# Antenna HW#8 Bonus Paper

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# Summary of "The field equivalence principle: Illustration of establishment of Non-intuitive Null fields"

## Introduction

The field equivalence principle is one of the **fundamental theorems** and has **numerous applications** in electromagnetic theory. In this article **Love's** and **Schelkunoff's** forms of **Equivalence principle** are also introduced. The examples consist of **plane waves** in **two half-space regions separated** by an **infinite planar surface**.

## Love's Equivalence Principle

Considering two regions denoted by I, II, and separated by the boundary S, which can be an arbitrary mathematical surface. Next, we consider the following assumptions about these defined regions.

- **Sources**: Region I, contains electric and magnetic current densities $J_1, M_1$. Region II is considered source-free.

- **Material**: Region II is free space and region I contains some material with electric permittivity $\varepsilon$ , and magnetic permeability $\mu$.

- **Problem**: we want to find the field caused by the inner excitation sources in the free space.

The figure below show's an illustration of this problem.

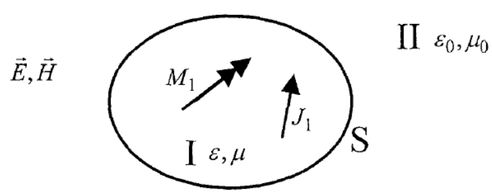


**Figure 2. The geometry of the problem.**

Love suggested that the **sources within** the **boundary** can be **replaced** by **electric** and **magnetic currents** <u>on it.</u> Moreover, the **material filling the region withing the boundary is replaced by free space**. The **excitation** is now **modeled** by the **tangential components** on the **surface** of the **boundary**. N is the normal vector between the tow regions and points directly towards region II.

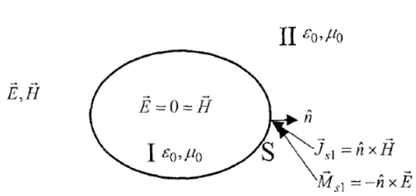


**Figure 3. Love's equivalence for Region II.**

It is stated that the tangential **current components on the surface produce a null field in Region I** which is not easily acceptable.

The mapping we discussed is known as **Love's equivalence for Region II** It means **we are changing properties of region I, and the boundary**, by **holding region II fixed**

$$\boxed{\begin{aligned} \bar{J}_{s1} &= \hat{n} \times \vec{H} \\ \bar{M}_{s1} &= -\, \hat{n} \times \vec{E} \end{aligned}}$$

The **equivalent problem** involving changes on region II and the boundary may also be set up.

In this model of the main problem**, we fill region II, with the material placed inside region I**. As region II had contained no independent excitation source, we should **set the surface currents** such that **the imply a null field in region II**.

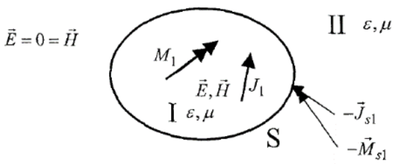


**Figure 4. Love's equivalence for Region I.**

This equivalence is called **the Equivalence for region I,** meaning **properties of region I** is **maintained** in the described model.

$$\bar{J}_{s1} = -\hat{n} \times \vec{H}$$

$$\bar{M}_{s1} = \hat{n} \times \vec{E}$$

Both the models described above and the original problem can be solved equivalently for each other.

The equivalence principle is based on the theorem of **"uniqueness of response"** in electromagnetic theory. Cause

1. Each region satisfies maxwell's equation
2. Tangential components satisfy the boundary conditions

## Schelkunoff's Equivalence Principle

Variants of the equivalence are proposed for different applications in modeling electromagnetic problems.

One **popular variant is to place PEC in the region of null fields**. This is completely fine causing the existence of null fields isn't at all related to the material placed in the corresponding region.

Placing a PEC, lets us use the **image principle**, and model it with another equivalent surface current. Now the whole problem is reduced to a **superposition of 2 surface currents.**

The tangential electric currents models on the surface are **impressed currents** and thus don't produce radiation

The fact stated above is proven using "Lorentz reciprocity theorem".

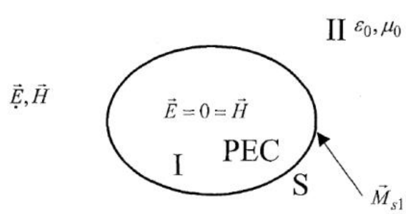


**Figure 5. Schelkunoff's equivalence for Region II, with a PEC in Region I.**

- The problem is reduced to find the fields caused by the magnetic currents placed on the surface.

The similar equivalence modelling can be done by placing a PMC and eliminating Magnetic currents as well.

## Example #1: Equivalence Principle for a homogenous medium

The simplest example we can consider is the wave propagation in free space. By dividing the homogenous by an arbitrary plane. We know the solution to this problem known as "wave propagation through free space":

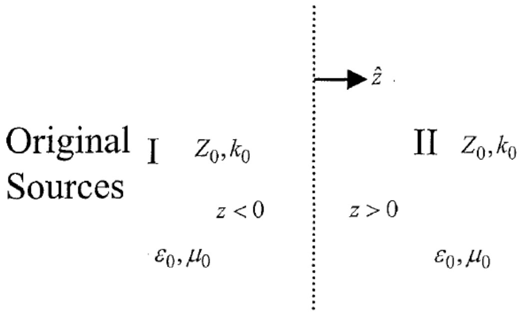


**Figure 6. Plane-wave propagation in a homogeneous region.**

$$\vec{E}^{inc} = \hat{x} E_0 e^{-jk_0 z}$$
$$\vec{H}^{inc} = \hat{y} \frac{E_0}{Z_0} e^{-jk_0 z}. \tag{1}$$

In Equation (1), $k_0 = 2\pi/\lambda_0$ (where $\lambda_0$ is the free-space wavelength) is the wavenumber, and $Z_0$ ($= \sqrt{\mu_0/\varepsilon_0}$) is the intrinsic impedance of free space.

### Love's Equivalence for region II:

Region I should contain a null field and an equivalent electric and magnetic current should be placed on the plane.

### Love's Equivalence for region I:

Respectively, Region II should contain a null field and an equivalent electric and magnetic current should be placed on the plane.

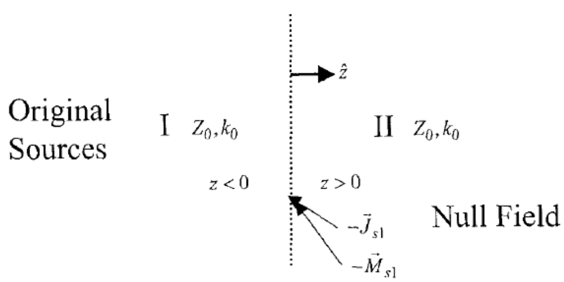


**Figure 8. Love's Equivalence Principle applied to Region I.**

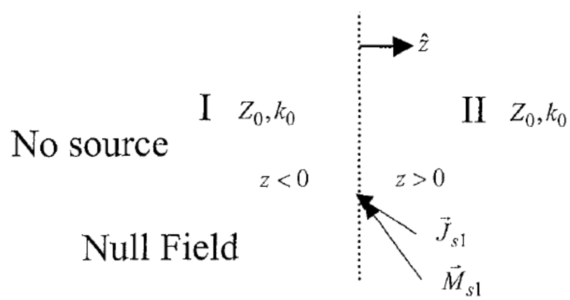


**Figure 7. Love's Equivalence Principle applied to Region II.**

$$\vec{J}_{s1} = \hat{n} \times \vec{H} = \hat{z} \times \hat{y} \frac{E_0}{Z_0} = -\hat{x} \frac{E_0}{Z_0},$$

$$\vec{M}_{s1} = -\hat{n} \times \vec{E} = -\hat{z} \times \hat{x} E_0 = -\hat{y} E_0.$$

### Fields created by electric current:

$z < 0$

$$\vec{E} = \hat{x} \frac{E_0}{2} e^{jk_0 z}$$
$$\vec{H} = -\hat{y} \frac{E_0}{2Z_0} e^{jk_0 z}$$

$z > 0$

$$\vec{E} = \hat{x} \frac{E_0}{2} e^{-jk_0 z}$$
$$\vec{H} = \hat{y} \frac{E_0}{2Z_0} e^{-jk_0 z}$$

### Fields created by magnetic current:

$z < 0$

$$\vec{E} = -\hat{x} \frac{E_0}{2} e^{jk_0 z}$$
$$\vec{H} = \hat{y} \frac{E_0}{2Z_0} e^{jk_0 z}$$

$z > 0$

$$\vec{E} = \hat{x} \frac{E_0}{2} e^{-jk_0 z}$$
$$\vec{H} = \hat{y} \frac{E_0}{2Z_0} e^{-jk_0 z}$$

By applying the superposition theorem, we obtain a net electric and magnetic field for each region and the filed nullity in the other region is obtained.

| $z < 0$ | $z > 0$ |
|---|---|
| $\vec{E} = 0$ | $\vec{E} = \vec{E}^{inc}$ |
| $\vec{H} = 0$ | $\vec{H} = \vec{H}^{inc}$ |

**Schelkunoff's Equivalence for region I:**

Replacing free space by PEC in region II, we obtain the null field in region II. Using image principle, we can replace the PEC with the image of the magnetic equivalent current and calculate the desired fields using superposition of magnetic currents as electric currents cancel out.

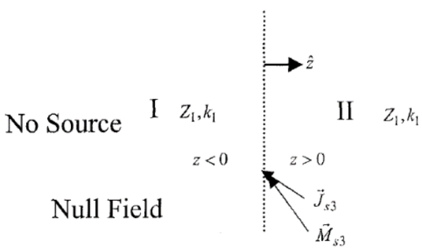


**Figure 12. The equivalence for Region II.**

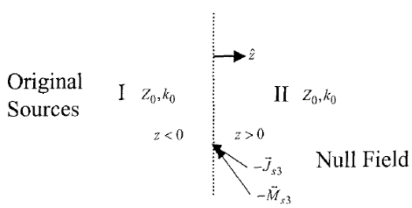


**Figure 13. The equivalence for Region I.**

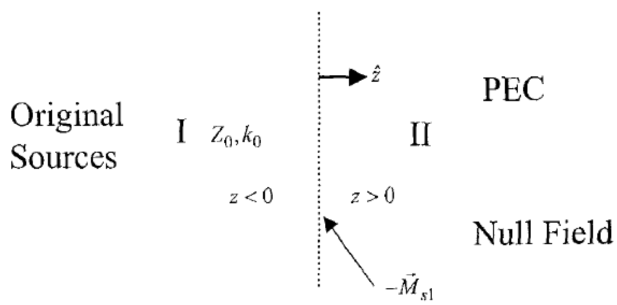


**Figure 10. Schelkunoff's Equivalence Principle applied to Region I.**

$$\vec{E}^{ref} = -\hat{x}E_0 e^{jk_0 z}$$

$$\vec{H}^{ref} = \hat{y}\frac{E_0}{Z_0}e^{jk_0 z}$$

$$\vec{E}\left(-2\vec{M}_{s1}\right) = \hat{x}E_0 e^{jk_0 z}$$

$$\vec{H}\left(-2\vec{M}_{s1}\right) = -\hat{y}\frac{E_0}{Z_0}e^{jk_0 z}$$