ADA Assignment 2

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Master's Theorem

If $f(n) \in \Theta(n^d)$ or $f(n) = c * n^d$ where $d \geq 0$ in recurrence

$T(n) = aT(n/b) + f(n)$ then,

$$T(n) \in \begin{cases} \Theta(n^d) & \text{if } a < b^d \\ \Theta(n^d \log n) & \text{if } a = b^d \\ \Theta(n^{\log_b a}) & \text{if } a > b^d \end{cases}$$

1. $T(n) = 8T\left(\frac{n}{2}\right) + 1000n^2$

$\rightarrow$ $a = 8$, $b = 2$, $f(n) = 1000n^2 = c * n^d$

$\therefore d = 2$

$b^d = 2^2 = 4$

Hence $a > b^d$

$\therefore T(n) \in \Theta(n^{\log_b a})$

$\log_b a = \log_2 8 = 3$

$\therefore T(n) \in \Theta(n^3)$

2. $T(n) = 2T\left(\frac{n}{2}\right) + n^2$

$\rightarrow$ $a = 2$, $b = 2$, $f(n) = n^2 = c * n^d$

$\therefore d = 2$

$b^d = 2^2 = 4$

Hence $a < b^d$

$\therefore T(n) \in \Theta(n^d)$

~~log~~ $T(n) \in \Theta(n^2)$

3. $T(n) = 2T\left(\frac{n}{2}\right) + 10n$

$\rightarrow$ $a = 2$, $b = 2$, $f(n) = 10n = c * n^d$

$\therefore$ $d = 1$

$\therefore$ $b^d = 2^1 = 2$

Hence $a = b^d$

$\therefore$ $T(n) \in \Theta(n^d \log n)$

~~log n~~ $n^d = n^1 = n$

$\therefore$ $T(n) \in \Theta(n \log n)$