Homework 18

Sahana Sarangi

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Problem 1.2: Sarah can bicycle a loop around the north part of Lake Washington in 2 hours and 40 minutes. If she could increase her average speed by 1 km/hr, it would reduce her time around the loop by 6 minutes. How many kilometers long is the loop?

Solution: We can let x be Sarah's speed. 2 hours and 40 minutes is equivalent to $\frac{8}{3}$ hours. We know that distance is equal to rate multiplied by time. If Sarah can bike around the lake in $\frac{8}{3}$ at a rate of x km/hr, we can express the length of the loop as $\frac{8}{3}x$. We also know that if Sarah increases her speed by 1 km/hr, it will take her six minutes less to bike around the loop. We can express this speed as $x+1$. 6 minutes is equivalent to $\frac{1}{10}$ of an hour, so Sarah's time at this speed will be $\frac{8}{3} - \frac{1}{10} = \frac{80}{30} - \frac{3}{30} = \frac{77}{30}$ hours. Using the same formula that distance is equal to rate time time, we can express the length of the loop as $\frac{77}{30}(x+1)$. Now that we have two expressions for the length of the loop, we know that they must be equal, so we can set them equal to each other:

$$\frac{8}{3}x = \frac{77}{30}(x+1)$$

Simplifying:

$$x = \frac{77}{3}$$

We now know that when Sarah takes $\frac{8}{3}$ hours to bike around the loop, she bikes at a rate of $\frac{77}{3}$ km/hr. To find the distance of the loop, we can use the fact that distance equals rate times time again. Therefore, the $\boxed{\text{distance around the loop would be } \frac{77}{3} \cdot \frac{8}{3} = \frac{616}{9} \text{ kilometers.}}$

Problem 1.4: The Eiffel Tower has a mass of 7.3 million kilograms and a height of 324 meters. Its base is square with a side length of 125 meters. The steel used to make the Tower occupies a volume of 930 cubic meters. Air has a density of 1.225 kg per cubic meter. Suppose the Tower was contained in a cylinder. Find the mass of the air in the cylinder. Is this more or less than the mass of the Tower?

Solution: To solve this problem, we can recall that density is equal to mass divided by volume. First, we can find the mass of the cylinder. We know the height is 324 and the base of the Eiffel tower is inscribed in the base of the cylinder. If the square base has side length 125 meters, we can use the 45-45-90 triangle theorem to find that the diagonal length of the square base (diameter of the cylinder base) is $125\sqrt{2}$. This means the radius of the cylinder is half of this, or $62.5\sqrt{2}$. We know that the volume of a cylinder is its radius squared times its height times π. Therefore, the volume of the cylinder will be $325 \cdot \left(62.5\sqrt{2}\right)^2 \cdot \pi = 325 \cdot 7812.5 \cdot \pi = 2531250\pi$ cubic meters.

We are given that the Eiffel tower has a volume of 930 cubic meters. We know that the volume of the air is the volume of the entire cylinder minus the volume of the Eiffel tower. Therefore, the volume of the air is $2531250\pi - 930$ cubic meters. Now, we can find the mass of the air. We are given that air has a density of 1.225 kg per cubic meter. If mass is density times volume, we know that the mass of the air is $1.225 \cdot (2531250\pi - 930) = \boxed{3100781.25\pi - 1139.25} \approx 9740252.35$ kg. We know that the mass of the Eiffel tower is 7300000 kilograms. Comparing this 7300000 and the approximate value we got for the mass of the air, we can see that the $\boxed{\text{mass of the air is greater than the mass of the tower.}}$

Reflection answers: Comparing my homework 1 solutions with the new ones, I think I have improved in the efficiency of my problem solving. This time around I picked a less time-consuming and hard way of solving the problem. Last time I made a linear equation to represent distance/rate/time which was correct, but not as straightforward as the way I solved it this time. My mathematical writing has also improved because it no longer takes me several hours to type up a homework and this time did not take me 3 pages to explain these 2 problems. I now condense my solutions to leave out things that are unecessary. I no longer explain tedious details that take time and space to type out (such as explaining cross multiplication), and instead am able to describe more in less words. Also, I don't show too many intermediate steps of algebra that are obvious and don't need a lot of explaining. I can improve my problem solving by taking some time to consider whether my approach is the most efficient way to solve the problem before typing up the solution. I can improve my mathematical writing by improving my clarity, which I have gotten feedback on in past homeworks. Sometimes my wording is confusing or needs to be explained in more detail.