Homework 14

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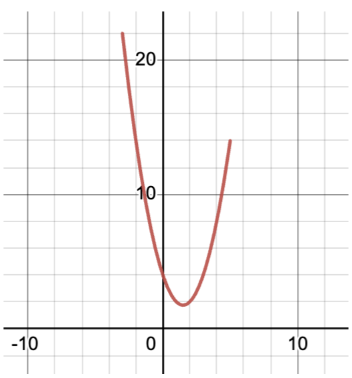
Problem 7.3:

Part (a): Sketch the graph of the function $f(x) = x^2 - 3x + 4$ on the interval $-3 \leq x \leq 5$. What is the maximum value of $f(x)$ on that interval? What is the minimum value of $f(x)$ on that interval?

Part (b): Sketch the graph of the function $f(x) = x^2 - 3x + 4$ on the interval $2 \leq x \leq 7$. What is the maximum value of $f(x)$ on that interval? What is the minimum value of $f(x)$ on that interval?

Part (c): Sketch the graph of the function $g(x) = -(x+3)^2 + 3$ on the interval $0 \leq x \leq 4$. What is the maximum value of $g(x)$ on that interval? What is the minimum value of $g(x)$ on that interval?

Part (a) Solution: Graph of the function:



We know that this quadratic function is increasing away from the vertex, as the a value is positive. Therefore, we can plug in both $x = -3$ and $x = 5$ into the formula and compare the two to find the maximum $f(x)$ value. Plugging in $x = -3$:

$$f(x) = (-3)^2 - 3(-3) + 4$$
$$f(x) = 22$$

Now, plugging in $x = 5$:

$$f(x) = 5^2 - 3(5) + 4$$
$$f(x) = 14$$

Because we got a larger value for $f(x)$ when plugging in $x = -3$, we can say the maximum value of $f(x)$ on this interval is $\boxed{22.}$

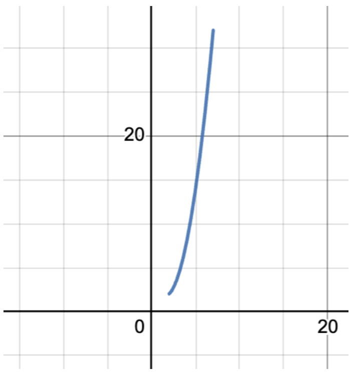
Because this function increases away from the vertex, the vertex is also the minimum value for $f(x)$. We can complete the square to write this equation in vertex form:

$$f(x) = x^2 - 3x + 2.25 - 2.25 + 4$$

$$f(x) = (x - 1.5)^2 + 1.75$$

The y-coordinate of the vertex is 1.75, meaning that the minimum value for $f(x)$ on this interval is $\boxed{1.75.}$

Part (b) Solution: Graph of this function:



This quadratic equation is also increasing away from the vertex because a is positive, so we can plug in $x = 2$ and $x = 7$ into the formula and compare the values. Plugging in $x = 2$:

$$f(x) = 2^2 - 3(2) + 4$$
$$f(x) = 2$$

Now, plugging in $x = 7$:

$$f(x) = 7^2 - 3(7) + 4$$
$$f(x) = 32$$

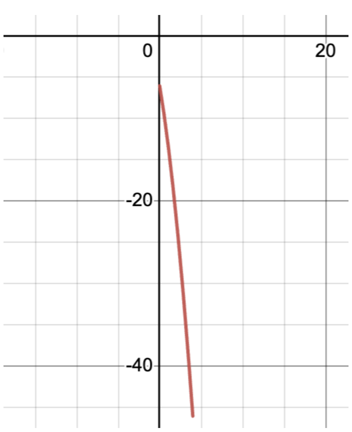
Because we got a larger value for $f(x)$ when plugging in $x = 7$, the maximum value for $f(x)$ is $\boxed{32.}$

To find the minimum in part (a), we first completed the square to write the function in vertex form. As the part (b) function is the exact same, we know that vertex form we solved for in part (a) is applicable here too. We found that the minimum was 1.75 when $x = 1.5$. However, $x = 1.5$ is farther left than the lower bound of the domain, -2, meaning that the minimum of the function is actually when $x = -2$. Substituting -2 for x into the function:

$$f(x) = (2)^2 - 3(2) + 4$$
$$f(x) = 2$$

Therefore, the minimum of the function on this interval is $\boxed{2.}$

Part (c) Solution: Graph of this function:



The a value of this function is negative, meaning that the parabola is facing downwards and is decreasing away from the vertex. We are given the formula of the function in vertex form, so we can say the vertex is $(-3, 3)$. However, because there is a restriction on the domain, 3 is not the maximum of the function. The vertex has an x-value of -3, but this is less than the lower bound of the domain (0). Therefore, we can find the maximum of the function by substituting 0 for x into the formula:

$$g(x) = -(0+3)^2 + 3$$
$$g(x) = -6$$

Therefore, the maximum of this function is $\boxed{-6.}$

Because this function decreases away from the vertex, the minimum value of this function will be when x is the upper boundary of the domain (4). We can substitute 4 for x in the formula:

$$g(x) = -(4+3)^2 + 3$$
$$g(x) = -46$$

Therefore, the minimum of this function is $\boxed{-46.}$

Problem 7.4: If the graph of the quadratic function $f(x) = x^2 + dx + 3d$ has its vertex on the x-axis, what are the possible values of d? What if $f(x) = x^2 + 3dx - d^2 + 1$?

Solution: If the vertex of this function is on the x-axis, it means that the y-coordinate (or k) of its vertex is equal to 0. To find the possible values of d, we will need to rewrite the equation $f(x) = x^2 + dx + 3d$ in vertex form. Recall that vertex form is

$$f(x) = a(x-h)^2 + k$$

We already know that $a = 1$, as the coefficient for the x^2 term is 1. To write the equation in vertex form, we will need to complete the square:

$$f(x) = x^2 + dx + 0.25d^2 - 0.25d^2 + 3d$$

Simplifying:

$$f(x) = (x + 0.5d)^2 - 0.25d^2 + 3d$$

In this equation, $\left(-0.25d^2 + 3d\right)$ is the k value. If the k value must be 0, we can set this expression equal to 0 and solve for d:

$$-0.25d^2 + 3d = 0$$
$$\boxed{d = 0,\ d = 12}$$

We can use the same process to find values for d when the equation is $f(x) = x^2 + 3dx - d^2 + 1$. We can start by rewriting this equation in vertex form:

$$f(x) = x^2 + 3dx + 2.25d^2 - 2.25d^2 - d^2 + 1$$

Simplifying:

$$f(x) = (x + 1.5d)^2 - 3.25d^2 + 1$$

Our k value here is $\left(-3.25d^2 + 1\right)$. k must be 0, so we can set this expression equal to 0 and solve for d:

$$-3.25d^2 + 1 = 0$$
$$\boxed{d = \frac{2}{\sqrt{13}},\ d = -\frac{2}{\sqrt{13}}}$$

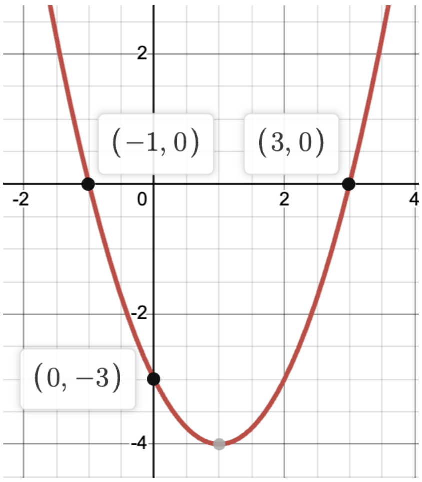
Problem 7.6: Sketch the graph of $y = x^2 - 2x - 3$. Label the coordinates of the x and y intercepts of the graph. In the same coordinate system, sketch the graph of $y = |x^2 - 2x - 3|$, give the multipart rule and label the x and y intercepts of the graph.

Solution: We can find the x-intercepts of the graph by solving for x when $y = 0$:

$$0 = x^2 - 2x - 3$$

$$x = -1,\ x = 3$$

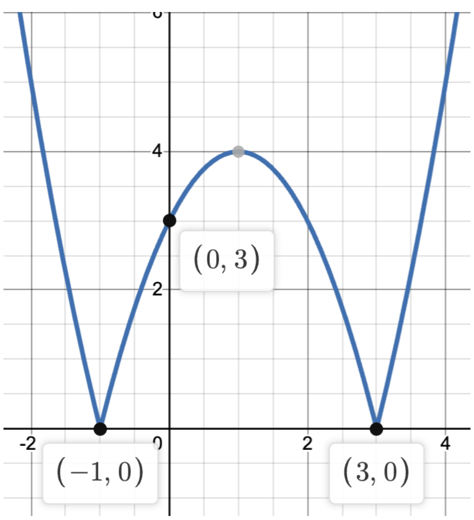
The y-intercept of this graph is when $x = 0$, so the y-intercept is at $(0, -3)$. The graph of the function $y = x^2 - 2x - 3$ is:



The graph of the function $y = |x^2 - 2x - 3|$ is an absolute value function, meaning that it can be written as two different equations:

$$y = x^2 - 2x - 3 \text{ and } y = -x^2 + 2x + 3$$

To start, we can find the graph of the function $y = |x^2 - 2x - 3|$:



The x-intercepts of this graph are $(-1, 0)$ and $(3, 0)$. The y-intercept is $(0, -3)$. We can see that this graph is that of the equation $y = x^2 - 2x - 3$ when $x \leq -1$ and $x \geq 3$. We can confirm this by looking at the x-intercepts and stretch of the two sides of the quadratic. If we wrote the quadratic equation in factored form, it would be $y = (x+1)(x-3)$ which is equivalent to $y = x^2 - 2x - 3$.

The graph of this function is $y = -x^2 + 2x + 3$ when $-1 < x < 3$. We can confirm this by writing the equation of the graph on this domain in vertex form using the vertex and stretch of this part of this quadratic, which would be $y = -(x-1)^2 + 4$. This is equivalent to $y = -x^2 + 2x + 3$. Therefore, the multipart rule for this function is

$$\boxed{|x^2 - 2x - 3| = \begin{cases} x^2 - 2x - 3 \text{ if } x \leq -1 \\ -x^2 + 2x + 3 \text{ if } -1 < x < 3 \\ x^2 - 2x - 3 \text{ if } x \geq 3 \end{cases}}$$