Homework 4

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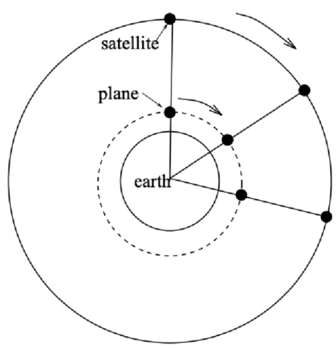
Problem 15.6: An aircraft is flying at the speed of 500 mph at an elevation of 10 miles above the earth, beginning at the North pole and heading South along the Greenwich meridian. A spy satellite is orbiting the earth at an elevation of 4800 miles above the earth in a circular orbit in the same plane as the Greenwich meridian. Miraculously, the plane and satellite always lie on the same radial line from the center of the earth. Assume the radius of the earth is 3960 miles.



(a) When is the plane directly over a location with latitude $74°30'18''$ N for the first time?

(b) How fast is the satellite moving?

(c) When is the plane directly over the equator and how far has it traveled?

(d) How far has the satellite traveled when the plane is directly over the equator?

Part (a) Solution: A location with latitude $74°30'18''$ N is $\left(74 + \frac{30}{60} + \frac{18}{3600}\right)$ degrees above the equator. This is the same as $\frac{268218}{3600}$ degrees above the equator, or 15.495 degrees below the North pole. The plane is traveling at a linear velocity of 500 mph on a circular orbit that is $3960 + 10 = 3970$ miles away from the Earth's center because the plane is traveling 10 miles above the surface of the earth. This means the radius of the plane's circular orbit is 3970 miles. We know that angular velocity is the same as linear velocity divided by radius, so the plane's angular velocity is $\frac{500}{3970}$ radians, or $\left(\frac{18000}{794\pi}\right)^\circ$ per hour. If this is the plane's angular velocity, then for it to travel 15.495 degrees to reach latitude $74°30'18''$ N it would take $\boxed{\frac{15.495 \cdot 794\pi}{18000} \approx 2.15 \text{ hours.}}$

Part (b) Solution: If the plane and the satellite manage to stay on the same radial line at all times, then they must have the same angular velocity which is $\frac{500}{3970}$ radians per hour. The satellite is traveling 4800 miles above the surface of the Earth, which is $4800 + 3960 = 8760$ miles above the center of the Earth. This means the radius of the circular orbit of the satellite is 8760 miles. Linear velocity is just radius multiplied by angular velocity, so the satellite's linear velocity is $\boxed{\frac{500 \cdot 8760}{3970} = \frac{438000}{397} \text{ mph.}}$

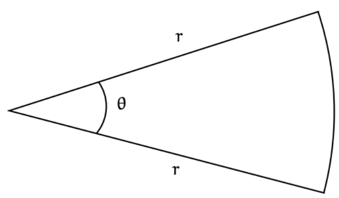
Part (c) Solution: When the plane is directly above the equator, it will have traveled 90 degrees. The plane's angular velocity is $\left(\frac{18000}{794\pi}\right)^\circ$ per hour, so to travel 90 degrees (and be directly above the equator) it would take $\boxed{\frac{90\cdot794\pi}{18000} = \frac{71460\pi}{18000} \text{ hours.}}$ We know the plane's linear velocity is 500 mph, so after $\frac{71460\pi}{18000}$ hours the plane would have traveled $\boxed{\frac{500\cdot71460\pi}{18000} = \frac{35730000\pi}{18000} \text{ miles.}}$

Part (d) Solution: The satellite's linear velocity is $\frac{438000}{397\cdot500}$ of the plane's linear velocity, so if the plane travels $\frac{35730000\pi}{18000}$ miles, the satellite will have traveled

$$\boxed{\frac{438000}{397\cdot500}\cdot\frac{35730000\pi}{18000} \text{ miles}}$$

when the plane reaches the equator.

Problem 15.8: Matilda is planning a walk around the perimeter of Wedge Park, which is shaped like a circular wedge, as shown below. The walk around the park is 2.1 miles, and the park has an area of 0.25 square miles. If θ is less than 90 degrees, what is the value of the radius, r?



Solution: For this problem, we can consider this park to be a sector of a circle. We can let the length of the curved side of the park be x. If the perimeter of the park is 2.1 miles, then we can say $2r + x = 2.1$ and $x = 2.1 - 2r$. The ratio of the area of the sector (0.25) to the area of the whole circle ($r^2\pi$) would be equal to the ratio of the length of the curved side of the sector ($2.1 - 2x$) to the perimeter of the whole circle ($2r\pi$). We then have the following equation:

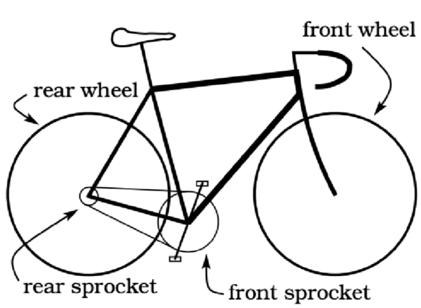
$$\frac{0.25}{r^2\pi} = \frac{2.1 - 2r}{2r\pi}$$

Solving for r, we have $r = \frac{1.05+\sqrt{1.1025-1}}{2} \approx 0.685$, $r = \frac{1.05-\sqrt{1.1025-1}}{2} \approx 0.365$. However, when $r = \frac{1.05-\sqrt{1.1025-1}}{2}$, we have the equation

$$\frac{\theta}{360} = \frac{0.25}{\left(\frac{1.05-\sqrt{1.1025-1}}{2}\right)^2\pi}$$

Solving for θ, we have $\theta \approx 215.126$. We are looking for a value of θ that is less than 90 degrees, so we know that r cannot be $\frac{1.05-\sqrt{1.1025-1}}{2}$. The answer is $\boxed{r = \frac{1.05+\sqrt{1.1025-1}}{2}.}$

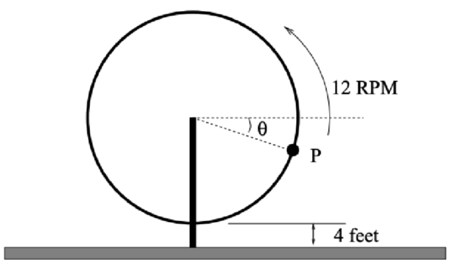
Problem 16.2: You are riding a bicycle along a level road. Assume each wheel is 26 inches in diameter, the rear sprocket has a radius of 3 inches and the front sprocket has a radius of 7 inches. How fast do you need to pedal (in revolutions per minute) to achieve a speed of 35 mph?



Solution: To travel 35 mph, the rear wheel's linear velocity would have to be 184800 feet per hour (fph). Because angular velocity is linear velocity divided by radius (which for the rear wheel is 13 inches, or $\frac{13}{12}$ feet), the rear wheel would need to have an angular velocity of $\frac{2217600}{13}$ radians per hour. Because the rear sprocket and the rear wheel are concentric, the rear sprocket would also have an angular velocity of $\frac{2217600}{13}$ radians per hour. Linear velocity is angular velocity multiplied by radius (which for the rear sprocket is $\frac{3}{12}$ feet), the rear sprocket's linear velocity is $\frac{2217600}{52}$ fph.

The front sprocket and rear sprocket are connected by a belt, so they must have the same linear velocity of $\frac{2217600}{52}$ fph. The front spocket's angular velocity in revolutions per hour is just the linear velocity divided by its perimeter ($\frac{14}{12}\pi$), which is $\frac{2217600 \cdot 12\pi}{52 \cdot 14}$. Because we are asked for how fast we need to pedal in revolutions per minute and not revolutions per hour, we have to divide this angular velocity by 60. We then know that we have to pedal $\boxed{\frac{2217600 \cdot 12\pi}{52 \cdot 14 \cdot 60} \approx 194 \text{ revolutions per minute}}$ to go 35 mph.

Problem 16.5: John has been hired to design an exciting carnival ride. Tiff, the carnival owner, has decided to create the worlds greatest ferris wheel. Tiff isn't into math; she simply has a vision and has told John these constraints on her dream: (i) the wheel should rotate counterclockwise with an angular speed of 12 RPM; (ii) the linear speed of a rider should be 200 mph; (iii) the lowest point on the ride should be 4 feet above the level ground.



(a) Find the radius of the ferris wheel.

(b) Once the wheel is built, John suggests that Tiff should take the first ride. The wheel starts turning when Tiff is at the location P, which makes an angle θ with the horizontal, as pictured. It takes her 1.3 seconds to reach the top of the ride. Find the angle θ.

(c) Poor engineering causes Tiff's seat to fly off in 6 seconds. Describe where Tiff is located (an angle description) the instant she becomes a human missile.

Part (a) Solution: Radius is linear velocity divided by angular velocity. The linear velocity is 200 mph, and the angular velocity is $12 \cdot 2\pi = 24\pi$ radians per minute, or 1440π radians per hour. So the ferris wheel's radius is $\boxed{\frac{200}{1440\pi} = \frac{5}{36\pi} \text{ miles.}}$

Part (b) Solution: 1.3 seconds is $\frac{1.3}{60}$ of a minute. This means that in 1.3 seconds the wheel would have taken $\frac{1.3}{60} \cdot 12 = \frac{15.6}{60}$ revolutions. θ is the angular distance between point P and the eastern most point of the circle (counterclockwise). Traveling from the eastern most point of the circle to the top of the circle counterclockwise is traveling $\frac{1}{4}$ of the whole circle, or $\frac{1}{4}$ of a revolution. Hence the difference between the eastern most point of the circle to the top counterclockwise and point P to the top counterclockwise is $\frac{15.6}{60} - \frac{1}{4} = \frac{0.6}{60} = \frac{1}{100}$ revolutions. θ is then $\frac{1}{100}$ revolutions, which is equivalent to $\boxed{\frac{1}{100} \cdot 2\pi = \frac{\pi}{50} \text{ radians.}}$

Part (c) Solution: 6 seconds is $\frac{1}{10}$ of a minute, so in 6 seconds the ferris wheel would have completed $\frac{1}{10} \cdot 12 = \frac{6}{5}$ revolutions. One revolution is 2π radians, so $\frac{6}{5}$ of a revolution is $\frac{6}{5} \cdot 2\pi = \frac{12\pi}{5}$ radians. $\frac{12\pi}{5}$ radians is two full rotations and $\frac{2\pi}{5}$ radians left over. So if Tiff starts at point P, she will make two full rotations from P and then travel another $\frac{2\pi}{5}$ radians counterclockwise from P before she flies off. So she will be $\boxed{\frac{2\pi}{5} \text{ radians counterclockwise from P.}}$