# Homework 10

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**Problem 14.6:** Find the linear-to-linear function whose graph has $y = 6$ as a horizontal asymptote and passes through $(0, 10)$ and $(3, 7)$.

**Solution:** The standard form of a linear to linear rational function is $y = \frac{ax+b}{x+c}$ where $a$, $b$, and $c$, are constants. We are given that the horizontal asymptote is $y = 6$, so our value of $a$ is 6. Our function is then $y = \frac{6x+b}{x+c}$. We are also given two coordinates: $(0, 10)$ and $(3, 7)$. Plugging these two coordinates into our function, we can get the following equations:

$$7 = \frac{18 + b}{3 + c}$$

$$10 = \frac{0 + b}{0 + c}$$

Solving the first equation for $b$, we have $b = 3 + 7c$. Solving the second equation for $b$, we have $10c = b$. Setting these two equations for $b$ equal to each other, we have $3 + 7c = 10c$. Solving for $c$, we have $c = 1$. Plugging this value of $c$ into our second equation, we have $10 = \frac{b}{1}$. Solving for $b$, we get $b = 10$. Plugging these values for $a$, $b$, and $c$, into our rational function, we get

$$\boxed{y = \frac{6x + 10}{x + 1}}$$

**2022 Practice Final Problem 3:** Let $f$ be an unknown one-to-one function, and let $g$ be the function

$$g(x) = f(2f^{-1}(x) + 1)$$

Find a formula for $g^{-1}(x)$ (in terms of $f$ and $f^{-1}$).

**Solution:** To find the inverse of $g$, we can start by switching $g(x)$ and $x$ in the equation for $g(x)$. For simplicity, we can let $y = g(x)$ and rewrite the equation using $y$:

$$x = f(2f^{-1}(y) + 1)$$

Now, we need to solve this equation for $y$. We can start by applying $f^{-1}$ to both sides:

$$f^{-1}(x) = 2f^{-1}(y) + 1$$

Simplifying:

$$\frac{f^{-1}(x) - 1}{2} = f^{-1}(y)$$

Now, we can apply $f$ to both sides of the equation. This would make our formula for $g^{-1}(x)$ be

$$f\left(\frac{f^{-1}(x) - 1}{2}\right) = y$$

or

$$\boxed{g^{-1}(x) = f\left(\frac{f^{-1}(x) - 1}{2}\right)}$$

**2022 Practice Final Problem 7:** The population of the world used to be growing exponentially, but lately it's been leveling out. One way to model this is with the following function:

$$P(t) = 8.5^{\frac{t-90}{t+10}}$$

where t is the year and $P(t)$ is the population in year $t$, measured in billions. Based on this model, when will there be 8 billion people in the world?

**Solution:** To find when there are 8 billion people, we can set $P(t)$ equal to 8:

$$8 = 8.5^{\frac{t-90}{t+10}}$$

Writing this as a logarithm:

$$\log_{8.5}(8) = \frac{t-90}{t+10}$$

Simplifying:

$$t\log_{8.5}(8) + 10\log_{8.5}(8) = t - 90$$

$$t(\log_{8.5}(8) - 1) = -10\log_{8.5}(8) - 90$$

Solving for $t$, the population will be 8 billion after

$$\boxed{t = \frac{-10\log_{8.5}(8) - 90}{\log_{8.5}(8) - 1} \text{ years.}}$$

**2022 Practice Final Problem 8:** Rewrite the expression

$$\log_4 x - \log_{16} x$$

as a single logarithm.

**Solution:** Using the change of base formula to write $\log_{16} x$ as a logarithm with base 4, we have

$$\log_4 x - \frac{\log_4 x}{\log_4 16}$$

Simplifying:

$$\log_4 x - \frac{\log_4 x}{2}$$

$$\frac{2\log_4 x}{2} - \frac{\log_4 x}{2}$$

$$\boxed{\frac{1}{2}\log_4 x}$$

**2019 Practice Final Problem 5:** Find all solutions to the equation

$$\log_5(x+2) + \log_5(x+3) = \log_5(2x+4)$$

**Solution:** Using the logarithmic rule $\log_a x + \log_a y = \log_a (xy)$, we can rewrite this as

$$\log_5 ((x+2)(x+3)) = \log_5 (2x+4)$$

Because logarithms are one to one, this equation is the same as

$$(x+2)(x+3) = 2x+4$$

Solving for $x$:

$$x^2 + 3x + 2 = 0$$

$$x = -1,\ x = -2$$

From these solutions, the solution $x = -2$ is extraneous. If $x = -2$ is a solution, then when we substitute $-2$ for $x$ into the initial equation, the logarithm $\log_5 x + 2$ will be $\log_5 0$. The argument of a logarithm must be greater than 0 (as a number to the power of something cannot be less than or equal to 0). Hence, $x = -2$ is not a solution and the answer is $\boxed{x = -1.}$