Homework 11

Sahana Sarangi

November 13th, 2023

Problem 6.2: For each of the following functions, graph $f(x)$ and $g(x) = |f(x)|$, and give the multipart rule for $g(x)$.

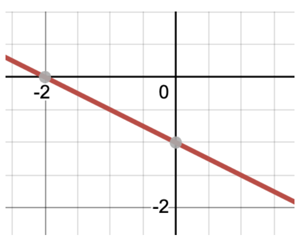
Part (a): $f(x) = -0.5x - 1$

Part (b): $f(x) = 2x - 5$

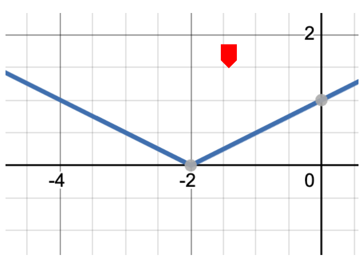
Part (c): $f(x) = x + 3$

Part (a) Solution:

Graph of $f(x) = -0.5x - 1$:



Graph of $g(x) = |f(x)|$:



The graph of $g(x)$ is an absolute value function. However, unfortunately, we cannot write the formula for $g(x)$ as the formula for an absolute value function. We can split the graph of this function by drawing a vertical line that goes through the point $(-2, 0)$. This results in two different lines. We can start by taking the left-hand side line. The slope of this line is $-\frac{1}{2}$ and it intersects the point $(-2, 0)$. This means we can write the equation for this line in point slope form as $y = -\frac{1}{2}(x + 2)$. However, we know that the line stops once it hits the x-axis, or it does not have any points that have an x-value that is greater than -2. This means that the domain for this line would be defined as $x \leq -2$. The right-hand side line has a slope of $\frac{1}{2}$ and intersects the point $(-2, 0)$. This means the equation for the line in point slope form is $y = \frac{1}{2}(x + 2)$. This line does not have any points that have x-values that are less than -2. This means that the domain of this part would be $x \geq -2$. However, the point when $x = -2$ is already included in the domain of the line $y = -\frac{1}{2}(x + 2)$, meaning that the domain of right-hand side line is $x > -2$. This means the multipart rule for $g(x)$ is

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$$g(x) = \begin{cases} -\frac{1}{2}(x+2) & \texttt{if} \quad x \le -2 \\ \frac{1}{2}(x+2) & \texttt{if} \quad x > -2 \end{cases}$$

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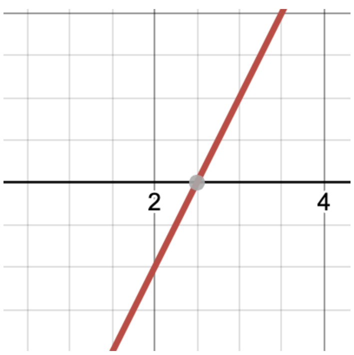
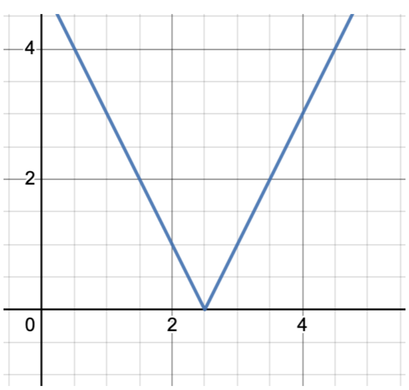
Part (b) Solution:

Graph of $f(x) = 2x - 5$:



Graph of $g(x) = |f(x)|$:



The formula for $g(x)$ is made of two different lines when you split the graph vertically by the line $x = 2.5$. The line with the positive slope has a slope of 2 and intersects the point $(2.5, 0)$. Therefore, the equation for this line in point slope form is $y = 2(x - 2.5)$. This line does not have any points that have an x-value that is less than 2.5, meaning that the domain for this line is $x \ge 2.5$. The line with the negative slope has a slope of -2 and intersects the point $(2.5, 0)$. This means the equation for this line in point slope form is $y = -2(x - 2.5)$. This line has no x-values that are greater than 2.5, meaning the domain for this line is also $x \le 2.5$. However, the point when $x = 2.5$ is already included in the domain of the line $y = 2(x - 2.5)$, which means the domain of $y = -2(x - 2.5)$ is $x < 2.5$. Therefore, the multipart rule for $g(x)$ is

$$g(x) = \begin{cases} 2x - 5 & \texttt{if} \quad x \geq 2.5 \\ -2x + 5 & \texttt{if} \quad x < 2.5 \end{cases}$$

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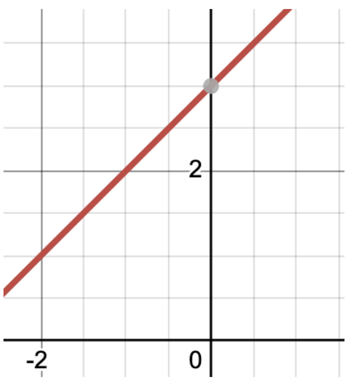
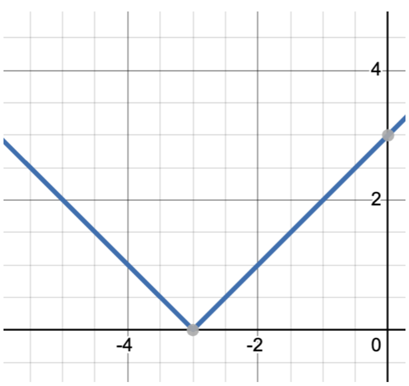
Part (c) Solution:

Graph of $f(x) = x + 3$:



Graph of $g(x) = |f(x)|$:



The graph of $g(x)$ is made of two lines: one with a positive slope and another with a negative slope. The line with the negative slope has a slope of -1 and intersects the point $(-3, 0)$. This means the equation for this line in point slope form is $y = -(x + 3)$. This line also does not have any points that have x-values greater than -3, meaning that the domain for this line is $x \leq -3$. The line with a positive slope has a slope of 1 and intersects the point $(-3, 0)$. This means that the equation for this line in point slope form is $y = x + 3$. This line does not have any x-values that are less than -3, meaning that the domain of this line would be $x \geq -3$. However, the point where $x = -3$ is already included in the domain for $y = -(x + 3)$, meaning that the domain of $y = x + 3$ is $x > -3$. Therefore, the multipart rule for $g(x)$ is

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$$g(x) = \begin{cases} -(x+3) & \texttt{if } x \leq -3 \\ x+3 & \texttt{if } x > -3 \end{cases}$$

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Problem 6.3: Solve each of the following equations for x.

Part (a): $g(x) = 17$, where $g(x) = |3x+5|$.

Part (b): $f(x) = 1.5$, where

$$f(x) = \begin{cases} 2x & \text{if } x < 3 \\ 4 - x & \text{if } x \geq 3 \end{cases}$$

Part (c): $h(x) = -1$, where

$$h(x) = \begin{cases} -8 - 4x & \text{if } x \leq -2 \\ 1 + \frac{1}{3}x & \text{if } x > -2 \end{cases}$$

Part (a) Solution: To solve this, we can substitute 17 for $g(x)$ in $g(x) = |3x+5|$:

$$17 = |3x+5|$$

Now, we can solve for x:

$$3x + 5 = 17,\ 3x + 5 = -17$$

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$$x = 4,\ x = -\frac{22}{3}$$

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Part (b) Solution: To solve this, we can set both $2x$ equal to 1.5 to find x:

$$2x = 1.5$$

$$x = 0.75$$

We know that the domain when $f(x) = 2x$ is $x < 3$. When we solved for x, we found that $x = 0.75$. Because this value fits the domain, we know that

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Part (c) Solution: The first formula for $h(x)$ is $-8-4x$. We can try to find x when $h(x) = -1$ by setting $-8-4x$ equal to -1:

$$-8 - 4x = -1$$
$$x = -\frac{7}{4}$$

We know that the domain of $h(x) = -8-4x$ is $x \le -2$. When solving for x when $h(x) = -8-4x$, we found that $x = -\frac{7}{4}$. This does not fit the domain $x \le -2$, meaning that $h(x)$ does not equal -1 when $h(x) = -8-4x$. We can now try solving for x when $h(x) = 1 + \frac{1}{3}x$. Substituting -1 for $h(x)$:

$$-1 = 1 + \frac{1}{3}x$$

Solving for x:

$$x = -6$$

The domain of $h(x) = 1 + \frac{1}{3}x$ is $x > -2$. $x = -6$ does not fit in this domain, which means that $h(x)$ doesn't equal -1 when $h(x) = 1 + \frac{1}{3}x$.

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( Therefore, there are no real solutions for x when h(x) = -1. )
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Problem 6.4:

Part (a): Let $f(x) = x + |2x-1|$. Find all solutions to the equation $f(x) = 8$.

Part (b): Let $g(x) = 3x - 3 + |x+5|$. Find all values of a which satisfy the equation $g(a) = 2a + 8$.

Part (c): Let $h(x) = |x| - 3x + 4$. Find all solutions to the equation $h(x-1) = x - 2$.

Part (a) Solution: To solve for x, we can substitute 8 for $f(x)$ in the equation $f(x) = x + |2x-1|$:

$$8 = x + |2x - 1|$$

Solving for x:

$$8 - x = |2x - 1|$$
$$2x - 1 = 8 - x,\ 2x - 1 = x - 8$$

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Part (b) Solution: We can start by plugging a into $g(x)$, which would result in

$$g(a) = 3a - 3 + |a + 5|$$

Now that we have two equations for $g(a)$, we can set them equal to each other to solve for a.

$$3a - 3 + |a + 5| = 2a + 8$$

Simplifying:

$$|a + 5| = -a + 11$$

$$a + 5 = -a + 11,\ a + 5 = a - 11$$

Solving the equation

$$a + 5 = -a + 11$$

Will result in $a = 3$. However, if we simplify the equation

$$a + 5 = a - 11,$$

it will result in $5 = -11$, which is false. Therefore, the second equation has no real solutions, meaning that the only real solution for a is

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Part (c) Solution: We can find $h(x-1)$ by substituting $x - 1$ for x in the equation $h(x) = |x| - 3x + 4$:

$$h(x - 1) = |x - 1| - 3(x - 1) + 4$$

Simplifying:

$$h(x - 1) = |x - 1| - 3x + 7$$

Now that we have two equations for $h(x - 1)$, we can set them equal to each other and solve for x:

$$|x - 1| - 3x + 7 = x - 2$$

Solving for x:

$$|x - 1| = 4x - 9$$

$$x - 1 = 4x - 9,\ x - 1 = 9 - 4x$$

$$x = \frac{8}{3},\ x = 2$$

However, $x = 2$ is not a solution. When plugging 2 back into $h(x-1) = x - 2$, we should get a result of 0. When plugging 2 back into $h(x-1) = |x-1| - 3x + 7$, we get a result of 2. Because we are not getting the same output from both equations when $x = 2$, 2 is not a solution. The only solution is

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