K-Means Problem

By – Prof. Ansar Sheikh

K-Means Algorithm for Clustering

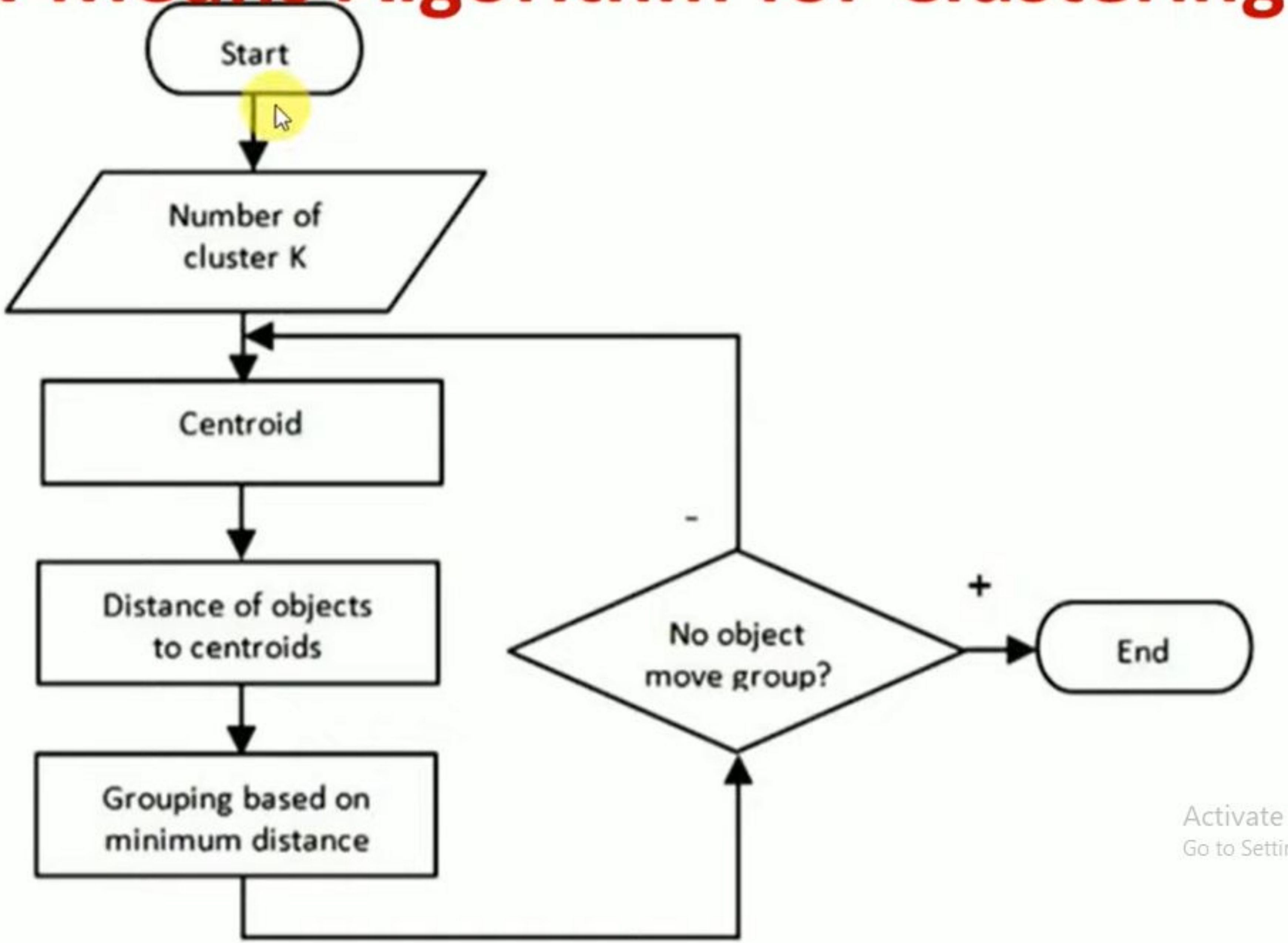
- kMeans algorithm is an unsupervised learning algorithm
- Given a data set of items, with certain features, and values for these features, the algorithm will categorize the items into k groups or clusters of similarity.
- To calculate the similarity, we can use the Euclidean distance, Manhattan distance, Hamming distance, Cosine distance as measurement.

K-Means Algorithm for Clustering

Here is the pseudocode for implementing a K-means algorithm.

```
Input: Algorithm K-Means (K number of clusters, D list of data points)
1. Choose K number of random data points as initial centroids (cluster centers).
2. Repeat till cluster centers stabilize:
    a. Allocate each point in D to the nearest of Kth centroids.
    b. Compute centroid for the cluster using all points in the cluster.
```

K-Means Algorithm for Clustering


Start
Number of cluster K
Centroid
Distance of objects to centroids
Grouping based on minimum distance
No object move group?
-
+
End

Advantages and Disadvantages of K-Means Algorithm

Advantages of K-Means Algorithm

1. K-means algorithm is simple, easy to understand, and easy to implement.
2. It is also efficient, in which the time taken to cluster K-means rises linearly with the number of data points.
3. No other clustering algorithm performs better than K-means.

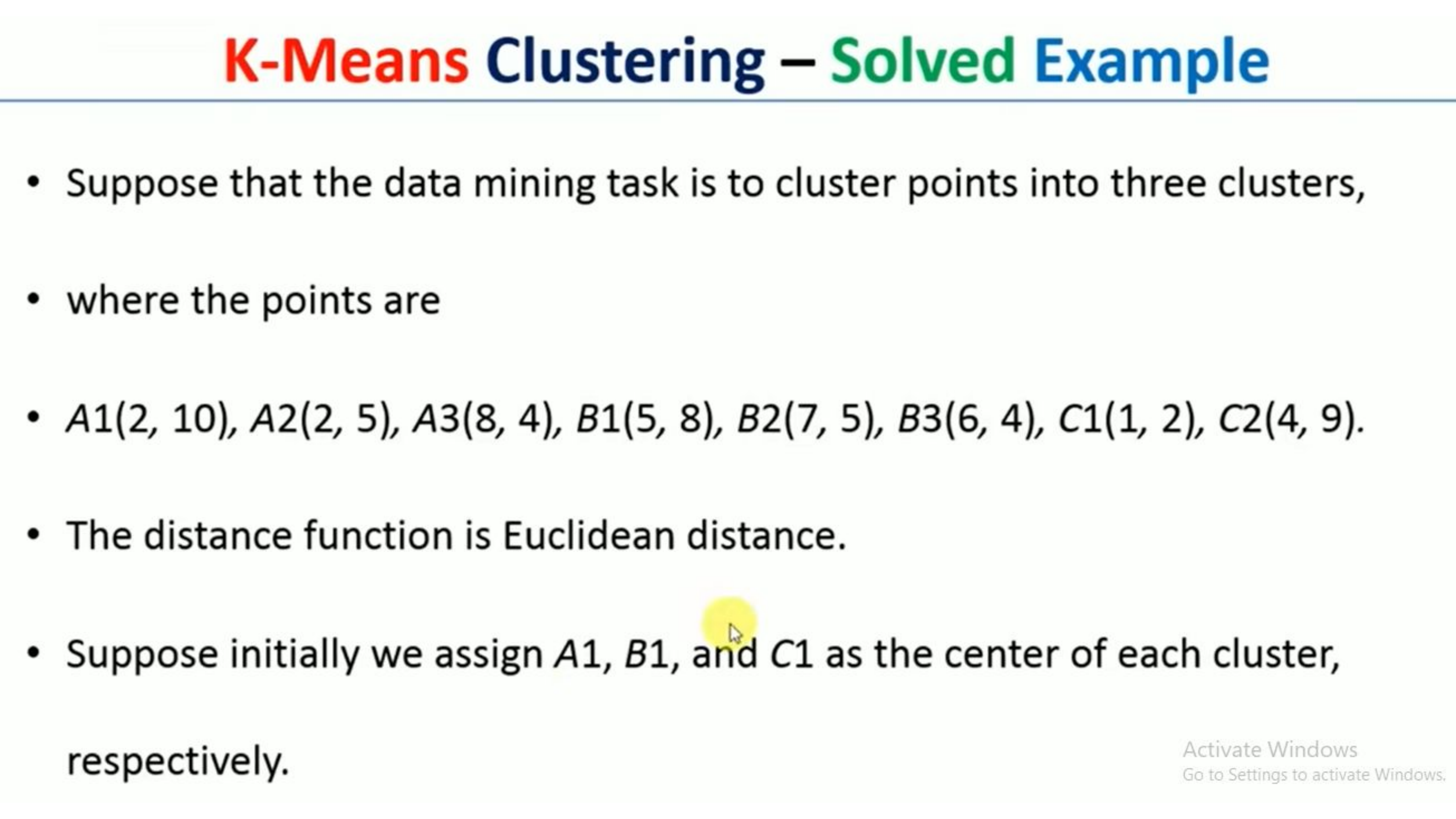
Disadvantages of K-Means Algorithm

1. The user needs to specify an initial value of K.
2. The process of finding the clusters may not converge.
3. It is not suitable for discovering clusters that are not hyper ellipsoids or hyper spheres).

K-Means Clustering

A1(2, 10), A2(2, 5),
A3(8, 4), B1(5, 8),
B2(7, 5), B3(6, 4),
C1(1, 2), C2(4, 9)

Solved
Example



K-Means Clustering – Solved Example

- Suppose that the data mining task is to cluster points into three clusters,
- where the points are
- *A*1(2, 10), *A*2(2, 5), *A*3(8, 4), *B*1(5, 8), *B*2(7, 5), *B*3(6, 4), *C*1(1, 2), *C*2(4, 9).
- The distance function is Euclidean distance.
- Suppose initially we assign *A*1, *B*1, and *C*1 as the center of each cluster, respectively.

K-Means Clustering – Solved Example

Data Points			Distance to			Cluster	New Cluster
A1	2	10					
A2	2	5					
A3	8	4					
B1	5	8					
B2	7	5					
B3	6	4					
C1	1	2					
C2	4	9					

K-Means Clustering – Solved Example

Initial Centroids:

A1: (2, 10)

B1: (5, 8)

C1: (1, 2)

Data Points			Distance to						Cluster	New Cluster
			2	10	5	8	1	2		
A1	2	10								
A2	2	5								
A3	8	4								
B1	5	8								
B2	7	5								
B3	6	4								
C1	1	2								
C2	4	9								

K-Means Clustering – Solved Example

Initial Centroids:

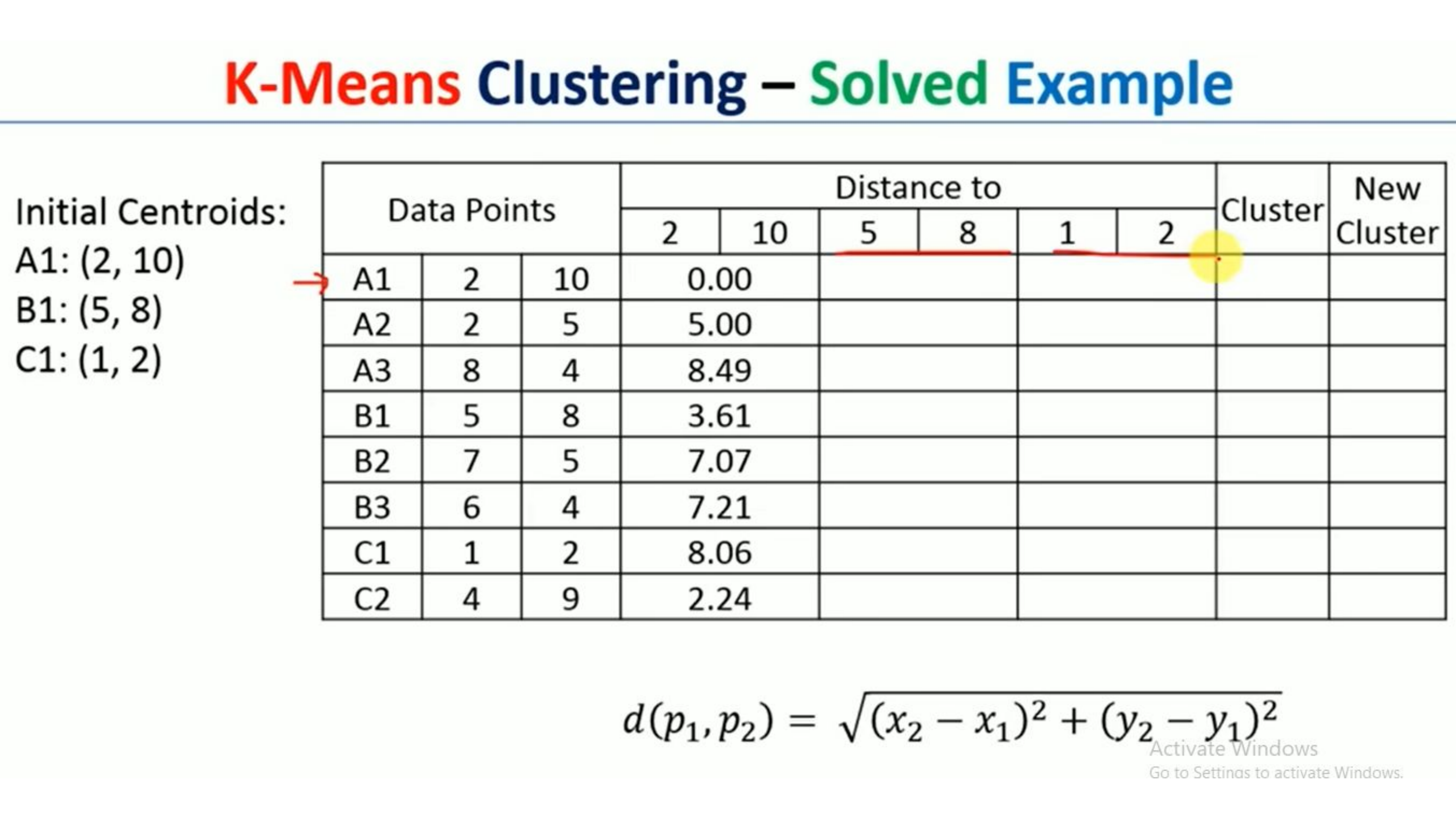
A1: (2, 10)

B1: (5, 8)

C1: (1, 2)

Data Points			Distance to						Cluster	New Cluster
			2	10	5	8	1	2		
A1	2	10								
A2	2	5								
A3	8	4								
B1	5	8								
B2	7	5								
B3	6	4								
C1	1	2								
C2	4	9								

$$d(p_1, p_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



K-Means Clustering – Solved Example

Initial Centroids:

A1: (2, 10)

B1: (5, 8)

C1: (1, 2)

Data Points			Distance to						Cluster	New Cluster
			2	10	5	8	1	2		
A1	2	10	0.00							
A2	2	5	5.00							
A3	8	4	8.49							
B1	5	8	3.61							
B2	7	5	7.07							
B3	6	4	7.21							
C1	1	2	8.06							
C2	4	9	2.24							

$$d(p_1, p_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

K-Means Clustering – Solved Example

Initial Centroids:

A1: (2, 10)

B1: (5, 8)

C1: (1, 2)

Data Points			Distance to			Cluster	New Cluster
			2 10	5 8	1 2		
A1	2	10	0.00	3.61	8.06		
A2	2	5	5.00	4.24	3.16		
A3	8	4	8.49	5.00	7.28		
B1	5	8	3.61	0.00	7.21		
B2	7	5	7.07	3.61	6.71		
B3	6	4	7.21	4.12	5.39		
C1	1	2	8.06	7.21	0.00		
C2	4	9	2.24	1.41	7.62		

$$d(p_1, p_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

K-Means Clustering – Solved Example

Initial Centroids:

A1: (2, 10)

B1: (5, 8)

C1: (1, 2)

Data Points			Distance to						Cluster	New Cluster
			2	10	5	8	1	2		
A1	2	10	0.00		3.61		8.06		1	
A2	2	5	5.00		4.24		3.16		3	
A3	8	4	8.49		5.00		7.28		2	
B1	5	8	3.61		0.00		7.21		2	
B2	7	5	7.07		3.61		6.71		2	
B3	6	4	7.21		4.12		5.39		2	
C1	1	2	8.06		7.21		0.00		3	
C2	4	9	2.24		1.41		7.62		2	

$$d(p_1, p_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

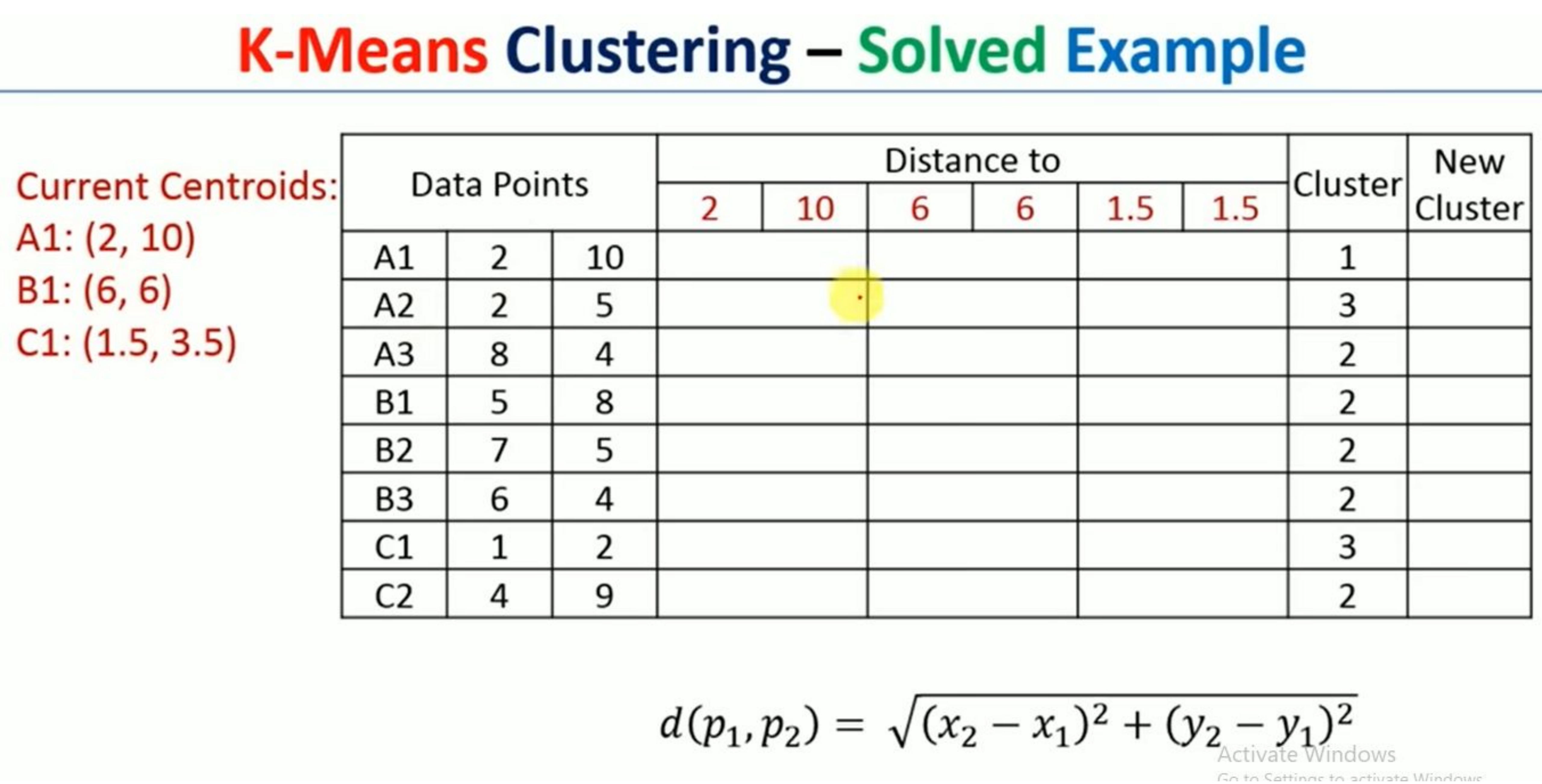
K-Means Clustering – Solved Example

Initial Centroids:
A1: (2, 10)
B1: (5, 8)
C1: (1, 2)

New Centroids:
A1: (2, 10)
B1: (6, 6)
C1: (1.5, 3.5)

Data Points			Distance to						Cluster	New Cluster
			2	10	5	8	1	2		
A1	2	10	0.00		3.61		8.06		1	
A2	2	5	5.00		4.24		3.16		3	
A3	8	4	8.49		5.00		7.28		2	
B1	5	8	3.61		0.00		7.21		2	
B2	7	5	7.07		3.61		6.71		2	
B3	6	4	7.21		4.12		5.39		2	
C1	1	2	8.06		7.21		0.00		3	
C2	4	9	2.24		1.41		7.62		2	

$$d(p_1, p_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



K-Means Clustering – Solved Example

Current Centroids:

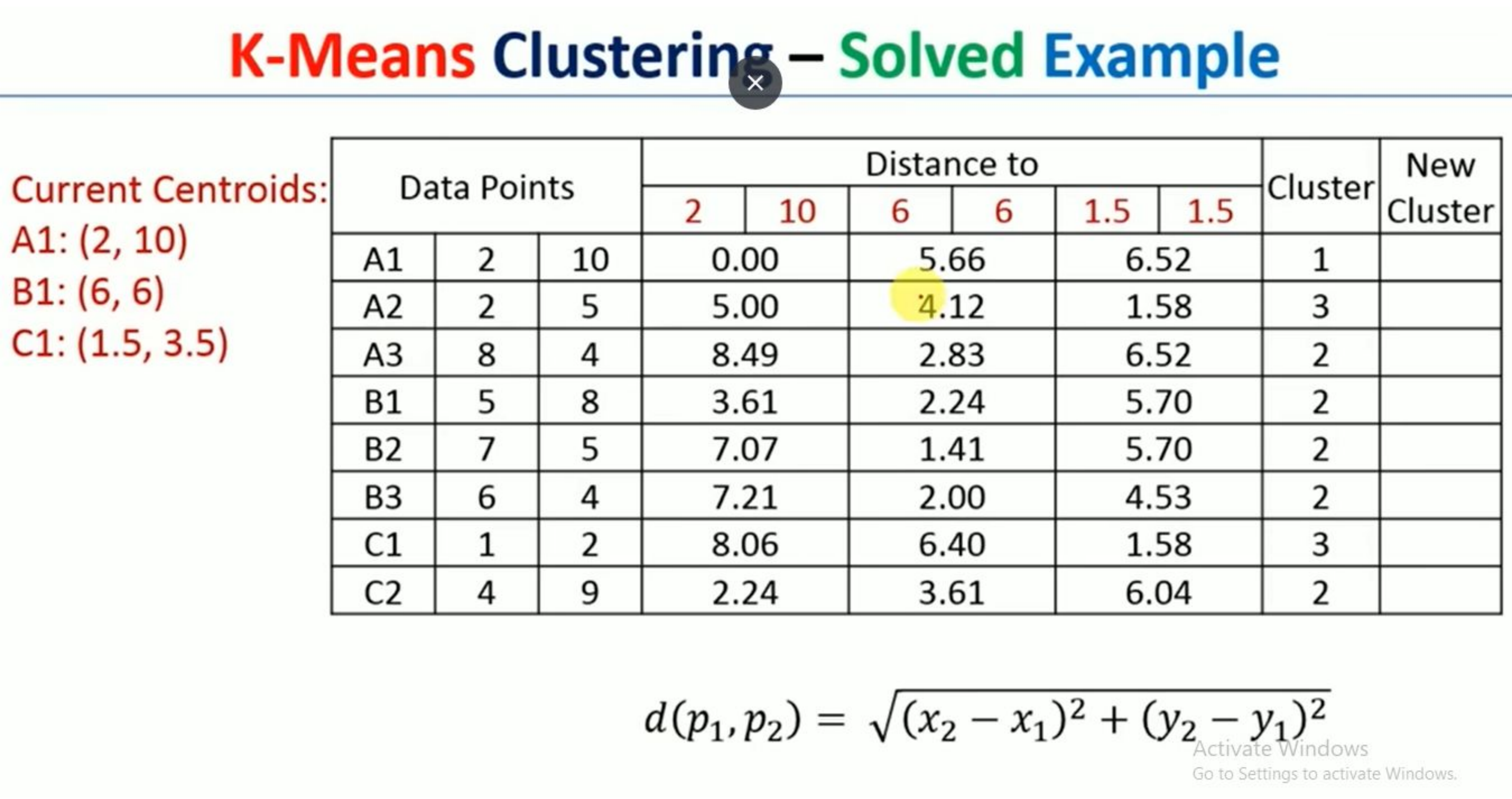
A1: (2, 10)

B1: (6, 6)

C1: (1.5, 3.5)

Data Points			Distance to						Cluster	New Cluster
			2	10	6	6	1.5	1.5		
A1	2	10							1	
A2	2	5							3	
A3	8	4							2	
B1	5	8							2	
B2	7	5							2	
B3	6	4							2	
C1	1	2							3	
C2	4	9							2	

$$d(p_1, p_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



K-Means Clustering – Solved Example

Current Centroids:

A1: (2, 10)

B1: (6, 6)

C1: (1.5, 3.5)

Data Points			Distance to						Cluster	New Cluster
			2	10	6	6	1.5	1.5		
A1	2	10	0.00		5.66		6.52		1	
A2	2	5	5.00		4.12		1.58		3	
A3	8	4	8.49		2.83		6.52		2	
B1	5	8	3.61		2.24		5.70		2	
B2	7	5	7.07		1.41		5.70		2	
B3	6	4	7.21		2.00		4.53		2	
C1	1	2	8.06		6.40		1.58		3	
C2	4	9	2.24		3.61		6.04		2	

$$d(p_1, p_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

K-Means Clustering – Solved Example

Current Centroids:
A1: (2, 10)
B1: (6, 6)
C1: (1.5, 3.5)

Data Points			Distance to						Cluster	New Cluster
			2	10	6	6	1.5	1.5		
A1	2	10	0.00		5.66		6.52		1	1
A2	2	5	5.00		4.12		1.58		3	3
A3	8	4	8.49		2.83		6.52		2	2
B1	5	8	3.61		2.24		5.70		2	2
B2	7	5	7.07		1.41		5.70		2	2
B3	6	4	7.21		2.00		4.53		2	2
C1	1	2	8.06		6.40		1.58		3	3
C2	4	9	2.24		3.61		6.04		2	1

$$d(p_1, p_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

K-Means Clustering – Solved Example

Current Centroids:

A1: (2, 10)

B1: (6, 6)

C1: (1.5, 3.5)

New Centroids:

A1: (3, 9.5) ✓

B1: (6.5, 5.25) ✓

C1: (1.5, 3.5) ✓

Data Points			Distance to						Cluster	New Cluster
			2	10	6	6	1.5	1.5		
A1	2	10	0.00		5.66		6.52		1	1
A2	2	5	5.00		4.12		1.58		3	3
A3	8	4	8.49		2.83		6.52		2	2
B1	5	8	3.61		2.24		5.70		2	2
B2	7	5	7.07		1.41		5.70		2	2
B3	6	4	7.21		2.00		4.53		2	2
C1	1	2	8.06		6.40		1.58		3	3
C2	4	9	2.24		3.61		6.04		2	1

$$d(p_1, p_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

K-Means Clustering – Solved Example

Current Centroids:
A1: (3, 9.5)
B1: (6.5, 5.25)
C1: (1.5, 3.5)

Data Points			Distance to			Cluster	New Cluster
A1	2	10				1	
A2	2	5				3	
A3	8	4				2	
B1	5	8				2	
B2	7	5				2	
B3	6	4				2	
C1	1	2				3	
C2	4	9				1	

$$d(p_1, p_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

K-Means Clustering – Solved Example

Current Centroids:

A1: (3, 9.5)

B1: (6.5, 5.25)

C1: (1.5, 3.5)

Data Points			Distance to						Cluster	New Cluster
			3	9.5	6.5	5.25	1.5	3.5		
A1	2	10	1.12		6.54		6.52		1	
A2	2	5	4.61		4.51		1.58		3	
A3	8	4	7.43		1.95		6.52		2	
B1	5	8	2.50		3.13		5.70		2	
B2	7	5	6.02		0.56		5.70		2	
B3	6	4	6.26		1.35		4.53		2	
C1	1	2	7.76		6.39		1.58		3	
C2	4	9	1.12		4.51		6.04		1	

$$d(p_1, p_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

K-Means Clustering – Solved Example

Current Centroids:

A1: (3, 9.5)

B1: (6.5, 5.25)

C1: (1.5, 3.5)

Data Points			Distance to						Cluster	New Cluster
			3	9.5	6.5	5.25	1.5	3.5		
A1	2	10	1.12		6.54		6.52		1	1
A2	2	5	4.61		4.51		1.58		3	3
A3	8	4	7.43		1.95		6.52		2	2
B1	5	8	2.50		3.13		5.70		2	1
B2	7	5	6.02		0.56		5.70		2	2
B3	6	4	6.26		1.35		4.53		2	2
C1	1	2	7.76		6.39		1.58		3	3
C2	4	9	1.12		4.51		6.04		1	1

$$d(p_1, p_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

K-Means Clustering – Solved Example

Current Centroids:
A1: (3, 9.5)
B1: (6.5, 5.25)
C1: (1.5, 3.5)

Data Points			Distance to						Cluster	New Cluster
			3	9.5	6.5	5.25	1.5	3.5		
A1	2	10	1.12		6.54		6.52		1	1
A2	2	5	4.61		4.51		1.58		3	3
A3	8	4	7.43		1.95		6.52		2	2
B1	5	8	2.50		3.13		5.70		2	1
B2	7	5	6.02		0.56		5.70		2	2
B3	6	4	6.26		1.35		4.53		2	2
C1	1	2	7.76		6.39		1.58		3	3
C2	4	9	1.12		4.51		6.04		1	1

New Centroids:
A1: (3.67, 9)
B1: (7, 4.33)
C1: (1.5, 3.5)

$$d(p_1, p_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

K-Means Clustering – Solved Example

Current Centroids:

A1: (3.67, 9)

B1: (7, 4.33)

C1: (1.5, 3.5)

Data Points			Distance to						Cluster	New Cluster
			3.67	9	7	4.33	1.5	3.5		
A1	2	10							1	
A2	2	5							3	
A3	8	4							2	
B1	5	8							1	
B2	7	5							2	
B3	6	4							2	
C1	1	2							3	
C2	4	9							1	

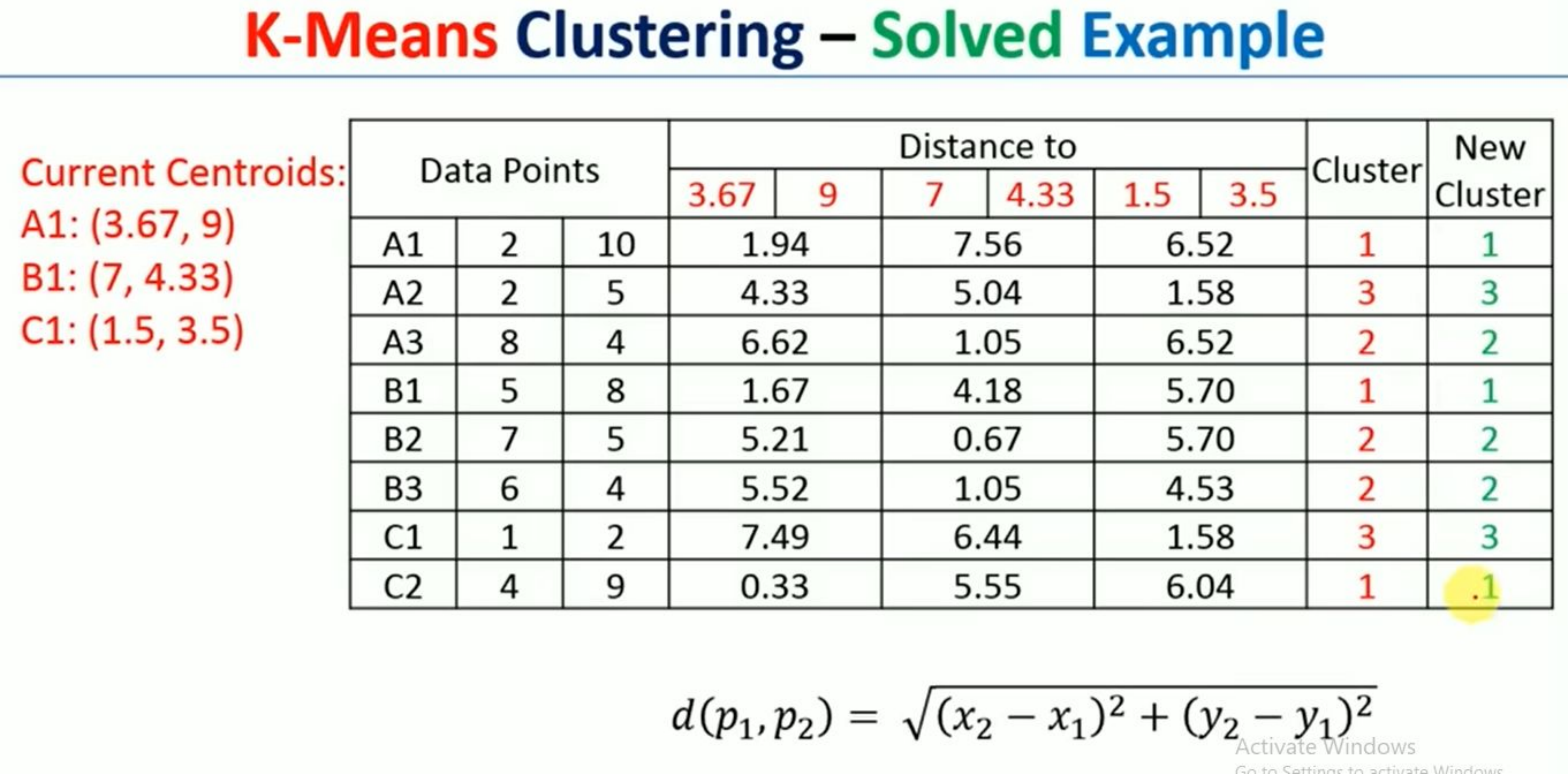
$$d(p_1, p_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

K-Means Clustering – Solved Example

Current Centroids:
A1: (3.67, 9)
B1: (7, 4.33)
C1: (1.5, 3.5)

Data Points			Distance to						Cluster	New Cluster
			3.67	9	7	4.33	1.5	3.5		
A1	2	10	1.94		7.56		6.52		1	
A2	2	5	4.33		5.04		1.58		3	
A3	8	4	6.62		1.05		6.52		2	
B1	5	8	1.67		4.18		5.70		1	
B2	7	5	5.21		0.67		5.70		2	
B3	6	4	5.52		1.05		4.53		2	
C1	1	2	7.49		6.44		1.58		3	
C2	4	9	0.33		5.55		6.04		1	

$$d(p_1, p_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$



K-Means Clustering – Solved Example

Current Centroids:

A1: (3.67, 9)

B1: (7, 4.33)

C1: (1.5, 3.5)

Data Points			Distance to			Cluster	New Cluster
			3.67 9	7 4.33	1.5 3.5		
A1	2	10	1.94	7.56	6.52	1	1
A2	2	5	4.33	5.04	1.58	3	3
A3	8	4	6.62	1.05	6.52	2	2
B1	5	8	1.67	4.18	5.70	1	1
B2	7	5	5.21	0.67	5.70	2	2
B3	6	4	5.52	1.05	4.53	2	2
C1	1	2	7.49	6.44	1.58	3	3
C2	4	9	0.33	5.55	6.04	1	1

$$d(p_1, p_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Thank
you