Connections 2

Sahana Sarangi

October 16th, 2023

Problem C: A high-speed train is traveling along the equator at 550 miles per hour, towards the west. The radius of the Earth at the equator is 3960 miles. Aboard the train is a snack cart, which is moving backwards from the front of the train at a rate of 4 feet per second. On top of the snack cart is a record player, on which sits a record with a radius of 12 inches. The record is spinning so that it completes a full rotation 33 times every minute. On top of the record are two beetles, a blue one and a red one. Both of them are walking along the surface of the record, so that their paths form straight lines on the record. At 2:00:00 PM, the red beetle is walking due north, the blue beetle is walking due east, and the red beetle is two inches north and three inches west of the blue beetle. Both beetles walk at 1.5 inches per second. When will the distance between the beetles be exactly 12 inches?

Solution: After $-\frac{5-\sqrt{287}}{3}$ seconds, or approximately just before 2:00:04 PM.

Generalized Problem: A high-speed train is traveling along the equator at 550 miles per hour, towards the west. The radius of the Earth at the equator is 3960 miles. Aboard the train is a snack cart, which is moving backwards from the front of the train at a rate of 4 feet per second. On top of the snack cart is a record player, on which sits a record with a radius of 12 inches. The record is spinning so that it completes a full rotation 33 times every minute. On top of the record are two beetles, a blue one and a red one. Both of them are walking along the surface of the record, so that their paths form straight lines on the record. At 2:00:00 PM, the red beetle is walking due south, the blue beetle is walking due west, and the red beetle is three inches north and three inches west of the blue beetle. Both beetles walk at r inches per second. What are possible values of r so that the beetles meet before the record makes one full rotation?

Solution: We can start by finding how long it takes the record to make one full rotation. We know that it makes 33 rotations per minute. Because a minute has 60 seconds, we can say that it takes the record $\frac{60}{33}$ seconds, or $\frac{20}{11}$ seconds to make one full rotation. Because the beetles have to meet before the record makes one full rotation, we know that the beetles will have to meet before $\frac{20}{11}$ seconds. We know that when the beetles meet, they will be 0 inches apart. We also know that the red beetle is three inches north and three inches west of the blue beetle. Therefore, using the Pythagorean Theorem, we can express the distance between them as

$$\sqrt{(3)^2+(3)^2} =$$
$$3\sqrt{2}\,\text{inches.}$$

Now that we know the distance between the beetles, we can write an equation using the distance formula. To write this equation we have to remember that distance is equal to rate multiplied by time. We also know that r is the beetles' speed, and we can use the variable t to represent the time they have been walking for.

$$\sqrt{13} = \sqrt{(rt)^2+(rt)^2}$$

Because the question is asking for possible values of r, we have to solve this equation for t first. After solving for t, we know that whatever expression we get as a result must be less than $\frac{20}{11}$ because the time the beetles take to meet must be less than the time it takes for the record to make one full rotation.

Solving for t:

$$13 = r^2t^2 + r^2t^2$$
$$13 = t^2(2r^2)$$
$$t = \sqrt{\frac{13}{2r^2}}$$

As described earlier, we can use this expression in an inequality such that this expression will be less than $\frac{20}{11}$. This would look like:

$$\sqrt{\frac{13}{2r^2}} < \frac{20}{11}$$

Solving for r:

$$\frac{13}{2r^2} < \frac{400}{121}$$
$$\frac{13}{2r^2} - \frac{400}{121} < 0$$
$$\frac{1573 - 800r^2}{442r^2} < 0$$
$$\frac{800(\sqrt{\frac{1573}{800}} - r)(r + \sqrt{\frac{1573}{800}})}{442r^2} < 0$$
$$\frac{800(r - \sqrt{\frac{1573}{800}})(r + \sqrt{\frac{1573}{800}})}{442r^2} > 0$$
$$r < -\sqrt{\frac{1573}{800}},\ r > \sqrt{\frac{1573}{800}}$$

Because r represents the speed at which the beetles are walking, it is unreasonable for r to be a negative value. Therefore, the values for r that satisfy the condition that the beetles must meet before the record makes one full rotation are $\boxed{\text{all real numbers that are greater than } \sqrt{\tfrac{1573}{800}}.}$