Homework 10

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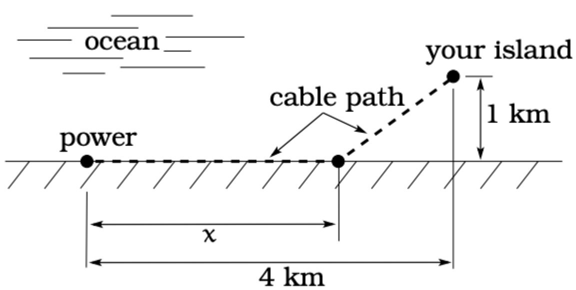
Problem 5.7: After winning the lottery, you decide to buy your own island. The island is located 1 km offshore from a straight portion of the mainland. There is currently no source of electricity on the island, so you want to run a cable from the mainland to the island. An electrical power sub-station is located 4 km from your island's nearest location to the shore. It costs \$50,000 per km to lay a cable in the water and \$30,000 per km to lay a cable over the land.



Part (a): Explain why we can assume the cable follows the path indicated in the picture; i.e. explain why the path consists of two line segments, rather than a weird curved path AND why it is OK to assume the cable reaches shore to the right of the power station and the left of the island.

Part (b): Let x be the distance downshore from the power sub-station to where the cable reaches the land. Find a function $f(x)$ in the variable x that computes the cost to lay a cable out to your island.

Part (c): Make a table of values of $f(x)$, where $x = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots, \frac{7}{2}, 4$. Use these calculations to estimate the installation of minimal cost.

Part (a) Solution: We can assume the cable consists of two line segments instead of a curved path because the problem explicitly states the price of laying a cable in the water and cable over land, suggesting that the cable runs along both land and water (forming two line segments). Also, cable usually takes the shortest/simplest distance between two points. Taking a curved path is just unnecessary because it would be a waste of cable (as a curved path would be longer) and therefore money. It is okay to assume the cable reaches shore to the right of the power station and the left of the island because the right/left directions do not matter. Choosing to have the cable reach the shore to the left of the power station and the right of the island would simply result in the mirror image of the picture provided, and therefore not make any difference.

Part (b) Solution: To find the cost of laying down a cable to the island, we have to find the cost of laying down the cable on land and the cost of laying down the cable on the water separately. The problem states that laying down the cable on land costs \$30000 per km. If the distance downshore from the power sub-station to where the cable reaches land is x, then the cost of laying down the cable on the land would be \$30000x. Now, we have to find the cost of laying down the cable path on the water. The horizontal distance between where the cable reaches land to the island is $4 - x$ km. We know that the vertical distance from the point where the cable reaches land to the island is 1 km. Now, we can use the distance formula to find the length of the cable laid on the water. The length would be $\sqrt{(4-x)^2+1}$. The cost of laying down a cable on land is \$50000 per km, so the cost of laying down cable for this distance would be \$50000 $\cdot\sqrt{(4-x)^2+1}$. The cost of laying the cable out to the island is the sum of the cost of laying the cable on land and the cost of laying the cable on water, meaning that the function to compute the total cost would be $\boxed{f(x) = 30000x + 50000\sqrt{(4-x)^2+1}.}$

Part (c) Solution:

x	$f(x)$
0	≈ 206155.28
0.5	≈ 197002.75
1	≈ 188113.88
1.5	≈ 179629.12
2	≈ 171803.40
2.5	≈ 165138.78
3	≈ 160710.68
3.5	≈ 160901.70
4	≈ 170000

The minimal cost from this table is \$160710.68, when $x = 3$. The closest x-values to 3 are 2.5 and 3.5, meaning that the minimum cost of installation must be when $\boxed{2.5 < x < 3.5.}$

Problem 5.8: This problem deals with the "mechanical aspects" of working with the rule of a function. For each of the functions listed in (a)-(c), calculate: $f(0)$, $f(-2)$, $f(x+3)$, $f(\heartsuit)$, $f(\heartsuit + \triangle)$.

Part (a): The function $f(x) = \frac{1}{2}(x-3)$ on the domain of all real numbers.

Part (b): The function $f(x) = 2x^2 - 6x$ on the domain of all real numbers.

Part (c): The function $f(x) = 4\pi^2$.

Part (a) Solution: We can find $f(0)$ by substituting 0 for x. Therefore, $f(0) = \frac{1}{2}(0-3) = -1.5$. We can repeat the same process to find $f(-2)$. Substituting -2 for x, we can say $f(-2) = \frac{1}{2}(-2-3) = -2.5$. Substituting $x+3$ for x, we can say $f(x+3) = \frac{1}{2}(x+3-3) = 0.5x$. Substituting $\heartsuit$ for x, we can say $f(\heartsuit) = \frac{1}{2}(\heartsuit - 3) = 0.5\heartsuit - 1.5$. Substituting $\heartsuit + \triangle$ for x, we can say $f(\heartsuit + \triangle) = 0.5\heartsuit + 0.5\triangle - 1.5$.

Part (b) Solution: Substituting 0 for x, we can say $f(0) = 2(0)^2 - 6(0) = 0$. Substituting -2 for x, we can say $f(-2) = 2(-2)^2 - 6(-2) = 20$. Substituting $x+3$ for x, we can say $f(x+3) = 2(x+3)^2 - 6(x+3) = 2x^2 + 6x$. Substituting $\heartsuit$ for x, we can say $f(\heartsuit) = 2(\heartsuit)^2 + 6(\heartsuit)$. Substituting $\heartsuit + \triangle$ for x, we can say $f(\heartsuit + \triangle) = 2\heartsuit^2 + 4\heartsuit \cdot \triangle + 2\triangle^2 - 6\heartsuit - 6\triangle$.

Part (c) Solution: In the function $f(x) = 4\pi^2$, there is no x in the formula for the function. This means that we cannot substitute 0, -2, $x+3$, $\heartsuit$. or $\heartsuit + \triangle$ for x. For the function of all of these values, the result will be $4\pi^2$.

Problem 5.9: Which of the curves in Figure 5.14 represent the graph of a function? If the curve is not the graph of a function, describe what goes wrong and how you might "fix it." When you describe how to "fix" the graph, you are allowed to cut the curve into pieces and such that each piece is the graph of a function. Many of these problems have more than one correct answer.

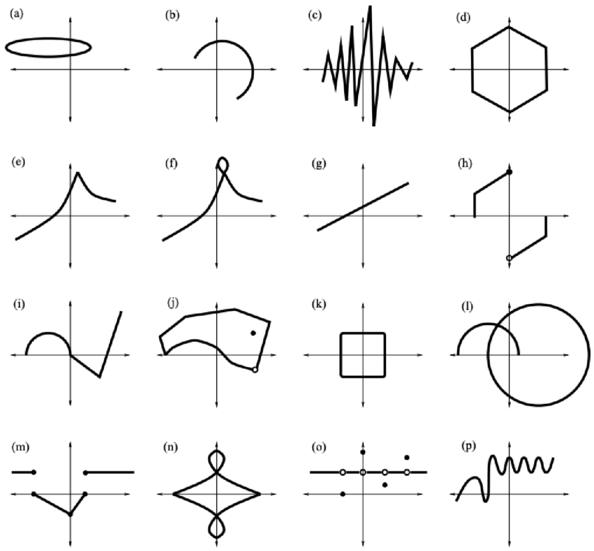


Figure (a): Figure (a) is not a function because there are x-values which appear to have more than one y-value. It could be fixed by cutting the ellipse in half horizontally. We could then move either half to the left or to the right in a way such that no x-values would have more than one y-value.

Figure (b): Not a function because there are multiple y-values for some of the x-values to the right of the y-axis. We could fix this by taking the points on the graph below the x-axis and moving them to the right so that no x-values have more than one y-value.

Figure (c): This is a function.

Figure (d): This is not a function because every single x-value that have more than one y-value. This can be fixed by removing the two segments of the hexagon that are vertical lines. Then, we could move either the top two line segments or bottom two line segments to either the left or right so that all x-values have at max one y-value.

Figure (e): Is a function.

Figure (f): Not a function because some x-values have more than one y-value. We could fix this by removing the loop at the top of the graph.

Figure (g): Is a function.

Figure (h): This is not a function because of the vertical line segments. We can fix this by removing the vertical segments entirely.

Figure (i): Is a function.

Figure (j): This is not a function because there are multiple y-values for certain x-values. We can fix this by first removing the point in the middle of the shape. Then we can cut the shape by drawing a horizontal line that intersects the left-most vertex of the shape. We can then shift the bottom half of the shape to the left or right so that no x-values have more than one y-value.

Figure (k): Not a function because there are multiple y-values for certain x-values. We can fix this by removing the vertical line segments of the graph. Then, we can shift either the top or bottom horizontal line segment to the left or right so that the resulting graph passes the vertical line test.

Figure (l): Not a function because there are multiple y-values for some of the x-values. We can fix this by first moving the smaller semicircle to the left so that it does not intersect the circle. Then, we can cut the circle horizontally so that it forms two semicircles. We can then move either semicircle to the right so no x-values have more than one y-value.

Figure (m): This is not a function because there are three x-values that have two y-values. We can fix this by changing the closed-dot ends of the two line segments to open dots instead.

Figure (n): This is not a function because there are multiple y-values for some of the x-values. We can fix this by first removing the loops at the top and bottom of the graph. Then, we can take the resulting graph and cut it down the middle horizontally. We can move either of these pieces to the right or left so that no x-values have more than one y-value.

Figure (o): Is a function.

Figure (p): Is a function.