Homework 19

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Hard Practice Final Problem 2: A USPS truck and a UPS truck depart from the same location. The USPS truck heads in a direction 7 degrees east of north; the UPS truck goes in a direction 7 degrees north of west. The UPS truck leaves fifteen minutes earlier than the USPS truck, and drives 5 mph faster. The USPS truck leaves at 2:00 PM and drives at 45 miles per hour. When will the trucks be 20 miles apart?

Solution: The trucks are traveling on lines that are perpendicular to each other, so we can use the Pythagorean theorem to calculate the distance between them. We can let t represent the time since the USPS truck has left. The USPS truck travels at a rate of 45 miles per hour, so we can express the USPS's distance from its starting point as $45t$. The UPS truck travels 5 miles per hour faster, or 50 miles per hour. The UPS truck has also left 15 minutes, or $\frac{1}{4}$ of an hour earlier. This means the UPS truck has already traveled $\frac{50}{4}$ miles before the USPS truck starts, or before $t = 0$. Therefore, we can express the UPS truck's distance from its starting point as $\frac{25}{2} + 50t$. Now, we can plug these two parameters into the Pythagorean theorem to find the distance between them:

$$d^2 = (45t)^2 + \left(\frac{25}{2} + 50t\right)^2$$

We are trying to find when the distance between them is 20 miles, so we can rewrite our equation as :

$$400 = (45t)^2 + \left(\frac{25}{2} + 50t\right)^2$$

Solving for t:

$$400 = 2025t^2 + \frac{625}{4} + 1250t + 2500t^2$$

$$\frac{975}{4} - 4525t^2 - 1250t = 0$$

$$t = -\frac{50 - 11\sqrt{79}}{362},\ t = -\frac{50 + 11\sqrt{79}}{362}$$

Time $-\frac{50+11\sqrt{79}}{362}$ is approximately -0.4, while the time $-\frac{50-11\sqrt{79}}{362}$ is approximately 0.13. We know that time can only be positive, so the trucks will be 20 miles apart after $\boxed{-\frac{50-11\sqrt{79}}{362} \text{ hours.}}$

Hard Practice Final Problem 12: A farmer is building a field adjacent to a river. Deer have been eating his crops, so he wants to fence it off. Since the river's too fast for deer to swim in it, he doesn't need to fence off that side. Fencing costs 12 dollars per foot, and he has 6000 dollars to spend. If he wants his field to be rectangular, with one side along the river, what's the greatest area he can enclose?

Solution: We can let x be the length of the shorter side of the rectangle in feet. If fencing costs 12 dollars a foot, the cost of building one of the shorter sides of the rectangle is $12x$ and the cost of building two shorter sides is $24x$. Therefore, the farmer only has $6000 - 24x$ dollars to build the last side of the fencing. We know that the area of the rectangle is its length times its width, so we can express the area (y) as $y = 12x(6000 - 24x)$. Expanding this equation yields $y = 72000x - 288x^2$. Now, we can find the x-coordinate of the vertex of this quadratic:

$$-\frac{b}{2a} = \frac{72000}{576} = 125$$

This means that to maximize the area of the rectangle, the length of the shorter side of the rectangle must be 125 feet. We can find the optimal cost of the longer side of the rectangle by plugging in 125 for x in the expression that represents its price $(6000 - 24x)$. This would result in the price being $6000 - 3000 = 3000$. Therefore, the optimal cost of building the longer side of the rectangle is 3000 dollars. We can recall that the price of the fencing is 12 dollars per foot. This means the length of the longer side should be $\frac{3000}{12} = 250$ feet. Area is length times width, so the maximum area of the rectangle is $\boxed{125 \cdot 250 = 31250 \text{ square feet.}}$