Homework 3

Sahana Sarangi

October 12th, 2023

Problem 2.2: Start with two points $M = (a, b)$and $N = (s, t)$ in the xy-coordinate system. Let d be the distance between these two points. Answer these questions and make sure you can justify your answers:

Part (a): TRUE or FALSE: $d = \sqrt{(a-s)^2 + (b-t)^2}$

Part(b): TRUE or FALSE: $d = \sqrt{(a-s)^2 + (t-b)^2}$

Part (c): TRUE or FALSE: $d = \sqrt{(s-a)^2 + (t-b)^2}$

Part (d): Suppose M is the beginning point and N is the ending point; recall Equation 2.2 and Figure 2.10 on Page 19. What is Δx? What is Δy?

Part (e): Suppose N is the beginning point and M is the ending point; recall Equation 2.2 and Figure 2.10 on Page 19. What is Δx? What is Δy?

Part (f): If $\Delta x = 0$, what can you say about the relationship between the positions of the two points M and N? If $\Delta y = 0$, what can you say about the relationship between the positions of the two points M and N? (Hint: Use some specific values for the coordinates and draw some pictures to see what is going on.)

Part (a) Solution: We know that the distance formula is

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

where d is the distance, x_1 and x_2 are the x-coordinates of the given points, and y_1 and y_2 are the y-coordinates of the given points. Knowing that the points we are given are (a, b) and (s, t), we can plug these values into the distance formula to find the distance between them. This would give us:

$$d = \sqrt{(a-s)^2 + (b-t)^2}$$

This matches the equation given, so we can say that the answer is $\boxed{\text{TRUE.}}$

Part (b) Solution: The distance formula equation we solved for earlier does not match the one given for this problem. The first binomial, $(a - s)$, remains consistent. The second binomial, which we found to be $(b - t)$ is instead flipped in the problem to say $(t - b)$. However, when we have a binomial squared, the order in which the terms are listed does not matter. This means that $(b - t)^2$ would be the same value as $(t - b)^2$. We can prove this by expanding each:

$$(b-t)^2 = b^2 - 2tb + t^2$$

$$(t-b)^2 = t^2 - 2tb + b^2$$

Because these values are the same, we can say the answer to Part (b) is $\boxed{\text{TRUE.}}$

Part (c) Solution: In the distance equation given in the problem, we see the binomials $(s - a)^2$ and $(t - b)^2$. Both of these are the opposite of the binomials we found in the distance equation we created in Part (a). As we found earlier, we know that flipping the order of the variables in a binomial does not change the value of its square. Therefore, we know that the answer to Part (c) is $\boxed{\text{TRUE.}}$

Part (d) Solution: On page 18, $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$. If we know that M is defined as (x_1, y_1) and N is defined as (x_2, y_2), we can plug in those values to find Δx and Δy. Therefore,

$$\boxed{\Delta x = s - a}$$

and

$$\boxed{\Delta y = t - b}$$

Part (e) Solution: Changing whether M or N is the starting point only changes the order in which the variables are written. This means that the point M would now be (x_2, y_2) and point N would now be (x_1, y_1). Therefore,

$$\boxed{\Delta x = a - s}$$

and

$$\boxed{\Delta y = b - t}$$

Part (f) Solution: If $\Delta x = 0$, then the x-values of the coordinates are the same, meaning that the two points are on a vertical line. If $\Delta y = 0$, then the y-values of the coordinates are the same, meaning that the two points are on a horizontal line.

Problem 2.3: Steve and Elsie are camping in the desert, but have decided to part ways. Steve heads North, at 6 AM, and walks steadily at 3 miles per hour. Elsie sleeps in, and starts walking West at 3.5 miles per hour starting at 8 AM. When will the distance between them be 25 miles?

Solution: To solve this problem, we would use Pythagorean theorem. We know that the distance between them (25 miles) will be equal to the square root of Steve's distance squared added to Elsie's distance squared. We know that Steve's speed is 3 mph. We don't know the time that Steve has been walking for, so we can express this as x. Therefore, we can say that Steve's distance is $3x$ miles. For Elsie, we know that she walks 3.5 mph and has been walking for two hours less than Stevie (8 AM is two hours after 6 AM). Therefore, we can say her distance is $(3.5(x - 2))$ miles. Using this information, we can plug these into the Pythagorean Theorem and write the equation

$$25 = \sqrt{(3x)^2 + (3.5(x - 2))^2}$$

Squaring both sides to solve for x:

$$625 = 9x^2 + 12.25x^2 - 49x + 49$$
$$21.25x^2 - 49x - 576 = 0$$

Using the quadratic formula, we can say

$$x = \frac{98 + 2\sqrt{51361}}{85}, \; x = \frac{98 - 2\sqrt{51361}}{85}$$

We know that Steve's time, or x, cannot be a negative number, so only the first solution can be correct. $\frac{98+2\sqrt{51361}}{85}$ hours after 6 AM would be $\boxed{\text{just after 12:29 PM.}}$

Problem 2.6: Allyson and Adrian have decided to connect their ankles with a bungee cord; one end is tied to each person's ankle. The cord is 30 feet long, but can stretch up to 90 feet. They both start from the same location. Allyson moves 10 ft/sec and Adrian moves 8 ft/sec in the directions indicated.

Part (a): Where are the two girls located after 2 seconds?

Part (b): After 2 seconds, will the slack in the bungee cord be used up?

Part (c): Determine when the bungee cord first becomes tight; i.e. there is no slack in the line. Where are the girls located when this occurs?

Part (d): When will the bungee cord first touch the corner of the building? (Hint: Use a fact about "similar triangles".)

Part (a) Solution: We know that Allyson moves 10 ft/sec and Adrian moves 8 ft/sec, meaning that in two seconds $\boxed{\text{Allyson will have moved 20 feet north and Adrian will have moved 16 feet west.}}$

Part (b) Solution: The slack of the cord is used up once the girls are more than 30 feet apart. If after two seconds, Allyson has moved 20 ft and Adrian has moved 16 ft, we can use the Pythagorean Theorem to find the distance between them. We can say that the distance, d can be expressed as

$$d = \sqrt{20^2 + 16^2} = \sqrt{656}\,\text{ft}$$

Because they are only $\sqrt{656}$ feet apart, $\boxed{\text{after two seconds they have not yet used up the slack in the bungee cord.}}$

Part (c) Solution: The bungee cord first becomes tight when the girls are 30 feet apart. If we represent the time they have been moving for as x, using the distance formula we can write this equation:

$$30 = \sqrt{(10x)^2 + (8x)^2}$$

Solving for x:

$$900 = 100x^2 + 64x^2$$
$$x^2 = \frac{900}{164}$$
$$x = \sqrt{\frac{900}{164}}$$

Therefore, the bungee cord will become tight $\boxed{\sqrt{\tfrac{900}{164}} \text{ seconds after they start moving.}}$

Allyson will be $\boxed{10\sqrt{\tfrac{900}{164}} \text{ north of the starting position}}$ and Adrian will be $\boxed{8\sqrt{\tfrac{900}{164}} \text{ west of the starting point}}$

Part (d) Solution: We can say that three triangles can be formed when the bungee cord touches the corner of the building. Only two of these are necessary to use similar triangles to find when the cord will touch the corner. One of these triangles has vertices at the corner of the building, A, Allyson's location, C, and the point on the building directly left of her position, B. Another triangle has a vertex at point C, a vertex at the starting point D, and a vertex at Adrien's location, E. We can say that $\triangle ABC$ is similar to $\triangle CDE$ because both are right triangles and $\angle BAC$ is equivalent to $\angle ECD$ because of Alternate Interior Angles Theorem. If the distance from point D to point C is the distance Allyson has traveled, then we can say $\overline{\text{CD}} = 10t$, where t is the time Allyson and Adrien have been traveling for. Because the distance from point E to the point on the building directly north of it is 30 feet in length, we can say that $\overline{\text{AB}} = 10t - 30$.

We can define the distance between the starting point and Adrien's position, $\overline{\text{CD}}$, as $8t$. We are also given that the length of $\overline{\text{BC}}$ is 20 feet. Because these triangles are similar, we know that their corresponding side lengths will be proportional. We can write this proportion as:

$$\frac{\overline{\text{AB}}}{\overline{\text{CD}}} = \frac{\overline{\text{BC}}}{\overline{\text{DE}}}$$

Plugging in the values that we know, we can write this proportion as:

$$\frac{10t - 30}{10t} = \frac{20}{8t}$$

Solving for t:

$$10t(20) = 8t(10t - 30)$$
$$80t^2 - 440t = 0$$
$$40t(2t - 11) = 0$$
$$t = 0,\ t = 5.5$$

The solution $t = 0$ is not feasible because the bungee cord is not touching the building 0 seconds after Allyson and Adrien start running. Therefore, the $\boxed{\text{bungee cord will touch the building after 5.5 seconds.}}$