Homework 2

Sahana Sarangi

October 9th, 2023

Problem 1.5: Marathon runners keep track of their speed using units of pace = minutes/mile.

Part (a): Lee has a speed of 16 ft/sec; what is his pace?

Part (b): Allyson has a pace of 6 min/mile; what is her speed?

Part (c): Adrienne and Dave are both running a race. Adrienne has a pace of 5.7 min/mile and Dave is running 10.3 mph. Who is running faster?

Solution Part (a): In order to find Lee's pace, we have to convert his speed of 16 ft/sec into minutes per mile. We can start by "flipping" the fraction to write 16 ft/sec in seconds per foot. We could write this as

$$\frac{1\,\text{sec}}{16\,\text{ft}} = \left(\frac{1}{16}\right) \cdot \frac{\text{sec}}{\text{ft}}$$

To convert seconds per foot into minutes per mile, we can multiply $\left(\frac{1}{16}\right) \cdot \frac{\text{sec}}{\text{ft}}$ by each unit conversion. We can start by converting the expression into minutes per foot. To do this, we would write

$$\left(\frac{1}{16}\right) \cdot \frac{\text{sec}}{\text{ft}} \cdot \frac{\text{minutes}}{60\,\text{sec}}$$

Now, to convert from minutes per foot into minutes per mile, we would multiply by the unit conversion for miles to feet. The expression would then look like:

$$\left(\frac{1}{16}\right) \cdot \frac{\text{sec}}{\text{ft}} \cdot \frac{\text{minutes}}{60\,\text{sec}} \cdot \frac{5280\,\text{ft}}{\text{mile}}$$

Simplifying this expression, we can say that Lee's pace is:

$$\frac{5280\,\text{minutes}}{960\,\text{mile}} =$$

$$\boxed{\frac{5.5\,\text{minutes}}{\text{mile}}}$$

Solution Part (b): To find Allyson's speed, we have can start by converting 6 minutes/mile to miles per minute.

$$\frac{6\,\text{minutes}}{\text{mile}} = \left(\frac{1}{6}\right) \cdot \frac{\text{miles}}{\text{minute}}$$

Similarly to part (a), we can convert miles per minute into feet per second by multiplying by each unit conversion. Multiplying by the feet to miles conversion and minutes to seconds conversion, the expression could be written as

$$\left(\frac{1}{6}\right)\cdot\frac{\text{miles}}{\text{minute}}\cdot\frac{5280\,\text{ft}}{\text{miles}}\cdot\frac{\text{minute}}{60\,\text{sec}}$$

Simplifying this expression, we can say that Allyson's speed is:

$$\frac{5280\,\text{ft}}{360\,\text{sec}} =$$

$$\boxed{\frac{\frac{44}{3}\,\text{ft}}{\text{sec}}}$$

Solution Part (c): To determine whether Adrienne or Dave is running faster, we can compare their speeds. To do this, we can compare their speeds in miles per hour. Because Dave's speed is already given in miles per hour, we only have to convert Adrienne's. "Flipping" the fraction like the other parts of the problem, we can convert Adrienne's pace to miles per minute as such:

$$\frac{5.7\,\text{minutes}}{\text{mile}} = \left(\frac{1}{5.7}\right)\cdot\frac{\text{miles}}{\text{minute}}$$

Multiplying the expression by the unit conversion for minutes to hours, we can write it as:

$$\left(\frac{1}{5.7}\right)\cdot\frac{\text{miles}}{\text{minute}}\cdot\frac{60\,\text{minutes}}{\text{hour}}$$

Simplifying, Adrienne's speed would be

$$\frac{60\,\text{miles}}{5.7\,\text{hours}} \approx \frac{10.53\,\text{miles}}{\text{hour}}$$

Because Adrienne's speed of approximately 10.53 miles per hour is greater than Dave's speed of 10.3 miles per hour, $\boxed{\text{Adrienne is running faster.}}$

Problem 1.8: The famous theory of relativity predicts that a lot of weird things will happen when you approach the speed of light $c = 3 \times 10^8$ m/sec. For example, here is a formula that relates the mass m_σ (in kg) of an object at rest and its mass when it is moving at a speed v:

$$m = \frac{m_\sigma}{\sqrt{1-\frac{v^2}{c^2}}}.$$

Part (a): Suppose the object moving is Dave, who has a mass of $m_\sigma = 66$ kg at rest. What is Dave's mass at 90% of the speed of light? At 99% of the speed of light? At 99.9% of the speed of light?

Part (b): How fast should Dave be moving to have a mass of 500 kg?

Solution Part (a): To find Dave's mass at 90% of the speed of light, we can start by calculating that 90% of the speed of light is:

$$3 \times 10^8 \cdot 0.9 = 2.7 \times 10^8\,\text{m/sec}$$

We can now plug in this value and Dave's resting mass into the formula given and say that his mass at 90% of the speed of light is

$$\boxed{\frac{66}{\sqrt{1-\frac{(2.7\times10^8)^2}{(3\times10^8)^2}}}\,\text{kg}}$$

We can repeat this process to find Dave's mass at 99% of the speed of light. We can express 99% of the speed of light as:

$$3 \times 10^8 \times 0.99 = 2.97 \times 10^8$$

We can now plug in this value and Dave's resting mass into the formula. Dave's mass at 99% of the speed of light is

$$\boxed{\frac{66}{\sqrt{1 - \frac{(2.97 \times 10^8)^2}{(3 \times 10^8)^2}}} \text{ kg}}$$

To find Dave's mass at 99.9% of the speed of light, we can find 99.9% of the speed of life:

$$3 \times 10^8 \times 0.999 = 2.997 \times 10^8$$

We can plug in this value and Dave's resting mass into the original formula to get that his mass at the speed of life is:

$$\boxed{\frac{66}{\sqrt{1 - \frac{(2.997 \times 10^8)^2}{(3 \times 10^8)^2}}} \text{ kg}}$$

Solution Part (b): To find how fast Dave has to be moving to have a mass of 500 kg, we would plug in 500 kg for m, 66 kg for m_σ and the speed of light (3×10^8) for c in the formula given. This would make the equation look like:

$$500 = \frac{66}{\sqrt{1 - \frac{v^2}{(3 \times 10^8)^2}}}$$

$$\frac{66}{500} = \sqrt{1 - \frac{v^2}{(3 \times 10^8)^2}}$$

$$-\left(\left(\frac{66}{500}\right)^2 - 1\right) = \frac{v^2}{(3 \times 10^8)^2}$$

$$-\left(\left(\frac{66}{500}\right)^2 - 1\right) \cdot 9 \cdot 10^{16} = v^2$$

And we can say Dave's speed would be:

$$\boxed{\sqrt{-\left(\left(\frac{66}{500}\right)^2 - 1\right) \cdot 9 \cdot 10^{16}} \, \frac{\text{m}}{\text{sec}}}$$

Problem 1.10: *Aleko's Pizza* has delivered a beautiful 16 inch diameter pie to Lee's dorm room. The pie is sliced into 8 equal sized pieces, but Lee is such a non conformist he cuts off an edge as pictured. John then takes one of the remaining triangular slices. Who has more pizza and by how much?

Solution: To start this problem, we can find the area of the sector in which the shaded area (Lee's portion) resides. We know that the diameter of the circle is 16 inches, so the radius of the circle would be 8 inches. Using the formula πr^2 to find the area of the pizza, where r is the radius of the circle, we can say the area of the pizza is 64π square inches. The sector of the circle where the shaded area is marked is $\frac{2}{8}$ of the entire circle because the sector is made up of two out of the eight slices of the pizza. Therefore, the area of the sector is

$$\frac{2}{8} \cdot 64\pi = 16\pi \,\text{square inches.}$$

To find the area of Lee's portion, we have to find the area of the unshaded region of the sector (twice John's portion). To find the area of the unshaded region, we have to find the central angle of the sector. Because we know that the pizza is split up into 8 slices, the sector consists of 2 of those slices, and the pizza is a total of 360 degrees, we can find the sector's central angle by writing:

$$360^\circ \cdot \frac{1}{4} = 90^\circ$$

Because the central angle of the sector is 90°, we know that the unshaded region of the sector is a right triangle. Therefore we can use the formula $\frac{bh}{2}$, where b is the length of the base of the triangle and h is the length of the height of the triangle to find the area of the triangle. In this case, both the height and base of the triangle are radii and are 8 inches long. Substituting 8 for both b and h, we can say the area of the unshaded region is

$$\frac{8 \cdot 8}{2} = 32 \,\text{square inches.}$$

As per the diagram, we know that John's portion of the pizza is half of the unshaded region of the sector. Therefore, the area of John's portion is 16 square inches.

We can represent Lee's portion of the pizza (the shaded region) as the area of the unshaded region subtracted from the total area of the sector. Therefore, the area of Lee's portion would be:

$$(16\pi - 32) \,\text{square inches.}$$

Because $(16\pi - 32)$ square inches is greater than John's portion of 16 square inches, we can say that $\boxed{\text{Lee's portion of the pizza was greater than John's by } (16\pi - 48) \text{ square inches.}}$