1.1 Physical quantities and measurement techniques

FOCUS POINTS

- ★ Describe how to measure length, volume and time intervals using simple devices.
- ★ Know how to determine the average value for a small distance and a short time interval.
- ★ Understand the difference between scalar and vector quantities, and give examples of each.
- ★ Calculate or determine graphically the resultant of two perpendicular vectors.

This topic introduces the concept of describing space and time in terms of numbers together with some of the basic units used in physics. You will learn how to use simple devices to measure or calculate the quantities of length, area and volume. Accurate measurements of time will be needed frequently in the practical work in later topics and you will discover how to choose the appropriate clock or timer for the measurement of a time interval. Any single measurement will not be entirely accurate and will have an error associated with it. Taking the average of several measurements, or measuring multiples, reduces the size of the error.

Many physical quantities, such as force and velocity, have both magnitude and direction; they are termed vectors. When combining two vectors to find their resultant, as well as their size, you need to take into account any difference in their directions.

Units and basic quantities

Before a measurement can be made, a standard or *unit* must be chosen. The size of the quantity to be measured is then found with an instrument having a scale marked in the unit.

Three basic quantities we measure in physics are **length**, **mass** and **time**. Units for other quantities are based on them. The SI (Système International d'Unités) system is a set of metric units now used in many countries. It is a decimal system in which units are divided or multiplied by 10 to give smaller or larger units.

Measuring instruments on the flight deck of a passenger jet provide the crew with information about the performance of the aircraft (see Figure 1.1.1).

▲ **Figure 1.1.1** Aircraft flight deck

Powers of ten shorthand

This is a neat way of writing numbers, especially if they are large or small. The example below shows how it works.

$$4000 = 4 \times 10 \times 10 \times 10 = 4 \times 10^3$$
$$400 = 4 \times 10 \times 10 = 4 \times 10^2$$
$$40 = 4 \times 10 = 4 \times 10^1$$
$$4 = 4 \times 1 = 4 \times 10^0$$
$$0.4 = 4/10 = 4/10^1 = 4 \times 10^{-1}$$
$$0.04 = 4/100 = 4/10^2 = 4 \times 10^{-2}$$
$$0.004 = 4/1000 = 4/10^3 = 4 \times 10^{-3}$$

The small figures 1, 2, 3, etc. are called **powers of ten**. The power shows how many times the number has to be multiplied by 10 if the power is greater than 0 or divided by 10 if the power is less than 0. Note that 1 is written as 10^0.

This way of writing numbers is called **standard notation**.

Length

The unit of length is the **metre** (m) and is the distance travelled by light in a vacuum during a specific time interval. At one time it was the distance between two marks on a certain metal bar. Submultiples are:

1 decimetre (dm) $= 10^{-1}$ m

1 centimetre (cm) $= 10^{-2}$ m

1 **millimetre** (mm) $= 10^{-3}$ m

1 **micrometre** (µm) $= 10^{-6}$ m

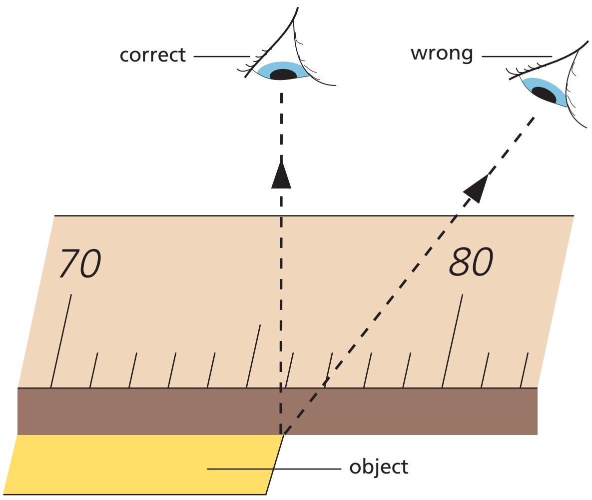
1 **nanometre** (nm) $= 10^{-9}$ m

A multiple for large distances is

1 **kilometre** (km) $= 10^3$ m ($\frac{5}{8}$ mile approx.)

1 **gigametre** (Gm) $= 10^9$ m = 1 billion metres

Many length measurements are made with rulers; the correct way to read one is shown in Figure 1.1.2. The reading is 76 mm or 7.6 cm. Your eye must be directly over the mark on the scale or the thickness of the ruler causes a parallax error.



▲ **Figure 1.1.2** The correct way to measure with a ruler

To obtain an average value for a small distance, multiples can be measured. For example, in ripple tank experiments (Topic 3.1), measure the distance occupied by five waves, then divide by 5 to obtain the average wavelength.

Significant figures

Every measurement of a quantity is an attempt to find its true value and is subject to errors arising from limitations of the apparatus and the experimenter. The number of figures, called **significant figures**, given for a measurement indicates how accurate we think it is and more figures should not be given than are justified.

For example, a value of 4.5 for a measurement has two significant figures; 0.0385 has three significant figures, 3 being the most significant and 5 the least, i.e. it is the one we are least sure about since it might be 4 or it might be 6. Perhaps it had to be estimated by the experimenter because the reading was between two marks on a scale.

When doing a calculation your answer should have the same number of significant figures as the measurements used in the calculation. For example, if your calculator gave an answer of 3.4185062, this would be written as 3.4 if the measurements had two significant figures. It would be written as 3.42 for three significant figures. Note that in deciding the least significant figure you look at the next figure to the right. If it is less than 5, you leave the least significant figure as it is (hence 3.41 becomes 3.4), but if it equals or is greater than 5 you increase the least significant figure by 1 (round it up) (hence 3.418 becomes 3.42).

If a number is expressed in standard notation, the number of significant figures is the number of digits before the power of ten. For example, 2.73×10^3 has three significant figures.

Test yourself

1 How many millimetres are there in these measurements?
  a 1 cm
  b 4 cm
  c 0.5 cm
  d 6.7 cm
  e 1 m
2 What are these lengths in metres?
  a 300 cm
  b 550 cm
  c 870 cm
  d 43 cm
  e 100 mm
3 a Write the following as powers of ten with one figure before the decimal point:
  100 000 3500 428 000 000 504 27 056
  b Write out the following in full:
  10^3 2×10^6 6.92×10^4 1.34×10^2 10^9
4 a Write these fractions as powers of ten:
  1/1000 7/100 000 1/10 000 000 3/60 000
  b Express the following decimals as powers of ten with one figure before the decimal point:
  0.5 0.084 0.000 36 0.001 04

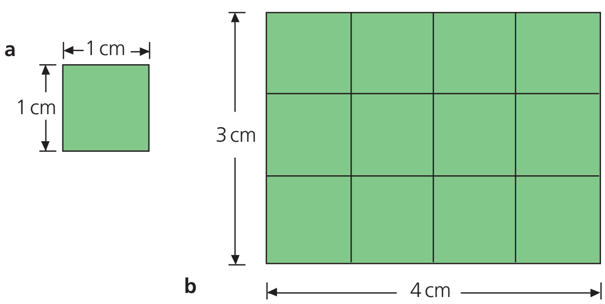
Area

The **area of the square** in Figure 1.1.3a with sides 1 cm long is 1 square centimetre ($1\,\text{cm}^2$). In Figure 1.1.3b the rectangle measures 4 cm by 3 cm and has an area of $4 \times 3 = 12\,\text{cm}^2$ since it has the same area as twelve squares each of area $1\,\text{cm}^2$. The area of a square or rectangle is given by

$$\text{area} = \text{length} \times \text{breadth}$$

The SI unit of area is the square metre (m^2) which is the area of a square with sides 1 m long. Note that

$$1\,\text{cm}^2 = \frac{1}{100}\,\text{m} \times \frac{1}{100}\,\text{m} = \frac{1}{10\,000}\,\text{m}^2 = 10^{-4}\,\text{m}^2$$



▲ **Figure 1.1.3**

Sometimes we need to know the area of a triangle. It is given by

$$\text{area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$$

The **area of a circle** of radius r is πr^2 where $\pi = 22/7$ or 3.14; its circumference is $2\pi r$.

Worked example

Calculate the area of the triangles shown in Figure 1.1.4.

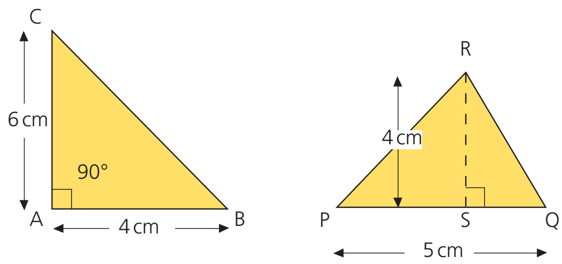
a $\text{area of triangle} = \frac{1}{2} \times \text{base} \times \text{height}$

so $\text{area of triangle ABC} = \frac{1}{2} \times \text{AB} \times \text{AC}$

$= \frac{1}{2} \times 4\,\text{cm} \times 6\,\text{cm} = 12\,\text{cm}^2$

b $\text{area of triangle PQR} = \frac{1}{2} \times \text{PQ} \times \text{SR}$

$= \frac{1}{2} \times 5\,\text{cm} \times 4\,\text{cm} = 10\,\text{cm}^2$



▲ **Figure 1.1.4**

Now put this into practice

1 Calculate the area of a triangle whose base is 8 cm and height is 12 cm.
2 Calculate the circumference of a circle of radius 6 cm.

Volume

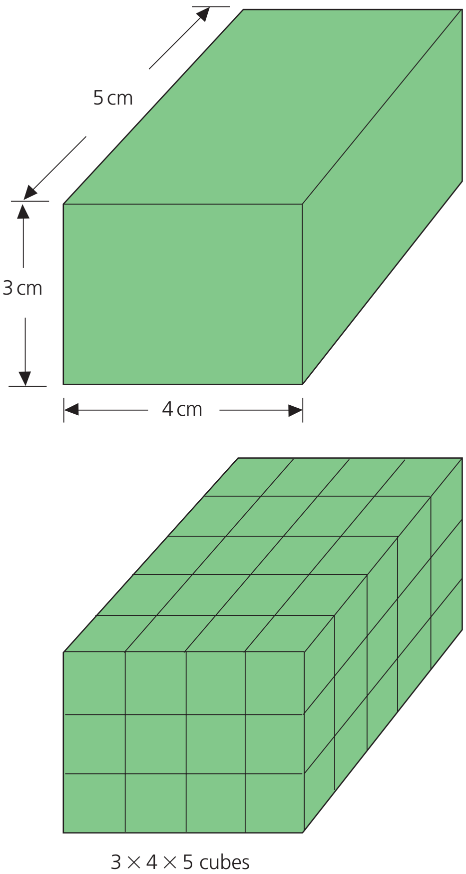
Volume is the amount of space occupied. The unit of volume is the **cubic metre** (m^3) but as this is rather large, for most purposes the **cubic centimetre** (cm^3) is used. The volume of a cube with 1 cm edges is $1\,\text{cm}^3$. Note that

$$1\,\text{cm}^3 = \frac{1}{100}\,\text{m} \times \frac{1}{100}\,\text{m} \times \frac{1}{100}\,\text{m}$$

$$= \frac{1}{1\,000\,000}\,\text{m}^3 = 10^{-6}\,\text{m}^3$$

For a regularly shaped object such as a rectangular block, Figure 1.1.5 shows that

$$\text{volume} = \text{length} \times \text{breadth} \times \text{height}$$

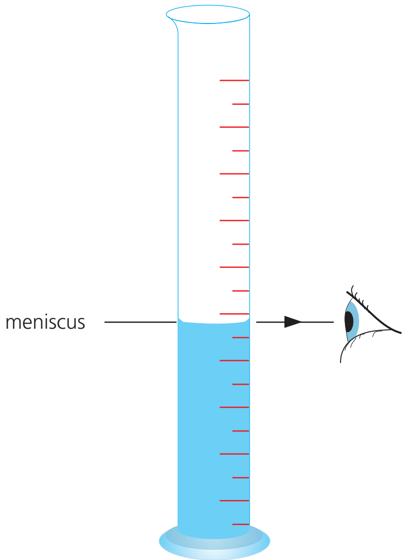


▲ **Figure 1.1.5**

The **volume of a cylinder** of radius r and height h is $\pi r^2 h$.

The volume of a liquid may be obtained by pouring it into a measuring cylinder (Figure 1.1.6). When making a reading the cylinder must be upright and your eye must be level with the bottom of the curved liquid surface, i.e. the **meniscus**. The meniscus formed by mercury is curved oppositely to that of other liquids and the top is read.

Measuring cylinders are often marked in millilitres (ml) where 1 ml = 1 cm^3; note that 1000 cm^3 = 1 dm^3 (= 1 litre).



▲ **Figure 1.1.6** A measuring cylinder

Worked example

a Calculate the volume of a block of wood which is 40 cm long, 12 cm wide and 5 cm high in cubic metres.

$$\begin{aligned}\text{volume } V &= \text{length} \times \text{breadth} \times \text{height}\\ &= 40\,\text{cm} \times 12\,\text{cm} \times 5\,\text{cm}\\ &= 2400\,\text{cm}^3\\ &= 2400 \times 10^{-6}\,\text{m}^3\\ &= 2.4 \times 10^{-3}\,\text{m}^3\end{aligned}$$

b Calculate the volume of a cylinder of radius 10 mm and height 5.0 cm in cubic metres.

volume of cylinder $V = \pi r^2 h$

$r = 10\,\text{mm} = 1.0\,\text{cm}$ and $h = 5.0\,\text{cm}$

so $V = \pi r^2 h$

$$\begin{aligned}&= \pi \times (1.0\,\text{cm})^2 \times 5.0\,\text{cm}\\ &= 16\,\text{cm}^3 = 16 \times 10^{-6}\,\text{m}^3 = 1.6 \times 10^{-5}\,\text{m}^3\end{aligned}$$

Now put this into practice

1. Calculate the volume of a rectangular box which is 30 cm long, 25 cm wide and 15 cm high in cubic metres.
2. Calculate the volume of a cylinder of radius 50 mm and height 25 cm in cubic metres.

Time

The unit of time is the **second** (s), which used to be based on the length of a day, this being the time for the Earth to revolve once on its axis. However, days are not all of exactly the same duration and the second is now defined as the time interval for a certain number of **energy** changes to occur in the caesium atom.

Time-measuring devices rely on some kind of constantly repeating oscillation. In traditional clocks and watches a small wheel (the balance wheel) oscillates to and fro; in digital clocks and watches the oscillations are produced by a tiny quartz crystal. A swinging pendulum controls a pendulum clock.

To measure an interval of time in an experiment, first choose a timer that is precise enough for the task. A stopwatch is adequate for finding the period in seconds of a pendulum (see Figure 1.1.7 opposite), but to measure the speed of sound (Topic 3.4), a clock that can time in milliseconds is needed. To measure very short time intervals, a digital clock that can be triggered to start and stop by an electronic signal from a microphone, photogate or mechanical switch is useful. Tickertape timers or dataloggers are often used to record short time intervals in motion experiments. Accuracy can be improved by measuring longer time intervals. Several oscillations (rather than just one) are timed to find the period of a pendulum; the average value for the period is found by dividing the time by the number of oscillations. Ten ticks, rather than single ticks, are used in tickertape timers.

Test yourself

5 The pages of a book are numbered 1 to 200 and each leaf is 0.10 mm thick. If each cover is 0.20 mm thick, what is the thickness of the book?

6 How many significant figures are there in a length measurement of
- **a** 2.5 cm
- **b** 5.32 cm
- **c** 7.180 cm
- **d** 0.042 cm?

7 A rectangular block measures 4.1 cm by 2.8 cm by 2.1 cm. Calculate its volume giving your answer to an appropriate number of significant figures.

8 What type of timer would you use to measure the period of a simple pendulum? How many oscillations would you time?

Practical work

Period of a simple pendulum

For safe experiments/demonstrations related to this topic, please refer to the *Cambridge IGCSE Physics Practical Skills Workbook* that is also part of this series.

In this investigation you have to make time measurements using a stopwatch or clock. A motion sensor connected to a datalogger and computer could be used instead of a stopwatch for these investigations.

Attach a small metal ball (called a bob) to a piece of string, and suspend it as shown in Figure 1.1.7 opposite. Pull the bob a small distance to one side, and then release it so that it oscillates to and fro through a small angle.

Find the time for the bob to make several complete oscillations; one oscillation is from A to O to B to O to A (Figure 1.1.7). Repeat the timing a few times for the same number of oscillations and work out the average.

1. The time for one oscillation is the **period** T. Determine the period of your pendulum.
2. The **frequency** f of the oscillations is the number of complete oscillations per second and equals $1/T$. Calculate a value for f for your pendulum.
3. Comment on how the amplitude of the oscillations changes with time.
4. Plan an investigation into the effect on T of (i) a longer string and (ii) a larger bob.
5. What procedure would you use to determine the period of a simple pendulum?
6. In Figure 1.1.7 if the bob is first released at B, give the sequence of letters which corresponds to one complete oscillation.

7 Explain where you would take measurements from to determine the length of the pendulum shown in Figure 1.1.7.

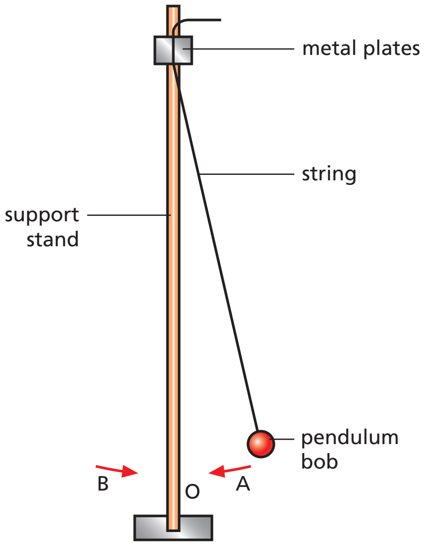


Figure 1.1.7

Systematic errors

Figure 1.1.8 shows a part of a ruler used to measure the height of a point P above the bench. The ruler chosen has a space before the zero of the scale. This is shown as the length x. The height of the point P is given by the scale reading added to the value of x. The equation for the height is

$$\text{height} = \text{scale reading} + x$$

$$\text{height} = 5.9 + x$$

By itself the scale reading is not equal to the height. It is too small by the value of x.

This type of error is known as a **systematic error**. The error is introduced by the system. A half-metre ruler has the zero at the *end of the ruler* and so can be used without introducing a systematic error.

When using a ruler to determine a height, the ruler must be held so that it is vertical. If the ruler is at an angle to the vertical, a systematic error is introduced.

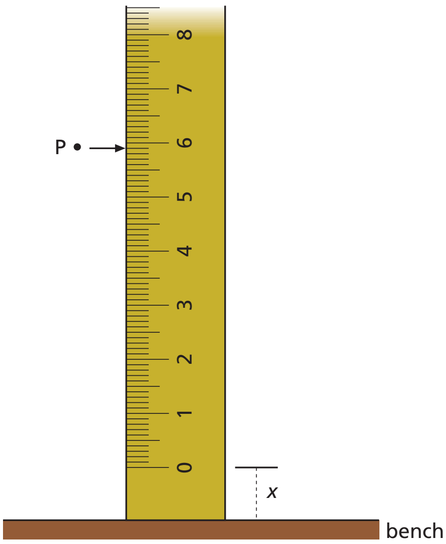


Figure 1.1.8

Going further

Vernier scales and micrometers

Lengths can be measured with a ruler to a precision of about 0.5 mm. Some investigations may need a more precise measurement of length, which can be achieved by using vernier calipers (Figure 1.1.9) or a micrometer screw gauge.

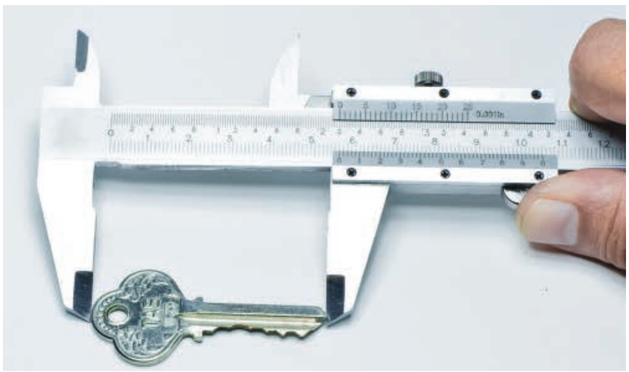


Figure 1.1.9 Vernier calipers in use

Vernier scale

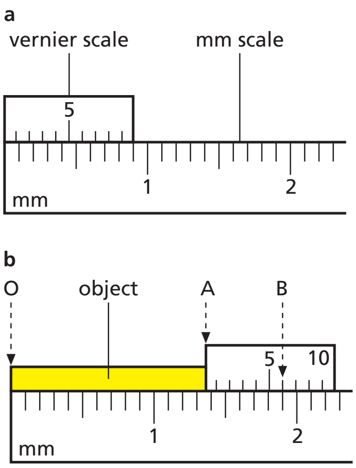
The calipers shown in Figure 1.1.9 use a vernier scale. The simplest type enables a length to be measured to 0.01 cm. It is a small sliding scale which is 9 mm long but divided into ten equal divisions (Figure 1.1.10a) so

$$\begin{aligned} 1 \text{ vernier division} &= \frac{9}{10}\,\text{mm} \\ &= 0.9\,\text{mm} \\ &= 0.09\,\text{cm} \end{aligned}$$

One end of the length to be measured is made to coincide with the zero of the millimetre scale and the other end with the zero of the vernier scale. The length of the object in Figure 1.1.10b is between 1.3 cm and 1.4 cm. The reading to the second place of decimals is obtained by finding the vernier mark which is exactly opposite (or nearest to) a mark on the millimetre scale. In this case it is the 6th mark and the length is 1.36 cm, since

$$\begin{aligned} OA &= OB - AB \\ OA &= (1.90\,\text{cm}) - (6 \text{ vernier divisions}) \\ &= 1.90\,\text{cm} - 6(0.09)\,\text{cm} \\ &= (1.90 - 0.54)\,\text{cm} \\ &= 1.36\,\text{cm} \end{aligned}$$

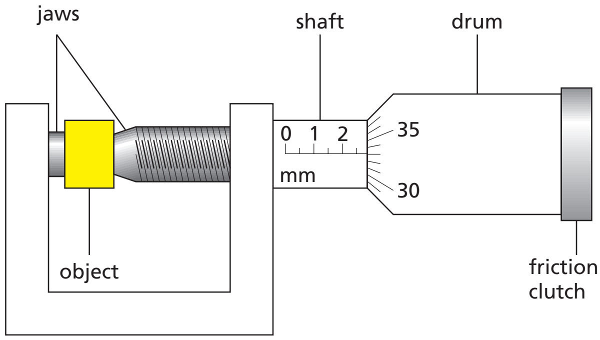
Vernier scales are also used on barometers, travelling microscopes and spectrometers.



▲ **Figure 1.1.10** Vernier scale

Micrometer screw gauge

This measures very small objects to 0.001 cm. One revolution of the drum opens the flat, parallel jaws by one division on the scale on the shaft of the gauge; this is usually mm, i.e. 0.05 cm. If the drum has a scale of 50 divisions round it, then rotation of the drum by one division opens the jaws by 0.05/50 = 0.001 cm (Figure1.1.11). A friction clutch ensures that the jaws exert the same force when the object is gripped.



▲ **Figure 1.1.11** Micrometer screw gauge

The object shown in Figure 1.1.11 has a length of

$$\begin{aligned} &2.5\,\text{mm on the shaft scale} + 33 \text{ divisions on the drum scale} \\ &= 0.25\,\text{cm} + 33(0.001)\,\text{cm} \\ &= 0.283\,\text{cm} \end{aligned}$$

Before making a measurement, check to ensure that the reading is zero when the jaws are closed. Otherwise the zero error must be allowed for when the reading is taken.

Scalars and vectors

Length and time can be described by a single number specifying size, but many physical quantities have a directional character.

A **scalar** quantity has magnitude (size) only. Time is a scalar and is completely described when its value is known. Other examples of scalars are distance, speed, time, mass, pressure, energy and **temperature**.

A **vector** quantity is one such as force which is described completely only if both its size (magnitude) and direction are stated. It is not enough to say, for example, a force of 10 N, but rather a force of 10 N acting vertically downwards. Gravitational field strength and electric field strength are vectors, as are weight, velocity, acceleration and momentum.

A vector can be represented by a straight line whose length represents the magnitude of the quantity and whose direction gives its line of action. An arrow on the line shows which way along the line it acts.

Scalars are added by ordinary arithmetic; vectors are added geometrically, taking account of their directions as well as their magnitudes. In the case of two vectors F_X and F_Y acting at right angles to each other at a point, the magnitude of the resultant F, and the angle θ between F_X and F can be calculated from the following equations:

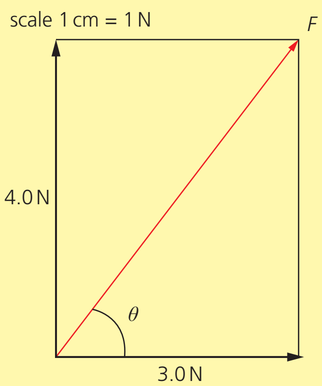
$$F = \sqrt{{F_X}^2 + {F_Y}^2}, \quad \tan\theta = \frac{F_Y}{F_X}$$

The resultant of two vectors acting at right angles to each other can also be obtained graphically.

Worked example

Calculate the resultant of two forces of 3.0 N and 4.0 N acting at right angles to each other.

Let $F_X = 3.0$ N and $F_Y = 4.0$ N as shown in Figure 1.1.12.



▲ **Figure 1.1.12** Addition of two **perpendicular** vectors

Then

$$F = \sqrt{F_X^2 + F_Y^2} = \sqrt{3.0^2 + 4.0^2} = \sqrt{9 + 16} = \sqrt{25} = 5.0\,\text{N}$$

and $\tan\theta = \dfrac{F_Y}{F_X} = \dfrac{4.0}{3.0} = 1.3$

so $\theta = 53°$.

The resultant is a force of 5.0 N acting at 53° to the force of 3.0 N.

Graphical method

The values for F and θ can be found graphically by drawing the vectors to scale on a piece of graph paper as shown in Figure 1.1.12.

First choose a scale to represent the size of the vectors (1 cm could be used to represent 1.0 N).

Draw the vectors at right angles to each other. Complete the rectangle as shown in Figure 1.1.12 and draw the diagonal from the origin as shown. The diagonal then represents the resultant force, F. Measure the length of F with a ruler and use the scale you have chosen to determine its size. Measure the angle θ, the direction of the resultant, with a protractor.

Check that the values for F and θ you obtain are the same as those found using the algebraic method.

Now put this into practice

1. Calculate the following square roots.
   a $\sqrt{6^2 + 8^2}$
   b $\sqrt{5^2 + 7^2}$
   c $\sqrt{2^2 + 9^2}$
2. Calculate
   a tan 30°
   b tan 45°
   c tan 60°.
3. Calculate the resultant of two forces of 5.0 N and 7.0 N which are at right angles to each other.
4. At a certain instant a projectile has a horizontal velocity of 6 m/s and a vertical velocity of 8 m/s.
   a Calculate the resultant velocity of the projectile at that instant.
   b Check your answer to a by a graphical method.

Revision checklist

After studying Topic 1.1 you should know and understand the following:

✓ how to make measurements of length and time intervals, minimise the associated errors and use multiple measurements to obtain average values

✓ the difference between scalars and vectors and recall examples of each.

After studying Topic 1.1 you should be able to:

✓ write a number in powers of ten (standard notation) and recall the meaning of standard prefixes

✓ measure and calculate lengths, areas and volumes of regular objects and give a result with the correct units and an appropriate number of significant figures

✓ determine by calculation or graphically the resultant of two vectors at right angles.

Exam-style questions

1 A chocolate bar measures 10 cm long by 2 cm wide and is 2 cm thick.

a Calculate the volume of one bar. [3]

b How many bars each 2 cm long, 2 cm wide and 2 cm thick have the same total volume? [3]

c A pendulum makes 10 complete oscillations in 8 seconds. Calculate the time period of the pendulum. [2]

[Total: 8]

2 a A pile of 60 sheets of paper is 6 mm high. Calculate the average thickness of a sheet of the paper. [2]

b Calculate how many blocks of ice cream each 10 cm long, 10 cm wide and 4 cm thick can be stored in the compartment of a freezer measuring 40 cm deep, 40 cm wide and 20 cm high. [5]

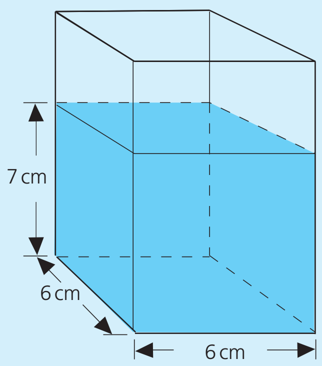
[Total: 7]

3 A Perspex container has a 6 cm square base and contains water to a height of 7 cm (Figure 1.1.13).

a Calculate the volume of the water. [3]

b A stone is lowered into the water so as to be completely covered and the water rises to a height of 9 cm. Calculate the volume of the stone. [4]

[Total: 7]



▲ **Figure 1.1.13**

4 a State the standard units of length and time. [2]

b A measurement is stated as 0.0125 mm. State the number of significant figures. [1]

c Write down expressions for

i the area of a circle [1]

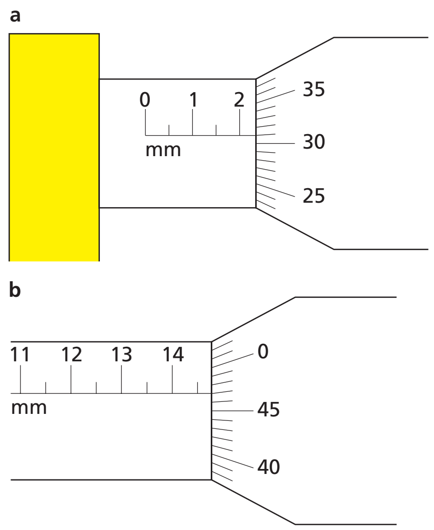
ii the circumference of a circle [1]

iii the volume of a cylinder. [2]

[Total: 7]

Going further

5 What are the readings on the micrometer screw gauges in Figures 1.1.14a and 1.1.14b?



▲ **Figure 1.1.14**

[Total: 4]

6 a Select which of the following quantities is a vector.

A length
B temperature
C force
D time [1]

b Two forces of 5 N and 12 N act at right angles to each other.
Using a piece of graph paper determine the magnitude and direction of the resultant force graphically. State the scale you use to represent each vector. You will need a protractor to measure the angle the resultant makes with the 5 N force. [7]

[Total: 8]