MTE 360 Automatic Control Systems: Lab #3

Identification and P-Control of Cascaded Systems

In Lab 3 & 4, the plant is a flexible drive system. In Lab 3, you will first assess the performance and stability limitations, due to the inherent mechanical flexibility, by implementing a simple P-controller. Next, you will proceed to identify the dynamics of the flexible drive system and determine the key physical parameters using time and frequency domain identification techniques. In Lab 4, you will use the model you identified in Lab 3 to design several advanced control techniques to stabilize and accurately control the position.

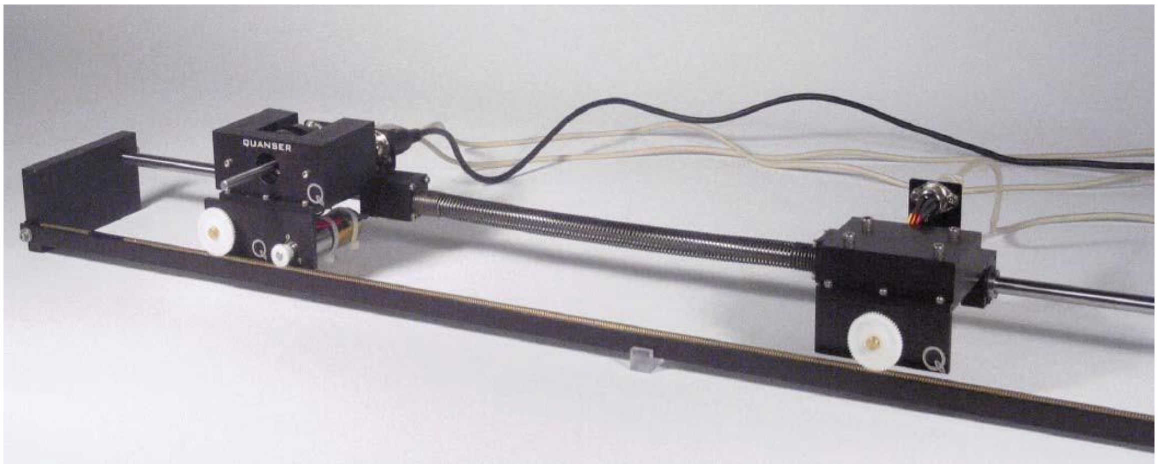

Figure 1: Flexible drive system.

The structure of this lab handout is listed as follows.

Part 1 is an introduction to modeling and joint control of a flexible drive system pair.

Part 2 is the experiment to conduct during your lab session.

Part 3 is a series of analysis procedures to be completed to finish up the lab report.

<u>This is an intensive lab compared to earlier ones. It is imperative that the students keep up to date with the material covered in class in order to be able to complete this lab. Every group is responsible for analyzing their own data.</u>

<u>There is no pre-lab for lab #3 but the post-lab is trickier than the other labs and should not be left to last minute.</u>

<u>Consider completing your report earlier and discussing it with the TAs well before your report deadline.</u>

1 Introduction

1.1 Experimental Setup:

The experimental setup is shown in Figure 1. An additional mass has been attached to the motor driven cart used in Labs 1 and 2. The spring has relatively low stiffness and negligible damping, which causes the two masses to vibrate when external forces are applied to either mass. The secondary mass is instrumented with a rotary encoder which measures the linear position through a gear that is in contact with the rack. The vibrations resulting from the spring's flexibility make the stabilization of this drive more challenging, especially when the goal is to accurately control the position of the second cart. When a proportional (P-type) control loop is closed using this cart's position as the feedback, the gain can only be increased by a modest amount before the structural vibrations of the drive start interacting with the controller dynamics and cause closed loop instability. This presents a severe limitation in terms of the achievable performance and stability characteristics, a problem often encountered in applications such as flexible robotics, complex object/tools manipulation, and satellite control. In this laboratory, you will follow a systematic approach in designing your controller, so each factor that limits stability or performance will be handled one at a time.

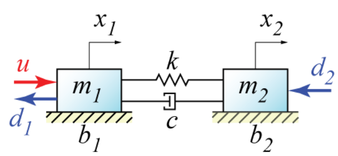

Figure 2: Model of flexible drive.

A simplified physical model of the flexible drive is shown in Figure 2. The system parameters are scaled to consider the control signal as the equivalent actuation force. $u\ [V]$ represents the actuation input (i.e. the control signal); x_1 and $x_2\ [mm]$ are the position of carts 1 and 2, respectively; m_1 and $m_2\ [V/(mm/sec^2)]$ are the control signal equivalent mass values. $k\ [V/mm]$ is the spring's stiffness that couples the two masses; b_1 and $b_2\ [V/(mm/sec)]$ are the viscous damping coefficients acting on m_1 and m_2, respectively; $C\ [V/(mm/sec)]$ is the spring's inherent damping, which is negligible in comparison to the damping introduced by b_1 and b_2 (hence you can assume that $C = 0$ in this setup); d_1 and d_2 represent external disturbances, like Coulomb friction, which act on the two masses. The position response for the two masses has been derived as:

$$\left.\begin{aligned}
x_1(s) &= \underbrace{\frac{\frac{1}{m_1 m_2}\left[m_2 s^2 + (b_2 + C)s + k\right]}{s(s^3 + a_1 s^2 + a_2 s + a_3)}}_{G_1(s)} \cdot [u(s) - d_1(s)] - \frac{\frac{1}{m_1 m_2}[Cs + k]}{s(s^3 + a_1 s^2 + a_2 s + a_3)} \cdot d_2(s) \\
x_2(s) &= \underbrace{\frac{\frac{1}{m_1 m_2}[Cs + k]}{s(s^3 + a_1 s^2 + a_2 s + a_3)}}_{G_2(s)} \cdot [u(s) - d_1(s)] - \frac{\frac{1}{m_1 m_2}\left[m_1 s^2 + (b_1 + C)s + k\right]}{s(s^3 + a_1 s^2 + a_2 s + a_3)} \cdot d_2(s)
\end{aligned}\right\} \quad (1)$$

where,

$$a_1 = \frac{m_1 b_2 + m_2 b_1 + (m_1 + m_2)C}{m_1 m_2}, \quad a_2 = \frac{b_1 b_2 + (b_1 + b_2)C + (m_1 + m_2)k}{m_1 m_2}, \quad a_3 = \frac{(b_1 + b_2)k}{m_1 m_2} \quad (2)$$

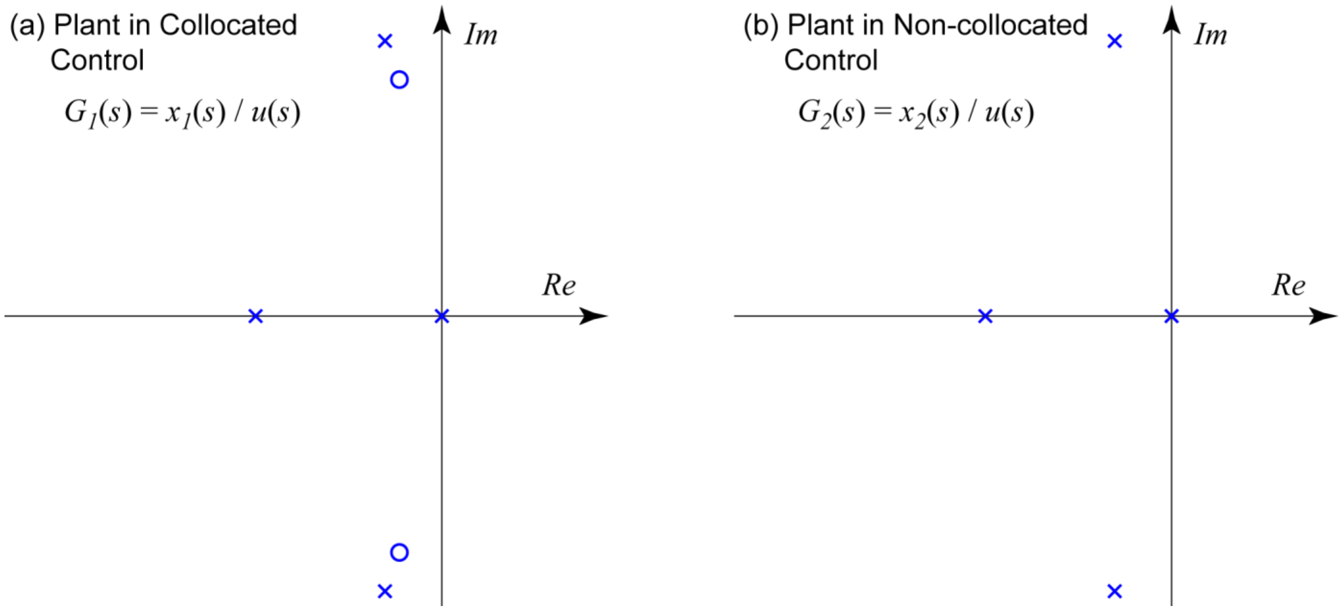

Figure 3: Pole-zero maps for control of (a) Collocated (b) Non-collocated drive models.

Assuming that d_1 and d_2 are primarily caused by Coulomb friction (i.e. $d_1 = d_{1\_coul} \cdot sgn(\dot{x_1})$, $d_2 = d_{2_{coul}} \cdot sgn(\dot{x_2})$, when a step voltage is applied at the input, $u(s) = u_0/s$, it can be verified from the Final Value Theorem that both masses will travel at the steady state velocity of:

$$v_{1ss} = v_{2ss} = \frac{u_0 - (d_{1\_coul} + d_{2\_coul})}{b_1 + b_2} \tag{3}$$

1.2 Collocated versus Non-collocated Control:

When the feedback is obtained directly from the point of actuation, this is described as "collocated control". On the other hand, when there is additional dynamics between the actuator and the sensor, this leads to "non-collocated control", which is usually more challenging in terms of stabilizing the closed loop response. Considering Figure 2, when the actuation force is applied to m_1 and the position loop is closed using the measurement of x_1, this results in collocated control. When x_2 is used as the feedback while the actuation is applied at m_1, this is then a non-collocated control case. Representative pole-zero maps comparing the plant in both cases have been presented in Figure 3. In collocated control, the effect of the poorly damped complex poles is reasonably attenuated by the zeros that occur in the vicinity of these poles (see the numerator of $G_1(s)$ in Equation 1). In non-collocated control, there are no complex conjugate zeros to lesson the effect of poorly damped poles (i.e. see the numerator of $G_2(s)$ in Equation 1), thus leading to considerable oscillations of the drive when the measurement of x_2 is used in closing the feedback loop.

The ultimate goal of labs 3 and 4 is to accurately control the position of the 2nd cart, which is a non-collocated control problem. In the following, you will first intuitively see the performance limitation and stability issues encountered in this type of arrangement, when only P-control is used.

2 In-Lab for Lab #3

WARNING: Be very careful in executing the following sections: 2.1, 2.2, 2.3, 2.4. Some of the experiments may AND WILL result in instability! Be prepared to press the STOP button quickly to prevent the setup from crashing. You will also need to collect signals at the same time, even when the system is unstable.

2.0 General tips
To save data of a scope to the workspace.
Remember to call ‘clear all’ command in the command prompt between the experiments.

2.1 Non-collocated P-Control (Stable Case):

Close the feedback loop using measurement from Encoder 2. The signals have already been calibrated to be in $[mm]$. Use the Simulink file: *p_control_x2.slx*, shown in Figure 4. Use the given Matlab file *load_trajectory.m*, to load the position commands from the file *trajectory_data.mat*. The loaded *s* and *t* values will be used in the repeating sequence block. Use a small controller gain (such as $K_p = 0.05$ $[V/mm]$) that will yield a stable and vibration-free response. Record the tracking signals for a period of 4 $[sec]$ (i.e., two forward and backward movements). Capture the profiles for time (t $[sec]$), control input (u $[V]$) command position (x_r $[mm]$), cart 1 position ($x_1 [mm]$), and cart 2 position (x_2 $[mm]$). After you save the data from the real-time scopes to the Matlab workspace, modify and use the file: *plot_data_p_control.m* to process, plot and save your data.

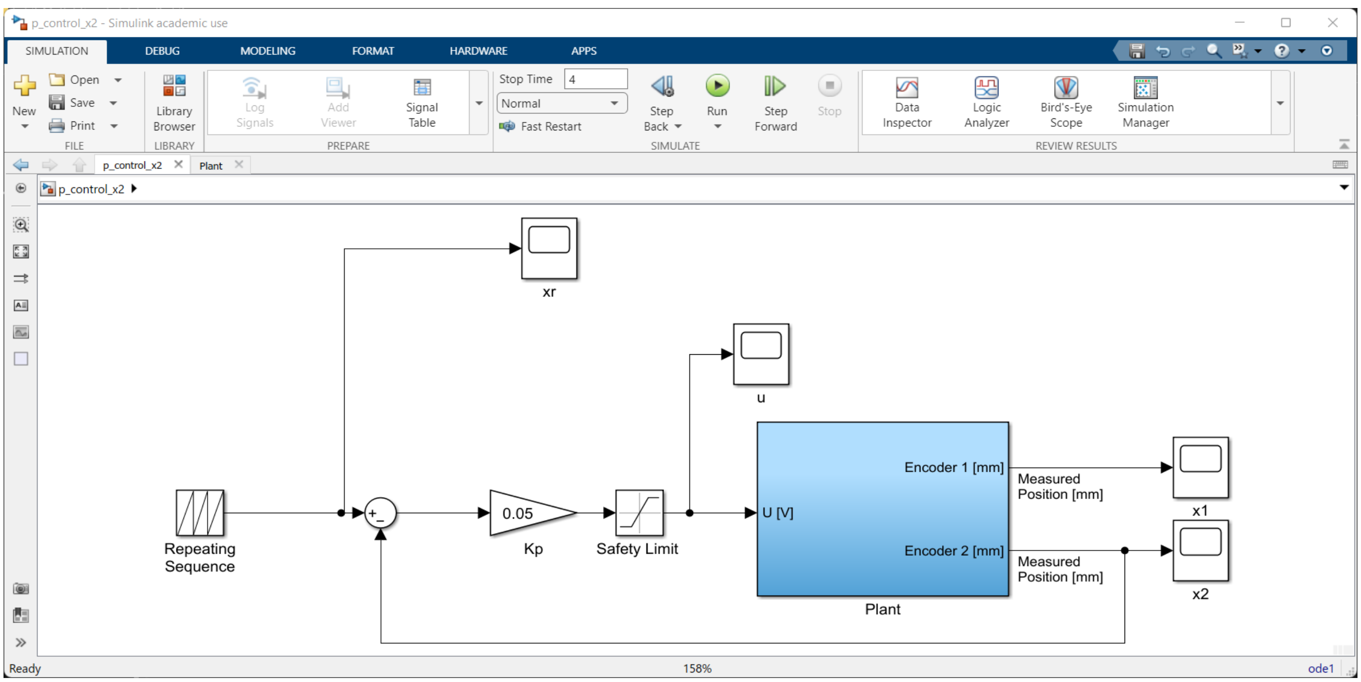

Figure 4: Real-time file for performing P-control using 2nd cart position (a non-collocated control scenario).

2.2 Non-collocated P-control (Critically Stable Case):

Gradually increase the controller gain until the drive response starts becoming vibratory. **The moment you notice instability, decrease the gain to recover stability (interpret instability when the response amplitude increases).** Report the values of K_p for which you obtained the critically stable system. Capture the tracking signals and save the data for this critically stable case.

2.3 Non-collocated P-Control (Unstable Case):

Select a value for the controller gain K_p that is a little above the critically stable limit. **This will result in an unstable case.** After starting the experiment, capture data for a short period, less than 4 $[sec]$, and **be sure to press STOP before the setup physically crashes or travels too close to one end.** Save your data for the unstable case.

2.4 Collocated P-control:

This time, close the position loop using x_1 for feedback. Apply the same gain you had used in Part 2.3, which gave an unstable result in non-collocated control. Save the data for this collocated P-control case. Use the Simulink file: *p_control_x1.slx* to perform the experiment and modify the Matlab script file *plot_data_p_control.m* to display and save the results. <u>Ensure to base your tracking error calculation on the position of the 2nd cart</u> ($e = x_r - x_2$)

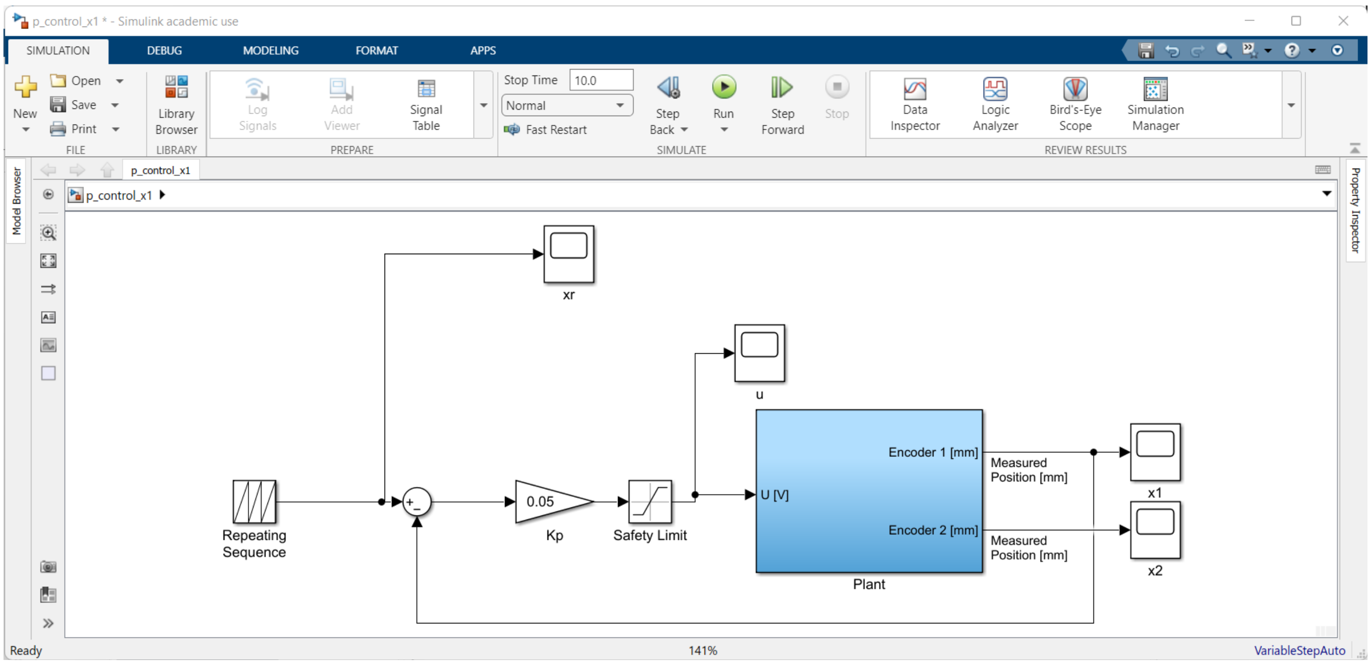

Figure 5: Real-time file for performing P-control using 1st cart position (a collocated control scenario).

2.5 Step Response Measurement:

Similar to lab 1, step response measurements will be used to determine the parameter values for the system model given in Equation 1. It is a good idea to take several measurements $(2-3)$ for both step responses (± 1.5 $[V]$ and ± 2 $[V]$) to improve the reliability of the measurements by averaging, incorrect parameters will have a significant effect on the performance of the controller you will design in lab 4.

Measure the open-loop velocity step response of both masses by applying a square wave excitation with ± 1.5 $[V]$ amplitude and 0.5 $[Hz]$ frequency. Capture time (t $[sec]$), excitation input (u $[V]$), cart 1 velocity (v_1 $[mm/\sec]$), and cart 2 velocity (v_2 $[mm/\sec]$) data for a duration of $\mathbf{8}$ $[\boldsymbol{sec}]$. Modify and use the Matlab file: *plot_velocity_step_response.m* to display and save your data.

Repeat the same experiment with ± 2 $[V]$ amplitude and 0.5 $[Hz]$ frequency. You need both for identification.

Be very careful as the setup will travel larger strokes at faster speeds in this case ($\pm 2[V]$). Make sure that the drive does not reach its travel limits, or crash to the end of the guide.

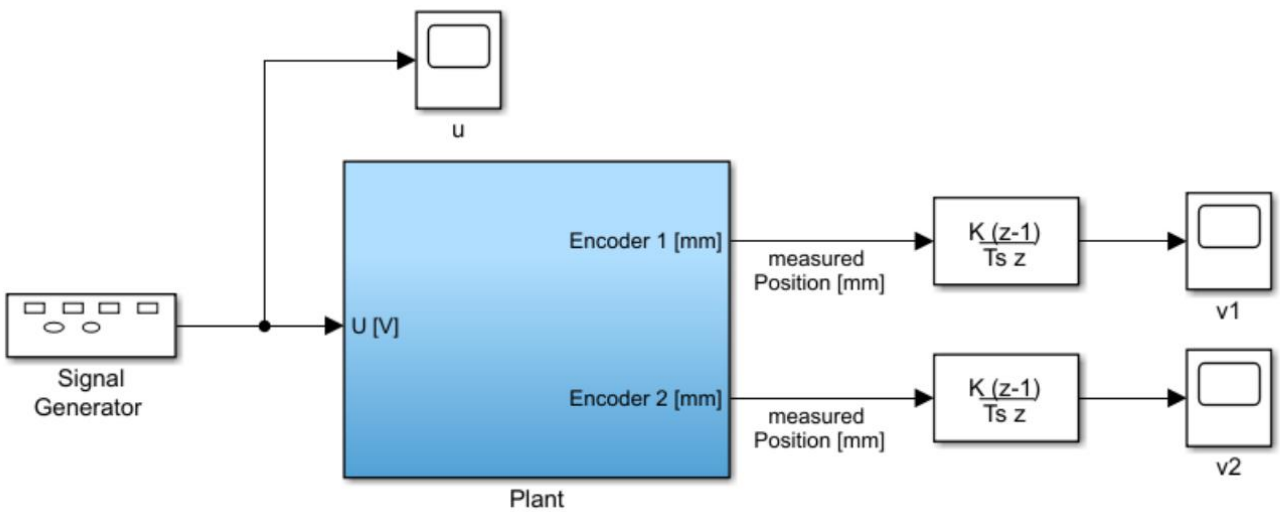

Figure 6: Real-time file for measuring open-loop velocity step response from both masses.

Save the data you recorded with both excitation amplitudes.

2.6 Frequency Response Measurement:

Measure the open-loop velocity step response of both masses by applying a sinusoidal excitation with ± 2.5 $[V]$ amplitude. Test the following frequencies.

Frequency [Hz]	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	8.0	12.0	16.0	20.0

For each excitation frequency, capture time (t $[sec]$), excitation input (u $[V]$), cart 1 velocity ($v_1[mm/sec]$), and cart 2 velocity ($v_2[mm/sec]$) data for a duration of $\mathbf{8}$ $[\boldsymbol{sec}]$.

3 Post-Lab for Lab #3

3.1 Time and Frequency Domain Identification:

In this section, you will identify the dynamic parameters of the flexible drive model, i.e., Equation 1 and 2. In order to estimate the physical parameters of the drive, with its relatively complex dynamics, you will need to make a few practical assumptions (as stated in the following).

Assumptions:

1. The mass ratio between the two carts is $\alpha = m_2/m_1 = 0.54$ (confirmed by measurement)
2. The ratios between viscous and Coulomb friction values affecting both carts have been estimated to be: $\beta = b_2/b_1 = 0.1$, $\gamma = d_{2\_coul}/d_{1\_coul} = 0.1$
3. The damping effect in the spring is negligible in comparison to the viscous and Coulomb friction acting on the two masses, hence $C = 0$

The transfer function denominator in Equation 1 can be expressed in the form of:

$$s(s+p)(s^2 + 2\zeta\omega_n s + {\omega_n}^2) = s[s^3 + (2\zeta\omega_n + p)s^2 + ({\omega_n}^2 + 2\zeta\omega_n)s + p{\omega_n}^2] \tag{4}$$

The poles comprise of an integrator ($p_1 = 0$), which converts the velocity response into position, a real pole at $p_2 = -p$, and a pair of complex conjugate poles with a natural frequency of ω_n $[rad/\text{sec}]$ and a damping ratio ζ [ ] (i.e. $p_{3,4} = -\zeta\omega_n \pm i\omega_d$, where $\omega_d = \omega_n\sqrt{1-\zeta^2}$). Considering the pole-zero maps in Figure 3, it can be assumed that the velocity response of the 1st mass (i.e. $sG_1(s)$) will be dominated by the real pole (p_2), since the complex conjugate poles will be cancelled out by the nearby zeros. On the other hand, the velocity response of the 2nd mass (that is $sG_2(s)$ in Laplace domain) will contain both the effect of the real pole (p_2), as well as the complex conjugate pole pair ($p_{3,4}$). In addition, Equation 3 indicates the combined influence of a step input signal (u_0), the total damping ($b_T = b_1 + b_2$), and total Coulomb friction ($d_{T\_coul} = d_{1\_coul} + d_{2\_coul}$) on the steady state velocity response.

3.2 Parameter Identification from Step Response:

By analyzing the two step response measurements you obtained in Part 2.5, determine the total viscous friction $b_T = b_1 + b_2$ and Coulomb friction $d_{T\_coul} = d_{1\_coul} + d_{2\_coul}$ in the drive system. You may use the steady state velocity formula, given in Equation 3. From the assumptions made regarding friction distribution ($\beta = b_2/b_1$, $\gamma = d_{2\_coul}/d_{1\_coul}$), determine the individual values for viscous and Coulomb friction (b_1, b_2, d_{1_coul}, d_{2_coul}).

$$v_{1ss} = v_{2ss} = \frac{u_0 - (d_{T\_coul})}{b_T} \tag{5}$$

<u>Hint:</u> *The response for two different amplitudes was collected ($u_0 = \pm 1.5[V]$ and $u_0 = \pm 2[V]$) meaning Equation (5) be used to solve for b_T and d_{T_coul}.*

Considering the poles which determine the velocity response, p_2 (the real pole) is mainly influenced by the total viscous friction ($b_T = b_1 + b_2$) and total mass ($m_T = m_1 + m_2$) of the drive. In fact, if the spring between the two masses had infinite stiffness, the whole system would behave like a single rigid body mass sliding on a guideway with Coulomb friction, which was the model in Lab 2. Hence, you can assume that the follow relation approximately holds (Note: this is <u>not</u> an exact formula!):

$$p \cong \frac{b_T}{m_T} = \frac{b_1+b_2}{m_1+m_2} \tag{6}$$

We had mentioned earlier that the velocity response of the first cart (i.e. $v_1(s)/u(s) = sG_1(s)$) would be dominated mainly by the real pole at $-p$, since the effect of the complex poles are attenuated by the complex zeros (Figure 3a). In this case, the dynamics of $sG_1(s)$ can be approximated with a first order transfer function. With this knowledge, determine the value of the real pole ($-p$) considering the step response for ± 2 $[V]$ input excitation.

Hint: Consider the time required to reach 63% of the output range. Recall Lab #1

After having calculated the real pole, find the value of total mass (m_T) and individual mass values (m_1 and m_2). Use the mass distribution ratio $\alpha = m_2/m_1 = 0.54$ provided in the assumptions in Section 3.

The velocity response of the 2nd mass ($sG_2(s)$) will have a strong oscillatory component, due to the complex conjugate poles ($p_{3,4} = -\zeta\omega_n \pm j\omega_d$) which are not attenuated by nearby zeros (Figure 3b).

The oscillations will occur in the step response at the damped natural frequency of $\omega_d = \omega_n\sqrt{1-\zeta^2}$. By observing the second cart's velocity response, recorded with ± 1.5 $[V]$ of step excitation, determine the value of the damped natural frequency ω_d. You will now determine the undamped natural frequency (ω_n) and damping ratio (ζ) by trial and error:

Step 1: Guess a value for ζ (try $\zeta = 0.01 \ldots 0.20$)
Step 2: Calculate ω_n based on the measured ω_d and assumed ζ
Step 3: Estimate the spring stiffness (k) using the approximate formula: $k \cong m_1 m_2 {\omega_n}^2/(m_1 + m_2)$
Step 4: Now that all parameters (m_1, m_2, b_1, b_2, k, C) are known, construct the transfer function denominator in Equation 1
Step 5: Check the natural frequency and damping of the open loop poles. You may use the *damp*() function in Matlab for this. Reassign ζ to the damping ratio of the complex poles
Step 6: If ζ has converged (i.e., the value did not change much) then stop. Otherwise, go back to Step 2 and iterate.

Present the final values of ζ, ω_n, and k, which you converge on.

3.3 Model Validation with Time Domain Simulation:

You will now verify the model you identified by simulating its response to the excitation captured during the step response experiment. The Simulink file, shown in Figure 7, has already been provided for you on the course website (*flexible_drive.mdl*).

Apply the input signal u $[V]$ which was recorded during the step response experiment with ± 1.5 $[V]$ amplitude and run the simulation for the duration of the captured data. Overlay the simulated velocity profiles for v_1 and v_2 on top of the experimentally recorded data in Part 2.5. If you see major discrepancies between the measured and simulated step response, you may slightly adjust the real pole location $-p$ and re-compute the other parameters using the procedure explained in Part 3.2.

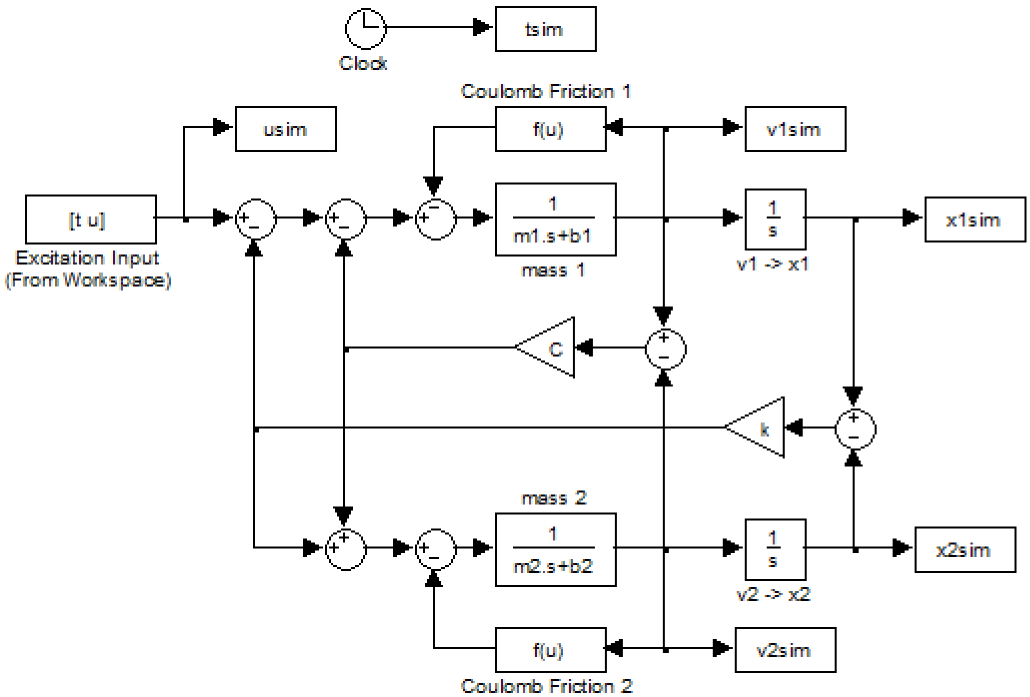

Figure 7: Simulink model of the flexible drive (*provided on the course website*).

3.4 Model Validation with Frequency Domain Data:

Frequency domain identification usually complements the information obtained from time response data. In this lab, you will use frequency domain data to validate your model. Since the controller design in Lab #4 will be carried out using frequency domain techniques, it is very important to have an analytical model that accurately captures the frequency response of the experimental setup.

Process the measurements you collected in Part 2.6 using the Matlab function *fit_sine_wave*() (posted on Learn). This function can be used to find the best fitting amplitude and phase values for sinusoidal signals which are corrupted by noise. Construct a table in the format shown on the next page, using the amplitude and phase estimates obtained for individual input voltage and output velocity signals. <u>You may need to adjust phase values by $-360°$ to get the correct value; use your judgment.</u>

Present the Bode plots of the theoretical velocity response for both carts (i.e. $sG1(s)$ and $sG2(s)$ from Equation (1)) in two separate graphs, neglecting the effect of Coulomb friction (i.e. neglecting $d1(s)$ and $d2(s)$) in your model. In other words, the transfer functions to be plotted become:

$$\left.\begin{aligned} v_1(s) = sG_1(s) = \frac{\frac{1}{m_1 m_2}[m_2 s^2 + (b_2 + c)s + k]}{(s^3 + a_1 s^2 + a_2 s + a_3)} u(s) \\ v_2(s) = sG_2(s) = \frac{\frac{1}{m_1 m_2}[cs + k]}{(s^3 + a_1 s^2 + a_2 s + a_3)} u(s) \end{aligned}\right\} \tag{7}$$

Use a logarithmic frequency range of $10^{-1} \dots 10^3$ $[rad/sec]$, with 1000 data points. You may use Matlab's logspace() function for this. Present the Bode plots with logarithmic scales for gain $[(mm/sec)/V]$ (Not decibels) and frequency $[rad/sec]$ axes, and a linear scale for the phase $[deg]$. In generating the data for your plots, you may either use Matlab's bode() function, or substitute values of $j\omega$ directly into the transfer function numerator and denominator. If you take the latter approach, be sure to use the unwrap() function in order to process the phase shift correctly.

PS% The detail of MATLAB functions can be found in MATLAB Help Documentation.

Frequency [Hz]	Gain v_1 / u [(mm/sec)/V]	Phase $\angle(v_1 / u)$ [deg]	Gain v_2 / u [(mm/sec)/V]	Phase $\angle(v_2 / u)$ [deg]
0.5				
1.0				
1.5				
2.0				
2.5				
3.0				
3.5				
4.0				
4.5				
5.0				
8.0				
12.0				
16.0				
20.0				

3.5 Report Content:

Include the plots from each of Parts 2.1, 2.2, 2.3, 2.4 in the specified format using the provided *plot_data_p_control* file:

1. Commanded (xr) and actual position trajectory of cart 1 (x1) versus t
2. Commanded (xr) and actual position trajectory of cart 1 (x2) versus t
3. Cart 2 tacking error: $e = x_r - x_2$ versus t.
4. Control signal: u versus t.

Based on the results you observe in these plots, briefly discuss the following:

- What is the effect of changing the P-control gain in non-collocated control?
- What are the advantages and disadvantages of collocated control with respect to non-collocated control in terms of achieving closed-loop stability and positioning accuracy for m_2?

Plot the step responses for **both** amplitudes from Part 2.5 in the specified format using the provided *plot_velocity_step_response* file:

1. Cart 1 velocity v_1 and Cart 2 velocity v2 versus t (on top)
2. Input signal: u versus t (at the bottom)

For Part 2.6 a plot for all frequencies is not required. Instead, provide $1 - 4$ plots, following the format for Part 2.5, that can be used to highlight visual observations about how the system responds as the input frequency changes, referencing the natural frequency, phase lag between the carts, and amplitude.

For Part 3.2, describe the methodology used (including any averaging) and show some plots which demonstrate how the values were collected from the responses. Show the final values for b_T, b_1, b_2, d_{coul_T}, d_{coul_1}, d_{coul_2}, p, m_T, m_1, m_2, ω_d, k, ζ, ω_n, C in a table. Provide comments if any were adjusted and the reason.

For Part 3.3, overlay the simulated velocity profiles for v_1 and v_2 on top of the experimentally recorded data in Part 2.5. Use a different line style, thickness, or preferably color to distinguish the simulated velocity response from the experimentally recorded ones. . The plots should be in the following format:

1) Simulated and experimental $v1$ versus t (on the top).
2) Simulated and experimental $v2$ versus t (in the middle).
3) Input signal u versus t (at the bottom).

Compare the simulation and experimental results and state two observations.

For Part 3.4, include the table of gain and phase values obtained from *fit_sine_wave*(). Show a bode plot for both the theoretical response of $sG1(s)$ and $sG2(s)$ from Equation (7) using the identified parameters from Part 3.2, with the overlaid experimental values as discrete elements from the table. Remember to set the units of the axes correctly as mentioned in Part 3.4. Compare the theoretical and measured frequency response functions (FRF's); this just means to compare the bode plots to the points and state two observations for each plot.

Report format and submission: (please check laboratory general rules and report guidelines first)