October 5 Homework

Sahana Sarangi

October 5th, 2023

Problem 1.2: Sarah can bicycle a loop around the north part of Lake Washington in 2 hours and 40 minutes. If she could increase her average speed by 1 km/hr, it would reduce her time around the loop by 6 minutes. How many kilometers long is the loop?

Solution: Let x be the length of the bicycle loop in kilometers, and let y be the speed of Sarah's bicycle in kilometers per hour. Distance is equal to rate multiplied by time, and we can represent Sarah's time to bike around the loop as $\frac{8}{3}$ hours. Therefore, we can express the length of the loop as

$$x = \frac{8}{3}y$$

We know that if she increases her speed by 1 kilometer per hour, her time will decrease by six minutes. We can represent this reduced time as $\frac{77}{30}$ hours and increased speed as $(y+1)$. Using this information, we can write the equation

$$\frac{x}{y+1} = \frac{77}{30}$$

We can substitute the value for x in the first equation, $\frac{8}{3}y$, for the value of x in the second equation. This gives us

$$\frac{\frac{8}{3}y}{y+1} = \frac{77}{30}$$

We can use cross multiplication to solve for the value of y:

$$\frac{8}{3}y \cdot 30 = 77 \cdot (y+1)$$
$$80y = 77y + 77$$
$$3y = 77$$
$$y = \frac{77}{3}$$

To find the distance around the loop, we would plug in the value for y, $\frac{77}{3}$, into the first equation. The equation would then look like

$$x = \frac{8}{3} \cdot \frac{77}{3} = \frac{616}{9}$$

Therefore, knowing the value of x is the distance in kilometers of the loop, $\boxed{\text{the loop would be } \frac{616}{9} \text{ km long.}}$

Problem 1.3: The density of lead is 11.34 g/cm^3 and the density of aluminum is 2.69 g/cm^3. Find the radius of lead and aluminum spheres each having a mass of 50 kg.

Solution: The formula for the density of an object is

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

We can start by solving for the radius of the lead sphere. Because the formula for the volume of a sphere is $V = \frac{4}{3}\pi r^3$, where r is the radius and V is the volume of the sphere, we can solve for the formula for the radius of a sphere as

$$V \cdot \left(\frac{3}{4}\right) \cdot \left(\frac{1}{\pi}\right) = r^3$$

and cube-rooting both sides of the equation gets us

$$r = \left(\frac{3V}{4\pi}\right)^{\left(\frac{1}{3}\right)}$$

After this, we can plug in the density and mass given in the problem into the equation that solves for the density of an object. Doing this, we get

$$11.34\,\mathrm{g/cm^3} = \frac{50\,\mathrm{kg}}{V}$$

Converting kilograms to grams to ensure all values are in the same units, we get

$$11.34 = \frac{50000}{V}$$

Solving this, we can say

$$V = \frac{50000}{11.34}$$

We can now substitute this value for the value of the volume into the formula for the radius. Doing this can give us the radius of the lead sphere, expressed as:

$$\boxed{r = \left(\frac{3 \cdot \frac{50000}{11.34}}{4\pi}\right)^{\frac{1}{3}}\ \mathrm{cm}}$$

To find the radius of the aluminum sphere, we would repeat the same process. Starting with plugging in the values for density and mass of the sphere given in the problem, we can write the density equation as

$$2.69\,\mathrm{g/cm^3} = \frac{50\,\mathrm{kg}}{V}$$

Converting all values to grams, the equation would be written as

$$2.69 = \frac{50000}{V}$$

Solving for V, we would get

$$V = \frac{50000}{2.69}$$

We can now substitute this value for the value of the volume in the equation solving for the radius of the sphere. Doing this, we can say the radius of the aluminum sphere is

$$\boxed{r = \left(\frac{3 \cdot \frac{50000}{2.69}}{4\pi}\right)^{\frac{1}{3}}\ \mathrm{cm}}$$

Problem 1.4: The Eiffel Tower has a mass of 7.3 million kilograms and a height of 324 meters. Its base is square with a side length of 125 meters. The steel used to make the Tower occupies a volume of 930 cubic meters. Air has a density of 1.225 kg per cubic meter. Suppose the Tower was contained in a cylinder. Find the mass of the air in the cylinder. Is this more or less than the mass of the Tower?

Solution: The approach to this problem is to first find the volume of the cylinder. To find the volume of the air, we would subtract the volume of the Eiffel Tower from the volume of the cylinder. Then, using the volume of the air and density of the air (given) we can solve for the mass of the air.

To start, we can remember that

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

To find the volume of the cylinder, we have to use the formula $\text{CylinderVolume} = \pi r^2 h$, where Cylinder-Volume is the volume, r is the radius of the base of the cylinder, and h is the height of the cylinder. To find the radius of the base, we can remember that the diameter of the base is equal to the length of a diagonal of the square base of the Tower. We know that the side lengths of the Tower's base are 125 meters and the radius the cylinder's base will be half its diameter, so we can write

$$r \cdot 2 = \sqrt{(125)^2 + (125)^2}$$

Then we can solve for r:

$$r = \frac{\sqrt{(125)^2 + (125)^2}}{2}$$

$$r = \frac{125\sqrt{2}}{2}$$

Now, we can plug in the value for the cylinder's height and radius into the equation to find the cylinder's volume. Doing this gives us

$$\text{CylinderVolume} = \pi \cdot \left(\frac{125\sqrt{2}}{2}\right)^2 \cdot 324$$

Solving for CylinderVolume, we can get

$$\text{CylinderVolume} = 2531250\pi \, \text{m}^3$$

We are given that the volume of the Eiffel Tower is $930 \, \text{m}^3$. To find the volume of the air inside the cylinder, we have to subtract the volume of the Eiffel Tower from the volume of the cylinder. Using the variable AirVolume to represent the volume of the air, we can solve for this value:

$$\text{AirVolume} = 2531250\pi - 930$$

Using the first equation mentioned, the formula to find density, we can calculate the mass of the air inside the cylinder. We know that the density of the air is $1.225 \, \text{kg/m}^3$ and the volume of the air is $(2531250\pi - 930) \, \text{m}^3$. Using the variable AirMass to represent the mass of the air, we can write the equation as

$$1.225 = \frac{\text{AirMass}}{2531250\pi - 930}$$

Solving for the variable AirMass, the mass of the air can be written as

$$\boxed{\text{AirMass} = 1.225 \cdot (2531250\pi - 930) \, \text{kg}}$$

The mass of the air can be simplified approximately to $9740252.35 \, \text{kg}$. Because the mass of the Eiffel Tower is $7300000 \, \text{kg}$, $\boxed{\text{the mass of the air inside the cylinder is greater than the mass of the Eiffel Tower.}}$