# Proofs name: Sahand Dler

## 1. Proof:

A× (B ∩ C) = (A × B) ∩ (A × C)

Proof:  
Let (a, x) ∈ A × (B ∩ C).

Then a ∈ A and x ∈ B ∩ C, meaning x ∈ B and x ∈ C.  
Therefore, (a, x) ∈ A × B and (a, x) ∈ A × C.  
Hence, (a, x) ∈ (A × B) ∩ (A × C).  
  
Conversely,  
 Let (a, x) ∈ (A × B) ∩ (A × C).  
 Then (a, x) ∈ A × B and (a, x) ∈ A × C.  
 Thus, a ∈ A, x ∈ B, and x ∈ C.  
 Hence, x ∈ B ∩ C and (a, x) ∈ A × (B ∩ C).  
  
Thus, A × (B ∩ C) = (A × B) ∩ (A × C).

## 2. Proof:

A × (B ∪ C) = (A × B) ∪ (A × C)

Proof:  
 Let (a, x) ∈ A × (B ∪ C).  
 Then a ∈ A and x ∈ B ∪ C, meaning x ∈ B or x ∈ C.  
 If x ∈ B, then (a, x) ∈ A × B.  
 If x ∈ C, then (a, x) ∈ A × C.  
 Thus, (a, x) ∈ (A × B) ∪ (A × C).  
  
Conversely,  
 Let (a, x) ∈ (A × B) ∪ (A × C).  
 Then (a, x) ∈ A × B or (a, x) ∈ A × C.  
 Thus, a ∈ A and x ∈ B or x ∈ C, which implies x ∈ B ∪ C.  
 Hence, (a, x) ∈ A × (B ∪ C).  
  
Thus, A × (B ∪ C) = (A × B) ∪ (A × C).

## 3. Proof:

(A × B) ∩ (C × D) = (A ∩ C) × (B ∩ D)

Proof:  
 Let (x, y) ∈ (A × B) ∩ (C × D).  
 Then (x, y) ∈ A × B and (x, y) ∈ C × D.  
 Thus, x ∈ A, y ∈ B, x ∈ C, and y ∈ D.  
 Hence, x ∈ A ∩ C and y ∈ B ∩ D.  
 Therefore, (x, y) ∈ (A ∩ C) × (B ∩ D).  
  
Conversely,  
Let (x, y) ∈ (A ∩ C) × (B ∩ D).  
 Then x ∈ A ∩ C and y ∈ B ∩ D, meaning x ∈ A, x ∈ C, y ∈ B, and y ∈ D.  
 Thus, (x, y) ∈ A × B and (x, y) ∈ C × D.  
 Hence, (x, y) ∈ (A × B) ∩ (C × D).  
  
Thus, (A × B) ∩ (C × D) = (A ∩ C) × (B ∩ D).

## 4. Proof:

(A × B) ∪ (C × D) ⊆ (A ∪ C) × (B ∪ D)

Proof:  
 Let (x, y) ∈ (A × B) ∪ (C × D).  
 Then (x, y) ∈ A × B or (x, y) ∈ C × D.  
 If (x, y) ∈ A × B, then x ∈ A and y ∈ B.  
 Since x ∈ A ⊆ A ∪ C and y ∈ B ⊆ B ∪ D, (x, y) ∈ (A ∪ C) × (B ∪ D).  
 If (x, y) ∈ C × D, then similarly, (x, y) ∈ (A ∪ C) × (B ∪ D).  
  
Thus, (A × B) ∪ (C × D) ⊆ (A ∪ C) × (B ∪ D).