L04 Edges, Corners, and Blobs

Edge Detection

Slides from Svetlana Lazebnik

Edge Detection

Winter in Kraków photographed by Marcin Ryczek

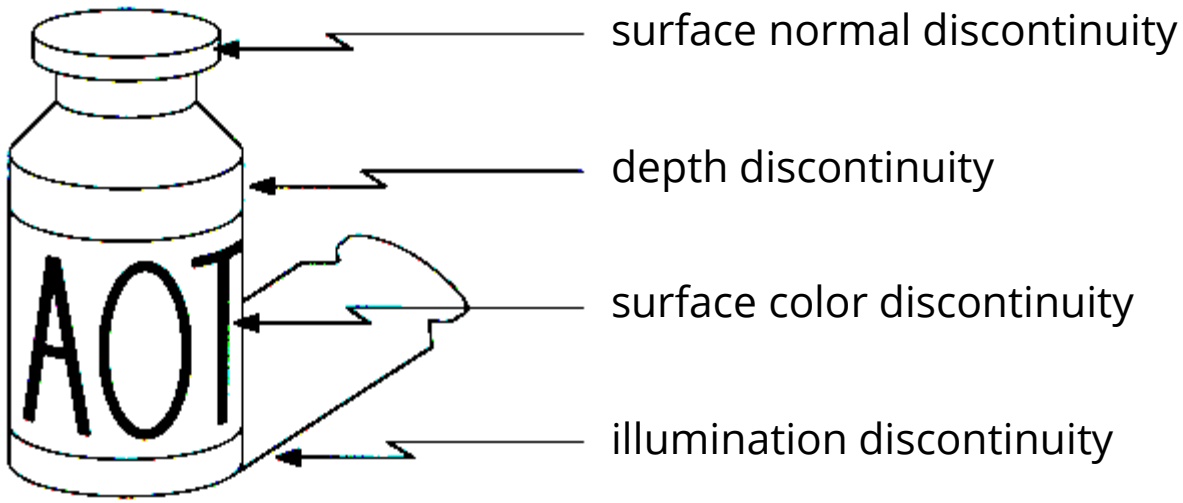
Edge Detection

- **Goal:** Identify sudden changes (discontinuities) in an image
  - Intuitively, most semantic and shape information from the image can be encoded in the edges
  - More compact than pixels
- **Ideal:** artist's line drawing (but artist is also using object-level knowledge)

Source: D. Lowe

Origin of Edges

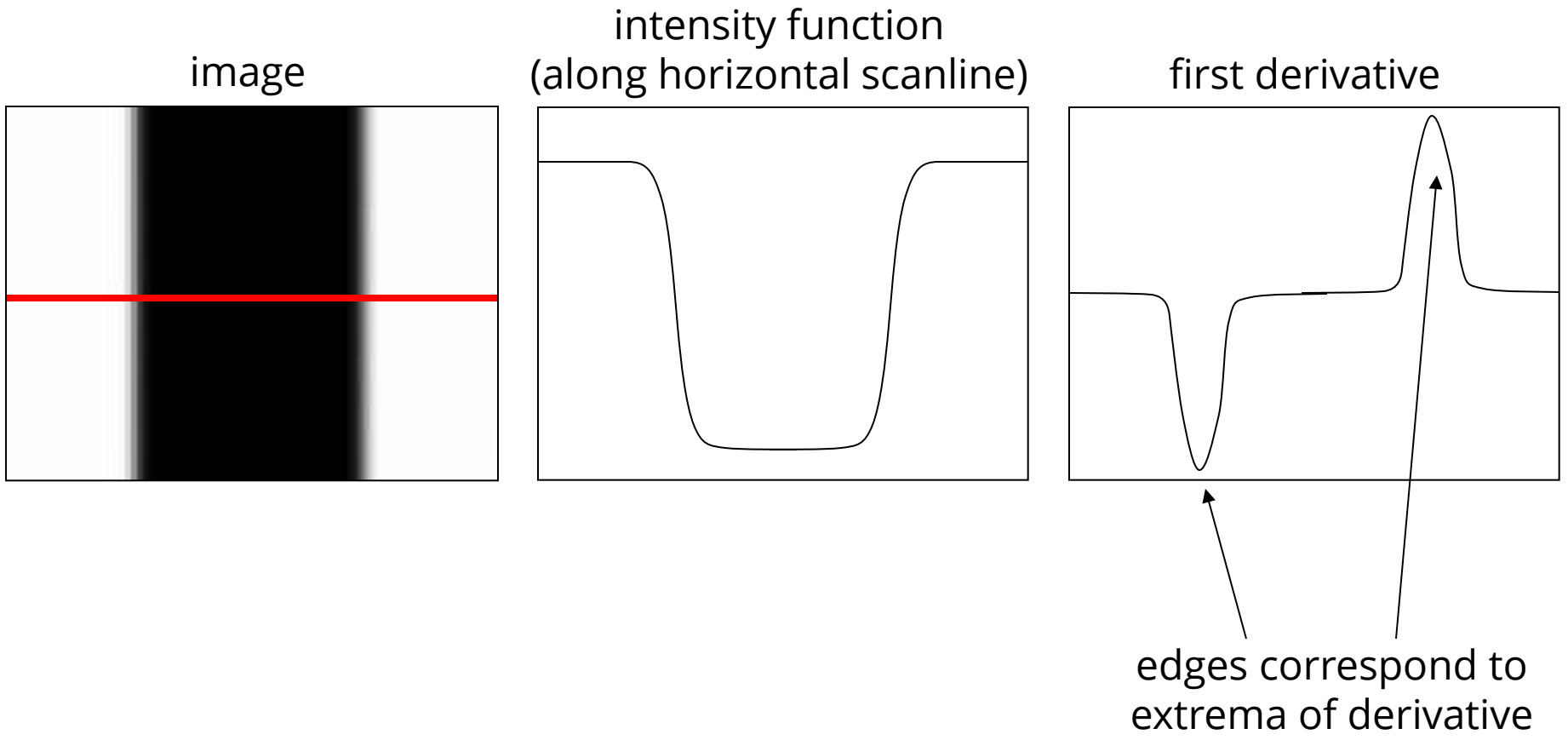
Edges are caused by a variety of factors:



Source: Steve Seitz

Edge Detection

An edge is a place of rapid change in the image intensity function



Derivatives with Convolution

For 2D function f(x,y), the partial derivative is:

$$\frac{\partial f(x,y)}{\partial x} = \lim_{\varepsilon \to 0} \frac{f(x+\varepsilon, y) - f(x,y)}{\varepsilon}$$

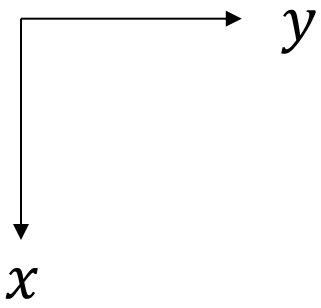
For discrete data, we can approximate using finite differences:

$$\frac{\partial f(x,y)}{\partial x} \approx \frac{f(x+1, y) - f(x,y)}{1}$$

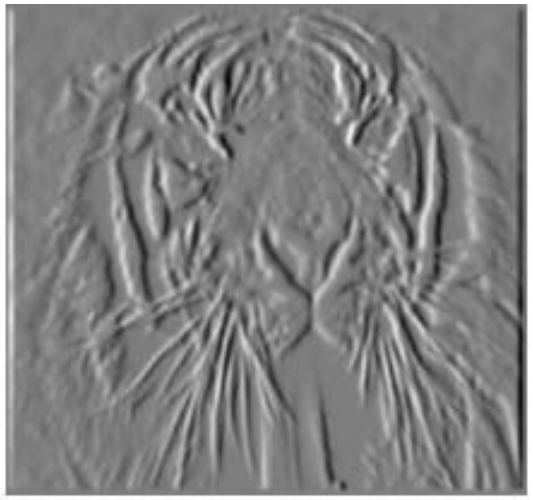
To implement the above as convolution, what would be the associated filter?

Source: K. Grauman

Partial Derivatives of an Image

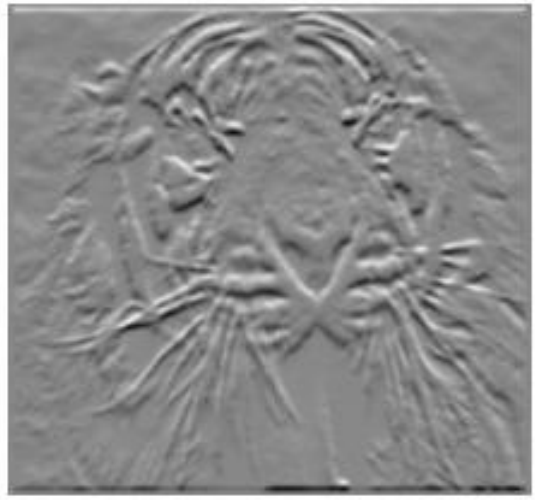


$$\frac{\partial f(x,y)}{\partial y}$$



-1	1

$$\frac{\partial f(x,y)}{\partial x}$$



-1
1

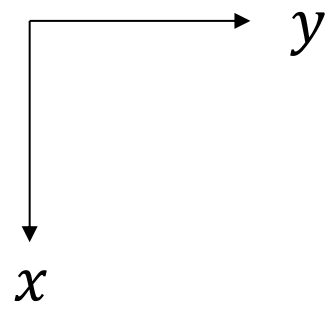
or

1
-1

Which shows changes with respect to x?

Finite Difference Filters

Other approximations of derivative filters



Prewitt

$M_x =$

-1	-1	-1
0	0	0
1	1	1

$M_y =$

-1	0	1
-1	0	1
-1	0	1

Sobel

$M_x =$

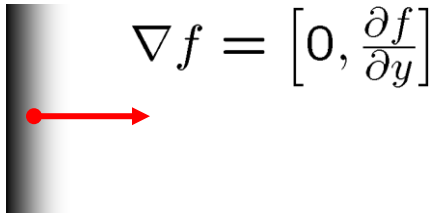
-1	-2	-1
0	0	0
1	2	1

$M_y =$

-1	0	1
-2	0	2
-1	0	1

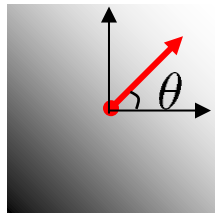
Image Gradient

The gradient of an image: $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$



$\nabla f = \left[0, \frac{\partial f}{\partial y}\right]$

$\nabla f = \left[\frac{\partial f}{\partial x}, 0\right]$



$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

The gradient points in the direction of most rapid increase in intensity.

How does this direction relate to the direction of the edge?

The gradient direction is given by $\theta = \tan^{-1}\left(\frac{\partial f}{\partial y} / \frac{\partial f}{\partial x}\right)$

The edge strength is given by the gradient magnitude $\|\nabla f\| = \sqrt{\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2}$

Source: Steve Seitz

Application: Gradient-Domain Image Editing

Goal: solve for pixel values in the target region to match gradients of the source region while keeping background pixels the same

sources/destinations

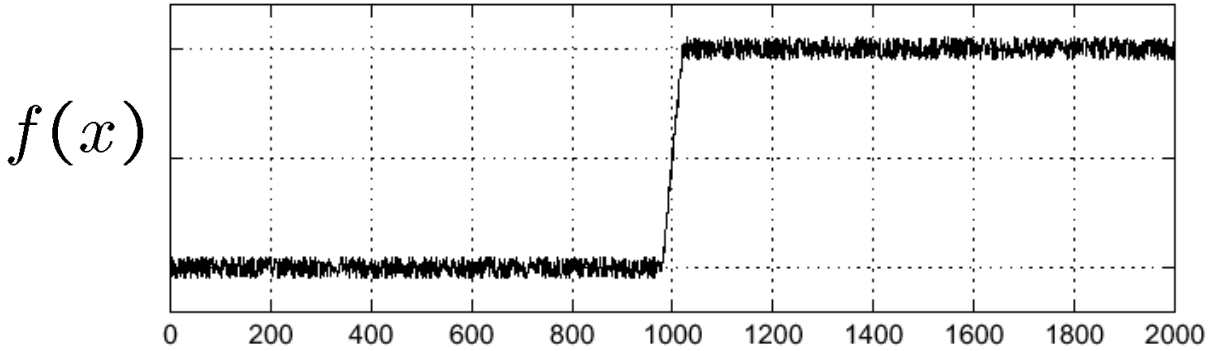
cloning

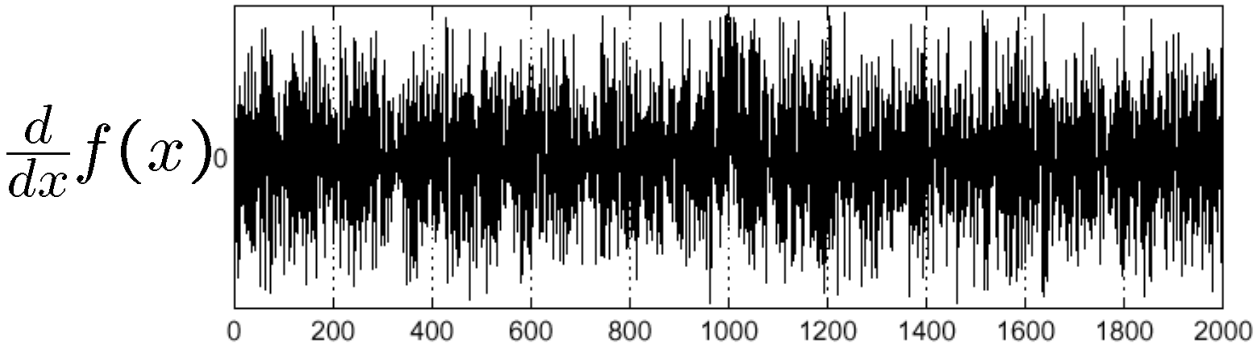
seamless cloning

P. Perez, M. Gangnet, A. Blake, Poisson Image Editing, SIGGRAPH 2003

Effects of Noise

Consider a single row or column of the image

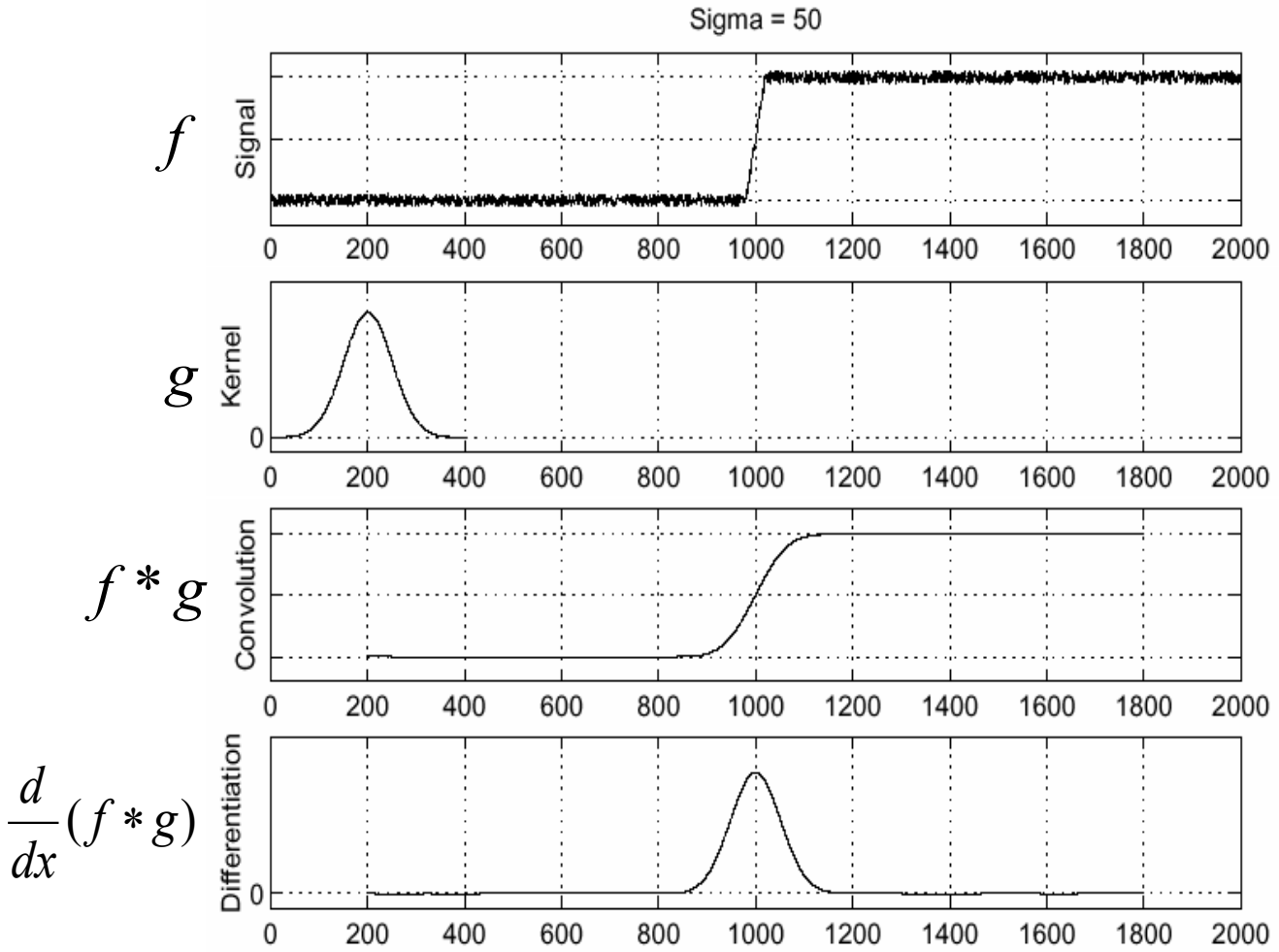




Where is the edge?

Source: S. Seitz

Solution: Smooth First



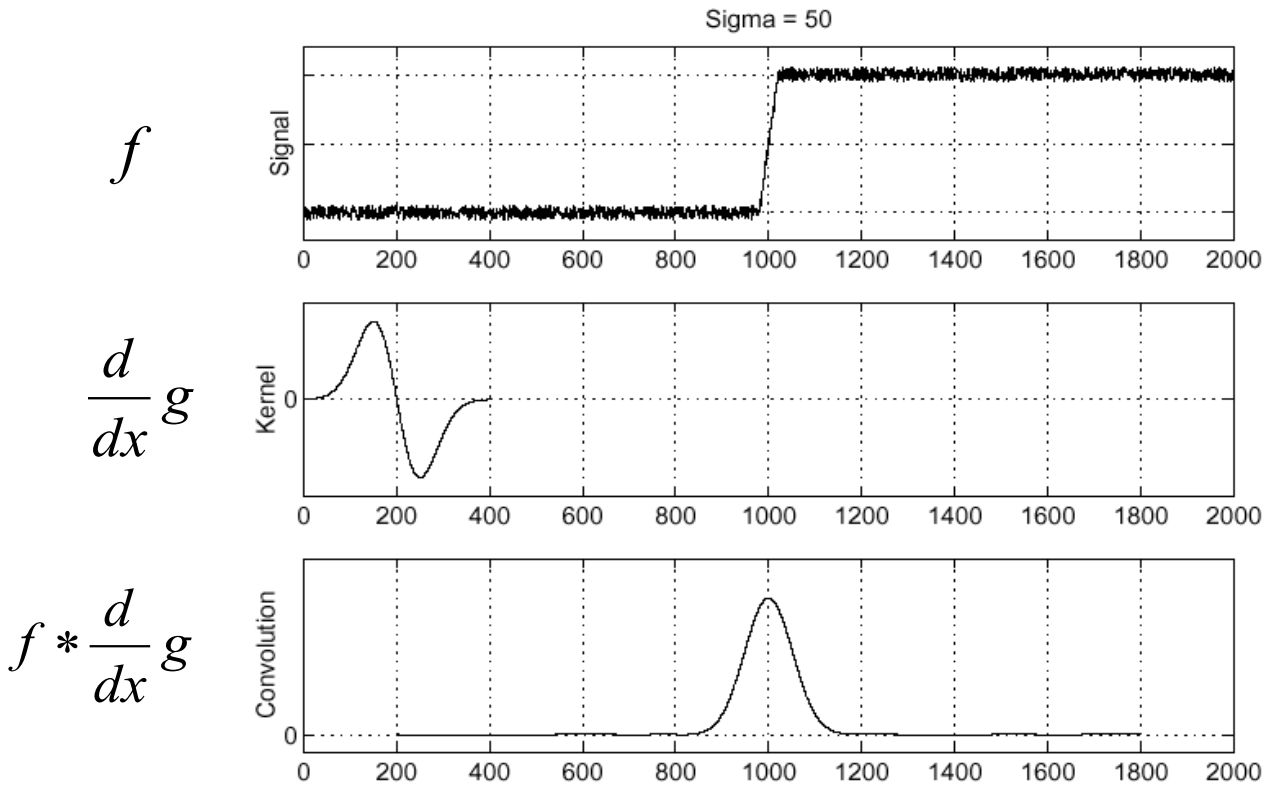
To find edges, look for peaks in $\frac{d}{dx}(f * g)$

Source: S. Seitz

Derivative Theorem of Convolution

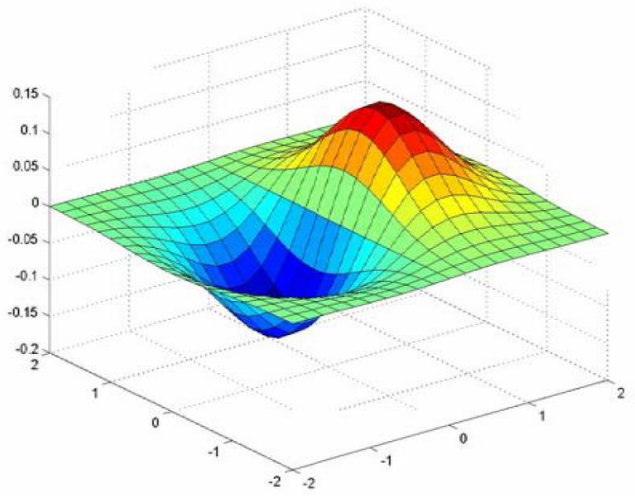
- Differentiation is convolution, and convolution is associative:

- This saves us one operation: $$\frac{d}{dx}(f * g) = f * \frac{d}{dx} g$$

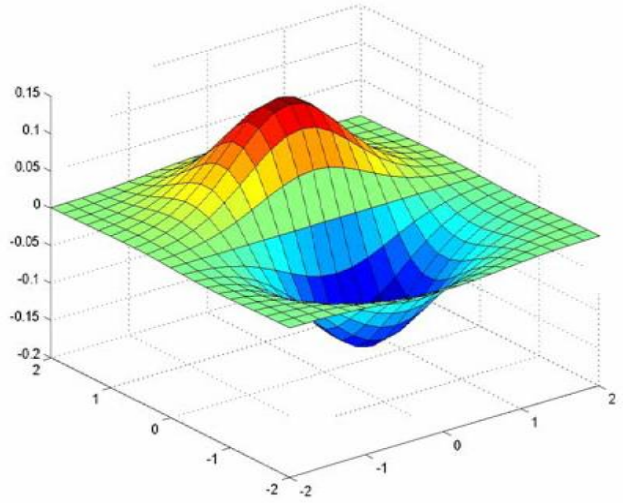


Source: S. Seitz

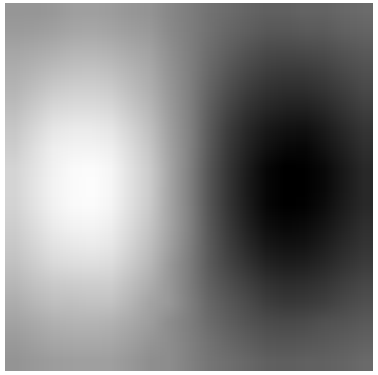
Derivative of Gaussian Filters

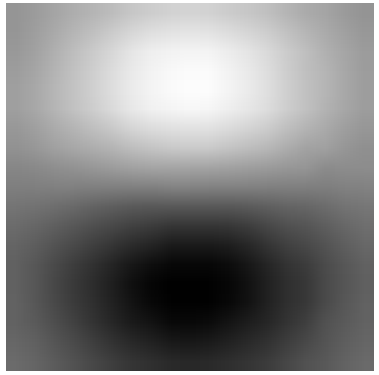


x-direction



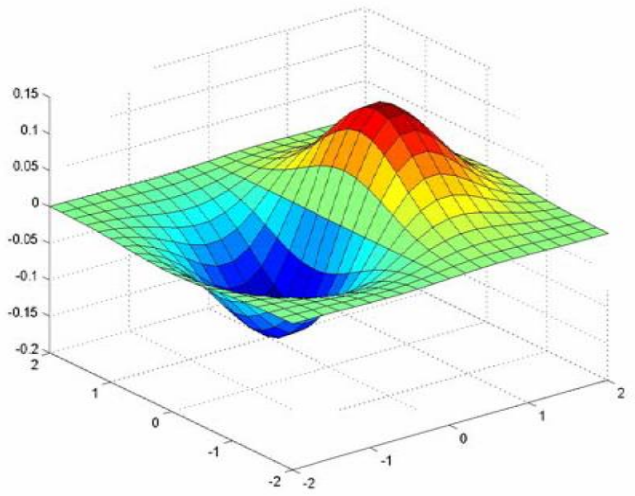
y-direction



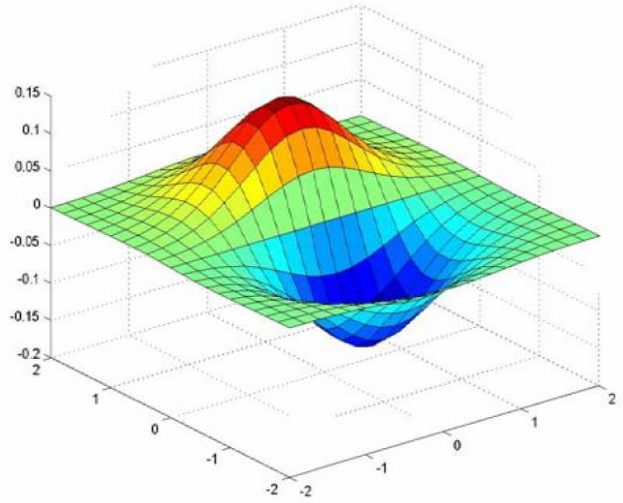


Which one finds horizontal/vertical edges?

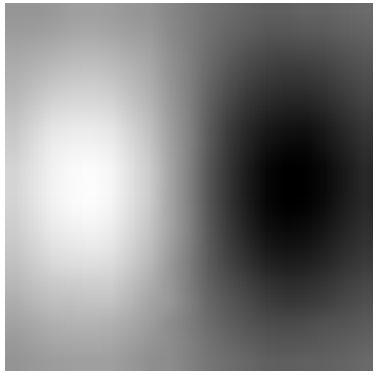
Derivative of Gaussian Filters

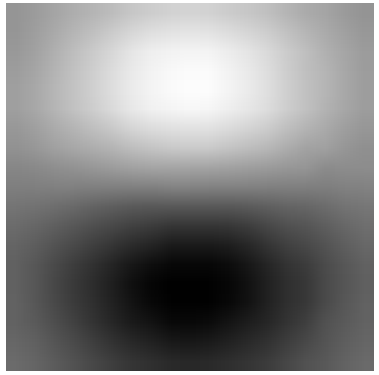


x-direction



y-direction





Are these filters separable?

Recall: Separability of the Gaussian Filter

$$G_\sigma(x,y) = \frac{1}{2\pi\sigma^2}\exp^{-\frac{x^2+y^2}{2\sigma^2}}$$

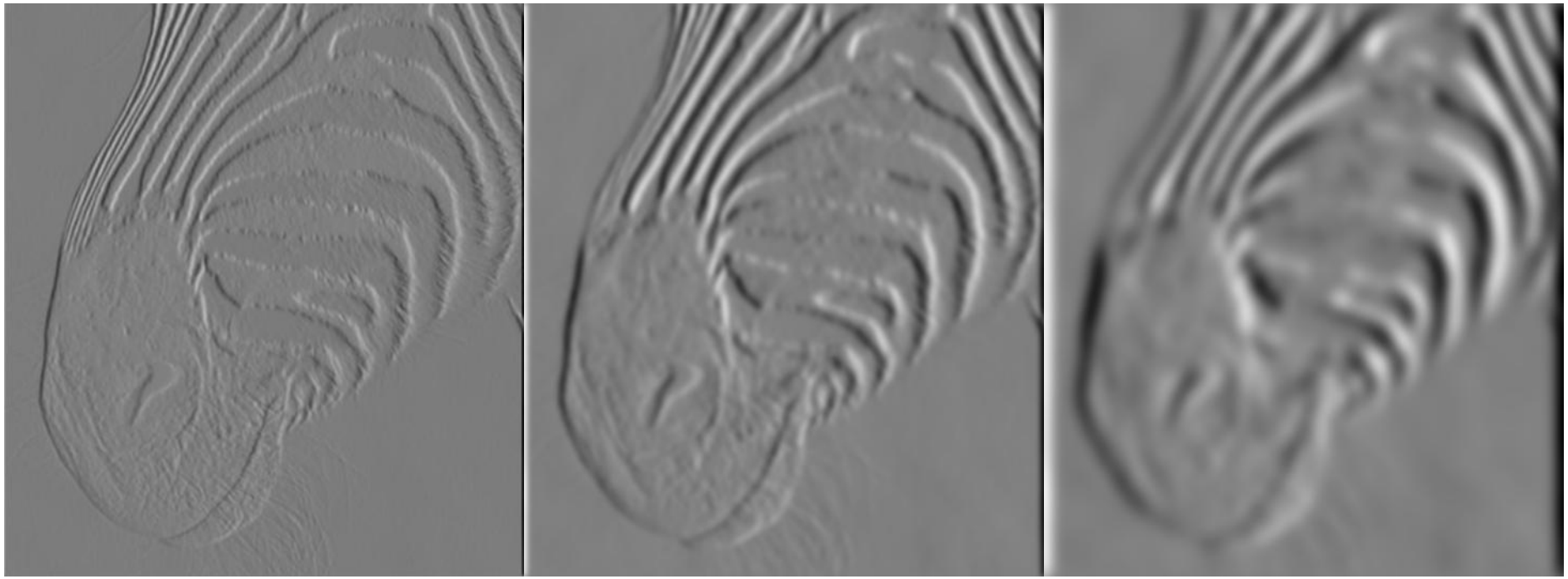
$$= \left(\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{x^2}{2\sigma^2}}\right)\left(\frac{1}{\sqrt{2\pi}\sigma}\exp^{-\frac{y^2}{2\sigma^2}}\right)$$

The 2D Gaussian can be expressed as the product of two functions, one a function of x and the other a function of y

In this case, the two functions are the (identical) 1D Gaussian

Source: D. Lowe

Scale of Gaussian Derivative Filter



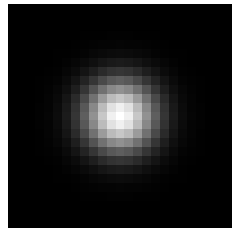
1 pixel 3 pixels 7 pixels

Smoothed derivative removes noise, but blurs edge. Also finds edges at different "scales"

Source: D. Forsyth

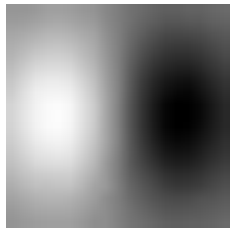
Review: Smoothing vs. Derivative Filters

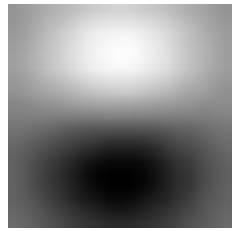
Smoothing filters



- Gaussian: remove “high-frequency” components; “low-pass” filter
- Can the values of a smoothing filter be negative?
- What should the values sum to?
  - **One:** constant regions are not affected by the filter

Derivative filters





- Derivatives of Gaussian
- Can the values of a derivative filter be negative?
- What should the values sum to?
  - **Zero:** no response in constant regions
- High absolute value at points of high contrast

The Canny Edge Detector

1. Filter image with derivative of Gaussian
2. Find magnitude and orientation of gradient
3. **Non-maximum suppression**:
   - Thin wide “ridges” down to single pixel width
4. **Linking and thresholding** (**hysteresis**):
   - Define two thresholds: low and high
   - Use the high threshold to start edge curves and the low threshold to continue them

J. Canny, ***A Computational Approach To Edge Detection***, IEEE Trans. Pattern Analysis and Machine Intelligence, 8:679-714, 1986.

The Canny Edge Detector

original image

Slide credit: Steve Seitz

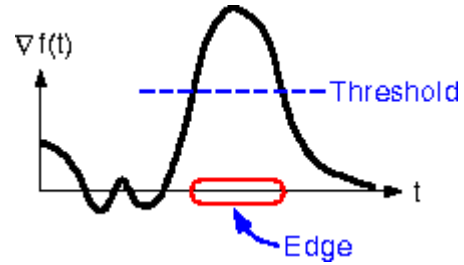
The Canny Edge Detector

norm of the gradient

The Canny Edge Detector

thresholding

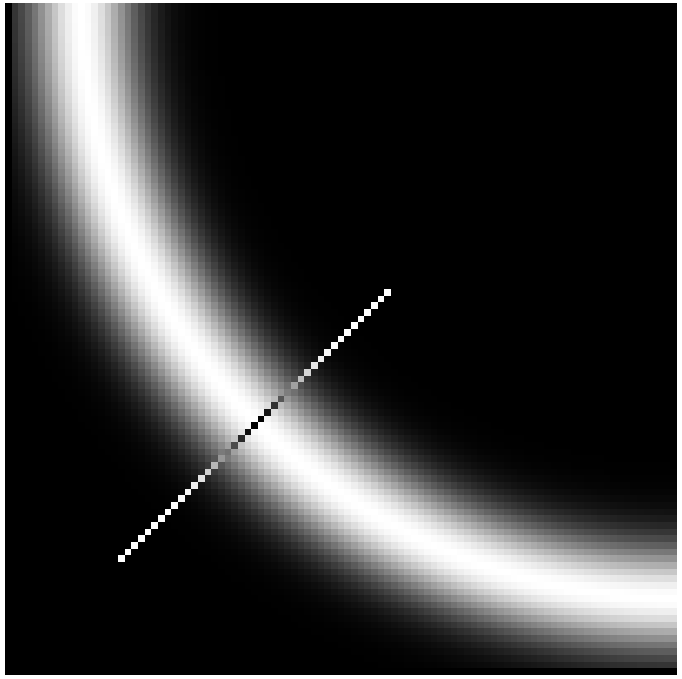
The Canny Edge Detector

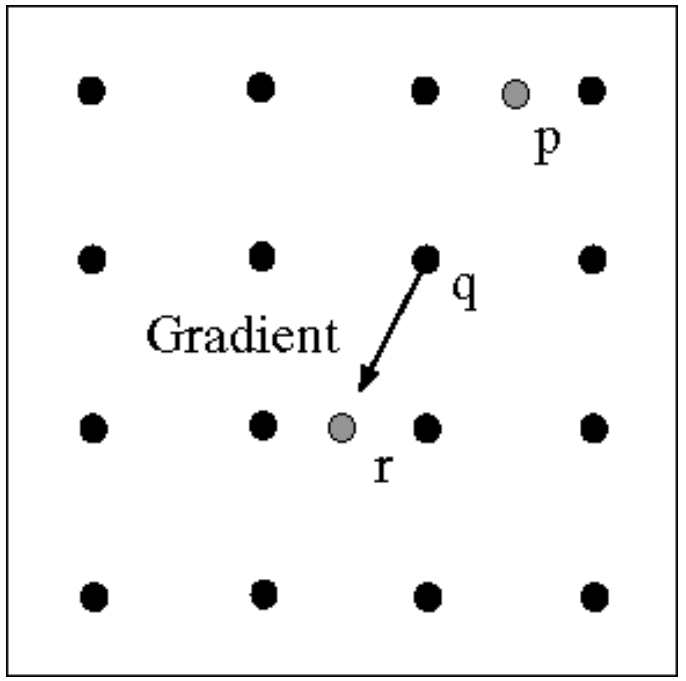


How to turn these thick regions of the gradient into curves?

thresholding

Non-Maximum Suppression





Check if pixel is local maximum along gradient direction, select single max across width of the edge

- requires checking interpolated pixels p and r

The Canny Edge Detector

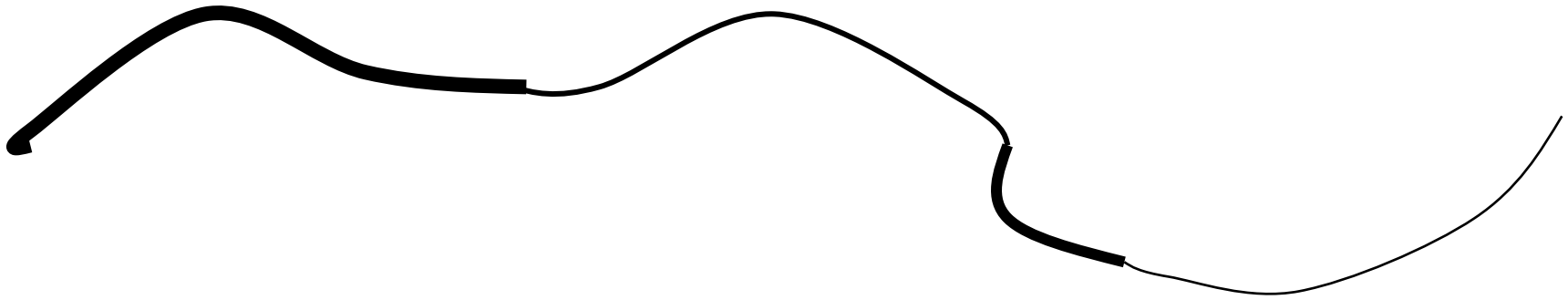
Problem: pixels along this edge didn't survive the thresholding

thinning

(non-maximum suppression)

Hysteresis Thresholding

Use a high threshold to start edge curves, and a low threshold to continue them.



Source: Steve Seitz

Hysteresis Thresholding

original image

high threshold
(strong edges)

low threshold
(weak edges)

hysteresis threshold

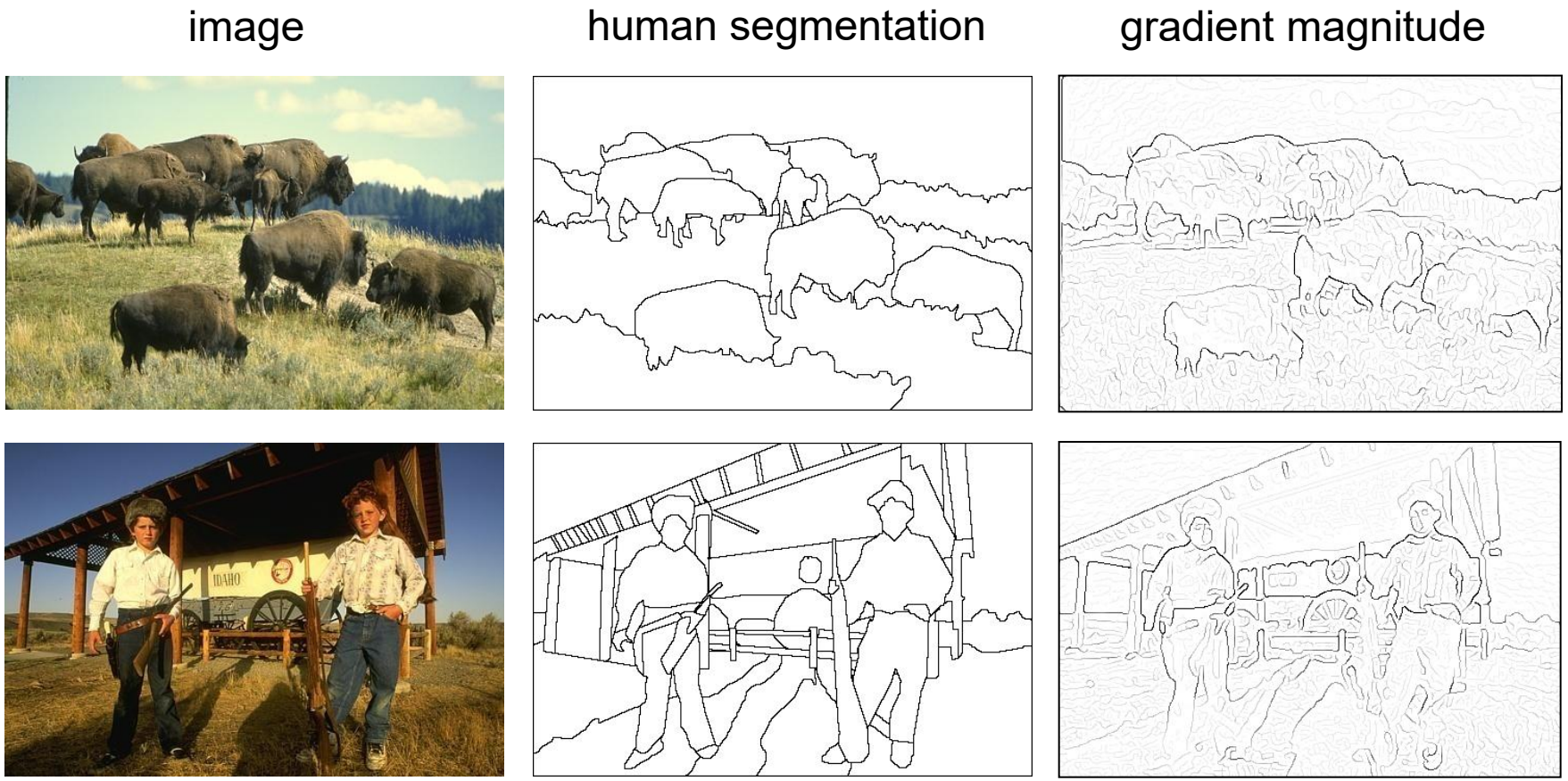
Source: L. Fei-Fei

Recap: Canny Edge Detector

1. Compute x and y gradient images
2. Find magnitude and orientation of gradient
3. **Non-maximum suppression**:
   - Thin wide “ridges” down to single pixel width
4. **Linking and thresholding** (**hysteresis**):
   - Define two thresholds: low and high
   - Use the high threshold to start edge curves and the low threshold to continue them

J. Canny, ***A Computational Approach To Edge Detection***, IEEE Trans. Pattern Analysis and Machine Intelligence, 8:679-714, 1986.

Image Gradients vs. Meaningful Contours

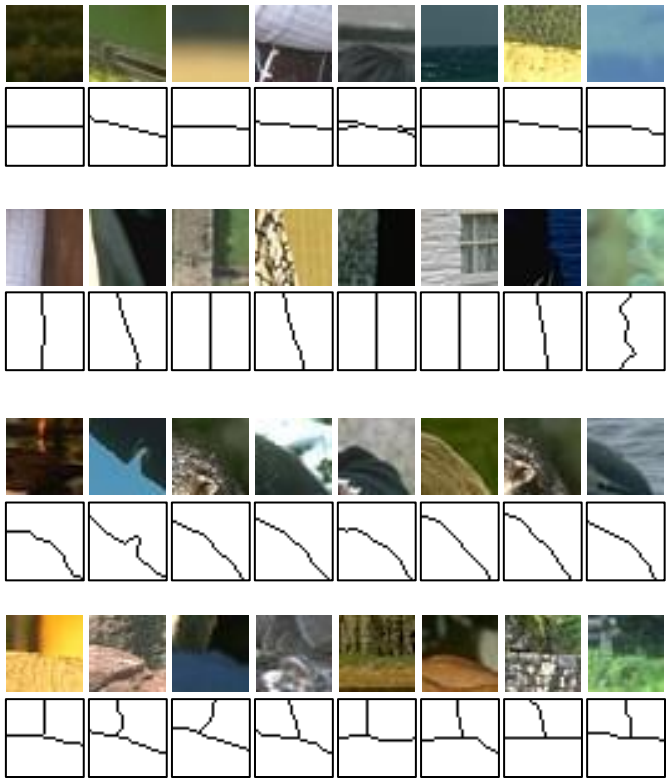


Berkeley segmentation database:

http://www.eecs.berkeley.edu/Research/Projects/CS/vision/grouping/segbench/

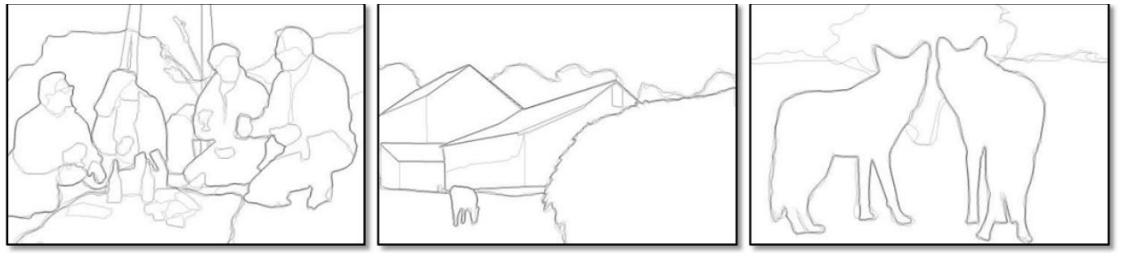
Data-Driven Edge Detection

Training data



Input images

Ground truth



Output

P. Dollar and L. Zitnick, Structured forests for fast edge detection, ICCV 2013

Corners

Slides are from Svetlana Lazebnik

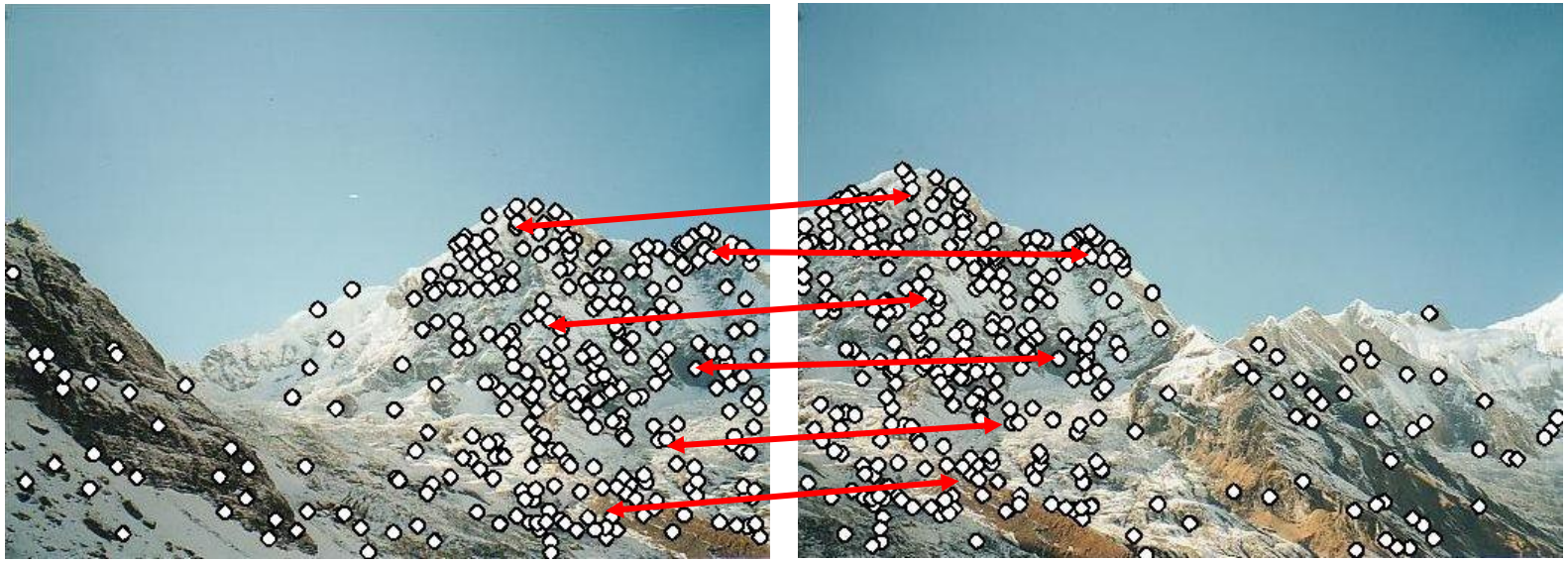
Feature Extraction: Corners

Why Extract Features?

- Motivation: panorama stitching
  - We have two images – how do we combine them?

Why Extract Features?

- Motivation: panorama stitching
  - We have two images – how do we combine them?



Step 1: extract features

Step 2: match features

Why Extract Features?

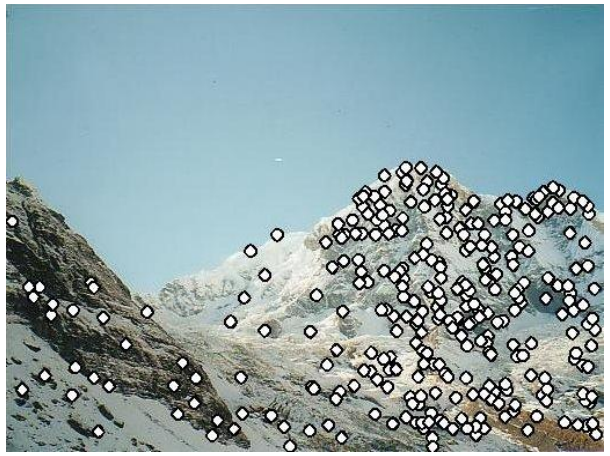
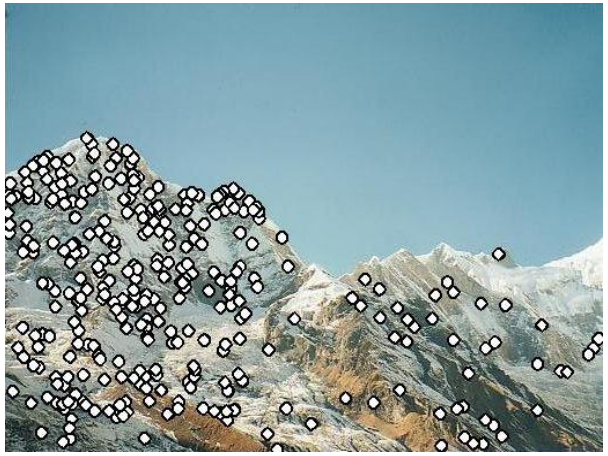
- Motivation: panorama stitching
  - We have two images – how do we combine them?

Step 1: extract features

Step 2: match features

Step 3: align images

Characteristics of Good Features

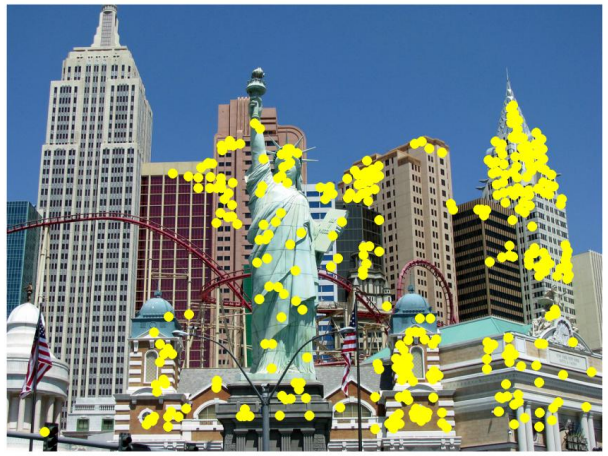



- Repeatability
  - The same feature can be found in several images despite geometric and photometric transformations
- Saliency
  - Each feature is distinctive
- Compactness and efficiency
  - Many fewer features than image pixels
- Locality
  - A feature occupies a relatively small area of the image; robust to clutter and occlusion

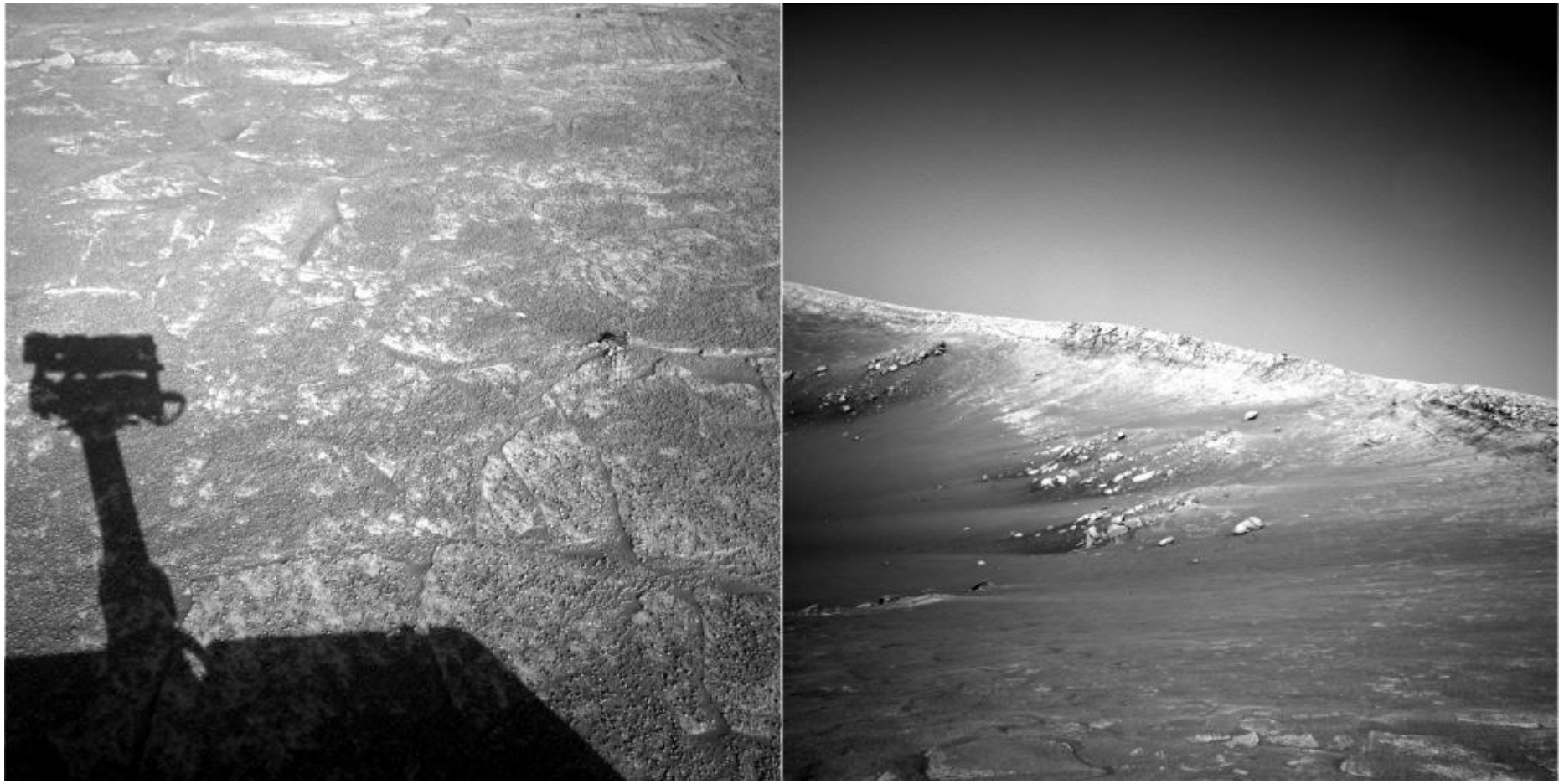
Applications

Feature points are used for:

- Image alignment
- 3D reconstruction
- Motion tracking
- Robot navigation
- Indexing and database retrieval
- Object recognition

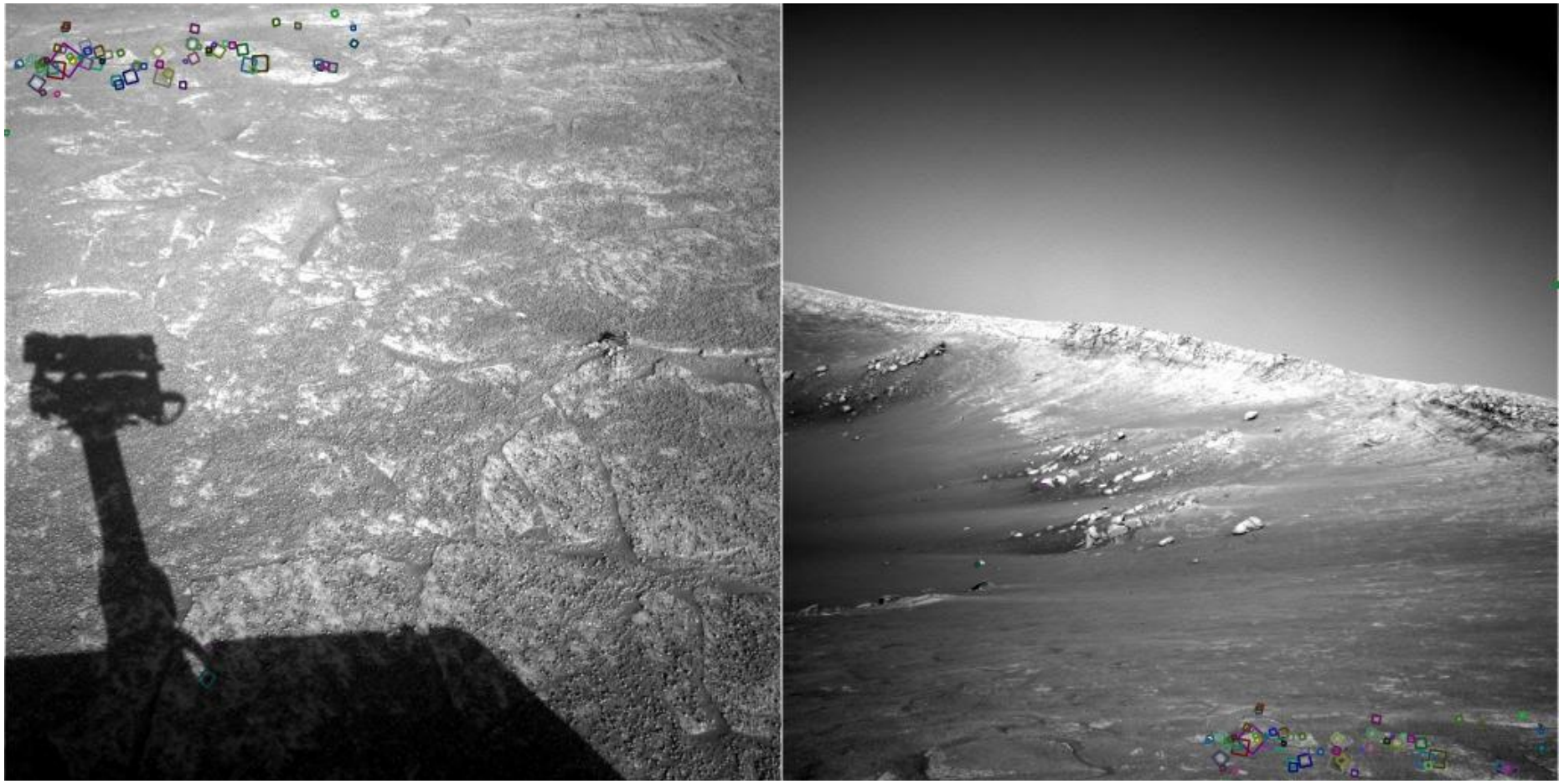


A Hard Feature Matching Problem



NASA Mars Rover images

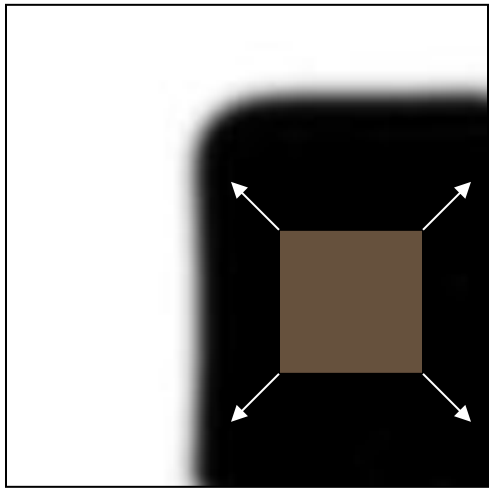
Answer Below (Look for Tiny Colored Squares …)



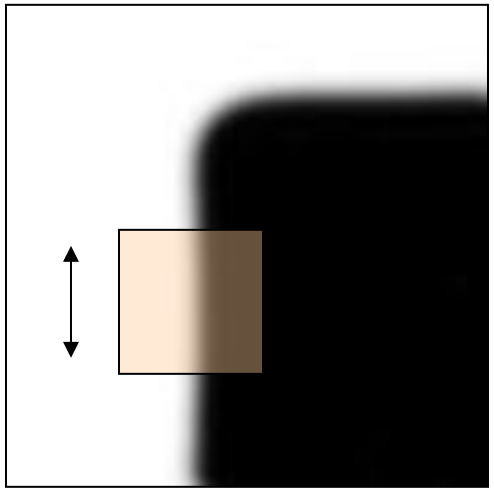
NASA Mars Rover images
with SIFT feature matches
Figure by Noah Snavely

Corner Detection: Basic Idea

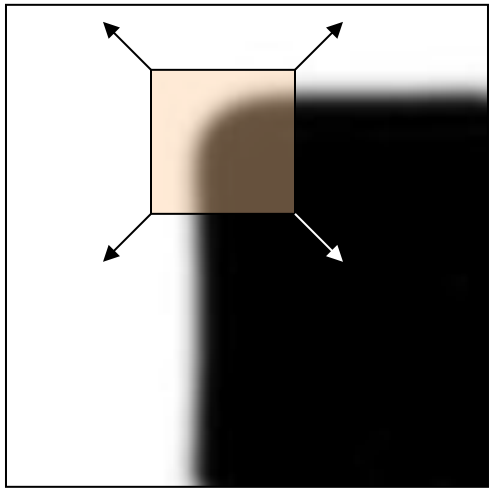
We should easily recognize the point by looking through a small window
Shifting a window in *any direction* should give *a large change* in intensity



"flat" region:
no change in all directions



"edge":
no change along the edge direction



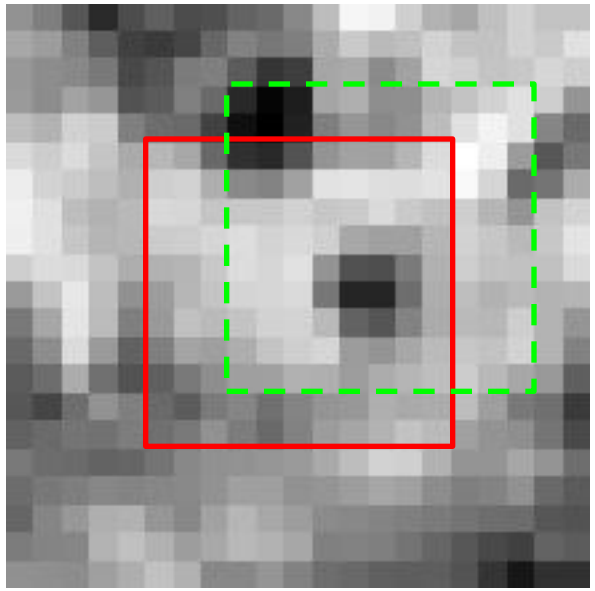
"corner":
significant change in all directions

Corner Detection: Mathematics

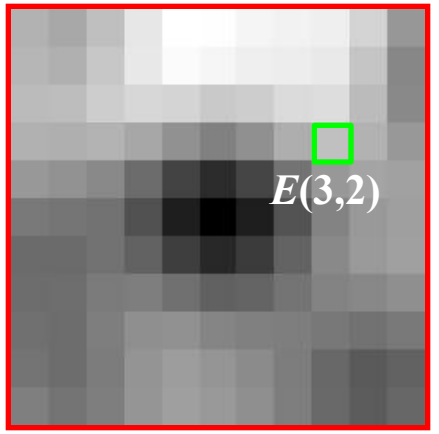
Change in appearance of window *W* for the shift [*u,v*]:

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$

$I(x, y)$



$E(u, v)$



Corner Detection: Mathematics

Change in appearance of window *W* for the shift [*u,v*]:

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$

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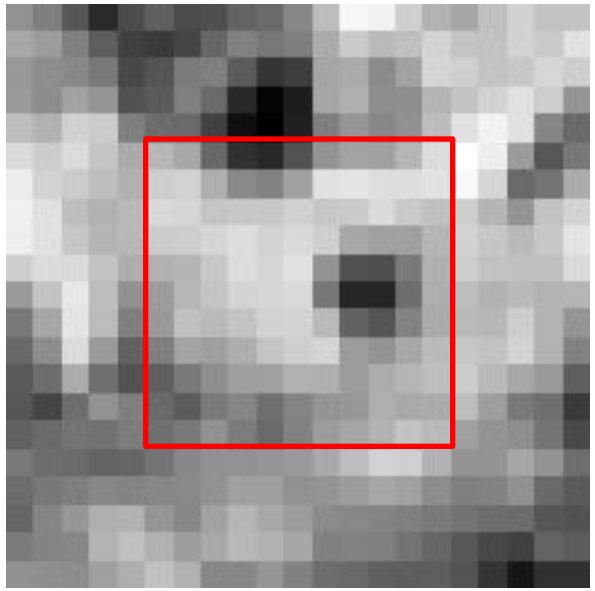
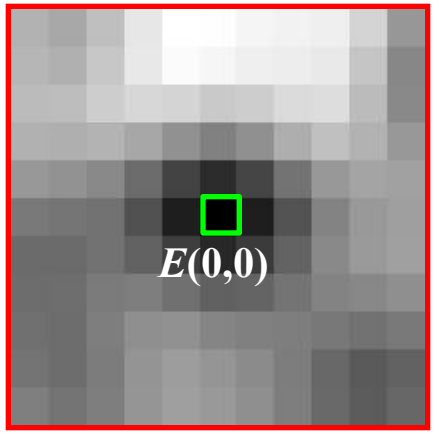


$E(u, v)$



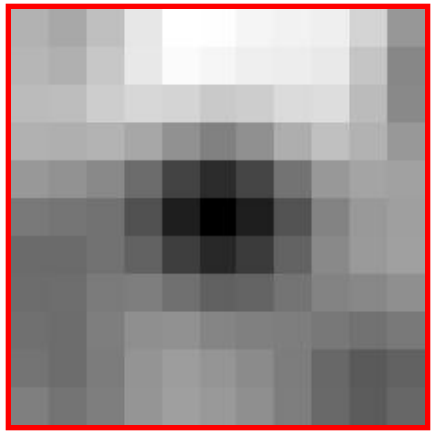
Corner Detection: Mathematics

Change in appearance of window *W* for the shift [*u,v*]:

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$

We want to find out how this function behaves for small shifts

$E(u, v)$



Corner Detection: Mathematics

- First-order Taylor approximation for small motions [*u*, *v*]:

$$I(x+u, y+v) \approx I(x,y) + I_x u + I_y v$$

- Let's plug this into *E*(*u*,*v*):

$$E(u,v) = \sum_{(x,y)\in W} [I(x+u, y+v) - I(x,y)]^2$$

Corner Detection: Mathematics

The quadratic approximation can be written as

$$E(u,v) \approx [u \quad v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

where *M* is a second moment matrix computed from image derivatives:

$$M = \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix}$$

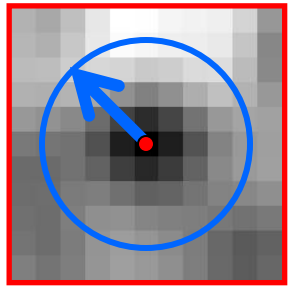
(the sums are over all the pixels in the window *W*)

Interpreting the Second Moment Matrix

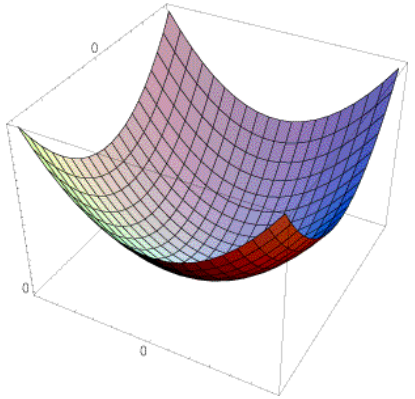
- The surface $E(u,v)$ is locally approximated by a quadratic form. Let's try to understand its shape.
  - Specifically, in which directions does it have the smallest/greatest change?

$$E(u,v) \approx \begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix}$$

$E(u, v)$



$$M = \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix}$$



Interpreting the Second Moment Matrix

First, consider the axis-aligned case (gradients are either horizontal or vertical)

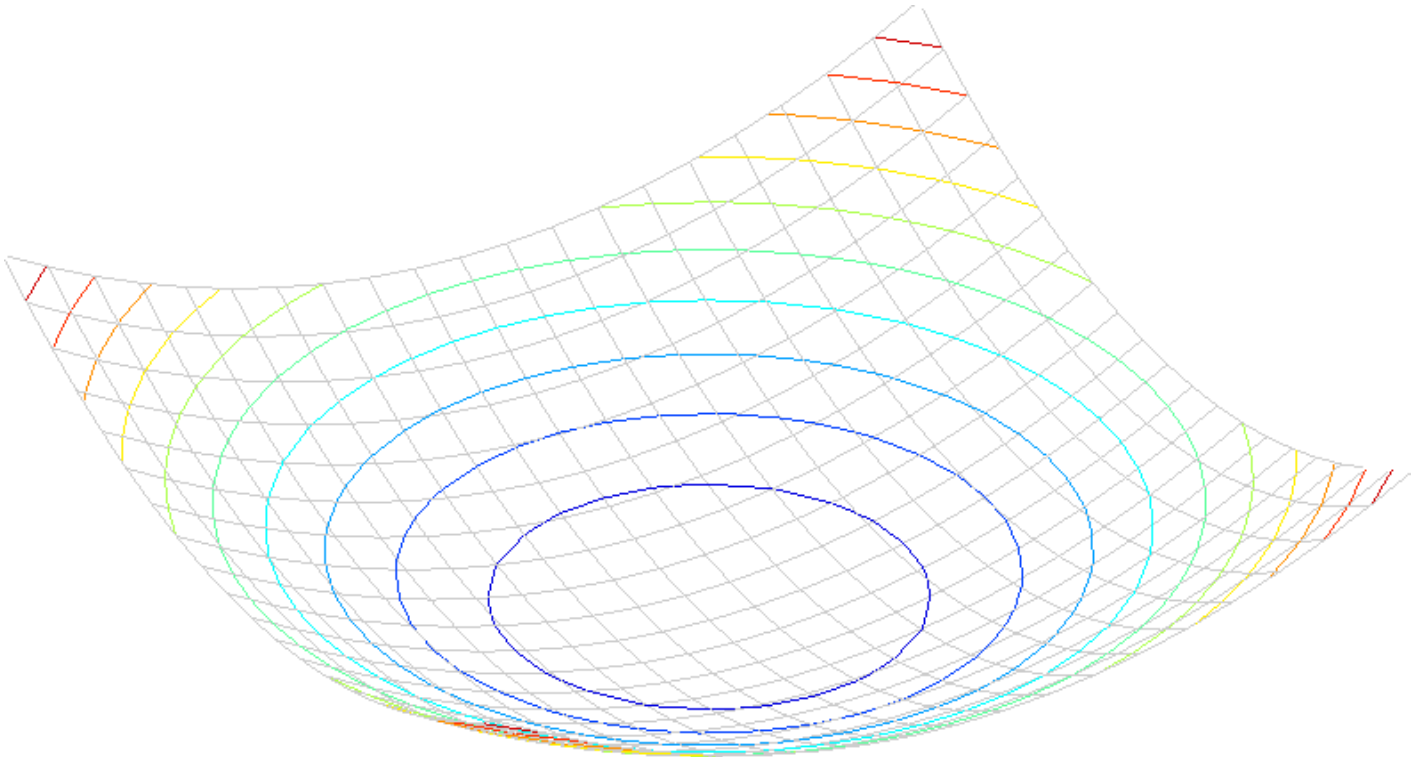
$$M = \begin{bmatrix} \sum_{x,y} I_x^2 & \sum_{x,y} I_x I_y \\ \sum_{x,y} I_x I_y & \sum_{x,y} I_y^2 \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$$

If either *a* or *b* is close to 0, then this is **not** a corner, so look for locations where both are large.

Interpreting the Second Moment Matrix

Consider a horizontal "slice" of $E(u, v)$: $\begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$

This is the equation of an ellipse.



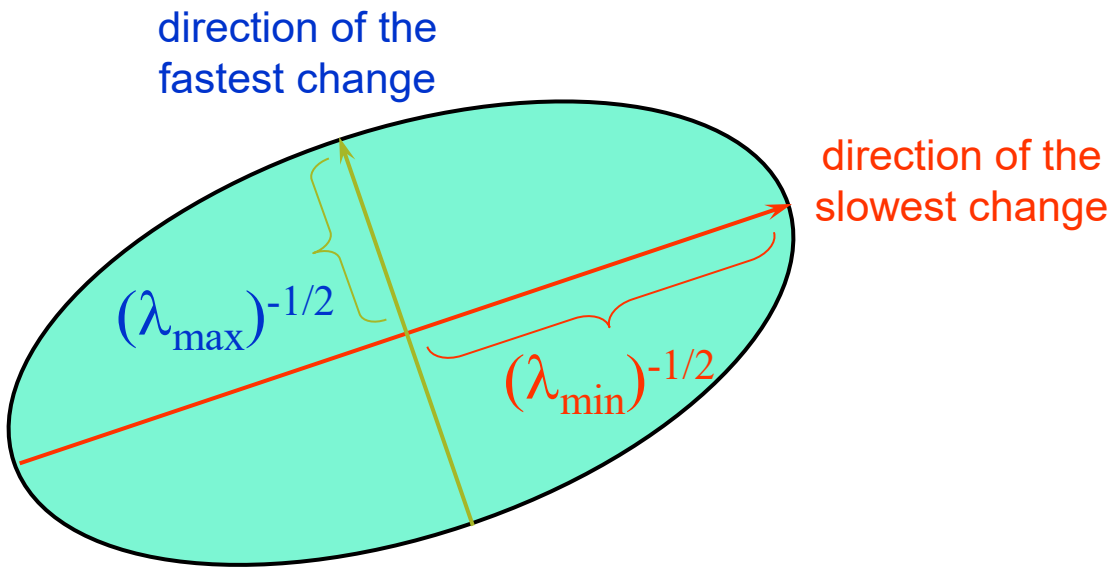
Interpreting the Second Moment Matrix

Consider a horizontal "slice" of $E(u, v)$: $\begin{bmatrix} u & v \end{bmatrix} M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$

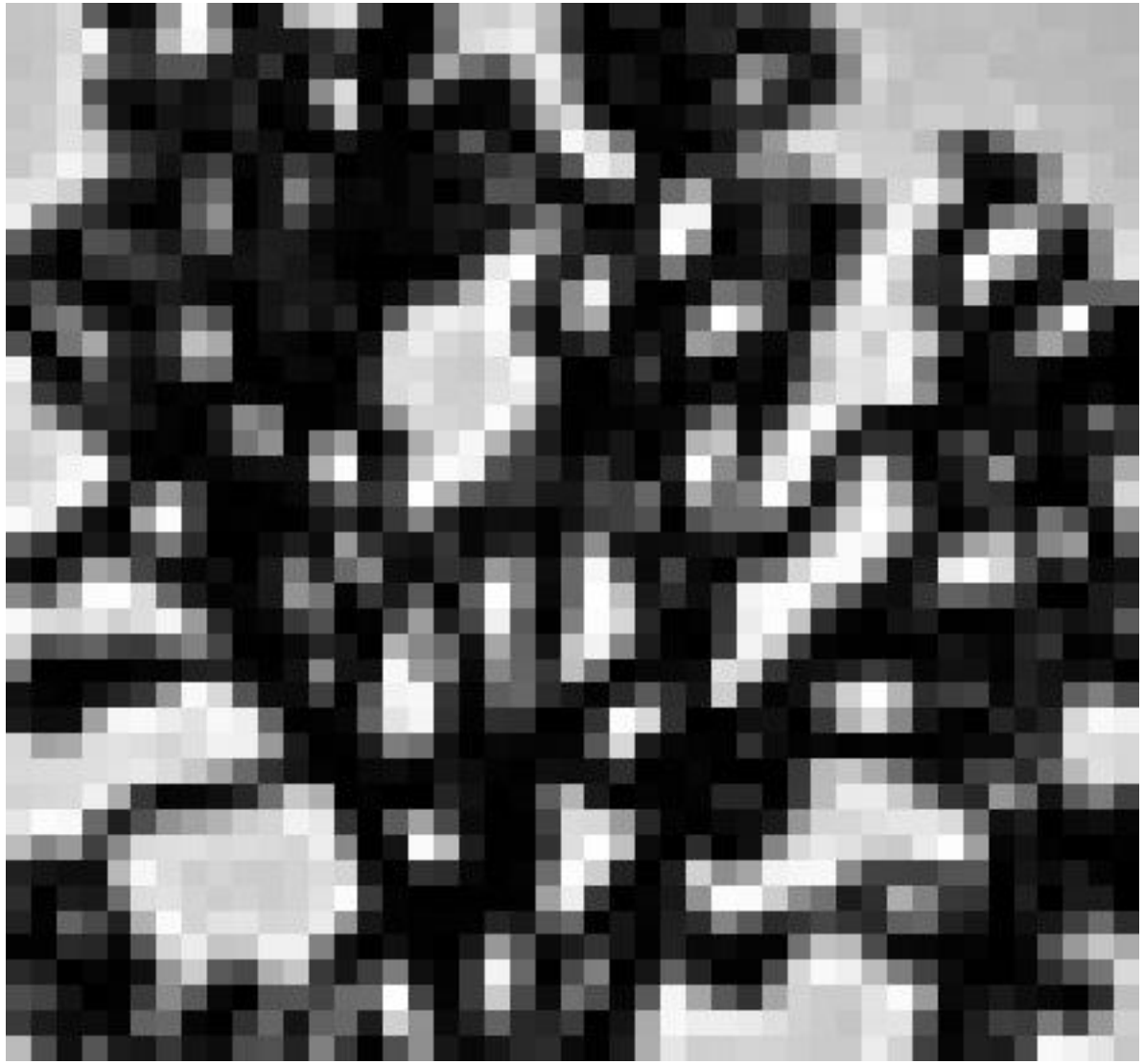
This is the equation of an ellipse.

Diagonalization of M: $M = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$

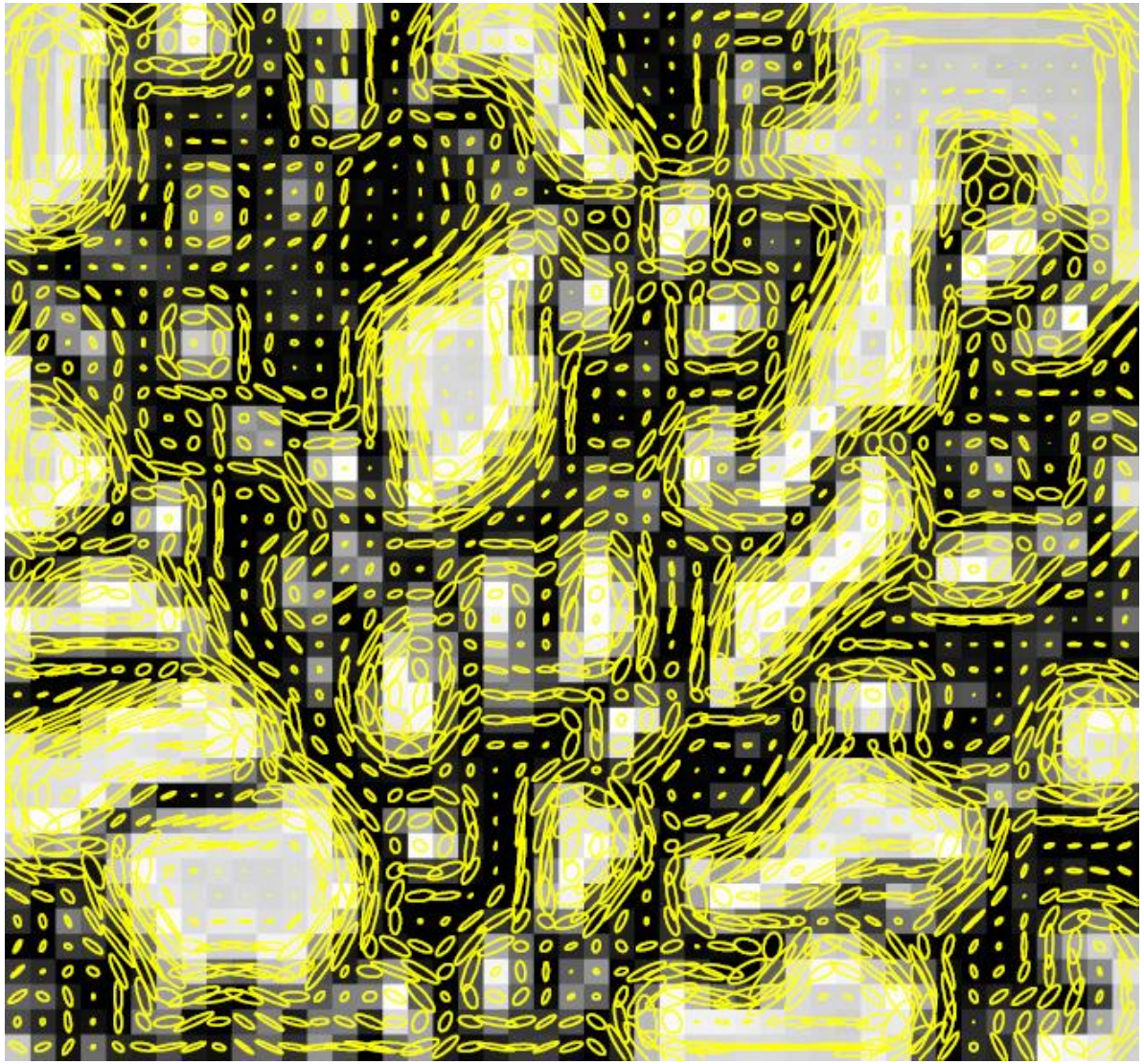
The axis lengths of the ellipse are determined by the eigenvalues and the orientation is determined by R



Visualization of Second Moment Matrices

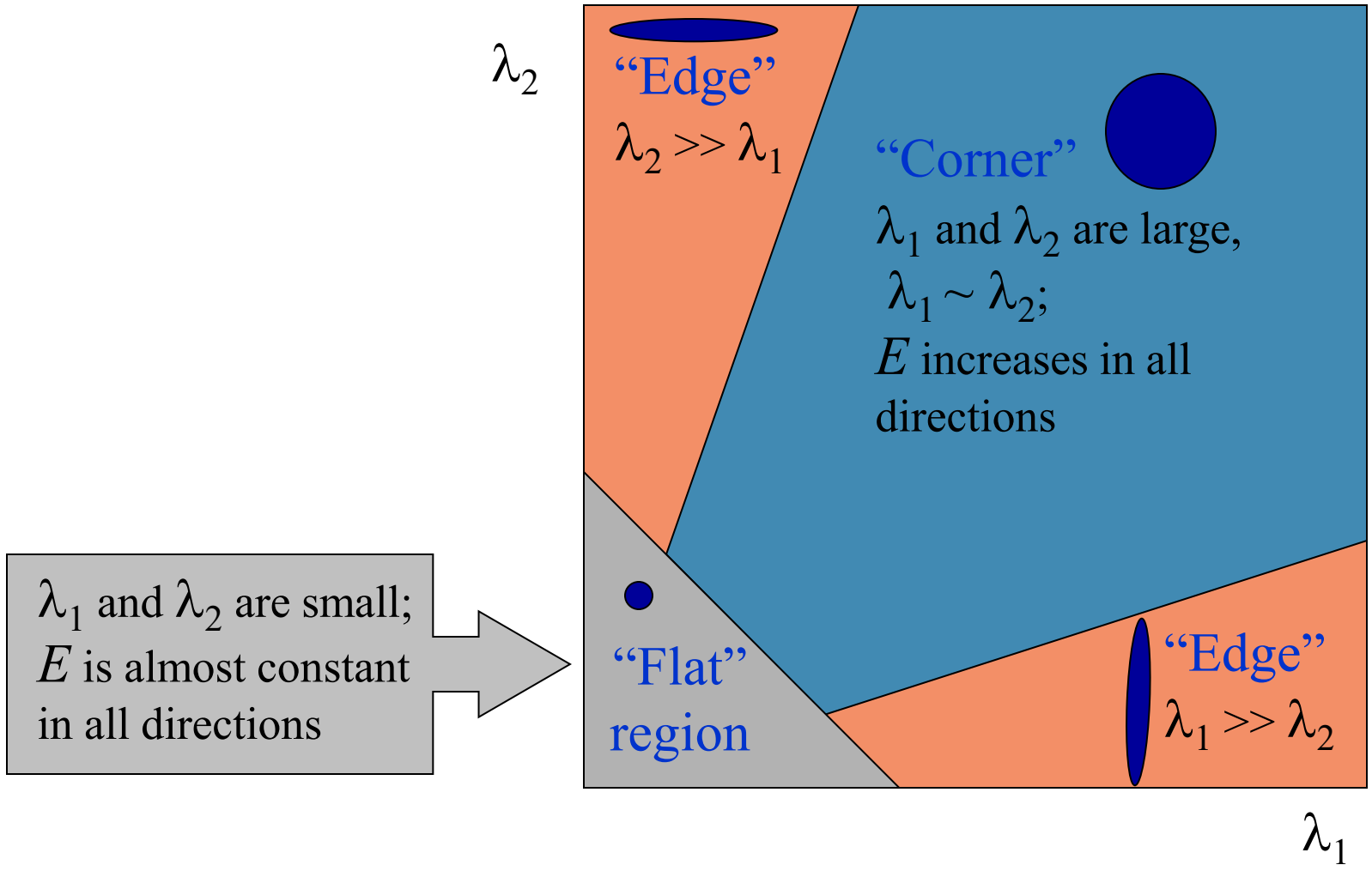


Visualization of Second Moment Matrices



Interpreting the Eigenvalues

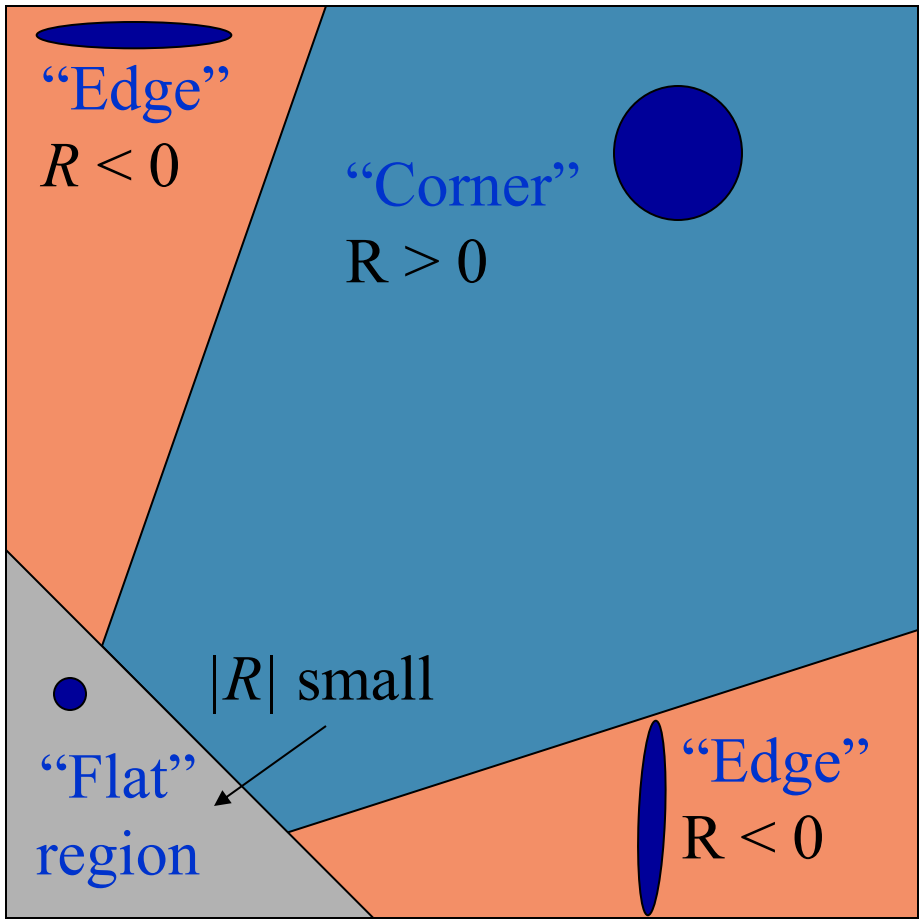
Classification of image points using eigenvalues of *M*:



Corner Response Function

$$R = \det(M) - \alpha \,\mathrm{trace}(M)^2 = \lambda_1\lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

α: constant (0.04 to 0.06)



The Harris Corner Detector

1. Compute partial derivatives at each pixel
2. Compute second moment matrix *M* in a Gaussian window around each pixel:

$$M = \begin{bmatrix} \sum_{x,y} w(x,y) I_x^2 & \sum_{x,y} w(x,y) I_x I_y \\ \sum_{x,y} w(x,y) I_x I_y & \sum_{x,y} w(x,y) I_y^2 \end{bmatrix}$$

C.Harris and M.Stephens. "A Combined Corner and Edge Detector."
Proceedings of the 4th Alvey Vision Conference: pages 147—151, 1988.

The Harris Corner Detector

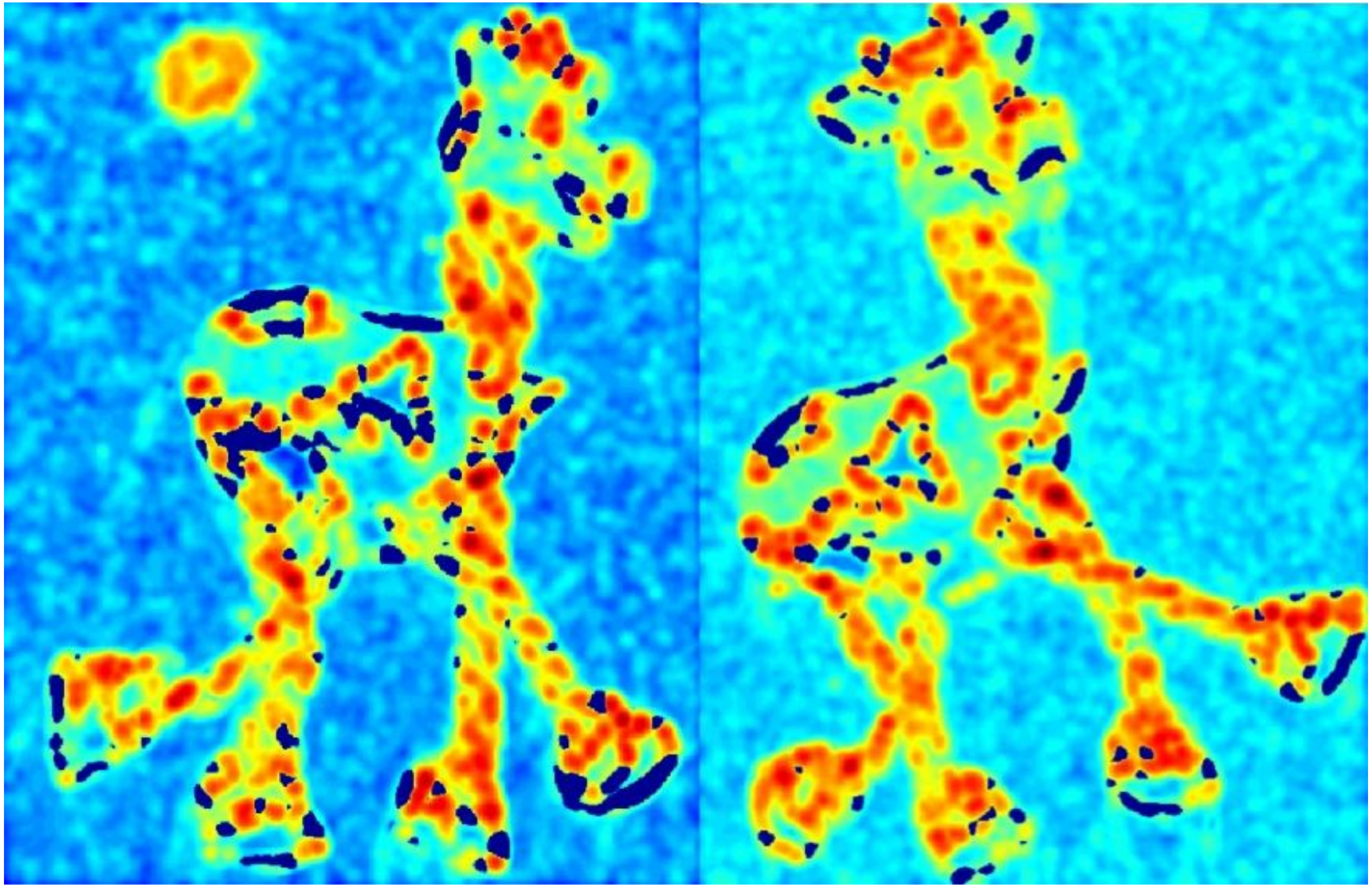
1. Compute partial derivatives at each pixel
2. Compute second moment matrix *M* in a Gaussian window around each pixel
3. Compute corner response function *R*

C. Harris and M. Stephens. "A Combined Corner and Edge Detector." *Proceedings of the 4th Alvey Vision Conference*: pages 147—151, 1988.

Harris Detector: Steps

Harris Detector: Steps

Compute corner response R



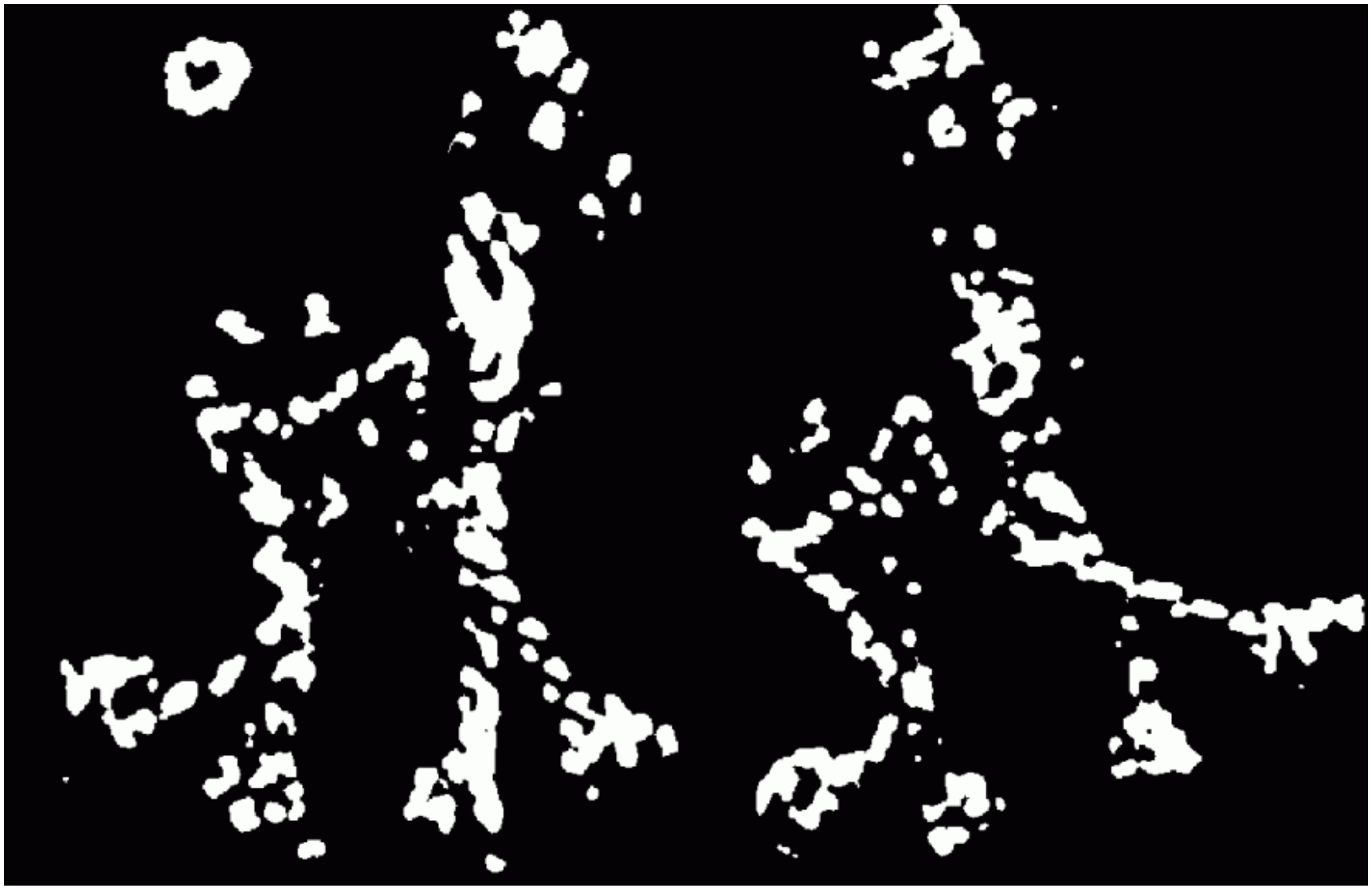
The Harris Corner Detector

1. Compute partial derivatives at each pixel
2. Compute second moment matrix *M* in a Gaussian window around each pixel
3. Compute corner response function *R*
4. Threshold *R*
5. Find local maxima of response function (nonmaximum suppression)

C.Harris and M.Stephens. "A Combined Corner and Edge Detector."
Proceedings of the 4th Alvey Vision Conference: pages 147—151, 1988.

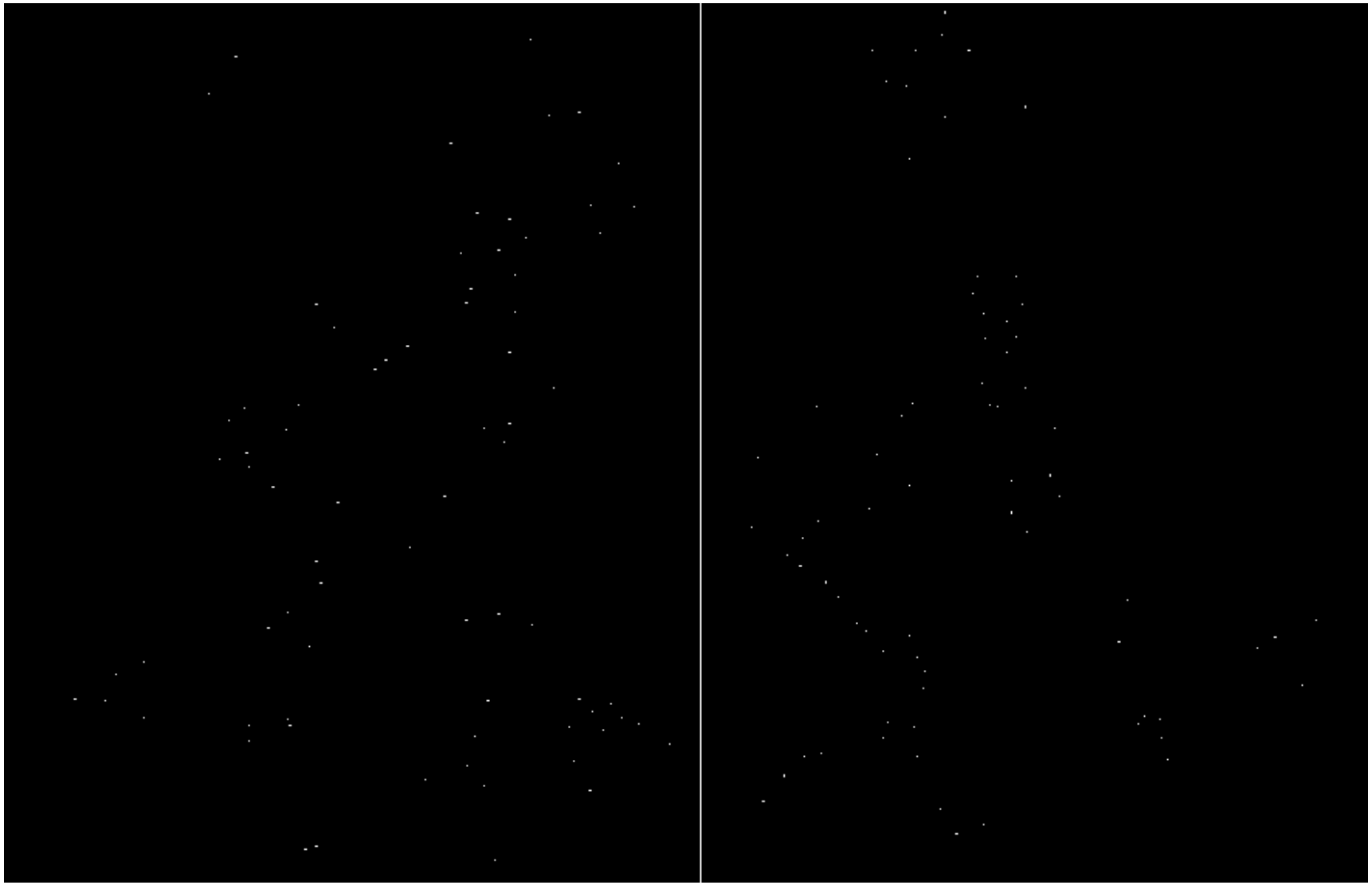
Harris Detector: Steps

Find points with large corner response: R > threshold



Harris Detector: Steps

Take only the points of local maxima of *R*



Harris Detector: Steps

Invariance and Covariance

We want corner locations to be *invariant* to photometric transformations and *covariant* to geometric transformations

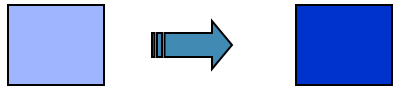
- **Invariance:** image is transformed and corner locations do not change
- **Covariance:** if we have two transformed versions of the same image, features should be detected in corresponding locations

Activity

Is Harris corner detection invariant to

1. Affine intensity changes $I \quad a\,I + b$
2. Image translation
3. Image rotation
4. Image scaling

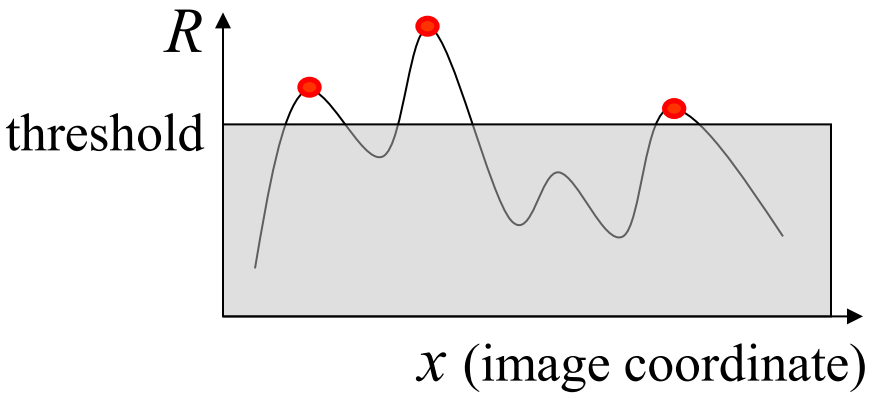
Affine Intensity Change

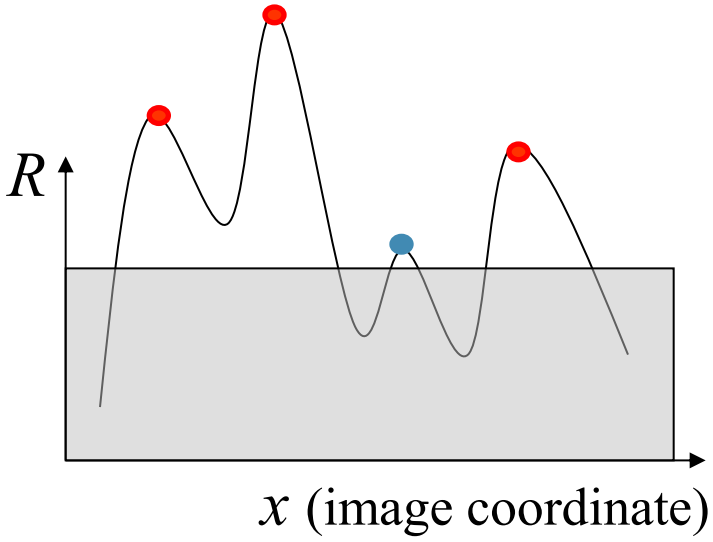


$I \quad a I + b$

Only derivatives are used => invariance to intensity shift $I \quad I + b$

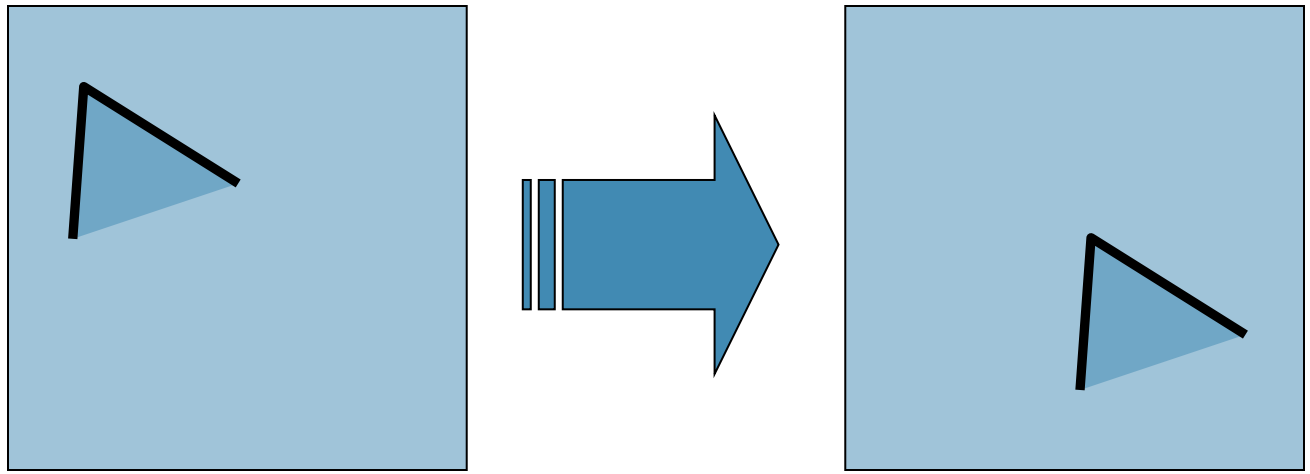
Intensity scaling: $I \quad a I$





Partially invariant to affine intensity change

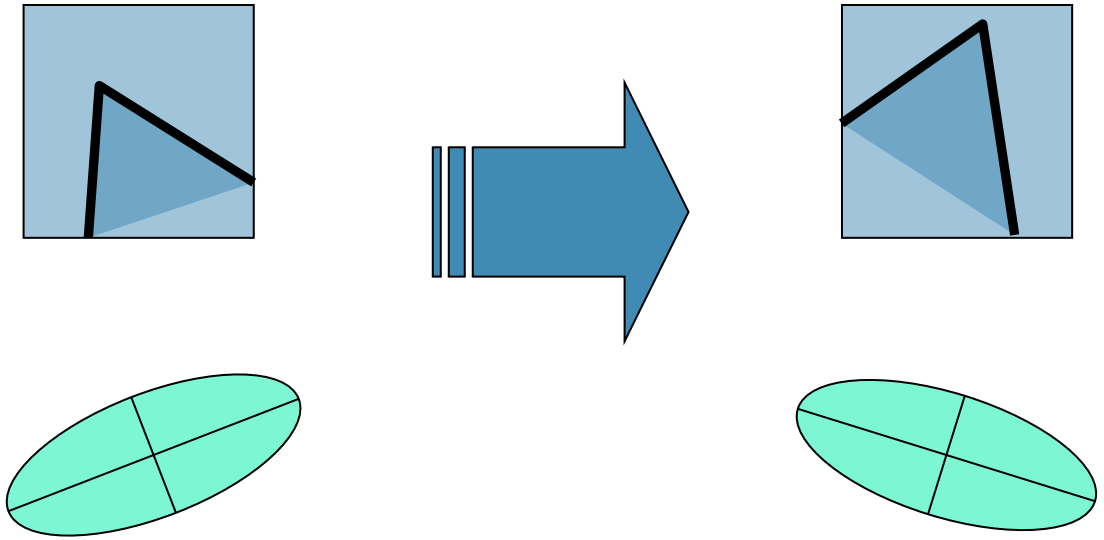
Image Translation



Derivatives and window function are shift-invariant

Corner location is covariant w.r.t. translation

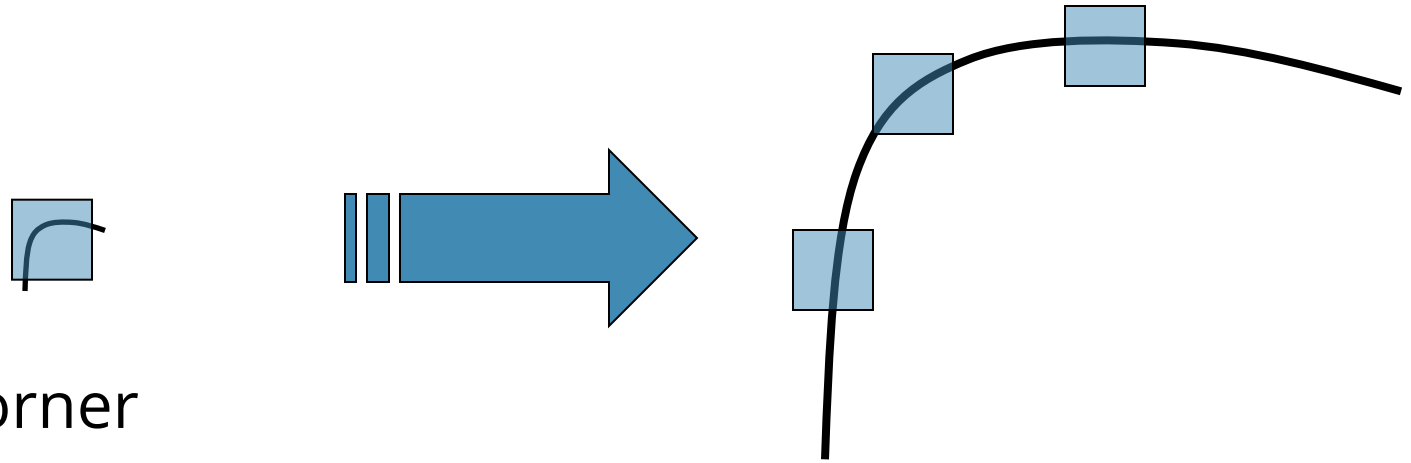
Image Rotation



Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner location is covariant w.r.t. rotation

Scaling



Corner

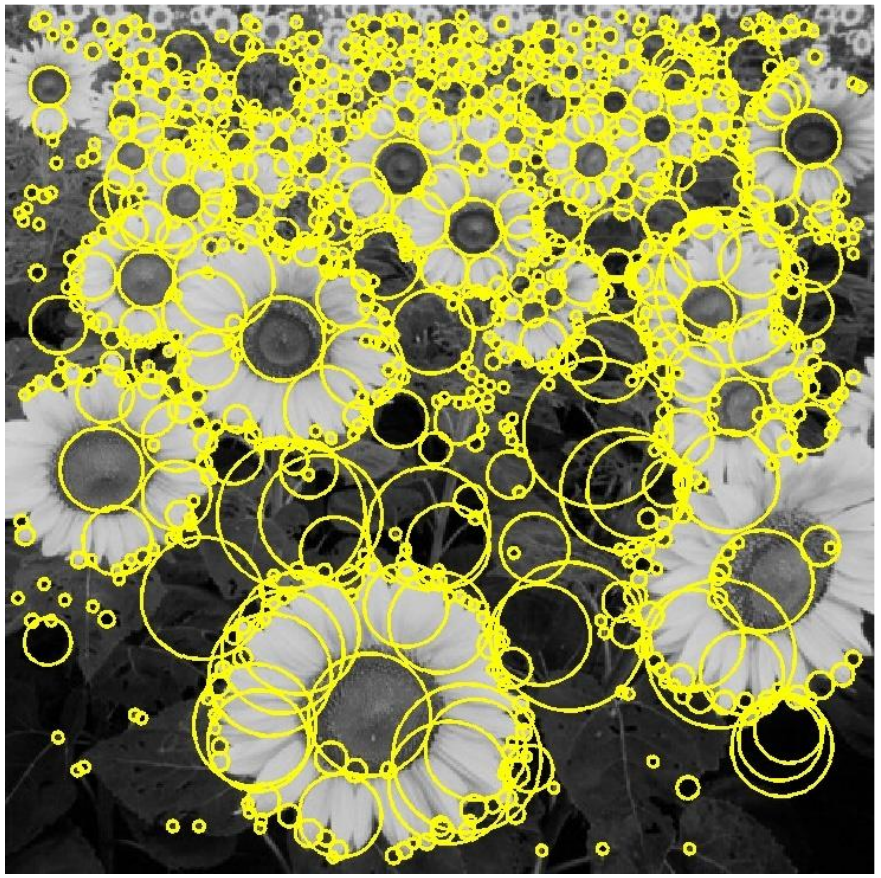
All points will be classified as edges

Corner location is not covariant to scaling!

Blob Detection

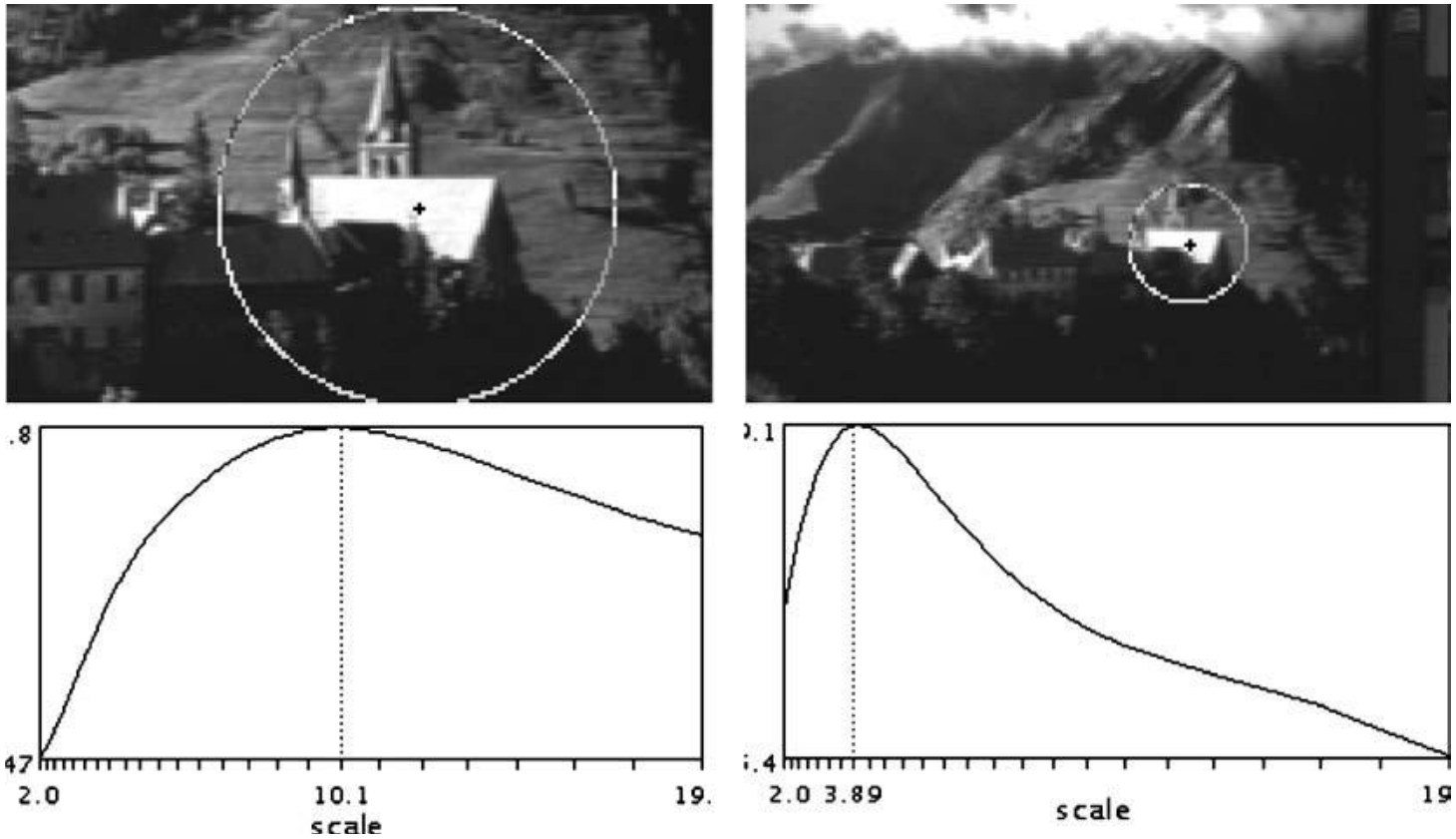
Slides from Svetlana Lazebnik

Blob Detection



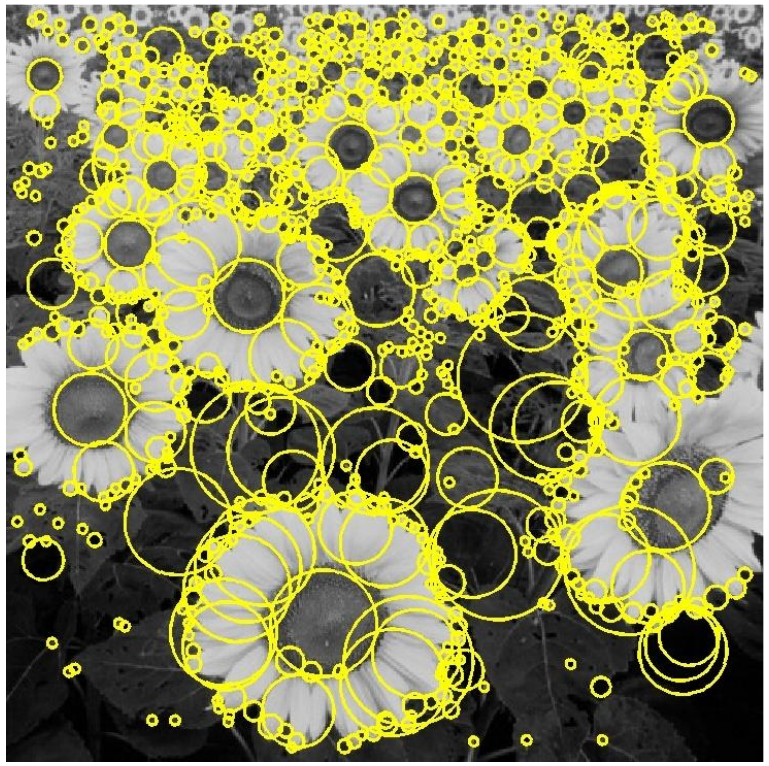
Feature Detection with Scale Selection

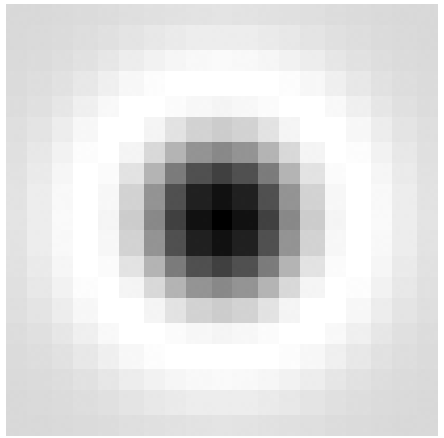
We want to extract features with characteristic scale that is *covariant* with the image transformation



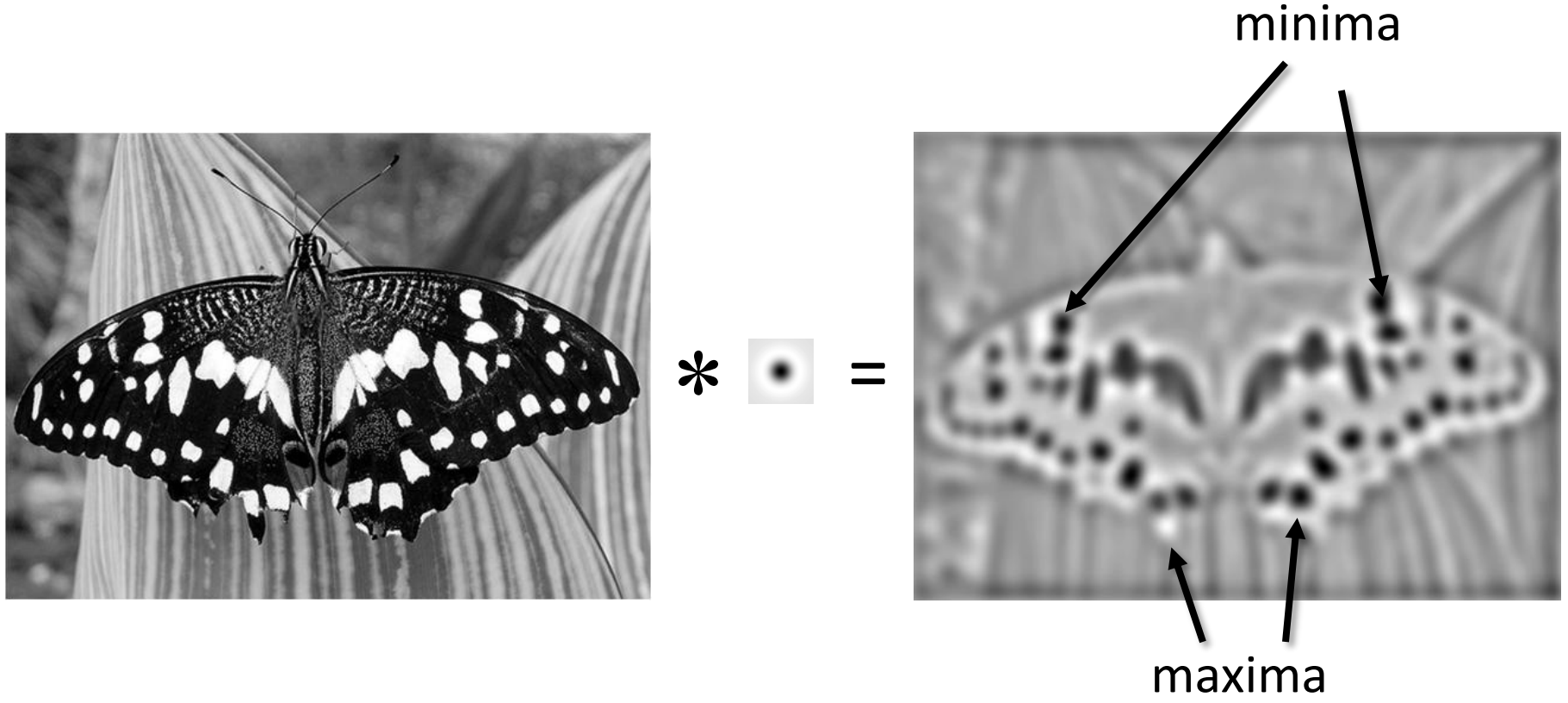
Blob Detection: Basic idea

To detect blobs, convolve the image with a “blob filter” at multiple scales and look for extrema of filter response in the resulting *scale space.*





Blob Detection: Basic Idea

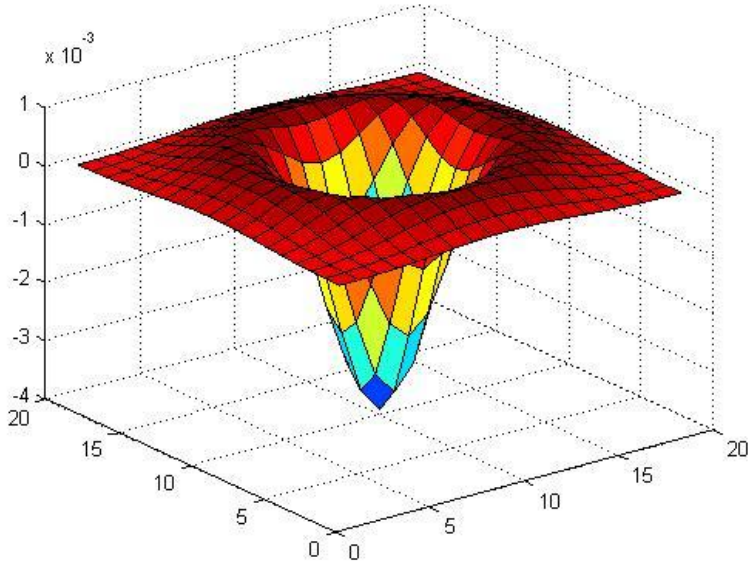


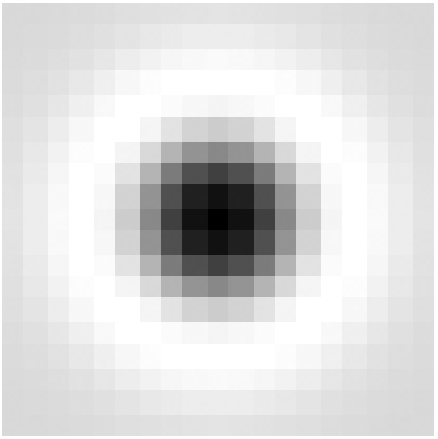
Find maxima *and minima* of blob filter response in space *and scale*

Source: N. Snavely

Blob Filter

Laplacian of Gaussian: Circularly symmetric operator for blob detection in 2D

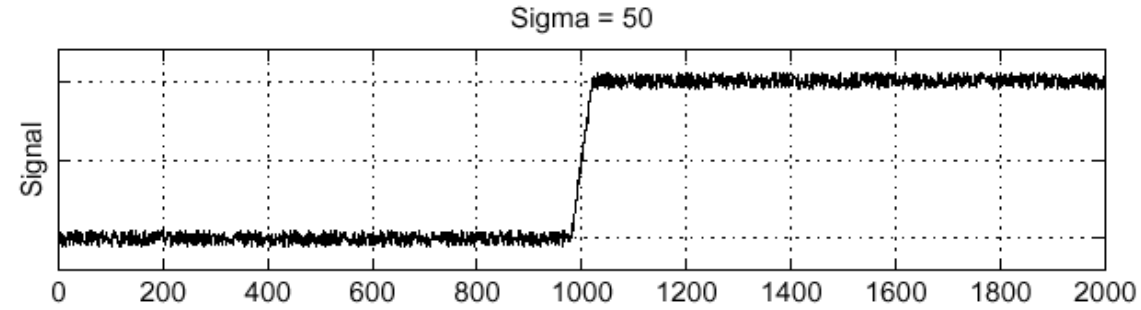




$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$$

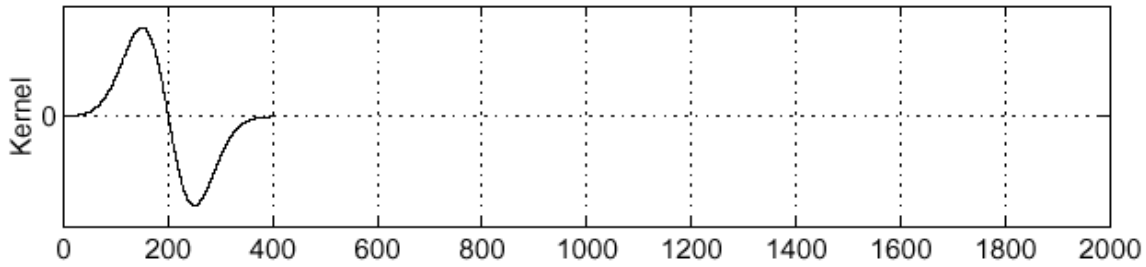
Recall: Edge Detection

f



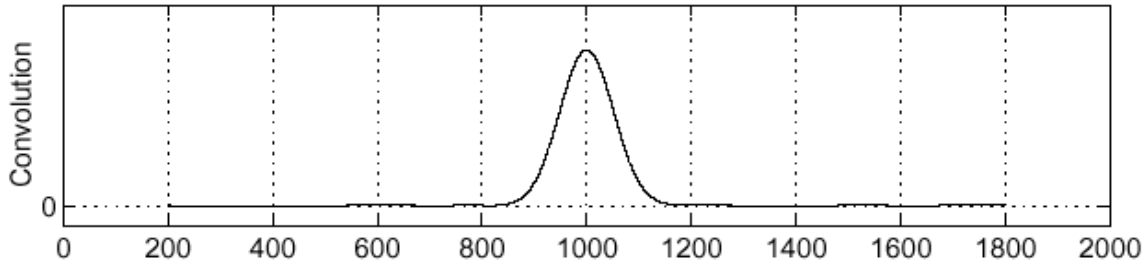
Edge

$\frac{d}{dx} g$



Derivative of Gaussian

$f * \frac{d}{dx} g$

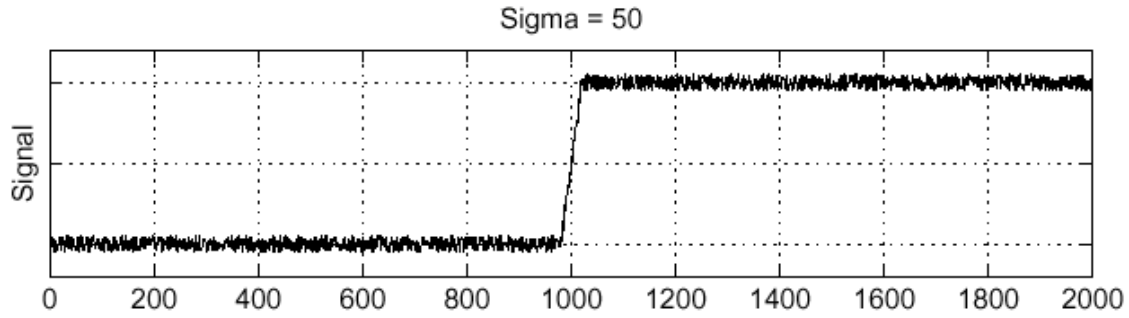


Edge = maximum of derivative

Source: S. Seitz

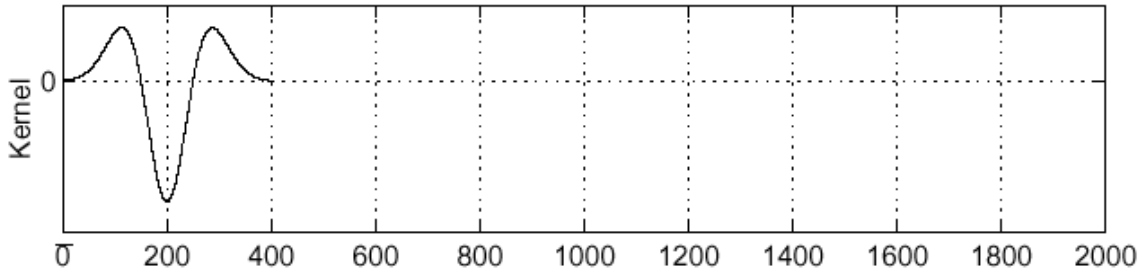
Edge Detection, Take 2

f



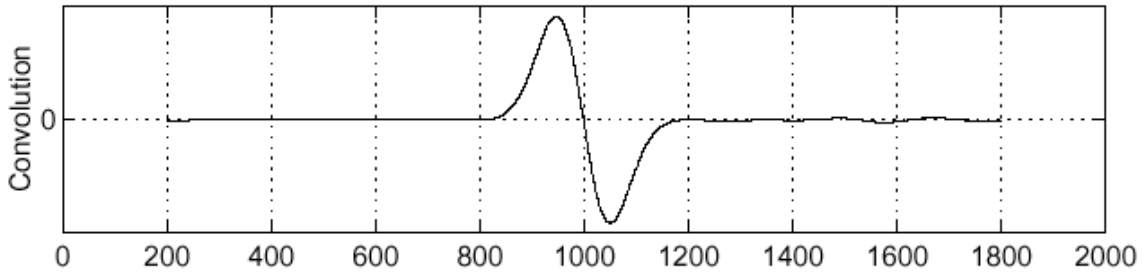
Edge

$\frac{d^2}{dx^2} g$



Second derivative of Gaussian (Laplacian)

$f * \frac{d^2}{dx^2} g$

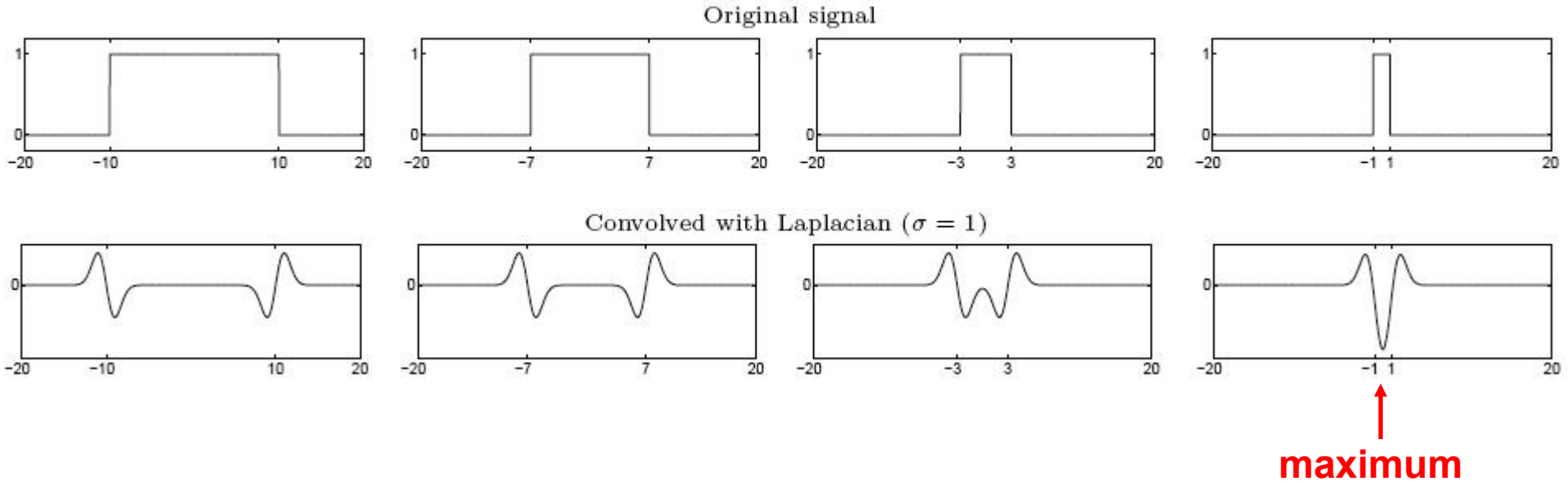


Edge = zero crossing of second derivative

Source: S. Seitz

From Edges to Blobs

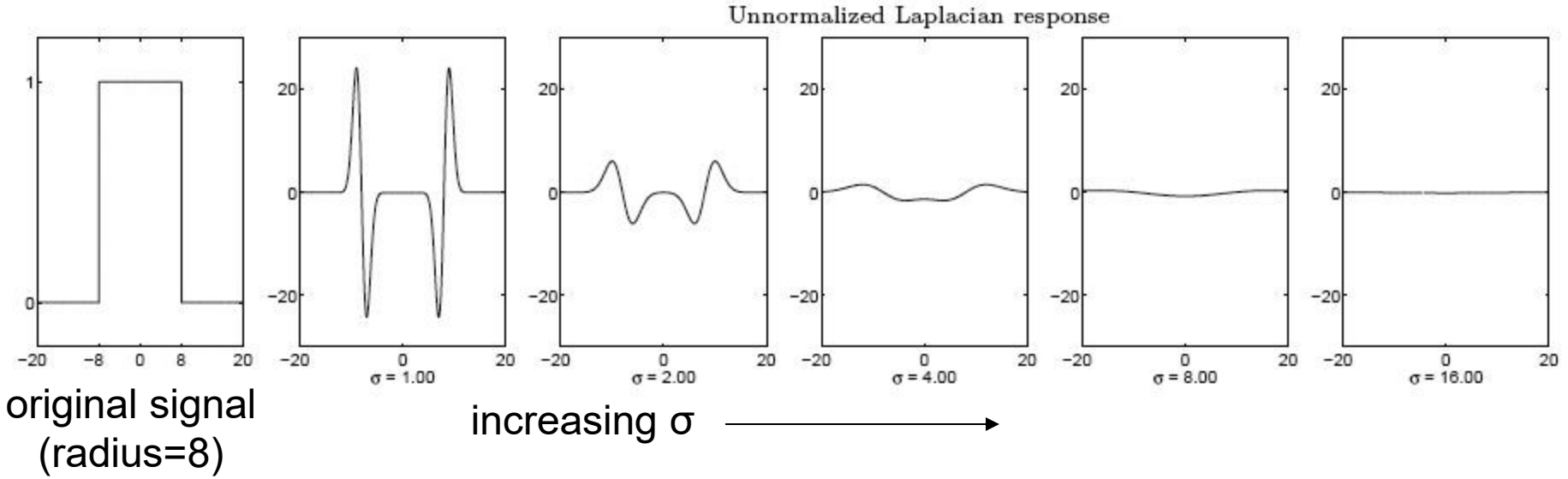
- Edge = ripple
- Blob = superposition of two ripples



Spatial selection: the magnitude of the Laplacian response will achieve a maximum at the center of the blob, provided the scale of the Laplacian is "matched" to the scale of the blob

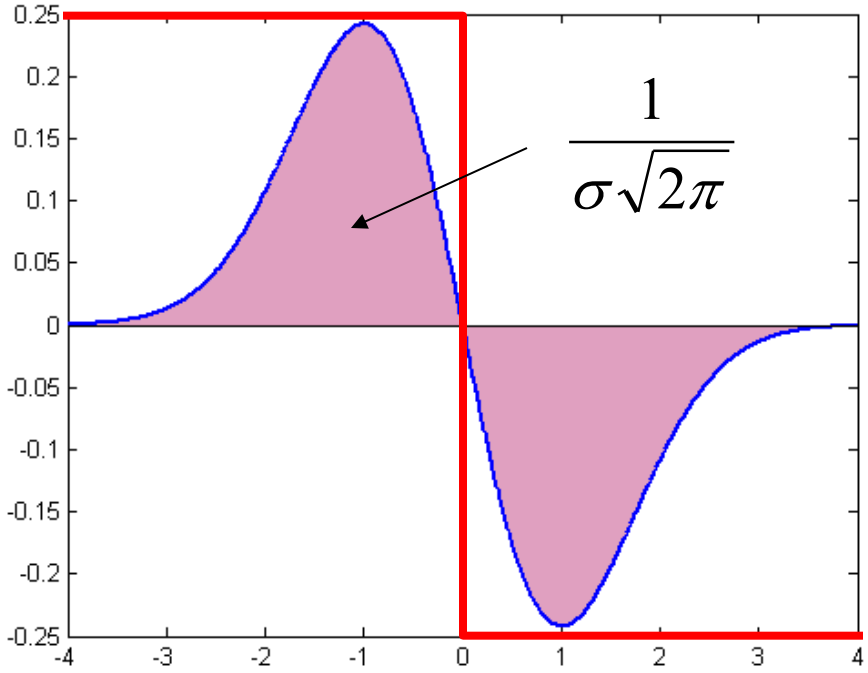
Scale Selection

- We want to find the characteristic scale of the blob by convolving it with Laplacians at several scales and looking for the maximum response
- However, Laplacian response decays as scale increases:



Scale Normalization

The response of a derivative of Gaussian filter to a perfect step edge decreases as σ increases



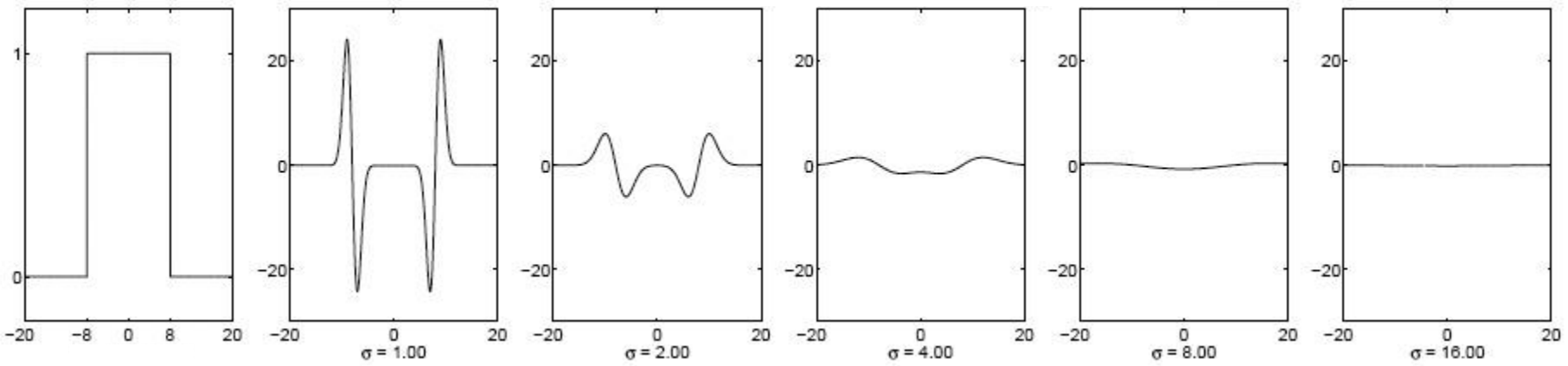
Scale Normalization

- The response of a derivative of Gaussian filter to a perfect step edge decreases as σ increases
- To keep response the same (scale-invariant), must multiply Gaussian derivative by σ
- Laplacian is the second Gaussian derivative, so it must be multiplied by σ^2

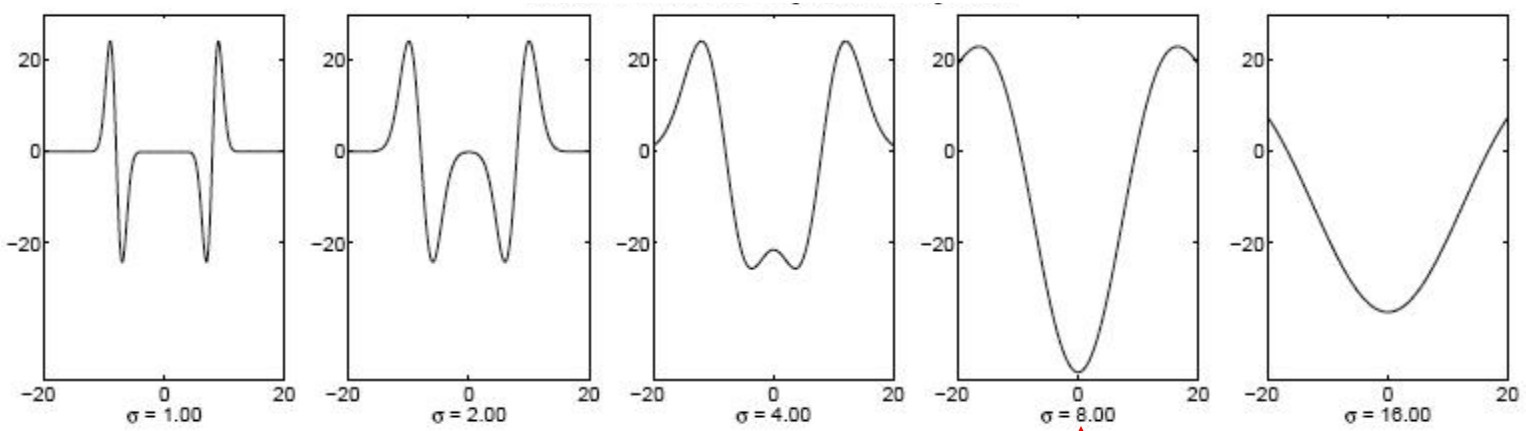
Effect of Scale Normalization

Original signal

Unnormalized Laplacian response

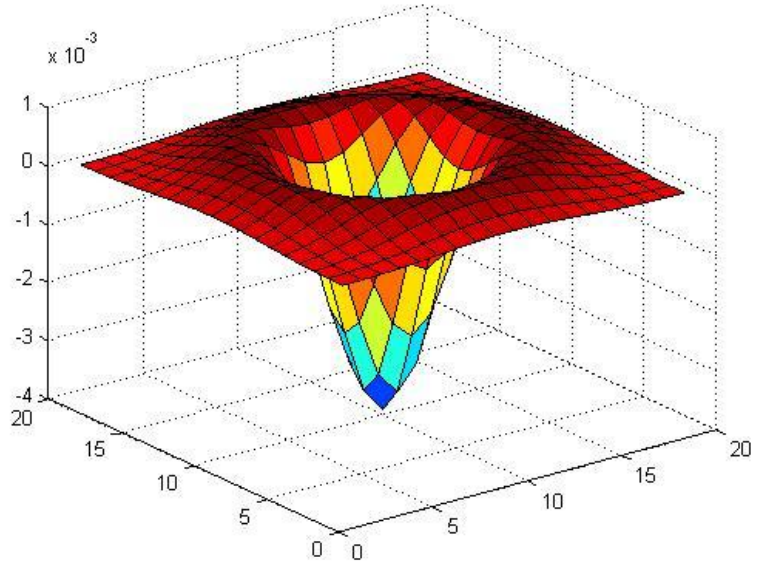


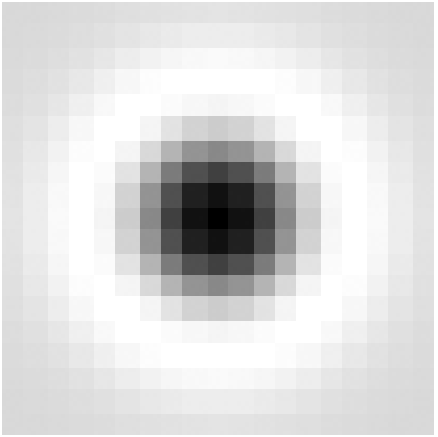
Scale-normalized Laplacian response



Blob Detection in 2D

Laplacian of Gaussian: Circularly symmetric operator for blob detection in 2D

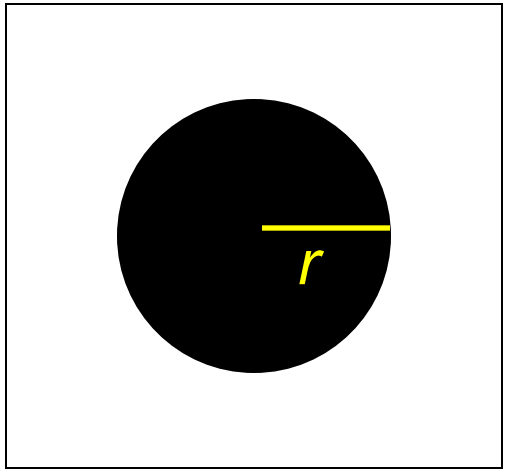




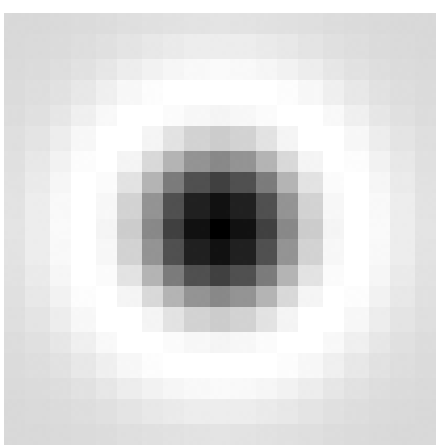
Scale-normalized: $$\nabla^2_{\text{norm}} g = \sigma^2 \left(\frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2} \right)$$

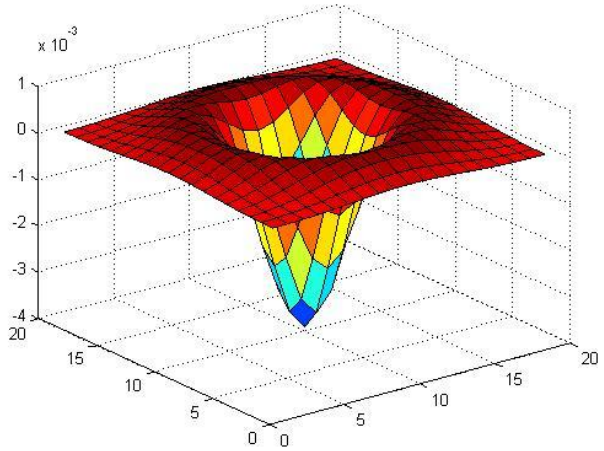
Scale Selection

At what scale does the Laplacian achieve a maximum response to a binary circle of radius r?



image





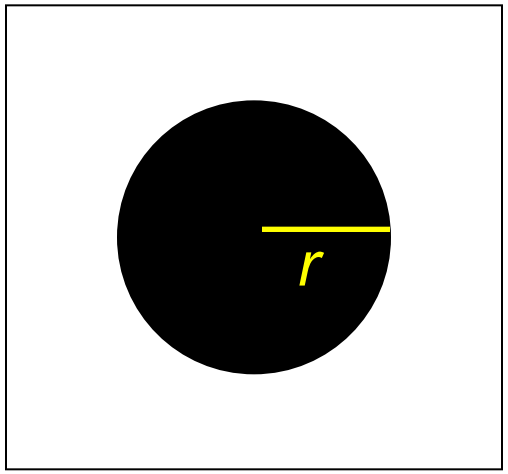
Laplacian

Scale Selection

- At what scale does the Laplacian achieve a maximum response to a binary circle of radius r?
- To get maximum response, the zeros of the Laplacian have to be aligned with the circle
- The Laplacian is given by (up to scale):

$$(x^2 + y^2 - 2\sigma^2)\, e^{-(x^2+y^2)/2\sigma^2}$$

- Therefore, the maximum response occurs at $\sigma = r/\sqrt{2}.$



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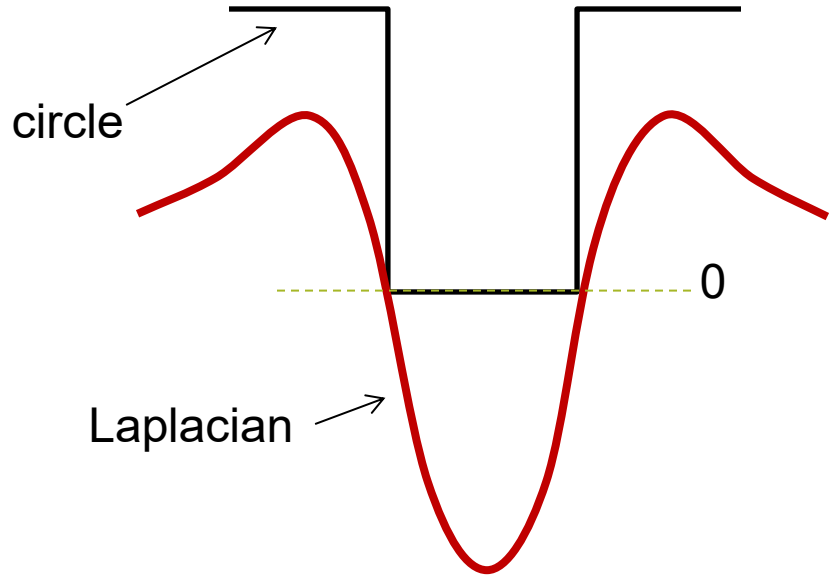
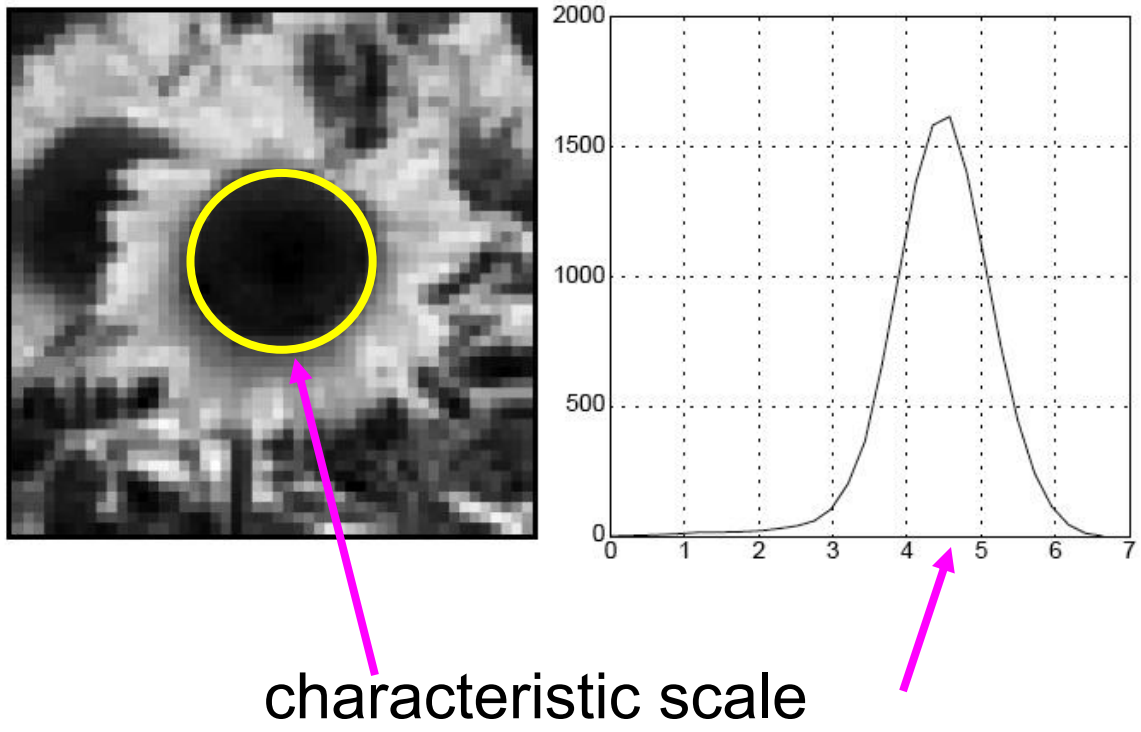


Characteristic Scale

We define the characteristic scale of a blob as the scale that produces peak of Laplacian response in the blob center



T. Lindeberg (1998). "Feature detection with automatic scale selection." *International Journal of Computer Vision* **30** (2): pp 77--116.

Scale-Space Blob Detector

1. Convolve image with scale-normalized Laplacian at several scales

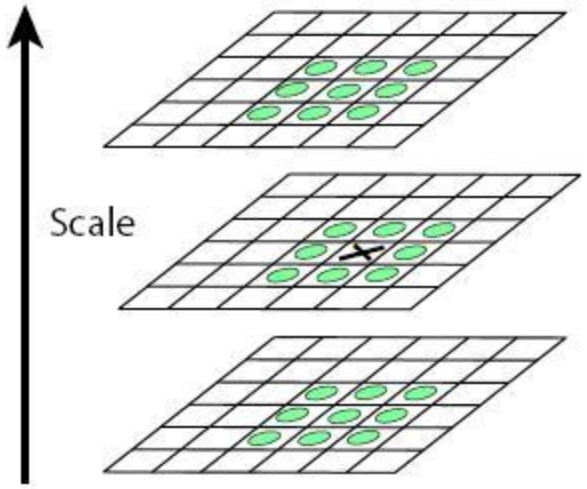
Scale-Space Blob Detector: Example

Scale-Space Blob Detector: Example

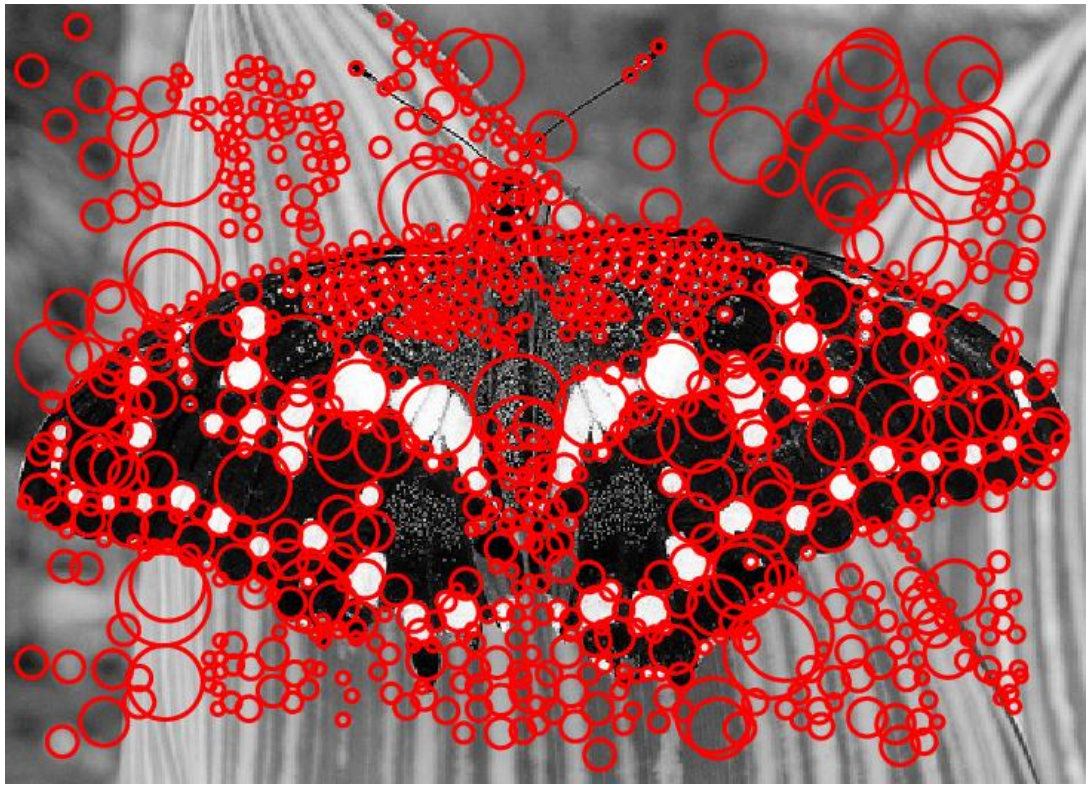
sigma = 11.9912

Scale-Space Blob Detector

1. Convolve image with scale-normalized Laplacian at several scales
2. Find maxima of squared Laplacian response in scale-space



Scale-Space Blob Detector: Example



Coin Detection Problem.

Assume that we are given an image containing more than one each of quarters, dimes, nickels, and pennies. Also, you are given template images of these coins in a standard 300x300 canvas.

Your task is to detect each coin and tell me the amount of money represented in the image.

Then, tell me what the assumptions are.

Efficient Implementation

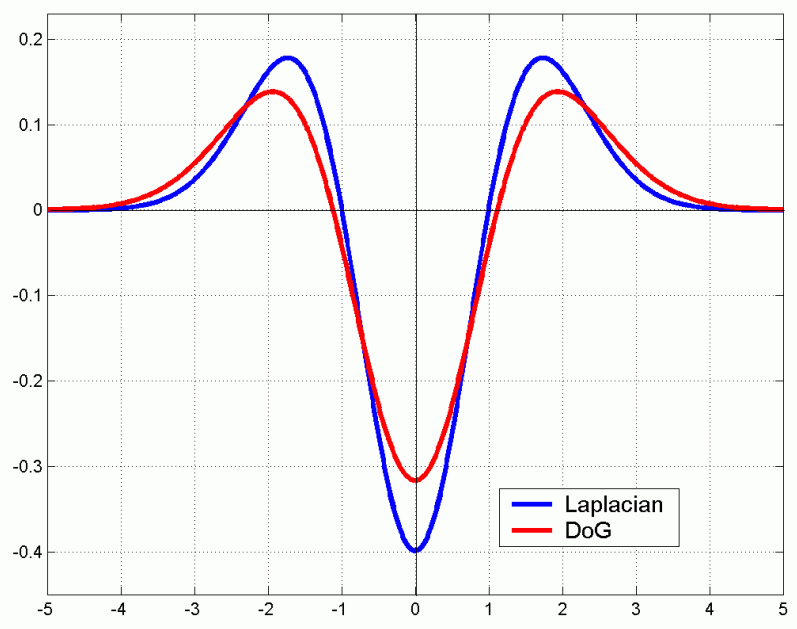
Approximating the Laplacian with a difference of Gaussians:

$$L = \sigma^2 \left(G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma) \right)$$

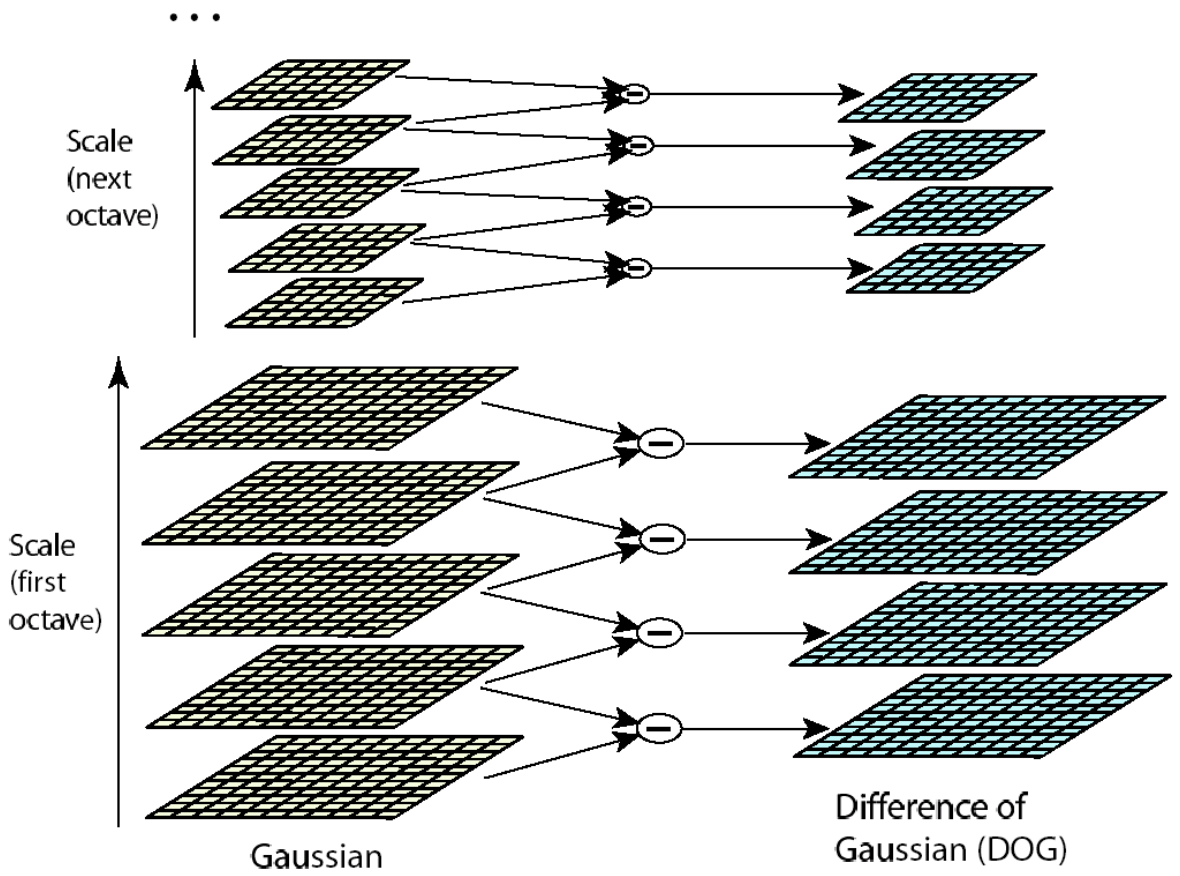
(Laplacian)

$$DoG = G(x, y, k\sigma) - G(x, y, \sigma)$$

(Difference of Gaussians)



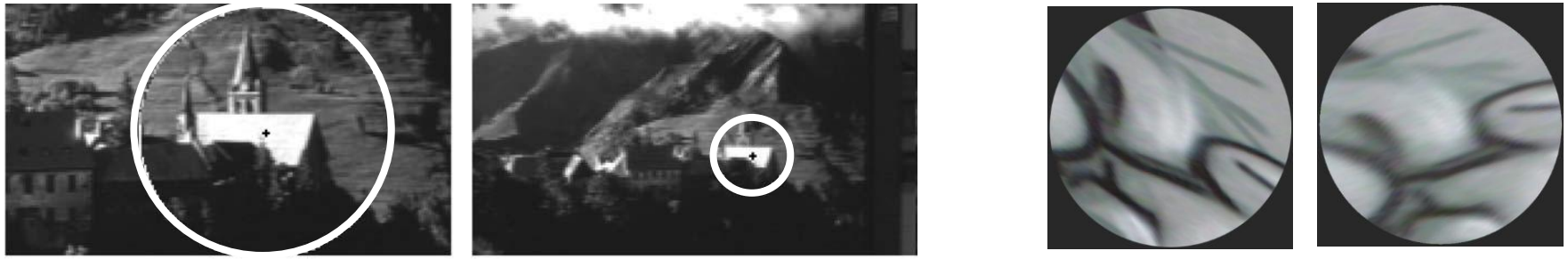
Efficient Implementation



David G. Lowe. "Distinctive image features from scale-invariant keypoints." *IJCV* 60 (2), pp. 91-110, 2004.

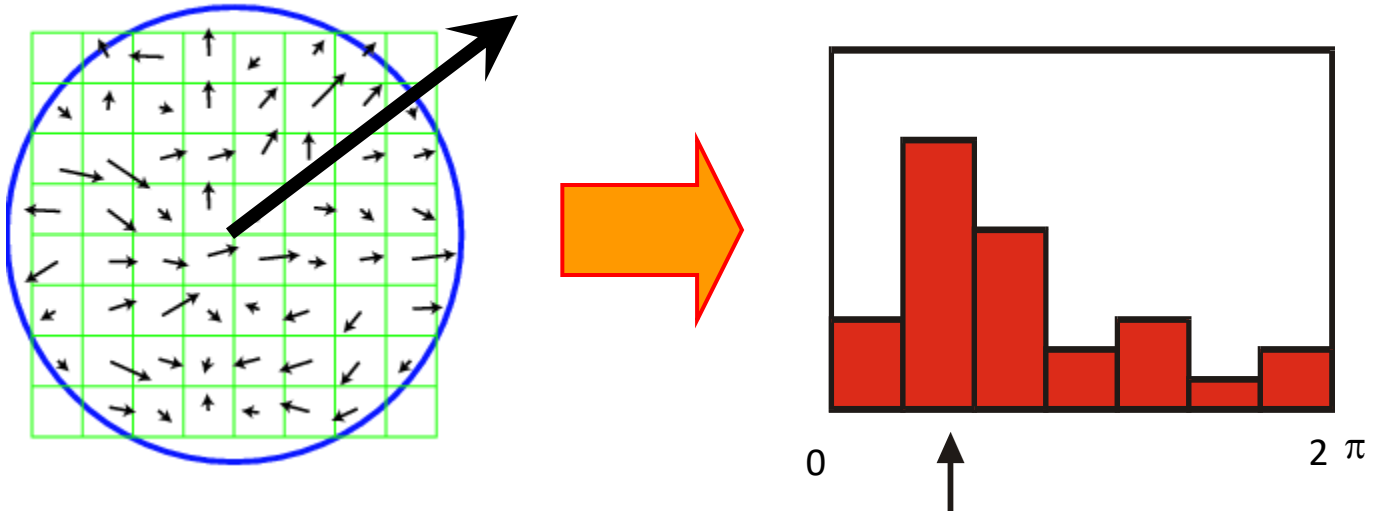
From Feature Detection to Feature Description

- Scaled and rotated versions of the same neighborhood will give rise to blobs that are related by the same transformation
- What to do if we want to compare the appearance of these image regions?
  - *Normalization*: transform these regions into same-size circles
  - Problem: rotational ambiguity



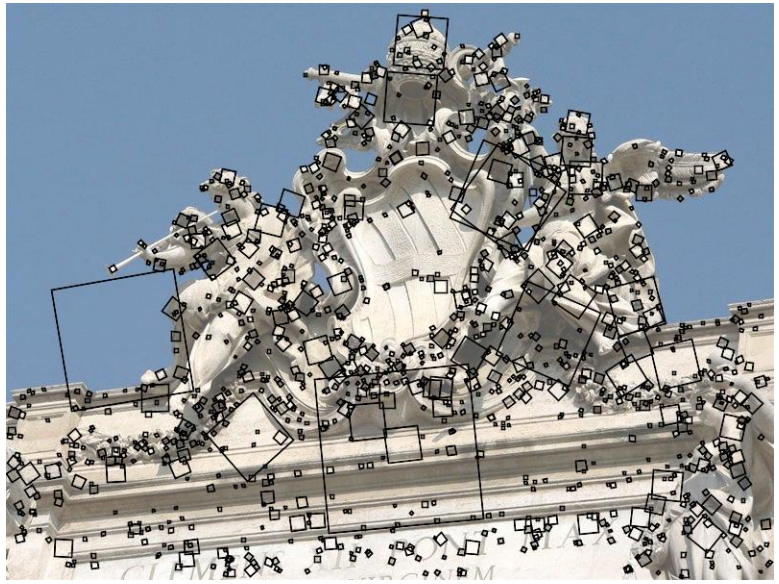
Eliminating Rotation Ambiguity

- To assign a unique orientation to circular image windows:
  - Create histogram of local gradient directions in the patch
  - Assign canonical orientation at peak of smoothed histogram



SIFT Features

Detected features with characteristic scales and orientations:



David G. Lowe. "Distinctive image features from scale-invariant keypoints." *IJCV* 60 (2), pp. 91-110, 2004.

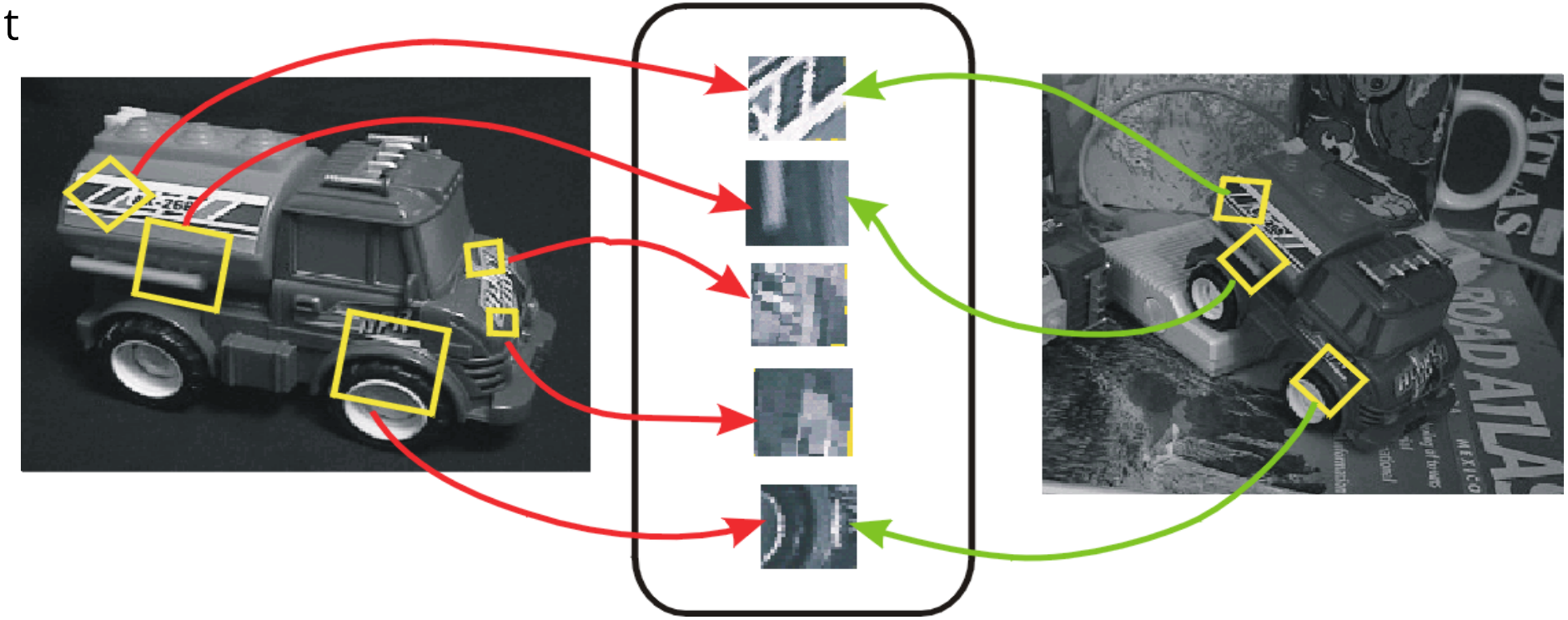
From Feature Detection to Feature Description

Detection is *covariant*:

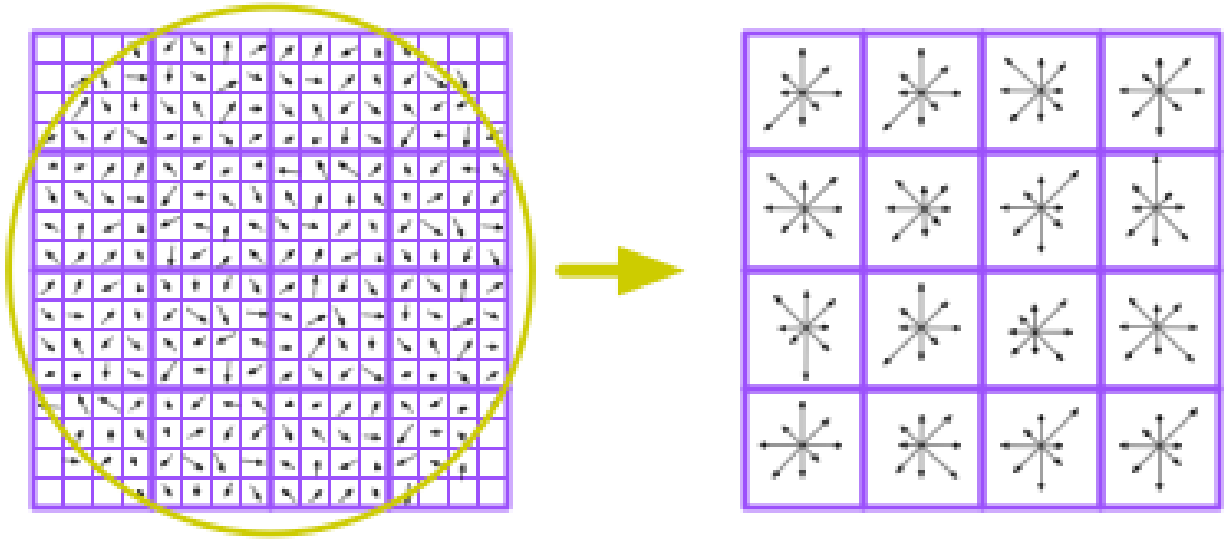
features(transform(image)) = transform(features(image))

Description is *invariant*:

feat



SIFT Descriptors



David G. Lowe. "Distinctive image features from scale-invariant keypoints." *IJCV* 60 (2), pp. 91-110, 2004.

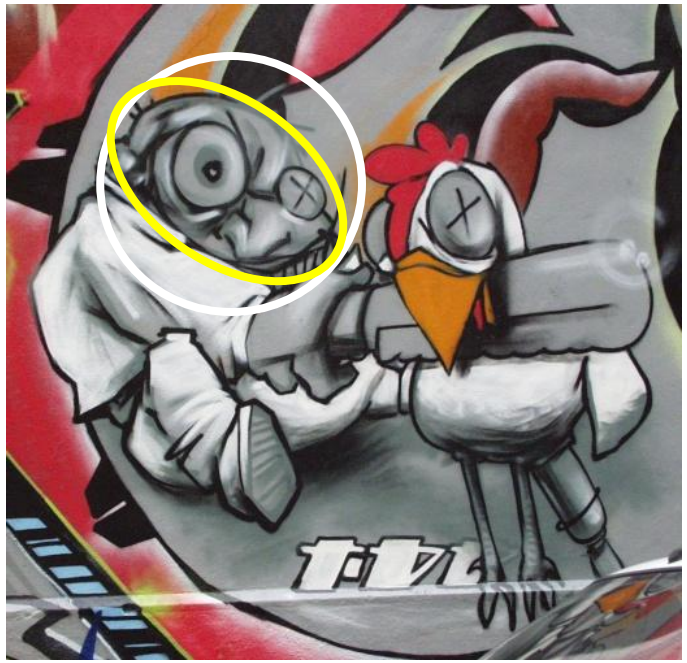
Properties of SIFT

Extraordinarily robust detection and description technique

- Can handle changes in viewpoint
  - Up to about 60 degree out-of-plane rotation
- Can handle significant changes in illumination
  - Sometimes even day vs. night
- Fast and efficient—can run in real time
- Lots of code available

Affine adaptation

Affine transformation approximates viewpoint changes for roughly planar objects and roughly orthographic cameras



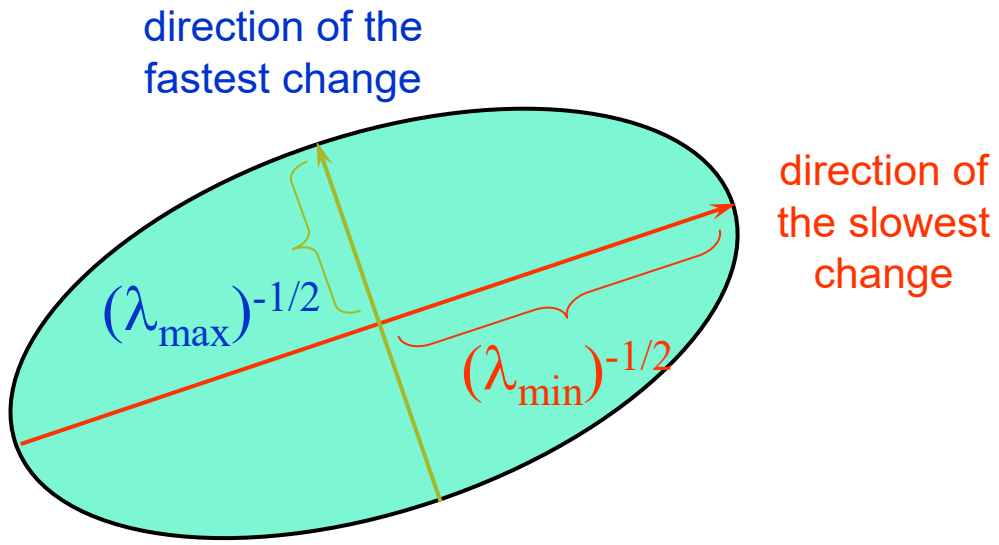
Affine Adaptation

Consider the second moment matrix of the window containing the blob:

$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix} = R^{-1} \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} R$$

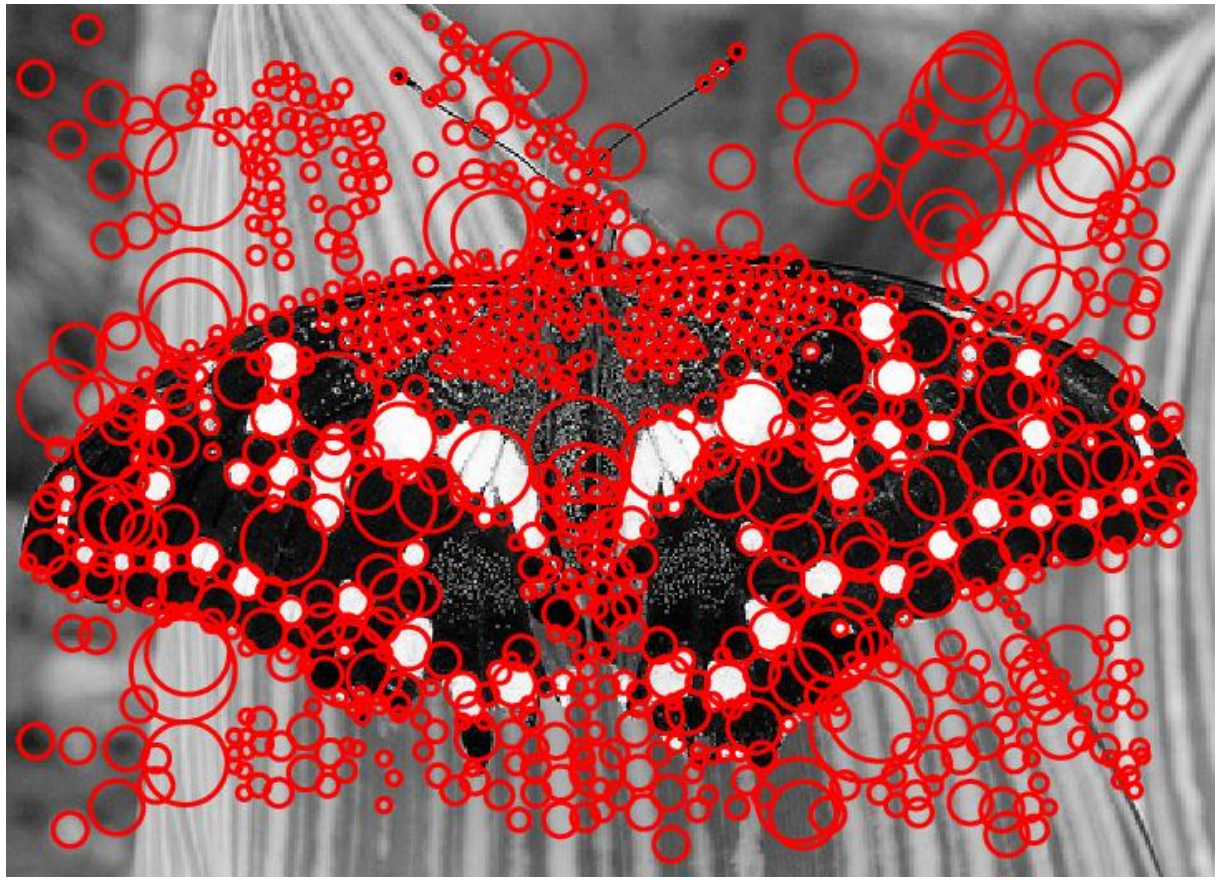
Recall:

$$[u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix} = \text{const}$$



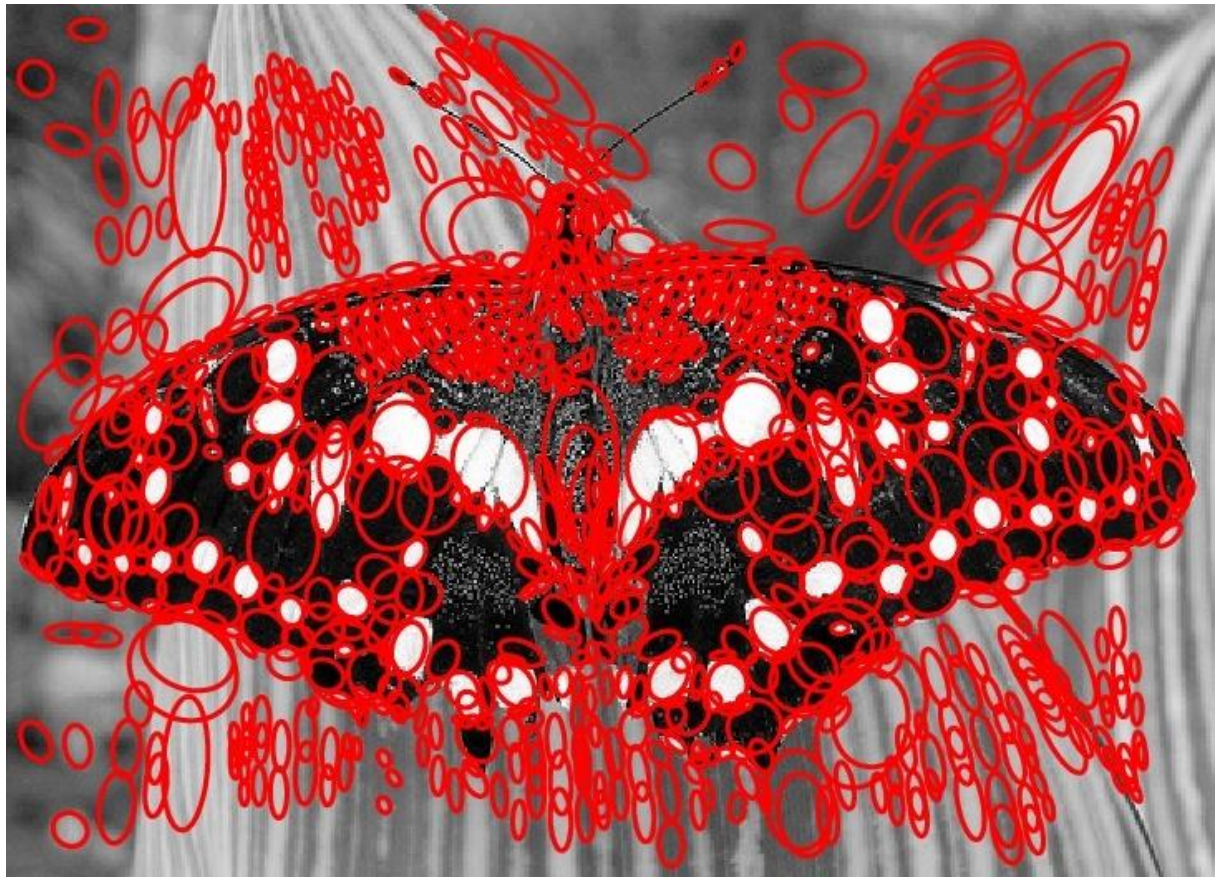
This ellipse visualizes the "characteristic shape" of the window

Affine Adaptation Example



Scale-invariant regions (blobs)

Affine Adaptation Example



Affine-adapted blobs