

Module - II

Part - II

Microwave Transmission Lines

Microwave Frequencies

The term microwave frequencies is used for those wavelengths measured in centimeters from 30 cm to 1mm.

- Microwave indicates the wavelengths in the micron ranges
- Microwave frequencies are up to Infrared and visible-light regions and hence refers to from 1 GHz to 10^6 GHz.
- The Institute of Electrical and Electronics Engineers (IEEE) recommended microwave band designations as shown in below table.

Designation	Frequency (GHz)
VHF	0.5 – 1.0
L	1.0 – 2.0
S	2.0 – 4.0
C	4.0 – 8.0
X	8.0 – 12.4
Ku	12.4 – 18.0
K	18.0 – 26.5
Ka	26.5 – 40.0
V	40.0 – 75.0
W	75.0 – 110.0
D	110.0 – 170.0

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Microwave Devices:

- In the late 1930s it became evident that as the wavelength approached the physical dimensions of the vacuum tubes, the electron transit angle, interelectrode capacitance and lead inductance appeared to limit the operation of vacuum tubes in microwave frequencies.
- In 1939 W.L Hahn and G.F. Metcalf proposed a theory for velocity modulation for microwave tubes.
- Since then the concept of microwave tubes has deviated from that of conventional vacuum tubes as a result of the application of new principles in the amplification and generation of microwave energy.
- Microwave generation and amplification were accomplished by means of velocity-modulation theory. Microwave solid-state-devices such as tunnel diodes, Gunn diodes, transferred electron devices (TEDs) and avalanche transit-time devices have been developed to perform these functions.
- The common characteristic of all microwave solid-state devices is the negative resistance that can be used for microwave oscillation and amplification.

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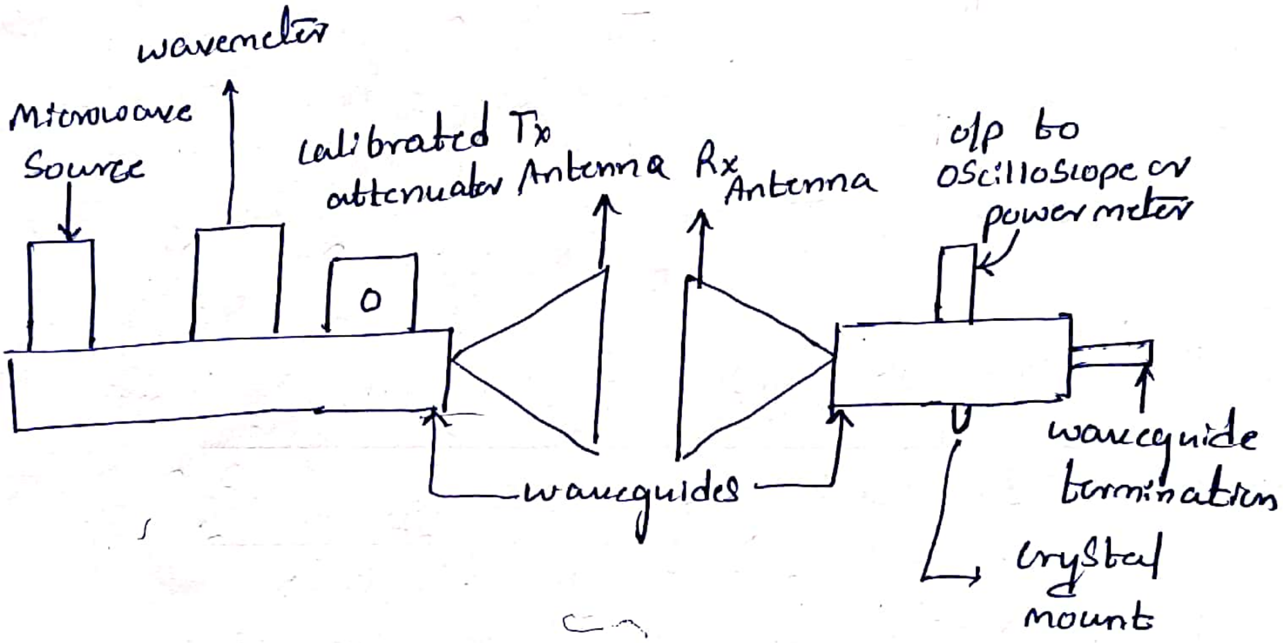
Microwave Systems

- A microwave system consists of a transmitter subsystem, including a microwave oscillator, waveguides and a transmitting antenna and a receiver subsystem that includes a receiving antenna, transmission line or waveguide, a microwave amplifier and a receiver.
- figure shows a typical microwave system.



Microwave System.

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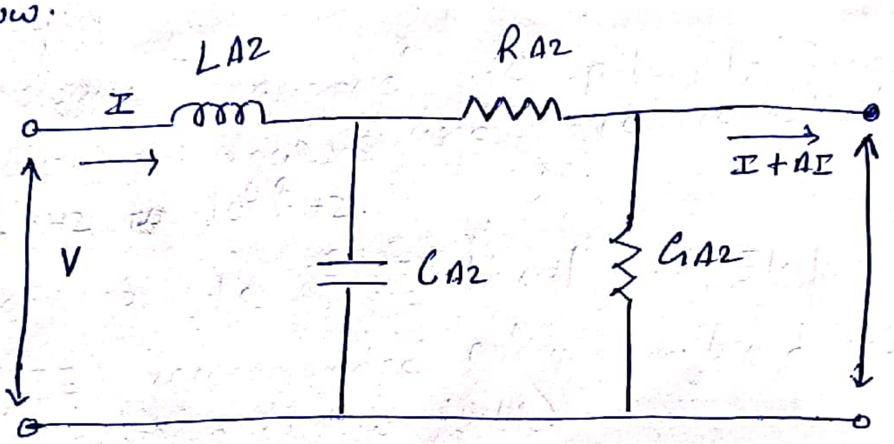
Transmission Line equations and Solutions

* At RF and microwave frequencies, the line parameters are distributed along the whole length of the line in the direction of propagation (z-direction).

* A small length of the line can be represented by an equivalent symmetrical T-networks with constant parameters R, L, G and C per unit length as shown below.



R → Resistance per unit length which takes into account the ohmic loss in the line conductor

L → Inductance per unit length which takes into account the magnetic energy storage around the conductor

G → Conductance per unit length which takes into account the dielectric loss between the line conductors.

C → Capacitance per unit length which appears due to two conductors at different potential and represents the electric energy storage.

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* The line voltage per unit length decrease as

$$\Delta V = -(R\Delta z + j\omega L\Delta z)I$$

$$\lim_{\Delta z \to 0} \frac{\Delta V}{\Delta z} = -(R + j\omega L)I$$

$$\frac{dV}{dz} = -(R + j\omega L)I \longrightarrow (1)$$

* The line current per unit length decrease

$$\Delta I = -(G\Delta z + j\omega C\Delta z)V$$

$$\lim_{\Delta z \to 0} \frac{\Delta I}{\Delta z} = -(G + j\omega C)V$$

$$\frac{dI}{dz} = -(G + j\omega C)V \longrightarrow (2)$$

* $\frac{dV}{dz} = -(R + j\omega L)I = -ZI$

$\frac{dI}{dz} = -(G + j\omega C)V = -YV$

where $\left.\begin{array}{l} Z = R + j\omega L \\ Y = G + j\omega C \end{array}\right\}$ are series impedance and shunt admittance

* Eliminating I by differentiating Eq (1)

$$\frac{d^2V}{dz^2} = -(R + j\omega L)\frac{dI}{dz}$$

Substituting $\frac{dI}{dz}$ from Eq (2)

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$$\frac{d^2V}{dz^2} = +(R+j\omega L)(G+j\omega C)V$$

$$\frac{d^2V}{dz^2} = \gamma^2 V \longrightarrow (3)$$

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$$\frac{d^2I}{dz^2} = \gamma^2 I \longrightarrow (4)$$

Let us assume sinusoidal variation of Voltage and current solutions for (3) and (4) Equations

$$V = V(z,t) = V(z)\, e^{j\omega t}$$
$$I = I(z,t) = I(z)\, e^{j\omega t}$$

$$V(z,t) = V(z)\, e^{j\omega t} = \left(V_1 e^{-\gamma z} + V_2 e^{+\gamma z}\right) e^{j\omega t} \longrightarrow (5)$$

where V_1 and V_2 are constants

γ = propagation constant

$$\gamma = \sqrt{ZY} = \sqrt{(R+j\omega L)(G+j\omega C)}$$
$$= \alpha + j\beta$$

α – Attenuation constant

β – Phase constant.

$$I(z,t) = I(z)\, e^{j\omega t} = \left(I_1 e^{-\gamma z} + I_2 e^{+\gamma z}\right) e^{j\omega t} \longrightarrow (6)$$

For lossless medium $\alpha = 0$ then

$$V(z,t) = \left(V_1 e^{-(\alpha+j\beta)z} + V_2 e^{+(\alpha+j\beta)z}\right) e^{j\omega t}$$
$$= V_1 e^{-j\beta z} e^{j\omega t} + V_2 e^{j\beta z} e^{j\omega t}$$
$$= V_1 e^{j(\omega t-\beta z)} + V_2 e^{j(\omega t+\beta z)}$$
$$= V_1 \cos(\omega t-\beta z) + V_2 \cos(\omega t+\beta z) \longrightarrow (7)$$

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* WKT

$$\vec{E} = E_0 e^{-\alpha z} \cos(\omega t - \beta z)\,\hat{x}$$

For lossless medium $\alpha = 0$

$$\vec{E} = E_0 \cos(\omega t - \beta z)\,\hat{x} \longrightarrow (8)$$

* Comparing Equation (7) and (8) we can arrive at the conclusion that voltage flowing in transmission line will have nature of wave.

* From Eq (7)

first term in the right hand side represents the wave travelling in +z direction

Second term in the right hand side represents the wave traveling in -z direction

* |||ly we can follow for the current solutions in Eq (6).

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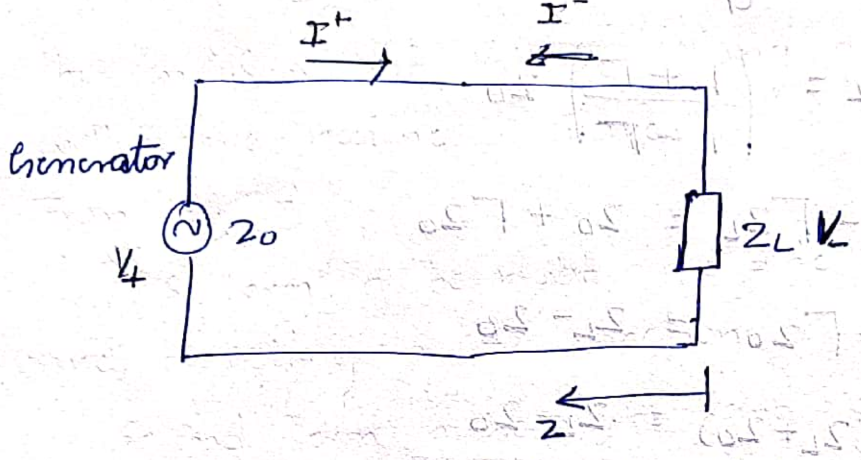
Reflection Co-efficient

* We know the voltage and current equations of transmission line which are as follows

$$\left.\begin{aligned} V(z,t) &= V^+ e^{-\gamma z} + V^- e^{+\gamma z} \\ I(z,t) &= I^+ e^{-\gamma z} + I^- e^{+\gamma z} \end{aligned}\right\} \rightarrow (1)$$



Transmission line of length 'l' terminated in a load impendence of Z_L and characteristic impendence Z_0

* WKT $\quad Z_0 = \dfrac{V^+}{I^+} \quad \Big| \quad -Z_0 = \dfrac{V^-}{I^-} \quad \Big\} \rightarrow (2)$

Substituting Eq (2) in Eq (1)

we have

$$I(z,t) = \frac{V^+}{Z_0} e^{-\gamma z} - \frac{V^-}{Z_0} e^{+\gamma z}$$

* At $z=0$. The ratio of voltage to the current at the receiving end is the load impedance.

$$Z_L = \frac{V}{I} = \frac{V^+ + V^-}{V^+ - V^-} \times Z_0$$

$$Z_L = \frac{V^+(1 + V^-/V^+)}{V^+(1 - V^-/V^+)} \times Z_0 \longrightarrow (3)$$

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* Reflection co-efficient (Γ) can be defined as ratio of reflected voltage to incident voltage.

Reflected voltage/current / Incident voltage/current

$$\boxed{\Gamma = \frac{V^-}{V^+}} \rightarrow (4)$$

Reflected voltage at load / Incident voltage at load

* Substituting Eq (4) in Eq (3) we have

$$Z_L = \left(\frac{1+\Gamma}{1-\Gamma}\right) Z_0$$

$$Z_L - \Gamma Z_L = Z_0 + \Gamma Z_0$$

$$\Gamma Z_L + \Gamma Z_0 = Z_L - Z_0$$

$$\Gamma (Z_L + Z_0) = Z_L - Z_0$$

$$\boxed{\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

* If $Z_L = Z_0$ $\Gamma = 0$ No reflection

If $Z_L \neq Z_0$ $\Gamma \neq 0$ Reflection

↳ Maximum power will be transfered to the load without any reflection and the line is said to be matched terminated.

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Transmission co-efficient (T)

* It is defined as ratio of amplitude of transmitted wave or voltage or current to the amplitude of incident wave or voltage or current.

$$T = \frac{\text{transmitted wave or voltage or current}}{\text{Incident wave or voltage or current}}$$

$$T = \frac{V_{tr}}{V_{inc}} = \frac{I_{tr}}{I_{inc}}$$

* WKT $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$

$$\frac{V^-}{V^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$V_{tr} = V^+ + V^-$ (Travelling wave at the receiving end)

$$V_{tr} = V^+ + V^+\left[\frac{Z_L - Z_0}{Z_L + Z_0}\right]$$

$$V_{tr} = V^+\left[1 + \frac{Z_L - Z_0}{Z_L + Z_0}\right]$$

$$= V^+\left[\frac{Z_L + Z_0 + Z_L - Z_0}{Z_L + Z_0}\right]$$

$$V_{tr} = V^+\left[\frac{2Z_L}{Z_L + Z_0}\right]$$

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$$\frac{V_{tr}}{V^+} = \frac{2Z_L}{Z_L+Z_0}$$

$$\boxed{\tau = \frac{2Z_L}{Z_L+Z_0}}$$

* Relationship b/w Reflection co-efficient and (Γ) Transmission Co-efficient (τ)

Consider $1+\Gamma = 1+\frac{Z_L-Z_0}{Z_L+Z_0}$

$$1+\Gamma = \frac{Z_L+Z_0+Z_L-Z_0}{Z_L+Z_0}$$

$$1+\Gamma = \frac{2Z_L}{Z_L+Z_0}$$

$$\boxed{1+\Gamma = \tau}$$

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Standing Wave.

* A Standing wave in a transmission line is a wave in which the distribution of current, voltage or field strength is formed by the superposition of two waves of the same frequency propagating in opposite directions.

* The effect is a series of nodes and antinodes at fixed points along the transmission line.

* The general solutions of the transmission-line equation consist of two waves traveling in opposite directions with unequal amplitude and one of equation can be written as

$$V = V^{+} e^{-\alpha z} e^{-j\beta z} + V^{-} e^{\alpha z} e^{j\beta z}$$

$$= V^{+} e^{-\alpha z}\left[\cos\beta z - j\sin\beta z\right] + V^{-} e^{\alpha z}\left[\cos\beta z + j\sin\beta z\right]$$

$$= \left[V^{+} e^{-\alpha z} + V^{-} e^{\alpha z}\right]\cos\beta z - j\left(V^{+} e^{-\alpha z} - V^{-} e^{\alpha z}\right)\sin\beta z$$

* With no loss it is assumed that $V^{+} e^{-\alpha z}$ and $V^{-} e^{\alpha z}$ are real.

Then voltage-wave equation can be expressed

$$V_s = V_0 e^{-j\phi}$$

This is called the equation of the voltage standing wave.

Where

$$V_0 = \left[\left(V^{+} e^{-\alpha z} + V^{-} e^{\alpha z}\right)^2 \cos^2\beta z + \left(V^{+} e^{-\alpha z} - V^{-} e^{\alpha z}\right)^2 \sin^2\beta z\right]^{1/2}$$

which is called the standing wave pattern of the voltage wave or the amplitude of the standing wave.

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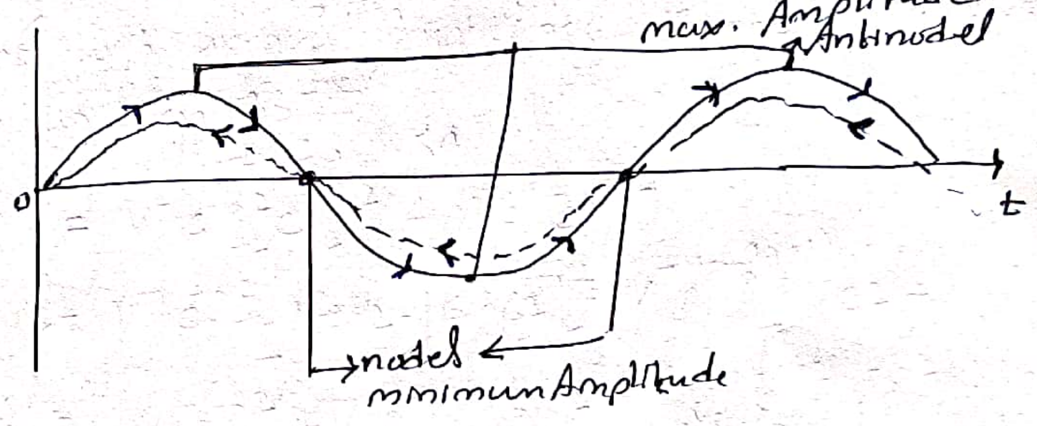
$$\phi = \arctan\left(\frac{V_+ e^{-\alpha z} - V^- e^{\alpha z}}{V_+ e^{-\alpha z} + V^- e^{\alpha z}} \tan(\beta z)\right)$$

which is called the phase pattern of the standing wave.

* when the transmission line is not terminated in its characteristic impendence Z_0, then there will be reflection of waves.

Below figure represents the standing waves on a line



* Standing wave Ratio (ρ)

It is defined as the ratio of maximum voltage or current to minimum voltage or current on a line having standing waves is called as "Standing wave Ratio" → SWR and denoted by ρ and given by

$$\rho = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}} \quad \Big\} \text{ magnitudes.}$$

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Relationship between standing wave ratio (ρ) and Reflection Co-efficient (Γ)

* WKT, the voltage and current at any point on the line is given by

$$V(z) = \frac{V_L(Z_L+Z_0)}{2Z_L}\left[e^{\gamma z}+\Gamma e^{-\gamma z}\right] \rightarrow (1)$$

$$I(z) = \frac{I_L(Z_L+Z_0)}{2Z_0}\left[e^{\gamma z}-\Gamma e^{-\gamma z}\right] \rightarrow (2)$$

* For a lossless medium $\alpha=0$

then $\gamma = \alpha + j\beta$

$\gamma = j\beta$

and Let $\Gamma = |\Gamma| e^{j\theta}$

$|\Gamma| \rightarrow$ magnitude of reflection co-efficient

$\theta \rightarrow$ phase angle of reflection co-efficient.

Let $\frac{V_L(Z_L+Z_0)}{2Z_L} = V'$

* Substituting γ, Γ and V' in equation (1) we have

$$V(z) = V'\left[e^{j\beta z} + |\Gamma| e^{j\theta} e^{-j\beta z}\right]$$

$$= V' e^{j\beta z}\left[1+|\Gamma| e^{-j(2\beta z-\theta)}\right]$$

Considering only the magnitude on both sides we get

$$|V(z)| = |V'|\left[1+|\Gamma| e^{-j(2\beta z-\theta)}\right] \rightarrow (3)$$

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* The voltage $|V(z)|$ has maximum amplitude when two waves are in phase.

$$2\beta z - \theta = 2\pi n.$$

where $n = 0, 1, 2 \ldots$

$|V(z)| = |V_{max}|$ and $e^{-j2n\pi} = 1$ for all n.

So $|V_{max}| = |V'|\left[1 + |\Gamma| e^{-j2n\pi}\right]$

$$|V_{max}| = |V'|\left[1 + |\Gamma|\right] \longrightarrow (4)$$

* Similarly, the voltage has minimum amplitude when two wave are out of phase.

$$2\beta z - \theta = (2n+1)\pi$$

where $n = 0, 1, 2 \ldots$

$|V(z)| = |V_{min}|$ and $e^{-j(2n+1)\pi} = -1$ for all n.

So $|V_{min}| = |V'|\left[1 + |\Gamma| e^{-j(2n+1)\pi}\right]$

$$|V_{min}| = |V'|\left[1 - |\Gamma|\right] \longrightarrow (5)$$

* Substituting Equ (4) and Equ (5) in Standing wave Ratio ie $\rho = \dfrac{|V_{max}|}{|V_{min}|}$

$$\rho = \frac{|V_{max}|}{|V_{min}|} = \frac{|V'|\left[1 + |\Gamma|\right]}{|V'|\left[1 - |\Gamma|\right]}$$

$$\boxed{\rho = \frac{1 + |\Gamma|}{1 - |\Gamma|}}$$

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* Only the magnitude of reflection co-efficient Γ is to be used while calculating ρ and not it's phase angle.

The SWR is only a real quantity and not a complex quantity.

~~Smith~~ SMITH CHART

* The Smith chart is a graphical aid designed in solving problems with transmission lines and matching circuits.
* It can be used to simultaneously display multiple parameters including impedances, admittances, reflection co-efficients, constant gain contours etc.
* It is most frequently used at within the unity radius region.
* It consists of a plot of the normalized impedance or admittance with the angle and magnitude of a generalized complex reflection co-efficient in a unity circle.
* The chart is applicable to the analysis of a lossless line as well as a lossy line.

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Construction of Smith Chart:-

* It is constructed using voltage reflection co-efficient Equation ie

$$\frac{Z_{in}}{Z_0} = \frac{1+|\Gamma|\,\underline{/\theta-2\beta z}}{1-|\Gamma|\,\underline{/\theta-2\beta z}} \longrightarrow (1)$$

Input impedance Z_{in} when divided by Z_0 is called "Normalized input impedance"

* Let normalized input impedance be given by

$$\frac{Z_{in}}{Z_0} = r+jx \longrightarrow (2)$$

The quantity $|\Gamma|\,\underline{/\theta-2\beta z}$ is a complex quantity and let it be given by

$$|\Gamma|\,\underline{/\theta-2\beta z} = U+jV \longrightarrow (3)$$

* Substituting Eq (3) and Eq (2) in Eq (1) we have.

$$r+jx = \frac{1+U+jV}{1-U-jV}$$

By Rationalization

$$r+jx = \frac{(1+U)+jV}{(1-U)-jV} \times \frac{(1-U)+jV}{(1-U)+jV}$$

$$= \frac{(1+jV)^2-U^2}{(1-U)^2+V^2}$$

$$= \frac{(1-V^2-U^2)}{(1-U)^2+V^2} + j\frac{2V}{(1-U)^2+V^2}$$

* Equating real and imaginary parts we have

$$(4) \longleftarrow r = \frac{1-V^2-U^2}{(1-U)^2+V^2} \qquad x = \frac{2V}{(1-U)^2+V^2} \longrightarrow (5)$$

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Constant Resistance Circles.

* From Eq 4 we have

$$r[1-2u+u^2+v^2] = 1-v^2-u^2$$

$$r-2ur+rv^2+ru^2 = 1-v^2-u^2$$

$$u^2(1+r)-2ur+v^2(1+r) = 1-r$$

Dividing throughout by $(1+r)$ we get

$$u^2-\frac{2ur}{1+r}+v^2 = \left(\frac{1-r}{1+r}\right)$$

adding $\left(\frac{r}{1+r}\right)^2$ to both sides we get

$$u^2-2u\left(\frac{r}{1+r}\right)+\left(\frac{r}{1+r}\right)^2+v^2 = \frac{1-r}{1+r}+\left(\frac{r}{1+r}\right)^2$$

$$\left(u-\frac{r}{1+r}\right)^2+v^2 = \frac{1-r^2+r^2}{1+r^2}$$

$$\left(u-\frac{r}{1+r}\right)^2+v^2 = \left(\frac{1}{1+r}\right)^2 \longrightarrow 6$$

* Equation 6 represents the equation of a circle of radius $\left(\frac{1}{1+r}\right)$ and centre at $\left(\frac{r}{1+r},0\right)$

This equation represents normalized resistance circles.

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Constant Reactance Circles.

* From Equation (5) we have

$$x = \frac{2v}{(1-u)^2 + v^2}$$

$$(u^2+v^2-2u+1)\,x = 2v$$

$$(u^2-2u+1)\,x + v^2 x - 2v = 0$$

Dividing by x throughout, we get

$$(u-1)^2 + v^2 - \frac{2v}{x} = 0$$

Adding $\frac{1}{x^2}$ to both sides, we get

$$(u-1)^2 + \left(v - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2 \longrightarrow (7)$$

* Equation (7) represents the equation of a circle of radius $(1/x)$ and centre at $(1, \frac{1}{r})$

This equation represents normalized reactance circles.

* The superposition of these constant-resistance circles and constant-reactance circles forms SMITH CHART.

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Constant VSWR (ρ) Circles

* From constant resistance circles Equation i.e.

$$\left(u - \frac{r}{1+r}\right)^2 + v^2 = \left(\frac{1}{1+r}\right)^2$$

when $v=0$ we have

$$\left(u - \frac{r}{1+r}\right)^2 = \left(\frac{1}{1+r}\right)^2$$

$$u - \frac{r}{r+1} = \pm\left(\frac{1}{1+r}\right)$$

$$u = \frac{r \pm 1}{1+r}$$

Either $u = 1$ OR $u = \frac{r-1}{r+1}$

Consider $u = \frac{r-1}{r+1}$

$$ur + u = r - 1.$$

$$u + 1 = r - ru$$

$$u + 1 = r(1-u)$$

$$r = \frac{u+1}{1-u}$$

$$\boxed{r = \frac{1+u}{1-u}}$$

WKT $|\Gamma| \underline{/\theta - 2\beta z} = u + jv = u$ Since $v=0$

$$u = |\Gamma|$$

Then we get

$$r = \frac{1+|\Gamma|}{1-|\Gamma|} = \rho$$

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Applications and Properties of Smith Chart.

1) Smith Chart consists of resistance and reactance circles. Hence the given load impedance Z_L has to be divided by Z_0 before entering in the Smith chart. This process is called normalization and the ratio (Z_L/Z_0) is called 'normalized impedance'.

2) Plotting of an impedance: Any complex impedance can be shown by a single point on the Smith chart.

This point is the intersection of $r = \frac{R_L}{Z_0}$ circle and $x = j\frac{X_L}{Z_0}$ arc of a circle.

Ex Let $Z_L = 120 + j160\Omega$ and $Z_0 = 400\Omega$

Normalizing we get $r = R_L/Z_0 = 120/400 = 0.3$

$x = X_L/Z_0 = 160/400 = 0.4$

The point 'A' in the Smith chart shows the point corresponding to a normalized impendance $0.3 + j0.4$ on the chart.

3) VSWR Determination :- With 'O' as centre and 'OA' as radius, a circle is drawn which cuts the horizontal axis at 'r'. This gives the VSWR.

4) Determination of Γ in magnitude and direction:

→ The line OA is the extended in the Smith chart to meet the outer circle at point 'C'. The angle at point C = θ = 133.2 is the phase angle of the reflection co-efficient.

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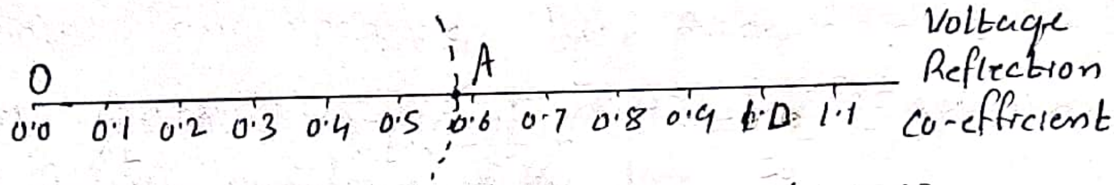
→ To find magnitude of Γ, the linear scale at the bottom of the chart is referred which is marked 'Voltage reflection co-efficient'.

With 0.0 as centre and radius exactly equal to OA, an arc is cut on the linear scale as shown below. The value of $|\Gamma|$ is read as 0.59 corresponding to point A.



The reflection co-efficient $\Gamma = 0.59 \angle 133.2^\circ$

5) Location of V_{max} and V_{min} : The constant ρ circles intersects the central horizontal axis at point D and B as shown in Smith chart.

The point 'D' corresponds to voltage minima and 'B' to voltage maxima.

Line Impedance at V_{max} = 3.91

Line Impedance at V_{min} = 0.26

6) Open and Short circuited line :-

At point F on the horizontal axis $r = \infty$ and $X = \infty$ which represents the open circuit termination of the line.

At point E we have $r = 0$ and $X = 0$ which represents short circuit termination.

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7) Movement along periphery of the chart:

→ Movement in clockwise direction from point E results in number of wavelengths toward generator and movement in anti-clockwise direction from point E in wavelengths toward load.

→ One Full rotation corresponds to $\lambda/2$.

8) Matched load: Consider the circle $r=1$

$$Z_0 = Z_L \quad \text{or} \quad R_L = Z_0$$

The resistive component of the load is equal to characteristic impedance of the line.

This represents the condition of no reflection.

9) Conversion of impedance to admittance.

→ To find the admittance of an impedance at point A; the point 'A' is rotated through constant ρ-circle by an amount $\lambda/4$ which is equivalent to $180°$.

→ The point is diametrically opposite to A has a value, which is the admittance corresponding to the impendance at point 'A'

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SINGLE - STUB IMPEDANCE MATCHING

* Impendance matching is one of the important aspects of high frequency circuit analysis. To avoid reflections and for maximum power transfer the circuits have to be impedance matching.

* Transmission line sections can be used for the purpose of impedance matched. one of method or Technique is single-stub impedance matching.

* A stub is a short-circuited section of a transmission line connected in parallel to the main transmission line.

* A stub of appropriate length is placed at some distance from the load such that the impendance seen beyond the stub is equal to the characteristic impedance.

* Suppose we have a load impedance Z_L connected to a transmission line with characteristic impedance Z_0 as shown in figure. The objective here is that no reflection should be seen by the generator.

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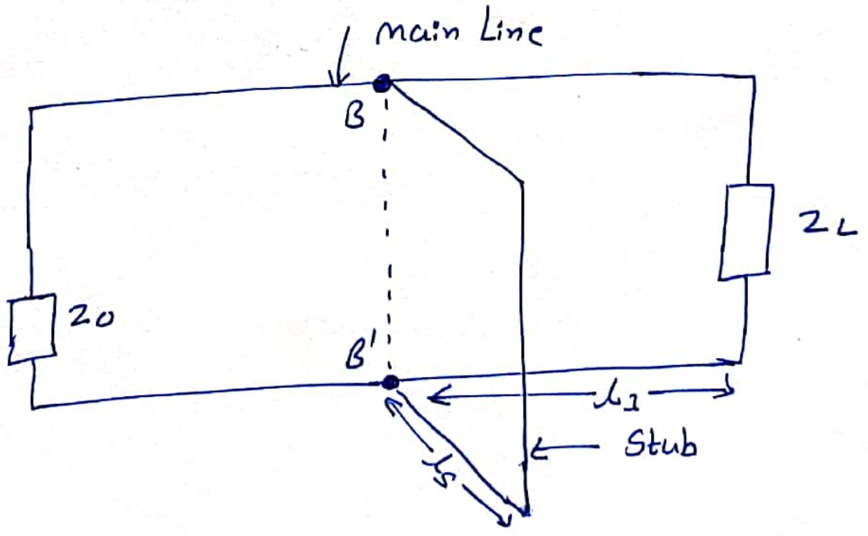

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* This can be achieved by adding a stub to the main line such that reflected wave from the short-circuit end of the stub and the reflected wave from the load on the main line completely cancel each other at point B to give no net reflected wave beyond point B towards the generator.

* Since we have a parallel connection of transmission lines, it is more convenient to solve the problem using admittances rather than impedances.

$$y = \frac{1}{Z_L}$$

$$y = g + jb$$

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