Homework 5

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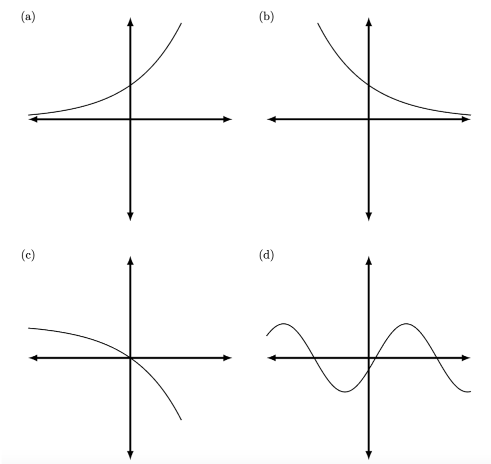
February 5th, 2024

Midterm Practice Problem 6: Consider the function

$$g(x) = 9 \cdot 4^{1-2x}$$

Which of the following graphs is most likely to be the graph of g? Briefly (in one or two sentences) explain your reasoning.



Solution: To solve this problem, we can first convert $g(x)$ into standard exponential form:

$$g(x) = 9 \cdot 4 \cdot 4^{-2x} = 36 \cdot \left(4^{-2}\right)^x = 36\left(\frac{1}{16}\right)^x$$

This is an exponential decay function that is positive because $a > 0$ and $0 < b < 1$. The only option that contains a graph that is both positive and an exponential decay function is $\boxed{\text{option (b).}}$

Midterm Practice Problem 3: Let $f(x) = \frac{1}{2x+3}$. Find $f^{-1}(x)$:

Solution: To start, we can swap $f(x)$ and x in this function. For simplicity, we can let $y = f(x)$ and rewrite this equation using y:

$$x = \frac{1}{2y+3}$$

Solving for y will give us the inverse function:

$$2xy = 1 - 3x$$

$$y = \frac{1-3x}{2x}$$

Replacing y with $f^{-1}(x)$, we can say $\boxed{f^{-1}(x) = \frac{1-3x}{2x}.}$

2020 Practice Midterm Problem 2: Let $f(x) = \frac{2}{x-3}$ and $g(x) = \sqrt{4-x}$. Find the domain of $f(g(x))$.

Solution: To solve this, we can first find $f(g(x))$. We can do this by substituting $g(x)$ for x in the functionn f:

$$f(g(x)) = \frac{2}{\sqrt{4-x}-3}$$

We know that we cannot divide any number by 0, so $\sqrt{4-x}-3 \neq 0$. To find the restriction on x, we can solve the equation $\sqrt{4-x}-3 = 0$. The value of x that satisfies this equation is -5. Therefore, our first restriction on x is that $x \neq -5$.

x is also under the square root, so we must consider that as well. We know that the value under a square root must be greater than or equal to 0 (so that we aren't square rooting a negative number), so $4-x \geq 0$. The solution to this inequality is $x \leq 4$. Therefore, the domain of $f^{-1}(x)$ is $\boxed{x \leq 4 \text{ and } x \neq -5.}$

2020 Midterm Practice Problem 3: Let $f(x) = \frac{5}{x^3-2}$. Find $f^{-1}(x)$.

Solution: To find the inverse, we can first swap $f(x)$ and x. For simplicity, we can let $y = f(x)$ and rewrite the equation using y:

$$x = \frac{5}{y^3-2}$$

Solving for y will give us the inverse function:

$$xy^3 - 2x = 5$$

$$y^3 = \frac{5+2x}{x}$$

$$y = \sqrt[3]{\frac{5+2x}{x}}$$

Replacing y with $f^{-1}(x)$, we can say that $\boxed{f^{-1}(x) = \sqrt[3]{\frac{5+2x}{x}}.}$