10. $\dfrac{0.482 + 1.6}{0.024 \times 1.83}$

11. $\dfrac{8.52 - 1.004}{0.004 - 0.0083}$

12. $\dfrac{1.6 - 0.476}{2.398 \times 41.2}$

13. $\left(\dfrac{2.3}{0.791}\right)^7$

14. $\left(\dfrac{8.4}{28.7 - 0.47}\right)^3$

15. $\left(\dfrac{5.114}{7.332}\right)^5$

16. $\left(\dfrac{4.2}{2.3} + \dfrac{8.2}{0.52}\right)^3$

17. $\dfrac{1}{8.2^2} - \dfrac{3}{19^2}$

18. $\dfrac{100}{11^3} + \dfrac{100}{12^3}$

19. $\dfrac{7.3 - 4.291}{2.6^2}$

20. $\dfrac{9.001 - 8.97}{0.95^3}$

21. $\dfrac{10.1^2 + 9.4^2}{9.8}$

22. $(3.6 \times 10^{-8})^2$

23. $(8.24 \times 10^4)^3$

24. $(2.17 \times 10^{-3})^3$

25. $(7.095 \times 10^{-6})^{\frac{1}{3}}$

26. $\sqrt[3]{\left(\dfrac{4.7}{2.3^2}\right)}$

Checking answers

Here are five calculations, followed by sensible checks.

a) $22.2 \boxed{\div} 6 = 3.7$ check $3.7 \boxed{\times} 6 = 22.2$

b) $31.7 \boxed{-} 4.83 = 26.87$ check $26.87 \boxed{+} 4.83 = 31.7$

c) $42.8 \boxed{\times} 30 = 1284$ check $1284 \boxed{\div} 30 = 42.8$

d) $\sqrt{17} = 4.1231$ check 4.1231^2

e) $3.7 + 17.6 + 13.9$ check $13.9 + 17.6 + 3.7$ (add in reverse order)

Calculations can also be checked by rounding numbers to a given number of significant figures.

f) $\dfrac{6.1 \times 32.6}{19.3} = 10.3\ (\text{to 3 s.f.})$

Check this answer by rounding each number to one significant figure and estimating.

$$\frac{6.1 \times 32.6}{19.3} \approx \frac{6 \times 30}{20} = \frac{180}{20} = 9$$

This is close to 10.3
so the actual answer probably is 10.3

Tip

'≈' means 'approximately equal to'

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Exercise 1.9C

1. Use a calculator to work out the following then check the answers as indicated.

a) $92.5 \times 20 = \square$ Check $\square \div 20 = \square$

b) $14 \times 328 = \square$ Check $\square \div 328 = \square$

c) $63 - 12.6 = \square$ Check $\square + 12.6 = \square$

d) $221.2 \div 7 = \square$ Check $\square \times 7 = \square$

e) $384.93 \div 9.1 = \square$ Check $\square \times 9.1 = \square$

f) $13.71 + 25.8 = \square$ Check $\square - 25.8 = \square$

g) $95.4 \div 4.5 = \square$ Check $\square \times 4.5 = \square$

h) $8.2 + 3.1 + 19.6 + 11.5 = \square$ Check $11.5 + 19.6 + 3.1 + 8.2 = \square$

i) $\sqrt{39} = \square$ Check $\square^2 = 39$

j) $3.17 + 2.06 + 8.4 + 16 = \square$ Check $16 + 8.4 + 2.06 + 3.17 = \square$

2. The numbers below are rounded to one significant figure to *estimate* the answer to each calculation. Match each question below to the correct estimated answer.

A $\boxed{21.9 \times 1.01}$ P $\boxed{10}$

B $\boxed{\dfrac{19.82^2}{(18.61 + 22.3)}}$ Q $\boxed{5}$

C $\boxed{7.8 \times 1.01}$ R $\boxed{0.5}$

D $\boxed{\dfrac{\sqrt{98.7}}{8.78 + 11.43}}$ S $\boxed{8}$

E $\boxed{\dfrac{21.42 + 28.6}{18.84 - 8.99}}$ T $\boxed{20}$

3. Given that $281 \times 36 = 10\,116$, work out the following without using a calculator:

a) $10\,116 \div 36$ **b)** $10\,116 \div 281$ **c)** 28.1×3.6

4. Mavis is paid a salary of \$49 620 per year.
Work out a rough estimate for her weekly pay.
Give your answer correct to one significant figure.

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5. In 2011, the population of France was 61 278 514 and the population of Greece was 9 815 972. Roughly how many times bigger was the population of France compared to the population of Greece?

Tip

Round the numbers to one significant figure.

6. Estimate, correct to one significant figure:

a) $41.56 \div 7.88$

b) $\dfrac{5.13 \times 18.777}{0.952}$

c) $\dfrac{1}{5}$ of £14892

d) $\dfrac{0.0974 \times \sqrt{104}}{1.03}$

e) $\dfrac{6.84^2 + 0.983}{5.07^2}$

f) $\dfrac{2848.7 + 1024.8}{51.2 - 9.98}$

g) $\dfrac{2}{3}$ of £3124

h) $18.13 \times (3.96^2 + 2.07^2)$

Time and money

You can use your calculator to work out problems involving time and money. Money can just be treated as a decimal written correct to 2 decimal places.

When working with time, the main thing is to remember that there are 60 minutes in an hour, not 100. This means that 2 hours and 25 minutes is not written as 2.25 hours, since 2.25 hours means $2\frac{1}{4}$ hours, which is 2 hours and 15 minutes.

If you did want to enter 2 hours and 25 minutes into your calculator, the easiest way to do this would be to use the 'hours, minutes and seconds' button, which usually looks like $\boxed{\circ\ '\ ''}$.

To enter 2 hours and 25 minutes, press 2 $\boxed{\circ\ '\ ''}$ 25 $\boxed{\circ\ '\ ''}$.

If the time you want to enter also contains seconds, you can press the button again.

For example, to enter 3 hours 14 minutes and 8 seconds you would press 3 $\boxed{\circ\ '\ ''}$ 14 $\boxed{\circ\ '\ ''}$ 8 $\boxed{\circ\ '\ ''}$.

If you then want to convert that time into a decimal, you can do this using the $\boxed{S \Leftrightarrow D}$ button.

The $\boxed{\circ\ '\ ''}$ button can also be used in reverse to convert a decimal into hours, minutes and seconds. For example, if you type in 2.18 and press $\boxed{\circ\ '\ ''}$, the calculator will display 2˚10'48", which means that 2.18 hours is the same as 2 hours, 10 minutes and 48 seconds.

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Example

How many hours, minutes and seconds is 3.65 hours?

3.65 $\boxed{\circ\ ,\ ,,}$ $\boxed{=}$ 3°36’36”

So 3.65 hours is 3 hours 36 minutes and 36 seconds.

Exercise 1.9D

1. Convert into a decimal number of hours:

a) 2 hours 13 minutes and 6 seconds **b)** 5 hours 42 minutes and 38 seconds

c) 1 hour 6 minutes and 50 seconds

2. Convert into hours, minutes and seconds:

a) 2.64 hours **b)** 3.88 hours **c)** 8.29 hours

3. Work out in hours, minutes and seconds:

a) 1.54 hours + 2.83 hours **b)** 9.82 hours + 4.72 hours

4. How many cents are there in 5.3 dollars?

5. Eric buys 34 cakes at $1.42 each and 28 drinks at $0.89 each.
How much did he spend in total?

6. Eight people share a prize of $215 000 between them.
How much do they each get?

7. Four people split a restaurant bill of $34.40 between them.
How much do they each have to pay?

8. Three singers book 6.75 hours of studio time that they share equally.
How many hours and minutes do they each get?

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Revision exercise 1

You may use a calculator only for Questions **15** and **16**.

1. Evaluate:

a) $148 \div 0.8$ **b)** $0.024 \div 0.000\,16$

c) $(0.2)^2 \div (0.1)^3$ **d)** $2 - \frac{1}{2} - \frac{1}{3} - \frac{1}{4}$

e) $1\frac{3}{4} \times 1\frac{3}{5}$ **f)** $\dfrac{1\frac{1}{6}}{1\frac{2}{3} + 1\frac{1}{4}}$

2. a) $-7 + 24$ **b)** $+5 - +18$

c) $+14 + -9$ **d)** $-20 - -10$

3. a) $-4 \times (+9)$ **b)** $-3 \times (-7)$

c) $+70 \div (-7)$ **d)** $-45 \div (-9)$

4. On each bounce, a ball rises to $\frac{4}{5}$ of its previous height. To what height will it rise after the third bounce, if dropped from a height of 250 cm?

5. A man spends $\frac{1}{3}$ of his salary on accommodation and $\frac{2}{5}$ of the remainder on food.
What fraction is left for other purposes?

6. Express 0.054 73:

a) correct to three significant figures

b) correct to three decimal places

c) in standard form.

7. Evaluate $\frac{2}{3} + \frac{4}{7}$, to three decimal places.

8. Evaluate:

a) $121^{\frac{1}{2}}$ **b)** 7^{-1}

c) $9^{-\frac{1}{2}}$ **d)** $16^{\frac{3}{2}}$

e) $\left(4^{\frac{1}{2}}\right)^3 \times 36^{\frac{1}{2}}$ **f)** 12^0

9. Evaluate, giving the answers in standard form:

a) $3600 \div 0.00012$ **b)** $\dfrac{3.33 \times 10^4}{9 \times 10^{-1}}$

c) $(30\,000)^3$

10. Given that

$$t = 2\pi\sqrt{\left(\frac{l}{g}\right)}$$

find the value of t, to three significant figures, when $l = 2.31$ and $g = 9.81$

11. a) From the following numbers, write down:

1 3 8 9 10

i) the prime number

ii) a multiple of 5

iii) two square numbers

iv) two factors of 32.

b) Find two numbers m and n from the list such that $m = \sqrt{n}$ and $n = \sqrt{81}$.

c) If each of the numbers in the list can be used once, find p, q, r, s, t such that $(p + q)r = 2(s + t) = 36$.

12. The value of t is given by

$$t = 2\pi\sqrt{\left(\frac{2.31^2 + 0.9^2}{2.31 \times 9.81}\right)}.$$

Without using a calculator, and using suitable approximate values for the numbers in the formula, find an estimate for the value of t. (To earn the marks in this question you must show the various stages of your working.)

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13. Baichu's heart has beat at an average rate of 72 beats per minute throughout his life. Baichu is sixty years old. How many times has his heart beat during his life? Give the answer in standard form correct to 2 significant figures.

14. Estimate giving the answers to 1 significant figure. Do not use a calculator.

a) $(612 \times 52) \div 49.2$

b) $(11.7 + 997.1) \times 9.2$

c) $\sqrt{\left(\dfrac{91.3}{10.1}\right)}$

d) $\pi\sqrt{5.2^2 + 18.2}$

15. Evaluate giving the answers to 4 significant figures:

a) $\dfrac{0.74}{0.81 \times 1.631}$ **b)** $\sqrt{\left(\dfrac{9.61}{8.34 - 7.41}\right)}$

c) $\left(\dfrac{0.741}{0.8364}\right)^4$ **d)** $\dfrac{8.4 - 7.642}{3.333 - 1.735}$

16. Evaluate giving the answers to 3 significant figures:

a) $\sqrt[3]{(9.61 \times 0.0041)}$ **b)** $\left(\dfrac{1}{9.5} - \dfrac{1}{11.2}\right)^3$

c) $\dfrac{15.6 \times 0.714}{0.0143 \times 12}$ **d)** $\sqrt[4]{\left(\dfrac{1}{5 \times 10^3}\right)}$

17. Write down the reciprocals of:

a) 20 **b)** $\dfrac{1}{3}$ **c)** 0.4 **d)** 1.5

18. The number of cells in a bacterial culture doubles every hour.

At the end of the first hour there are 2^6 cells. How many cells will there be at the end of the third hour?

19. Rationalise the denominators of these fractions.

a) $\dfrac{1}{\sqrt{5}}$ **b)** $\dfrac{2}{1 + \sqrt{5}}$ **c)** $\dfrac{\sqrt{2} + 1}{\sqrt{2} - 1}$

20. Write down the nth term formulae for the following sequences:

a) $-1, 3, 7, 11, \ldots$

b) $2, 7, 14, 23, \ldots$

c) $3, 10, 29, 66, \ldots$

d) $4, 16, 64, 256, \ldots$

21. The width of a particular book is 2 cm when measured to the nearest cm.

Zac needs to build a bookshelf to hold 50 of these books.

What are the upper and lower bounds for the length of the shelf?

22. Convert into decimals:

a) 43% **b)** 20% **c)** 3%

d) 12.5% **e)** 115%

23. Convert into decimals:

a) $\dfrac{1}{4}$ **b)** $\dfrac{2}{5}$ **c)** $\dfrac{7}{8}$ **d)** $\dfrac{3}{7}$ **e)** $\dfrac{15}{6}$

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Examination-style exercise 1

NON-CALCULATOR SECTION

1. Calculate $\frac{3^2}{2^6}$

a) giving your answer as a fraction [1]

b) giving your answer as a decimal. [1]

2. Work out the exact value of

$$1 + \frac{2}{4 + \frac{8}{16 + 32}}$$ [2]

3. Write down:

a) an irrational number between 1 and 2 [1]

b) a prime number between 70 and 80. [1]

4. At 07:20 Mrs Smith bought 150 bagels at a retail shop for 54 cents each. 155 minutes later she sold them all to a supermarket for 85 cents each.

a) What was the time when she sold the bagels? [1]

b) Calculate her total profit. [1]

5. Write down the next term in each of the following sequences.

a) 12.4, 9.4, 6.4, 3.4, 0.4, ... [1]

b) 3, 5, 9, 15, 23, ... [1]

6. a) The formula for the nth term of the sequence 2, 15, 48, 110, 210, ... is

$$\frac{n(n+1)(3n-1)}{2}$$

Find the 9th term. [1]

b) The nth term of the sequence 12, 19, 28, 39, 52, ... is $(n+2)^2 + 3$

Write down the formula for the nth term of the sequence 19, 26, 35, 46, 59, ... [1]

7. To raise money for charity, Julie walks 48 km, correct to the nearest kilometre, every day for 5 days.

Copy and complete the statement for the distance, d km, she walks during those 5 days: .... $\leqslant d$ km $<$ .... [1]

8. The distance between London and Chicago is 3900 km correct to the nearest 100 km.

A businessman travelled from London to Chicago and then back to London.

He did this four times in a year.

Between what limits is the total distance he travelled?

Write your answer as ....km $\leqslant$ total distance travelled $<$ ....km. [2]

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9. A rectangle has sides of length 8.6 cm and 4.3 cm correct to one decimal place.
Calculate the lower bound for the area of the rectangle as accurately as possible. [2]

10. Write $\frac{\sqrt{2}+1}{\sqrt{8}-2}$ as $\frac{a}{b}+\sqrt{c}$, where a, b and c are integers. [2]

CALCULATOR SECTION

11. In 2021 there were 58 thousand taxis in London, correct to the nearest thousand.
If the average distance travelled by each taxi in one day was 120 km correct to two significant figures, work out the upper bound for the total distance travelled by all the taxis in one day, correct to the nearest million km. [2]

12. The mass of the Earth is roughly $\frac{1}{320}$ of the mass of the planet Jupiter.
The mass of the Earth is 5.97×10^{24} kilograms. Calculate the mass of the planet Jupiter, giving your answer in standard form, correct to 2 significant figures. [3]

13. Use your calculator to work out $\sqrt{(9 + 5 \times 184^{0.1})}$ [1]

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2 Algebra 1

Isaac Newton (1642–1727) was an English scientist and mathematician, and a prominent figure in the Scientific Revolution of the 17th century. He went to Trinity College Cambridge in 1661 and by the age of 23 he had made three major discoveries: the nature of colours, calculus and the law of gravitation. He used his version of calculus to give the first satisfactory explanation of the motion of the Sun, the Moon and the stars. Because he was extremely sensitive to criticism, Newton was always very secretive, but he was eventually persuaded to publish his discoveries in 1687.

- Substitute into expressions and formulae.
- Simplify expressions and expand brackets.
- Construct and solving linear equations including those where x appears in the denominator as part of a linear expression.
- Solve simultaneous equations.

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2.1 Substitution

In algebra, letters are used to represent numbers. These letters are called *variables*.

Mathematical expressions are made up of one or more terms and operations. A term may be a number, a variable or a combination of both. The expression $5x^2 - 6x + 7$ has three terms:

$5x^2$, $-6x$ and 7

You can evaluate an expression by replacing the variables in the expression with specific values. This is called *substitution*.

For example, when $x = -1$, the expression $5x^2 - 6x + 7$ is evaluated:

$$\begin{aligned} 5(-1)^2 - 6(-1) + 7 &= 5 \times 1 + 6 + 7 \\ &= 18 \end{aligned}$$

Example

When $a = 3$, $b = -2$, and $c = 5$, find the value of:

a) $3a + b$ **b)** $ac + b^2$ **c)** $\dfrac{a+c}{b}$ **d)** $a(c - b)$

a) $$\begin{aligned} 3a + b &= (3 \times 3) + (-2) \\ &= 9 - 2 \\ &= 7 \end{aligned}$$

b) $$\begin{aligned} ac + b^2 &= (3 \times 5) + (-2)^2 \\ &= 15 + 4 \\ &= 19 \end{aligned}$$

c) $$\begin{aligned} \frac{a+c}{b} &= \frac{3+5}{-2} \\ &= \frac{8}{-2} \\ &= -4 \end{aligned}$$

d) $$\begin{aligned} a(c - b) &= 3[5 - (-2)] \\ &= 3(7) \\ &= 21 \end{aligned}$$

Note that working *down* the page makes the steps easy to read and easy to follow.

Tip

When substituting, remember to always use BIDMAS.

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Exercise 2.1A

Evaluate the following.

For Questions **1** to **12**, $a = 3$, $c = 2$, $e = 5$.

1. $3a - 2$ **2.** $4c + e$ **3.** $2c + 3a$ **4.** $5e - a$

5. $e - 2c$ **6.** $e - 2a$ **7.** $4c + 2e$ **8.** $7a - 5e$

9. $c - e$ **10.** $10a + c + e$ **11.** $a + c - e$ **12.** $a - c - e$

For Questions **13** to **24**, $h = 3$, $m = -2$, $t = -3$.

13. $2m - 3$ **14.** $4t + 10$ **15.** $3h - 12$ **16.** $6m + 4$

17. $9t - 3$ **18.** $4h + 4$ **19.** $2m - 6$ **20.** $m + 2$

21. $3h + m$ **22.** $t - h$ **23.** $4m + 2h$ **24.** $3t - m$

For Questions **25** to **36**, $x = -2$, $y = -1$, $k = 0$.

25. $3x + 1$ **26.** $2y + 5$ **27.** $6k + 4$ **28.** $3x + 2y$

29. $2k + x$ **30.** xy **31.** xk **32.** $2xy$

33. $2(x + k)$ **34.** $3(k + y)$ **35.** $5x - y$ **36.** $3k - 2x$

Tip

$2x^2$ means $2(x^2)$

$(2x)^2$ means 'work out $2x$ and *then* square it'

$-7x$ means $-7(x)$

$-x^2$ means $-(x^2)$

Example

When $x = -2$, find the value of:

a) $2x^2 - 5x$ **b)** $(3x)^2 - x^2$

a) $2x^2 - 5x = 2(-2)^2 - 5(-2)$
$= 2(4) + 10$
$= 18$

b) $(3x)^2 - x^2 = (3 \times -2)^2 - 1(-2)^2$
$= (-6)^2 - 1(4)$
$= 36 - 4$
$= 32$

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Exercise 2.1B

If $x = -3$ and $y = 2$, evaluate:

1. x^2	**2.** $3x^2$	**3.** y^2	**4.** $4y^2$
5. $(2x)^2$	**6.** $2x^2$	**7.** $10 - x^2$	**8.** $10 - y^2$
9. $20 - 2x^2$	**10.** $20 - 3y^2$	**11.** $5 + 4x$	**12.** $x^2 - 2x$
13. $y^2 - 3x^2$	**14.** $x^2 - 3y$	**15.** $(2x)^2 - y^2$	**16.** $4x^2$
17. $(4x)^2$	**18.** $1 - x^2$	**19.** $y - x^2$	**20.** $x^2 + y^2$
21. $x^2 - y^2$	**22.** $2 - 2x^2$	**23.** $(3x)^2 + 3$	**24.** $11 - xy$
25. $12 + xy$	**26.** $(2x)^2 - (3y)^2$	**27.** $2 - 3x^2$	**28.** $y^2 - x^2$
29. $x^2 + y^3$	**30.** $\dfrac{x}{y}$	**31.** $10 - 3x$	**32.** $2y^2$
33. $25 - 3y$	**34.** $(2y)^2$	**35.** $-7 + 3x$	**36.** $-8 + 10y$
37. $(xy)^2$	**38.** xy^2	**39.** $-7 + x^2$	**40.** $17 + xy$
41. $-5 - 2x^2$	**42.** $10 - (2x)^2$	**43.** $x^2 + 3x + 5$	**44.** $2x^2 - 4x + 1$
45. $\dfrac{x^2}{y}$			

Example

When $a = -2$, $b = 3$, $c = -3$, evaluate:

a) $\dfrac{2a\,(b^2 - a)}{c}$ **b)** $\sqrt{(a^2 + b^2)}$

a) $(b^2 - a) = 9 - (-2)$

$= 11$

$$\therefore \frac{2a\,(b^2 - a)}{c} = \frac{2 \times (-2) \times (11)}{-3}$$

$$= \frac{-44}{-3}$$

$$= \frac{44}{3}$$

$$= 14\frac{2}{3}$$

b) $\sqrt{(a^2 + b^2)} = \sqrt{(-2)^2 + (3)^2}$

$= \sqrt{4 + 9}$

$= \sqrt{13}$

Tip

In mathematics, the $\therefore$ symbol means 'therefore'.

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Exercise 2.1C

Evaluate the following expressions.

For Questions **1** to **16**, $a = 4,\ b = -2,\ c = -3$.

1. $a(b + c)$ **2.** $a^2\,(b - c)$ **3.** $2c(a - c)$ **4.** $b^2\,(2a + 3c)$

5. $c^2(b - 2a)$ **6.** $2a^2(b + c)$ **7.** $2(a + b + c)$ **8.** $3c(a - b - c)$

9. $b^2 + 2b + a$ **10.** $c^2 - 3c + a$ **11.** $2b^2 - 3b$ **12.** $\sqrt{\left(a^2 + c^2\right)}$

13. $\sqrt{\left(ab + c^2\right)}$ **14.** $\sqrt{\left(c^2 - b^2\right)}$ **15.** $\dfrac{b^2}{a} + \dfrac{2c}{b}$ **16.** $\dfrac{c^2}{b} + \dfrac{4b}{a}$

For Questions **17** to **32**, $k = -3,\ m = 1,\ n = -4$.

17. $k^2(2m - n)$ **18.** $5m\sqrt{\left(k^2 + n^2\right)}$ **19.** $\sqrt{(kn + 4m)}$

20. $kmn(k^2 + m^2 + n^2)$ **21.** $k^2m^2(m - n)$ **22.** $k^2 - 3k + 4$

23. $m^3 + m^2 + n^2 + n$ **24.** $k^3 + 3k$ **25.** $m(k^2 - n^2)$

26. $m\sqrt{(k - n)}$ **27.** $100k^2 + m$ **28.** $m^2(2k^2 - 3n^2)$

29. $\dfrac{2k + m}{k - n}$ **30.** $\dfrac{kn - k}{2m}$ **31.** $\dfrac{3k + 2m}{2n - 3k}$

32. $\dfrac{k + m + n}{k^2 + m^2 + n^2}$

For Questions **33** to **48**, $w = -2,\ x = 3,\ y = 0,\ z = -\dfrac{1}{2}$

33. $\dfrac{w}{z} + x$ **34.** $\dfrac{w + x}{z}$ **35.** $y\left(\dfrac{x + z}{w}\right)$ **36.** $x^2\,(z + wy)$

37. $x\sqrt{\left(x + wz\right)}$ **38.** $w^2\sqrt{\left(z^2 + y^2\right)}$ **39.** $2(w^2 + x^2 + y^2)$ **40.** $2x(w - z)$

41. $\dfrac{z}{w} + x$ **42.** $\dfrac{z + w}{x}$ **43.** $\dfrac{x + w}{z^2}$ **44.** $\dfrac{y^2 - w^2}{xz}$

45. $z^2 + 4z + 5$ **46.** $\dfrac{1}{w} + \dfrac{1}{z} + \dfrac{1}{x}$ **47.** $\dfrac{4}{z} + \dfrac{10}{w}$ **48.** $\dfrac{yz - xw}{xz - w}$

49. Find $K = \sqrt{\left(\dfrac{a^2 + b^2 + c^2 - 2c}{a^2 + b^2 + 4c}\right)}$ when $a = 3,\ b = -2,\ c = -1$.

50. Find $W = \dfrac{kmn(k + m + n)}{(k + m)(k + n)}$ when $k = \dfrac{1}{2},\ m = -\dfrac{1}{3},\ n = \dfrac{1}{4}$

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When a calculation is repeated many times, it is often helpful to use a formula. An example of a scientific formula is the formula for converting between degrees Celsius and degrees Fahrenheit. An example of a mathematical formula is the one for calculating the volume of a sphere.

Example 1

Use the formula $F = \frac{9}{5}C + 3$ to convert 45 °C to degrees Fahrenheit.

If $C = 45$, then $F = \frac{9}{5} \times 45 + 32\ 113$ °F.

Tip

Rearranging the formula to convert degrees Fahrenheit to degrees Celsius will be covered in Chapter 8: Changing the subject of a formula.

Example 2

Use the formula $V = \frac{4}{3}\pi r^3$ to calculate the volume of a sphere with diameter 12 cm.

Leave your answer in terms of π.

The diameter is 12 cm, so the radius is 6 cm.

So $V = \frac{4}{3}\pi \times 6^3 = 288\pi\ \text{cm}^3$

Exercise 2.1D

1. The final speed v of a car is given by the formula $v = u + at$.

   [u = initial speed, a = acceleration, t = time taken]

   Find v when $u = 15$ m/s, $a = 0.2$ m/s^2, $t = 30$ s.

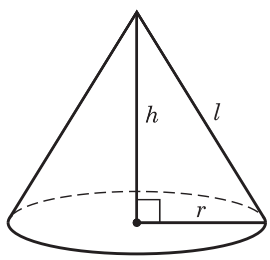
2. The period T of a simple pendulum is given by the formula $T = 2\pi\sqrt{\left(\frac{l}{g}\right)}$, where l is the length of the pendulum and g is the gravitational acceleration. Find T when $l = 0.65$ m, $g = 9.81$ m/s^2 and $\pi = 3.142$.

3. The total surface area A of a cone is related to the radius r and the slant height l by the formula $A = \pi r(r + l)$. Find A when $r = 7$ cm and $l = 11$ cm.

4. The sum S of the squares of the integers from 1 to n is given by $S = \frac{1}{6}n\ (n + 1)\ (2n + 1)$. Find S when $n = 12$.

Tip

The period of a pendulum is the time it takes to complete one full cycle: a left swing and a right swing.



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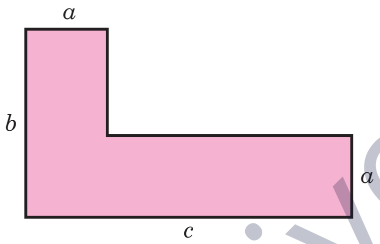
5. The acceleration a of a train is found using the formula
$a = \frac{v^2 - u^2}{2s}$.
Find a when $v = 20$ m/s, $u = 9$ m/s and $s = 2.5$ m.

6. Einstein's famous equation relating energy, mass and the speed of light is $E = mc^2$.
Find E when $m = 0.0001$ kg and $c = 3 \times 10^8$ m/s.

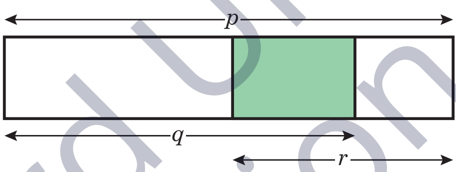
7. The distance s travelled by an accelerating rocket is given by $s = ut + \frac{1}{2}at^2$.
Find s when $u = 3$ m/s, $t = 100$ s and $a = 0.1$ m/s^2.

8. Find a formula for the area of the shape below, in terms of a, b and c.



Tip
You can find out more about area in Chapter 5.

9. Find a formula for the length of the shaded part below, in terms of p, q and r.



2.2 Brackets and simplifying

A term outside a pair of brackets multiplies each of the terms inside the brackets. This is the *distributive law*.

Example 1

$3(x - 2y) = 3x - 6y$

Example 2

$2x(x - 2y + z) = 2x^2 - 4xy + 2xz$

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Example 3

$7y - 4(2x - 3) = 7y - 8x + 12$

In general, like terms can be added:

x terms can be added to x terms

y terms can be added to y terms

x^2 terms can be added to x^2 terms

But they must not be mixed.

Example 4

$$2x + 3y + 3x^2 + 2y - x = 2x - x + 3y + 2y + 3x^2$$
$$= x + 5y + 3x^2$$

You can rearrange the expression to group together like terms.

Example 5

$$7x + 3x(2x - 3) = 7x + 6x^2 - 9x$$
$$= 6x^2 - 2x$$

Exercise 2.2A

Simplify these expressions as far as possible.

1. $3x + 4y + 7y$

2. $4a + 7b - 2a + b$

3. $3x - 2y + 4y$

4. $2x + 3x + 5$

5. $7 - 3x + 2 + 4x$

6. $5 - 3y - 6y - 2$

7. $5x + 2y - 4y - x^2$

8. $x^2 - 2 + 3x + x^2 + 7$

9. $2x - 7y - 2x - 3y$

10. $4a + 3a^2 - 2a$

11. $1 + 7a - 8a^2 + 6 + a^2$

12. $x^2 + 3x^2 - 4x^2 + 5x$

13. $\frac{3}{a} + b + \frac{7}{a} - 2b$

14. $\frac{4}{x} - \frac{7}{y} + \frac{1}{x} + \frac{2}{y}$

15. $\frac{m}{x} + \frac{2m}{x}$

16. $\frac{5}{x} - \frac{7}{x} + \frac{1}{2}$

17. $\frac{3}{a} + b + \frac{2}{a} + 2b$

18. $\frac{n}{4} - \frac{m}{3} - \frac{n}{2} + \frac{m}{3}$

19. $x^3 + 7x^2 - 2x^3$

20. $(2x)^2 - 2x^2$

21. $(3y)^2 + x^2 - (2y)^2$

22. $(2x)^2 - (2y)^2 - (4x)^2$

23. $5x - 7x^2 - (2x)^2$

24. $\frac{3}{x^2} + \frac{5}{x^2}$

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Expand the brackets and collect like terms to simplify each expression.

25. $3x + 2(x + 1)$
26. $5x + 7(x - 1)$
27. $7 + 3(x - 1)$
28. $9 - 2(3x - 1)$
29. $3x - 4(2x + 5)$
30. $5x - 2x(x - 1)$
31. $7x + 3x(x - 4)$
32. $4(x - 1) - 3x$
33. $5x(x + 2) + 4x$
34. $3x(x - 1) - 7x^2$
35. $3a + 2(a + 4)$
36. $4a - 3(a - 3)$
37. $3ab - 2a(b - 2)$
38. $3y - y(2 - y)$
39. $3x - (x + 2)$
40. $7x - (x - 3)$
41. $5x - 2(2x + 2)$
42. $3(x - y) + 4(x + 2y)$
43. $x(x - 2) + 3x(x - 3)$
44. $3x(x + 4) - x(x - 2)$
45. $y(3y - 1) - (3y - 1)$
46. $7(2x + 2) - (2x + 2)$
47. $7b(a + 2) - a(3b + 3)$
48. $3(x - 2) - (x - 2)$

Two pairs of brackets

To expand two pairs of brackets, multiply each term in the first pair of brackets by each term in the second pair.

Example 1

Expand $(x + 5)(x + 3)$

$$\begin{aligned}(x + 5)(x + 3) &= x(x + 3) + 5(x + 3) \\ &= x^2 + 3x + 5x + 15 \\ &= x^2 + 8x + 15\end{aligned}$$

(Multiply each term in the second bracket by x and by 5.)

Example 2

$$\begin{aligned}(2x - 3)(4y + 3) &= 2x(4y + 3) - 3(4y + 3) \\ &= 8xy + 6x - 12y - 9\end{aligned}$$

Example 3

$$\begin{aligned}3(x + 1)(x - 2) &= 3[x(x - 2) + 1(x - 2)] \\ &= 3[x^2 - 2x + x - 2] \\ &= 3x^2 - 3x - 6\end{aligned}$$

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Exercise 2.2B

Expand the brackets and simplify:

1. $(x + 1)(x + 3)$
2. $(x + 3)(x + 2)$
3. $(y + 4)(y + 5)$
4. $(x - 3)(x + 4)$
5. $(x + 5)(x - 2)$
6. $(x - 3)(x - 2)$
7. $(a - 7)(a + 5)$
8. $(z + 9)(z - 2)$
9. $(x - 3)(x + 3)$
10. $(k - 11)(k + 11)$
11. $(2x + 1)(x - 3)$
12. $(3x + 4)(x - 2)$
13. $(2y - 3)(y + 1)$
14. $(7y - 1)(7y + 1)$
15. $(3x - 2)(3x + 2)$
16. $(3a + b)(2a + b)$
17. $(3x + y)(x + 2y)$
18. $(2b + c)(3b - c)$
19. $(5x - y)(3y - x)$
20. $(3b - a)(2a + 5b)$
21. $2(x - 1)(x + 2)$
22. $3(x - 1)(2x + 3)$
23. $4(2y - 1)(3y + 2)$
24. $2(3x + 1)(x - 2)$
25. $4(a + 2b)(a - 2b)$
26. $x(x - 1)(x - 2)$
27. $2x(2x - 1)(2x + 1)$
28. $3y(y - 2)(y + 3)$
29. $x(x + y)(x + z)$
30. $3z(a + 2m)(a - m)$

Be careful with an expression like $(x - 3)^2$.
It is not $x^2 - 9$ or even $x^2 + 9$.

$$\begin{aligned}(x - 3)^2 &= (x - 3)(x - 3)\\ &= x(x - 3) - 3(x - 3)\\ &= x^2 - 6x + 9\end{aligned}$$

Another common mistake occurs with an expression like $4 - (x - 1)^2$.

A common error is to forget that to multiply a set of brackets by -1, you need to change the sign of *all* terms inside the brackets.
The following work is correct.

$$\begin{aligned}4 - (x - 1)^2 &= 4 - 1(x - 1)(x - 1)\\ &= 4 - 1(x^2 - 2x + 1)\\ &= 4 - x^2 + 2x - 1\\ &= 3 + 2x - x^2\end{aligned}$$

Using a bracket here helps to get the signs correct.

Exercise 2.2C

Expand the brackets and simplify:

1. $(x + 4)^2$
2. $(x + 2)^2$
3. $(x - 2)^2$
4. $(2x + 1)^2$
5. $(y - 5)^2$
6. $(3y + 1)^2$

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7. $(x + y)^2$
8. $(2x + y)^2$
9. $(a - b)^2$
10. $(2a - 3b)^2$
11. $3(x + 2)^2$
12. $(3 - x)^2$
13. $(3x + 2)^2$
14. $(a - 2b)^2$
15. $(x + 1)^2 + (x + 2)^2$
16. $(x - 2)^2 + (x + 3)^2$
17. $(x + 2)^2 + (2x + 1)^2$
18. $(y - 3)^2 + (y - 4)^2$
19. $(x + 2)^2 - (x - 3)^2$
20. $(x - 3)^2 - (x + 1)^2$
21. $(y - 3)^2 - (y + 2)^2$
22. $(2x + 1)^2 - (x + 3)^2$
23. $3(x + 2)^2 - (x + 4)^2$
24. $2(x - 3)^2 - 3(x + 1)^2$

Three pairs of brackets

To expand three pairs of brackets, expand the first two pairs of brackets, and then multiply this result by the third pair.

Example

$$
\begin{aligned}
(x+1)(x+2)(x+3) &= [x(x+2) + 1(x+2)](x+3) \\
&= [x^2 + 2x + x + 2](x+3) \\
&= (x^2 + 3x + 2)(x+3) \\
&= x(x^2 + 3x + 2) + 3(x^2 + 3x + 2) \\
&= x^3 + 3x^2 + 2x + 3x^2 + 9x + 6 \\
&= x^3 + 6x^2 + 11x + 6
\end{aligned}
$$

Exercise 2.2D

Expand the brackets and simplify.

1. $(x + 2)(x - 3)(x - 4)$
2. $(x - 1)(x + 2)(x - 5)$
3. $(x + 6)(x - 3)(x + 5)$
4. $(2x - 1)(x + 1)(x - 1)$
5. $(3x + 1)(2x + 1)(x - 2)$
6. $(x + 2)(4x - 3)(2x + 3)$
7. $(6x - 5)(2x + 7)(3x - 8)$
8. $(x + 1)^2(x - 4)$
9. $(x - 3)(x - 2)^2$
10. $(x - 1)(2x + 3)^2$
11. $(x - 1)^3$
12. $(3x + 2)^3$
13. $(x - 2)^3 - (x + 1)^3$
14. $(x + 3)^3 - (x - 4)^3$
15. $(2x + 1)^3 + 3(x + 1)^3$

2.3 Solving linear equations

If an equation contains only one variable, and the highest power of that variable is 1, then the equation is a *linear equation*. In this section you are going to solve linear equations.

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Here are some examples, illustrating a few of the techniques you may use.

- If the x term is negative, add an x term with a positive coefficient to both sides of the equation.

Example 1

Solve $4 - 3x = 2$

$4 = 2 + 3x$ (Add $3x$ to both sides.)

$2 = 3x$ (Subtract 2 from both sides.)

$\frac{2}{3} = x$ (Divide both sides by 3.)

- If there are x terms on both sides, collect them on one side and then simplify.

Example 2

Solve $2x - 7 = 5 - 3x$

$2x + 3x = 5 + 7$ (Add $3x$ to both sides.)

$5x = 12$

$x = \frac{12}{5} = 2\frac{2}{5}$ (Divide both sides by 5 and simplify.)

- If there is a fraction in the x term, multiply out to simplify the equation.

Example 3

Solve $\frac{2x}{3} = 10$

$2x = 30$ (Multiply both sides by 3.)

$x = \frac{30}{2} = 15$ (Divide both sides by 2 and simplify.)

Exercise 2.3A

Solve:

1. $2x - 5 = 11$ **2.** $3x - 7 = 20$ **3.** $2x + 6 = 20$ **4.** $5x + 10 = 60$

5. $8 = 7 + 3x$ **6.** $12 = 2x - 8$ **7.** $-7 = 2x - 10$ **8.** $3x - 7 = -10$

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9. $12 = 15 + 2x$

10. $5 + 6x = 7$

11. $\frac{x}{5} = 7$

12. $\frac{x}{10} = 13$

13. $7 = \frac{x}{2}$

14. $\frac{x}{2} = \frac{1}{3}$

15. $\frac{3x}{2} = 5$

16. $\frac{4x}{5} = -2$

17. $7 = \frac{7x}{3}$

18. $\frac{3}{4} = \frac{2x}{3}$

19. $\frac{5x}{6} = \frac{1}{4}$

20. $-\frac{3}{4} = \frac{3x}{5}$

21. $\frac{x}{2} + 7 = 12$

22. $\frac{x}{3} - 7 = 2$

23. $\frac{x}{5} - 6 = -2$

24. $4 = \frac{x}{2} - 5$

25. $10 = 3 + \frac{x}{4}$

26. $\frac{a}{5} - 1 = -4$

27. $100x - 1 = 98$

28. $7 = 7 + 7x$

29. $\frac{x}{100} + 10 = 20$

30. $1000x - 5 = -6$

31. $-4 = -7 + 3x$

32. $2x + 4 = x - 3$

33. $x - 3 = 3x + 7$

34. $5x - 4 = 3 - x$

35. $4 - 3x = 1$

36. $5 - 4x = -3$

37. $7 = 2 - x$

38. $3 - 2x = x + 12$

39. $6 + 2a = 3$

40. $a - 3 = 3a - 7$

41. $2y - 1 = 4 - 3y$

42. $7 - 2x = 2x - 7$

43. $7 - 3x = 5 - 2x$

44. $8 - 2y = 5 - 5y$

45. $x - 16 = 16 - 2x$

46. $x + 2 = 3.1$

47. $-x - 4 = -3$

48. $-3 - x = -5$

49. $-\frac{x}{2} + 1 = -\frac{1}{4}$

50. $-\frac{3}{5} + \frac{x}{10} = -\frac{1}{5} - \frac{x}{5}$

Equations with brackets

Example

Solve $x - 2(x - 1) = 1 - 4(x + 1)$ (Expand the brackets.)

$x - 2x + 2 = 1 - 4x - 4$ (Be careful to get the sign of each term correct.)

$x - 2x + 4x = 1 - 4 - 2$ (Add $4x$ to both sides.)

$3x = -5$ (Simplify.)

$x = -\frac{5}{3}$ (Divide both sides by 3.)

Exercise 2.3B

Solve:

1. $x + 3(x + 1) = 2x$

2. $1 + 3(x - 1) = 4$

3. $2x - 2(x + 1) = 5x$

4. $2(3x - 1) = 3(x - 1)$

5. $4(x - 1) = 2(3 - x)$

6. $4(x - 1) - 2 = 3x$

7. $4(1 - 2x) = 3(2 - x)$

8. $3 - 2(2x + 1) = x + 17$

9. $4x = x - (x - 2)$

10. $7x = 3x - (x + 20)$

11. $5x - 3(x - 1) = 39$

12. $3x + 2(x - 5) = 15$

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13. $7 - (x + 1) = 9 - (2x - 1)$

14. $10x - (2x + 3) = 21$

15. $3(2x + 1) + 2(x - 1) = 23$

16. $5(1 - 2x) - 3(4 + 4x) = 0$

17. $7x - (2 - x) = 0$

18. $3(x + 1) = 4 - (x - 3)$

19. $3y + 7 + 3(y - 1) = 2(2y + 6)$

20. $4(y - 1) + 3(y + 2) = 5(y - 4)$

21. $4x - 2(x + 1) = 5(x + 3) + 5$

22. $7 - 2(x - 1) = 3(2x - 1) + 2$

23. $10(2x + 3) - 8(3x - 5) + 5(2x - 8) = 0$

24. $2(x + 4) + 3(x - 10) = 8$

25. $7(2x - 4) + 3(5 - 3x) = 2$

26. $10(x + 4) - 9(x - 3) - 1 = 8(x + 3)$

27. $5(2x - 1) - 2(x - 2) = 7 + 4x$

28. $6(3x - 4) - 10(x - 3) = 10(2x - 3)$

29. $3(x - 3) - 7(2x - 8) - (x - 1) = 0$

30. $5 + 2(x + 5) = 10 - (4 - 5x)$

31. $6x + 30(x - 12) = 2\left(x - 1\frac{1}{2}\right)$

32. $3\left(2x - \frac{2}{3}\right) - 7(x - 1) = 0$

33. $5(x - 1) + 17(x - 2) = 2x + 1$

34. $6(2x - 1) + 9(x + 1) = 8\left(x - 1\frac{1}{4}\right)$

35. $7(x + 4) - 5(x + 3) + (4 - x) = 0$

36. $0 = 9(3x + 7) - 5(x + 2) - (2x - 5)$

37. $10(2.3 - x) - 0.1(5x - 30) = 0$

38. $8\left(2\frac{1}{2}x - \frac{3}{4}\right) - \frac{1}{4}(1 - x) = \frac{1}{2}$

39. $(6 - x) - (x - 5) - (4 - x) = -\frac{x}{2}$

40. $10\left(1 - \frac{x}{10}\right) - (10 - x) - \frac{1}{100}(10 - x) = 0.05$

Example

Solve $(x + 3)^2 = (x + 2)^2 + 3^2$

$$
\begin{aligned}
(x + 3)(x + 3) &= (x + 2)(x + 2) + 9 \\
x^2 + 6x + 9 &= x^2 + 4x + 4 + 9 \\
6x + 9 &= 4x + 13 \\
2x &= 4 \\
x &= 2
\end{aligned}
$$

Exercise 2.3C

Solve:

1. $x^2 + 4 = (x + 1)(x + 3)$

2. $x^2 + 3x = (x + 3)(x + 1)$

3. $(x + 3)(x - 1) = x^2 + 5$

4. $(x + 1)(x + 4) = (x - 7)(x + 6)$

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5. $(x - 2)(x + 3) = (x - 7)(x + 7)$

6. $(x - 5)(x + 4) = (x + 7)(x - 6)$

7. $2x^2 + 3x = (2x - 1)(x + 1)$

8. $(2x - 1)(x - 3) = (2x - 3)(x - 1)$

9. $x^2 + (x + 1)^2 = (2x - 1)(x + 4)$

10. $x(2x + 6) = 2(x^2 - 5)$

11. $(x + 1)(x - 3) + (x + 1)^2 = 2x(x - 4)$

12. $(2x + 1)(x - 4) + (x - 2)^2 = 3x(x + 2)$

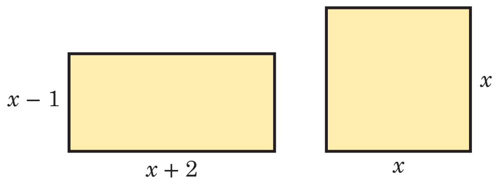
13. $(x + 2)^2 - (x - 3)^2 = 3x - 11$

14. $x(x - 1) = 2(x - 1)(x + 5) - (x - 4)^2$

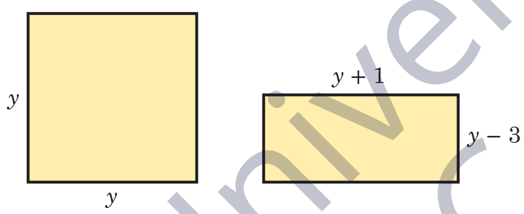
15. $(2x + 1)^2 - 4(x - 3)^2 = 5x + 10$

16. $2(x + 1)^2 - (x - 2)^2 = x(x - 3)$

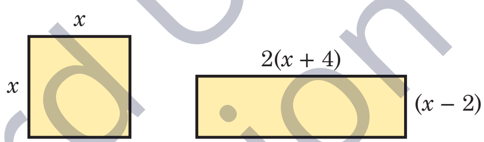
17. The area of the rectangle here exceeds the area of the square by 2 cm^2.
Find x.



18. The area of the square exceeds the area of the rectangle by 13 m^2.
Find y.



19. The area of the square is half the area of the rectangle. Find x.



Equations involving fractions

When solving equations involving fractions, multiply both sides of the equation by a suitable number or letter to eliminate the fractions.

Example 1

Solve $\dfrac{5}{x} = 2$

$5 = 2x$ (Multiply both sides by x.)

$\dfrac{5}{2} = x$ (Divide both sides by 2.)

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Example 2

Solve $\frac{x+3}{4} = \frac{2x-1}{3}$

$$12 \times \frac{(x+3)}{4} = 12 \times \frac{(2x-1)}{3}$$ (Multiply both sides by 12.)

$$3(x+3) = 4(2x-1)$$ (Or you can cross multiply.)

$$3x + 9 = 8x - 4$$

$$13 = 5x$$ (Subtract $3x$, not $8x$, so that the x term is positive.)

$$\frac{13}{5} = x$$

$$x = 2\frac{3}{5}$$

Example 3

Solve $\frac{5}{(x-1)} + 2 = 12$

$$\frac{5}{(x-1)} = 10$$ (2 and 12 are like terms so combine them first.)

$$5 = 10(x-1)$$

$$5 = 10x - 10$$

$$15 = 10x$$

$$\frac{15}{10} = x$$

$$x = 1\frac{1}{2}$$

Exercise 2.3D

Solve:

1. $\frac{7}{x} = 21$ **2.** $30 = \frac{6}{x}$ **3.** $\frac{5}{x} = 3$

4. $\frac{9}{x} = -3$ **5.** $11 = \frac{5}{x}$ **6.** $-2 = \frac{4}{x}$

7. $\frac{x}{4} = \frac{3}{2}$ **8.** $\frac{x}{3} = \frac{5}{4}$ **9.** $\frac{x+1}{3} = \frac{x-1}{4}$

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10. $\frac{x+3}{2} = \frac{x-4}{5}$
11. $\frac{2x-1}{3} = \frac{x}{2}$
12. $\frac{3x+1}{5} = \frac{2x}{3}$
13. $\frac{8-x}{2} = \frac{2x+2}{5}$
14. $\frac{x+2}{7} = \frac{3x+6}{5}$
15. $\frac{1-x}{2} = \frac{3-x}{3}$
16. $\frac{2}{x-1} = 1$
17. $\frac{x}{3} + \frac{x}{4} = 1$
18. $\frac{x}{3} + \frac{x}{2} = 4$
19. $\frac{x}{2} - \frac{x}{5} = 3$
20. $\frac{x}{3} = 2 + \frac{x}{4}$
21. $\frac{5}{x-1} = \frac{10}{x}$
22. $\frac{12}{2x-3} = 4$
23. $2 = \frac{18}{x+4}$
24. $\frac{5}{x+5} = \frac{15}{x+7}$
25. $\frac{9}{x} = \frac{5}{x-3}$
26. $\frac{4}{x-1} = \frac{10}{3x-1}$
27. $\frac{-7}{x-1} = \frac{14}{5x+2}$
28. $\frac{4}{x+1} = \frac{7}{3x-2}$
29. $\frac{x+1}{2} + \frac{x-1}{3} = \frac{1}{6}$
30. $\frac{1}{3}(x+2) = \frac{1}{5}(3x+2)$
31. $\frac{1}{2}(x-1) - \frac{1}{6}(x+1) = 0$
32. $\frac{1}{4}(x+5) - \frac{2x}{3} = 0$
33. $\frac{4}{x} + 2 = 3$
34. $\frac{6}{x} - 3 = 7$
35. $\frac{9}{x} - 7 = 1$
36. $-2 = 1 + \frac{3}{x}$
37. $4 - \frac{4}{x} = 0$
38. $5 - \frac{6}{x} = -1$
39. $7 - \frac{3}{2x} = 1$
40. $4 + \frac{5}{3x} = -1$
41. $\frac{9}{2x} - 5 = 0$
42. $\frac{x-1}{5} - \frac{x-1}{3} = 0$
43. $\frac{x-1}{4} - \frac{2x-3}{5} = \frac{1}{20}$
44. $\frac{4}{1-x} = \frac{3}{1+x}$
45. $\frac{x+1}{4} - \frac{x}{3} = \frac{1}{12}$
46. $\frac{2x+1}{8} - \frac{x-1}{3} = \frac{5}{24}$

2.4 Problems solved by linear equations

Step 1 Let the unknown quantity be x (or any other letter) and state the units (where appropriate).

Step 2 Express the given statement in the form of an equation. Do not include the units in the equation.

Step 3 Solve the equation for x and give the answer in *words*. (Do not finish by just writing '$x = 3$'.)

Step 4 Check your solution using the initial problem (not your equation).

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Example 1

The sum of three consecutive whole numbers is 78. Find the numbers.

Let the smallest number be x; then the other numbers are $(x + 1)$ and $(x + 2)$.

Form an equation:

$$x + (x + 1) + (x + 2) = 78$$
$$3x + 3 = 78$$

Solve: $3x = 75$

$x = 25$

In words:

The three numbers are 25, 26 and 27.

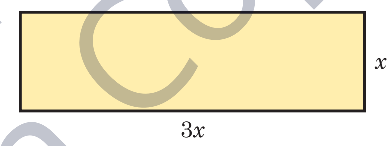
Check: $25 + 26 + 27 = 78$

Example 2

The length of a rectangle is three times its width. If the perimeter is 36 cm, find the width.

Let the width of the rectangle be x cm.

Then the length of the rectangle is $3x$ cm.



Form an equation.

$x + 3x + x + 3x = 36$ or $2(x + 3x) = 36$

Solve: $8x = 36$

$x = \frac{36}{8}$

$x = 4.5$

In words:

The width of the rectangle is 4.5 cm

Check: If width $= 4.5$ cm

length $= 13.5$ cm

perimeter $= 36$ cm

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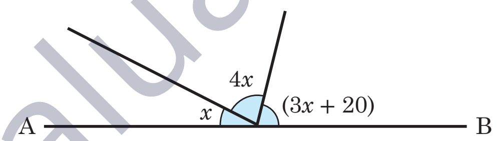
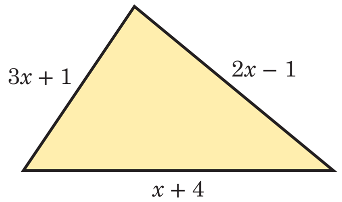
Exercise 2.4A

Solve each problem by forming an equation. The first questions are easy but should still be solved using an equation, in order to practise the method.

1. The sum of three consecutive numbers is 276. Find the numbers.
2. The sum of four consecutive numbers is 90. Find the numbers.
3. The sum of three consecutive odd numbers is 177. Find the numbers.
4. Find three consecutive even numbers which add up to 1524.
5. When a number is doubled and then added to 13, the result is 38. Find the number.
6. When a number is doubled and then added to 24, the result is 49. Find the number.
7. When 7 is subtracted from three times a certain number, the result is 28. What is the number?
8. The sum of two numbers is 50. The second number is five times the first. Find the numbers.
9. Two numbers are in the ratio 1:11 and their sum is 15. Find the numbers.
10. The length of a rectangle is twice the width. If the perimeter is 20 cm, find the width.
11. The width of a rectangle is one third of the length. If the perimeter is 96 cm, find the width.
12. If AB is a straight line, find x.



13. If the perimeter of the triangle is 22 cm, find the length of the shortest side.



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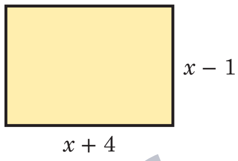
14. If the perimeter of the rectangle is 34 cm, find x.



15. The difference between two numbers is 9. Find the numbers if their sum is 46.

16. The three angles in a triangle are in the ratio 1 : 3 : 5. Find them.

17. The three angles in a triangle are in the ratio 3 : 4 : 5. Find them.

18. The product of two consecutive odd numbers is 10 more than the square of the smaller number. Find the smaller number.

19. The product of two consecutive even numbers is 12 more than the square of the smaller number. Find the numbers.

20. The sum of three numbers is 66. The second number is twice the first and six less than the third. Find the numbers.

21. The sum of three numbers is 28. The second number is three times the first and the third is 7 less than the second. What are the numbers?

22. David's mass is 5 kg less than Sucha's, who in turn is 8 kg lighter than Paul. If their total mass is 197 kg, how heavy is each person?

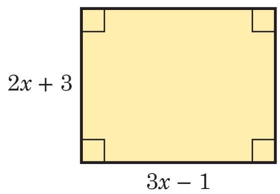
23. Nilopal is 2 years older than Devjan who is 7 years older than John. If their combined age is 61 years, find the age of each person.

24. Kim has four times as many marbles as Ava. If Kim gave 18 to Ava they would have the same number. How many marbles has each person?

25. Mukat has five times as many books as Usha. If Mukat gives 16 books to Usha, they will each have the same number. How many books does each girl have?

26. The result of multiplying a number by 3 is the same as adding 12 to it. What is the number?

27. Find the area of the rectangle if the perimeter is 52 cm.



28. The result of multiplying a number by 3 and subtracting 5 is the same as doubling the number and adding 9. Find the number.

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29. Two girls have \$76 between them. If the first girl gave the second girl \$7, they would each have the same amount of money. How much does each girl have?

30. A tennis racket costs \$12 more than a hockey stick. If the price of the two is \$31, find the cost of the tennis racket.

Example

A man leaves home at 16:42 and walks to a library, 6 km away, arriving at 17:30. He walked part of the way at 5 km/h and then, realising the time, he ran the rest of the way at 10 km/h. How far did he run?

Let the distance he ran be x km.
Then the distance he walked = $(6 - x)$ km.

Time taken to walk $(6 - x)$ km at 5 km/h $= \dfrac{(6-x)}{5}$ hours.

Time taken to run x km at 10 km/h $= \dfrac{x}{10}$ hours.

Tip

Use the formula $\text{time} = \dfrac{\text{distance}}{\text{speed}}$.

$$\text{Total time taken} = 48 \text{ minutes}$$
$$= \frac{48}{60} \text{ hour} = \frac{4}{5} \text{ hour}$$

$$\therefore \frac{(6-x)}{5} + \frac{x}{10} = \frac{4}{5}$$

Multiply by 10:

$$2(6 - x) + x = 8$$
$$12 - 2x + x = 8$$
$$4 = x$$

He ran a distance of 4 km.

Check:

Time to run 4 km $= \dfrac{4}{10} = \dfrac{2}{5}$ hour

Time to walk 2 km $= \dfrac{2}{5}$ hour

Total time taken $= \left(\dfrac{2}{5} + \dfrac{2}{5}\right) = \dfrac{4}{5}$ hour

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Exercise 2.4B

1. Every year a man is paid \$500 more than the previous year. If he receives \$17 800 over four years, what was he paid in the first year?
2. Samir buys x cans of soda at 80 cents each and $(x + 4)$ cans of soda at 85 cents each. The total cost was \$8.35. Find x.
3. The length of a straight line segment ABC is 5 m. If AB : BC = 2 : 5, find the length of AB.
4. The opposite angles of a cyclic quadrilateral are $(3x + 10)°$ and $(2x + 20)°$. Find the angles.
5. The interior angles of a hexagon are in the ratio 1 : 2 : 3 : 4 : 5 : 9. Find the angles.
6. A woman is 32 years older than her son. Ten years ago, she was three times as old as her son was at that time. Find the current age of the woman and her son.
7. A bus is travelling with 48 passengers. When it arrives at a stop, x passengers get off and 3 get on. At the next stop half the passengers get off and 7 get on. There are now 22 passengers. Find x.
8. A bus is travelling with 52 passengers. When it arrives at a stop, y passengers get off and 4 get on. At the next stop one-third of the passengers get off and 3 get on. There are now 25 passengers. Find y.
9. In a regular polygon with n sides, each interior angle is $\left(180 - \frac{360}{n}\right)$ degrees. How many sides does a polygon have if each interior angle is 156°?
10. Consider the equation $\frac{k}{x} = 12$ where k is any number between 20 and 65 and x is a positive integer. What are the possible values of x?
11. Mahmoud runs to a marker and back in 15 minutes. His speed on the way to the marker is 5 m/s and his speed on the way back is 4 m/s. Find the distance to the marker.
12. A car completes a journey in 10 minutes. For the first half of the journey the speed was 60 km/h and for the second half the speed was 40 km/h. How far is the journey?

Tip

Opposite angles of a cyclic quadrilateral add to 180°.

Tip

Interior angles of a hexagon add to 720°.

Tip

Formulae for speed, distance and time are studied in detail in section 3.6 on page 111.

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2.5 Simultaneous equations

To find the value of two unknowns in a problem, *two* different equations must be given that relate the unknowns to each other. These two equations are called *simultaneous* equations. There are two algebraic ways to solve simultaneous equations: the substitution method and the elimination method. You can also solve simultaneous equations graphically.

Substitution method

This method is used when one equation contains a unit quantity of one of the unknowns, as in equation (2) of the example below.

Example

$3x - 2y = 0 \qquad (1)$

$2x + y = 7 \qquad (2)$

Label the equations so that the working is made clear.

In this case, write y in terms of x from equation (2).

Substitute this expression for y into equation (1) and solve to find x.

Find y from equation (2) using this value of x.

$2x + y = 7 \qquad (2)$

$y = 7 - 2x$

Substituting into (1)

$3x - 2(7 - 2x) = 0$

$3x - 14 + 4x = 0$

$7x = 14$

$x = 2$

Substituting into (2)

$2 \times 2 + y = 7$

$y = 3$

The solutions are $x = 2$, $y = 3$.

$3 \times 2 - 2 \times 3 = 0$

$2 \times 2 + 3 = 7$

These values of x and y are the only pair which simultaneously satisfy *both* equations.

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Exercise 2.5A

Use the substitution method to solve these simultaneous equations.

1. $2x + y = 5$
$x + 3y = 5$

2. $x + 2y = 8$
$2x + 3y = 14$

3. $3x + y = 10$
$x - y = 2$

4. $2x + y = -3$
$x - y = -3$

5. $4x + y = 14$
$x + 5y = 13$

6. $x + 2y = 1$
$2x + 3y = 4$

7. $2x + y = 5$
$3x - 2y = 4$

8. $2x + y = 13$
$5x - 4y = 13$

9. $7x + 2y = 19$
$x - y = 4$

10. $b - a = -5$
$a + b = -1$

11. $a + 4b = 6$
$8b - a = -3$

12. $a + b = 4$
$2a + b = 5$

13. $3m = 2n - 6\frac{1}{2}$
$4m + n = 6$

14. $2w + 3x - 13 = 0$
$x + 5w - 13 = 0$

15. $x + 2(y - 6) = 0$
$3x + 4y = 30$

16. $2x = 4 + z$
$6x - 5z = 18$

17. $3m - n = 5$
$2m + 5n = 7$

18. $5c - d - 11 = 0$
$4d + 3c = -5$

It is useful at this point to revise the operations of addition and subtraction with negative numbers.

Example

Simplify:

a) $-7 + (-4) = -7 - 4 = -11$

b) $-3x + (-4x) = -3x - 4x = -7x$

c) $4y - (-3y) = 4y + 3y = 7y$

c) $3a + (-3a) = 3a - 3a = 0$

Exercise 2.5B

Evaluate:

1. $7 + -6$

2. $8 + -11$

3. $5 - +7$

4. $6 - -9$

5. $-8 + -4$

6. $-7 - -4$

7. $10 + -12$

8. $-7 - +4$

9. $-10 - +11$

10. $-3 - -4$

11. $4 - +4$

12. $8 - -7$

13. $-5 - +5$

14. $-7 - -10$

15. $16 - +10$

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