MTE 360 Automatic Control Systems: Lab #2

PD / PID Control, Steady State Error and Stability Analysis

In this laboratory, you will

1) Design and implement PD and PID controllers,
2) Determine the stability limits for your design,
3) Conduct steady-state error analysis and analyze the effect of disturbance inputs.

- ❖ Every group will use separate controller design specifications, which is provided on the last page of this document.
- ❖ Every group must work independently, communication between groups is not allowed.

1- Proportional – Derivative (PD) Controller

A PD controlled cart system is modeled in Figure 1. x_r and x denote the command and actual (measured) positions[1], respectively. The tracking error is equal to $e = x_r - x$. The control signal (u) is the motor input voltage generated proportional to the value of the tracking error, and its time derivative, through K_p and K_d gains, respectively. In the estimated cart model, m and b represent the equivalent inertia and viscous damping, respectively. Since we do not know the exact values for m and b, we should use our previously obtained estimations to approximate the cart model (from Lab#1).

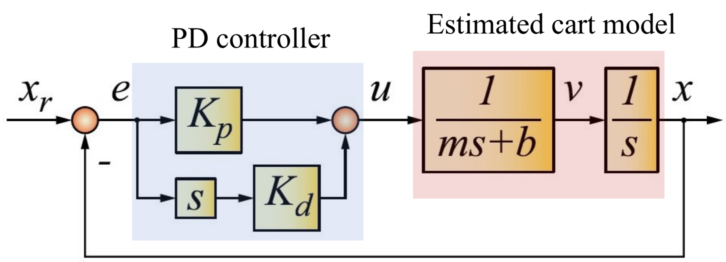

Figure 1: PD controlled servo system.

A. [Before the lab session] Using the estimated k_v and τ_v values found in Lab 1, determine the values of the equivalent inertia (m) and damping (b) in the estimated drive model.

B. [Before the lab session] Find the close loop transfer function of the above diagram. Compare it with the transfer function of an ideal second order system as

$$\frac{X(s)}{X_r(s)} = \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}.$$

Determine the values of K_p and K_d which result in the given (see page 6) natural frequency ω_n and damping ratio ζ for the closed-loop poles. The specifications for each group have been posted on the last page.
Hint: Consider the resemblance between the closed-loop transfer function denominator, and that of an ideal second order system.

[1] In the provided model, the x_r and x values are always in millimetres.

C. [After the lab session] Simulate the closed-loop step response for the system given in Figure 1. Repeat the same simulation for the ideal second order transfer function. Plot the input command (x_r), and the measured position (x) of both systems overlaid on top of each other in the same graph. Answer the following questions in order

1. What is the behaviour of an ideal second order system with the given natural frequency of ω_n and damping ratio of ζ in terms of <u>rise time</u>, <u>overshoot</u>, and the <u>response initial slope</u>? (Provide analytical response)
2. How do you expect changes of P and D gains affect the abovementioned criteria? (Provide analytical response)?
3. Do your observations from simulation of the ideal second order system match your analysis? (Answer with referencing to the obtained graph)
4. Do your observations from simulation of the closed loop system in Figure 1 match your analysis? (Answer with referencing to the obtained graph)

D. [During the lab session] Apply the designed PD controller in Part 1.B to the cart setup. Use a PD controller block according to the following steps:

a. Add a PID controller to your model from Simulink/Continuous library.
b. Open the block parameters (by double clicking on the block).
c. Change the Controller type from PID to PD.
d. Set the obtained proportional and derivative coefficients.
e. Leave the filter coefficient as default (N = 100).
f. Make sure to add a saturation block after the PID controller with upper and lower limits of ± 7.

Your implementation should look like Figure 2. Simulate the closed-loop step response by applying a square wave input with ±10 [mm] amplitude at 0.5 [Hz] frequency. Capture 8.0 [sec] of data containing the reference position (x_r), actual position (x), error (e), and control (u) signal (you will later plot them).

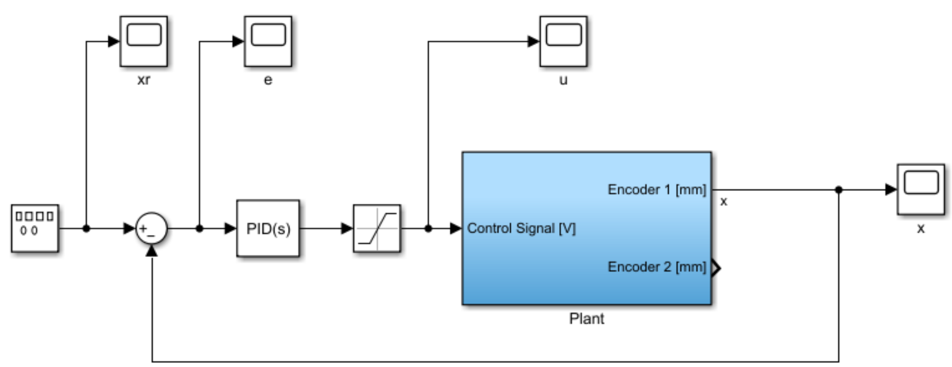

Figure 2- Implementation of a PD controller on the cart setup.

E. [During the lab session] Apply the position command shown in Figure 3 to your closed-loop system implemented in previous part following these steps:

a. Download the “Trajectory.mat” file from the Lab folder in LEARN and load it to your workspace.
b. Add a “From Workspace” block to your simulation.
c. Double click on the block and insert “Trajectory” under the “data” parameter.
d. Use the block output as your new input to the mode.

During the experiment, monitor and save the commanded and actual position, tracking error, and control signal ($u(t)$) profiles. Capture data for 8.0 [sec] and save the data for x_r , x , e, and u .

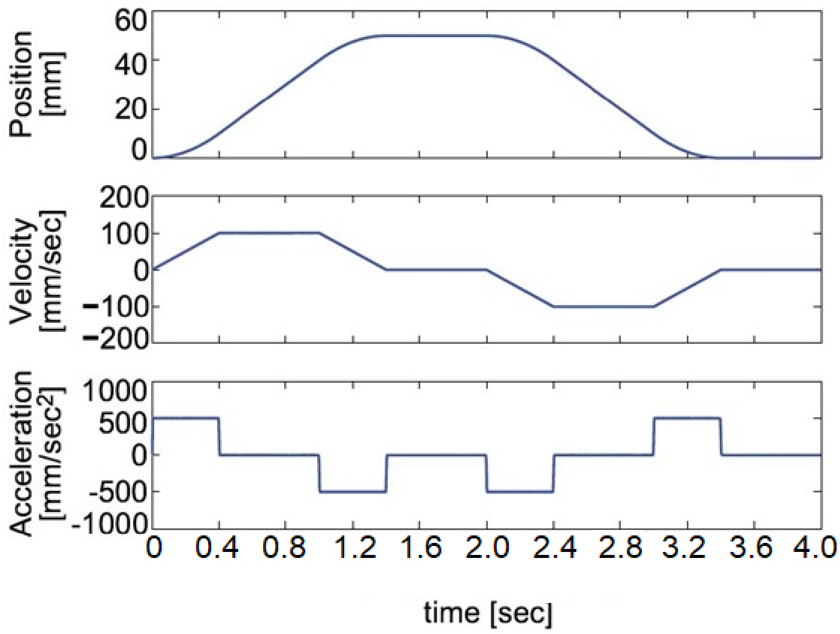

Figure 3: Command trajectory for tracking experiments.

F. [After the lab session] Repeat tasks 1.D and 1.E with the estimated cart model from Figure 1. For each input case present a 3x1 subplots with common time axis[2]. Which single factor causes the main difference between the responses of actual and the estimated plants?

2 - Proportional – Integral – Derivative (PID) Control and Stability Analysis

Integral action is frequently used in control systems to eliminate steady-state tracking error. By adding an integrator with a gain $K_i[v/(mm.s)]$ to the PD controller, a Proportional – Integral – Derivative (PID) controller is obtained, as shown in Figure 4:

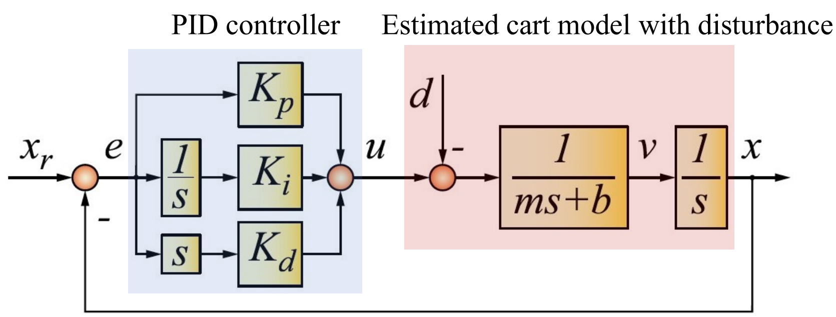

Figure 4: PID controlled drive system.

[2] Generate a 3x1 subplot, and in the first graph, plot x_r as well as the x for the actual and estimated plants overlaid on top of each other. In the second graph, plot the error of actual and estimated plants. In the third graph, plot the control output (u) of the actual and estimated plants. Repeat the whole procedure for the other input.

You will soon learn about an approach to tune PID controllers semi-automatically, but here we will focus on the effect of PID controller on stability, transient response and the tracking error, while the parameters of PID controllers are prescribed.

A. [Before the lab session] Using Routh's stability criterion, derive the mathematical expressions for the PID gains (K_p, K_d, and K_i) which will guarantee the stability of the closed-loop system in Figure 4 (consider d=0).

B. [Before the lab session] Using the PD gains you computed in Part 1.B, determine the upper limit for the integral gain ($K_{i,\max}$) , required to hold stability.

C. [During the lab session] Implement and test your PID controller on the cart system and estimated plants using the smooth trajectory in Figure 3 as the input. To have a reasonable amount of integral action, while also guaranteeing stability, set $K_i = 0.3K_{i,\max}$. Capture the profiles for x_r , x , e, and u for a duration of 8.0 [sec].

D. [After the lab session] Simulate the step response of the estimated system with PID controller with $K_i = \alpha K_{i,\max}$ for $\alpha = 0.3, 0.7, 1, 1.2$. Hint: Use the simulation you used in Part 1.D (Figure 2). Remove the saturation, change the controller type to PID, and set the integral gain. Set the Derivative filter coefficient (N) greater than 10000.

Present the results in a 3x1 subplots[3] with common time axis for each α. Comment on how performance of the system is affected by α as you increase it. Support your comments with control theory and your engineering judgment.

E. [After the lab session] Show the simulation and experimental results on top of each other, as described in section 1.F. Compare PID and PD controller performance using the obtained results in section 1.F. Elaborate on the similarities and differences between the two cases. Comment on the contribution of integral action to the servo performance. Does integral control provide better tracking and disturbance rejection? You may use the results of Part 3.D.

F. [After the lab session] Plot pole-zero maps of the closed loop system with PD and PID controllers with $K_i = 0.3K_{i,\max}$. Comment on the pole and zero locations, also referring to natural frequency and damping ratio values. You may use the MATLAB commands "pzmap()" and "damp()". How the closed loop pole locations are affected with the introduction of integral action?

3 –Disturbance and Steady State Error Analysis [Optional post lab activity with 20 bonus points]

So far you did not include the effect of the disturbance (static friction) in your estimated model simulation. A more realistic model would also include the effect of friction as a second input, as shown in Figure 4. In the figure, d [v] represents the equivalent disturbance, which mainly originates from the Coulomb friction in the drive mechanism.

[3] Generate a 3x1 subplot. In the first graph, plot xr as well as the x for each α overlaid on top of each other. In the second and the third graphs, plot the tracking error and control output for each α overlaid, respectively.

Considering that the magnitude of Coulomb friction should ideally be constant (d_c [v]), the disturbance model can be expressed as:

$$\mathrm{d} = \mathrm{d_c}.\,\mathrm{sgn}(\mathrm{v})\ ,\ \text{where}\ \mathrm{sgn}(\mathrm{v}) = \begin{cases} +1 & \text{if } \mathrm{v} > 0 \\ 0 & \text{if } \mathrm{v} = 0 \\ -1 & \text{if } \mathrm{v} < 0 \end{cases}$$

In the following steps, you will analyze how the disturbance will affect the performance of PD and PID controller. Then, you will identify the disturbance average value. Finally, you will enhance the accuracy of the estimated plant by adding the identified disturbance.

A. Express the drive position x as a function of trajectory command (x_r) and disturbance input d in in Figure 4 in the following form:

$$X(s) = G_{x_r \to x}(s)X_r(s) + G_{d \to x}(s)D(s)$$

B. Using the above equation, express the tracking error e as a function of the command position (x_r) and disturbance d in the following form:

$$E(s) = G_{x_r \to e}(s)X_r(s) + G_{d \to e}(s)D(s)$$

C. We define $e_{ss_{traj}}$ as the steady state error due to the trajectory input (computed when the disturbance is zero). Similarly, $e_{ss_{fric}}$ is the steady state error due to the disturbance input (computed when the input trajectory is zero). What is the expressions for $e_{ss_{traj}}$ and $e_{ss_{fric}}$ in case of step and ramp trajectory inputs? (Consider step function for disturbance in both cases)

D. Repeat Part 3.C for a PD controller. Then, compare the PD and PID controllers' performance in terms of steady state error in presence of disturbance. Support your response with help of the simulation results in sections 1.F and 2.C.

E. Carefully observe the value of tracking error during <u>constant velocity</u> motion in the data collected from Part 1.E (with the smooth trajectory). Use the above analysis to determine the average value of Coulomb friction (d_c [v]) in the cart setup. Explain your methodology in the calculations. Also indicate the theoretical value of $e_{ss_{traj}}$.

F. Incorporate the effect of Coulomb friction into your estimated setup simulation (1.F), as shown in Figure 5. Overlay the collected data from the estimated and actual cart system experiments once more similar to what you did in 1.F. Comment on how modeling the Coulomb friction has affected the simulation results by comparing your results with the results of 1.F.

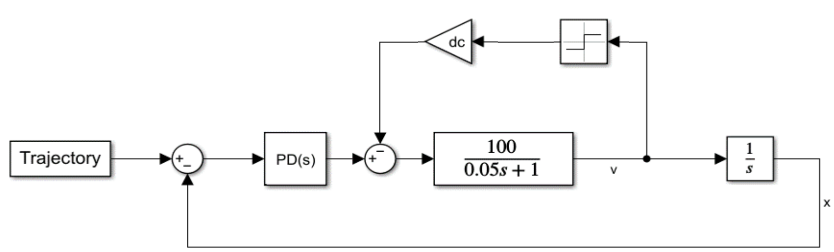

Figure 5: Simulink model of PD controlled system with Coulomb friction.

Report format and submission: (please check laboratory general rules and report guideline first) Every group will submit one electronic report onto LEARN.

MTE 360 Lab 2 - PD Controller Design Specifications

Do not forget multiplying your natural frequency (ω_n) with '2π' .For example, if ω_n is given 5 [Hz], it is equivalent to 10π [rad/s].

Group 1		Group 2		Group 3		Group 4		Group 5	
ω_n [Hz] 7.0	ζ [ ] 0.7	ω_n [Hz] 7.0	ζ [ ] 0.6	ω_n [Hz] 8.0	ζ [ ] 0.8	ω_n [Hz] 9.0	ζ [ ] 0.5	ω_n [Hz] 10.0	ζ [ ] 0.7

Group 6		Group 7		Group 8		Group 9		Group 10	
ω_n [Hz] 6.0	ζ [ ] 0.5	ω_n [Hz] 7.5	ζ [ ] 0.6	ω_n [Hz] 8.5	ζ [ ] 0.8	ω_n [Hz] 9.5	ζ [ ] 0.5	ω_n [Hz] 6.0	ζ [ ] 0.7

Group 11		Group 12		Group 13		Group 14		Group 15	
ω_n [Hz] 9.5	ζ [ ] 0.7	ω_n [Hz] 8.0	ζ [ ] 0.6	ω_n [Hz] 9.0	ζ [ ] 0.7	ω_n [Hz] 10.0	ζ [ ] 0.5	ω_n [Hz] 6.5	ζ [ ] 0.7

Group 16		Group 17		Group 18		Group 19		Group 20	
ω_n [Hz] 7.5	ζ [ ] 0.5	ω_n [Hz] 8.5	ζ [ ] 0.6	ω_n [Hz] 9.5	ζ [ ] 0.8	ω_n [Hz] 6.5	ζ [ ] 0.8	ω_n [Hz] 7.5	ζ [ ] 0.8

Group 21		Group 22		Group 23		Group 24		Group 25	
ω_n [Hz] 8.0	ζ [ ] 0.7	ω_n [Hz] 8.5	ζ [ ] 0.5	ω_n [Hz] 7.0	ζ [ ] 0.8	ω_n [Hz] 6.0	ζ [ ] 0.9	ω_n [Hz] 10.0	ζ [ ] 0.6

Group 26		Group 27		Group 28		Group 29		Group 30	
ω_n [Hz] 9.0	ζ [ ] 0.8	ω_n [Hz] 6.0	ζ [ ] 0.8	ω_n [Hz] 8.5	ζ [ ] 0.7	ω_n [Hz] 8.5	ζ [ ] 0.9	ω_n [Hz] 6.5	ζ [ ] 0.9