explain how the time of impact affects the force acting in a collision.

Exam-style questions

1 A truck A of mass 500 kg moving at 4 m/s collides with another truck B of mass 1500 kg moving in the same direction at 2 m/s.

a Write down an expression for momentum. [1]

b Calculate the momentum of truck A before the collision. [2]

c Calculate the momentum of truck B before the collision. [2]

d Determine the common velocity of the trucks after the collision. [4]

[Total: 9]

2 The velocity of an object of mass 10 kg increases from 4 m/s to 8 m/s when a force acts on it for 2 s. Write down the

a initial momentum of the object [2]

b final momentum of the object [2]

c momentum gained in 2 s [2]

d value of the force [2]

e impulse of the force. [2]

[Total: 10]

3 A rocket of mass 10 000 kg uses 5.0 kg of fuel and oxygen to produce exhaust gases ejected at 5000 m/s.

a Define momentum. [1]

b Calculate the backward momentum of the ejected gas. [2]

c Explain what is meant by the principle of conservation of momentum. [2]

d Calculate the increase in velocity of the rocket. [3]

[Total: 8]

4 A boy hits a stationary billiard ball of mass 30 g head on with a cue. The cue is in contact with the ball for a time of 0.001 s and exerts a force of 50 N on it.

a Calculate the acceleration of the ball during the time it is in contact with the cue. [2]

b Work out the impulse on the ball in the direction of the force. [2]

c Calculate the velocity of the ball just after it is struck. [2]

d Give *two* ways by which the velocity of the ball could be increased. [2]

[Total: 8]