Que) For the given eq^n $y_1 = x_1 \oplus x_2 \oplus x_3$
$y_2 = x_1$. Find whether the S-box is linear or non linear. Represent the relationship using matrices

→ The above set of eq^n represent linear S-box.

$$y_1 = a_{1,1}x_1 \oplus a_{1,2}x_2 \oplus a_{1,3}x_3$$
$$y_2 = a_{1,1}x_1 \oplus 0 \oplus 0$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Que) In an S box with 3 inputs and 2 output we have the eq^n as

$y_1 = x_1 x_2$

$y_2 = x_1 + x_2 x_3$

Identify the type of S-box & show justification

→ The above set of eq^n represent non-linear S-box because it does not have any replationship between input and outputs.

same bits

Que) S box may or maynot be invertible. Justify your answer.

For an invertible S-box the number of input bits should be equal with number of output bits.

Following shows an example:

Eg. I/O relation of 2×2 S-box is shown in the following table. Show/ Design S-box table.

leftmost ↓ \	0	1 → Rightmost
0	01	11
1	10	00

Soln:

{ For the above table, the leftmost bit of the given input defines the row and the remaining rightmost bits define the column }

The above table can be represented as:

I/P ⇒	00	01	10	11
O/P ⇒	01	11	10	00
	1	3	2	0
	0	1	2	3

swap content and indices

3	0	2	1

3	0	2	1

I/p ⇒ 00 01 10 11

O/p ⇒ 11 00 10 01

Invertible S box :

	0	1
0	11	00
1	10	01

Que] Find the invertible S box for given values

	00	01	10	11
0	011	101	111	100
1	000	010	001	110

I/p ⇒ 000 001 010 011 100 101 110 111

O/p ⇒ 011 101 111 100 000 010 001 110

3	5	7	4	0	2	1	6
0	1	2	3	4	5	6	7

6	6	5	3	3	1	7	2

4	6	7	4

3	5	7	4	0	2	1	6

Que] Write
Block method
① Transposition P&box / D-box
② S box → linear
→ non linear
③ Ex-OR operation [Other component]

iii]

	00	01	10	11
0	110	001	010	011
1	100	111	101	000

ii]

	0	1
0	00	10
1	11	01

GF(2^n) : Galois Finite field
↳ Finite field
→ Finite no. of element

Pg 160
It means finite range
For eg. no. of bits = 64
Range : 2^{64}

X-OR Operation uses GF(2^n)
Properties : ① Closure
② Associativity
③ Commutativity
④ Existence of identity
⑤ Existence of inverse

To avoid carry & borrow use X-OR

④ ⇒ $x \oplus (00\ldots0) = x$
⑤ ⇒ $x \oplus x = (0000\ldots0)$
③ ⇒ $x \oplus y \leftrightarrow y \oplus x$

Other Component

→ X-OR → Circular shift (L & R)

→ Complement → Swap

→ Inverse → Split & combine [162]

Invertibility [Circular Shift]

A circular left shift opr. is inverse of circular right ship

Que) a] Show the result of 3 bit circular ~~left~~ right shift on word $(10011011)_2$

→

1 0 0 1 1 0 1 1 → 100

1 0 0 1 1 0 1 1

1 1 0 0 1 1 0 1

1 1 1 0 0 1 1 0

0 1 1 1 0 0 1 1

b] Show the result of 3 bit circular ~~right~~ left shift on word resulting from Part (a)

→

1 0 0 1 1 0 1 1

0 0 1 1 0 1 1 1

0 1 1 0 1 1 1 0

1 1 0 1 1 1 0 0

c] Compare the result of Part b with original word in Part a

→

Que 16] a] Swap the word $(10011011)_2$
b] Swap the word resulting from Part a
c] Compare the result of Part a and Part b to show that swapping is a self invertible operation

Que 17] Find result of following operation
a] (01001101) ⊕ (01001101)
b] (01001101) ⊕ (10110010)
c] (01001101) ⊕ (00000000)
d] (01001101) ⊕ (11111111)

→ a]
$$\begin{array}{r} 01001101 \\ \oplus\ 01001101 \\ \hline 00000000 \end{array}$$

b]
$$\begin{array}{r} 01001101 \\ \oplus\ 10110010 \\ \hline 11111111 \end{array}$$

c]
$$\begin{array}{r} 01001101 \\ 00000000 \\ \hline \end{array}$$

Que) A 6×2 S-box is X-Ored the odd no. of bit to get the left bit of O/p. Similarly perform X-OR operation on the even numbered bits to get the right bit of the O/p. Perform the mentioned on the following i/p string.

① 110010

② 101101

→ 5 4 3 2 1 0 odd bits

① 1 1 0 0 1 0 $1 \oplus 0 \oplus 1$

$= 0 \oplus 1 = 0$

even bits, $1 \oplus 0 \oplus 0 = 1$ And

$0 \oplus 0$ 01

5 4 3 2 1 0

② 1 0 1 1 0 1

odd bits, $1 \oplus 1 \oplus 0 = 1$

even bits, $0 \oplus 1 \oplus 1 =$

Que) The leftmost bit of 4×3 S-box rotates the other 3 bits. If the leftmost bit is 0 then other bits are rotated to right 1 bit. Similarly, if leftmost bit is 1 then the 3 other bits are rotated to the left 1 bit perform the above and generate the output for given input

① 1011 110

② 0110 011 0

Product Cipher

→ It is generally implemented in Modern block cipher
→ It is the product of substitution unit and transposition unit using other component.
→ Benefit of creating Product cipher is to apply on Modern Block Cipher.

Properties :
① Diffusion ⇒ Hides the relation between CT & PT.
② Confusion ⇒ Confuse the relationship between key & CT

NOTES

- Product Cipher is a combination of substitution, transposition, other component.
- It was designed by Shanon to generate complex Modern Block Cipher
- To build / design product cipher Shanon uses two imp properties

① Diffusion : It hides the relationship between CT & PT
② Confusion : This property confuses the attacker to find relation between key & CT.

The diffusion and confusion properties are applied based on the number of rounds i.e. number of iteration

n-bit PT ⇒ n-bit key
⇓
n-bit CT

} depends on rounds

MT : Middle text

It is generated at intermediate between PT & CT.

PT
MI
CT

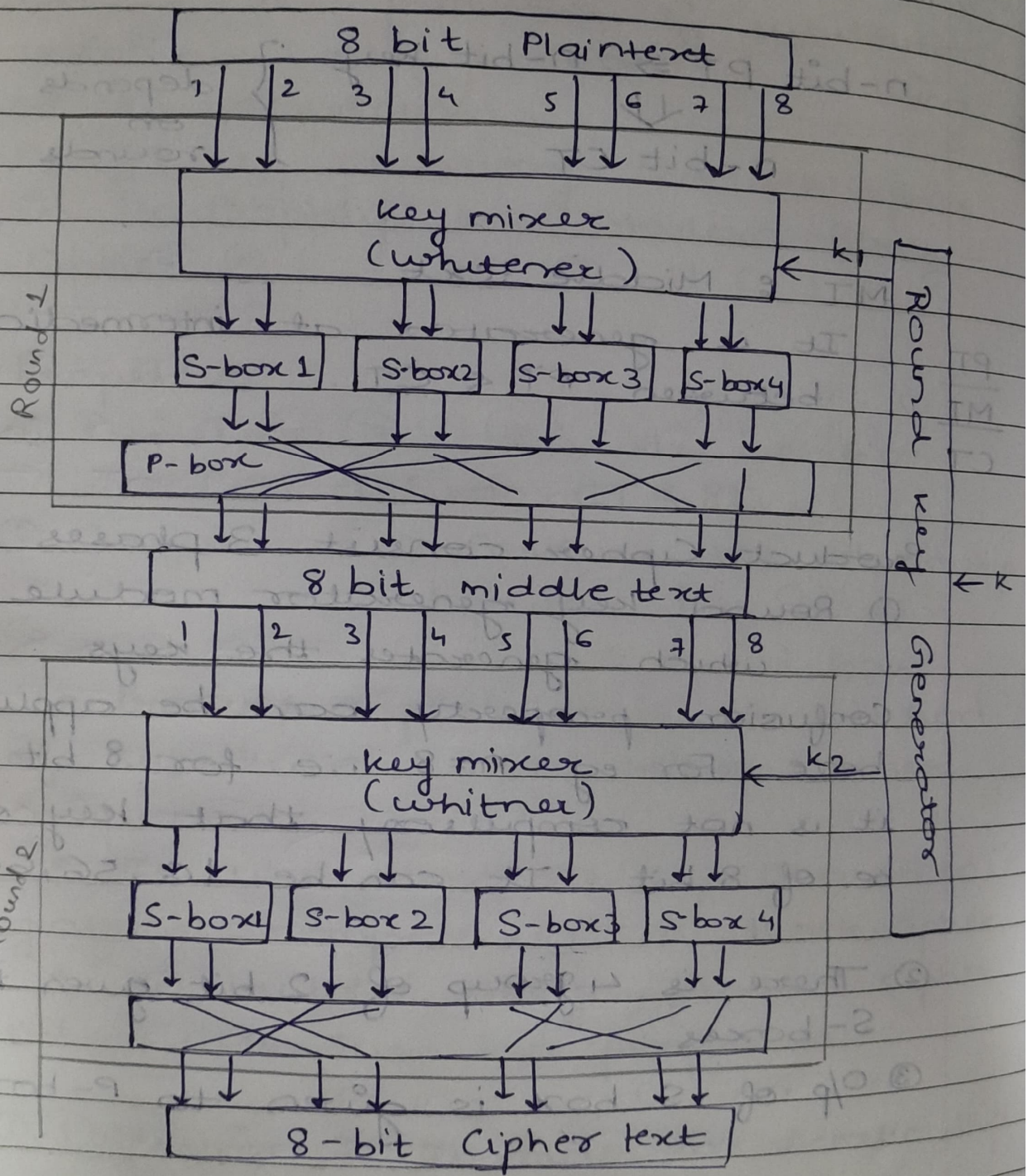
Product Cipher consist 3 phases :

① Round key generator module ⇒ which generates the keys
Confusion property can be applied here. For eg n=8 i.e for 8 bit PT it is not compulsory that key should be of 8 bit. It can be 16, 56, 64, etc.

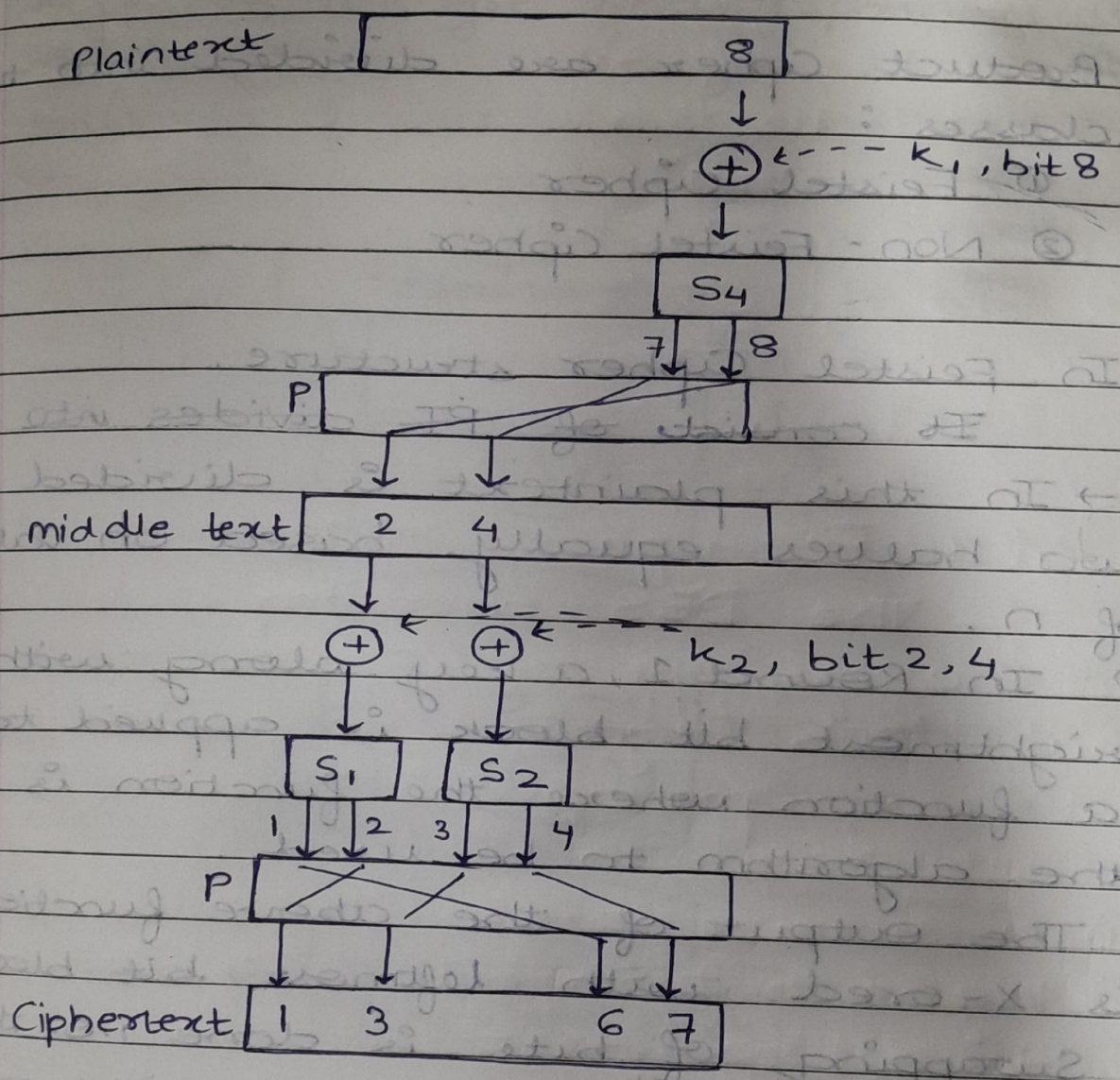
② There is 4 group of 2 bit given to S-boxes.

③ O/p of S box is given to P-box

Following figure shows product cipher of 2 rounds :



Following figures shows diffusion and confusion in block cipher



In the given figure, the 8th bit of PT is X-ored with 8th bit of key k_1. The X-ored value is given as input to 1×2 box, so we get 2 output 7 & 8 bit. The output of S box is given as input to P box. It gives middle text 2, 4 It is O/p of Round 1.

Summary : The 8th bit of PT affects 1, 3, 6, 7 bit of CT

Classes of Product Cipher :

Product Cipher are divided into two classes :

① Feistel cipher

② Non - Feistel Cipher

In Feistel Cipher structure,

It ~~consists of~~ PT divides into

→ In this plaintext is divided into two halves equally based on value of n.

→ In Round 1, a key along with rightmost bit block is applied to a function where the function is the algorithm to be used.

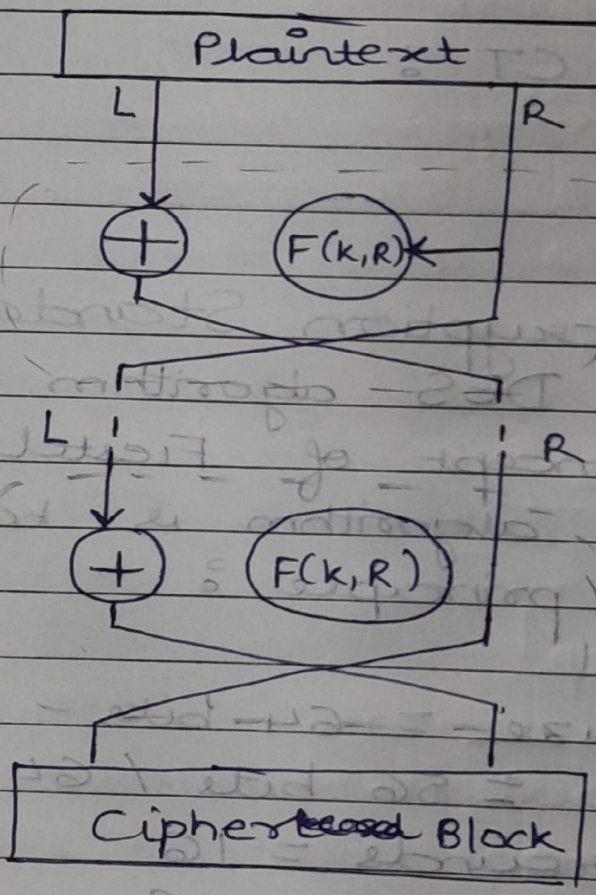
→ The output of the above function is X-ored with leftmost bit block

→ Swapping of bits is done i.e. the leftmost bit block now becomes right most bit block and similarly the right most bit block becomes leftmost bit block.

Thus, round 1 gets completed in above 4 steps. Thus the step 4 along with round key value becomes the input for next round.

Thus after the number of rounds completed depending on the type of algorithm we get final Ciphertext block.

Following figure shows the above process



Block Cipher designed principle are based on the following :

① Block Size : Length of the PT is equal to length of CT (depends on Algo)

② key Size : The given key size is known as master key size, depends on algorithm.

③ Number of rounds : Conversion from PT to CT

④ Number of subkeys count : We need to generate subkey from master key

⑤ Size of CT :

8-10m

* Data Encryption Standard Algo:

The DES algorithm is based on the concept of Fiestel structure. Thus, DES algorithm is based on following principle :

① Block size = 64 bits
② key size = 56 bits / 64 bits
③ No. of rounds = 16
④ No. of subkey = 16 where each subkey size = 48 bits
⑤ Size of ciphertext = 64 bits

- Initial permutation : reordering of bits
- 64 bits is given to round 1

Round 1 have 2 i/p :

① 64 bit reordered PT
② K_1

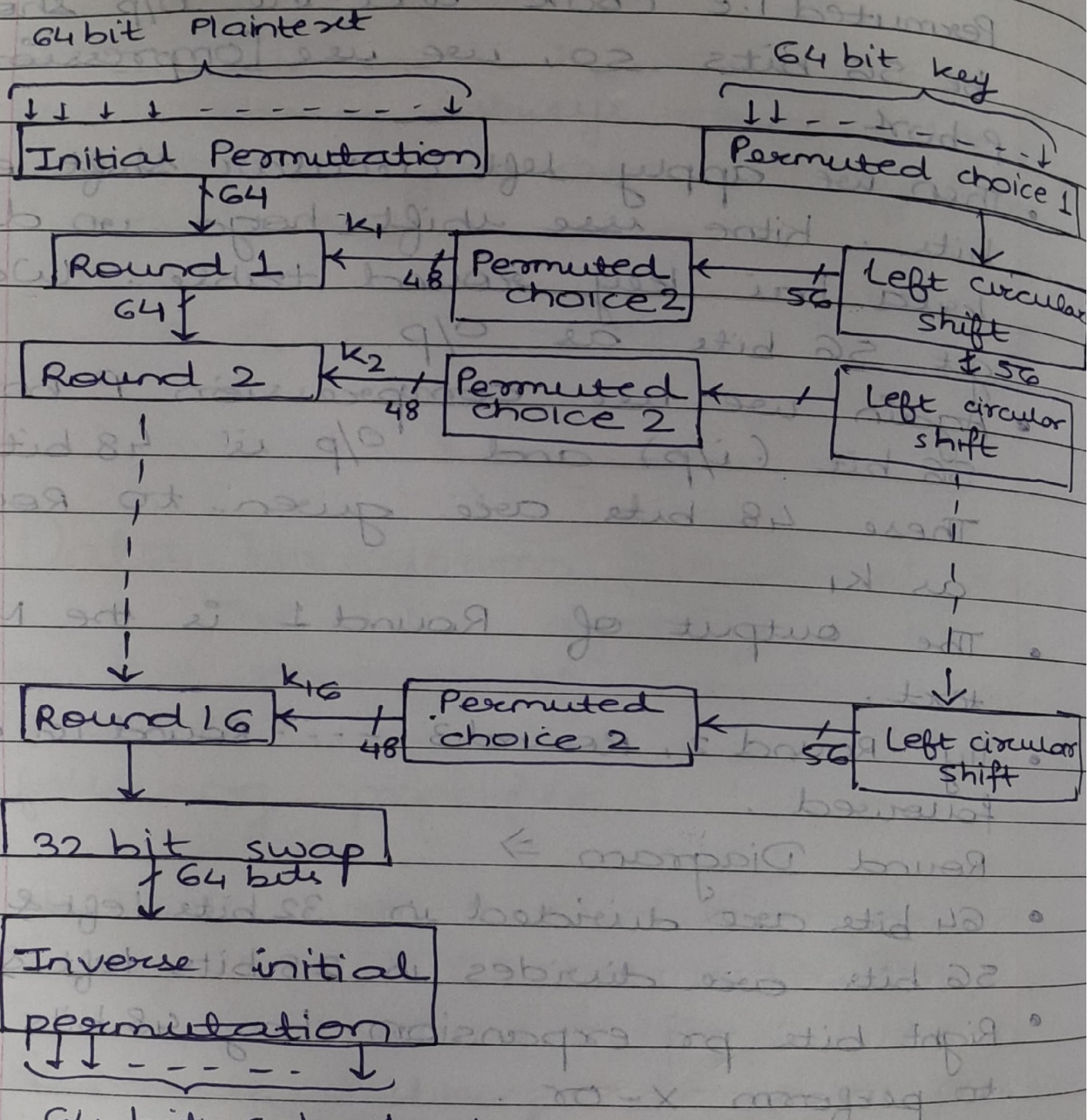
- We have 64 bit key which is given to Permuted i.e Pbox and the o/p should be 56 bits so, we use Compression P-box.
- Then we apply left circular shift on 56 bits. Kitne ~~use~~ shift hoga wo depend karta hai key round table. We get 56 bits as o/p.
- Again we use compression P box 56 bit (i/p) and o/p is 48 bits These 48 bits are given to Round 1 as K_1
- The output of Round 1 is the Middle-text.
- IIly Round 2, Round 3, ---- Round 16 are followed.

Round Diagram ⇒

- 64 bits are divided in 32 bits left & right.
- 56 bits are divides in 28 bits left & right.
- Right bits pr expansion bcz 48 bits chahiye to perform X-or.
- X-OR ka o/p is given to S box (32 bits)
- Then permutation i.e reordering (32 bits)
- Then XOR and swapping.
- In case of key, we perform left shift bcz we are doing encryption and do right shift in case of decryption

Block diagram of DES algorithm :

Fig 1



Following fig shows DES single round function process :

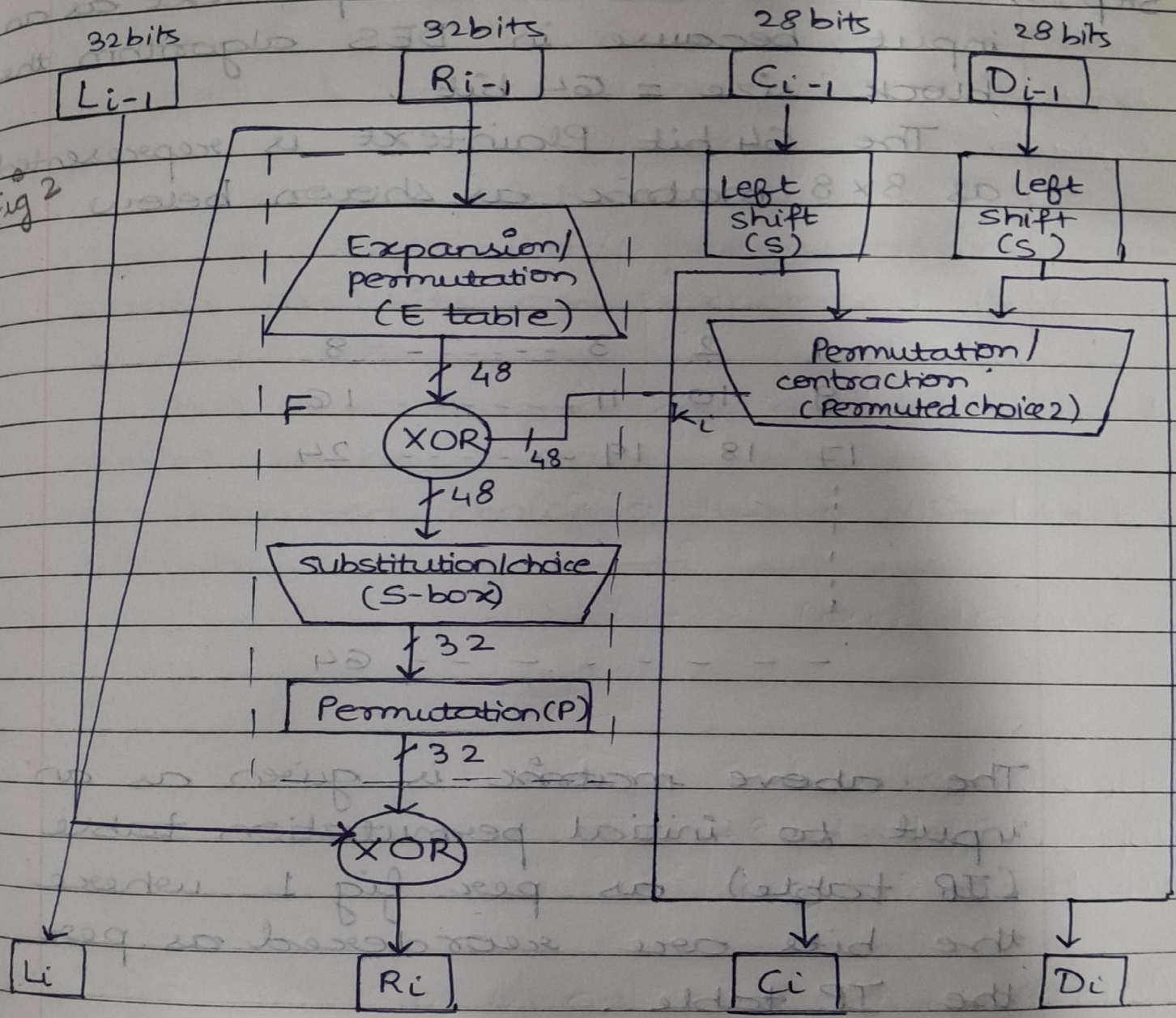


Fig 2

DES Encryption process :

Step 1 ⇒ Consider the 64 bit plaintext as an input because in DES algorithm the block size = 64 bit.

The 64 bit Plaintext is represented as 8×8 matrix as shown below.

1	2	3	.....	8
9	10	11	.....	16
17	18	19	.....	24
⋮				
.				
-	-	-	-	64

The above matrix is given as an input to initial permutation table (IP table) as per fig 1 where the bits are reordered as per the IP table.

According to the IP table, the first bit position of the IP table represented by 58th bit of PT.

Ily second bit position of IP table is reordered with 50th bit of PT and so on.

In this way 64 bits are reordered

After IP it is given to the round 1, according to fig 2.

Thus, the PT is now divided into two equal halves each of 32 bits block size.

Now, the right half 32 bits is given to expansion permutation table where the bits are expanded from 32 to 48 bits.

Thus the output of the EP table is needed to be X-ored with the output of Permuted choice 2 table. For this, we have to generate subkeys.

* Blowfish Algorithm :

- It is an symmetric key algorithm
- The size of the plaintext is 64 bits. It uses the concept of Fiestal structure
- It has variable key size as compare to DES (56 bits) (32 to 448 bit)
- Blowfish was developed by Bruce Schiner in 1993
- Total no. of rounds = 16 to convert the plaintext to ciphertext.
- It is an alternative DES algorithm
- It is divided into 2 parts :
  1. Generation of subkeys
  2. Data Encryption / Decryption process

① GENERATION OF SUBKEYS :

→ As the key size is variable that is of the size (32 bits to 448 bit) it is divided into 14 subkeys i.e. $k_1, k_2 \ldots k_{14}$ ($1 \leq n \leq 14$) each of 32 bit size.

→ Blowfish introduces the concept of p-array and S-boxes. Thus there are total 18 p-arrays numbered from p_1 to p_{18} or p_0 to p_{17} followed by 4 boxes of size 8×32.

→ Each Sbox has total 256 entries that is (0-255) which is defined as

$S_{1,0}$ $S_{1,1}$ ---- $S_{1,255}$

$S_{2,0}$. $S_{2,1}$ ---- $S_{2,255}$

0^{th} entry ↵ ↳ 255^{th} entry

→ Each S-box has 8 bit input & 32 bit output thus in 8 bit input 1st, 7^{th}, 8^{th} bit represents the row number.

→ Whereas (2,3,4,5,6) bit represents the column number. The intersection of these gives skew value.

→ Initialize the

In Fig 2,
32 bits are given as input
i.e 8 bit to each S box (4-S boxes)

→ The function F in Fig A is as follows as shown in Fig B

① Divide the 32 bit left block into four 8 bit subblock i.e. 8 bit S-box named as a, b, c, d.

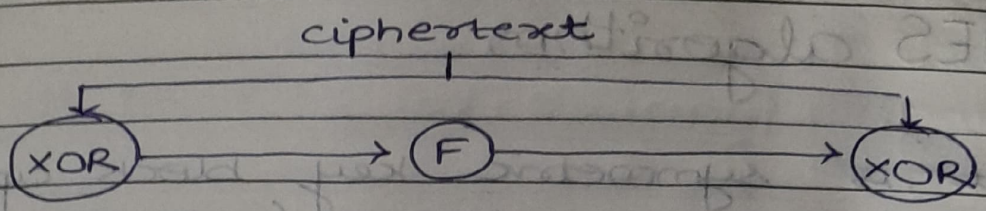
② Compute $F[a,b,c,d] = ((S_{1,a} + S_{2,b})$ x-or $S_{3,c}) + S_{4,d}$

Assume, $a = 10$, $b = 95$, $c = 37$, $d = 191$
then $F(a,b,c,d) = ((S_{1,10} + S_{2,95})$ x-or $S_{3,37}) + S_{4,191}$

$S_{1,10}$: 1st S box ki 10th value
$S_{2,95}$: 2nd S box ki 95th entry
[Each S-box has total 256 entries]

Following figure shows the Blowfish Decryption process :

Atul kate ch3 Pg 123-130



Que) Justify how to generate the subkey values using Blowfish Encryption algorithm

subkey : P array and S array.

Initial values → P_1 – P_{18} } 32 bit
S_1 – S_4 } fp of π

Refer ch 3 atul kahte pt c,d,e

* AES algorithm

→ It is symmetric key block cipher method
→ Developed by NIST (National institute of Standards and Technology) in Dec 2001
→ It's a non-Feistal structure
→ Block size of PT and CT is 128 bits
→ NO. of rounds : 10/12/14 bits

Fig 7.1 Fig 1. General design of AES encryption cipher

→ 128 bit PT is given to pre-round transformation [also called preprocessing where add round key operation is performed. Here 128 PT X-OR k_0
Round key = 128.

N_R = Number of rounds

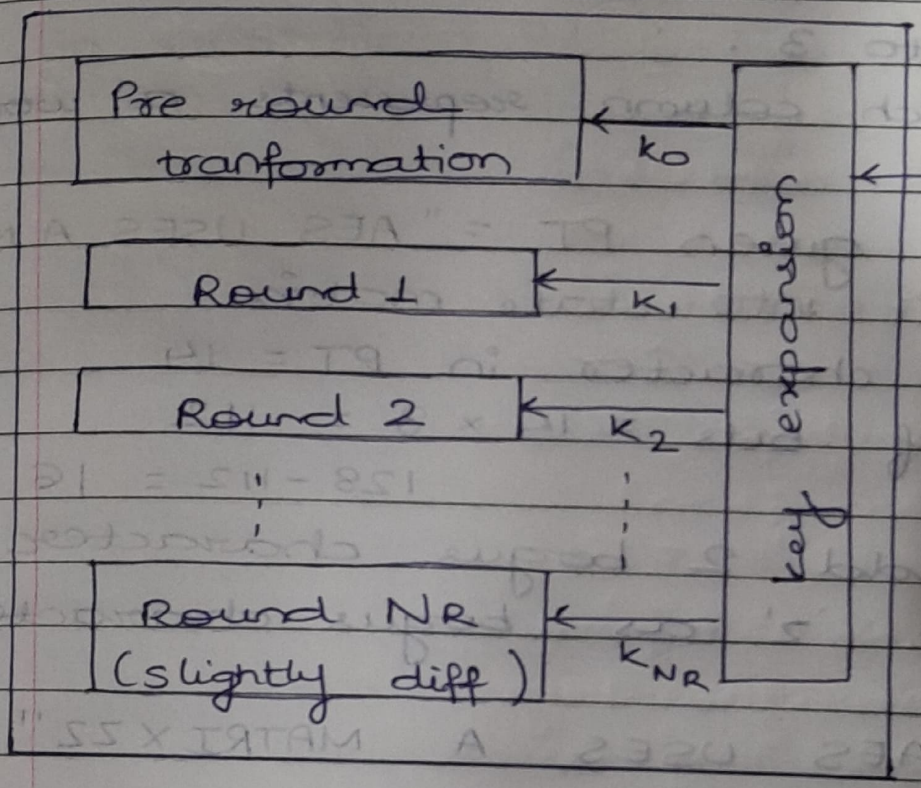
Imp → The number of round key generated by key expansion algo. is always one more than the number of rounds.

In other words, we have

NO. of round keys = $N_r + 1$

We refer round keys as $k_0, k_1, k_2 \ldots k_n$

Fig. General design of AES encryption



NR	key size
10	128
12	192
14	256

Data units used in AES

There are 5 data units

⑤ bit
① byte = 8 bits
② word = 4 bytes
③ block = 16 bytes = 128 bits
④ State matrix = 4 words

The block is converted in state matrix!

$$\begin{bmatrix} S_{0,0} & S_{0,1} & S_{0,2} & S_{0,3} \\ S_{1,0} & S_{1,1} & S_{1,2} & S_{1,3} \\ S_{2,0} & S_{2,1} & S_{2,2} & S_{2,3} \\ S_{3,0} & S_{3,1} & S_{3,2} & S_{3,3} \end{bmatrix} \downarrow \begin{bmatrix} w_0 & w_1 & w_2 & w_3 \end{bmatrix}$$

Define state matrix (2m)

It is a 4×4 matrix where rows ranges from 0 to 3 and column ranges from 0 to 3.

Each column represents a word

Que) For the given PT = "AES USES A MATRIX" convert it into state matrix.

Total character in PT = 14

∴ No of bits = 14 × 8 = 112

128 - 112 = 16

So we add 2 bogus character

Consider 'z' as bogus character

PT = "AES USES A MATRIXZZ"

16 × 8 = 128 bits

char to int values

↳ hexdec

ASCII → 00 to 25 → hexdec

A	E	S	U	E	S	A	M	A	T	R	I	X	Z	Z
00	04	18	20	04	18	00	12	00	19	17	08	23	25	25

Integer to Hex

00 04 18 20 04 18 00 12 00 19 17 08

00 04 12 14 04 02 00 0C 00 03 01 08

23 25 25

07 09 09

state matrix :

$$\begin{bmatrix} 00 & 04 & 00 & 08 \\ 04 & 02 & 03 & 07 \\ 12 & 00 & 01 & 09 \\ 14 & 0C & 08 & 09 \end{bmatrix}$$

que] Let the PT = "NET THE R THIS IS AN EXERCISE". Convert in state matrix

16 × 8 = 128 bits

T H I S I S A N E X E R C I S E

19 7 8 18 8 18 0

Unit : 3 Assymetric key

In symmetric key cryptography, the secret key must be shared between the two users whereas in case of assymetric key cryptography, the key is generated and kept as a personal secret.

Assymetric

↙ ↘

public key (encryption) | private key (decryption)

It is the responsiblity of the receiver to generate the keys.

The public key is distributed by the receiver and private key is kept by the receiver

Chapter 9 : Mathematics for assymetric key

positive integer

- Number 1 → exactly one divisor
- Prime → exactly two divisor
- Composite → more than two divisor

To check if number is prime ?

→ check number is divisible by all primes less less than $\sqrt{n}$ n

For eg 97

$\sqrt{97} \Rightarrow 9$ (we will consider only floor value)

2, 3, 5, 7 } check these.

→ Euler's Phi - Function

Denoted by $\phi(n)$ which is also called as Euler's totient function plays a very important role in cryptography

The function finds the number of integers that are both smaller than n and relatively prime to n.

Z_n is ller to $\phi(n)$

The following helps to find the value of $\phi(n)$:

① $\phi(1) = 0$

② $\phi(p) = p - 1$ if p is a prime

③ $\phi(m \times n) = \phi(m) \times \phi(n)$ if m and n are relatively prime

④ $\phi(p^e) = p^e - p^{e-1}$ if p is a prime

We can combine the above four rules to find value of $\phi(n)$. For example, if n can be factored as

$$n = p_1^{e_1} \times p_2^{e_2} \times \cdots \times p_k^{e_k}$$

then we combine the third and the forth rule to find

$$\phi(n) = (p_1^{e_1} - p_1^{e_1-1}) \times (p_2^{e_2} - p_2^{e_2-1}) \times \cdots \times (p_k^{e_k} - p_k^{e_k-1})$$

① $\phi(13) = 13-1 = 12$ [Using 2nd property]

② $\phi(10) = \phi(2 \times 5)$

By using property ③ where m and n are relative coprime

$\phi(2) \times \phi(5) = 1 \times 4 = 4$

③ $\phi(240)$

$240 = 2^4 \times 3 \times 5$

2	240
2	120
3	60
2	20
5	10
2	2
	1

Using property 4

$(2^4 - 2^3) \cdot \phi(3) \times \phi(5)$

16
- 8

$\phi(8) \times \phi(3) \times \phi(5)$

$(2^3 - 2^2) \times \phi(3) \times (5)$

$(2^2 - 2^1) \times \phi(3) \times \phi(5)$

$\phi(1) \times \phi(3) \times \phi(5)$

$1 \times 2 \times 4 = 8$

$$\begin{array}{r} 23 \\ \times\ 23 \\ \hline 529 \end{array}$$

Que) Find values of

① $\phi(529)$
② $\phi(582)$
③ $\phi(580)$
④ $\phi(500)$
⑤ $\phi(5101)$

→ $\phi(529) = \phi(23^2)$

$= 23^2 - 23^1$

$= 529 - 23$

$= 506$

23	529
23	23
	1

Que) ① $\phi(29) \Rightarrow 29-1 = 28$

② $\phi(32) \Rightarrow \phi(2^5)$

$= 2^5 - 2^4 = 32 - 16$

$= 16$

③ $\phi(80) \Rightarrow 2^4 \times 5$

$(2^4 - 2^3) \times \phi(5)$

$(16 - 8) \times 4$

8×4

$= 32$

2	80
2	40
2	20
2	10
	5

④ $\phi(100) \Rightarrow 2^2 \times 5^2$

2	100
2	50
	25

* Fermat's Little Theorem

It plays a very important role in number theory and cryptography. We introduce two versions of theorem here :

First Version :

The first version says that if p is a prime and a is an integer such that p does not divide a, then

$$\boxed{a^{p-1} \equiv 1 \bmod p}$$

i.e $a^{p-1} \bmod p = 1$

Second Version :

The second version removes the condition on a. It says that if p is a prime, and a is an integer then

Format mei ek to power wala p chota nora chahiye p se or equal but in que ② it is greater (12>11) so we apply rules

$$\boxed{a^p = a \bmod p}$$

que) $6^{10} \bmod 11$

$6^{11-1} \bmod 11 = 1$

② $3^{12} \bmod 11 \quad (3^{11} \times 3) \bmod 11$

$(a \times b) \bmod n = ((a \bmod n) \times (b \bmod n)) \bmod n$

$= ((3^{11} \bmod 11)(3 \bmod 11)) \bmod 11$

$= (3 \times 3) \bmod 11$

↳ using second version

$= 9 \bmod 11$

$= 9$

* Application of Fermat's little theorem

It is helpful for the following:

① To find a solution for modular exponentiation if modulus is a prime number

② Used to find the solution for calculating of multiplicative inverses if modulus is a prime number.

following equation shows the concept to find the multiplicative inverse as given below :

If p is a prime and a is an integer such that p does not p does not divide a ($p \nmid a$), then

$$a^{-1} \bmod p = a^{p-2} \bmod p$$

Que) $8^{-1} \bmod 7$

→ $8^{5} \bmod 7$

$5 < 7$

power < p then we can use Fast exponentiation (square and multiply method)

Represent x in binary $x = 5$

$5 = (101)_2$

$8^5 \bmod 7$

Let $x = 5 = (101)_2$

$= (8^4 \cdot 8^1) \bmod 7$

$= ((8^4 \bmod 7)(8^1 \bmod 7)) \bmod 7)$

$= (1 \times 1) \bmod 7$

$= 1 \bmod 7$

$= 1$

Ques $8^{-1} \bmod 17$

$8^{15} \bmod 17$

$15 < 17$, we use format exponentiation (square and multiply method)

Let $x = 15 = (1111)_2$

$8^8 \bmod 17 = 1$

$8^4 \bmod 17 = 16$

$8^2 \bmod 17 = 13$

$8 \bmod 17 = 8$

$1 \times 16 \times 13 \times 8 = 1664$

$1664 \bmod 17 = 15$

We can solve this using EEA to check the ans.

* Euler's Theorem

Pg 257

It can be thought of as a generalization of Fermat's little theorem. The modulus in the Fermat theorem is a prime, the modulus in Euler's theorem is an integer. We introduce two versions:

First Version :

It is similar to first version of Fermat's little theorem. If a and n are coprime, then $\boxed{a^{\phi(n)} \equiv 1 \pmod{n}}$

Second Version :

It is similar to second version of Fermatt's little theorem ; it removes the condition than a and n should be coprime. If $n = p \times q$, $a < n$, and k is an integer then

$$a^{k \times \phi(n)+1} \equiv a \pmod{n}$$

Thus the Euler's theorem is used to solve problem related to modular exponentiation and to find multiplicative inverse of the number

Multiplicative Inverse : Euler's theorem can be used to find multiplicative inverses modulo a prime ; Euler's theorem can be used to find multiplicative inverses modulo a composite. If n and a are coprime then $a^{-1} \bmod n = a^{\phi(n)-1} \bmod n$

$5^{-1} \bmod 396 = 5^{\phi(396)-1} \bmod 396$

① $20^{62} \bmod 77$

$(20 \bmod 77)\ (20^{\phi(77)+1} \bmod 77)$

① $20^{62} \bmod 77$

Using second version,

$a = 20 \quad n = 77$

$\gcd(20, 77) = 1$

$\phi(77) = \phi(7 \times 11)$

$= \phi(7) \times \phi(11)$ (using 3rd prop)

$= 6 \times 10$

$= 60$

$20^{[60 \cdot 2]} \bmod 77$

$\Rightarrow [20^{60} \bmod 77] \times [20^{2} \bmod 77]$

② $6^{24} \bmod 35$

$a = 6 \quad n = 35$

$\phi(35) = \phi(7 \times 5)$

$= \phi(7) \times \phi(5)$

$= 6 \times 4$

$= 24$

Using first version,

$6^{24} \bmod 35 = 1$

③ Proof that $11^{\phi(10)} \bmod 10 = 1$

→ $a = 11 \quad n = 10$

$\phi(n) = \phi(10) = \phi(5 \times 2)$

$= 4 \times 1$

$= 4$

$11^{4} \bmod 10 \Rightarrow 1$ [calculator]

④ $7^{-1} \bmod 15$

$n = 15$ $\quad \phi(n) = \phi(15) = \phi(3 \times 5)$

$= 2 \times 4 = 8$

$7^{8-1} \bmod 15$

$7^7 \bmod 15 \Rightarrow$

Using square and binary function & by applying modular property

$7^7 \bmod 15 = 13$

* Chinese Remainder Theorem

In cryptography we use the concept of congruence instead of equality thus the two integers are congruent, if we use the congruent operator ($\equiv$) and add the phrase (mod n) to the right side of the congruence to define the value of modulus that makes the relationship valid. So in general, congruence eq^n is shown by

$x \equiv a \pmod n$

$-8 \bmod 10$

$(10-8)$

Z

↓

contains all -ve, 0, +ve values

Z_n

↓

contains 0 to n-1 values.

The congruence operator is used as equality in cryptography, but both are not same because it shows Many to one relationship between each other.

For eg. (one, many)

$$2 \equiv 12 \pmod{10}$$
$$2 \equiv 22 \pmod{10}$$
$$2 \equiv 222 \pmod{10}$$
$$2 \equiv -8 \pmod{10}$$
$$2 \equiv 2 \pmod{10}$$
$$2 \equiv \vdots$$

The CRT is used to solve a set of congruent equation with one variable but different moduli, which are relative prime, as shown below

$$x_0 \equiv a_1 \pmod{m_1}$$
$$x \equiv a_2 \pmod{m_2}$$
$$\vdots$$
$$x \equiv a_k \pmod{m_k}.$$

The CRT states that above eq^n has unique solution if moduli are relatively prime.

Thus to solve the congruent set of equation there are various methods available like CRT, Quadratic Reduce method, Discrete logarithmic method.

Following shows the steps:

① Find $M = m_1 \times m_2 \times \cdots m_k$. This is the common modulus

② Find $M_1 = M/m_1$, $M_2 = M/m_2$, ... $M_k = M/m_k$.

③ Find the multiplicative inverse of $M_1, M_2, \cdots M_k$ using corresponding module $(m_1, m_2, \cdots m_k)$. Call the inverses $M_1^{-1}, M_2^{-1}, M_3^{-1}, \cdots M_k^{-1}$

④ The solution to simultaneous eq is

$$x = (a_1 \times M_1 \times M_1^{-1} + a_2 \times M_2 \times M_2^{-1} + \cdots + a_k \times M_k \times M_k^{-1}) \bmod M$$

Foreg $x \equiv 2 \pmod 3$

$x \equiv 3 \pmod 5$

$x \equiv 2 \pmod 7$

→ $m_1 = 3$, $m_2 = 5$, $m_3 = 7$

According to CRT algo,

$M = 3 \times 5 \times 7 = 3 \times 35 = 105$

$M_1 = \frac{105}{3} = 35$ $M_2 = \frac{105}{5} = 21$ $M_3 = \frac{105}{7} = 15$

To find M_1^{-1}, M_2^{-1}, M_3^{-1}

$M_1^{-1} = 35^{-1} \bmod 3$ (Using Fermat)

$= 35^1 \bmod 3$

$1 < 3$, $x = 1 = 0001$

35 mod Ans : 23

HW

Find value of x for given set of congruent eqn.

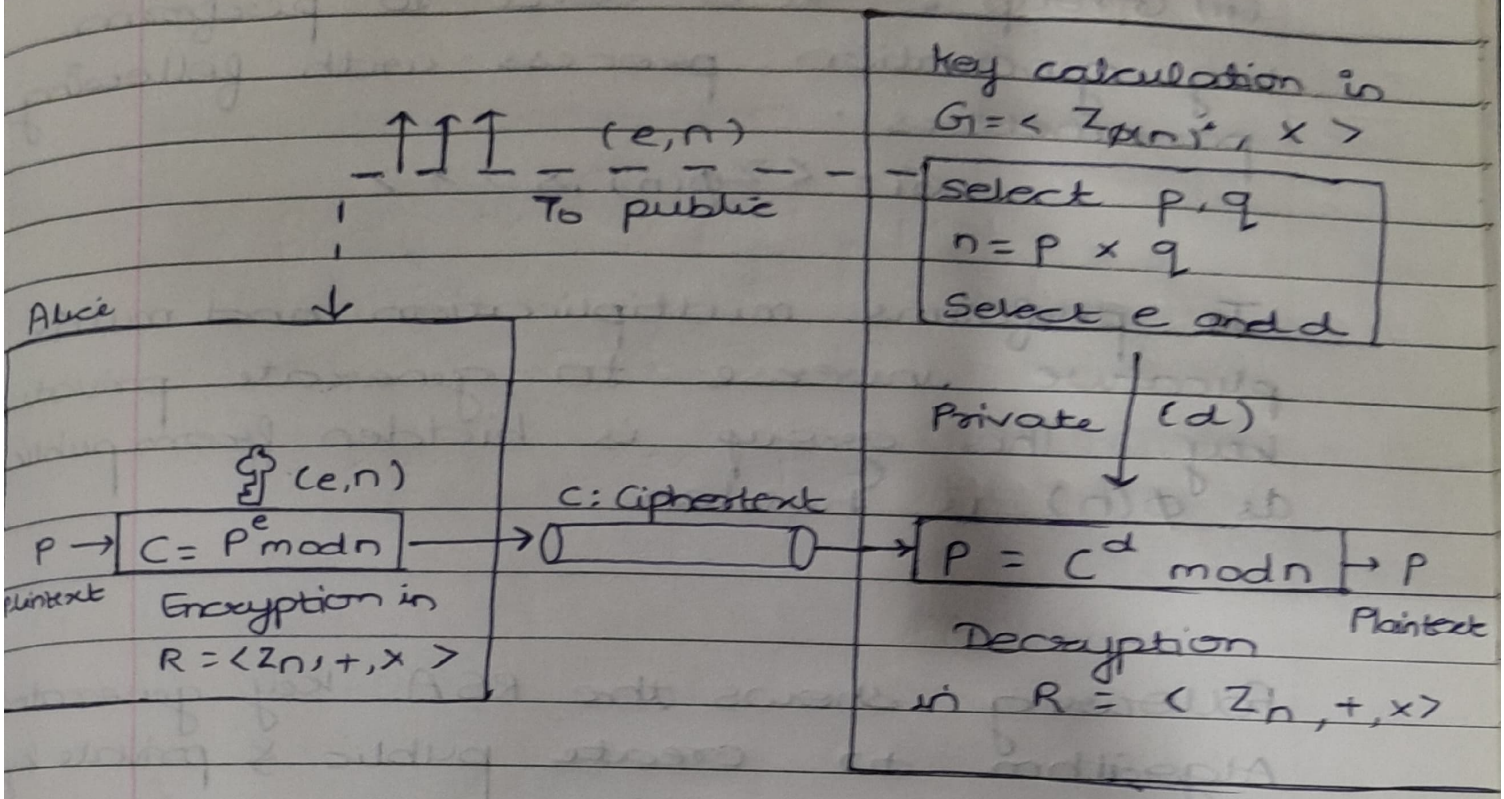
$x \equiv 3 \pmod 7$

$x \equiv 3 \pmod{13}$

$x \equiv 0 \pmod{12}$

* RSA crypto system

- The most common public key algorithm is RSA cryptosystem which was invented by 3 inventors : Rivest, Shamir and Adleman.
- RSA uses two exponents 'e' & 'd' where 'e' is used to create Public key and 'd' is used to generate Public key
- RSA uses modular exponentiation method to perform encryption and decryption.
- Assuming PT = Plaintext and CT = Ciphertext, therefore encryption is performed using $\boxed{CT = PT^e \bmod n}$
- Hy, the decryption process uses $PT = CT^d \bmod n$ to generate PT.
- The modulus 'n' should be a large number to improve security during key generation process.
- Following fig shows encryption, decryption and key generation process



From the above fig RSA uses two algebraic structures known as a ring and a group

(i) Ring : This structure is used to perform encryption and decryption with the following parameters

$$R = \langle Z_n, +, \times \rangle$$

Thus the ring perform two arithmetic operation : Addition and multiplication The ring is public as the modulus is public. Thus Alice can send message to Bob to perform encryption.

The reverse is used to perform the decryption at Bob side.

It is also known as commutative ring

(ii) Group : G is used to perform key generation process with following parameters.

$$G = \langle Z_{\phi(n)}, \times, \times \rangle$$

It performs multiplication and multiplicative inverse to generate private key. This group is hidden from public as $\phi(n)$ is hidden.

Following shows the RSA key generation Algorithm to create public & private key After key generation Bob announces the ~~key~~ tuple e,n as the public key to be used by Alice for performing encryption. similarly, Bob keeps the integer d as private key.

Imp

The recommended size for each prime number 'p' and 'q' is 512 bits i.e 154 decimal digits and the size of modulus 'n' is 1024 bits i.e 309 digits to make RSA algorithm more secure

- RSA key Generation Algorithm :

```
RSA - key - Generation
{
    Select two large prime p & q such p≠q
    n ← p × q
    ϕ(n) ← (p-1) × (q-1)
    Select e such that 1 < e < ϕ(n) and e is
    coprime to ϕ(n)
    d ← e^-1 mod ϕ(n)        // d is inverse of e modulo ϕ(n)
    Public_key ← (e, n)      // To be announced publically
    Private_key ← d          // To be kept secret
    return Public_key & Private_key
}
```

- RSA encryption :

```
RSA - Encryption (P, e, n)      // P is PT in Zn & P<n
{
    C ← Fast_exponentiation (P, e, n)
    return C
}
```

- RSA decryption :

```
RSA_Decryption (C, d, n)
{
    P ← Fast_exponentiation (C, d, n)
    return P
}
```

Unit 4 :
Key Management

→ How to generate a session key and distribute it

(session key: two user ke bich mei jab communication ho rha hai tab key use krne.)

→ Symmetric key
↳ Alice —key₁→ Bob
Bob —key₂→ Alice

For n no. of user,
No. of session key for bidirectional = $n(n-1)$

No. of session key for unidirectional = $\frac{n(n-1)}{2}$

Key Distribution center
Fig 15.1 forouzan

It is centralised server which contain database

KDC → db (database)
→ username entity
→ Password
→ key

When Alice communicates with bob
1. Alice send request to KDC
2. KDC checks authentication
3. KDC sends request to bob and in this way they commu
4. If bob agrees session key is created between these two

Types of KDC
① Flat multiple KDC
② Hierarchial multiple KDC (15.2 & 15.3)

→ Write KDC approach ?
→ Summary of KDC.
→ Working of KDC ? → syllabus otherwise not syllabus

- KDC uses Needham - Schroeder Protocol

There are chances of replay attack which is improved using Needham Protocol.

Fig 15.6 In this session ko log in and log out time diya jayega.
Session will be expired.
i.e Session time out hoga.
i.e. We will use time out function.

nonce → number's once.
↓
pseudo random no. which can be used only one time.

* Kerberos.

It is network authentication protocol which is used in KDC.
Version available are 4 & 5.

→ Write a short note on kerberos
Fig 15.7.

Kerberos

AS (Authentication server) → only verify

TGS (Ticket granted server) → only generating session key

Real server (client)

These were two responsibilities of KDC which are distributed in two server.

→ Write a short note on kerboros and explain with example. 10 marks
(2 diagrams)

Kerboros version 5.

Symmetric key KDC

① Using KDC (using kerboros)

② without using KDC (using Diffie-Hellman key agreement protocol)

A session key can be created using two ways

* Diffie Hellman method
Fig 15.9.

The values of p and g are public. It means that we are using it through internet.

As shown in the above figure let the two parties choose two random numbers 'p' and 'g' where 'p' is a large prime number of the order 1024 bits i.e maximum 300 decimal digit..

Similarly, the second number 'g' is a generator of the order p-1 in the group $< Z_p^*, \times >$ $\times$: multiplicative inverse

These two values are public values and can be send through the internet. Following are the steps to be followed

① Alice choose a large random number x such that $0 \leq x \leq p-1$ and calculates $R_1 = g^x \bmod p$

② Bob chooses another large random number y such that $0 \leq y \leq p-1$ and calculates $R_2 = g^y \bmod p$.

③ Alice sends R_1 to Bob. Note that Alice does not send the value of x; she sends only R_1

④ Bob sends R_2 to Alice. Again, note that Bob does not send the value of y, he sends only R_2

⑤ Alice calculates $K = (R_2)^x \bmod p$

⑥ Bob calculates $K = (R_1)^y \bmod p$

From the above steps K is symmetric key for the session.

$$K = (g^x \bmod p)^y \bmod p$$
$$= (g^y \bmod p)^x \bmod p$$
$$= g^{xy} \bmod p$$

Que) Let $p = 23$, $g = 7$, $x = 3$, $y = 6$
Use Diffie Hellman key exchange algorithm and get value of session key K

→

$$R_1 = g^x \bmod p$$
$$= 7^3 \bmod 23$$
$$= 21$$

$$R_2 = g^y \bmod p$$
$$= 7^6 \bmod 23$$
$$= 4$$

$$K = (R_2)^x \bmod p$$
$$= (4)^3 \bmod 23$$
$$= 18$$

Assignment Que :

① In Diffie-Hellman Protocol, let $g = 7$, $p = 23$, $x = 3$, $y = 5$

(A) What is value of symmetric key K

(B) What is value of R_1 and R_2

② In the Diffie Hellman protocol what happens if x and y have same value i.e Alice and Bob choose same number

③ Are R_1 and R_2 the same. Do the session keys calculated by Alice and Bob have the same values. Use an example to prove your claim

Ans 1 ⇒ $g = 7$, $p = 23$, $x = 3$, $y = 5$

$R_1 = g^x \bmod p$

$= 7^3 \bmod 23$

$= 21$

$R_2 = g^y \bmod p$

$= 7^5 \bmod 23$

$= 17$

$K = (R_1)^x \bmod p$

$= (21)^5 \bmod 23$

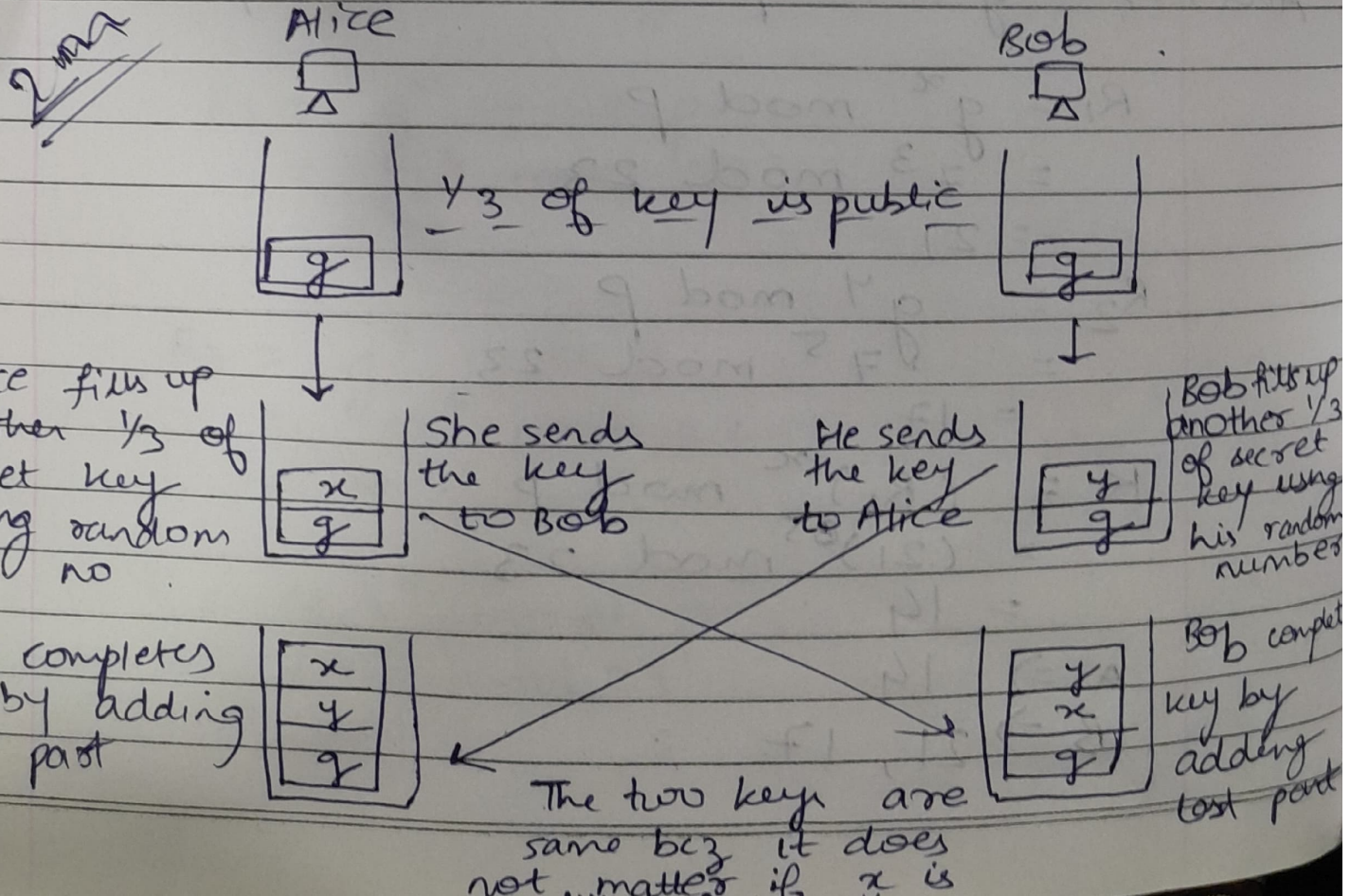
$= 14$

A ⇒ 14

B ⇒ 21, 17

* Analysis of Diffie Hellman Algo.

As shown in the following figure the session key between Alice and Bob is made up of 3 parts g, x, y. The first part i.e 'g' is made public whereas the other two parts must be added by Alice and Bob. As the two keys are same, Alice cannot find the value 'y' used by Bob because the calculation is done in modulo p. Illy Bob cannot find the value 'x' used by Alice for the reason same. Thus to know the values of x and y Discrete logarithm method is used.



* Public key Distribution. X.509 certificate

- In public key cryptography, everyone has access to everyones public key; public keys are available to the public
- Public keys like secret key or session key needs to be distributed publically

① Public Announcement
→ Announcing a public key
② Trusted certer
③ Centralized Trusted center.
④ Certification Authority

* X.509 certificate format.
Fig 15.17

As each certificate have different formats the ITU has designed X.509 certificate format in a structured way accepted by internet. It is a well known protocol called as ASN.1 (Abstract syntax Notation)

ITU: International telecommunication Union

bytes wale diagram google se dekhna.

Following fig shows X.509 (1518 bytes)

Pg 482