Homework 4

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Problem 2.7: Brooke is located 5 miles out from the nearest point A along a straight shoreline in her seakayak. Hunger strikes and she wants to make it to Kono's for lunch; see picture. Brooke can paddle 2 mph and walk 4 mph.

Part (a): If she paddles along a straight line course to the shore, find an expression that computes the total time to reach lunch in terms of the location where Brooke beaches the boat.

Part (b): Determine the total time to reach Kono's if she paddles directly to the point "A".

Part (c): Determine the total time to reach Kono's if she paddles directly to Kono's.

Part (d): Do you think your answer to (b) or (c) is the minimum time required for Brooke to reach lunch?

Part (e): Determine the total time to reach Kono's if she paddles directly to a point on the shore half way between point "A" and Kono's. How does this time compare to the times in parts (b) and (c)? Do you need to modify your answer to part (d)?

Solution Part (a): To solve, we can start by finding the distance between Brooke and the location where she beaches the boat as well as the distance between the location where she beaches the boat and Kono's. We can define the distance between point A and the location where Brooke beaches the boat as x. Therefore, using the Pythagorean Theorem, we can define the distance between Brooke and the place where she beaches the boat as

$$\sqrt{5^2 + x^2}$$

Because the distance between point A and Kono's is 6 miles, we can define the distance between the location where she beaches the boat and Kono's as

$$6 - x$$

The distance formula is $d = rt$ where d is the distance, r is the rate, and t is the time. We know that Brooke's paddling speed is 2 mph and walking speed is 4 mph, and we just solved for both distances. Therefore, we can write the total time it takes to get to Kono's as

$$\boxed{(\frac{\sqrt{5^2 + x^2}}{2} + \frac{6 - x}{4}) \text{ hours.}}$$

Solution Part (b): To find the total time to reach Kono's if Brooke paddles directly to point A, we have to find the distance from Brooke to point A and the distance from point A to Kono's. The distance from Brooke to point A is given as 5 miles and the distance from point A to Kono's is given as 6 miles. Using the distance formula, we can say Brooke's total time is each distance divided by the rate at which she travels that distance. Therefore, her total time would be

$$\boxed{(\frac{5}{2}+\frac{6}{4})\text{ hours.}}$$

Solution Part (c): To find the time it takes Brooke to paddle directly to Kono's, we have to find her distance from Kono's using Pythagorean Theorem. Because Brooke is 5 miles from point A and point A is 6 miles from Kono's, we can express Brooke's distance from Kono's as

$$\sqrt{5^2+6^2}=\sqrt{61}\text{ miles.}$$

We know that Brooke's paddling speed is 2 mph. Using the distance formula, we can find that her time to travel directly to Kono's will be

$$\boxed{\frac{\sqrt{61}}{2}\text{ hours.}}$$

Solution Part (d): I do not think my answer to Part (b) or (c) is the minimum time required for Brooke to reach Kono's.

An example:
Using the answer I got in Part (a), I know that if I substituted the number 2 for x in my answer, Brooke's time would then be:

$$\frac{\sqrt{25+2^2}}{2}+\frac{6-2}{4}$$

Which is equal to

$$(\frac{\sqrt{29}}{2}+1)\text{ hours.}$$

Although $(\frac{\sqrt{29}}{2}+1)$ hours isn't the least amount of time Brooke can take to get to Kono's, it's less than the values I got in both parts (b) and (c), meaning that $\boxed{\text{the answers for (b) and (c) are not the minimum time required.}}$

Solution Part (e): The point that is halfway between point A and Kono's is 3 miles out from point A, because the distance between the two is 6 miles. As we defined in Part (a), the variable x represents the distance from point A and where Brooke beaches her boat. Therefore, we can use the expression/solution of Part (a) to solve this problem. Substituting 3 for x, we can say that the time she takes in hours can be represented as

$$\frac{\sqrt{5^2+3^2}}{2}+\frac{6-3}{4}$$

Simplifying:

$$\boxed{(\frac{\sqrt{34}}{2}+\frac{3}{4})\text{ hours.}}$$

$(\frac{\sqrt{34}}{2}+\frac{3}{4})$ hours is $\boxed{\text{less than the times we got in Part (b) and (c).}}$

$\boxed{\text{I do not have to modify my answer to Part (d)}}$ because I already proved that my answers to Part (b) and (c) are not the minimum time required for Brooke to get to Kono's.

Problem 2.8: A spider is located at the position $(1, 2)$ in a coordinate system, where the units on each axis are feet. An ant is located at the position $(15, 0)$ in the same coordinate system. Assume the location of the spider after t minutes is $s(t) = (1 + 2t, 2 + t)$ and the location of the ant after t minutes is $a(t) = (15 - 2t, 2t)$.

Part (a): Sketch a picture of the situation, indicating the locations of the spider and ant at times $t = 0, 1, 2, 3, 4, 5$ minutes. Label the locations of the bugs in your picture, using the notation $s(0), s(1), ..., s(5), a(0), a(1), ..., a(5)$.

Part (b): When will the x-coordinate of the spider equal 5? When will the y-coordinate of the ant equal 5?

Part (c): Where is the spider located when its y-coordinate is 3?

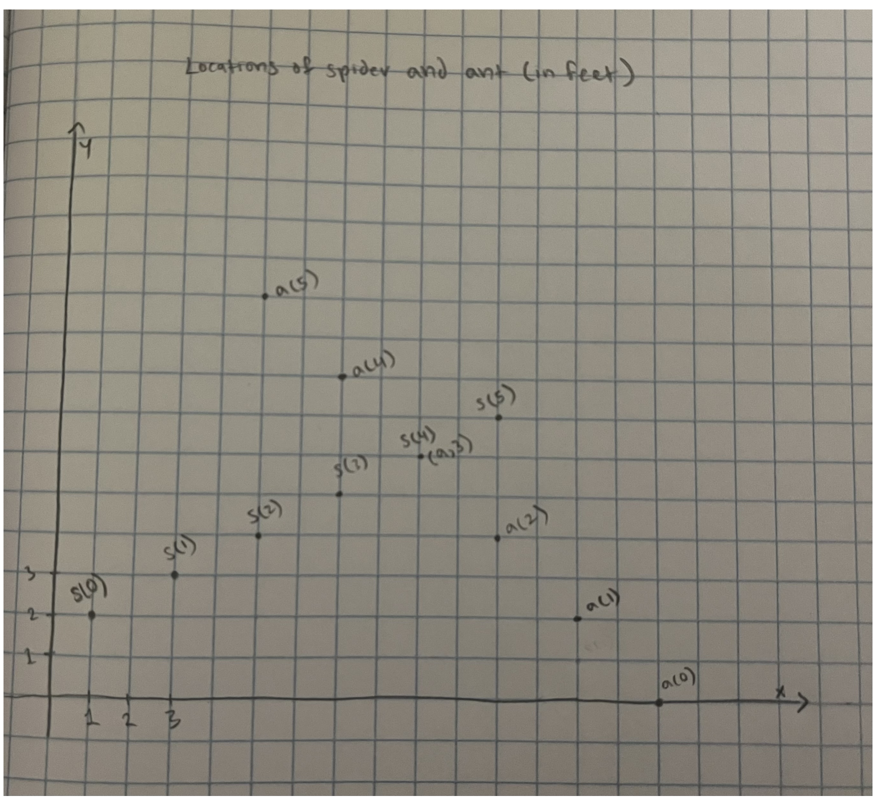
Part (d): Where is each bug located when the y-coordinate of the spider is twice as large as the y-coordinate of the ant?

Part (e): How far apart are the bugs when their x-coordinates coincide? Draw a picture, indicating the locations of each bug when their x-coordinates coincide.

Part (f): A sugar cube is located at the position $(9, 6)$. Explain why each bug will pass through the position of the sugar cube. Which bug reaches the sugar cube first?

Part (g): Find the speed of each bug along its line of motion; which bug is moving faster?

Part (a) Solution:



Part (b) Solution: The x-coordinate of the spider $\boxed{\text{will equal 5 after two minutes.}}$ The y-coordinate of the ant $\boxed{\text{will equal 5 after 2.5 minutes}}$ because the ant will reach this point halfway between points $a(2)$ and $a(3)$ (see drawing).

Part (c) Solution: The spider will be located at point $(3, 3)$ when its y-coordinate is 3 (see drawing).

Part (d) Solution: To solve this problem, we can use the expression given that defines $s(t)$ and $a(t)$. Because the y-coordinate of the spider is twice as large as the y-coordinate of the ant, we can write:

$$2 + t = 2 \cdot 2t$$

Solving for t:

$$t = \frac{2}{3}$$

This means that the y-coordinate of the spider will be twice as large as the y-coordinate of the ant after $\frac{2}{3}$ minutes. Plugging in $\frac{2}{3}$ for t in both functions, $\boxed{\text{the location of the ant will be at } (\frac{41}{3}, \frac{2}{3})}$ and $\boxed{\text{the location of the spider will be at } (\frac{7}{3}, \frac{8}{3}).}$

Part (e) Solution: For each of their x-coordinates to coincide, the $x-$values have to be equal to each other. This means we can write:

$$1 + 2t = 15 - 2t$$

and

$$t = \frac{7}{2}$$

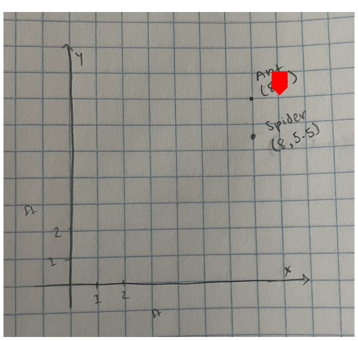
The distance between the two bugs when their x-coordinates coincide will be the distance between their y-coordinates because they will be located on the same vertical line. We can plug in $\frac{7}{2}$ into their y-coordinates.

Ant's y-coordinate: $\frac{7}{2} \cdot 2 = 7$

Spider's y-coordinate: $\frac{7}{2} + 2 = \frac{11}{2}$

To find the distance between the two, we can subtract $\frac{11}{2}$ from 7 to get that $\boxed{\text{they are } \frac{3}{2} \text{ feet apart.}}$

Picture:



Part (f) Solution: Each bug will pass through the point (9,6) because if we set each of their x-coordinates equal to 9 and each of their y-coordinates equal to 6, when we solve for the value of t for each bug, they will be equivalent. For example, when solving for t in the spider's x-coordinate and y-coordinate, the value of t will be the same for each. To determine which bug will pass through the sugar cube first, we have to find the when each one will pass through it. To do this, we have to solve for t for each bug.

Spider:

$$1 + 2t = 9$$
$$t = 4$$

Ant:

$$15 - 2t = 9$$
$$t = 3$$

Because the value of t is greater for the spider, we know that $\boxed{\text{the ant will reach the sugar cube first}}$ because it takes less time to get to the sugar cube.

Part (g) Solution: To solve this problem, we can use the distance formula. To find speed, we have to know distance and time. We can pick any two random points for each bug, and then calculate the distance between the two. The difference in t values used the calculate each of the bugs' coordinates is their time. We can start by calculating the speed of the spider. Using the image in Part (a), we can use coordinates (1, 2) and (3, 3). Because coordinate (1, 2) is defined as $s(0)$ and coordinate (3, 3) is defined as $s(1)$, we can say that the spider's time it takes to travel between these two points is 1 minute. Using the distance formula, we can say that the distance between these two points is

$$\sqrt{(1-3)^2 + (2-3)^2}$$

Simplifying:

$$(\sqrt{5})\,\text{feet.}$$

Because distance is equal to speed multiplied by time, we can express the spider's speed as

$$\frac{\sqrt{5}}{1}\ \text{feet per minute.}$$

To find the speed of the ant, we can repeat this process. We have to find two points that the ant reaches 1 minute apart from each other. We can use the points (15, 0) and (13, 2). Because (15, 0) is defined as $a(0)$ and (13, 2) is defined as $a(1)$, we know that the time the ant takes to travel the distance between the two is 1 minute. Using the distance formula, we can find the distance between the two.

$$\sqrt{(15-13)^2 + (0-2)^2}$$

Simplifying, the distance between the two points is:

$$2\sqrt{2}\,\text{feet.}$$

Therefore, we can say the ant's speed is

$$\frac{2\sqrt{2}}{1}\ \text{feet per minute.}$$

Because $2\sqrt{2}$ is greater than $\sqrt{5}$, we can say that $\boxed{\text{the ant is faster than the spider.}}$

Problem 2.11: Here is a list of some algebra problems with "solutions." Some of the solutions are correct and some are wrong. For each problem, determine: (i) if the answer is correct, (ii) if the steps are correct, (iii) identify any incorrect steps in the solution (noting that the answer may be correct but some steps may not be correct).

Part (a):

$$\begin{aligned} &\text{if}\, x \neq 1, \\ \frac{x^2-1}{x+1} &= \frac{x^2+(-1)1}{x+1} \\ &= \frac{x^2}{x} + \frac{-1}{1} \\ &= x-1 \end{aligned}$$

Part (b):

$$\begin{aligned} (x+y)^2-(x-y)^2 &= (x^2+y^2)-x^2-y^2 \\ &= 0 \end{aligned}$$

Part (c):

$$\begin{aligned} &\text{If}\, x \neq 4 \\ \frac{9(x-4)^2}{3x-12} &= \frac{3^2(x-4)^2}{3x-12} \\ &= \frac{(3x-12)^2}{3x-12} \\ &3x-12 \end{aligned}$$

Part (a) Solution: We can start by slightly simplifying the equation to say

$$\frac{x^2-1}{x+1} = \frac{x^2-1}{x+1}$$

The problem states that x is not equal to -1. Therefore, dividing (x^2-1) by $(x+1)$ will give us $(x-1)$. The solution given is $(x-1)$, so $\boxed{\text{the solution is correct.}}$ However, the $\boxed{\text{steps in the solution are incorrect}}$. Specifically, the $\boxed{\text{step that is incorrect is the second line in the solution.}}$ On the right side of the equation, the fraction

$$\frac{x^2-1}{x+1} = \frac{x^2+(-1)1}{x+1}$$

was simplified into

$$= \frac{x^2}{x} + \frac{-1}{1}$$

The fraction cannot be split like this, because the denominator $(x+1)$ cannot be split into x and 1 in different fractions.

Part (b) Solution: To find whether the solution is correct, we can simplify the equation.

$$\begin{aligned}(x+y)^2-(x-y)^2 &= (x^2+y^2)-x^2-y^2\\ x^2+2xy+y^2-x^2+2xy+y^2 &= x^2+y^2-x^2-y^2\\ &= 0\end{aligned}$$

The problem states the solution to the equation is 0, and we found that this $\boxed{\text{solution is correct}}$. The $\boxed{\text{steps to solve this problem are correct}}$ because the right side of the equation has been simplified to 0 correctly.

Part (c) Solution: To find whether the solution given is correct, we can solve the equation:

$$\begin{aligned}\frac{9(x-4)^2}{3x-12} &= \frac{3^2(x-4)^2}{3x-12}\\ &= \frac{9(x-4)^2}{3x-12}\\ &= 3(x-4)\\ &= 3x-12\end{aligned}$$

This solution matches, so $\boxed{\text{the solution given is correct.}}$ The $\boxed{\text{steps described in the problem are correct}}$.